UiO : Institutt for teoretisk astrofysikk
Universitetet i Oslo

Rosseland Centre for Solar Physics

Norwegian Olympiad on Astronomy and Astrophysics

Problem set (and solutions) for Round II

2023/2024 school year

Date: Any day during week 4 (22nd – 26th January 2024)

Allowed aids: Calculator, pencil/pen and physical constants and formulas given below. Time: 90 minutes

This is a multiple choice problem set. There are four possible answers for each problem - A, B, C og D. Use the answer sheet at the end of the problem set to mark the letter corresponding to your chosen answer. There is only a single correct answer for each problem and all problems yield the same number of points. Zero points are given for a problem if more than one answer is marked. Wrong answers do not yield negative points.

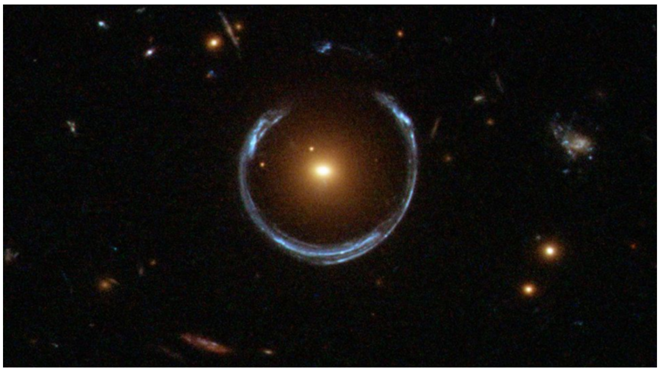
The problem set has 12 pages, and there are 20 problems.

Good luck!

Constants and formulas:

- Apparent magnitude: $m = -2.5 log\ (F/F_0)$
- Hubble's law: $v = H_0 d$, $H_0 \approx$ 73 km/s/Mpc
- Doppler formula: $v/c = (\lambda - \lambda_0)/\lambda_0$
- The Rayleigh criterion: = $1.22 \cdot \lambda/D$ rad
- Speed of light = 299792458 m/s
- 1 parsec (pc) ≈ 3.26 light years
- 1 Astronomical Unit (AU) ≈ $1.5 \cdot 10^{11}$ m
- Newton's law of gravity: $F_G = GmM/r^2$, $G \approx 6.67 \cdot 10^{-11}$ m^3/kg/s^2
- Wien law: $\lambda_{max} = b/T$, $b \approx 2.9 \cdot 10^6$ nm·K
- Stefan-Boltzmann's law: $F = \sigma T^4$, $\sigma \approx 5.67 \cdot 10^{-8}$ W/m^2/K^4

1) What is this?



   a) An exoplanet with a dusty trail orbiting a distant star in another solar system.
   b) The asteroid belt in the Solar System, viewed from the far-away Voyager spacecraft.
   c) **A foreground galaxy bending and focusing the light of a background galaxy.**
   d) A supermassive black hole surrounded by its event horizon.

   Solution: c) Light emitted from a distant blue galaxy in the background is bent by the curved spacetime due to the mass of another yellow galaxy in the foreground, effectively functioning as a lens. It's called an Einstein ring!

2) The Pole Star has a fixed position in the night because…
   a) the Earth has zero velocity relative to the Pole Star.
   b) the Earth lies on the rotation axis of the Pole Star.
   c) the Earth and the Pole Star both have the same velocity inside the Milky Way.
   d) **the Pole Star lies on the rotation axis of the Earth**.

   Solution: d) The Pole Star lies on the rotation axis of the Earth, so its position does not change with Earth's rotation as the other stars.

3) What is the Doppler effect useful for?
   a) Measure the radial velocity of a star or astronomical object.
   b) Measure the dust content of a galaxy.
   c) Find water on Mars.
   d) Predict solar eclipses.

   Solution: a) It is used to measure the radial velocity of a star or astronomical object.

