UiO : Institutt for teoretisk astrofys
Universitetet i Oslo

Rosseland Centre for Solar Physics

Norwegian Olympiad on Astronomy and Astrophysics

Problem set (and solutions) for Round 2

2022/2023 school year

Date: Any date during week 5 (30 January – 5 February 2023)

Allowed aids: Calculator, pencil/pen and physical constants and formulas given below. Time: 90 minutes

This is a multiple choice problem set. There are four possible answers for each problem - A, B, C og D. Use the answer sheet at the end of the problem set to mark the letter corresponding to your chosen answer. There is only a single correct answer for each problem and all problems yield the same number of points. Zero points are given for a problem if more than one answer is marked. Wrong answers do not yield negative points.

The problem set has 6 pages, and there are 20 problems.

Good luck!

Constants and formulas:

- Apparent magnitude: $m = -2.5\log(F/F_0)$
- Hubble's law: $v = H_0 d$, $H_0 \approx 73$ km/s/Mpc
- Doppler formula: $v/c = (\lambda - \lambda_0)/\lambda_0$
- The Rayleigh criterion: $= 1.22 \cdot \lambda/D$ rad
- Speed of light c = 299792458 m/s
- 1 parsec (pc) ≈ 3.26 light years ≈ 206 265 astronomical unit (AU)
- Newton's law of gravity: $F_G = G\,mM/r^2$, $G \approx 6.67\cdot10^{-11}$ m^3/kg/s^2
- Wien's law: $\lambda_{max} = b/T$, $b \approx 2.9 \cdot 10^6$ nm·K
- Stefan-Boltzmann's law: $F = \sigma T^4$, $\sigma \approx 5.67\cdot10^{-8}$ W/m^2/K^4
- $M_\oplus \approx 5.9 \cdot 10^{24} kg$, $R_\oplus \approx 6400\ km$

1) The Andromeda galaxy is also known as M31. What does the "M" stand for in this case?

a) Méchain
b) Messier
c) Minkowski
d) Melotte

Solution: b. The Andromeda galaxy is the 31st entry in a catalogue of diffuse "comet-like" celestial objects compiled by French astronomer Charles Messier (1730-1817). Some of the objects (such as the Andromeda galaxy) were previously known, others were discovered by Messier himself and his colleague Pierre Méchain.

2) Your friend Sophie is on a backpacking trip around the world and sends you a message about the wonderful objects she is seeing in the night sky, including the Magellanic Clouds, the Coalsack and the Carina Nebula. Which of the following destinations in her itinerary could the message be sent from?

a) California
b) Japan
c) Chile
d) Morocco

Solution: c. These objects are all in the far southern sky and are only well visible from near the equator or further south. The Magellanic Clouds have e.g. a declination of approximately -70 degrees, and are only visible above the horizon from a latitude further south than (90 - 70) = 20 degrees North. Chile is the only of these destinations which is far enough south.

3) Which constellation is shown below:

a) Leo (the lion)
b) Boötes (the bear keeper)
c) Auriga (the charioteer)
d) Cetus (the sea monster)

Solution: c.

4) Recently, a gigantic sunspot four times our planet's size could be observed with the naked eye from Earth (with proper eye-protection, of course). How do sunspots occur?

a) The spots form because of the light from those areas of the Sun being completely absorbed before coming to Earth.
b) Celestial objects pass between the Sun and Earth such that the parts of the Sun where we notice the sunspots are occluded.
c) The Sun's strong magnetic field causes there to be areas that are colder than their surroundings and appear black.
d) The Sun is a yellow dwarf and is already halfway into its lifecycle, the sunspots being proof of its reduced hydrogen supply.

Solution: c. Even though we don't know all the intricacies of the process that makes sunspots happen, we are certain that it has something to do with the Sun's magnetic field.

5) Given a measurement of the *irradiance* (radiated power per area) of a star, which two laws can you use to most accurately determine the wavelength that contributes the most to the star's intensity spectrum?

a) The Rayleigh-Jeans law and the Stefan-Boltzmann law
b) The Stefan-Boltzmann law and Wien's displacement law
c) Kepler's third law and Fourier's law of heat conduction
d) Wien's displacement law and Newton's law of cooling

Solution: b. Given a star's irradiance P, one can estimate its temperature T from the Stefan-Boltzmann law $P = \sigma T^4$, and subsequently its peak wavelength from Wien's displacement law $\lambda_{max} = b/T$. These laws are both consequences of Planck's law that governs the radiation from black bodies, like stars.

