

Norwegian Olympiad on Astronomy and Astrophysics



Final round problems 2022/2023

Date: Friday, March 24th 2023

Allowed aids: calculator, pen/pencil, constants and formulas given below

Time: max. 150 minutes

Part 1 (15 multiple choice questions): There are four possible answers for each problem - A, B, C and D. Use the answer sheet to mark your chosen answer. There is only one single correct answer for each problem and all problems yield the same number of points. Zero points are given for a problem if more than one answer is marked. Wrong answers do not yield negative points.

Part 2 (4 open questions): Explain your reasoning (please write clearly), and write your name on the sheets that you hand in!

Good luck!

Constants and formulas:

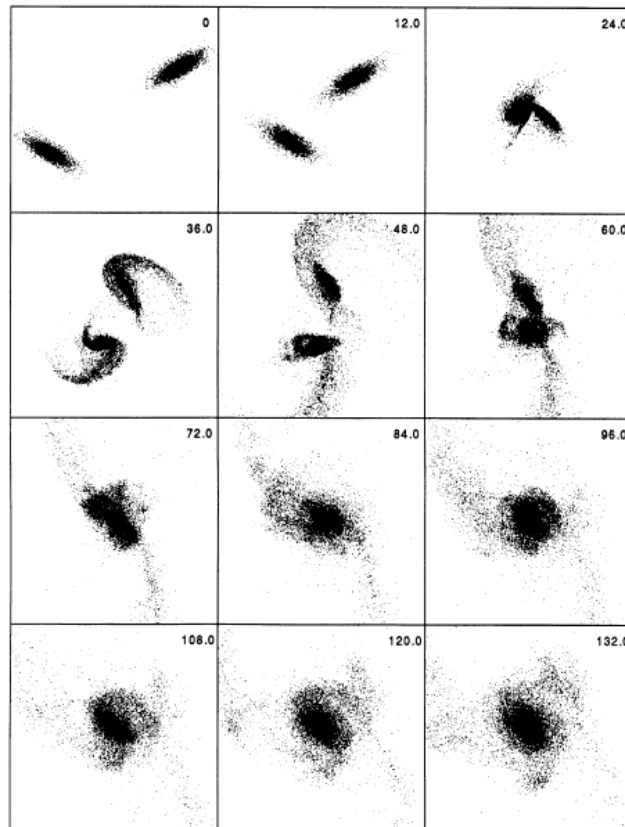
- 1 parsec (pc) ≈ 3.26 light years $\approx 206\,265$ AU
- Newton's law of gravity: $F = GmM/r^2$, $G \approx 6.67 \cdot 10^{-11} \text{ m}^3/\text{kg}/\text{s}^2$
- Kepler's third law: $a^3/P^2 = G(M + m)/4\pi^2$
- Wien's displacement law: $\lambda_{\text{max}} = b/T$, $b \approx 2.9 \cdot 10^6 \text{ nm} \cdot \text{K}$
- Stefan-Boltzmann's law: $F = \sigma T^4$, $\sigma \approx 5.67 \cdot 10^{-8} \text{ W}/\text{m}^2/\text{K}^4$
- Apparent magnitude: $m = -2.5 \log (F/F_0)$
- Hubble's law: $v = H_0 d$, $H_0 \approx 73 \text{ km}/\text{s}/\text{Mpc}$
- Earth's mass and radius: $5.97 \cdot 10^{24} \text{ kg}$ and 6371 km

Part 1: multiple choice questions (1 point per question)

- 1) $N \geq 2$ point masses, each with mass M/N , are spread uniformly around a ring with radius R (so that each adjacent pair spans $360^\circ/N$ of the circle). What total gravitational force F do they exert on a test mass m positioned a height h directly above the ring's center?
- a) $F = \frac{GMm}{h^2}$
 - b) $F = \frac{GMm}{R^2+h^2}$
 - c) $F = \frac{GMmh}{(R^2+h^2)^{3/2}}$
 - d) $F = \frac{GMm\sqrt{R^2+h^2}}{h^3}$
- 2) What would happen to the planets' orbits if the Sun suddenly turned into a black hole with Schwarzschild radius $R_s = 2GM_\odot/c^2$?
- a) They spiral inwards until reaching a new smaller stable orbit.
 - b) They spiral inwards and are swallowed by the black hole.
 - c) They do not change.
 - d) They are pushed out.
- 3) If the Earth stopped in its orbit, approximately how long would it take for it to crash into the Sun?
- a) 64.5 days
 - b) 129 days
 - c) 182.5 days
 - d) 365 days
- 4) This morning (competition day, 24. March 2023), we noticed a star rising at 03:40 sidereal time. At approximately what time will this star rise in ten days' time (3. April 2023)?
- a) 03:00
 - b) 03:20
 - c) 04:00
 - d) 04:20

- 5) Olbers' paradox poses the question of why our night sky is dark. Given that our universe is infinite and homogenous, every line of sight should end at the surface of the star, implying that the night sky should be very bright with not a single black spot between the stars. This is obviously false, and there are a number of explanations that try to resolve this paradox. Which of the following would **not** pose a valid explanation?
- a) The universe is too young for the light from distant stars to have reached us.
 - b) Given the constant expansion of the universe, light from distant stars is red-shifted to the point of non-visibility.
 - c) There is too much interstellar dust that absorbs the radiated light before it gets to us.
 - d) Stars' lifespans are too short for their light to significantly fill the empty spaces between one another.
- 6) How many stars on the entire sky have a negative apparent magnitude in the visible spectrum?
- a) 3
 - b) 4
 - c) 5
 - d) 6
- 7) A train was travelling eastwards along the 30th line of latitude with a velocity of 120km/h on a lovely sunny first day of spring, 20. March. Given that the day is 24^h long, approximately how long will a passenger on said train be able to see the sunlight?
- a) 11^h5^{min}
 - b) 12^h
 - c) 12^h35^{min}
 - d) 13^h10^{min}

- 8) What do the below snapshots from an N -body simulation from 1993 with $N = 16\,384$ particles demonstrate?

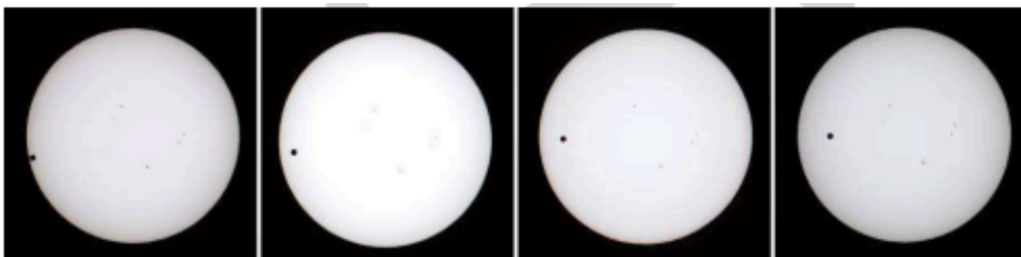


- a) Collision between two elliptical meteorites leading to a pile-up of space dust.
 - b) Merging of two spiral galaxies into an elliptical galaxy.
 - c) Formation of a supermassive black hole.
 - d) Collapse of a double-star system.
- 9) Why does a pole star appear stationary on the sky?
- a) Earth is not moving with respect to the pole star.
 - b) Earth has the rotation axis perpendicular to the pole star.
 - c) The pole star is almost along the axis of rotation of the earth.
 - d) Both Earth and the pole star have the same velocity in the Milky Way galaxy.

- 10) Which of the following phenomena would not work for estimating cosmic distances if our Universe wasn't expanding?
- a) Sometimes Venus can be seen transiting over the solar disc.
 - b) Stars with no proper motion appear to change their position in the sky when viewed six months apart.
 - c) Stars exhibit Doppler shift.
 - d) All supernovae of Type Ia have the same absolute brightness.
- 11) Multiple images are captured against the background stars over some time and combined into the image below. What does it show?



- a) Retrograde motion
 - b) A moon's orbit around its host planet.
 - c) A visual artifact due to Earth's rotation.
 - d) Proof of the Ptolemaic model.
- 12) What does the image below show?



- a) Sunspot activity.
- b) A planet passing in front of the Sun.
- c) View from the first Lagrange point of the white and black bodies.
- d) A crater on the Moon.

13) What does the image below show?



- a) The polar region on Mars
- b) Eclipse
- c) Occultation of Saturn
- d) Transit of Venus

14) Which of these sets of equations determines the position of a satellite m' at the Lagrange point from one massive body m orbiting a much more massive body M , where $M \gg m \gg m'$?

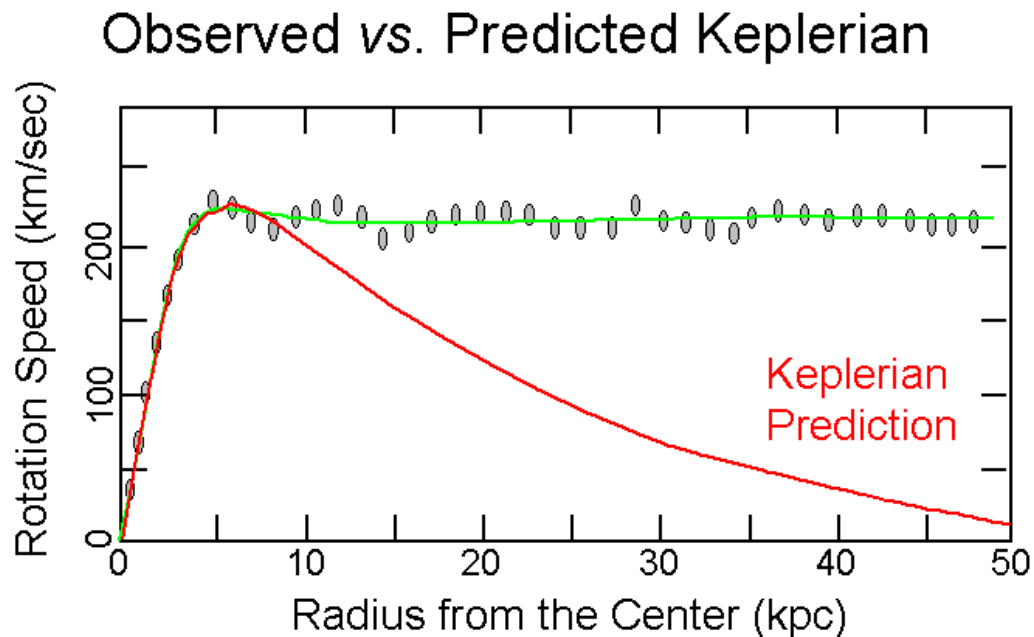
- a) Newton's second law on m' , and Kepler's third law on m' .
- b) Newton's second law on m' , and Kepler's third law on m .
- c) Newton's second law on M , and Kepler's third law on m' .
- d) Newton's second law on M , and Kepler's third law on m .

15) The Universe consists of matter, radiation and dark energy, each of which have dominated the Universe's energy density content in different eras. From the beginning of the Universe till today, in which order have they done so?

- a) Radiation $\rightarrow$ matter $\rightarrow$ dark energy
- b) Matter $\rightarrow$ dark energy $\rightarrow$ radiation
- c) Dark energy $\rightarrow$ matter $\rightarrow$ radiation
- d) Dark energy $\rightarrow$ radiation $\rightarrow$ matter

Part 2: Open Questions (points are specified per question)

- 1) This spherically averaged *galaxy rotation curve* shows observed orbital velocities v of stars and gas at distances r from a galactic center, compared to their predicted velocities from the masses of the observed stars.



- a) (0.5 pts) What is the mass M within $r < 8$ kpc in solar masses?
 - b) (1.0 pts) What is the predicted velocity profile $v(r)$ for $r > 8$ kpc, assuming most mass is located within 8 kpc?
 - c) (1.0 pts) What is the actual mass density profile $\rho(r)$ for $r > 8$ kpc?
 - d) (0.5 pts) What is the most widely accepted explanation of the discrepancy between the observed and predicted velocity profiles?
- 2) (3 pts) If you were an observer on the surface of Saturn, could you ever see a total solar eclipse caused by Titan, one of its moons? Elaborate your answer. Titan's diameter is given to be 5150 km, and it orbits Saturn at a distance of 1 222 000 km. Saturn's diameter is 120 000 km and it orbits the Sun at a distance of 9.5 AU. The Sun's angular diameter as seen from Earth is 0.5° .

3) Pulsars are a type of fast rotating, high density stars, which are known for their regular pulses of radiation. They are also sometimes called the 'lighthouses of the universe'. In this problem, we assume the pulsars to be spheres of uniform density, which are gravitationally bound.

a) (2 pts.) The rotation period of a pulsar was measured to be $P = 1.5$ ms. What limit does this put on its density?

b) (1 pt) The pulsar's mass is measured to be $1.5 M_{\odot}$.

What limit can we place on its radius?

4) On the following blank constellation map (on its own page):

a) (1 pt) Mark and name at least one constellation belonging to the Zodiac

b) (1pt) Mark at least two stars in the Aquila constellation with their Bayer designation (α , β , γ ...)

Student name:

