# Norwegian Olympiad on Astronomy and Astrophysics

## Final round problems (and solutions) 2023/2024

***Date:*** *Friday March 15th 2024*

***Allowed aids:*** *calculator, pen/pencil, constants and formulas given below*

***Time:*** *max. 150 minutes*

***Part 1 (17 multiple choice questions):*** *There are four possible answers for each problem - A, B, C and D. Use the answer sheet to mark your chosen answer. There is only one single correct answer for each problem, and all problems yield the same number of points. Zero points are given for a problem if more than one answer is marked. Wrong answers do not yield negative points.*

***Part 2 (4 open questions):*** *Explain your reasoning (please write clearly), and write your name on the sheets that you hand in! The number of points is displayed per problem.*

***Good luck!***

Constants and formulas:

- 1 AU = $1.5 \cdot 10^{11}$ m
- 1 parsec (pc) ≈ 3.26 light years ≈ 206 265 AU
- Newton's law of gravity: $F = GmM/r^2$, $G = 6.67 \cdot 10^{-11}$ $m^3/kg/s^2$
- Kepler's third law: $a^3/P^2 = G(M + m)/4\pi^2$
- Wien's displacement law: $\lambda_{max} = b/T$, $b \approx 2.9 \cdot 10^6$ nm · K
- Stefan-Boltzmann's law: $F = \sigma T^4$, $\sigma \approx 5.67 \cdot 10^{-8}$ $W/m^2/K^4$
- Apparent magnitude: $m = -2.5 \log_{10}(F/F_0)$
- Absolute magnitude: $M = m - 5 \log_{10}(d/(10\, pc))$
- Hubble's law: $v = H_0 d$, $H_0 \approx 70$ km/s/Mpc
- Earth's mass and radius: $5.97 \cdot 10^{24}$ kg and 6371 km
- The Sun's mass and radius: $2 \cdot 10^{30}$ kg and 696 340 km
- Earth-Moon distance: $3.8 \times 10^8$ m
- Speed of light: $c = 3 \cdot 10^8$ m/s
- Photon energy: $E = hf = hc/\lambda$, $h = 6.63 \cdot 10^{-34}\, Js$
- Schwarzschild radius: $R = 2GM/c^2$
- Spatial resolving power: $\phi \approx 1.22 \frac{\lambda}{D}$
- Elliptical eccentricity: $e = \sqrt{1 - (b/a)^2}$
- Redshift: $z = \lambda_{obs}/\lambda_{emit} - 1$
- Relativistic Doppler effect: $\frac{\lambda_{obs}}{\lambda_{emit}} = \sqrt{\frac{1+v/c}{1-v/c}}$

## Part 1: multiple choice questions (1 point per question)

1. Assuming the eye pupil has a diameter of 5mm, what is the minimum distance that can be resolved with our naked eyes on the Moon?
   a. around 10-50 km
   b. around 35-70 km
   c. around 70-100 km
   d. around 500-1000 km

**Solution:** b. The visible light spectrum's edges are $\lambda_{min} = 380$ nm and $\lambda_{max} = 750$ nm. Our pupils with diameter $D = 5$ mm can resolve angles $\phi \approx 1.22 \cdot \lambda/D$, corresponding to distances $\phi \cdot 3.8 \cdot 10^8\, m = \{35\, km,\ 70\, km\}$ on the Moon.

2. Calculate the total luminosity of a star whose surface temperature is 7500 K, and whose radius is 2.5 times that of our Sun. Give your answer in units of the solar luminosity $L_\odot$, assuming the surface temperature of the Sun is 5800 K.
   a. $L \approx 13\, L_\odot$
   b. $L \approx 17\, L_\odot$
   c. $L \approx 21\, L_\odot$
   d. $L \approx 25\, L_\odot$

**Solution:** b. $L \propto T^4$ by Stefan-Boltzmann's law, and $L \propto R^2$ because the Sun's surface area is $4\pi R^2$, so $L = (7500\, K/5800\, K)^4 \cdot (2.5\, R_\odot/R_\odot)^2\, L_\odot = 17.47\, L_\odot$.

3. Given that the Sun emits a black-body radiation at an effective temperature of 5800K, in which wavelength range is the power spectrum maximum?
   a. 565-590 nm (yellow)
   b. 590-625 nm (orange)
   c. 500-565 nm (green)
   d. 625-750 nm (red)

**Solution:** c. Using Wien's displacement law gives $\lambda_{max} = b/T \approx 500$ nm, which falls within the green wavelength band 495-570 nm.

4. Why is a sunspot darker than its surroundings?
   a. Nuclear fusion stops temporarily in that region.
   b. Interaction with dark matter.
   c. It is colder than its surroundings.
   d. It is more dense than its surroundings.