4) In the Moon's elliptical orbit around the Earth, where is its velocity smallest?
   a) **Apogee (furthest from the Earth)**
   b) Perigee (closest to the Earth)
   c) Focus point
   d) Lagrange point L3

   Solution: a)

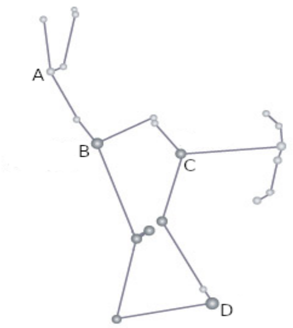
5) In the Sun-Earth system, we have five different so-called "Lagrange points". What are they?
   a) Regions in space between the Sun and the Earth where most space dust is accumulated.
   b) Possible locations of the Moon in space during a total solar eclipse.
   c) Equilibrium points where the gravitational pulls of the Sun and the Earth together make any small object's orbit around those points stable or quasi-stable.
   d) The five points needed to describe Earth's elliptical orbit around the Sun.

Solution: c)

6) Planets further away from the Sun are on almost circular orbits with …
   a) higher orbital velocities than those closer to the Sun.
   b) **lower orbital velocities than those closer to the Sun.**
   c) the same orbital velocities as those closer to the Sun.
   d) orbital velocities that follow no clear pattern.

Solution: b) Newton's law of gravity with centripetal acceleration gives $a = v^2/r \propto 1/r^2$, so $v \propto 1/\sqrt{r}$ decreases with distance. Alternatively, Kepler's third law $P^2 \propto r^3$ on a circular orbit gives $v = 2\pi r/P \propto 1/\sqrt{r}$.

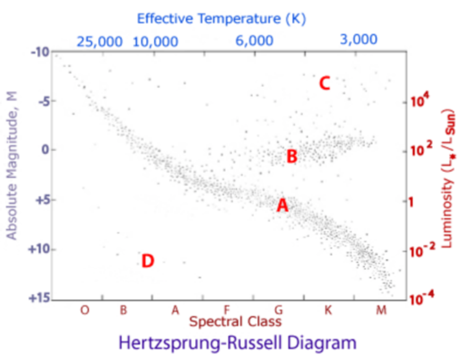
7) On the following depiction of the Orion constellation, which of the marked stars represent Betelgeuse and Rigel?



   a) B is Rigel, C is Betelgeuse
   b) A is Rigel, D is Betelgeuse
   c) C is Rigel, A is Betelgeuse
   d) D is Rigel, B is Betelgeuse

Solution: d)

8) Given the HR diagram below, where would we find Betelgeuse?



   a) A - the main series
   b) B - red giants
   c) **C – supergiants**
   d) D - white dwarves

Solution: c)

9) It's estimated that Betelgeuse's radius is 900 times that of our Sun's, and its luminosity is 126000 times larger. Determine Betelgeuse's surface temperature, given the Sun's to be 5778K.

a) ~1500 K
b) ~3700 K
c) ~5778 K
d) ~9200 K

Solution: b) Given the formula for luminosity to be $\sigma \cdot 4\pi R^2 T^4$, we can set up a ratio between the two luminosities and get the formula $T_B = T_{\odot}\sqrt{\frac{R_{\odot}}{R_B}}\left(\frac{L_B}{L_{\odot}}\right)^{\frac{1}{4}}$ which gives 3629K.

10) Which of these can be considered to be a "standard candle", i.e. an object whose brightness is known and can therefore be used to calculate cosmic distances?

a) **Cepheid stars**
b) Spiral galaxies
c) Black holes
d) Comets

Solution: a)

11) If light from a hot, opaque body passes through a cooler, thin gas, the observed spectrum will show

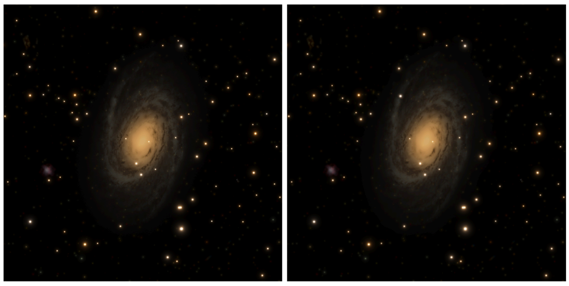
a) **narrow, dark lines on a bright background**
b) narrow, bright lines on a dark background
c) total darkness, since the gas absorbs all of the light of the source.
d) broad emission lines.

