

UNIVERSITY OF OSLO

Neutrino Mass, Seesaws and Left-Right Symmetry

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Fermion masses in the Standard Model

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$m f^2$



Fermion masses for dummies

$$\mathcal{L}_{\text{mass}} = mf^2$$

Fermion masses for dummies

$$\mathcal{L}_{\text{mass}} = mf^*f$$

Fermion masses for dummies

$$\mathcal{L}_{\text{mass}} = mf^\dagger f$$

Fermion masses for dummies

$$\mathcal{L}_{\text{Dirac}} = mf^\dagger \gamma_0 f$$

Fermion masses for dummies

$$\mathcal{L}_{\text{Dirac}} = mf^\dagger \gamma_0 f$$

$$\mathcal{L}_{\text{Majorana}} = mf^T f$$

Fermion masses for dummies

$$\mathcal{L}_{\text{Dirac}} = mf^\dagger \gamma_0 f$$

$$\mathcal{L}_{\text{Majorana}} = mf^T i\gamma_2 \gamma_0 f$$

Fermion masses for dummies

Chiral representation

$$f = \begin{pmatrix} f_L \\ f_R \end{pmatrix}$$
$$\gamma_0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}. \quad (1)$$

Fermion masses for dummies

$$\mathcal{L}_{\text{Dirac}} = m(f_L^\dagger f_R + f_R^\dagger f_L)$$

$$\mathcal{L}_{\text{Majorana}} = m(f_L^T \epsilon f_L - f_R^T \epsilon f_R)$$

The Standard Model

Gauge group:

$$\mathcal{G}_{\text{SM}} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \quad (2)$$

Quarks:

$$q_L^\alpha = \begin{pmatrix} u_L^\alpha \\ d_L^\alpha \end{pmatrix}, \quad u_R^\alpha, \quad d_R^\alpha. \quad (3)$$

Leptons:

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R. \quad (4)$$

The End?

The Higgs mechanism

Quiz

The ABEGHHK'tH mechanism

Higgs Mechanism

Higgs boson:

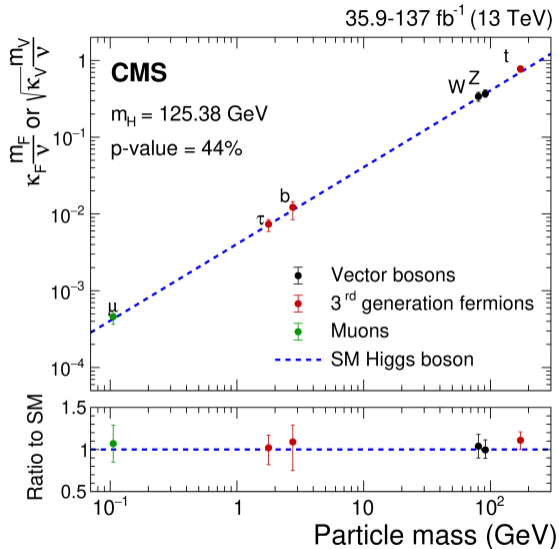
- Spin-0, Lorentz scalar
- $SU(2)_L$ doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ v + h \end{pmatrix}. \quad (5)$$

Dirac masses and Higgs couplings

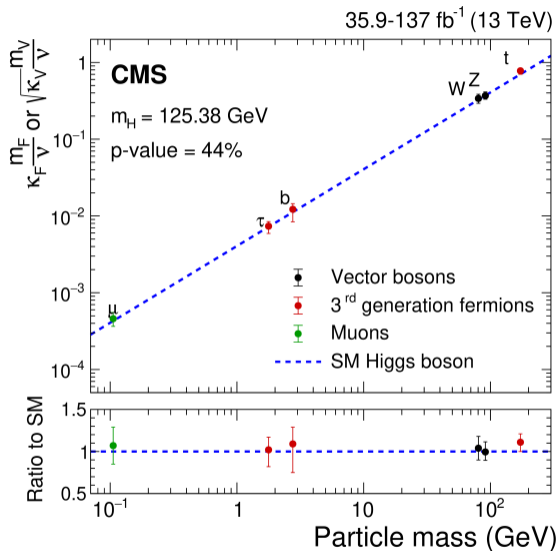
$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= y_e \ell_L^\dagger \Phi e_R \\ &= y_e (v + h) e_L^\dagger e_R \\ &= m_e e_L^\dagger e_R + \frac{m_e}{v} h e_L^\dagger e_R\end{aligned}\quad (6)$$

CMS (2020) [arXiv:2009.04363]



Higgs mechanism in the SM

- A single mass scale: $\langle \Phi \rangle = v$.
- Direct relation: $m \leftrightarrow y$.