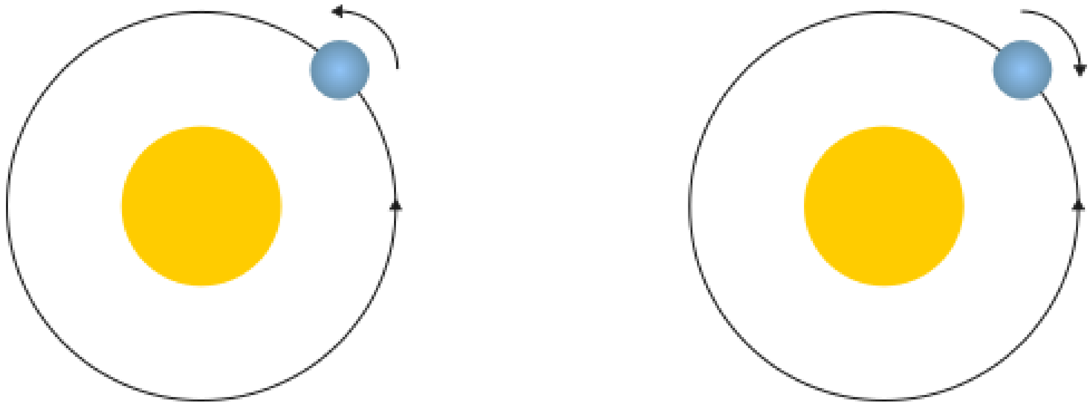
**Solution:** c.

5. In her backyard, a hobby astronomer observes the Moon with her 20 cm telescope that has an eyepiece with focal length 10 mm, and gets 100× magnification. What is the f-number (F#) of the telescope?
   a. 3

b. 5
c. 7
d. 9

**Solution:** b. $F\# = \frac{F}{D} = \frac{mag \cdot f}{D} = \frac{100 \cdot 10}{200} = 5$

6. Earth is a *prograde* planet because it orbits the Sun and rotates about its own axis in the same direction (left figure). In this case, a solar day is 24 hours, and a sidereal day is 23 hours and 56 minutes. How long would a *solar day* be if Earth was a *retrograde* planet, where either the orbit or rotation is reversed (right figure)?



a. 23 hours and 52 minutes
b. 23 hours and 56 minutes
c. 24 hours and 0 minutes
d. 24 hours and 4 minutes

**Solution:** a. The duration of a sidereal day is unchanged if Earth rotates in the opposite direction, so there are 366 sidereal days in one year regardless of the rotation direction. When Earth rotates and orbits in the same direction, there is one less solar day. In the opposite case, there is one more solar day. Thus,
$1\, yr = 365 \cdot T^{same}_{solar} = 366 \cdot T_{sidereal} = 367 \cdot T^{opposite}_{solar}$ and
$T^{opposite}_{solar} = \frac{365}{367} \cdot 24\, h = 23h + 52\, min$. Alternatively: a solar day is 4 minutes longer than a sidereal day in the prograde case, so it is 4 minutes shorter in the retrograde case.

7. The Solar System is around 8 kiloparsecs (kpc) away from the center of the Milky Way and moves with velocity 220 km/s relative to it. How many times has the Sun revolved around the center of the Galaxy since it was formed around 4.6 Gyr ago?
   a. Around 2 times
   b. Around 20 times
   c. Around 200 times
   d. Around 2000 times

**Solution:** b. The period of the Solar System's orbit around the Galactic Center is $P = 2\pi \cdot (8 \cdot 3.26 \cdot 10^3 \cdot 3 \cdot 10^5\, km/s \cdot yr) / (220\, km/s) = 223\, Myr$, so the number of orbits is $4.6\, Gyr / 223\, Myr \approx 20$.

8. What is the orbital period of a comet in the Solar system whose orbit has a semi-*minor* axis of 18 AU and eccentricity $\sqrt{3}\,/\,2$?
   a. 36 years
   b. 100 years
   c. 216 years
   d. 1800 years

**Solution:** c) From the eccentricity formula with $b = 18$ AU and $e = \sqrt{3}\,/\,2$, we find the semi-*major* axis $a = b(1 - e^2)^{-1/2} = 36\,AU$. From Kepler's third law, we then get $P^2 = 36^3\,yr^2 \Rightarrow P = 216\,yr$.

9. The angular diameter of a dense star cluster $d = 3\,kpc$ (kiloparsecs) away from us is $\delta = 25' = 1500''$, and its apparent magnitude is $m = 4.5$. Assume that the stars in the spherical cluster are similar to our Sun, and that they are uniformly distributed in the cluster. If the Sun's absolute magnitude is 4.7, determine the volumetric density of the stars in the cluster, i.e. how many stars per cubic parsec there are in the cluster. You might find the relation $1\,rad = 206265''$ useful.
   a. $5\,/\,pc^3$
   b. $20\,/\,pc^3$
   c. $80\,/\,pc^3$
   d. $120\,/\,pc^3$

