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Handling the spectral axis in CLASS

J. Pety[1,2] & S. Bardeau[1]

1. IRAM (Grenoble)
2. LERMA, Observatoire de Paris

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Abstract

An observer is interested by the physical properties (line frequencies, velocity of the gas) of a source in its own reference frame. However, the observations happen in a different frame, namely the topocentric frame for ground-based observations. This implies a change of frame to infer the physical properties in the source frame from the measured frequencies in the measurement frame. Several difficulties arise in astronomy: 1) the Earth ... To be completed.

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1.1 Natural expression of the spectral axis in radio-astronomy

Typical radio-astronomy spectrometers (*e.g.*, correlators or Fourier transform spectrometers) deliver the brightness along a given line of sight at regularly spaced frequencies *in the observatory frame*. The spectrum can be represented as a set of brightness temperatures $T(i)$, where the frequency axis in the observatory frame is defined as

$$f^{\rm obs}(i) = f^{\rm obs}_{\rm tuned} + (i - i_0)\,\delta f^{\rm obs}, \tag{1}$$

where $f^{\rm obs}(i)$ is the frequency at the channel i, $f^{\rm obs}_{\rm tuned}$ the frequency at the reference channel i_0, and $\delta f^{\rm obs}$ the channel spacing.

1.2 Various approximations of the Doppler effect

The Doppler effect expresses that a source emitting in its rest frame at a frequency $f^{\rm rest}$ will appear to emit at a frequency $f^{\rm obs}$ in the observation frame, which depends on the velocity of the source in the observation frame, $v^{\rm obs}$. Sections 1.3 and 1.4 will show how this property can be used to interpret the spectral axis measured in the observatory frame in terms of the property of the source in the source rest frame.

1.2.1 The Special Relativity formula

For sources lying at cosmological distances, General Relativity should be used to express the Doppler effect in order to take into account the effects of curvature of space. For a Euclidian cosmology where $q_0 = 0$, the Doppler effect can be expressed with Special Relativity.

In this case, the relation between $f^{\rm rest}$ and $f^{\rm obs}$ is given by

$$f^{\rm obs} = f^{\rm rest}\,\frac{\sqrt{c^2 - {v^{\rm obs}}^2}}{c + v^{\rm obs}_{\parallel}}, \tag{2}$$

with ${v^{\rm obs}}^2 = {v^{\rm obs}_{\parallel}}^2 + {v^{\rm obs}_{\perp}}^2$, where $v^{\rm obs}_{\parallel}$ and $v^{\rm obs}_{\perp}$ are respectively the velocity components along and perpendicular to the line of sight. $v^{\rm obs}_{\parallel}$ and $v^{\rm obs}_{\perp}$ are often called the radial and transverse velocity components.

1.2.2 Approximation 1: Neglecting the transverse Doppler effect when $v^{\rm obs} \ll c$

Astronomers have very little access to the velocity component perpendicular to the line of sight. They will thus assign the observed Doppler shift to the line-of-sight motion, *i.e.*, they will neglect the transverse Doppler effect.

The formula 2 indicates that the dependency on $v^{\rm obs}_{\perp}$ is of second order in v/c, while the dependency on $v^{\rm obs}_{\parallel}$ is only of first order in v/c. Hence, neglecting the transverse Doppler effect is correct as long as

$$(v^{\rm obs}_{\perp})^2 \ll c^2 - (v^{\rm obs}_{\parallel})^2. \tag{3}$$

In this case, the Doppler equation can be approximated to first order to

$$f^{\rm obs} = f^{\rm rest}\sqrt{\frac{c - v^{\rm obs}_{\parallel}}{c + v^{\rm obs}_{\parallel}}}. \tag{4}$$

In other words, the frequency change under a change of frame is attributed only to the radial velocity between the rest and observation frame. We note that condition 3 is enforced if the velocities are non-relativistic, *i.e.*

$$v^{\rm obs} \ll c. \tag{5}$$

1.2.3 Approximation 2: First order developments and velocity conventions

As formula 4 is mostly correct when $v^{\rm obs} \ll c$, we can as well continue to develop in $v_{\parallel}^{\rm obs}/c$ and limit ourselves to the first order term. However, there are two ways to do this development, starting either from the frequency Doppler formula (radio velocity convention) or from the wavelenght one (optical velocity convention).

The radio velocity convention Starting from Eq. 4, we obtain to first order in $v_{\parallel}^{\rm obs}/c$

$$f^{\rm obs} = f^{\rm rest}\left(1 - \frac{v_{\parallel}^{\rm obs}}{c}\right). \tag{6}$$

In this case, we can establish a linear velocity scale (see Sect. 1.3.2)

$$v^{\rm obs}(i) = v_{\rm sys}^{\rm obs} + (i - i_0)\,\delta v^{\rm obs} \quad \text{with} \quad \delta v^{\rm obs} = -c\frac{\delta f^{\rm obs}}{f_{\rm tuned}^{\rm rest}}. \tag{7}$$

This linear velocity scale is only a first order approximation, which is called the radio velocity convention. Indeed, it is well adapted to the radio spectrometer because 1) their natural output is a spectral axis regularly spaced in frequency and 2) the radio velocity convention gives a linear relation between the velocity and the frequency scale. This is why this convention is the default in CLASS.

The optical velocity convention Optical spectrometers (*e.g.*, grates) naturally delivers a spectral axis regularly spaced in wavelength. Eq. 4 can then be rewritten as

$$\lambda^{\rm obs} = \lambda^{\rm rest}\sqrt{\frac{c + v_{\parallel}^{\rm obs}}{c - v_{\parallel}^{\rm obs}}}. \tag{8}$$

The first order approximation gives

$$\lambda^{\rm obs} = \lambda^{\rm rest}\left(1 + \frac{v_{\parallel}^{\rm obs}}{c}\right). \tag{9}$$

Following the same path as in Sect. 1.3.2 but now in wavelenght, it is easy to show that we can establish a linear velocity scale proportional to the wavelength axis

$$v_{\rm opt}^{\rm obs}(i) = v_{\rm sys,opt}^{\rm obs} + (i - i_0)\,\delta v_{\rm opt}^{\rm obs} \quad \text{with} \quad \delta v_{\rm opt}^{\rm obs} = +c\frac{\delta\lambda^{\rm obs}}{\lambda_{\rm tuned}^{\rm rest}}. \tag{10}$$

This velocity scale is called the optical velocity convention. The relation between this velocity scale and the frequency spectral axis is non-linear. This is why it is not used in CLASS.

1.3 Interpreting the spectral axis in the local universe

Using the radio convention, the non-relativistic Doppler effect in the observatory frame can be written as

$$\frac{v^{\rm obs}}{c} = \frac{f^{\rm rest} - f^{\rm obs}}{f^{\rm rest}} = 1 - \frac{f^{\rm obs}}{f^{\rm rest}}, \tag{11}$$

where $v^{\rm obs}$ is the velocity of the source in the observatory frame, $f^{\rm obs}$ and $f^{\rm rest}$ are the frequency of the measured photon in the observatory and rest frame respectively. $v^{\rm obs}$ is positive if the source recesses and the rest frame is defined as the frame where the velocity of the emitting gas cell is zero. Introducing the doppler parameter $d^{\rm obs}$, we obtain

$$\frac{f^{\rm obs}}{f^{\rm rest}} = 1 + d^{\rm obs} \quad \text{with} \quad d^{\rm obs} \equiv -\frac{v^{\rm obs}}{c}. \tag{12}$$

1.3.1 Interpretation 1: At fixed $v^{\rm obs}$

Let's assume that the same gas cell in the source emits two lines at different frequencies. We are at fixed $v^{\rm obs}$ and thus $d^{\rm obs}$ because we consider the same gas cell. The frequency axes in the rest and observatory frames are thus given by

$$f^{\rm rest} = \frac{f^{\rm obs}}{1 + d^{\rm obs}}. \tag{13}$$

This means that the frequency separation between the two lines is different in the rest and observatory frames. The modeller has easily access to the rest frame frequencies of the line. It is thus important to display the spectrum frequency axis in the rest frequency axis. This can be achieved only for one velocity (The reasoning is here done at fixed $v^{\rm obs}$), which is by default assumed to be the systemic velocity of the source in the observatory frame, *i.e.*, the mean velocity of the source gas in the observatory frame, written $v^{\rm obs}_{\rm sys}$.

