UiO : Institutt for teoretisk astrofysikk
Universitetet i Oslo

Rosseland Centre for Solar Physics

Norwegian Olympiad on Astronomy and Astrophysics

Problem set (and solutions) for Round I

2022/2023 school year

Date: Any date during week 43-45 (24 October – 13 November 2022)

Allowed aids: Calculator, pencil/pen and physical constants and formulas given below. Time: 90 minutes

This is a multiple choice problem set. There are four possible answers for each problem - A, B, C og D. Use the answer sheet at the end of the problem set to mark the letter corresponding to your chosen answer. There is only a single correct answer for each problem and all problems yield the same number of points. Zero points are given for a problem if more than one answer is marked. Wrong answers do not yield negative points.

The problem set has 7 pages, and there are 25 problems.

Good luck!

Constants and formulas:

- Apparent magnitude: $m = -2.5\log(F/F_0)$
- Hubble's law: $v = H_0 d$, $H_0 \approx 73$ km/s/Mpc
- Doppler formula: $v/c = (\lambda - \lambda_0)/\lambda_0$
- The Rayleigh criterion: $= 1.22 \cdot \lambda/D$ rad
- Speed of light c = 299792458 m/s
- 1 parsec (pc) ≈ 3.26 light years
- Newton's law of gravity: $F_G = G\,mM/r^2$, $G \approx 6.67\cdot10^{-11}$ m^3/kg/s^2
- Wien's law: $\lambda_{max} = b/T$, $b \approx 2.9 \cdot 10^6$ nm·K
- Stefan-Boltzmann's law: $F = \sigma T^4$, $\sigma \approx 5.67\cdot10^{-8}$ W/m^2/K^4

1. Who made the first use of a telescope for astronomical discoveries?
    a. Nicolaus Copernicus
    b. Galileo Galilei
    c. Tycho Brahe
    d. Isaac Newton

Solution: b. Galileo Galilei

2. What is a planetary nebula?
    a. A shell of gas which is formed when a red giant star sheds its outer layers as it turns into a white dwarf.
    b. Fog on planets (e.g. morning fog observed along the limb of Mars).
    c. A cloud of gas and dust where planets form.
    d. The remnant of a supernova explosion.

Solution: a. A shell of gas which is formed when a red giant star sheds its outer layers as it turns into a white dwarf.

3. Which number is closest to the age of the Universe?
    a. 1.3 million years.
    b. 13 million years.
    c. 1.3 billion years.
    d. 13 billion years.

Solution: d) 13 billion years. If the students know the age of the Sun (5 billion years), they can easily realize that the only number which is larger than 5 is 13 billion years. "The Big Bang Theory" intro song can also be of help.

4. Which process occurs inside the cores of all main sequence stars?
    a. Fission of uranium into krypton and barium.
    b. Fusion of carbon into nitrogen and oxygen.
    c. Fusion of helium into carbon.
    d. Fusion of hydrogen into helium.

Solution: d. Fusion of hydrogen into helium

5. Which of the following is <u>not</u> a reason for using space telescopes?
    a. Telescopes in space produce sharper images by not being affected by the turbulence in the Earth's atmosphere.
    b. Telescopes in space can measure radiation at wavelengths which are blocked by the Earth's atmosphere.

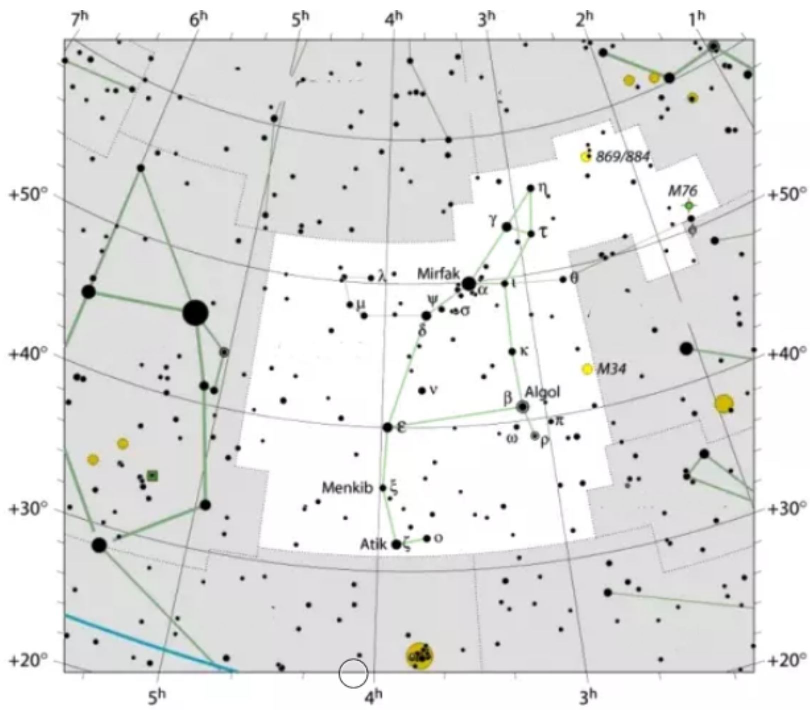
c. **The weightlessness in orbit makes space telescopes more sensitive than telescopes on the ground.**
d. Telescopes in space can observe objects continuously for months or years.