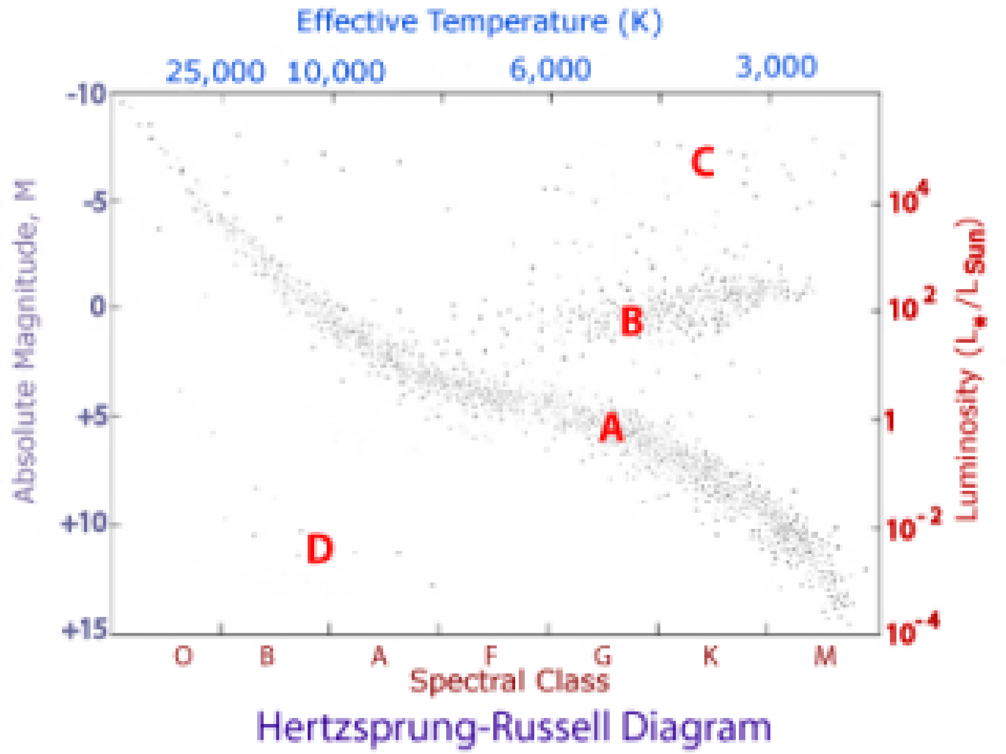
**Solution:** b. If our Sun was in the cluster in question, it would look like a star with apparent magnitude $m_0 = M_0 - 5 + 5\log_{10}(d/pc) = 4.7 - 5 + 5\,\log_{10}(3 \cdot 10^3) = 17.1$. If we denote the luminosity of a single star in the cluster as $L_0$, we can write $m_0 - m = 2.5\log\frac{NL_0}{L_0}$, where $N$ is the number of stars in the cluster. Solving for $N$ gives $N = 10^{(17.1-4.5)/2.5} = 1.095 \cdot 10^5$. Now, we need to determine the volume of the star cluster - since it's spherical, it's simply given by $V = \frac{4}{3}\pi R^3$, where the radius is determined by $R = \frac{1}{2}d\delta$, where $\delta$ needs to be expressed in radians. This gives $R = 10.9pc$ and the density is therefore $20.2\,pc^{-3}$.

10. Between a planet's conjunction (directly in line with the Sun as seen from Earth) and its opposition (directly opposite Earth from the Sun), its apparent magnitude was reduced by 0.85. Which planet is this?
   a. Jupiter
   b. Uranus
   c. Mars
   d. Venus

**Solution:** a) This question might seem very abstract to begin with, but we can realise that the relationship between the maximum and minimum distance of the planet from the Earth is given by $\frac{R_{max}}{R_{min}} = \sqrt{10^{0.4\cdot\Delta m}} = 1.5$. Realising that $R_{max} = R + 1AU$ and

$R_{min} = R - 1AU$, we get $R = 5AU$, which is the distance from the Sun to the planet. From this, we can conclude that the planet in question is Jupiter.

11. Based on the HR-diagram below, which of the following options is the correct ordering of luminosity from lowest to highest?




a. Red giants -> white dwarves -> supergiants
b. Red giants -> supergiants -> white dwarves
c. White dwarves -> red giants -> supergiants
d. Supergiants -> red giants -> white dwarves