Neutrino mass in the Standard Model

Neutrino mass in the Standard Model

$$\mathcal{G}_{\text{SM}} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \quad (7)$$

$$q_L^\alpha = \begin{pmatrix} u_L^\alpha \\ d_L^\alpha \end{pmatrix}, \quad u_R^\alpha, \quad d_R^\alpha, \quad (8)$$

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R, \quad (9)$$

Neutrino mass in the Standard Model

$$\mathcal{G}_{\text{SM}} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \quad (7)$$

$$q_L^\alpha = \begin{pmatrix} u_L^\alpha \\ d_L^\alpha \end{pmatrix}, \quad u_R^\alpha, \quad d_R^\alpha, \quad (8)$$

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R, \quad \nu_R. \quad (9)$$

Neutrino mass in the Standard Model

Right-handed neutrino is sterile if it exists, so either

ν_R does **not** exist \Rightarrow No Dirac mass

ν_R does exist \Rightarrow Both Dirac and Majorana mass

Majorana mass from scalar triplet

We could introduce a triplet

$$\Delta = \begin{pmatrix} \delta^0 & \delta^+ \\ \delta^+ & \delta^{++} \end{pmatrix} \quad (10)$$

coupling as

$$\mathcal{L}_{\text{Triplet}} = y_{\Delta} \ell_L^T \Delta \ell_L \quad (11)$$

Majorana mass from scalar triplet

Assume VEV for the neutral component

$$\langle \Delta \rangle = \begin{pmatrix} v_\Delta & 0 \\ 0 & 0 \end{pmatrix} \quad (12)$$

gives mass term

$$\mathcal{L}_{\text{Triplet}} = (y_\Delta v_\Delta) \nu_L^\top \nu_L \quad (13)$$

Majorana mass from scalar triplet

Small VEV:

- Light lepton number violating scalars

Big VEV:

- Corrections to Higgs mass

Neutrino mass in the Standard Model

$$\mathcal{G}_{\text{SM}} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \quad (14)$$

$$q_L^\alpha = \begin{pmatrix} u_L^\alpha \\ d_L^\alpha \end{pmatrix}, \quad u_R^\alpha, \quad d_R^\alpha, \quad (15)$$

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R, \quad \nu_R. \quad (16)$$

Neutrino mass in the Standard Model

Dirac mass

$$\begin{aligned}\mathcal{L}_{\text{Dirac},\nu} &= -y_\nu \ell_L^\dagger \epsilon \langle \Phi \rangle^* \nu_R \\ &= -m_D \nu_L^\dagger \nu_R\end{aligned}\tag{17}$$

Majorana mass

$$\mathcal{L}_{\text{Majorana},\nu} = -\frac{1}{2} m_M \nu_R^T \nu_R\tag{18}$$

Neutrino mass in the Standard Model

Neutrino mass matrix

$$\mathcal{L}_{\nu,\text{mass}} \sim (\nu_L \quad \nu_R) \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} \quad (19)$$

Mass eigenstates

$$\mathcal{L}_{\nu,\text{mass}} \sim (\nu \quad N) \begin{pmatrix} m_\nu & 0 \\ 0 & m_N \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}, \quad \text{where} \quad \begin{pmatrix} \nu \\ N \end{pmatrix} = U \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} \quad (20)$$

Seesaw mechanism

By assuming

$$m_D \ll m_M \quad (21)$$

we get

$$\begin{aligned} m_N &= m_M \\ m_\nu &= -\frac{m_D^2}{m_N} \end{aligned} \quad (22)$$



Introducing generations

The SM contains three generations of fermions

$$\nu_L \rightarrow \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \quad (23)$$

The masses turns into mass matrices

$$\begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix} \rightarrow \begin{pmatrix} 0 & M_D^T \\ M_D & M_M \end{pmatrix} \quad (24)$$

Introducing generations

$$\begin{aligned} M_N &= M_M \\ M_\nu &= -M_D^T \frac{1}{M_N} M_D \end{aligned} \tag{25}$$

Inverting the Seesaw

One generation:

$$m_\nu = -\frac{m_D^2}{m_N} \rightarrow m_D = i\sqrt{m_\nu m_N} = y_\nu v \quad (26)$$

Inverting the Seesaw

Three generations:

$$M_\nu = V_{\text{PMNS}}^* m_\nu V_{\text{PMNS}}^\dagger = -M_D^T \frac{1}{m_N} M_D. \quad (27)$$

To solve for M_D we notice that

$$I = (i m_\nu^{-\frac{1}{2}} V_{\text{PMNS}}^T M_D^T m_N^{-\frac{1}{2}}) (i m_N^{-\frac{1}{2}} M_D V_{\text{PMNS}} m_\nu^{-\frac{1}{2}}) \quad (28)$$

and it follows that

$$M_D = i \sqrt{m_N} O \sqrt{m_\nu} V_{\text{PMNS}}^\dagger, \quad \text{where } O \in O(3). \quad (29)$$