Solution: c. The weightlessness in space does not in itself make a telescope more sensitive. For each of the other alternatives there are several examples of previous or ongoing space missions, e.g. the Kepler mission detecting exoplanet transits by observing the same field continuously for several years.

6. Which theory is strongly supported by observations of galaxy redshifts and the cosmic background radiation?
   a. The theory of planetary formation.
   b. The theory of stellar evolution.
   c. **The Big Bang theory about the evolution of the Universe.**
   d. The theory used to explain the generation of gravitational waves observed by LIGO.

Solution: c. The Big Bang theory. (The gravitational waves detected with LIGO are generated by mergers of compact stellar remnants).

7. What constellation is depicted on the image below?
   a. Cassiopeia.
   b. **Perseus.**
   c. Hydra.
   d. Cepheus.



8. Which of the following objects/phenomena will <u>certainly not</u> be observed in the 21st century?
    a. Signs of biological activity on a planet other than the Earth.
    b. Gravitational waves generated by the collision of two neutron stars.
    c. A 0.10 solar mass red dwarf star which has left the main sequence.
    d. A star being ripped apart by tidal forces from a supermassive black hole.

Solution: c. The main sequence life time of a 0.10 solar mass red dwarf is much longer than the age of the Universe, so will not be feasible to observe such stars in a post-MS stage. Signs of biological activity may have already been observed (e.g., methane gas on Mars and phosphine gas on Venus) and is a major research goal for the coming decades. The two other phenomena have already been observed.

9. For the Earth, if the perihelion were 147 million km, approximately what will be the aphelion for the Earth?

(Note: Aphelion: Point farthest from the Sun in the orbit of a body about the Sun. Perihelion: Point nearest from the Sun in the orbit of a body about the Sun).
    a. About 2 times the Perihelion, 300 million km.
    b. About 3 times the Perihelion, 450 million km.
    c. **Slightly more than the perihelion, about 152 million km.**
    d. Exactly the same as the perihelion, 147 million km.

Solution: c. Slightly more than the perihelion, about 152 million km.

10. Which of the following developments in astronomy was only made possible by the invention of the telescope?
    a. The formulation of Kepler's laws of planetary motion.
    b. The discovery of sunspots.
    c. **The discovery of Saturn's rings.**
    d. The prediction of solar eclipses.

Solution: c. The discovery of Saturn's rings.

11. At noon on March 21st, an astronomer sticks a pole into the ground and measures the shadow to be the exact same length as the pole itself. Which of these latitudes must this have happened at?
    a. 0^o
    b. -90^o
    c. -60^o
    d. 45^o

Solution: d. March 21st is the day of Spring Equinox, meaning the Sun is able to reach the same altitude that the celestial equator reaches for a given latitude. The altitude is given by h = 90º–φ, where φ is the latitude. Given that the shadow and the pole are of the same length, the altitude must have been 45º, and the latitude must have been the same.

12. Why do planets have an approximately spherical shape, while asteroids have much more irregular shapes?
    a. Planets got their regular shapes right after they were formed, while asteroids are just lots of irregular rocks put together.
    b. Planets have gravity that's strong enough to force them into a spherical shape. This is not the case with asteroids.
    c. Asteroids move much faster than planets, not allowing them to stabilize their shapes.
    d. Asteroids are actually spherically shaped as well - it's the meteorites that fall to the Earth that are irregular in shape.

Solution: b. Asteroids are not nearly big enough to have gravitational forces that are so strong that they're pulled into a spherical shape.

13. Two stars are seen close to each other in the sky. Star A appears brighter than Star B. Which of the following statements could help explain the difference in their observed brightness? Assume both the stars to be perfect black bodies.
    a. Both the stars are identical except for the fact that star B is closer to us than star A.
    b. **Both the stars are at same distance, but star A appears yellow, while star B appears orange.**
    c. Both the stars are identical except for the fact that star A has smaller radius than star B.
    d. Both the stars are identical except for the fact that star A is less massive than star B.