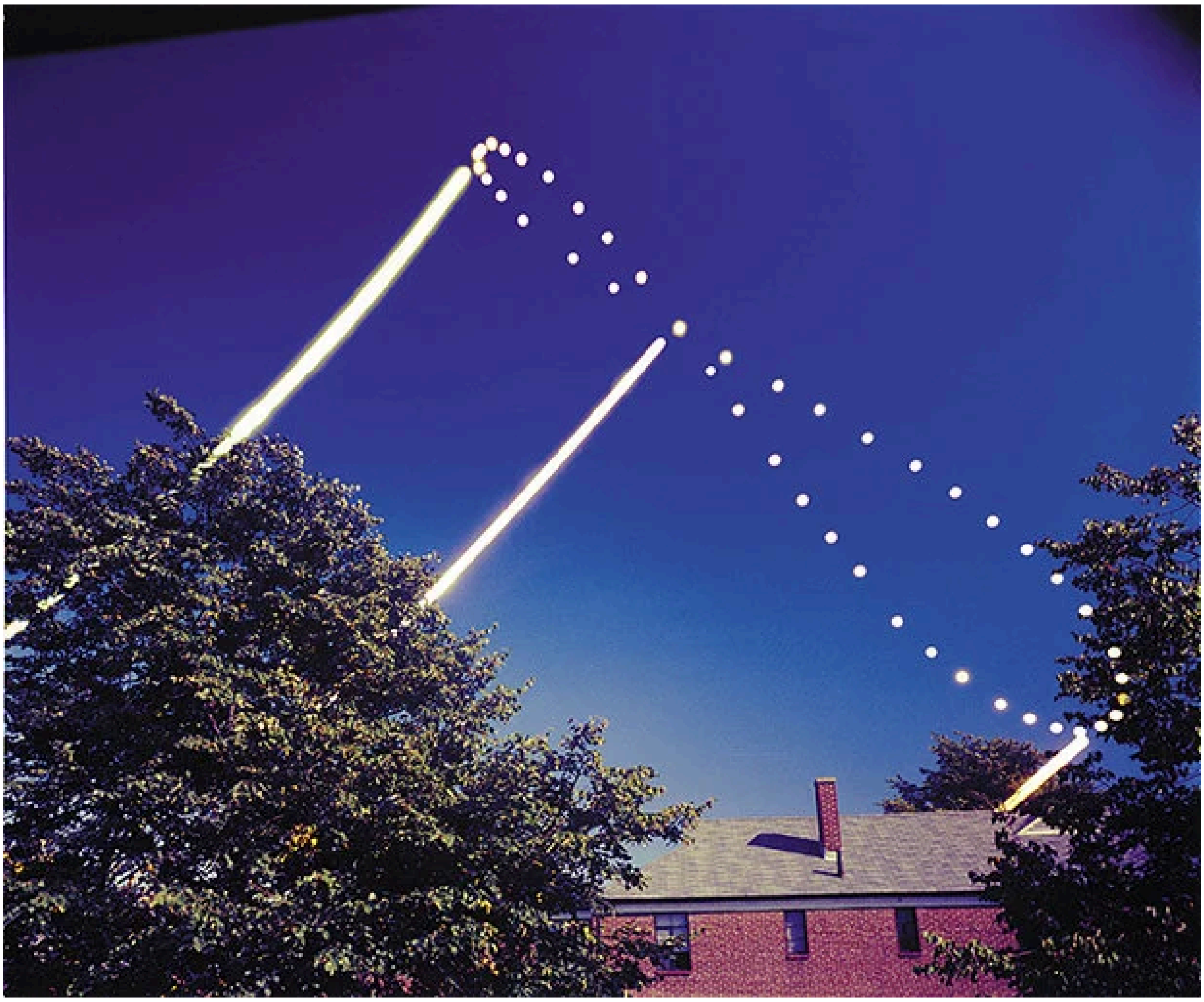
**Solution:** c) Corresponds to D -> B -> C in the diagram.

12. Evaluate the statement: “A photon can take a million years to get from the Sun’s core to its surface”:
    a. False - the speed of light is a constant and the photon takes a mere 2-3 seconds to reach the surface.
    b. True - photons don’t travel in a straight line in the sun because they collide with bigger more massive particles and in a sense get pushed around the core in a random fashion.
    c. Misleading - a single identifiable photon doesn’t bounce around, but what happens is that the photon starting from the core collides with a particle, is absorbed, and another photon is promptly emitted in a random fashion.
    d. False - all of the above are true, but the effect is minute and only increases the amount of time an average photon travels by half a second

**Solution:** c)

13. The photograph below this problem text is the first ever photographic recording of the so-called analemma that the Sun traces across the sky. Is this photograph real or

fake and why?



a. The photograph is fake, as evidenced by the three trails going across the sky.
b. The photograph is fake. There is no reason for the Sun to make a loop in the sky - a true analemma would be a straight line across the sky that corresponds to the celestial equator.
c. The photograph is real. The analemma has two components, north-south because of the change in Sun's declination as a result of Earth's tilt, and east-west which arises due to both the axial tilt and Earth's orbit's eccentricity.
d. The photograph is real. The analemma is due to Sun's movement across our sky during the day, and each exposure was taken an hour apart on the same day.

**Solution:** c) Those who might be confused by the trails might have their skepticism dissuaded if they realise that the photograph above is made by 44 exposures on a single frame of film and 3 long-exposure images which form the path of the Sun in the sky during a single day. The analemma is very much a real thing and in a sense gives us a graph of the declination against the equation of time.

14. If you stretch out your arm in a clenched fist and stretch out your index finger and pinky, approximately what angle will this span in the sky?
    a. 5 degrees
    b. 15 degrees
    c. 25 degrees

d. 30 degrees

**Solution:** b. This is a very useful rule of thumb for determining angular separation in the sky. In addition, the pinky spans about 1 degree, the index-middle-ring fingers together about 5, just a clenched fist 10, and a thumb and pinky 25.

15. The energy density of radiation, matter and dark energy in our (flat) universe evolves with the scale factor $a(t)$ as $\rho_r(t) \propto a(t)^{-4}$, $\rho_m(t) \propto a(t)^{-3}$ and $\rho_\Lambda(t) \propto a(t)^0$. Today $\Omega_r = \rho_r/\rho = 0.005$ % and $\Omega_m = \rho_m/\rho = 30$ %. What was the size ("radius") of the Universe, relative to today, when the densities of dark energy and matter were equal?
    a. 0.25
    b. 0.75
    c. 1.0
    d. 1.33