6) What is the Hertzsprung-Russell diagram?

a) A graph that illustrates the relationship between a star's mass and temperature.
b) A graph that illustrates the relationship between a star's luminosity and distance from Earth.
c) A graph that illustrates the relationship between a star's mass and luminosity.
d) A graph that illustrates the relationship between a star's luminosity and temperature.

Solution: d. A graph that illustrates the relationship between a star's luminosity and temperature.

7) Transit photometry is a method of detecting an exoplanet by means of analysing dips in its parent star's observed brightness when the planet passes in front of the star. What can this method tell us about the exoplanet?

a) Its radius.
b) Its mass.
c) Its temperature.
d) Its albedo.

Solution: a. The dip in the observed brightness is caused by the planet crossing in front of its parent star, and is dependent on their relative size. It follows naturally that it's the radius we can get using this method.

8) The cosmic microwave background radiation (CMBR), discovered by accident in 1965, is remnants of the radiation from:

a) The first moments after the Big Bang originating from the rapid expansion of the universe.
b) The formation of electrons, protons and neutrons.
c) The formation of the very first atoms, about 400 000 years after the Big Bang.

d) The formation of the very first stars, about 100 000 000 years after the Big Bang.

Solution: c. When the first ever hydrogen atoms formed after the dense plasma that filled the universe cooled enough for protons and electrons to combine, photons could start freely moving around instead of being scattered. The light coming from these photons represents what we today know as CMBR.

9) Alpha Persei's parallax is measured to be 6.4mas (milliarcseconds). Approximately how many times is the distance to Alpha Persei from Earth larger than from the Earth to the Sun?

a) 32 000 000
b) 1 320 000
c) 132 000 000
d) 280 000 000

Solution: a. Since we know that the distance between the Earth and the Sun is 1AU, we just need to express the 6.4mas in AU, and we have our number. That is done by doing the following: $1/(6.4*10^{-3})\cdot 206\,265 \approx 32\,000\,000$AU, which indeed is about 32 million times longer than the distance between the Earth and the Sun.

10) An astronaut is flying from the Sun to Sirius. Given that Sirius' luminosity is 22 times that of the Sun, and that Sirius' parallax is 0.373", how many light years will the astronaut have traveled before he notices the Sun and Sirius have the same brightness?

a) 1.53 ly
b) 2.38ly
c) 0.47ly
d) 0.73ly

Solution: a. Denoting the distance between the astronaut and the Sun as x, and the distance between the Sun and Sirius as $R = \frac{1}{0.373}\,pc$, we can write: $\frac{L_\odot}{x^2} = \frac{22L_\odot}{(R-x)^2}$, which gives us the following equation to solve:
$21x^2 + 2xR - R^2 = 0$, which gives solutions $0.47pc$ and $-0.73pc$. The negative solution is actually a valid distance as well, but it isn´t between Sirius and the Sun, so it is not relevant for this task. To get the distance in light years, we just multiply by 3.26.

11) The Earth's orbital velocity around the sun is 30 km/s. What is approximately the highest velocity a meteor can have relative to us as it encounters the Earth's atmosphere? You can assume that the meteor has been in a highly elliptical orbit around the sun, such that it started from a velocity close to zero at a distance much larger than the distance between the Earth and the sun.

a) 12 km/s
b) 30 km/s
c) 60 km/s
d) 72 km/s

Solution: d. A meteoroid which travels in a circular orbit around the sun in the opposite direction than the Earth would hit us head-on at a collision velocity of (30+30) km/s = 60 km/s, but this is still not the maximum collision velocity. An object which starts at almost zero velocity at a very large radius will attain a velocity at 1AU which can be calculated by how the loss in potential energy (which starts at almost zero) is transferred into kinetic energy at 1AU. Comparing this to the Earth's orbital velocity (calculated from the Newtonian force law with centripetal acceleration and the law of gravity) we find that the object can have a maximum velocity of $\sqrt{2} \times$ times the Earth's orbital velocity = 42 km/s. The relative velocity in a head-on collision is then (30+42) km/s = 72 km/s. Some students may also recall that the collision velocity of the Leonid meteor shower is 71 km/s, excluding alternatives a) - c). In this calculation we neglect the extra acceleration caused by the Earth's gravity.