As a convention, CLASS assumes that the tuned rest frequency and its corresponding observatory frequency at the source systemic velocity are associated to the common reference channel i_0. CLASS then displays the frequency axis in the rest frequency associated to the source systemic velocity through

$$f^{\rm rest}(i) = f^{\rm rest}_{\rm tuned} + (i - i_0)\,\delta f^{\rm rest} \quad \text{with} \quad \delta f^{\rm rest} = \frac{\delta f^{\rm obs}}{1 + d^{\rm obs}_{\rm sys}} \quad \text{and} \quad d^{\rm obs}_{\rm sys} = -\frac{v^{\rm obs}_{\rm sys}}{c}. \tag{14}$$

The plotted spectrum thus correctly displays the line at rest frequency positions, *i.e.*, the brightnesses of the gas whose velocity is equal to the systemic velocity in the observatory frame.

1.3.2 Interpretation 2: At fixed $f^{\rm rest}$

Let's assume that the measured spectrum is made of a line centered around $f^{\rm obs}_{\rm tuned}$ in the observatory frame. The associated rest frame frequency, *i.e.*, the frequency of the line at rest, will be $f^{\rm rest}_{\rm tuned}$. The observation is naturally set up so that the source frame is also the rest frame. This implies that $f^{\rm obs}_{\rm tuned}$, $f^{\rm rest}_{\rm tuned}$ and $v^{\rm obs}_{\rm sys}$ are linked through

$$\frac{f^{\rm obs}_{\rm tuned}}{f^{\rm rest}_{\rm tuned}} = 1 + d^{\rm obs}_{\rm sys} \quad \text{with} \quad d^{\rm obs}_{\rm sys} \equiv -\frac{v^{\rm obs}_{\rm sys}}{c}. \tag{15}$$

The modeler will use the Doppler effect to interpret the observed frequency difference $f^{\rm obs}(i) - f^{\rm obs}_{\rm tuned}$ as a local velocity difference (projected along the line of sight direction) around the systemic velocity of the source. Let's write $v^{\rm obs}(i)$ the velocity associated to $f^{\rm obs}(i)$. The velocity difference will be noted

$$\Delta v^{\rm obs}(i) \equiv v^{\rm obs}(i) - v^{\rm obs}_{\rm sys}. \tag{16}$$

In order to derive this velocity difference, we associate the velocity to each observed frequency through the Doppler effect, *i.e.*,

$$\frac{f^{\rm obs}(i)}{f^{\rm rest}_{\rm tuned}} = 1 + d^{\rm obs}(i) \quad \text{with} \quad d^{\rm obs}(i) \equiv -\frac{v^{\rm obs}(i)}{c}. \tag{17}$$

It is easy to deduce that

$$\Delta v^{\rm obs}(i) = c\,\frac{f^{\rm obs}_{\rm tuned} - f^{\rm obs}(i)}{f^{\rm rest}_{\rm tuned}}, \tag{18}$$

i.e., the local velocity difference is proportional to the observed frequency difference. We can define a linear velocity axis associated to the set of brightnesses, $T(i)$ as

$$v^{\rm obs}(i) = v^{\rm obs}_{\rm sys} + (i - i_0)\,\delta v^{\rm obs} \quad \text{with} \quad \delta v^{\rm obs} = -c\frac{\delta f^{\rm obs}}{f^{\rm rest}_{\rm tuned}}. \tag{19}$$

This velocity scale is only meaningful locally, i.e.*, as long as the brightnesses can be associated to the line rest frequency under consideration.*

1.4 Interpreting the spectral axis at high redshift

When the source is far from the observer, the radial velocity may become a sizable fraction of the light velocity (because of the universe expansion) and the approximations of section 1.2 are not valid anymore. However, Gordon et al. (1992) propose a solution based on the observational definition of the redshift.

1.4.1 The redshift: An observational quantity

In the first half of the twentieth century, optical astronomers noted that a source emitting in its rest frame at a wavelength λ^{rest} will appear to emit at a larger wavelength λ^{obs} in the observatory frame. To quantify this phenomenon, they introduced the observational notion of redshift, z, defined with

$$\lambda^{\mathrm{obs}} = \lambda^{\mathrm{rest}}\,(1+z). \tag{20}$$

The redshift is a positive quantity that can become extremely large. It is straightforward to reexpress the redshift using frequency instead of wavelength, *i.e.*,

$$f^{\mathrm{obs}} = \frac{f^{\mathrm{rest}}}{1+z}. \tag{21}$$

Some radio-astronomers used another definition of the redshift, called radio redshift, but this definition is now deprecated and the optical definition of the redshift is today universally used.

The redshift is of course linked to the Doppler effect. We however stress that the redshift is an observational quantity, independent of any mathematical expression of the Doppler effect. This is the key point to understand the solution of Gordon et al. (1992).

1.4.2 Splitting the change of frame

Gordon et al. (1992) proposed to split the problem of high redshift sources by combining a double change of frame with a change of interpretation of the spectral axis for each change of frame.

In the first change of frame, we consider a "local" observer in the source frame (*i.e.*, having the same systemic velocity viewed from Earth). This observer will interpret the change of velocities as changes in frequency for a fixed rest frequency chosen as reference, $f^{\mathrm{rest}}_{\mathrm{ref}}$ (see Section 1.3.2). Assuming that the velocity of the gas in the source frame are non-relativistic, he can use Eq. 18 to convert from velocities to frequencies in the source frame, *i.e.*, the rest frame. The only differences in this case are 1) that the observation frame is the source or rest frame, and 2) the notion of tuned frequency is replaced by the notion of reference frequency. There is no tuned frequency in this thought experiment. However, just as a convenience, we will note this reference frequency, $f^{\mathrm{rest}}_{\mathrm{tuned}}$. This yields

$$\Delta v^{\mathrm{sou}}(i) = c\,\frac{f^{\mathrm{rest}}_{\mathrm{tuned}} - f^{\mathrm{rest}}(i)}{f^{\mathrm{rest}}_{\mathrm{tuned}}}. \tag{22}$$

The second change of frame goes from the rest/source frame to the observatory frame. The key point here is that we are at fixed redshift. It could be said that we are at fixed v^{obs}, as in section 1.3.1, except that for high redshift source, the observational quantity is the redshift and not the source systemic velocity. We thus define the tuned redshift z_{tuned} as

$$z_{\mathrm{tuned}} \equiv \frac{f^{\mathrm{rest}}_{\mathrm{tuned}}}{f^{\mathrm{obs}}_{\mathrm{tuned}}} - 1. \tag{23}$$

With this definition, we write

$$f^{\mathrm{rest}}(i) = f^{\mathrm{obs}}(i)\,(1+z_{\mathrm{tuned}}) \quad \text{with} \quad \forall i, \quad z_{\mathrm{tuned}} = \text{constant}. \tag{24}$$

1.4.3 Interpretation 1: At fixed z and fixed f^{rest}

Combining Eq. 22 to 24, we yield

$$\Delta v^{\text{sou}}(i) = c\,(1 + z_{\text{tuned}})\,\frac{f^{\text{obs}}_{\text{tuned}} - f^{\text{obs}}(i)}{f^{\text{rest}}_{\text{tuned}}}, \tag{25}$$

i.e., the *local* velocity difference in the *source/rest* frame is proportional to the *observed* frequency difference in the *observatory* frame. We can define a linear velocity axis associated to the set of brightnesses, $T(i)$ as

$$v^{\text{sou}}(i) = (i - i_0)\,\delta v^{\text{sou}} \quad \text{with} \quad \delta v^{\text{sou}} = -c\,(1 + z_{\text{tuned}})\,\frac{\delta f^{\text{obs}}}{f^{\text{rest}}_{\text{tuned}}}. \tag{26}$$

This can be interpreted as follows. Let's assume that the measured spectrum is made of a line centered around $f^{\text{obs}}_{\text{tuned}}$ in the observatory frame. The associated rest frame frequency, *i.e.*, the frequency of the line at rest, will be $f^{\text{rest}}_{\text{tuned}}$. If the tuned frequency at channel i in the observatory frame is set to the redshifted frequency

$$f^{\text{obs}}_{\text{tuned}} = \frac{f^{\text{rest}}_{\text{tuned}}}{1 + z_{\text{tuned}}}, \tag{27}$$

the velocity axis defined in Eq. 26 can be interpreted as the local variation of the gas velocity in the rest/source frame for the line whose rest frequency is $f^{\text{rest}}_{\text{tuned}}$.