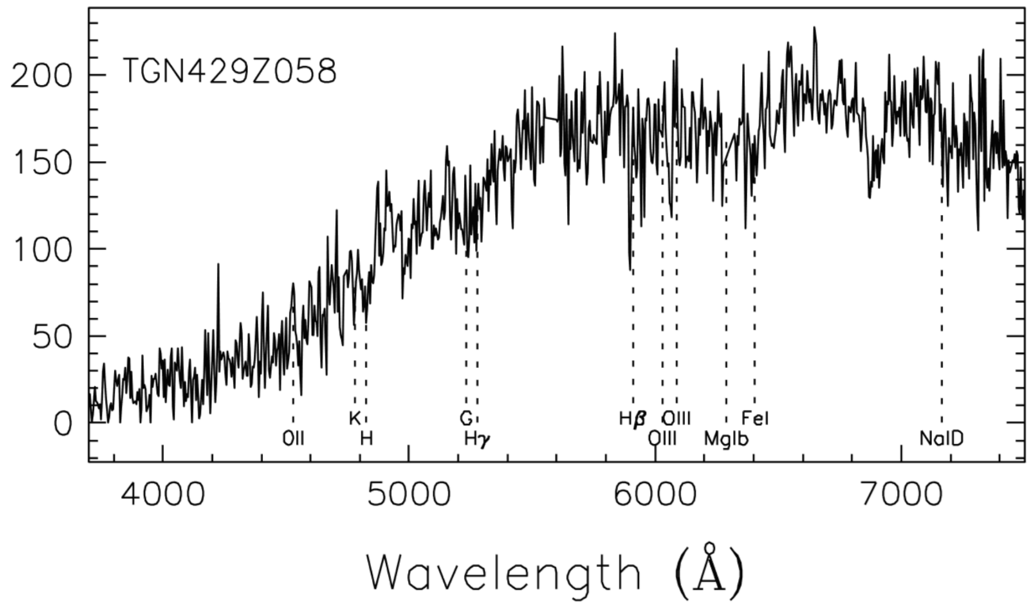
**Solution:** b. Today, we have $\Omega_\Lambda = \rho_\Lambda/\rho = 1 - \Omega_r - \Omega_m = 0.69995$. The scale factor at equality of dark energy and matter is obtained from $\Omega_m(t_0) \cdot a(t)^{-3} = \Omega_\Lambda(t_0)$, so $a(t) = (\Omega_m(t_0)/\,\Omega_\Lambda(t_0))^{1/3} \approx (0.3/0.69995)^{1/3} \approx 0.75$ .

16. We are pointing at a star with a refractor telescope. Its focal length is 100cm. We use an eyepiece with a focal length of 20 mm. However, in the current position of the telescope the distance between the lens of the eyepiece and the focal plane of the telescope is 30 mm. The angular distance from the observed star to its nearest star in the sky is greater than the angular resolution of the telescope. What would you expect to observe and why?
    a. A sharp image of the star because the telescope is properly focused.
    b. A blurry image of the star due to the angular resolution.
    c. A blurry image of the star because the telescope is not focused.
    d. A sharp image of the star because the angular resolution is good enough.

**Solution:** C. To get a focused image, the focal length of the eyepiece must coincide with the focal plan of the telescope.

17. Below is the observed spectrum of a galaxy, including its Hβ-line, which is emitted/absorbed when Hydrogen's energy level $E(n) =- 13.6\ eV/n^2$ changes from $n = 4$ to $n = 2$. The wavelength unit is $1\ \text{Å} = 10^{-10}$ m. Roughly how far away

from us is this galaxy?




a. 9 Mpc
b. 90 Mpc
c. 900 Mpc
d. 9000 Mpc

**Solution:** c. The observed Hβ wavelength is $\lambda_{obs} \approx 5900$ Å, and the emitted wavelength is $\lambda_{emit} = \frac{hc}{E(4)-E(2)} = \frac{hc}{13.6\ eV\cdot(2^{-2}-2^{-4})} = \frac{6.62\cdot10^{-34}\cdot3\cdot10^{8}}{13.6\cdot1.6\cdot10^{-19}\cdot(2^{-2}-2^{-4})} m = 4867$ Å. Then the galaxy's redshift is $z = \lambda_{obs}/\lambda_{emit} - 1 = 5900/4867 - 1 \approx 0.21$, and its recession velocity is $v = zc$. From Hubble's law, its distance from us is $d = \frac{v}{H_0} = \frac{zc}{H_0} = \frac{0.21\cdot3\cdot10^{5}\ km/s}{70\ km/(s\ Mpc)} = 900$ Mpc.

# Part 2: Open Questions (points are specified per question)

1. (1 point) The Voyager 1 space probe launched in 1977 is the most distant human-made object from Earth. As of January 2024, it was 136 AU away from Earth and moving with velocity 17 km/s relative to the Sun. Will Voyager 1 escape the Solar System?

**Solution:** Voyager 1's total mechanical energy is $E = m\left[\frac{1}{2}v^2 - \frac{GM}{r}\right] = m\left[\frac{1}{2}(17\cdot10^3)^2 - \frac{6.67\cdot10^{-11}\cdot2\cdot10^{30}}{136\cdot1.5\cdot10^{11}}\right]J = m\cdot1.4\cdot10^{8}J > 0$, so it will escape the Solar System (the answer does not change if one uses 135 AU or 137 AU to compensate for the distance between Earth and the Sun)