Casas, Ibarra (2001) [arXiv:hep-ph/0103065]

Right-handed neutrino in the Standard Model

- Mechanism to suppress m_ν
- Arbitrary mass scale m_M
- No direct relation between masses and couplings

The Left-Right Symmetric Model

The Standard Model

$$\mathcal{G}_{\text{SM}} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

$$q_L^\alpha = \begin{pmatrix} u_L^\alpha \\ d_L^\alpha \end{pmatrix}, \quad u_R^\alpha, \quad d_R^\alpha,$$
$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R, \quad \nu_R.$$

The Left-Right Symmetric Model

$$\mathcal{G}_{\text{LR}} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{\text{B-L}},$$

$$q_L^\alpha = \begin{pmatrix} u_L^\alpha \\ d_L^\alpha \end{pmatrix}, \quad q_R^\alpha = \begin{pmatrix} u_R^\alpha \\ d_R^\alpha \end{pmatrix},$$
$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \ell_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}.$$

Quiz

What are the weak hypercharges of the SM particles?

The Left-Right Symmetric Model

$$\mathcal{G}_{LR} = SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes Z_2,$$

$$q_L^\alpha = \begin{pmatrix} u_L^\alpha \\ d_L^\alpha \end{pmatrix}, \quad q_R^\alpha = \begin{pmatrix} u_R^\alpha \\ d_R^\alpha \end{pmatrix},$$
$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \ell_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}.$$

Discrete LR symmetry

Parity:

$$f_L \leftrightarrow f_R \quad (30)$$

Charge conjugation

$$\begin{aligned} f &\leftrightarrow f^c \\ f_L &\leftrightarrow \epsilon f_R^* \end{aligned} \quad (31)$$

New particles in the LR model

Three new $SU(2)_R$ gauge bosons

$$W_{R\pm}^\mu, W_{R0}^\mu \quad (32)$$

One "new" $U(1)_{B-L}$ gauge boson

$$B_{B-L}^\mu \quad (33)$$

New particles in the LR model

One scalar bidoublet

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}, \quad \Phi \rightarrow U_L \Phi U_R^\dagger \quad (34)$$

Two scalar triplets

$$\Delta_R = \begin{pmatrix} \delta_R^0 & \delta_R^+ \\ \delta_R^+ & \delta_R^{++} \end{pmatrix} \quad (35)$$

$$\Delta_R \rightarrow U_R \Delta_R U_R^\dagger, \quad \Delta_L \rightarrow U_L \Delta_L U_L^\dagger \quad (36)$$

New particles in the LR model

One scalar bidoublet

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}, \quad \Phi \rightarrow U_L \Phi U_R^\dagger \quad (34)$$

Two scalar triplets

$$\Delta_R = \begin{pmatrix} \delta_R^0 & \delta_R^+ \\ \delta_R^+ & \delta_R^{++} \end{pmatrix} \quad \Delta_L = \begin{pmatrix} \delta_L^0 & \delta_L^+ \\ \delta_L^+ & \delta_L^{++} \end{pmatrix} \quad (35)$$

$$\Delta_R \rightarrow U_R \Delta_R U_R^T, \quad \Delta_L \rightarrow U_L \Delta_L U_L^T \quad (36)$$

The scalar potential

$$\begin{aligned}
 V_C = & -\mu_1^2 \text{Tr } \Phi^\dagger \Phi - \mu_2^2 \left(e^{i\delta_1} \text{tr } \tilde{\Phi}^\dagger \Phi + h.c. \right) - \mu_3^2 \left(\text{tr } \Delta_L^\dagger \Delta_L + \text{tr } \Delta_R^\dagger \Delta_R \right) \\
 & + \lambda_1 \left(\text{tr } \Phi^\dagger \Phi \right)^2 + \lambda_2 \left(e^{i\delta_2} \left(\text{tr } \tilde{\Phi}^\dagger \Phi \right)^2 + h.c. \right) + \lambda_3 \text{tr } \tilde{\Phi}^\dagger \Phi \text{tr } \Phi^\dagger \tilde{\Phi} \\
 & + \lambda_4 \text{tr } \Phi^\dagger \Phi \left(e^{i\delta_3} \text{tr } \tilde{\Phi}^\dagger \Phi + h.c. \right) + \rho_1 \left(\left(\text{tr } \Delta_L^\dagger \Delta_L \right)^2 + \left(\text{tr } \Delta_R^\dagger \Delta_R \right)^2 \right) \\
 & + \rho_2 \left(\text{tr } \Delta_L \Delta_L \text{tr } \Delta_L^\dagger \Delta_L^\dagger + \text{tr } \Delta_R \Delta_R \text{tr } \Delta_R^\dagger \Delta_R^\dagger \right) \\
 & + \rho_3 \text{tr } \Delta_L^\dagger \Delta_L \text{tr } \Delta_R^\dagger \Delta_R + \rho_4 \left(e^{i\delta_4} \text{tr } \Delta_L \Delta_L \text{tr } \Delta_R^\dagger \Delta_R^\dagger + h.c. \right) \\
 & + (6 \text{ terms coupling } \Phi \text{ and } \Delta)
 \end{aligned} \tag{37}$$