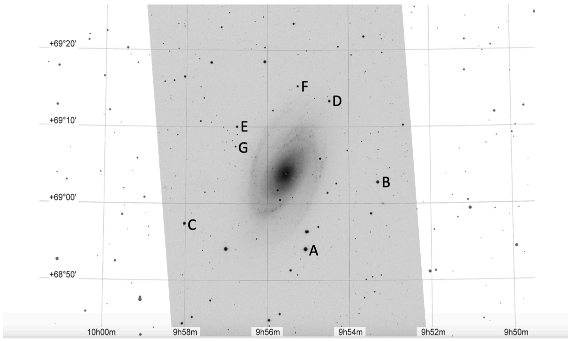
Solution: a) The light is absorbed at particular wavelengths corresponding to particular transitions from lower to higher energy levels in the atoms in the gas.

12) Below is a simulated image of a newly discovered supernova in one of the spiral arms of the galaxy Messier 81. The supernova is visible in the top right image; the top left image is a reference image without the supernova. Your task is to identify the supernova and estimate its apparent brightness (visual magnitude). You should use the set of comparison stars which are labeled by letters in the map below and have their magnitudes listed below (A=brightest; G=faintest).

A= 9.16
B=10.08

C=10.20
D=11.07
E=11.45
F=12.04
G=12.73





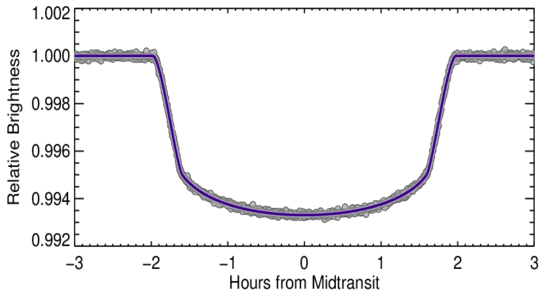
What is the visual magnitude of the supernova?

a) 9.20
b) 10.20
c) 12.20
d) 13.20

Solution: b) 10.20

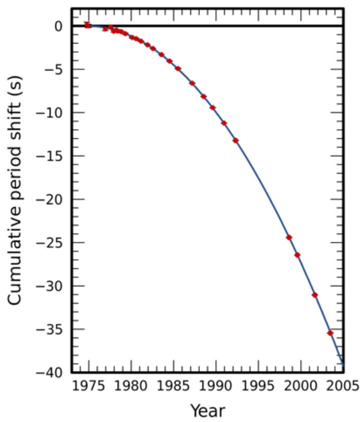
13) As an exoplanet passes directly between its host star and us, the brightness observed from the star (relative to its value before and after transit) falls as shown. What is the exoplanet's radius r in terms of the star's radius R?

a) $r = 0.00004\, R$
b) $r = 0.006\, R$
c) $\boldsymbol{r = 0.08\, R}$
d) $r = 0.3\, R$



Solution: c) The planet occludes the star and, at midtransit, its disk blocks $\pi r^2/\pi R^2 = 1 - 0.994 = 0.006$ of the light that reached us before and after the transit, so $r = \sqrt{0.006}\, R = 0.08\, R$.

14) PSR B1913+16 is a binary system consisting of two neutron stars, each with a mass of approximately 1.4 M_{sun} . Each component is orbiting a common centre of gravity with an orbital period of 7.75 hours. One of the neutron stars is a pulsar, and by measuring the systematic shifts of the arrival times of its pulses over the orbital period it is possible to make extremely precise measurements of its orbital motion. The red points in the plot below illustrate the measured change in orbital period across a time span of nearly 30 years. What causes this change?



a) Starquakes on the neutron stars decrease their moment of inertia which increases their rotational spin frequency, which in turn slows down the orbital speeds due to preservation of total angular momentum in the system.
b) The orbital period decreases as energy is emitted from the binary system in the form of gravitational waves with a total power of $7.35*10^{24}$ Watt, as predicted by Einstein's theory of general relativity.
c) A planet orbiting one of the neutron stars is perturbing the orbital movements of both neutron stars.
d) The mass of one of the neutron stars is decreasing due to mass ejection in strong stellar winds

Solution: b) PSR B1913+16, a.k.a. the "binary pulsar", is a famous system which provided ground-breaking evidence for the existence of gravitational waves. The dark blue curve in the plot above is the orbital change due to energy being emitted in the form of gravitational waves, as predicted from general relativity. The discoverers of PSR B1913+16, Russell Hulse and Joseph Taylor, were awarded the Nobel Prize in physics in 1993.