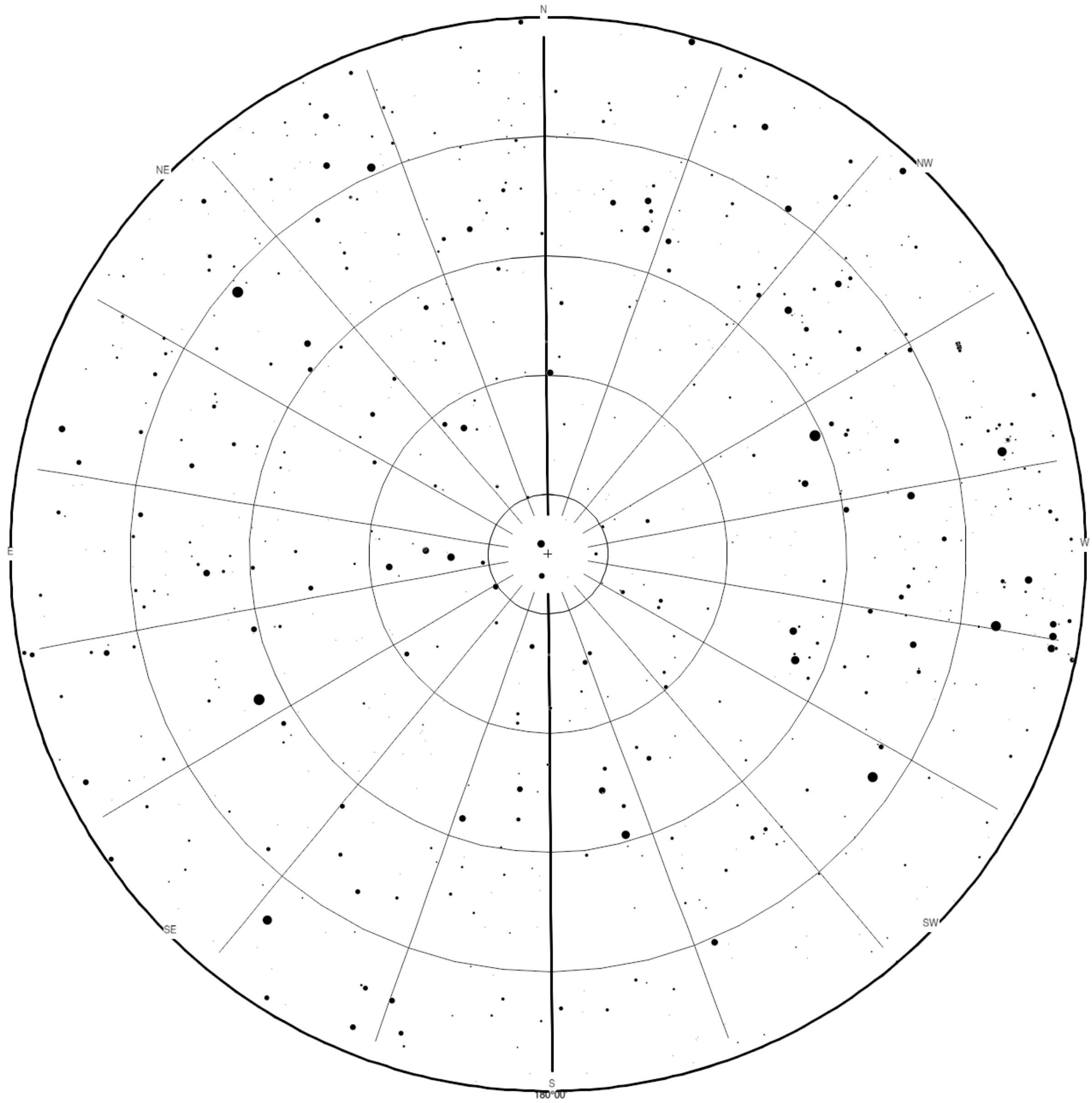
2. (2 points) Per is doing maintenance work outside his small private space station in outer space. Suddenly the rope that secured him to the station bursts! His 70 kg massive body is now separated 1 m from the space station and at rest relative to it. Luckily, he has a strong flashlight in his pocket that radiates light with a power of 10 W! Knowing of momentum conservation and that a photon with energy $E$ has momentum $p$ related through $E^2 = (pc)^2 + (mc^2)^2$, he shines the flashlight away

from the space station to propel himself towards it. How many hours does it take him to reach the space station?