Symmetry breaking

Neutral scalars acquire VEVs

$$\langle \Phi \rangle = \begin{pmatrix} v \cos \beta & 0 \\ 0 & v \sin \beta \end{pmatrix} \quad (38)$$

$$\Delta_R = \begin{pmatrix} v_R & 0 \\ 0 & 0 \end{pmatrix} \quad \Delta_L = \begin{pmatrix} v_L & 0 \\ 0 & 0 \end{pmatrix} \quad (39)$$

To recover the Standard Model

$$v_1^2 + v_2^2 = v^2 \quad (40)$$

$$v_R \gg v \gg v_L \propto \frac{v}{v_R} v \quad (41)$$

Gauge boson mixing

Neutral gauge bosons will mix, similarly as in the SM.

Find mass eigenstates with masses

$$\begin{aligned}m_A^2 &= 0 \\m_Z^2 &= m_{Z_{SM}}^2 \left(1 + \mathcal{O}\left(\frac{v^2}{v_R^2}\right)\right) \\m_{Z'}^2 &\propto v_R^2\end{aligned}\tag{42}$$

Gauge boson mixing

Also the charged gauge bosons will mix:

$$\begin{pmatrix} W_1^- \\ W_2^- \end{pmatrix} = \begin{pmatrix} 1 & -\xi \\ \xi^* & 1 \end{pmatrix} \begin{pmatrix} W_L^- \\ W_R^- \end{pmatrix}, \quad \text{with} \quad \xi \propto \frac{v^2}{v_R^2}, \quad (43)$$

and similarly

$$\begin{aligned} m_{W_1}^2 &= m_{W_{SM}}^2 \left(1 + \mathcal{O}\left(\frac{v^2}{v_R^2}\right) \right) \\ m_{W_2}^2 &= g^2 v_R^2 \end{aligned} \quad (44)$$

(CDF II, April 2022)

Yukawa couplings

Dirac term:

$$\mathcal{L}_{\text{Dirac}} = -Y \ell_L^\dagger \Phi \ell_R \quad (45)$$

Majorana term:

$$\mathcal{L}_{\text{Majorana}} = -Y_L \ell_L^T \Delta_L \ell_L - Y_R \ell_R^T \Delta_R \ell_R \quad (46)$$

Neutrino mass in the LR model

Dirac term:

$$\mathcal{L}_{\text{Dirac}} = -(v_1 Y) \nu_L^\dagger \nu_R \quad (47)$$

Majorana term:

$$\mathcal{L}_{\text{Majorana}} = -(\nu_L Y_L) \nu_L^T \nu_L - (\nu_R Y_R) \nu_R^T \nu_R \quad (48)$$

Neutrino mass in the LR model

Neutrino mass matrix

$$\mathcal{L}_{\nu,\text{mass}} \sim (\nu_L \quad \nu_R) \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} \quad (49)$$

Seesaw approximation

$$\begin{aligned} M_N &= M_R \\ M_\nu &= M_L - M_D^T \frac{1}{M_R} M_D \end{aligned} \quad (50)$$

Constraints from LR symmetry

Transformation of scalar fields:

$$\Phi \leftrightarrow \Phi^T, \quad \Delta_L \leftrightarrow \Delta_R^* \quad (51)$$

Constraints on Yukawa couplings:

$$Y = Y^T, \quad Y_L = Y_R^* \quad (52)$$

Neutrino mass in the LR model

Constraints on mass matrices

$$M_D = M_D^T, \quad M_L = \frac{V_L}{V_R} M_R^* \quad (53)$$

Neutrino mass

$$M_\nu = \frac{V_L}{V_R} M_N - M_D \frac{1}{M_N} M_D \quad (54)$$

Neutrino mass in the LR model

Neutrino mass

$$M_\nu = \frac{v_L}{v_R} M_N - M_D \frac{1}{M_N} M_D \quad (55)$$

Now we can solve directly for M_D

$$M_D = M_N \sqrt{\frac{v_L}{v_R} - \frac{1}{M_N} M_\nu} \quad (56)$$

Nemevšek, Senjanović, Tello (2013) [arXiv:1211.2837]

Summary

- Neutrino mass requires a new mass scale
- Seesaw mechanism explains light neutrinos through a high mass scale
- LR model connects the neutrino mass to the LR breaking scale
- We get a predictive relationship between couplings, masses and mixings

Summary

Neutrino mass could be a feature, not a bug!

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