12) In an artificially made Solar System, a new planet is put into place to orbit around the Sun with the same orbital radius as Earth. This planet is then painted completely black and moved to orbit with the orbital radius of Mars. Given that the planet's temperature hasn't changed in this process, and that the orbital radius of Mars is 1.5 that of the Earth, what was the planet's albedo, i.e. how much light did it reflect before we painted it black?

a) 0.67
b) 0.56
c) 0.44
d) 0.81

Solution: b. According to Stefan-Boltzmann's law, for the temperature to remain the same, the planet must absorb (and radiate) the same amount of energy. While it's on Earth's orbital radius, it absorbs the power given by $L_{sun}\frac{S(1-A)}{4\pi r_{\oplus}^2}$, where S is the planet's surface area, and A is its albedo. Since we paint it black after moving it to Mars, the planet absorbs $L_{sun}\frac{S}{4\pi r_M^2}$. Equating these two expressions and after some basic algebra, we get that $(1-A) = (r_{\oplus}/r_M)^2$. From here, it follows that $A = 0.56$.

13) The Subaru telescope in Hawaii has a primary mirror which is 8.2m in diameter, making it the largest optical telescope on Earth to use a non-segmented mirror. Assuming an average wavelength of light of 550nm, what is the smallest possible angle between two barely-resolvable point light sources?

a) **$8.18 \cdot 10^{-8}$ rad**
b) $6.7 \cdot 10^{-8}$ rad
c) $3.0 \cdot 10^{-8}$ rad
d) $1.5 \cdot 10^{-8}$ rad

Solution: a. Using the Rayleigh criterion, we can simply write the angle as
$\theta = 1.22 \cdot \frac{\lambda}{D}$, and inserting the values for the wavelength and the diameter gives us $8.18 \cdot 10^{-8}$ rad.

14) According to the Kepler's 3rd law of planetary motion, the square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit. How long does it take for Jupiter to complete one orbit around the Sun, in Earth years, given that it is approximately 5.2 astronomical units away from the Sun?

a) **11.86 years**
b) 2.28 years
c) 4.56 year
d) 140.60 years

Solution: a. Using Kepler's 3rd law of planetary motion $\frac{T_{Jupiter}^2}{T_{Earth}^2} = (\frac{a_{Jupiter}}{a_{Earth}})^3$
where T is the orbital period, a is the semi-major axis. Knowing that a_{Earth} = 1AU and T_{Earth}= 1 year, it results in Jupiter's period = 11.1 ~11.85 years.

15) Low Earth Orbit (LEO) satellites are typically between 160 to 2000 kilometers above the Earth's surface. These satellites are used for a variety of applications, such as Earth observation, weather forecasting, and telecommunications. What is the orbital velocity in km/s of a satellite placed in a Low Earth Orbit (LEO) at an altitude of 1600 km above the Earth's surface?

a) 0.7 km/s
b) 7.0 km/s
c) 70.0 km/s
d) 700.0 km/s

Solution: b. To calculate the velocity we can use the formula: $v = \sqrt{GM/r}$, where G is the gravitational constant, M is the mass of the Earth, and r is the distance of the satellite from the centre of the Earth. The radius of the Earth is approximately 6400 km, so at an altitude of 1600 km, the distance of the satellite from the centre of the Earth is 8000 km. Which results in $v = \sqrt{(6.67 \cdot 10^{-11} \cdot 5.9 \cdot 10^{24})/(8000 \cdot 10^{3})} = 7013.65\ m/s \approx$ 7.0 km/s.

