Norwegian Olympiad on Astronomy and Astrophysics

NORSK OLYMPIADE
I ASTRONOMI OG ASTROFYSIKK

Final round problems (and solutions) 2022/2023

Date: *Friday March 24th 2023*

Allowed aids: *calculator, pen/pencil, constants and formulas given below*

Time: *max. 150 minutes*

Part 1 (15 multiple choice questions): *There are four possible answers for each problem - A, B, C and D. Use the answer sheet to mark your chosen answer. There is only one single correct answer for each problem and all problems yield the same number of points. Zero points are given for a problem if more than one answer is marked. Wrong answers do not yield negative points.*

Part 2 (4 open questions): *Explain your reasoning (please write clearly), and write your name on the sheets that you hand in!*

Good luck!

Constants and formulas:

- 1 parsec (pc) ≈ 3.26 light years ≈206 265 AU
- Newton's law of gravity: $F = GmM/r^2$, $G \approx 6.67\cdot10^{-11}$ m³/kg/s²
- Kepler's third law: $a^3/P^2 = G(M + m)/4\pi^2$
- Wien's displacement law: $\lambda_{max} = b/T$, $b \approx 2.9\cdot10^{6}$ nm·K
- Stefan-Boltzmann's law: $F = \sigma T^4$, $\sigma \approx 5.67\cdot10^{-8}$ W/m²/K⁴
- Apparent magnitude: $m = -2.5\log(F/F_0)$

- Hubble's law: $v = H_0 d$, $H_0 \approx 73$ km/s/Mpc
- Earth's mass and radius: $5.97 \cdot 10^{24}$ kg and 6371 km

Part 1: multiple choice questions (1 point per question)

1) $N \geq 2$ point masses, each with mass M/N, are spread uniformly around a ring with radius R (so that each adjacent pair spans $360°/N$ of the circle). What total gravitational force F do they exert on a test mass m positioned a height h directly above the ring's center?
    a) $F = \frac{GMm}{h^2}$
    b) $F = \frac{GMm}{R^2+h^2}$
    c) $F = \frac{GMmh}{(R^2+h^2)^{3/2}}$
    d) $F = \frac{GMm\sqrt{R^2+h^2}}{h^3}$

    Solution: c) The magnitude of the force from each point mass is $f = G(M/N)m/(R^2 + h^2)$. Due to the overall symmetry of the configuration, only its vertical component $f \sin \theta$ contributes to the total force, where $\sin \theta = h/\sqrt{R^2 + h^2}$. Thus, the test particle feels the total force $F = Nf \sin \theta = GMmh/(R^2 + h^2)^{3/2}$.

2) What would happen to the planets' orbits if the Sun suddenly turned into a black hole with Schwarzschild radius $R_S = 2GM_\odot/c^2$?
    a) They spiral inwards until reaching a new smaller stable orbit.
    b) They spiral inwards and are swallowed by the black hole.
    c) They do not change.
    d) They are pushed out.

    Solution: c) The spatial curvature outside a spherically symmetric body depends only on its mass, not on whether it is a black hole, so the orbit does not change.

3) If the Earth stopped in its orbit, approximately how long would it take for it to crash into the Sun?
    a) 64.5 days

b) 129 days
c) 182.5 days
d) 365 days

Solution: a) Earth's free fall (2) towards the Sun is an "elliptical" orbit with zero semi-minor axis and semi-major axis equal to half the radius of the original orbit (1): $a_2 = a_1/2$. Kepler's law says $P_1^2/a_1^3 = P_2^2/a_2^3$, so $P_2 = 2^{-3/2}P_1$, but the crash takes only $P_2/2 = 2^{-5/2} \cdot 365\,d = 64.5\,d$.

4) This morning (competition day, 24. March 2023), we noticed a star rising at 03:40 sidereal time. At approximately what time will this star rise in ten days' time (3. April 2023)?
   a) 03:00
   b) 03:20
   c) 04:00
   d) 04:20

Solution: a) Two ways of solving this problem. Way 1: We know that a sidereal day is approximately 4min shorter than the synodic day. Over ten days, this difference accumulates to 40min, leading to the star rising 40min earlier, i.e. 03:00. Way 2: Given that a year is approximately 365 days, we can set up the following proportion: 10:365 = x:24, giving an x value of around 39.2min which can be approximated to 40min for the purposes of this task. The reason for the earlier rise is that the Sun's apparent movement is in the opposite direction of the apparent movement of the celestial sphere.

