



THE NORWEGIAN ASTRONOMY AND ASTROPHYSICS
OLYMPIAD COMMITTEE

IOAA NOTES

Part 6: Cosmology

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1 Other Celestial Objects

Cosmology is the study of the universe as a whole and how everything in it has come to be. Before we look at the history of the universe and how it might end, we will present some other important objects in the universe.

1.1 Binaries

Just like binary code consists of two components (zeros and ones), astronomical binaries consist of two stars (or two planets, but the stars are more interesting, and what we more usually refer to as binaries) that orbit a common centre of mass. Surprisingly enough, there are more binary systems than there are single stars from what we have observed!

For the two stars in a binary, the eccentricities of their orbits have the same value ($e_1 = e_2$). If the semi-major axes of the stars are a_1 and a_2 , and their masses are M_1 and M_2 , the following relationship holds:

$$M_1 a_1 = M_2 a_2 \quad (1)$$

From this relationship, we can conclude that the more massive star will have a smaller orbit.

Kepler's Third Law can be applied to binary systems as well, where the mass in question is $M_1 + M_2$, and the semi-major axis is the semi-major axis of what we call the *relative orbit*, which is simply $a_1 + a_2$:

$$G \frac{M_1 + M_2}{4\pi^2} = \frac{(a_1 + a_2)^3}{T^2} \quad (2)$$

Combining (1) and (2) allows us to calculate the masses of the stars in a binary if we know their semi-major axes and the orbital period around the centre of mass!

One important thing to note is that the total magnitude of a binary is *not* equal to the sum of the magnitudes of its components. If you want to calculate the total magnitude from knowing the components' magnitudes, you must first turn the magnitudes into luminosities, add the luminosities together, and then turn them back into magnitudes. This is a consequence of the fact that magnitudes are a logarithmic parameter.

There are different types of binaries based on how we observe them, and we'll present them all in very short points here. With the naked eye, if you look up at the night sky, the vast majority of binaries are not resolvable, meaning that you will see them as one bright spot in the sky.

Visual binaries are binaries that we can resolve using an optical instrument, such as a telescope. These binaries are either very close to us so that we can resolve them even though the components are not that separated, or the distance between the components is so vast that we can resolve it visually.

Spectroscopic binaries are configured so that we can't observe two separate stars visually, but if we look at the spectra coming from the stars at different times, we see periodic shifts in the spectral lines due to the Doppler shift. By observing spectral properties over a certain time period, we can with confidence say that we have two stars, rather than just one.

Eclipsing binaries are loved by olympiad question setters around the globe, since they pose a great opportunity for data analysis. Such binaries are special because their orbital plane is almost exactly in our line of sight (close to 90°), so when we observe the brightness-against-time graphs (light curves) coming from them, we can see eclipses. These eclipses can happen when the smaller star passes in front of the bigger from our point of view, and we observe a big dip in the light curve, or when the smaller star passes behind the bigger one, where we see a smaller dip in the light curve. Based on these dips, we can determine many important orbital parameters of the binary, and some properties of the stars - an extremely powerful method called photometry!

There are also multi-star systems with three, four, five or even six stars! The physics behind these is incredibly complex and a topic of modern astrophysical research, so you don't need to worry

about those just yet!

1.2 Clusters

Bigger collections of stars in the sky also exist, and we call those associations and clusters. Associations are groups of a couple of dozen stars, and we don't really focus on them all too much. Clusters, on the other hand, can be very interesting with their hundreds to hundreds-of-thousands of stars, and we differ between open and globular clusters.

Open clusters are groupings of fewer stars, usually in the order of a couple of hundred. The reason we call them open is that they are often irregularly shaped and quite spread out. As time progresses, these clusters slowly degenerate since the gravitational interaction between stars in the cluster is very weak and easily disturbed by gravitation from e.g. other clusters. Open clusters are young in the astronomical sense, in the order of magnitude of tens of millions of years. They are usually spread close to the galactic plane (which we will go into later on).

Globular clusters are often much further away from us and shine extremely brightly due to the up to several hundreds of thousands of stars present in them. We call them globular since they exhibit regular spherical shapes, and inside these spheres, we usually assume (with quite high precision) that the stars are uniformly distributed. Such clusters are extremely old and very stable due to the gravitational interactions between the stars contained within them.