1.4.4 Interpretation 2: At fixed z and fixed v^{sou}

Let's assume that the same gas cell in the source emits two lines at different frequencies. We are at fixed z_{tuned} because we consider the same gas cell. The frequency axes in the rest and observatory frames are thus given by

$$f^{\text{rest}} = f^{\text{obs}}\,(1 + z_{\text{tuned}}). \tag{28}$$

This means that the frequency separation between the two lines is different in the rest and observatory frames. The modeller has easily access to the rest frame frequencies of the line. It is thus important to display the spectrum frequency axis in the rest frequency axis. This can be achieved only for one redshift (The reasoning is here done at fixed z_{tuned}), which is by default assumed to be the systemic redshift of the source in the observatory frame, *i.e.*, the mean redshift of the source gas in the observatory frame.

As a convention, we can in addition assume that the tuned rest frequency and its corresponding observatory frequency at the source systemic velocity are associated to the common reference channel i_0. The frequency axis in the rest frame is then defined as

$$f^{\text{rest}}(i) = f^{\text{rest}}_{\text{tuned}} + (i - i_0)\,\delta f^{\text{rest}} \quad \text{with} \quad \delta f^{\text{rest}} = \delta f^{\text{obs}}\,(1 + z_{\text{tuned}}). \tag{29}$$

The plotted spectrum thus correctly displays the line at rest frequency positions, *i.e.*, the brightnesses of the gas at rest in the source frame.

1.4.5 Interpretation 3: At fixed f^{rest} and fixed v^{sou}

Let's assume that we are interested to detect a given bright line, *e.g.*, CO(1-0), at high redshift but the redshift of the source is unknown and/or their would be a forest of this line at different redshift for the same line of sight. We can also assume that the parcel of gas is at the same local velocity (an interesting particular case being when the parcel of gas is at rest) in the different source frames corresponding to the different redshift. We thus wish to associate a redshift axis to the set of brightnesses, $T(i)$. To do this, we use

$$z = \frac{f^{\text{rest}}_{\text{tuned}}}{f^{\text{obs}}} - 1, \tag{30}$$

and, after differentiation,

$$\delta z = -\frac{f_{\rm tuned}^{\rm rest}}{f_{\rm tuned}^{\rm obs}} \frac{\delta f^{\rm obs}}{f_{\rm tuned}^{\rm obs}} = -(1+z_{\rm tuned})^2 \frac{\delta f^{\rm obs}}{f_{\rm tuned}^{\rm rest}} = (1+z_{\rm tuned}) \frac{\delta v^{\rm sou}}{c}. \quad (31)$$

This yields

$$z(i) = \frac{1+z_{\rm tuned}}{1-(i-i_0)\,\frac{\delta z}{1+z_{\rm tuned}}} - 1 \quad (32)$$

$$\text{with} \quad z_{\rm tuned} = \frac{f_{\rm tuned}^{\rm rest}}{f_{\rm tuned}^{\rm obs}} - 1, \quad \delta z = (1+z_{\rm tuned}) \frac{\delta v^{\rm sou}}{c}, \quad \text{and} \quad \delta v^{\rm sou} = c\,(1+z_{\rm tuned}) \frac{\delta f^{\rm obs}}{f_{\rm tuned}^{\rm rest}}. \quad (33)$$

The sign in the definition of δz ensures that an increase of i implies an increase of $z(i)$. Moreover, the non-linear character of this axis comes from the fact that the spectral axis is regularly sampled in frequency unit while the adopted redshift definition is the optical one. We retrieve a linear axis

$$z(i) \simeq z_{\rm tuned} + (i-i_0)\,\delta z, \quad \text{only as long as} \quad (i-i_0)\,\delta z \ll (1+z_{\rm tuned}) \quad \text{or} \quad (i-i_0)\,\delta f^{\rm obs} \ll f_{\rm tuned}^{\rm obs}. \quad (34)$$

With current heterodyne detector in millimeter radioastronomy, the last condition is difficult to satisfy as the radiofrequence bandwidth can cover a significant fraction of the tune frequency, at least in the 3 mm band. The non-linear formula is thus useful in CLASS.

1.5 Validity of the radio velocity convention

1.5.1 Going from $f^{\rm obs}$ to $f^{\rm rest}$

In this specific transform, the velocity between the source/rest frame and the observatory frame (*i.e.*, the source systemic velocity in the observatory frame $v_{\rm sys}^{\rm obs}$) is fixed. This implies that the Doppler effect can be written as

$$\frac{f^{\rm rest}}{f^{\rm obs}} = {\rm f}(d_{\rm sys}^{\rm obs}) \quad \text{with} \quad d_{\rm sys}^{\rm obs} \equiv -\frac{v_{\rm sys}^{\rm obs}}{c}. \quad (35)$$

The first thing is that this transform is always linear. However, its physical interpretation in term of $v_{\rm sys}^{\rm obs}$ is different in the two following cases.

Case 1: High redshift universe This case is dealt with the redshift, *i.e.*,

$${\rm f}(d_{\rm sys}^{\rm obs}) = 1 + z(d_{\rm sys}^{\rm obs}). \quad (36)$$

As stated before, the redshift is before all an observational quantity. This means that the complex relationship between $d_{\rm sys}^{\rm obs}$ and z is hidden to the observer. This implies that the transformation between the observatory frequency scale and the rest one is straightforward and "exact" as a function of redshift, *i.e.*, independent of the interpretation as a function of $v_{\rm sys}^{\rm obs}$.

Case 2: Local universe In contrast to the previous case, the observer is used to express the Doppler as a function of the source systemic velocity in a local frame (observatory or Local Standard of Rest). Changing the frame to interpret the frequency axis must thus use a given approximation for the Doppler effect. As long as $d_{\rm vsys}^{\rm obs} \ll 1$, we can use the Special Relativity formula, *i.e.*,

$${\rm f}_{\rm SR}(x) = \sqrt{\frac{1-x}{1+x}}. \quad (37)$$

Using the radio convention is an additional approximation in which

$${\rm f}_{\rm RC}(x) = \frac{1}{1+x}. \quad (38)$$

The difference in the rest frequency between the two approximations is

$$f_{\rm SR}^{\rm rest} - f_{\rm RC}^{\rm rest} = f^{\rm obs}\,[\mathrm{f_{SR}}(d^{\rm obs}) - \mathrm{f_{RC}}(d^{\rm obs})]. \tag{39}$$

This just means that the same observed spectrum will be assigned two different systemic velocities, $v_{\rm SR}^{\rm obs}$ and $v_{\rm RC}^{\rm obs}$, depending on the level of approximation used. In other words, if an observer uses $v_{\rm SR}^{\rm obs}$ as systemic velocity to interpret a spectrum observed with the radio velocity convention, he will find that the lines appear at slightly different rest frequencies in a way that can be interpreted as a wrong systemic velocity. He will thus fit another systemic velocity and find $v_{\rm RC}^{\rm obs}$. The relation between the two associated doppler factors ($d_{\rm SR}^{\rm obs}$ and $d_{\rm RC}^{\rm obs}$) is of course

$$d_{\rm RC}^{\rm obs} = \sqrt{\frac{1+d_{\rm SR}^{\rm obs}}{1-d_{\rm SR}^{\rm obs}}} - 1, \tag{40}$$

and, using the Taylor expansion in $x = 0$,

$$\frac{d_{\rm RC}^{\rm obs} - d_{\rm SR}^{\rm obs}}{d_{\rm SR}^{\rm obs}} = \frac{d_{\rm SR}^{\rm obs}}{2} + O(d_{\rm SR}^{\rm obs}). \tag{41}$$

As $d_{\rm vsys}^{\rm obs} \ll 1$, this is a negligible difference in the systemic velocities, *i.e.*, the radio convention is good enough.