5) Olbers' paradox poses the question of why our night sky is dark. Given that our universe is infinite and homogenous, every line of sight should end at the surface of the star, implying that the night sky should be very bright with not a single black spot between the stars. This is obviously false, and there are a number of explanations that try to resolve this paradox. Which of the following would **not** pose a valid explanation?
   a) The universe is too young for the light from distant stars to have reached us.

b) Given the constant expansion of the universe, light from distant stars is red-shifted to the point of non-visibility.
c) There is too much interstellar dust that absorbs the radiated light before it gets to us.
d) Stars' lifespans are too short to significantly fill the empty spaces between one another.

Solution: c) Olbers' paradox is still a bit of an active discussion among cosmologists, but a combination of a), b) and d) with some other factors is the current explanation, though none are currently fully accepted. There is one big reason c) fails as an explanation - the amount of dust in space is not nearly enough to dampen the starlight in such a high degree. This explanation was actually proposed by the German astronomer H. W. M. Olbers who popularised the paradox.

6) How many stars on the entire sky have a negative apparent magnitude in the visible spectrum?
   a) 3
   b) 4
   c) 5
   d) 6

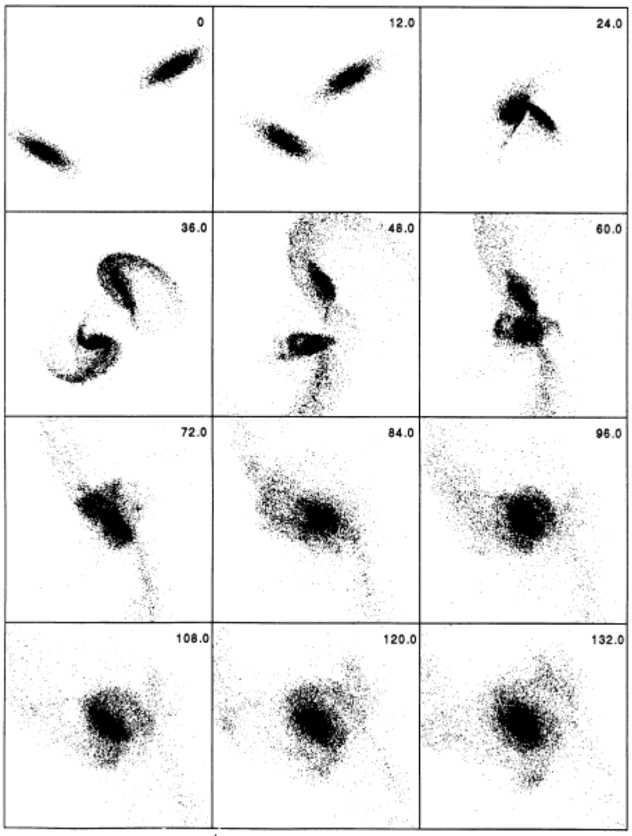
Solution: c) The Sun (-26.8), Sirius (-1.47), Canopus (-0.72), Arcturus (-0.06), and Alpha Centauri (-0.01)

7) A train was travelling eastwards along the 30th line of latitude with a velocity of 120km/h on a lovely sunny first day of spring, 20. March. Given that the day is 24^h long, approximately how long will a passenger on said train be able to see the sunlight?
   a) 11^h5^{min}
   b) 12^h
   c) 12^h35^{min}
   d) 13^h10^{min}

Solution: a)

The velocity of a point on Earth with respect to the planet's centre at a latitude φ is given as $v_0 = \frac{2\pi R_\oplus}{T}\cos\varphi$ (1), where $T = 24\text{h}$ is Earth's orbital period. Plugging in the values gives $1443\,\frac{\text{km}}{\text{h}}$. Given that the Earth rotates west-to-east, and the train is moving eastwards, its velocity relative to the centre of the Earth will be $120 + 1443 = 1563\,\frac{\text{km}}{\text{h}}$. Plugging this newfound velocity into (1) and solving for T', we get $T' = \frac{2\pi R_\oplus}{v}\cos\varphi \approx 22.16\text{h} \approx 22^{\text{h}}10^{\text{min}}$. The length of daylight can be inferred from the fact that the train is travelling on the first day of spring, i.e. spring equinox where the daytime is exactly equal to the nighttime. In the end, the sun will be visible for approximately $11^{\text{h}}5^{\text{min}}$

8) What do the below snapshots from an N-body simulation from 1993 with $N = 16\,384$ particles demonstrate?



a) Collision between two elliptical meteorites leading to a pile-up of space dust.
b) Merging of two spiral galaxies into an elliptical galaxy.
c) Formation of a supermassive black hole.
d) Collapse of a double-star system.