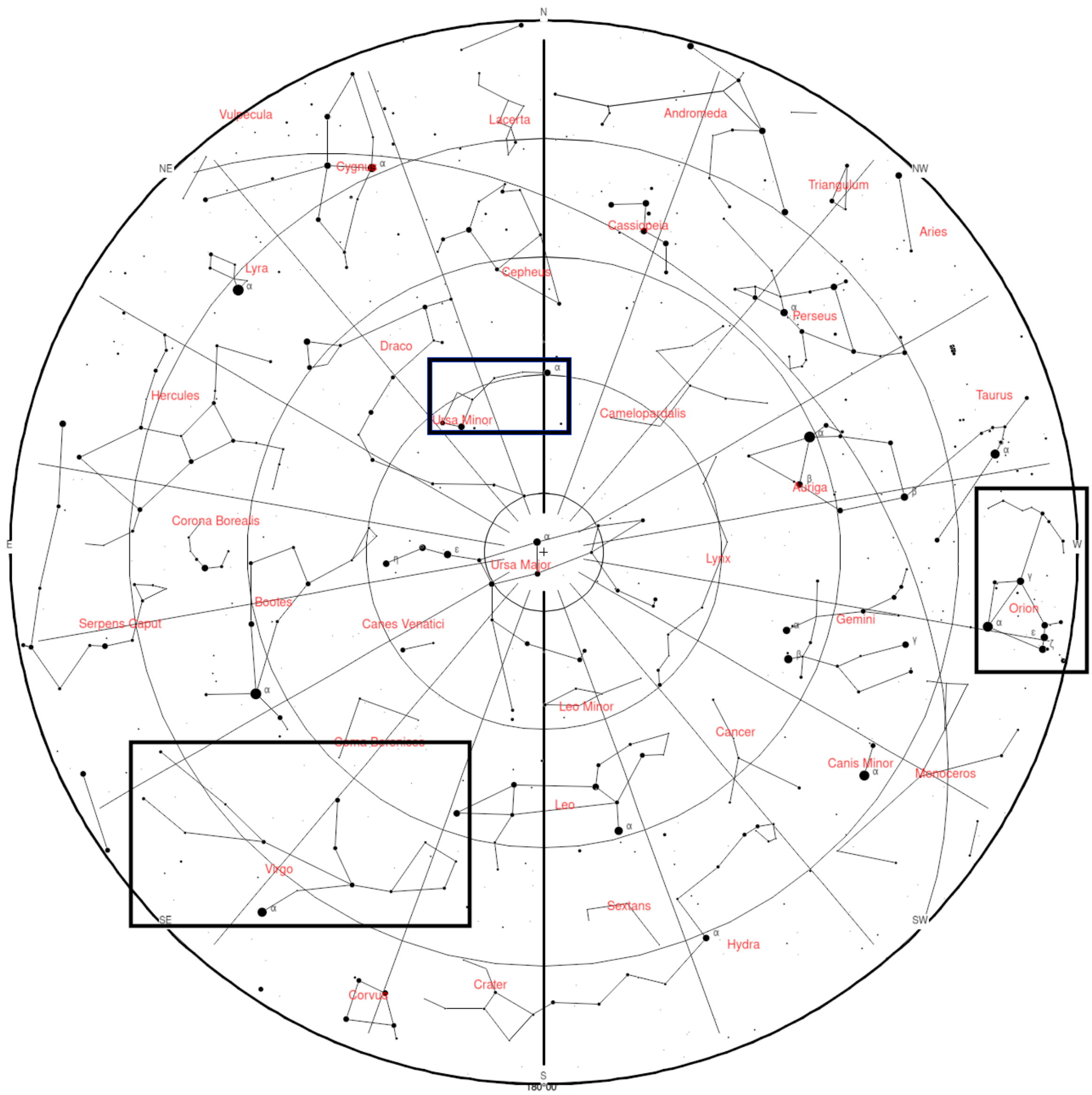
**Solution:** Momentum is conserved in the system with Per and the emitted photons, so the momentum change $dp/dt = (dE/dt)/c = P/c$, where $P = 10$ W, of emitted photons is transferred to Per, exerting the force $F = dp/dt = P/c$ by Newton's second law, corresponding to a constant acceleration $a = F/m = P/mc$. The time it takes to cover a distance $s$ with constant acceleration is

$t = \sqrt{2s/a} = \sqrt{2smc/P} = \sqrt{2 \cdot 1 \cdot 10 \cdot 3 \cdot 10^8/70}\, s = 64800\, s = 18\, h.$

3. The given star map is of Oslo's night sky at 00:00 10.03.2024.
   a. (4 x 0.5 points) Draw in the following constellations:
      i. Ursa Minor
      ii. Cassiopeia
      iii. Orion (hint: this constellation is not completely above the horizon)
      iv. Virgo
   b. (4 x 0.5 points) Mark the brightest star in each of the four constellations in question 3a with the appropriate Greek letter.
   c. (2 points) Approximately draw the line representing the galactic equator. Hint: the coordinate system of the star map is alt-az.