Nevertheless, the main use case is the following one. A "naive" observer got time at a given observatory that uses the radio convention to make a follow-up from an observation acquired in another observatory that uses the special relativity formula. The difference in rest frequency can easily be measurable as it is proportional to the tuned frequency. He probably does not understand the subtleties between the different approximations of the Doppler effect. He will then start to ask around what is wrong. This means that the problem is mainly an interface issue between the different observatories. The easiest solution to this problem would be to use the current standard in (radio-)observatories (What about ALMA, VLA, GBT, APEX). If most of them uses the special relativity formula, IRAM should probably also adopt the special relativity formula.

1.5.2 Going from the frequency to the velocity axis

The velocity scale is only meaningful locally (*i.e.*, at small values of $(i - i_0)\delta v$) around a given rest frequency, $f^{\rm rest}$. It is thus legitimate to use a linear relation between the frequency and the velocity axis. There are 3 possibilities depending on the approximation used to defined the systemic velocity. We stress that all these possibilities are equivalent for observations of the high redshift universe because, in this case, $v_{\rm sys}^{\rm obs}$ and thus $d_{\rm sys}^{\rm obs}$ are zero valued.

Solution 1: The radio convention is used to define both $v_{\rm sys}^{\rm obs}$ and $\delta v^{\rm obs}$ This is the simplest case, which is currently implemented in CLASS. It is defined as

$$v^{\rm obs}(i) = v_{\rm RC}^{\rm obs} + (i - i_0)\,\delta v_{\rm RC}^{\rm obs} \quad \text{with} \quad \delta v_{\rm RC}^{\rm obs} = -c\,\frac{\delta f^{\rm obs}}{f_{\rm tuned}^{\rm rest}}. \tag{42}$$

The radio convention is the result of the first order term of the Taylor expansion at $x = 0$ of

$$\frac{1}{\mathrm{f_{SR}}(x)} = \sqrt{\frac{1+x}{1-x}}. \tag{43}$$

To check the accuracy of the approximation on $\delta v_{\rm RC}^{\rm obs}$, let's use the next term in this Taylor expansion, *i.e.*,

$$\sqrt{\frac{1+x}{1-x}} \simeq 1 + x + \frac{x}{2}. \tag{44}$$

Differentiating this equation, we obtain the first order correction to the velocity channel width

$$\Delta\delta v^{\rm obs}_{\rm RC} = -\delta v^{\rm obs}_{\rm RC}\,\frac{d^{\rm obs}_{\rm RC}}{1+d^{\rm obs}_{\rm RC}}. \tag{45}$$

Let's now assume that we want an accumulated error over n channel to be less than a given tolerance T. This yields

$$v^{\rm obs}_{\rm sys} \leq \frac{c}{\frac{n}{T}+1} \tag{46}$$

This is the criterion to change from the local universe representation (using $v^{\rm obs}_{\rm sys}$) to the high redshift universe (using z). If we want a tolerance of one tenth of channel (*i.e.*, $T = 1/10$), we obtain

- $v^{\rm obs}_{\rm sys} \leq 3\,000\,{\rm km\,s^{-1}}$ for $n = 10$, *i.e.*, $1000\,{\rm km\,s^{-1}}$ wide line at $100\,{\rm km\,s^{-1}}$ resolution (*e.g.* local galaxies);
- $v^{\rm obs}_{\rm sys} \leq 300\,{\rm km\,s^{-1}}$ for $n = 100$, *i.e.*, $10\,{\rm km\,s^{-1}}$ wide line at $0.1\,{\rm km\,s^{-1}}$ resolution (*e.g.*, a local GMC);
- $v^{\rm obs}_{\rm sys} \leq 30\,{\rm km\,s^{-1}}$ for $n = 1000$, *i.e.*, $100\,{\rm km\,s^{-1}}$ wide line at $0.1\,{\rm km\,s^{-1}}$ resolution (*e.g.*, a Galactic outflow).

This does not seem enough.

Solution 2: Special relativity is used to define $v^{\rm obs}_{\rm sys}$ but the radio convention is kept to define $\delta v^{\rm obs}$ This gives

$$v^{\rm obs}(i) = v^{\rm obs}_{\rm SR} + (i-i_0)\,\delta v^{\rm obs}_{\rm RC} \quad \text{with} \quad \delta v^{\rm obs}_{\rm RC} = -c\,\frac{\delta f^{\rm obs}}{f^{\rm rest}_{\rm tuned}}. \tag{47}$$

The accuracy of the approximation around $v^{\rm obs}_{\rm SR}$ is the same as in the previous case.

Solution 3: Special relativity is used to define both $v^{\rm obs}_{\rm sys}$ and $\delta v^{\rm obs}$ If special relativity is used to define $v^{\rm obs}_{\rm sys}$, we would have a more consistent solution by linearizing the above equation at $x = -v^{\rm obs}_{\rm sys}/c$. It is easy to yield

$$v^{\rm obs}(i) = v^{\rm obs}_{\rm SR} + (i-i_0)\,\delta v^{\rm obs}_{\rm SR} \quad \text{with} \quad \delta v^{\rm obs}_{\rm SR} = -c\,\frac{\delta f^{\rm obs}}{f^{\rm rest}_{\rm tuned}}\,\left(1-d^{\rm obs}_{\rm sys}\right)^2\,\sqrt{\frac{1-d^{\rm obs}_{\rm sys}}{1+d^{\rm obs}_{\rm sys}}}. \tag{48}$$

I still need to check how the accuracy of the linearization is improved.

1.6 Handling the local and high redshift universe with the same formalism

In view of the importance of the high redshift studies in today radio-astronomy, it is desirable to handle the observation of high redshift sources in a better way than just replacing the rest frequency by the redshifted frequency at observation time as it is done today at IRAM. It would be better to generalize the formalism to handle both the local and high-redshift universe observations.

In this generalization, the spectral axis would be described with 3 parameters ($f^{\rm rest}$, $v^{\rm obs}_{\rm sys}$, z) with the convention that $v^{\rm obs}_{\rm sys} = 0$ when $z \neq 0$, and vice-versa. This would enable to keep the information about the redshift and the rest frequency when observing the high-redshift universe.

1.7 Using the radio velocity convention

In this framework, the generalization is straightforward. The frequency axis can then be described with

$$f^{\mathrm{rest}}(i) = f^{\mathrm{rest}}_{\mathrm{tuned}} + (i - i_0)\,\delta f^{\mathrm{rest}} \quad \text{with} \quad \delta f^{\mathrm{rest}} = \delta f^{\mathrm{obs}}\,\frac{1 + z_{\mathrm{tuned}}}{1 + d^{\mathrm{obs}}_{\mathrm{sys}}} \quad \text{and} \quad d^{\mathrm{obs}}_{\mathrm{sys}} = -\frac{v^{\mathrm{obs}}_{\mathrm{sys}}}{c}, \tag{49}$$

And the velocity axis with

$$v(i) = v^{\mathrm{obs}}_{\mathrm{sys}} + (i - i_0)\,\delta v \quad \text{with} \quad \delta v = -c\,(1 + z_{\mathrm{tuned}})\,\frac{\delta f^{\mathrm{obs}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}. \tag{50}$$

The velocity axis is given in the observation frame (*i.e.* $v \equiv v^{\mathrm{obs}}$ and $\delta v \equiv \delta v^{\mathrm{obs}}$) when $v^{\mathrm{obs}}_{\mathrm{sys}} \neq 0$, and in the source/rest frame (*i.e.* $v \equiv v^{\mathrm{sou}}$ and $\delta v \equiv \delta v^{\mathrm{sou}}$) when $z \neq 0$.

1.8 Using the special relativity formula

The generalization is slightly more complex. I will derive the formula once some choice is made about the question raised in the previous section.

2 Heterodyne receivers and signal/image frequency axes

2.1 USB and LSB frequency axes in the observatory frame

Heterodyne receivers mix a local oscillator frequency, $f^{\mathrm{obs}}_{\mathrm{LO}}$, to the astronomical signal to transfer the frequency axis from high radio frequencies, $f^{\mathrm{obs}}_{\mathrm{RF}}$, to low intermediate frequencies, $f^{\mathrm{obs}}_{\mathrm{IF}}$, which are then suitable for easy electronic processing. A consequence is that two bands of radio frequencies, namely the lower side band (LSB) and the upper side band (USB) are mixed in the observed spectrum, *i.e.*, the set of brightnesses, $T(i)$, are associated to one IF frequency axis and two RF frequency axes linked through

$$f^{\mathrm{obs}}_{\mathrm{IF}}(i) = SB_{\mathrm{sign}}\,\left[f^{\mathrm{obs}}_{\mathrm{RF}}(i) - f^{\mathrm{obs}}_{\mathrm{LO}}\right] \tag{51}$$

with $SB_{\mathrm{sign}} = -1$ for the RF frequency axis belonging to the lower side band (LSB) tuning and $SB_{\mathrm{sign}} = +1$ for the RF frequency axis belonging to the upper side band (USB). The measured spectra is thus in the IF axis and Eq. 1 must be modified as

$$f^{\mathrm{obs}}_{\mathrm{IF}}(i) = f^{\mathrm{obs}}_{\mathrm{IFtuned}} + (i - i_0)\,\delta f^{\mathrm{obs}}_{\mathrm{IF}}, \tag{52}$$

with

$$f^{\mathrm{obs}}_{\mathrm{IFtuned}} \equiv SB_{\mathrm{sign}}\,(f^{\mathrm{obs}}_{\mathrm{RFtuned}} - f^{\mathrm{obs}}_{\mathrm{LO}}) \equiv \mathrm{cste}. \tag{53}$$

The constancy of $f^{\mathrm{obs}}_{\mathrm{IFtuned}}$ is a property of the heterodyne receivers. Combining the last 3 equations, we yield

$$f^{\mathrm{obs}}_{\mathrm{RF}}(i) = (f^{\mathrm{obs}}_{\mathrm{LO}} + SB_{\mathrm{sign}}\,f^{\mathrm{obs}}_{\mathrm{IFtuned}}) + SB_{\mathrm{sign}}\,(i - i_0)\,\delta f^{\mathrm{obs}}_{\mathrm{IF}}. \tag{54}$$

2.2 Signal and image frequency axes in the observatory frame

The RF tuning frequency, $f^{\mathrm{obs}}_{\mathrm{RFtuned}}$, can be choosen in either band. This band is called the signal band. The other band is then called the image band. The associated RF frequencies are written $f^{\mathrm{obs}}_{\mathrm{sig}}$ and $f^{\mathrm{obs}}_{\mathrm{ima}}$. We obtain

$$f^{\mathrm{obs}}_{\mathrm{sig}}(i) = f^{\mathrm{obs}}_{\mathrm{sig,tuned}} + SB_{\mathrm{sign}}\,(i - i_0)\,\delta f^{\mathrm{obs}}_{\mathrm{IF}} \quad \text{with} \quad f^{\mathrm{obs}}_{\mathrm{sig,tuned}} \equiv f^{\mathrm{obs}}_{\mathrm{LO}} + SB_{\mathrm{sign}}\,f^{\mathrm{obs}}_{\mathrm{IFtuned}}, \tag{55}$$

and

$$f^{\mathrm{obs}}_{\mathrm{ima}}(i) = f^{\mathrm{obs}}_{\mathrm{ima,tuned}} - SB_{\mathrm{sign}}\,(i - i_0)\,\delta f^{\mathrm{obs}}_{\mathrm{IF}} \quad \text{with} \quad f^{\mathrm{obs}}_{\mathrm{ima,tuned}} \equiv f^{\mathrm{obs}}_{\mathrm{LO}} - SB_{\mathrm{sign}}\,f^{\mathrm{obs}}_{\mathrm{IFtuned}}. \tag{56}$$

The sign of the product $SB_{\mathrm{sign}}\,\delta f_{\mathrm{IF}}^{\mathrm{obs}}$ is arbitrary. Indeed, it only defines how the brightnesses are ordered in memory, *i.e.*, whether the frequency increases or decreases when i increases. It thus is simpler to assume that this product is always positive. This implies that the signal frequency increases and the image frequency decreases when i increases. In other words, we adopt the convention that the signal brightnesses are ordered by increasing frequency when i increases, *i.e.*, the product $SB_{\mathrm{sign}}\,\delta f_{\mathrm{IF}}^{\mathrm{obs}}$ is always positive.

As the absolute value of SB_{sign} is 1, we will greatly simplify the equations by replacing $SB_{\mathrm{sign}}\,\delta f_{\mathrm{IF}}^{\mathrm{obs}}$ with $\delta f_{\mathrm{IF}}^{\mathrm{obs}}$ with the convention that $\delta f_{\mathrm{IF}}^{\mathrm{obs}} > 0$ in Eqs 55 and 56. We can then rewrite them as

$$f_{\mathrm{sig}}^{\mathrm{obs}}(i) = f_{\mathrm{sig,tuned}}^{\mathrm{obs}} + (i - i_0)\,\delta f_{\mathrm{IF}}^{\mathrm{obs}} \quad \text{with} \quad f_{\mathrm{sig,tuned}}^{\mathrm{obs}} \equiv f_{\mathrm{LO}}^{\mathrm{obs}} + SB_{\mathrm{sign}}\,f_{\mathrm{IFtuned}}^{\mathrm{obs}}, \tag{57}$$

and

$$f_{\mathrm{ima}}^{\mathrm{obs}}(i) = f_{\mathrm{ima,tuned}}^{\mathrm{obs}} - (i - i_0)\,\delta f_{\mathrm{IF}}^{\mathrm{obs}} \quad \text{with} \quad f_{\mathrm{ima,tuned}}^{\mathrm{obs}} \equiv f_{\mathrm{LO}}^{\mathrm{obs}} - SB_{\mathrm{sign}}\,f_{\mathrm{IFtuned}}^{\mathrm{obs}}. \tag{58}$$

with

$$\delta f_{\mathrm{IF}}^{\mathrm{obs}} > 0. \tag{59}$$

2.3 Signal and image frequency axes in the rest frame

The discussion of Section 1.3.1 is still valid for both the signal and image frequency axes. Eq. 14 can easily be rewritten

$$f_{\mathrm{sig}}^{\mathrm{rest}}(i) = f_{\mathrm{sig,tuned}}^{\mathrm{rest}} + (i - i_0)\,\delta f^{\mathrm{rest}}, \tag{60}$$

and

$$f_{\mathrm{ima}}^{\mathrm{rest}}(i) = f_{\mathrm{ima,tuned}}^{\mathrm{rest}} - (i - i_0)\,\delta f^{\mathrm{rest}}, \tag{61}$$

with

$$\delta f^{\mathrm{rest}} = \frac{\delta f_{\mathrm{IF}}^{\mathrm{obs}}}{1 + d_{\mathrm{sys}}^{\mathrm{obs}}} \quad \text{and} \quad d_{\mathrm{sys}}^{\mathrm{obs}} = -\frac{v_{\mathrm{sys}}^{\mathrm{obs}}}{c}. \tag{62}$$

$f_{\mathrm{sig,tuned}}^{\mathrm{rest}}$ is simply the tuning frequency entered by the user. However, $f_{\mathrm{ima,tuned}}^{\mathrm{rest}}$ must be computed. The easiest way is to define the side band separation as

$$SB_{\mathrm{sep}}^{\mathrm{obs}} \equiv SB_{\mathrm{sign}}\left[f_{\mathrm{sig,tuned}}^{\mathrm{obs}} - f_{\mathrm{ima,tuned}}^{\mathrm{obs}}\right]. \tag{63}$$

It is straightfoward to show that it is constant and equal to

$$SB_{\mathrm{sep}}^{\mathrm{obs}} = \mathrm{cste} = 2\,f_{\mathrm{IFtuned}}^{\mathrm{obs}}. \tag{64}$$

In the rest (source) frame, the following relations hold

$$\frac{f_{\mathrm{sig,tuned}}^{\mathrm{obs}}}{f_{\mathrm{sig,tuned}}^{\mathrm{rest}}} = 1 + d_{\mathrm{sys}}^{\mathrm{obs}} \qquad \frac{f_{\mathrm{ima,tuned}}^{\mathrm{obs}}}{f_{\mathrm{ima,tuned}}^{\mathrm{rest}}} = 1 + d_{\mathrm{sys}}^{\mathrm{obs}}. \tag{65}$$

The side band separation in the rest frame is defined as

$$SB_{\mathrm{sep}}^{\mathrm{rest}} \equiv SB_{\mathrm{sign}}\left[f_{\mathrm{sig,tuned}}^{\mathrm{rest}} - f_{\mathrm{ima,tuned}}^{\mathrm{rest}}\right]. \tag{66}$$

It is then straightforward to show that

$$SB_{\mathrm{sep}}^{\mathrm{obs}} = SB_{\mathrm{sep}}^{\mathrm{rest}}\,\left[1 + d_{\mathrm{sys}}^{\mathrm{obs}}\right], \tag{67}$$

and

$$f_{\mathrm{ima,tuned}}^{\mathrm{rest}} = f_{\mathrm{sig,tuned}}^{\mathrm{rest}} - \frac{SB_{\mathrm{sep}}^{\mathrm{obs}}}{SB_{\mathrm{sign}}\,\left[1 + d_{\mathrm{sys}}^{\mathrm{obs}}\right]}. \tag{68}$$

3 Earth movements

3.1 Periodic shift and dilatation of the rest frequency axis

For a Earth-based observatory, the observed velocity of the source changes with time due to the rotation of the Earth around its axis and around the sun. This implies that the change from the observatory to the rest frame varies with time. The frequency scale in the observatory frame is fixed by the hardware. The frequency scale in the rest frame thus changes with time in two different ways, a shift and a dilatation. The tuned rest frequency is shifted, while the frequency resolution dilates the frequency axis around the tuned frequency.

If nothing is done, averaging spectra to increase the signal-to-noise ratio would imply first a resampling to align the observed frequency scales. It is desirable to avoid this because resampling has many unwanted side-effects, as round-off errors or introduced correlations of the noise levels between channels (see IRAM Memo 2009-4). We present here hardware solutions to this problem.

3.2 Stopping the periodic shift

As the systemic velocity of the source in the observatory frame varies with time in a predictable way, the radio-observatories tune the local oscillator frequency in order to stop the shifting of the observed frequency scale. For instance, the relation between the observatory frequency and the LSR (Local Standard of Rest) frequency is

$$f^{\rm lsr} = f^{\rm obs} + f^{\rm rest}\,\frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c}, \qquad (69)$$

because

$$f^{\rm obs} = f^{\rm rest}\left(1 - \frac{v_{\rm sys}^{\rm obs}}{c}\right) \quad \text{and} \quad f^{\rm lsr} = f^{\rm rest}\left(1 - \frac{v_{\rm sys}^{\rm lsr}}{c}\right). \qquad (70)$$

Thus adding $\left[SB_{\rm sign}\, f^{\rm rest}\,(v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr})/c\right]$ to the local oscillator frequency makes the signal frequency appear as if it was measured in the LSR frame. All the velocities are then expressed in the LSR frame instead of the observatory frame.

However, there is a single local oscillator frequency. Let's assume that the correction is done at the tuned signal frequency, *i.e.*,

$$f_{\rm sig,tuned}^{\rm lsr} = f_{\rm sig,tuned}^{\rm obs} + f_{\rm sig,tuned}^{\rm rest}\,\frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c}. \qquad (71)$$

The correction which is applied to another frequency is then

$$f^{\rm lsr}(i) = f^{\rm obs}(i) + f_{\rm sig,tuned}^{\rm rest}\,\frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c}, \qquad (72)$$

while the correction that should have been applied to another frequency is

$$f^{\rm lsr}(i) = f^{\rm obs}(i) + f^{\rm rest}(i)\,\frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c}. \qquad (73)$$

The difference (or error) is

$$\Delta f^{\rm lsr}(i) = (i - i_0)\,\delta f^{\rm rest}\,\frac{v_{\rm sys}^{\rm obs} - v_{\rm sys}^{\rm lsr}}{c}. \qquad (74)$$

The correction is thus exact only in $i = i_0$, *i.e.*, for the tuned signal frequency[1]. All the other signal and image frequencies oscillates with time with a frequency amplitude which linearly increases with the distance to the reference channel. In other words, this corrects only the global frequency shift, not the dilatation around the reference channel.

[1] In fact, there often are several local oscillators in a radio-observatory. At the Plateau de Bure Interferometer, two of them are used to stop both the signal and image tuned frequencies. This was useful at the time when the 1mm receivers were double-sideband and the sideband separation was obtained with Walsh phase modulation/demodulation.

Table 1: CLASS header parameters used to describe the frequency/velocity axes and their translation in this document.

CLASS name	Unit	Translation	Notation
nchan	—	Number of channel	—
rchan	—	Reference channel	i_0
restf	MHz	Tuned signal rest frequency	$f^{\mathrm{rest}}_{\mathrm{tuned,sig}}$
image	MHz	Tuned image rest frequency	$f^{\mathrm{rest}}_{\mathrm{tuned,ima}}$
fres	MHz	Observatory channel spacing	δf^{meas}
vres	$\mathrm{km\,s^{-1}}$	Velocity channel spacing	δv^{meas}
voff	$\mathrm{km\,s^{-1}}$	source systemic velocity	$v^{\mathrm{meas}}_{\mathrm{sys}}$
doppler	—	Doppler factor	$d^{\mathrm{meas}}_{\mathrm{sys}}$

3.3 Stopping the periodic dilatation

At IRAM-30m, G. Paubert proposes to stop the periodic dilatation in the rest frequency by adapting the channel spacing of the spectrometer frequency comb in real time. This in combination with LO tuning will stop all temporal variations of the frequency axis, removing the need for resampling when averaging spectra observed at different time of year.

This proposition is equal to a change of reference frame for the channel spacing. Due to hardware limitation, this change of frame can only be done from the observatory to the barycentric frame. This implies that adaptation of the frequency channel spacing when going from the barycentric frame to the source rest frame will still be needed.

4 CLASS implementation

4.1 The header parameters

Table 1 describes the CLASS header parameters used to describe the frequency/velocity axes and their translation in this document. The signal and image frequency axis are given in the source (rest) frame by

$$f^{\mathrm{rest}}_{\mathrm{sig}}(i) = f^{\mathrm{rest}}_{\mathrm{sig,tuned}} + (i - i_0)\,\delta f^{\mathrm{rest}}, \tag{75}$$

and

$$f^{\mathrm{rest}}_{\mathrm{ima}}(i) = f^{\mathrm{rest}}_{\mathrm{ima,tuned}} - (i - i_0)\,\delta f^{\mathrm{rest}}, \tag{76}$$

with

$$\delta f^{\mathrm{rest}} = \frac{\delta f^{\mathrm{meas}}_{\mathrm{IF}}}{1 + d^{\mathrm{meas}}_{\mathrm{sys}}} \quad \text{and} \quad d^{\mathrm{meas}}_{\mathrm{sys}} = -\frac{v^{\mathrm{meas}}_{\mathrm{sys}}}{c}, \tag{77}$$

where $\delta f^{\mathrm{meas}}_{\mathrm{IF}}$ is the frequency channel spacing , and $v^{\mathrm{meas}}_{\mathrm{sys}}$ is the source systemic velocity *in the measurement frame*. The corresponding velocity axis in the measurement frame is thus given by

$$v^{\mathrm{meas}}(i) = v^{\mathrm{meas}}_{\mathrm{sys}} + (i - i_0)\,\delta v^{\mathrm{meas}} \quad \text{with} \quad \delta v^{\mathrm{meas}} = -c\frac{\delta f^{\mathrm{meas}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}. \tag{78}$$

4.2 The MODIFY FREQUENCY command

This command is used to assign a new rest frequency to the current spectrum. The velocity axis is thus recomputed to set the source systemic velocity at this new rest frequency (see Section 1.3.2). For consistency the description of the image frequency axis must also be changed.