Solution: b)

9) Why does a pole star appear stationary on the sky?
a) Earth is not moving with respect to the pole star.
b) Earth has the rotation axis perpendicular to the pole star.

c) The pole star is almost along the axis of rotation of the earth.
d) Both Earth and the pole star have the same velocity in the Milky Way galaxy.

Solution: c)

10) Which of the following phenomena would not work for estimating cosmic distances if our Universe wasn't expanding??
    a) Sometimes Venus can be seen transiting over the solar disc.
    b) Stars with no proper motion appear to change their position in the sky when viewed six months apart.
    c) Stars exhibit Doppler shift.
    d) All supernovae of Type Ia have the same absolute brightness.

Solution: c) Using Doppler shift for measuring distance implies utilizing Hubble's law to find the distance to a certain galaxy, and that only holds if the theory of an expanding universe is true.

11) Multiple images are captured against the background stars over some time and combined into the image below. What does it show?

    a) Retrograde motion
    b) A moon's orbit around its host planet.
    c) A visual artifact due to Earth's rotation.
    d) Proof of the Ptolemaic model.

Solution: a) retrograde motion (e.g. of Mars as seen from Earth).

12) What does the image below show?

a) Sunspot activity.
b) A planet passing in front of the Sun.
c) View from the first Lagrange point of the white and black bodies.
d) A crater on the Moon.

Solution: b)

13) What does the image below show?

a) The polar region on Mars
b) Eclipse
c) Occultation of Saturn
d) Transit of Venus

Solution: c)

14) Which of these sets of equations determines the position of a satellite m' at the Lagrange point from one massive body m orbiting a much more massive body M, where $M \gg m \gg m'$?
   a) Newton's second law on m', and Kepler's third law on m'.
   b) Newton's second law on m', and Kepler's third law on m.
   c) Newton's second law on M, and Kepler's third law on m'.
   d) Newton's second law on M, and Kepler's third law on m.

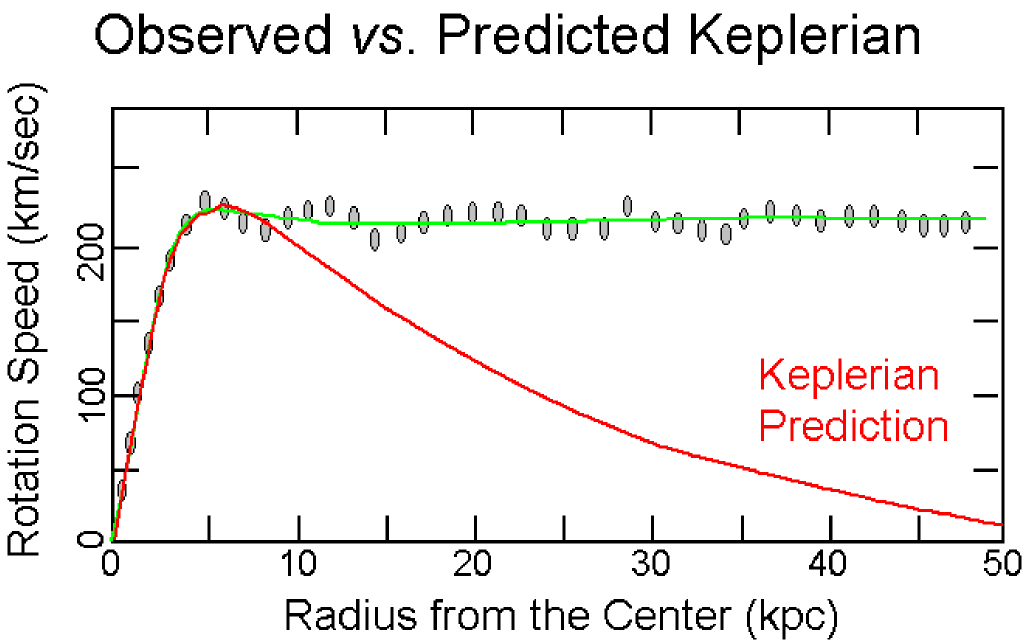
Solution: b) In this restricted three-body problem with $M \gg m \gg m'$, we can take $\{M, m, m'\}$'s accelerations to come from gravitational forces from $\{0, 1, 2\}$ other bodies. We can therefore ignore M's motion, use Kepler's third law (that applies to *two-body* systems) on m, while we must use Newton's "full" second law on m'.