Solution: Alternatives A, C and D would all make B brighter than A. Alternative B would be able to explain why A is brighter than B. If the two stars e.g. have similar size, then yellow star A will appear brighter at the same distance because it is intrinsically brighter as its surface temperature is hotter than star B (cf. Wien's law and Stefan-Boltzmann's law using the fact that yellow light has shorter wavelength than orange).

14. Based solely on color, which of the following stars is likely the hottest in terms of its surface temperature?

    a. A yellow dwarf
    b. A red dwarf
    c. A blue giant

d. All of them are approximately the same temperature

(Hint: look at a picture of a rainbow, the violet end has shorter wavelengths and the redder end has longer wavelengths).

Solution: c. A blue giant (refer to Wien's law).

15. If we ever make contact with aliens, which of our fundamental units is likely to match theirs? (In other words, which of these units is universally fundamental?)
    a. Kelvin
    b. Light year
    c. Astronomical Unit (A.U.)
    d. **None of these**

Solution: d. None of these. Alternatives b) and c) are linked to the Earth's orbit around the sun, while a) is linked to the difference between the boiling point and freezing point of water at the sea level pressure of the Earth's atmosphere.

16. An observer on Earth measures a planet's elongation (angular separation from the Sun) to be 100^o. Which of the following planets is not a possible candidate?
    a. Saturn
    b. Venus
    c. Jupiter
    d. Neptune

Solution: b. If a planet's elongation is > 90^o, it must be a superior planet. Venus is the only planet listed that's inferior and therefore cannot have this big of an elongation. Venus' greatest elongation is measured to be 48^o.

17. According to the standard model of cosmology, the three major components of the Universe are: dark energy, dark matter and ordinary matter. How did cosmologists come up with the idea of dark energy (a strange form of energy which has a negative pressure and has an anti-gravity behaviour)?
    a. Due to the fact that at large distances the objects recede from each other with the Hubble velocity which is against the nature of gravity.
    b. Because of the stability of large scale structures in the Universe which is not obtained using only normal form of matter.
    c. Because of the observations indicating that the Universe has entered an accelerating expansion phase which demands us having an energy acting opposite to gravity.

d. Because many gravitational effects cannot be explained by currently accepted theories of gravity unless more matter is present than can be seen.

Solution: c. Because of the observations indicating that the Universe has entered an accelerating expansion phase which demands us having an energy acting opposite to gravity.

18. An astronomer is observing the moon through a refracting telescope. How will the observed image change if the right half of the lens is covered and blocked?
   a. It won't change at all.
   b. We will still see the entirety of the Moon, but it will shine half as bright.
   c. We will only see the left half of The Moon.
   d. We will only see the right half of The Moon.

Solution: b. The uncovered half will still produce the same image as the entirety of the lens, but we are blocking half of the light rays that come to the lens, meaning that we'll get the same image, just that it will be twice as dim as compared to the original.

19. Assume that Earth and the Moon have circular orbits with radii $D_\odot = 150 \times 10^6$ km and $D_M = 384 \times 10^3$ km in the same plane. Take the Sun's radius to be $R_\odot = 696\ 000$ km, and the Moon's radius to be $R_M = 1\ 700$ km. During a solar eclipse, what is the maximum fraction of the Sun's area that the Moon can cover?
   a. 50 %
   b. 100 %
   c. 150 %
   d. 200 %

Solution: b. Viewed from Earth, the radii of the Sun and the Moon span the angles $\theta_\odot = \tan^{-1}(R_\odot / D_\odot)$ and $\theta_M = \tan^{-1}(R_M / D_M)$. With the given numbers we get $\theta_M / \theta_\odot = 1.0$, so the Moon covers almost exactly 100 % of the Sun when they are aligned with Earth. To simplify the calculation, one can use that $\tan^{-1}(R/D) \cong R/D$ when $R/D \ll 1$, as is the case for both the Sun and the Moon here. The result should be very well known to those who have seen (pictures of) a total solar eclipse!

20. You transmit a photon to your friend located in a galaxy far away through empty space. How does the photon's energy upon arrival compare to its value at departure?
   a. The photon's energy has increased upon arrival.
   b. The photon's energy is conserved and does not change.

c. **The photon's energy has decreased upon arrival.**
d. The photon is a quantum particle and as such has no energy.

Solution: c. The energy of a photon with wavelength λ is $E = hc/\lambda$. The Universe expands and stretches the wavelength λ as the photon propagates, so E decreases. The photon's energy is not conserved!