16) The Hubble's law states that the velocity at which a galaxy is receding from an observer increases proportionally with the distance of the galaxy from the observer. Additionally, galaxies are affected by the local gravity, which causes them to fall towards the center of their local group. This motion, as observed by a local observer, is known as the "peculiar velocity”. What is the velocity of a galaxy, as observed from Earth, located 1 Mpc away and moving towards the Earth with a peculiar velocity of 73 km/s?

a) The object appears to have 0 velocity relative to an observer on Earth
b) The object appears to be moving away from Earth at a velocity of 76 Km/s
c) The object appears to be moving towards Earth at a velocity of 76 Km/s
d) The object appears to be moving away from Earth at a velocity of 146 Km/s

Solution: a. $v = v_{peculiar} + H_0 \cdot d = -73\frac{km}{s} + 73\frac{km}{s} = 0$

17) Hydrogen is an element used a lot in spectroscopy because of its abundance in the universe combined with its particular emission spectrum. A spectral line used a lot is Hydrogen-α with a wavelength of 656 nm. We look at the emission spectrum of a galaxy and find the emission line to be at 670 nm. What can be the reason for this?

a) This is due to a measurement error - the instruments that recorded the 670 nm most probably weren't calibrated properly.

b) This is due to movement of the galaxy away from us at a velocity of about 6400 km/s.

c) This is due to movement of the galaxy towards us at a velocity of about 6270 km/s.

d) This is due to the galaxy's rotation around its own centre of mass at a velocity of about 6000 km/s.

Solution: b. The velocity can be expressed as $\frac{\Delta\lambda}{\lambda_0} \cdot c$ and turns out to be 6402 km/s. One can also just use basic Doppler effect knowledge and notice that the wavelength is higher and immediately conclude that the only correct option could be that it's moving away from us.

18) N-body simulations are a popular method for investigating the effects of gravity on computers. In its simplest form, N particles are placed in an initial configuration, and the gravitational forces between all pairs of particles are computed and used to update the particle positions over a certain number of time steps. If a computer uses the time $T_1 = 1\ hour$ to simulate $N_1 = 1\,000$ particles, roughly how much time T_2 would it spend simulating $N_2 = 1\,000\,000$ particles?

a) $T_2 = 3\ hours$

b) $T_2 = 1\,000\ hours = 41\ days$

c) $\boldsymbol{T_2 = 1\,000\,000\ hours = 114\ years}$

d) $T_2 = 1\,000\,000\,000\ hours = 114\ millennia$

Solution: c. The dominating contribution to the runtime of the simulation is the $N(N-1)$ operations needed to compute the force between all (ordered) pairs of particles. Other operations, such as updating the positions of all N particles, must only be performed once per particle, and are negligible when N is large. Assuming each operation takes the time t, the runtime of an N-body simulation is

$T = N\,(N-1)\,t$. Thus, $\frac{T_2}{T_1} = \frac{N_2(N_2-1)}{N_1(N_1-1)}$, so $T_2 = \frac{10^6(10^6-1)}{10^3(10^3-1)}\ h \approx 10^6\ h.$

19) In cosmology, the scale factor $a(t)$ accounts for the expansion of space: any distance $L(t_0)$ at the current time t_0 becomes $L(t) = a(t)L(t_0)$ at some other time t.

(Part I) How does the energy density $\epsilon = NE/V$ of a box with volume $V = L^3$ containing N matter particles with mass m and energy $E = mc^2$ depend on time?

a) $\epsilon(t) = \epsilon(t_0)$
b) $\epsilon(t) = \epsilon(t_0) \,/\, a(t)^1$
c) $\boldsymbol{\epsilon(t) = \epsilon(t_0) \,/\, a(t)^3}$
d) $\epsilon(t) = \epsilon(t_0) \,/\, a(t)^4$

Solution: c. Only the box' volume $V(t) = L(t)^3$ changes with time, so $\epsilon(t) = Nmc^2/L(t)^3 = Nmc^2/a(t)^3L(t_0)^3 = \epsilon(t_0)/a(t)^3$.

20) (Part II) How would the energy density of the box from the previous task depend on time if the box instead contained N photons with wavelength λ and energy $E = hc/\lambda$?

a) $\epsilon(t) = \epsilon(t_0)$
b) $\epsilon(t) = \epsilon(t_0) \,/\, a(t)^1$
c) $\epsilon(t) = \epsilon(t_0) \,/\, a(t)^3$
d) $\boldsymbol{\epsilon(t) = \epsilon(t_0) \,/\, a(t)^4}$

Solution: d. Both the box' volume $V(t) = L(t)^3$ and the photons' wavelength $\lambda(t)$ changes with time, so $\epsilon(t) = Nhc/\lambda(t)L(t)^3 = Nhc/a(t)\lambda(t_0)a(t)^3L(t_0)^3 = \epsilon(t_0)/a(t)^4$.