15) The Universe consists of matter, radiation and dark energy, each of which have dominated the Universe's energy density content in different eras. From the beginning of the Universe till today, in which order have they done so?
   a) Radiation → matter → dark energy
   b) Matter → dark energy → radiation
   c) Dark energy → matter → radiation
   d) Dark energy → radiation → matter

Solution: a) The Universe was first dominated by radiation, then matter, and then dark energy. Today, for example, the Universe consists of 30% matter, 70% dark energy and very little radiation, and there is only one alternative that ends with dark energy-domination.

Part 2: Open Questions (points are specified per question)

1) This spherically averaged *galaxy rotation curve* shows observed orbital velocities v of stars and gas at distances r from a galactic center, compared to their predicted velocities from the masses of the observed stars.



a) (0.5 pts) What is the mass M within $r < 8$ kpc in solar masses?
b) (1.0 pts) What is the predicted velocity profile $v(r)$ for $r > 8$ kpc, assuming most mass is located within 8 kpc?
c) (1.0 pts) What is the actual mass density profile $\rho(r)$ for $r > 8$ kpc?
d) (0.5 pts) What is the most widely accepted explanation of the discrepancy between the observed and predicted velocity profiles?

Solution: From $GMm/r^2 = mv^2/r$ or $P^2/r^3 = 4\pi^2/GM$ and $v = 2\pi r/P$, using $r = 8$ kpc and $v \approx 220$ km/s, we get $M = v^2 r/G \approx 9 \cdot 10^{10} M_{\odot}$. If M is constant for $r > 8$ kpc, we get $v = \sqrt{GM/r}$ there. If v is constant for $r > 8$ kpc, $M = v^2 r/G$ gives $dM = 4\pi r^2 \rho\, dr = v^2\, dr/G$, or $\rho = v^2/4\pi G r^2$. Dark matter is today's most widely accepted resolution to this discrepancy.

2) (3 pts) If you were an observer on the surface of Saturn, could you ever see a total solar eclipse caused by Titan, one of its moons? Elaborate your

answer. Titan's diameter is given to be 5150 km, and it orbits Saturn at a distance of 1 222 000 km. Saturn's diameter is 120 000 km and it orbits the Sun at a distance of 9.5 AU. The Sun's angular diameter as seen from Earth is 0.5°.

Solution:

For there to be a total solar eclipse, Titan's angular diameter as seen from Saturn must be greater than or equal to to the Sun's angular diameter as seen from Saturn.

Let R_T and R_S denote the respective radii of the celestial objects, and r_T and r_S denote the radii of their orbits. The maximum value of the satellite's angular diameter as seen from the planet can be written as:

$\delta_T = 2 \cdot \frac{180^o}{\pi} \cdot \frac{R_T}{r_T - r_S}$, and

$\delta_\odot = \frac{0.5^o}{r_S}$, where $[r_S] = AU$

Setting in the given values, we get that $\delta_T \approx 15.2'$ and $\delta_\odot \approx 3.15'$, meaning that the condition we set up at the start holds true.

3) Pulsars are a type of fast rotating, high density stars, which are known for their regular pulses of radiation. They are also sometimes called the 'lighthouses of the universe'. In this problem, we assume the pulsars to be spheres of uniform density, which are gravitationally bound.

   a) (2 pts.) The rotation period of a pulsar was measured to be P = 1.5 ms. What limit does this put on its density?

   b) (1 pt) The pulsar's mass is measured to be $1.5\,M_\odot$. What limit can we place on its radius?