21. You are sunbathing on a beach on Earth on a beautiful sunny day, feeling the intensity I_1 from the radiation of the Sun a distance d_1 away. Your friend is freezing to death on Uranus, feeling only the intensity $I_2 = I_1/400$. Her last wish is to know how far away she is from the Sun. Assuming that the Sun radiates uniformly in all directions, what is the distance d_2 between your friend on Uranus and the Sun?
    a. $d_2 = 20\ d_1$
    b. $d_2 = 400\ d_1$
    c. $d_2 = 600\ d_1$
    d. $d_2 = 8000\ d_1$

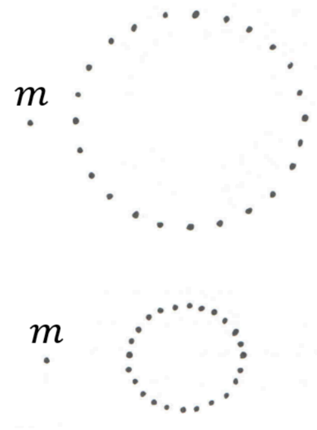
Solution: a. Denote the total power radiated by the Sun (its luminosity) by P. An observer placed on a fictional sphere with radius d centered at the Sun then measures the intensity $I = P/4\pi d^2$. Put in other words, any such sphere at *any* distance d from the Sun captures all its radiated power $P = 4\pi d^2 \cdot I$.
Thus, $P = 4\pi d_1^2 \cdot I_1 = 4\pi\ d_2^2 \cdot I_2$, so $d_2 = \sqrt{I_1/I_2}\ d_1 = \sqrt{400}\ d_1 = 20\ d_1$.

22. Neutron stars are the densest stellar objects we have observed in the Universe. They can have masses $M \leq 2\ M_\odot$ compressed down to radiuses $R \geq 10$ km, where $M_\odot \approx 300\ 000\ M_E$ is the Sun's mass and M_E is Earth's mass. Take Earth's radius to be $R_E \approx 6000$ km, and denote its surface gravitational acceleration by g_E. According to Newtonian theory, what is the gravitational acceleration on the surface of the most compact neutron stars?
    a. $2 \cdot 10^2\ g_E$
    b. $2 \cdot 10^5\ g_E$
    c. $2 \cdot 10^8\ g_E$
    d. $2 \cdot 10^{11}\ g_E$

Solution: d. According to Newton's law of gravity, the gravitational acceleration on the surface of a neutron star is

$$g = \frac{GM}{R^2} \leq \frac{G \cdot 2M_\odot}{(10\ km)^2} = \frac{G \cdot 2 \cdot 3 \cdot 10^5\ M_E}{(R_E/600)^2} = 6^3 \cdot 10^9\ \frac{GM_E}{R_E^2} = 2 \cdot 10^{11}\ g_E$$

23. Consider a small mass m located near a spherical cluster of masses. There is a gravitational force from the cluster on the mass m. Assume that the mass and the cluster are fixed and do not move due to the gravitational force. Imagine we slowly increase the radius of the cluster, while the mass m is always outside of the cluster. As a result of the expansion how does the gravitational force on mass m change?



a. It increases.
b. It decreases.
c. **It does not change.**
d. It depends on the rate of expansion.

Solution: c. The gravitational force doesn't change as a result of Gauss's law for gravity.

24. Today, the mass of the Sun is $M = 2 \cdot 10^{30}$kg, and it radiates away energy at a rate luminosity $L = 4 \cdot 10^{26}$ W. Mass and energy are related through Einstein's famous formula $E = M c^2$, where $c = 3 \cdot 10^8$ m/s is the speed of light. If the Sun kept radiating at its current rate until all its mass is depleted, how long would it take for it to lose all its mass?

a. $14 \cdot 10^8$ years
b. $14 \cdot 10^{10}$ years
c. **$14 \cdot 10^{12}$ years**
d. $14 \cdot 10^{14}$ years

Solution: c. $E = M c^2$, so $\Delta E/\Delta t = (\Delta M/\Delta t)\, c^2 = L$, so $\Delta t = \frac{M}{\Delta M/\Delta t} = \frac{Mc^2}{L} = 4.5 \cdot 10^{20} s = 14 \cdot 10^{12} years$

25. The expansion of the universe makes distant objects move away from us with certain velocity which is called Hubble velocity. The Hubble velocity can be calculated from the Hubble's law, $v = H_0 D$, where $H_0 \cong 70$ (km/s) / Mpc is the Hubble constant and D is the distance of the object from us. What is the distance which the Hubble velocity of objects is larger than the speed of light ($c = 3 \cdot 10^5$ km/s)?

a. 10^4 Mpc
b. 10^3 Mpc
c. 10^2 Mpc
d. 10 Mpc

Solution: a. 10^4 Mpc. $D = \frac{3\times10^5 (km/s)}{70\ (km/s/Mpc)} = \frac{3}{7} \times 10^4\ Mpc\ <\ 10^4\ Mpc$