This command exists because the line rest frequency is sometimes more uncertain than the spectrometer resolution, implying that the line appeared at velocity different from the source systemic velocity. Nowdays, the bandwidth of the receivers is moreover so large that many lines appear in the same spectrum. For this latter use case, the only way to associate the right velocity to different line rest frequency is to use this command.

4.2.1 Signal frequency axis

The signal frequency axis stays constant under this transform. Only its description changes. This can be written as

$$f_{\rm sig}^{\rm rest}(i) = f_{\rm sig,old}^{\rm rest} + (i - i_{0,\rm old})\,\delta f^{\rm rest}, \quad (79)$$
$$= f_{\rm sig,new}^{\rm rest} + (i - i_{0,\rm new})\,\delta f^{\rm rest}, \quad (80)$$

with

$$\delta f^{\rm rest} = \frac{\delta f_{\rm IF}^{\rm meas}}{1 + d_{\rm sys}^{\rm meas}}. \quad (81)$$

It is then easy to deduce that only the reference channel must be changed as

$$i_{0,\rm new} = i_{0,\rm old} + \frac{f_{\rm sig,new}^{\rm rest} - f_{\rm sig,old}^{\rm rest}}{\delta f_{\rm IF}^{\rm meas}} \left(1 + d_{\rm sys}^{\rm meas}\right) \quad (82)$$

4.2.2 Image frequency axis

The image frequency axis must also be kept fixed. This implies

$$f_{\rm ima}^{\rm rest}(i) = f_{\rm ima,old}^{\rm rest} - (i - i_{0,\rm old})\,\delta f^{\rm rest}, \quad (83)$$
$$= f_{\rm ima,new}^{\rm rest} - (i - i_{0,\rm new})\,\delta f^{\rm rest}, \quad (84)$$

We thus yield for the new tuned signal and image frequency the following relations

$$f_{\rm sig,new}^{\rm rest} = f_{\rm sig,old}^{\rm rest} + (i_{0,\rm new} - i_{0,\rm old})\,\delta f^{\rm rest}, \quad (85)$$
$$f_{\rm ima,new}^{\rm rest} = f_{\rm ima,old}^{\rm rest} - (i_{0,\rm new} - i_{0,\rm old})\,\delta f^{\rm rest}. \quad (86)$$

Adding them gives

$$f_{\rm ima,new}^{\rm rest} + f_{\rm sig,new}^{\rm rest} = f_{\rm sig,old}^{\rm rest} + f_{\rm ima,old}^{\rm rest}. \quad (87)$$

This relation can be used to computes the new image frequency. It can be interpreted as the LO tuning frequency is kept fixed under the transformation. But the side band separation changed.

4.2.3 Velocity axis

The reference channel of the velocity axis was already changed. It only remains to recompute the new velocity resolution associated to the new rest frequency

$$\delta v_{\rm new}^{\rm meas} = -c \frac{\delta f^{\rm meas}}{f_{\rm ima,new}^{\rm rest}}. \quad (88)$$

4.3 The `MODIFY VELOCITY` command

This command is used to assign a new source systemic velocity to the current spectrum. This command is used when the line rest frequency is known very accurately but the line appears shifted. This implies that the source systemic velocity is slightly offset (see Section 1.3.1).

4.3.1 Principle

Changing the source systemic velocity is a change of source reference frame. The signal and image frequency scales in the measurement frame must stay constant under this transform. However they change in the source reference frame. Indeed

$$f^{\text{meas}} = (1 + d_{\text{old}}^{\text{meas}})\, f_{\text{old}}^{\text{rest}} = (1 + d_{\text{new}}^{\text{meas}})\, f_{\text{new}}^{\text{rest}}, \tag{89}$$

with

$$d_{\text{old}}^{\text{meas}} = -\frac{v_{\text{sys,old}}^{\text{meas}}}{c} \quad \text{and} \quad d_{\text{new}}^{\text{meas}} = -\frac{v_{\text{sys,new}}^{\text{meas}}}{c}. \tag{90}$$

Hence

$$f_{\text{new}}^{\text{rest}} = f_{\text{old}}^{\text{rest}} \frac{1 + d_{\text{old}}^{\text{meas}}}{1 + d_{\text{new}}^{\text{meas}}}. \tag{91}$$

4.3.2 The signal frequency axis

The rest signal frequency must be kept fixed. Indeed, the user is trying to characterize a line at the rest signal frequency. But the rest frequency axis changed

$$f_{\text{sig,new}}^{\text{rest}}(i) = f_{\text{sig,tuned}}^{\text{rest}} + (i - i_{0,\text{new}})\, \delta f_{\text{new}}^{\text{rest}} \tag{92}$$

$$= \left[f_{\text{sig,tuned}}^{\text{rest}} + (i - i_{0,\text{old}})\, \delta f_{\text{old}}^{\text{rest}}\right] \frac{1 + d_{\text{old}}^{\text{meas}}}{1 + d_{\text{new}}^{\text{meas}}}. \tag{93}$$

This equality must be true whatever i. This implies that the reference channel and the frequency resolution must be recomputed with

$$i_{0,\text{new}} = i_{0,\text{old}} + \frac{f_{\text{sig,tuned}}^{\text{rest}}}{\delta f_{\text{old}}^{\text{meas}}} \left[d_{\text{new}}^{\text{meas}} - d_{\text{old}}^{\text{meas}}\right], \tag{94}$$

and

$$\delta f_{\text{new}}^{\text{rest}} = \delta f_{\text{old}}^{\text{rest}} \frac{1 + d_{\text{old}}^{\text{meas}}}{1 + d_{\text{new}}^{\text{meas}}}, \quad \textit{i.e.}, \quad \delta f_{\text{new}}^{\text{meas}} = \delta f_{\text{old}}^{\text{meas}}. \tag{95}$$

The last equation just indicates that the channel spacing, which were measured in the measurement frame, did not changed. Indeed, this is the source frame (and not the measurement frame) which is changed through this transformation. The equation for the computation of the rest frequency resolution from measurement frequency resolution is the usual one (Eq. 14).

4.3.3 The image frequency axis

Now that the reference channel has been computed to keep the rest signal frequency fixed, the only free parameter to compute the new image frequency axis is the tuned image frequency.

$$f_{\text{ima,new}}^{\text{rest}}(i) = f_{\text{ima,tuned,new}}^{\text{rest}} - (i - i_{0,\text{new}})\, \delta f_{\text{new}}^{\text{rest}} \tag{96}$$

$$= \left[f_{\text{ima,tuned,old}}^{\text{rest}} - (i - i_{0,\text{old}})\, \delta f_{\text{old}}^{\text{rest}}\right] \frac{1 + d_{\text{old}}^{\text{meas}}}{1 + d_{\text{new}}^{\text{meas}}}. \tag{97}$$

$$f_{\text{ima,new}}^{\text{rest}} = f_{\text{ima,old}}^{\text{rest}} \frac{1 + d_{\text{old}}^{\text{meas}}}{1 + d_{\text{new}}^{\text{meas}}} - f_{\text{sig,tuned}}^{\text{rest}} \frac{d_{\text{new}}^{\text{meas}} - d_{\text{old}}^{\text{meas}}}{1 + d_{\text{new}}^{\text{meas}}}. \tag{98}$$

4.3.4 The doppler factor

Note that the systemic velocity is often given in a different frame (LSR, heliocentric) than the measervatory frame. As long as the new systemic velocity is given in the same frame as the old systemic velocity, it is possible to compute the new doppler factor from the old one. Let's assume that the systemic velocity is given in the LSR frame. We have

$$d_{\rm old}^{\rm meas} = -\frac{v_{\rm lsr}^{\rm meas} + v_{\rm sys,old}^{\rm lsr}}{c} \quad \text{and} \quad d_{\rm new}^{\rm meas} = -\frac{v_{\rm lsr}^{\rm meas} + v_{\rm sys,new}^{\rm lsr}}{c}. \tag{99}$$

Subtracting both equation gives

$$d_{\rm new}^{\rm meas} = d_{\rm old}^{\rm meas} + \frac{v_{\rm sys,old}^{\rm lsr} - v_{\rm sys,new}^{\rm lsr}}{c}. \tag{100}$$

4.4 The MODIFY REDSHIFT command

This could be added.