15) How far away (in Mpc) is a galaxy whose radiation peak is at 650 nm that has the same radiation spectrum as a galaxy with 0 velocity relative to the Earth and whose radiation peak is at 550 nm? Note: you might find the formula for the relativistic redshift useful: $z = \sqrt{\frac{c+v}{c-v}} - 1$.

a) ~70 Mpc
b) ~350 Mpc
c) ~700 Mpc
d) ~7000 Mpc

Solution: c) The redshift can be expressed as $z = \frac{\lambda_1 - \lambda_0}{\lambda_0} = \frac{\lambda_1}{\lambda_0} - 1$. Equating this with the expression given in the problem text gives:

$$\frac{\lambda_1}{\lambda_0} - 1 = \sqrt{\frac{c + v}{c - v}} - 1 \Rightarrow \left(\frac{\lambda_1}{\lambda_0}\right)^2 \cdot (c - v) = (c + v)$$

Expanding this, and a little algebra gives:

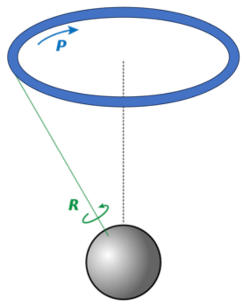
$$v = \frac{\left(\frac{\lambda_1}{\lambda_0}\right)^2 - 1}{\left(\frac{\lambda_1}{\lambda_0}\right)^2 + 1} \cdot c$$

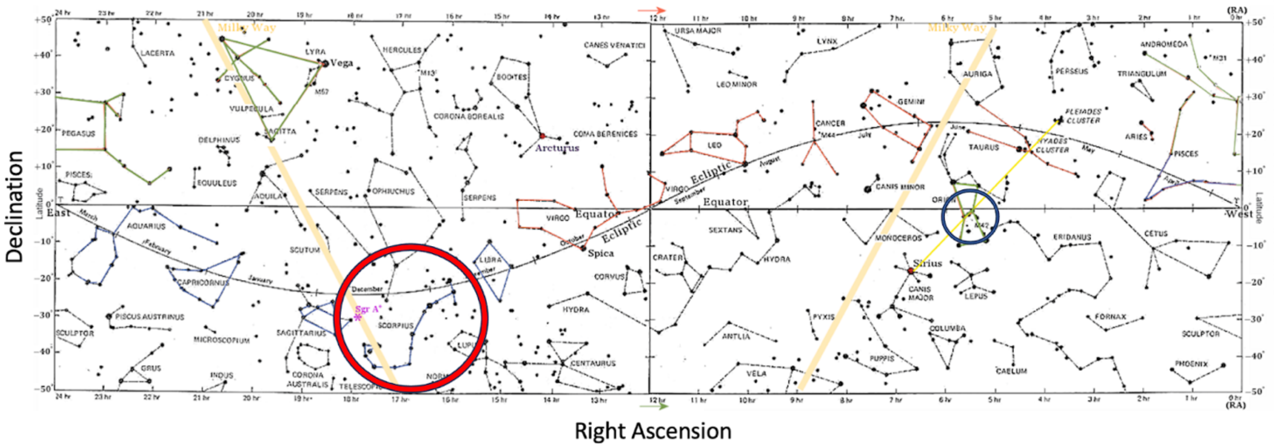
Using Hubble's law $v = H_0 d$ gives:

$$d = \frac{v}{H_0} \approx 691 Mpc \approx 700 Mpc$$

Remember that the velocity needs to be expressed in units of $\frac{km}{s}$ to be compatible with the Hubble constant given.

16) As illustrated in the figure, the Earth's axial precession results in a cyclical shift (P) of the orientation of the Earth's axis of rotation (R) with a period of approximately 26,000 years. The star map below shows both the ecliptic (curved black line) and the celestial equator (horizontal black line through the middle), illustrating the present location of the constellations with respect to the celestial equator.



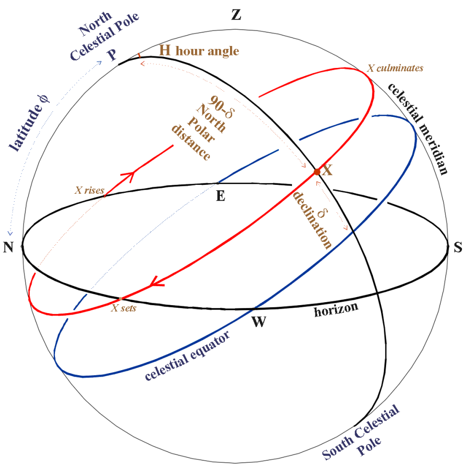


You build a time machine and set it to 13,000 years before the present epoch. When you arrive at this time in the past you notice that the night sky is rather different from what you are used to, even though your geographical latitude and longitude is still within present-day Norway. Over the next year you keep looking for some recognizable star patterns and constellations, particularly Orion's belt and Scorpius. Which of these would you be able to see?

a) Both Orion's belt and Scorpius
b) Neither Orion's belt nor Scorpius
c) Orion's belt, but not Scorpius
d) Scorpius, but not Orion's belt

Solution: d) 13,000 years is halfway through a full precession cycle of 26,000 years. This means that the constellations along the ecliptic which we now see as far north as possible (and thus high in our sky), at declinations +23 degrees, would have been as low in our sky as possible, at declinations -23 degrees, and vice versa. By looking at the present-day sky map above we can infer that Scorpius would have been much further north in the sky 13,000 years ago (and thus well visible from "Norway"), while Orion's belt (which the map shows to be presently located at the celestial equator, but about 23 degrees south of the location of the sun at summer solstice) would then have been approximately 23 degrees south of the location of the sun at *winter* solstice, i.e. at a declination of (-23-23)=-46 degrees. The southernmost stars we can see from the southernmost part of Norway (at latitude +58 degrees north) have declinations of (58-90)=-32 degrees. Hence, Orion's belt would not have been visible from our geographical location 13,000 years ago.

17) Listed in order of brightness below are the 22 brightest stars, with stellar designations, proper names and their celestial coordinates ("RA" = Right Ascension in hours and minutes, "Dec" = declination in degrees). The figure below may be helpful to illustrate the concept of declination. How many of these stars are visible from the latitude of Mandal (58 degrees north) ?



		RA	Dec
Alpha Canis Majoris	Sirius	06 45	-16.7
Alpha Carinae	Canopus	06 24	-52.7
Alpha Centauri	Rigil Kentaurus	14 40	-60.8
Alpha Bo�tis	Arcturus	14 16	+19.2
Alpha Lyrae	Vega	18 37	+38.8
Alpha Aurigae	Capella	05 17	+46.0
Beta Orionis	Rigel	05 15	-8.2
Alpha Canis Minoris	Procyon	07 39	+5.2
Alpha Eridani	Achernar	01 38	-57.2
Alpha Orionis	Betelgeuse	05 55	+7.4
Beta Centauri	Hadar	14 04	-60.4
Alpha Aquilae	Altair	19 51	+8.9
Alpha Crucis	Acrux	12 27	-63.1
Alpha Tauri	Aldebaran	04 36	+16.5
Alpha Virginis	Spica	13 25	-11.2
Alpha Scorpii	Antares	16 29	-26.4
Beta Geminorum	Pollux	07 45	+28.0
Alpha Piscis Austrini	Fomalhaut	22 58	-29.6
Beta Crucis	Mimosa	12 48	-59.7
Alpha Cygni	Deneb	20 41	+45.3
Alpha Leonis	Regulus	10 08	+12.0
Epsilon Canis Majoris	Adhara	06 59	-29.0

a) 11 stars
b) 6 stars
c) **16 stars**
d) 14 stars

Solution: c) The southernmost stars we can see from Mandal at latitude +58 degrees north have declinations of (58-90)=-32 degrees. There are 16 stars in the list with declinations >-32 degrees.

18) By studying absorption lines in a spectrum of the Andromeda galaxy you measure a blueshift corresponding to a radial velocity of -300 km/s, indicating that the Andromeda galaxy is moving towards us at this velocity. What is the origin of this phenomenon?

a) The Andromeda galaxy is actually moving away from us due to the expansion of the Universe, but this velocity measurement has not been corrected for the Earth's movement around the sun and the solar system's movement around the Milky Way center.
b) The universe has stopped expanding and is now contracting towards a future "Big Crunch"
c) The absorption lines in the spectrum are produced by high-velocity gas outflow from the Andromeda galaxy, and these blueshifted lines do not reliably measure the velocity of the galaxy itself, only the outflowing gas.
d) The gravitational attraction between galaxies belonging to the Local Group, such as the Andromeda galaxy and the Milky Way galaxy, is strong enough to overcome the expansion of the universe. The two galaxies are currently moving towards each other and will eventually merge.

Solution: d) Regarding alternative a): Even if the orbital velocity of the Earth (30 km/s) and the solar system's orbital velocity in the Milky Way (230 km/s) were both pointing directly towards Andromeda (they do not!), their combined velocity is not enough to explain the measurement. Regarding alternative c): The absorption lines are partially produced in stellar atmospheres and partially produced by interstellar gas, but in any case a non-active galaxy like Andromeda will not produce gas outflows as fast as 300 km/s.

19) A satellite orbits Jupiter in a perfectly circular orbit the same direction as Jupiter orbits itself (this is also called prograde motion). What is the maximum radius of this satellite's orbit so that an observer located at the Sun can see apparent retrograde motion at some point in time, i.e. that the satellite is moving in the opposite direction of Jupiter's rotation? Jupiter's mass is $1.9 \cdot 10^{27}$kg, the radius of its orbit around the Sun is 5.2AU, and the period it takes to do a full revolution around the Sun is approximately 12 years.

a) ~$3.14 \cdot 10^{6}$m
b) ~$\mathbf{7.50 \cdot 10^{8}}$m
c) ~$1.50 \cdot 10^{11}$m
d) ~$6.25 \cdot 10^{15}$m

Solution: b) To solve the problem, one must understand that the motion will appear to go in the opposite direction for an observer from the Sun if and only if the velocity of the satellite in orbit is greater than the velocity of Jupiter in orbit around the Sun.

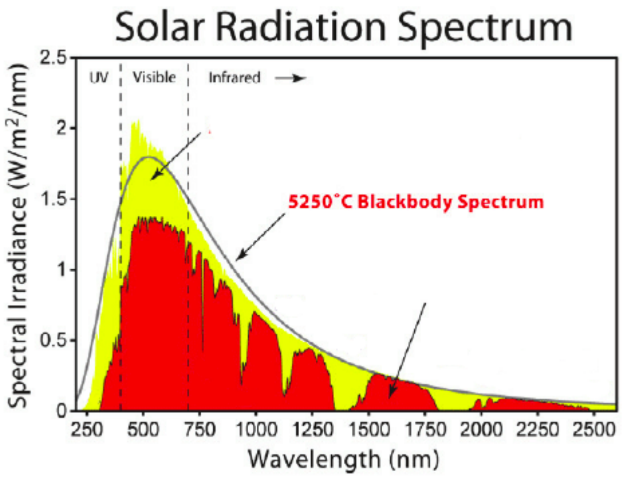
Jupiter's velocity is simply given by $v_J = \frac{2\pi \cdot 5.2AU}{12yr} \approx 13\frac{km}{s}$

The velocity of the satellite can be found by equating the gravitational force between Jupiter and the satellite with the centripetal force from the rotational motion.

$$G\frac{m_{sat}M_J}{{r_{sat}}^2} = \frac{m_{sat}{v_{sat}}^2}{r_{sat}} \Rightarrow r_{sat} = \frac{GM_j}{{v_{sat}}^2}$$

Since we know that $v_{sat} > v_J$, then we must have $r_{sat} < \frac{GM_j}{{v_J}^2} \approx 7.5 \cdot 10^{8}$m

20) Given that the black line in the image corresponds to the spectrum of a black body at 5250ºC, what do the yellow and the red spectra represent and why do we see them like this?



a) Earth's orbit around the Sun is not perfectly circular, so the spectra differ based on how close the sun is. The yellow spectrum is therefore when the Sun is closest to the Earth, and the red spectrum is observed at its furthest point.
b) Our atmosphere consists of gasses which absorb some of the light, leading to there being a difference in the spectra on top of the atmosphere and at sea level. The yellow spectrum is therefore what is observed in space before our atmosphere, and the red one is when the light passes completely through our atmosphere.
c) A total solar eclipse causes changes in the light observed, so the yellow spectrum is the non-eclipsed Sun, while the red spectrum is eclipsed.
d) The spectra are dependent on the solar activity that is approximately periodic with a period of 11 years. The yellow spectrum is therefore the spectrum at the peak in activity during these 11 years, and the red one is at the lowest of activities.

Solution: b)