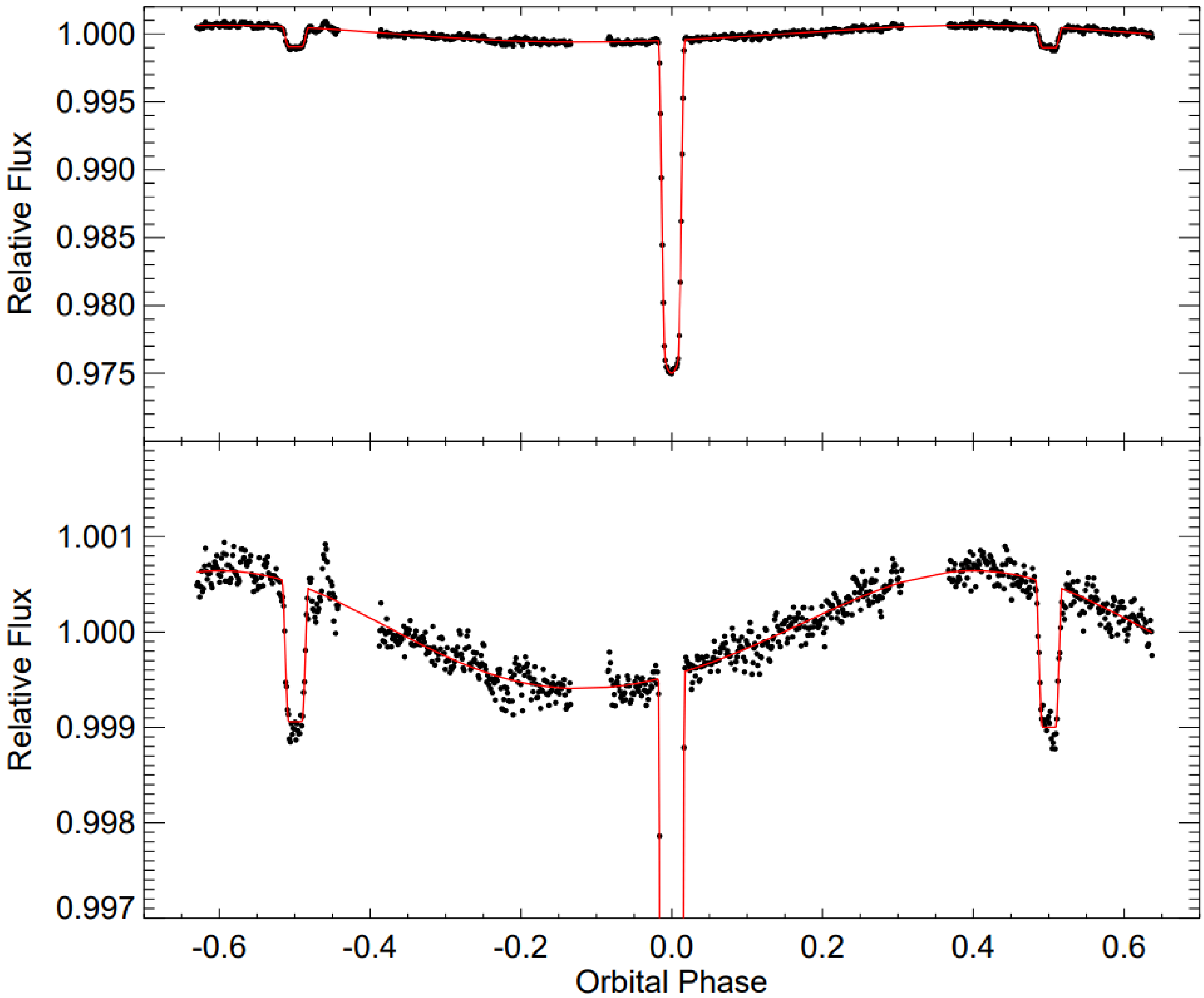

**Solution:** The constellations are marked with rectangles. The letter alpha is put at the brightest star in accordance with Bayer designation. Notice the curved line from NE to SW - this is the galactic equator. The point is to realise that this is a curve on the star map, going from the NE to the SW.



N
NE
NW
E
W
SE
SW
S
180°00
Vulpecula
Cygnus
Lacerta
Andromeda
Triangulum
Aries
Cassiopeia
Lyra
Cepheus
Perseus
Draco
Hercules
Ursa Minor
Camelopardalis
Taurus
Auriga
Corona Borealis
Ursa Major
Lynx
Orion
Bootes
Gemini
Serpens Caput
Canes Venatici
Leo Minor
Cancer
Coma Berenices
Canis Minor
Monoceros
Leo
Virgo
Sextans
Hydra
Crater
Corvus

4. The light curve shown on the image below is taken from research on exoplanet compositions. Both direct light from the star and reflections off the planet contribute. The host star of the exoplanet in question has a radius $R = 0.805R_\odot$ .

   a. (1 point) Explain the big difference noticed in the dips of the light curve. What do the smaller dips represent, and what does the larger one represent?
   b. (1 point) There is also a noticeable phase offset, meaning that the dips are happening after the light curve has reached a maximum/minimum. This can be explained as due to atmospheric circulation on the exoplanet from the pressure gradient of the dayside-to-nightside pressure. What kind of interaction between the planet and the star can cause this? (hint: our Moon exhibits the same interaction)
   c. (2 points) Because of the phenomenon from question b (that you do not need to have done to solve this problem), there is a difference between the dayside flux and the nightside flux emitted by the planet. Find the ratio between the dayside flux and the nightside flux from the light curve. (hint: normalise)
   d. (1 points) Finally, using the data from the curve, as well as the one given in the problem text, find the radius of the planet.



**Solutions:**

a. The big difference is the difference in transit vs. eclipse, i.e. the planet passing in front of its host star, and the planet passing behind it as observed by us. The big dip is the transit, the small ones are eclipses.

b. The existence of a big difference between the dayside and the nightside of the planet implies that the planet is tidally locked to its host star, just like the Moon is to us.

c. The ratio can be found by dividing the maximum normalised planet flux with the minimum normalised planet flux. Reading from the graph, the maximum relative flux is at about 1.0006, and the minimum is at 0.9994. These need to be normalised, which is done by subtracting the relative flux at the very bottom of the small dips, which is at approximately 0.9991

   i. $\frac{F_{planet, max}}{F_{planet, min}} = \frac{1.0006 - 0.9991}{0.9994 - 0.9991} \approx 5$

d. The transit depth is at $1 - 0.975 = 0.025$. This is proportional to $(\frac{R_{exoplanet}}{R_{host\ star}})^2$, so

$$R_{exoplanet} \approx R_{host\ star} \cdot \sqrt{transit\ depth} \approx 0.127 R_{\odot}$$