4.5 The CONSISTENCY command

This command is used to check whether all the spectra in the CLASS current index are consistent. In particular, checking their spectroscopic information ensures that their spectral axes are aligned before, *e.g.*, averaging the spectra. The consistency status helps CLASS commands or the end-user to decide whether further processing can be done channel by channel, or whether resampling is needed to realign the spectra channels.

The spectroscopic consistency aims at checking the alignment of both the signal frequency and the velocity axes. What should be done with the image frequency axis is unclear as it depends on the kind of data (*e.g.*, DSB spectra). One kind of X axes is considered to be aligned when the four following conditions are fulfilled:

1. The left edges of the X ranges covered by the spectra are the same to a given tolerance;
2. The right edges of the X ranges covered by the spectra are the same to the same tolerance;
3. The spectra have the same number of channels;
4. The resolution sign is the same.

The tolerance in the two first conditions is set to a fraction of channel (typically 10%). It is not yet customizable by the user. These two conditions ensure that the whole X range covered by all the spectra are identical within the accepted tolerance (including the dilatation due to the doppler effect that may vary from spectrum to spectrum). Combined with the third condition, this yields a identical resolution within the accepted tolerance. In particular, these three conditions tolerate a consistent shift in the reference channel and physical value at the reference channel. Finally, the last condition ensures that all the X axes increase or decrease in the same direction, channelwise. In the CLASS implementation, the tests 3 and 4 are done first, and only once for all kind of axes. The conditions 1 and 2 are then applied successively for the signal frequency and the velocity axes.

5 The situation at the IRAM-30m

5.1 Velocity conventions

For historical reasons, the velocity convention used to stop the periodic shift due to Earth rotation (see Sect. 3.2) is the optical convention at the IRAM-30m. Let's assume that the systemic velocity of the source is given in the radio convention as it is the natural convention for radio-observatories (see Sect. 1.2.3). The tuned frequency in the observatory frame is thus

$$f^{\rm obs}_{\rm tuned,opt} = \frac{f^{\rm rest}_{\rm tuned}}{1+\frac{v^{\rm obs}_{\rm sys}}{c}}, \tag{101}$$

while it should be

$$f^{\rm obs}_{\rm tuned,rad} = f^{\rm rest}_{\rm tuned}\left(1-\frac{v^{\rm obs}_{\rm sys}}{c}\right). \tag{102}$$

The output spectrum will appear shifted in the observatory frame by

$$f^{\rm obs}_{\rm tuned,rad} - f^{\rm obs}_{\rm tuned,opt} = f^{\rm rest}_{\rm tuned}\left(1-\frac{v^{\rm obs}_{\rm sys}}{c}-\frac{1}{1+\frac{v^{\rm obs}_{\rm sys}}{c}}\right) = -f^{\rm rest}_{\rm tuned}\left(\frac{v^{\rm obs}_{\rm sys}}{c}\right)^2\frac{1}{1+\frac{v^{\rm obs}_{\rm sys}}{c}}. \tag{103}$$

And the signal frequency in the rest frame is shifted by

$$f^{\rm rest}_{\rm sig,tuned,rad} - f^{\rm rest}_{\rm sig,tuned,opt} = -f^{\rm rest}_{\rm tuned}\frac{{d^{\rm obs}_{\rm sys}}^2}{1-{d^{\rm obs}_{\rm sys}}^2}. \tag{104}$$

At 115 GHz, this gives a shift of 3.2 kHz for a velocity of $50\,{\rm km\,s^{-1}}$ and 1.3 MHz for a velocity of $1000\,{\rm km\,s^{-1}}$. Hence both Galactic and extra-Galactic measurements are affected in a measurable way. Nevertheless, this can be measured only by comparison of the lines measured at the IRAM-30m with other radio-observatories.

We recommend that the 30m changes the formula it uses to stop the periodic shift due to Earth rotation and now use the radio convention.

5.2 Systemic velocity in different frames

The previous equations assume that the systemic velocity is given in the same frame as the frequency channel spacing. However, at the 30m the systemic velocity is usually given in the LSR frame (as requested by the user) and the channel spacing and the doppler factor are given in the observatory frame.

On one hand, the rest frequency scale is correctly computed by CLASS because the channel spacing and the doppler factor are given in the same frame. On the other hand, the velocity scale is only approximated because the source systemic velocity is given in another frame, most often the LSR frame.

The end-user (astronomer) is interested with the velocity scale in the LSR frame

$$v^{\rm lsr}(i) = v^{\rm lsr}_{\rm sys} + (i-i_0)\,\delta v^{\rm lsr} \quad \text{with} \quad \delta v^{\rm lsr} = -c\frac{\delta f^{\rm lsr}}{f^{\rm rest}_{\rm tuned}}. \tag{105}$$

But he actually gets

$$v^{\rm lsr}(i) = v^{\rm lsr}_{\rm sys} + (i-i_0)\,\delta v^{\rm obs} \quad \text{with} \quad \delta v^{\rm obs} = -c\frac{\delta f^{\rm obs}}{f^{\rm rest}_{\rm tuned}}. \tag{106}$$

The relative error can be expressed as

$$\frac{\delta v^{\rm obs}-\delta v^{\rm lsr}}{\delta v^{\rm lsr}} = \frac{\delta f^{\rm obs}}{\delta f^{\rm lsr}} - 1 = \frac{1+d^{\rm lsr}_{\rm sys}}{1+d^{\rm obs}_{\rm sys}} - 1. \tag{107}$$

We yield to first order in v/c

$$\frac{\delta v^{\rm obs}-\delta v^{\rm lsr}}{\delta v^{\rm lsr}} = \frac{v^{\rm obs}_{\rm sys}-v^{\rm lsr}_{\rm sys}}{c} \sim 10^{-4}. \tag{108}$$

If we tolerate an error of 1/10th of a channel on the velocity, this implies that the velocity scale is correct for about 1000 channels around the systemic velocity. This translates into $\sim 250\,\mathrm{km\,s^{-1}}$ at 1 mm for a spectral resolution of 195 kHz. This means that the interpretation of the velocity scale of Galactic observations is correct. On the other hand, this could be a problem for extra-galactic observations at extremely high spectral resolution, assuming that the observers do not first downsample their spectra to increase the signal-to-noise ratio per channel...

The easiest way out of the problem would be to write the parameters of the header in the LSR frame in the calibration software that writes the CLASS observation with

$$\delta f^{\mathrm{lsr}} = \delta f^{\mathrm{obs}} \frac{1 + d^{\mathrm{obs}}_{\mathrm{sys}}}{1 + d^{\mathrm{lsr}}_{\mathrm{sys}}}, \quad \delta v^{\mathrm{lsr}} = -c \frac{\delta f^{\mathrm{lsr}}}{f^{\mathrm{rest}}_{\mathrm{tuned}}}, \quad \text{and} \quad d^{\mathrm{lsr}} = 1 - \frac{v^{\mathrm{lsr}}}{c}. \tag{109}$$

In other words, we trick CLASS into thinking that the measurement frame is LSR, while the true measurement frame was the observatory.

```
***************************************************************************
Question: What about the image frequency? Should it be changed as in the
case of the MODIFY VELOCITY command?
***************************************************************************
```

6 Conclusions

Acknowledgement

The process to reach the final version of this memo took very long because the problem is extremely subtle even though it seems straightforward at first sight. We thank S. Guilloteau, H. Liszt, R. Lucas, G .Paubert, V. Piétu, and B. Winkel for useful discussions. Without them, we would probably still wander around.

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