Solution:

Solution:

(a) For a particle of mass (m) on the surface of the pulsar of mass M_P and radius r, the condition to stay attached to the pulsar will be

$$mr\omega^2 \leq \frac{GM_Pm}{r^2}$$
$$\omega^2 \leq \frac{GM_P}{r^3} = \frac{4\pi G\rho_p}{3}$$
$$\therefore \rho_p \geq \frac{3\omega^2}{4\pi G} = \frac{3\pi}{GT^2}$$
$$\rho_p \geq 6.28 \times 10^{16}\,\mathrm{kg\,m^{-3}}$$

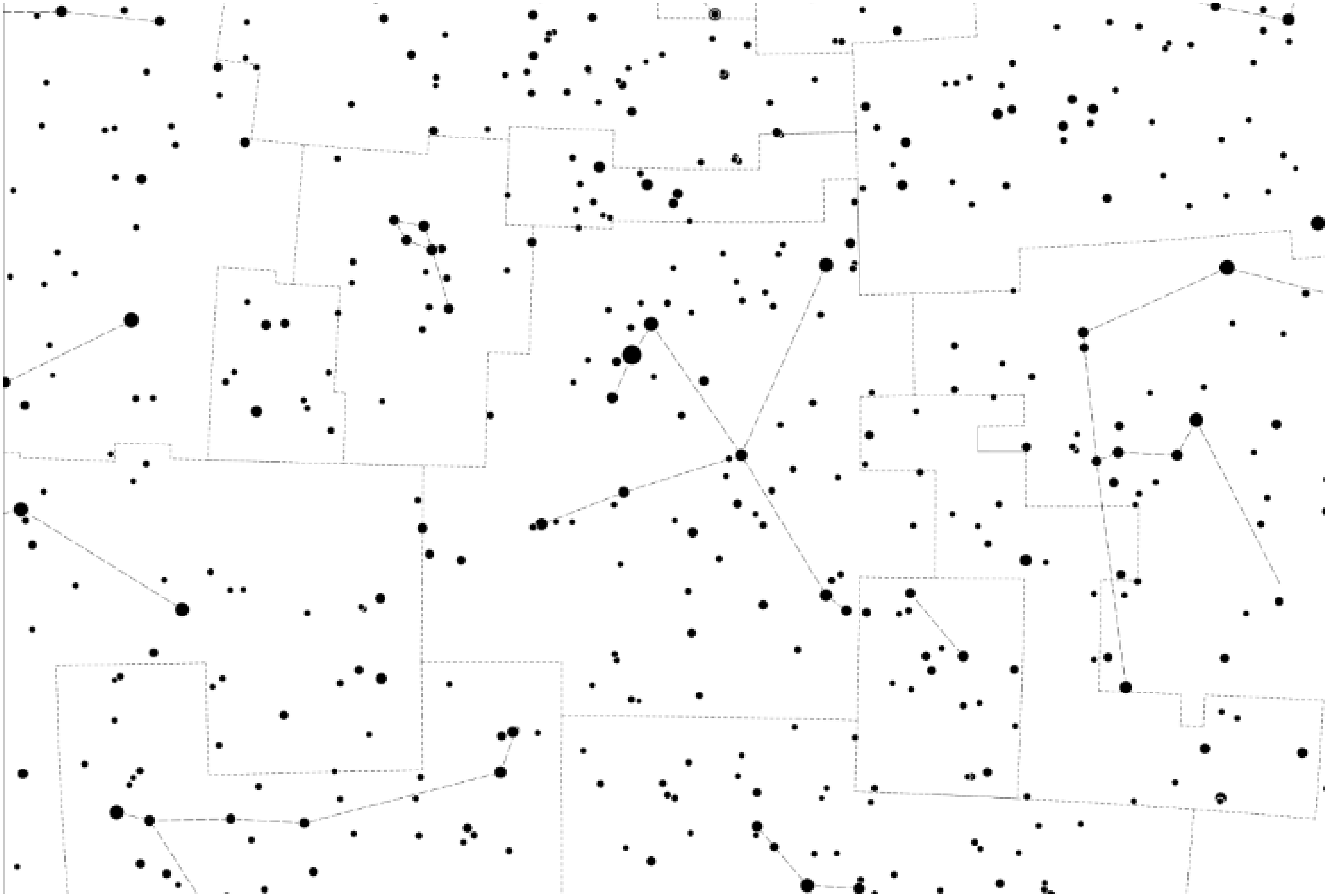
(b)

$$\rho_p = \frac{3M_P}{4\pi\ r^3}$$
$$r \leq 22.5\,\mathrm{km}$$

4) On the following blank constellation map (on its own page):

a) (1 pt) Mark and name at least one constellation belonging to the Zodiac
b) (1pt) Mark at least two stars in the Aquila constellation with their Bayer designation (α, β, γ…)

Student name:



Solution: