

Norwegian Olympiad on Astronomy and Astrophysics



Final round problems **(and solutions)** 2025/2026

Date: Friday March 20th 2026

Allowed aids: calculator, pen/pencil, ruler, planisphere, constants/formulas below

Time: 150 minutes

Part 1 (18 multiple choice questions): There are four possible answers for each problem - A, B, C and D. Use the answer sheet to mark your chosen answer. There is only one single correct answer for each problem, and all problems yield the same number of points. Zero points are given for a problem if more than one answer is marked. Wrong answers do not yield negative points.

Part 2 (1 observation question, 1 data analysis question + 1 open answers question): Explain your reasoning and write clearly! Write your name on the sheets that you hand in! The number of points is displayed per problem.

Good luck!

Constants and formulas:

- $1 \text{ AU} = 1.5 \cdot 10^{11} \text{ m}$
- $1 \text{ parsec (pc)} \approx 3.26 \text{ light years} \approx 206\,265 \text{ AU}$
- Newton's law of gravity: $F = GmM/r^2$, $G = 6.67 \cdot 10^{-11} \text{ m}^3/\text{kg/s}^2$
- Kepler's third law: $a^3/P^2 = G(M + m)/4\pi^2$
- Wien's displacement law: $\lambda_{\text{max}} = b/T$, $b \approx 2.9 \cdot 10^6 \text{ nm} \cdot \text{K}$
- Stefan-Boltzmann's law: $F = \sigma T^4$, $\sigma \approx 5.67 \cdot 10^{-8} \text{ W/m}^2/\text{K}^4$
- Boltzmann constant: $k_B = 1.38 \cdot 10^{-23} \text{ J/K}$
- Apparent magnitude: $m = -2.5 \log_{10}(F/F_0)$
- Absolute magnitude: $M = m - 5 \log_{10}(d/(10 \text{ pc}))$
- Hubble's law: $v = H_0 d$, $H_0 \approx 70 \text{ km/s/Mpc}$
- Earth's mass and radius: $5.97 \cdot 10^{24} \text{ kg}$ and 6371 km
- The Sun's mass and radius: $2 \cdot 10^{30} \text{ kg}$ and $696\,340 \text{ km}$
- Earth-Moon distance: $3.8 \times 10^8 \text{ m}$
- Speed of light: $c = 3 \cdot 10^8 \text{ m/s}$
- Photon energy: $E = hf = hc/\lambda$, $h = 6.63 \cdot 10^{-34} \text{ Js}$
- Schwarzschild radius: $R = 2GM/c^2$
- Spatial resolving power: $\phi \approx 1.22 \frac{\lambda}{D}$
- Elliptical eccentricity: $e = \sqrt{1 - (b/a)^2}$
- Redshift: $z = \lambda_{\text{obs}}/\lambda_{\text{emit}} - 1$
- Relativistic Doppler effect: $\frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} = \sqrt{\frac{1+v/c}{1-v/c}}$

Part 1: multiple choice questions (1 point per question)

- 1) Which of these coordinates is zero when a star reaches its upper culmination?
 - a) Altitude
 - b) Azimuth
 - c) **Hour Angle**
 - d) Declination

Solution: c) When a star reaches its upper culmination, it is crossing the local meridian, meaning that its hour angle is zero. In lower culmination, the hour angle is 12h. The right ascension of the star crossing the local meridian corresponds to the local sidereal time.

- 2) You are out observing the sky with a telescope, but have yet to set up a tracking system. Despite this, you wish to observe Bode's Galaxy, or M81 in the Messier catalogue, with a declination coordinate of approximately $\delta = 69^\circ$. Knowing that your telescope captures 1° of the sky in its FOV, approximately what is the maximum time you will be able to see M81 in the FOV without moving your telescope?



M81, Bode's Galaxy; credits: Adam Block/Mount Lemmon SkyCenter/University of Arizona

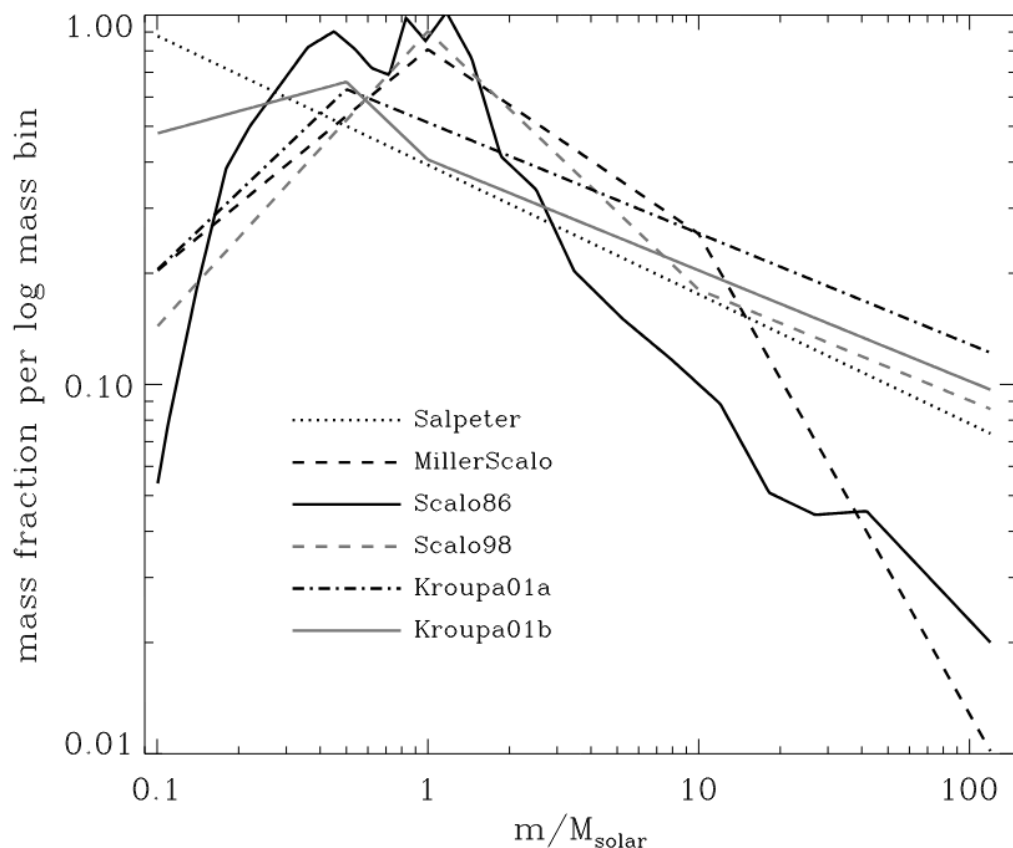
- a) 8 ms
- b) 4 s
- c) 4 min
- d) **11 min**

Solution: b) The sky makes one full rotation approximately every 24 hours. This would equate to a rotational speed of $\frac{360 \cdot 3600 \text{ arcsec}}{24 \cdot 3600 \text{ s}} \approx 15 \frac{\text{arcsec}}{\text{s}}$. The apparent drift of any celestial object depends on how close to the Celestial Equator it is: on the CE (i.e. declination 0), it is maximum, and at the celestial poles, it is minimum. The drift can be written on the form of $15 \frac{\text{arcsec}}{\text{s}} \cdot \cos(\delta)$, and if the galaxy in the ideal case "travels" along the diameter of the FOV, we get $t = \frac{1 \cdot 3600 \text{ arcsec}}{15 \frac{\text{arcsec}}{\text{s}} \cdot \cos(69^\circ)} \approx 670 \text{ s} \approx 11 \text{ min}$.

- 3) If you were to wish to do long exposure astrophotography, and thereby need to track M81, which of these mounts would be most suitable for this purpose?

- a) Equatorial
- b) Alt-az
- c) Dobsonian
- d) Tripod

- 4) (Alternative option for 4) One of the surprises discovered by the James Webb Space Telescope is that early galaxies seem too massive and mature for their age. A telescope cannot directly measure the mass of a distant galaxy; instead, we can estimate a galaxy's mass by measuring how much light its stellar population emits. But the observed brightness can be caused by few massive stars, many light stars, or an even mix. We therefore need to know the distribution of the stellar population, a so-called initial mass function (IMF). Which of the following IMFs can solve the massive early galaxy problem?



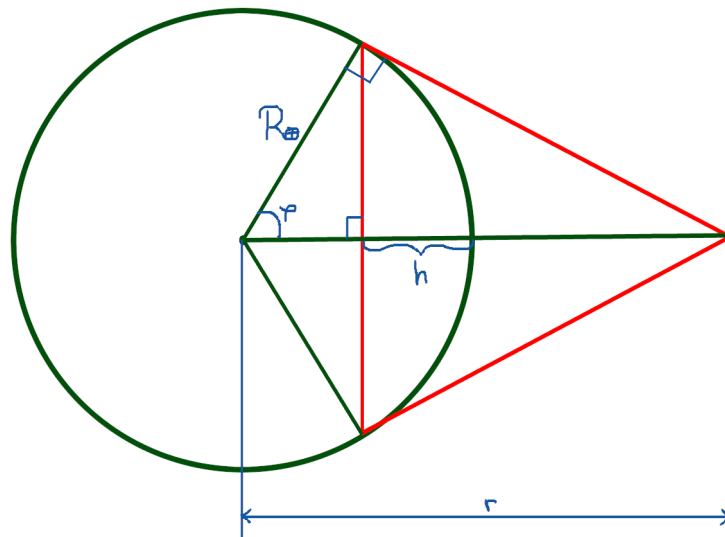
Different initial mass functions. The x-axis shows the mass of a star in units of solar masses and the y-axis shows the relative fraction of stars in different mass ranges. The figure is taken from Baldry & Glazebrook (2003).

- a) Top-light bottom-heavy IMF (fewer massive bright stars, more small faint stars)
- b) Top-heavy bottom-light IMF (more massive bright stars, fewer small faint stars)**
- c) A flat IMF (equal number of massive bright and small faint stars)
- d) Changing the IMF will not help; we need to change something else in our model

Solution: b) We need a top-heavy bottom-light IMF to solve the massive early galaxy problem. If the majority of observed light is emitted by massive bright stars, then you don't need that many stars and therefore stellar mass to explain the brightness.

- 5) From what percentage of the Earth's surface area is a geostationary satellite visible from at any given moment, assuming otherwise ideal conditions? Hint: the surface area of a spherical cap with height h , whose sphere has surface diameter R is $A = 2\pi R h$.
- 13%
 - 27%
 - 42%**
 - 67%

Solution: c)



The first step is to find the radius of the satellite's orbit. The key here is knowing that a geostationary satellite matches Earth's rotation exactly, meaning that it has a period of 24h. As such, we can use the centripetal

force to write: $G \frac{M_{\oplus} m}{r^2} = m \frac{v^2}{r}$, and with $v = \frac{2\pi r}{T}$, we get

$$r = \sqrt[3]{GM_{\oplus} \left(\frac{T}{2\pi}\right)^2} \approx 4.2 \cdot 10^7 \text{ m. The maximum latitude at which such a}$$

satellite can be observed from is $\varphi = \arccos \frac{R_{\oplus}}{r} \approx 81^\circ$. The surface area from which the satellite can be seen is described as a spherical cap that is made by making a cut between 81° N and 81° S , represented by the red vertical line. Its height can be found as $h = R_{\oplus} (1 - \cos(\varphi))$. The total

area therefore becomes $A = 2\pi R_{\oplus} h = 2\pi R_{\oplus}^2 (1 - \cos(\varphi))$. With the total surface area of the Earth being $A_{\oplus} = 4\pi R_{\oplus}^2$, the ratio becomes

$$\frac{1 - \cos(\varphi)}{2} \approx 42\%.$$

- 6) White dwarf stars are the final evolutionary stage for stars not massive enough to become neutron stars or black holes. There is no fusion taking place inside a white dwarf, so no thermal pressure acting against gravitational collapse. Instead, electron degeneracy pressure counteracts gravity. This pressure is relatively weak, so the white dwarf is extremely dense ($\rho_{WD} = 10^6 \rho_{\odot}$). The physics of electron degeneracy pressure yields a maximum mass for a white dwarf. What is this limit named?

(Question proposed by F. Neumann, winner of Astro-OL 2023 and 2024)

- a) **Chandrasekhar limit**
- b) Hubble limit
- c) Tolman-Oppenheimer-Volkoff (TOV) limit
- d) Schwarzschild radius

Solution: a). Chandrasekhar limit is correct. Hubble limit is the horizon of the sphere containing everything with Hubble-velocity ($v_H < c$) relative to Earth. TOV limit is the neutron star equivalent of the Chandrasekhar limit. Schwarzschild radius is the event horizon of a black hole.

- 7) Redshift is a common way to infer distances to galaxies, and is a dimensionless value representing how stretched out the light coming from those galaxies is. We say redshift because the cosmic expansion makes galaxies move away from us, which, similar to the Doppler effect, shifts the light to the redder ends of the spectrum. Do blueshifted galaxies exist and how?
- a) No, the cosmic expansion is extremely strong, overcoming all other forces and pulling galaxies away from us, so all galaxies have positive redshift values.
 - b) **Yes, some nearby galaxies, e.g. in the Local Group, have negative redshifts, i.e. are blueshifted, as their gravitational interactions with our galaxy manage to overcome the cosmic expansion and actually bring the galaxies closer to us.**
 - c) No, the definition of redshift does not allow values below 0, which only occurs for objects orbiting our galaxy.
 - d) Yes, the further away the galaxies are, the more likely it is they are moving towards us, as the expansion of the universe gets slower towards the edge of the universe.
- 8) You are observing a galaxy that appears to be colored red-orange (in its rest-frame). What does the red color tell you about the galaxy?
- a) The galaxy is predominantly made up of young massive stars
 - b) The galaxy is predominantly made up of young low-mass stars
 - c) **The galaxy is made up of massive stars with dusty gas or old low-mass stars**
 - d) There galaxy has a high star formation activity

Solution: c) The galaxy is old and has stopped forming stars, or is very dusty. Young massive stars are predominantly blueish and burn for only a short time, while small stars usually glow reddish and burn their fuel slowly over tens of billions of years. A red galaxy is therefore mostly made of old small stars and there is no more gas to form young stars that bring some blue light into the mix, or there are blue stars present obscured by dusty gas that absorbs the blue content of the light.

- 9) Knowing that the sidereal period of a planet is the time it takes to orbit the Sun fully relative to the fixed stars, and that the synodic period is the time it takes to get to the same position in relation to the Sun as seen from Earth, which of the following expressions connects the two periods of an inferior planet, i.e. a planet orbiting between the Earth and the Sun such as Mercury or Venus? (T_E is one Earth year)

- a) $\frac{1}{T_{synodic}} = \frac{1}{T_E} - \frac{1}{T_{sidereal}}$
 b) $T_{synodic} = T_{sidereal} - T_E$
 c) $\frac{1}{T_{synodic}} = \frac{1}{T_{sidereal}} - \frac{1}{T_E}$
 d) $T_{synodic} = T_E - T_{sidereal}$

Solution: c) The sidereal time is the period for a full revolution, i.e. 360° . An inferior planet is one that is between the Earth and the Sun, so Mercury or Venus. If we begin with the Sun, the inferior planet, and Earth in a straight line, once the planet has completed a single orbit, it will have to cover an additional angle θ to catch up to Earth and be aligned with the Sun again as seen from Earth. This means that the inferior planet moves faster than the Earth, and the difference in the angles travelled between the two planets must be equal to 360° . Since they both travel a synodic period, we can write:

$$T_{synodic} \cdot \frac{360^\circ}{T_{sidereal}} - T_{synodic} \cdot \frac{360^\circ}{T_E} = 360^\circ$$

Multiplying both sides by $\frac{1}{360^\circ \cdot T_{synodic}}$ gives:

$$\frac{1}{T_{synodic}} = \frac{1}{T_{sidereal}} - \frac{1}{T_E}$$

- 10) The critical density is the average matter density in the case of the universe having a flat geometry. If the first Friedmann equation has the form $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{\kappa c^2}{a^2}$, which of these expressions corresponds to the critical density?

- a) $\rho_{crit} = \frac{3a^2}{8\pi Gc^2}$
 b) $\rho_{crit} = \frac{3k_B^2}{8\pi G}$, where k_B is the Boltzmann constant
 c) $\rho_{crit} = \frac{3H_0^2}{8\pi G}$, where H_0 is the present value of the Hubble constant
 d) $\rho_{crit} = \frac{a^4}{(\kappa c^2)\pi}$

Solution: c) Here, the student must realise that the present Hubble constant is defined as the ratio of the time derivative of the scale factor and the scale factor itself. For a flat universe, the curvature parameter is zero, so the second term in the equation disappears, and the rest is just rearranging.

- 11) A very important star seen only in southern skies is Alpha Centauri. It is a triple star system, consisting of the stars Alpha Centauri A, Alpha Centauri B and Proxima Centauri, of which the first two make most of the contribution to Alpha Centauri's perceived brightness. If the apparent magnitude of Alpha Centauri A is 0.01, and the apparent magnitude of Alpha Centauri B is 1.33, what is the apparent magnitude of Alpha Centauri itself?

- a) -1.32
 b) **-0.27**
 c) 0.58
 d) 1.34

Solution: b) The key here is to understand that apparent magnitudes aren't simply additive due to being defined by logarithms. What is additive, however, is the brightness/flux of the stars when they are at the same distance from us, which can be concluded from the problem specifying that it is a triple star system. Using Pogson's equation, we can write:

$$F = F_0 \cdot 10^{-0.4(m-m_0)}. \text{ We know that the total flux will be } F_{tot} = F_A + F_B,$$

which when expanded gives the following equation:

$$F_0 \cdot 10^{-0.4(m_{tot}-m_0)} = F_0 \cdot 10^{-0.4(m_1-m_0)} + F_0 \cdot 10^{-0.4(m_2-m_0)}. \text{ All the terms}$$

have a common factor of $F_0 \cdot 10^{0.4m_0}$, so dividing by that on both sides

leaves us with $10^{-0.4m_{tot}} = 10^{-0.4m_1} + 10^{-0.4m_2}$. Finally,

$$m_{tot} = \frac{\log_{10}(10^{-0.4m_1} + 10^{-0.4m_2})}{-0.4} \approx -0.27, \text{ which is the actual apparent magnitude of Alpha Centauri.}$$

- 12) You go outside and observe the thin crescent New Moon in the Gemini constellation. What month could it possibly be?

- a) March/April
- b) May/June**
- c) September/October
- d) December/January

Solution: When the Moon is new, it is close to the Sun in the sky, meaning that the Sun must also be in Gemini. This occurs from late May to mid June.

- 13) Assume you are observing the sky with your brand-new space telescope to find a completely unknown galaxy. Because you are observing from space and have a state-of-the-art instrument, you are limited only by the ratio between the signal strength of the object you are observing and the intrinsic thermal noise in the telescope electronics, and this signal-to-noise ratio (SNR) is given by

$$SNR = \frac{tS_{object}}{\sqrt{tS_{object} + tS_{noise}}},$$

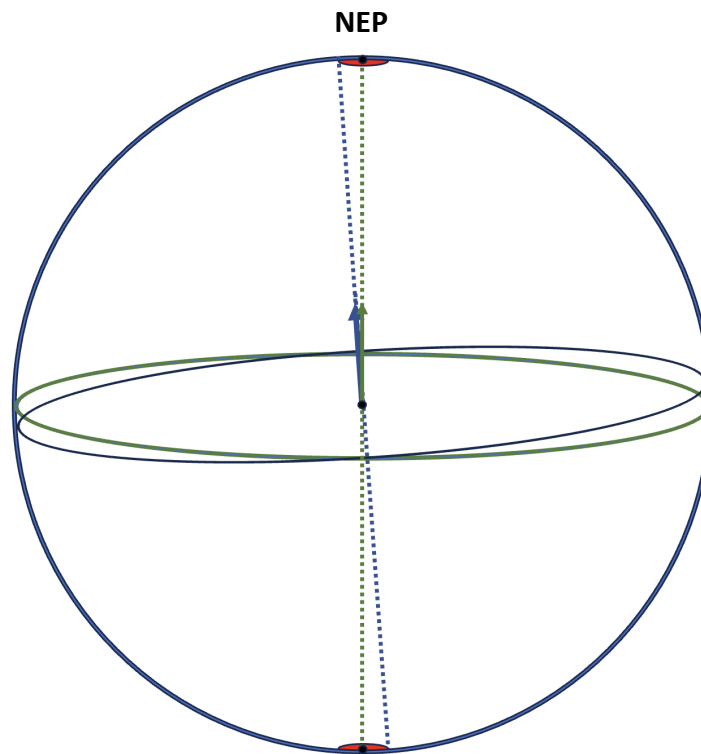
where t is the exposure time, S_{object} is the signal's strength, and S_{noise} is the thermal noise of the camera. Assume you can barely see the galaxy in the first image, but you want to increase the SNR by a factor of 5 to make a proper detection claim; how much do you need to increase the exposure time?

- a) 100 times longer
- b) 33.334 times longer
- c) 25 times longer**
- d) $2\pi^2$ times longer

Solution c) Because the SNR only depends on time and all other factors are constant, the $SNR \propto \sqrt{t}$. We then simply need to solve the equation:

$$SNR_{detection} = 5 \cdot SNR_0 \Rightarrow \sqrt{t_{detection}} = 5\sqrt{t_0} \Rightarrow t_{detection} = 25 t_0$$

giving an exposure time of 25 times the original exposure time



14) The interstellar comet 3I/ATLAS was formed around another star before our solar system existed and was subsequently expelled from its solar system. It is currently making its first and only visit to our solar system in a retrograde hyperbolic orbit. The inclination between its orbital plane and the Earth's orbital plane (the ecliptic plane) is just 4.89° , as illustrated in the figure above. An interstellar comet is expected to approach our solar system from a random direction. What is the probability that the inclination between its orbital plane and the ecliptic plane is 4.89° or less? [Hints: Include both prograde and retrograde comet orbits. Make small angle approximations $\sin x \approx x$ for small x .]

- a) 2.7%
- b) 0.72%
- c) **0.36%**
- d) 0.18%

Solution: The normal vector of the orbital plane of an interstellar comet will randomly point towards any direction on the celestial sphere with equal probability. An inclination equal to or less than 4.89° implies that the normal vector is pointing towards the red region on the top (for prograde orbits) or bottom (for retrograde orbits) of the figure. For small angle approximations ($\sin(\alpha) \cong \alpha$), and a locally flat region on the celestial sphere) the combined area (solid angle) of the red regions relative to the entire celestial sphere is then $2 \cdot \pi \cdot (4.89 \cdot \pi / 180)^2 / (4 \cdot \pi) = 0.0036$

15) The star Mintaka has celestial coordinates RA = 05h31m60s, Declination = -00°17'49".
On March 21st, for approximately how long will this star remain above the horizon after sunset?

- A) 1 hours and 30minutes
- B) 4 hours
- C) 5 hours and 30minutes**
- D) 7 hours

Solution: March 21st is the spring equinox, at which time the Sun has celestial coordinates (RA, Dec)=(0h,0°). Mintaka has the same declination as the Sun on this date, hence the difference in their setting times will be equal to the difference in their Right Ascension)

16) The observable Universe has an estimated mass of roughly $3.04 \cdot 10^{54}$ kg. Given this, what would be the Schwarzschild radius of a black hole whose mass equaled that of the observable Universe?

- a) 4.7 billion light-years
- b) 13.8 billion light-years
- c) 46.6 billion light-years
- d) 476 billion light-years**

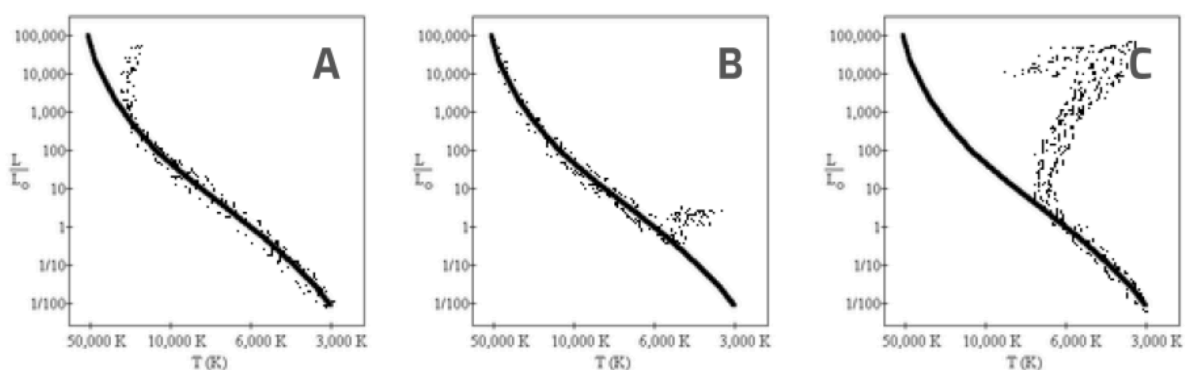
Solution: This is straightforward, although the result might be surprising, given that it exceeds the radius of the observable Universe (alternative c). Using the given formulas, and that one light year equals $c \cdot 365.25 \cdot 24 \cdot 3600$ s, we find

$$\frac{R}{1 \text{ ly}} = \frac{R}{365.25 \cdot 24 \cdot 3600 \text{ s} \cdot c} = \frac{2GM}{3.6525 \cdot 2.4 \cdot 3.6 \cdot 10^6 \text{ s} \cdot c^3} \approx \frac{2 \cdot 6.67 \cdot 10^{-11} \text{ m}^3 / (\text{kg s}^2) \cdot 3.04 \cdot 10^{54} \text{ kg}}{3.6525 \cdot 2.4 \cdot 3.6 \cdot 10^6 \text{ s} \cdot (3.00 \cdot 10^8 \text{ m/s})^3}$$

$$= \frac{2 \cdot 6.67 \cdot 3.04 \cdot 10^{13}}{3.6525 \cdot 2.4 \cdot 3.6 \cdot 3} \approx 4.76 \cdot 10^{11}$$

whereby

$$R \approx 476 \cdot 10^9 \text{ ly}$$



17) The plot above shows Hertzsprung-Russell diagrams for three star clusters A, B, and C.
Which of the following is listing these star clusters in order of increasing age ?

- A) A,C,B
- B) B,A,C**
- C) C,A,B

D) B,C,A

Solution: All the stars in a star cluster are normally formed at approximately the same time. In star cluster B, the least massive and hence slowest evolving stars have not yet reached the main sequence. In star cluster A only the very heaviest stars with shortest life spans have started leaving the main sequence, while in star cluster B more stars have had time to reach the main sequence.

18) Now we want to determine the age of star cluster A, based on the plot in the previous exercise (note that $L_{\odot}$ denotes the luminosity of the sun). To help us, an astronomer has determined the following empirical relations for main sequence stars: $L \sim M^{3.5}$ and $t \sim M^{-2.5}$, where L is the luminosity of the star, M its mass and t its life time on the main sequence. The sun will spend about 10 billion years on the main sequence. Based on this information and the HR diagram of cluster A, its age can be estimated as:

A) 7 billion years

B) 700 million years

C) 70 million years

D) 7 million years

Solution: The main sequence turnoff point is where the cluster deviates from the main sequence at the upper left. Reading the plot of star cluster A, the stars at this point have luminosities $\sim 1000 L_{\odot}$. Combining the two above equations gives $t \sim L^{-5/7}$ which yields $t = t_{\odot} \cdot 1000^{-5/7}$, where $t_{\odot}$ is the main sequence life time of the sun. This yields the age 70 million years for star cluster A.

Part 2: Observation, Data Analysis and open answer question

1) (6 points) On the sky chart of Trondheim on the 18th of February this year:

a) (0.5 points each) Mark the cardinal directions

Solution: See sky chart on next page. Reasonable deviations are accepted so long as the working is shown. Do not give marks for mixed up east and west.

b) (0.5 points for marking, 0.5 points for name) Which star is located at approximately altitude 30° and azimuth 210° (both proper name and Bayer designation are acceptable)? Mark it on the sky chart. Take azimuth 0 to be in cardinal north, and positive azimuth eastwards.

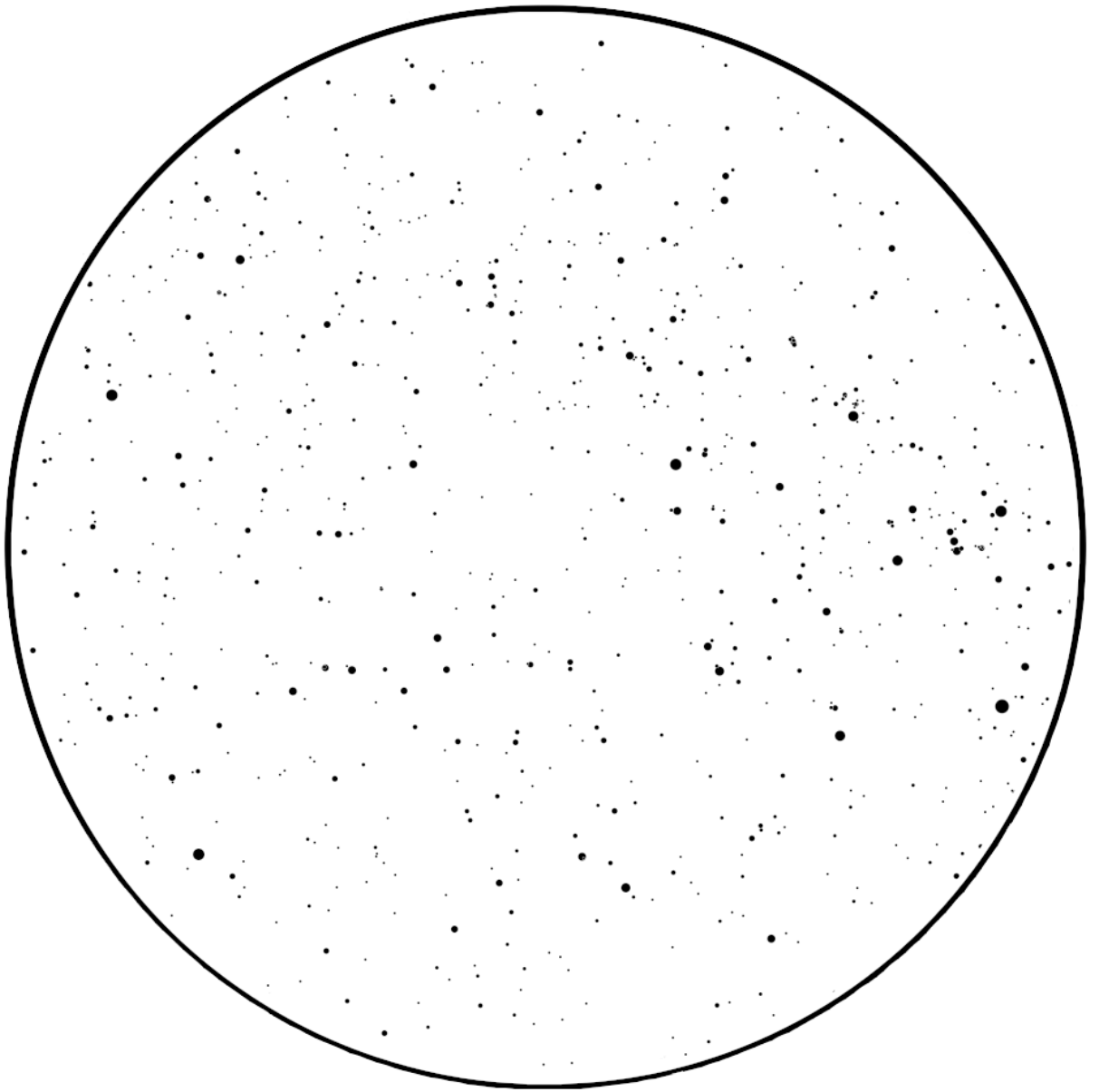
Solution: Betelgeuse or alpha Ori are both acceptable. See sky chart for marking. Since this depends on the previous problem, give points if marked azimuth corresponds to student's marking of north and east. (should discuss whether to give points if east is placed incorrectly in relation to north, but Betelgeuse is still marked; I am looking for an understanding of how azimuth works here).

c) (1 point each) Draw in the Ecliptic and Celestial Equator.

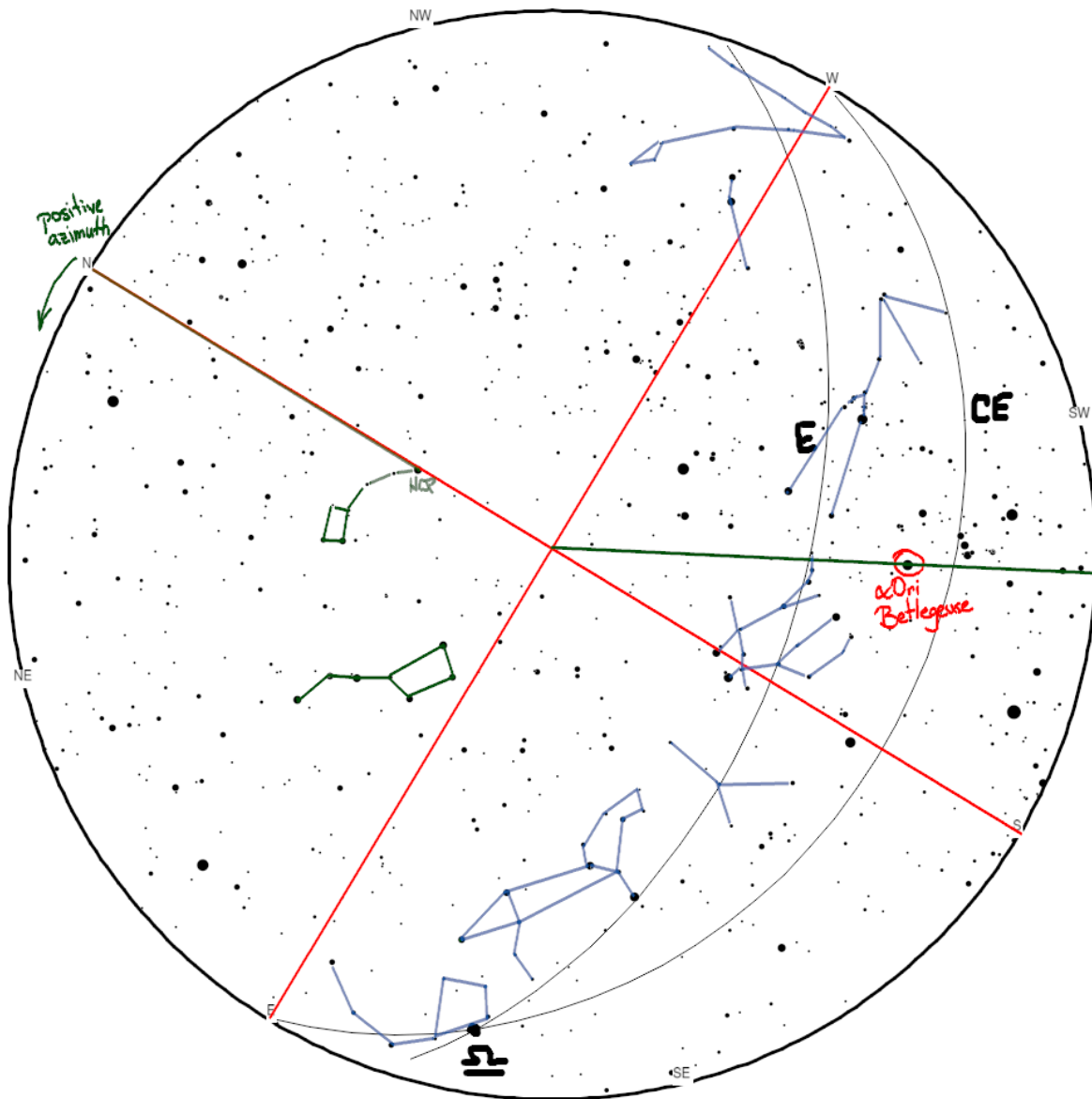
Solution: See sky chart. It is not expected that the curves match exactly; the Ecliptic should go through the zodiacal constellations, and the Celestial Equator should go from east to west and have a noticeable and reasonable curve, preferably so that the top point of the curve is as far from zenith as Polaris is high above the horizon. (possibly partial points for drawing from east to west with correct curve shape, just curving northwards instead of southwards)

d) (0.5 points each) What is the name of the intersection point between the two that is visible on this sky chart? In which constellation does it lie?

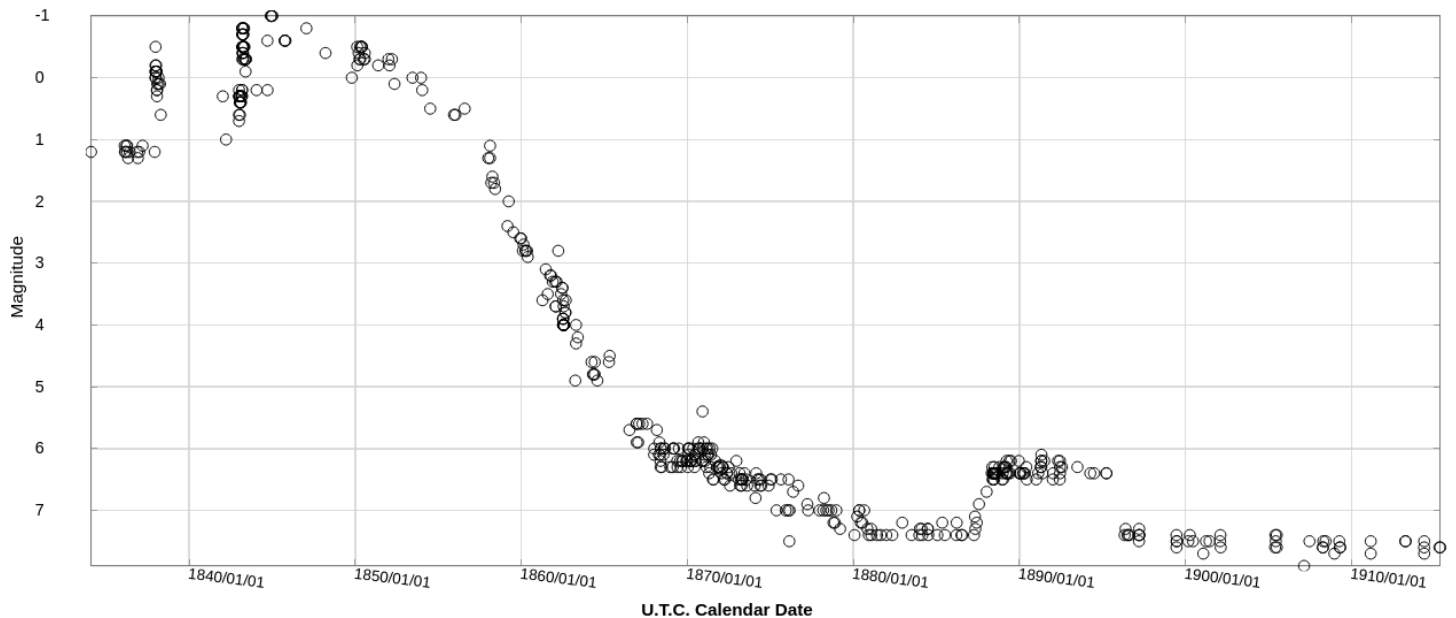
Solution: Autumnal Equinox (First point of Libra), lies in Virgo.



Solution sky chart (note: no need to draw in constellation lines, no need to mark non-cardinal directions, just NEWS suffice):



2) (10 points) Not all stars that exhibit interesting properties have been assigned a common name. One such star is η Car, a seemingly unremarkable star in the southern constellation Carina. The plot under this text shows an incredible increase in apparent brightness this star exhibited in the mid-1800s, termed “The Great Eruption”. At the peak, η Car became the second brightest star in the sky, just under Sirius. Its mass is estimated to be around $100M_{\odot}$ and it is located approximately 7500ly away from us.



- a) (1 point) Sir Arthur S. Eddington defined an upper limit for the luminosity of a star that is in hydrostatic equilibrium. This Eddington limit can be described by the expression $L_{Edd} = 3.2 \cdot 10^4 \cdot \left(\frac{M}{M_{\odot}}\right)L_{\odot}$, so if a star with mass M surpasses this luminosity, theory would state that its radiation pressure has become greater than the gravitational force holding the star together. What would be the result of the radiation pressure overcoming gravity?
- b) (1 point) Did η Car ever cross this theoretical limit during its “Great Eruption”? How about during the second “Lesser Eruption” around 1890? Comment on these findings in the context of your answer in a). Take the Sun’s absolute magnitude to be 4.83

Solution: 7500ly is approximately 2300pc (conversion given on front page). We can find the absolute magnitude of η Car by using the following relation, also provided in the beginning: $M_{\eta} = m_{\eta} - 5\log_{10}\left(\frac{d}{10pc}\right)$. For the “Great Eruption”, we can take $m_{\eta} \approx -1 \Rightarrow M_{\eta} = -12.80$, and for the “Lesser Eruption”

$m_{\eta} \approx 6.2 \Rightarrow M_{\eta} = -5.6$. Finally, we know that $\frac{L_{\eta}}{L_{\odot}} = 10^{0.4(M_{\odot} - M_{\eta})}$, so in the case of the “Great Eruption”, we have $L_{\eta, great} = 1.13 \cdot 10^8 L_{\odot}$ and

$L_{\eta, lesser} = 1.48 \cdot 10^4 L_{\odot}$. For the star assumed to have mass 100 times that of the Sun, the Eddington limit is $L_{Edd} = 3.2 \cdot 10^6 L_{\odot}$, which is obviously broken during

the “Great Eruption”, but not during the lesser one. A possible interpretation of this given that we can still see η Car in our skies is that the estimate of the mass of the star is wrong, or that the theoretical model simply hasn’t taken all possibilities into account, but give scores for any other explanation that makes sense in the context of the results.

- c) (7 points) In the 1990s, both amateur and professional observations started noting a steady brightening in η Car’s magnitude, hypothesised to be due to dissipation of a dust cloud located in front of the star. This brightening has continued on for several decades at approximately the same rate. You are provided with apparent magnitude data of η Car between the years 2020-2025¹. Use the provided millimetre paper to plot these points accordingly, and draw a best fit line through them.

Year	Apparent Magnitude [mag]
2020	4.27
2021	4.16
2022	4.04
2023	3.93
2024	3.82
2025	3.70

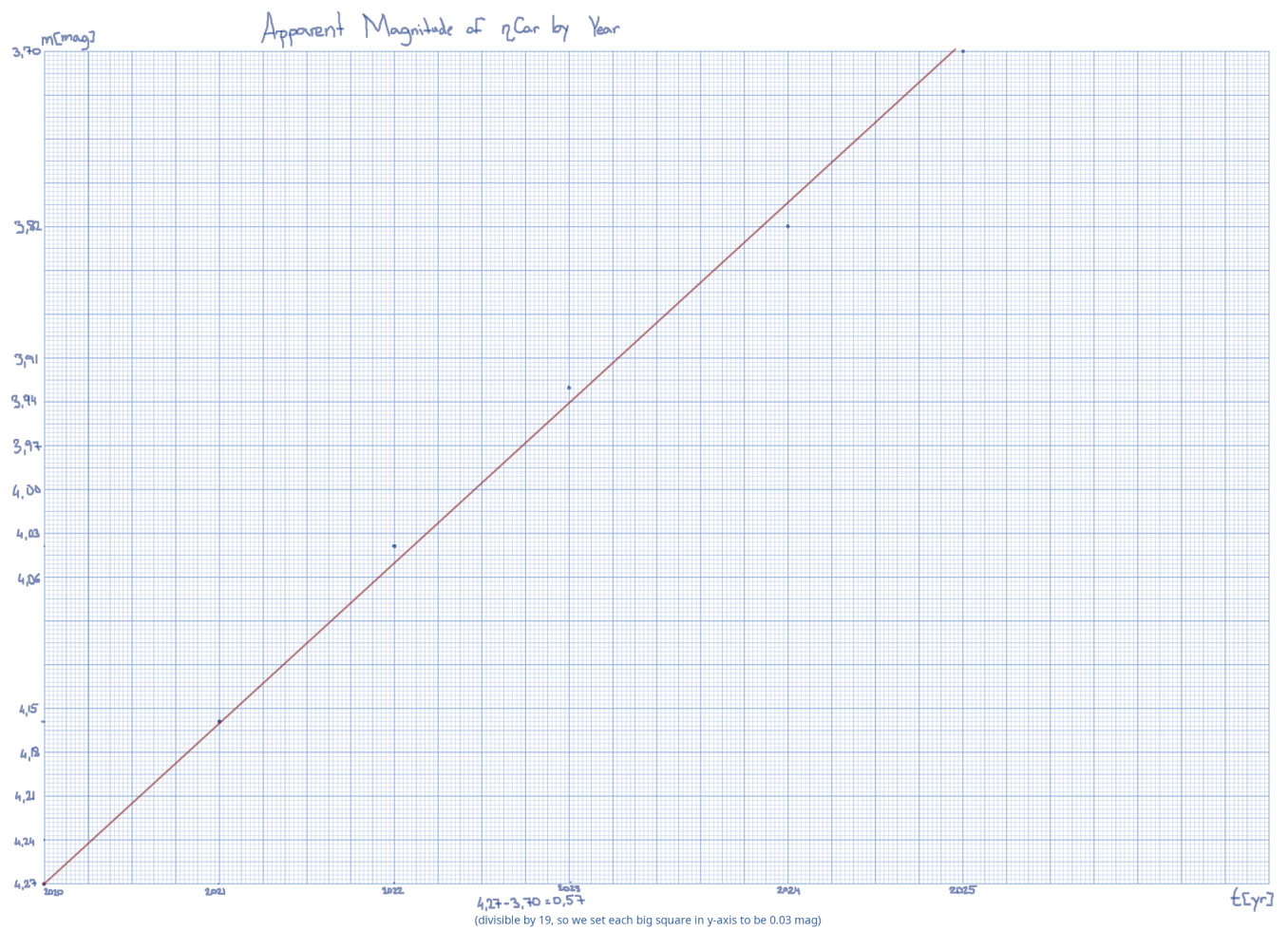
Solution: See attached drawing. Notice that magnitudes are plotted *decreasing* upwards, which should be accounted for in the scoring (they also have the graph from a), so they should be able to conclude how to set the axis up). Give points for: correct plotting, good eyeballing of best fit, having title, marking axes with units, marking ticks. This solution is just a suggestion, and any other reasonable ways to plot should be accepted, as long as they are legible and make sense. The slope of

the red line is: $a = \frac{3.94-4.27}{2023.2-2020} \approx -0.103$, and the intercept

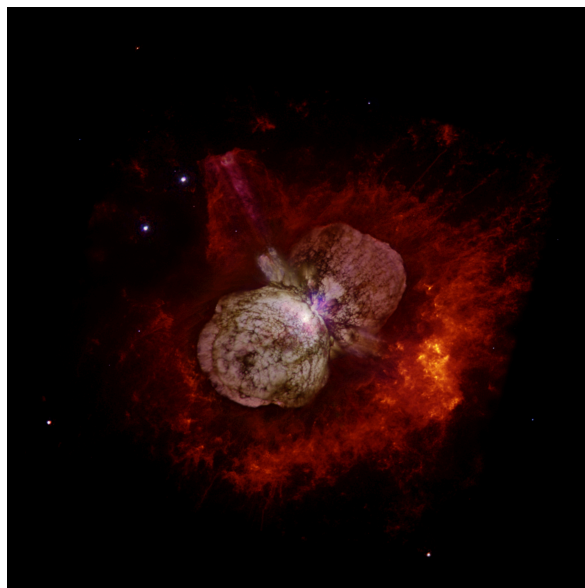
$b = 4.27 + 0.103 \cdot 2020 \approx 212.33$. **Scoring distribution:**

- 1 point for title+labelling axes
- 1 point for setting up linear axis scale
- 3 points for all correct points (0.5 points per correct point)
- 0.5 points if plot/points cover more than 50% of the paper
- 0.5 points for plotting magnitude decreasing upwards
- 1 point for drawing best fit line

¹ Data taken from <https://www.aavso.org/> and <https://arxiv.org/pdf/1901.00531>



- d) (1 point) η Car is surrounded by a nebula called Homunculus, making it a beautiful target to image. You can see one such photograph below this text, taken by the Hubble Space Telescope. Unfortunately, if η Car continues on its path of increasing magnitude, its brightness will completely outshine the nebula. From extinction calculations, this will happen when η Car reaches apparent magnitude somewhere between 3.34 and 2.44, making it about 10 times brighter than the nebula. According to your data analysis in c), give the earliest and latest year during which this can happen.



Solution: Using the regression result from the previous subproblem:

$m = -0.103t + 212.33$, we get $t = \frac{m-212.33}{-0.103}$. In the case of

$m = 3.34 \Rightarrow t \approx 2029$, and in the case of $m = 2.44 \Rightarrow t \approx 2037$ (rounding down since we're asking for the year *during* which this will happen. This is actually predicted to happen, and the results from <https://arxiv.org/pdf/1901.00531> calculate that it will be sometime between 2028 and 2036. Give points if the student correctly derives the expression for the year, regardless of whether the actual regression formula is good or not.

3) (4 points) The *Laser Interferometer Gravitational-wave Observatory* (LIGO) can detect gravitational waves from two coalescing black holes. By making extremely accurate measurements at two different locations, it is determined that two black holes were orbiting each other 250 times per second at the instance when their event horizons touched each other. What is the mass of the resulting, larger black hole that formed as a result of the collision? For this calculation you can assume that the black holes had zero spin and you can neglect the loss of black hole mass converted to energy of the gravitational waves.

Solution: For non-rotating black holes we can use the Schwarzschild equation to calculate the radius. Knowing the orbital frequency at the moment when the black holes touch we can even derive the semimajor axis of their orbital movement. We know $f = 250 \text{ Hz}$ and let r_1 and r_2 be the radii (event horizon = Schwarzschild radius in this case) for the two black holes with masses m_1 and m_2 . When the black holes touch each other, the semimajor axis is $a = \frac{R}{2} = \frac{r_1+r_2}{2}$ (1 point) and $M = m_1 + m_2$ is the combined mass (which is also the mass of the resulting black hole under our assumptions). We apply Kepler's third law

$$\frac{a^3}{T^2} = \frac{G(m_1+m_2)}{4\pi^2} \text{ (1 point) and get } \frac{R^3 f^2}{8} = \frac{GM}{4\pi^2} \text{ and note that } R = \frac{GM}{c^2} \text{ (1 point) to get}$$
$$\frac{(GM/c^2)^3 f^2}{8} = \frac{GM}{4\pi^2} \text{ which after a little algebra yields}$$
$$M = \frac{c^3}{2\pi G f} \approx 129.3 M_\odot \text{ (1 point).}$$