1.3 Black Holes

We touched on these very briefly in the previous handout, so we'll expand on them here somewhat. An important question is how we quantify when a star becomes a black hole at the end of its life. For this to happen, the entire mass has to collapse in a radius we call the Schwarzschild radius. This is a radius from which even light cannot escape the pull of the black hole, and it's why we call black holes black.

As hinted above, we get the Schwarzschild radius by looking at the formula for escape velocity, and setting the velocity in the equation to be the speed of light. Some simple algebra yields the following formula:

$$R_{Sch} = \frac{2GM}{c^2} \quad (3)$$

Outside of the Schwarzschild radius, things can escape, but again, not even light can escape from inside the radius. The boundary between where light can escape and where it can't is called the event horizon of the black hole. The centre of a black hole is referred to as the singularity, which essentially means we can't describe it mathematically using the knowledge we have at our disposition at this point in time.

Every black hole has mass, angular momentum and electric charge, with no other properties being observable. For IOAA, we look at non-rotating black holes, as rotating ones introduce a lot of complexity due to relativity considerations, so the only parameter of interest for us is the mass. For such non-rotating black holes, there is another limit we refer to as the innermost stable orbit radius. If an object is inside of this radius, it is guaranteed to fall into the black hole, and if it's outside, it will get ejected out. This radius is $3 \cdot R_{Sch}$.

1.4 Galaxies

These are the last celestial objects we'll cover for the time being. Galaxies are stars, planets, dust and gas clouds and dark matter all bound in a system by gravity. A lot of what we know about galaxies is based on the foundation laid out by Edwin Hubble in the 1920s. An important think he made is the so-called morphological classification of galaxies, or in other words, dividing them into groups based on their shapes. As such, we have four main groups: elliptical, spiral, lenticular (shaped like lenses, or flat disks) and irregular. You can see these types on Figure 1

Also note that this classification is based on what Hubble could observe visually - fainter galaxies that couldn't be observed as easily are also very diverse and are difficult to exactly sort into any of these categories.

Galaxy Zoo <https://www.zooniverse.org/projects/zookeeper/galaxy-zoo/> is a unique opportunity to see images of galaxies taken by the Euclid mission sent up in 2023, if you wish to see different types of galaxies and shapes (and maybe even make a small contribution to science by helping classify the galaxies). Go into the site and see what questions they ask you and use to classify galaxies! It's a good past-time activity. ;)

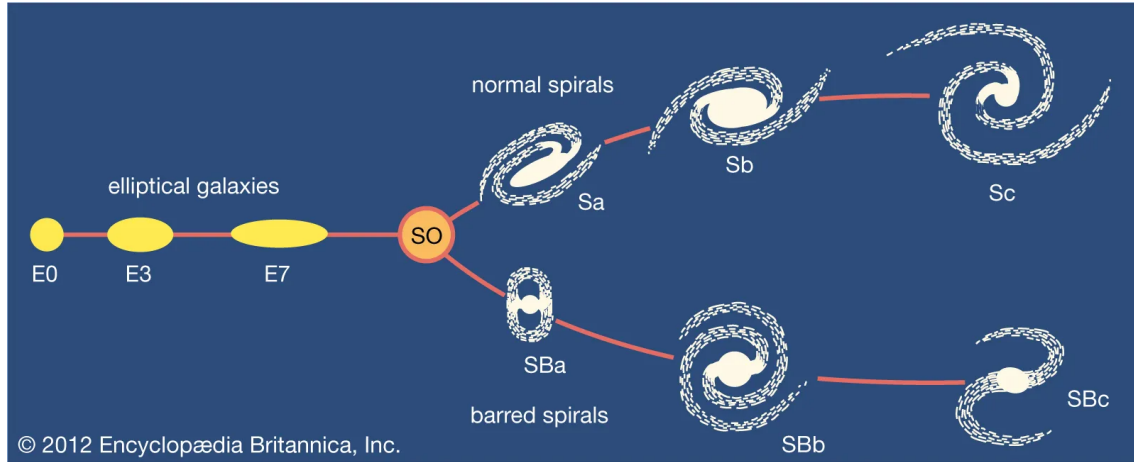


Figure 1: Hubble's Morphological Classification of galaxies.

1.4.1 Elliptical Galaxies

Elliptical galaxies are denoted by the letter E and a whole number from 0 to 7, which denotes the eccentricity. The number of the galaxy is found by using this formula, where b and a are the usual elliptic parameters:

$$n = 10 \cdot \left(1 - \frac{b}{a}\right) \quad (4)$$

As such, an E0 galaxy appears circular, while an E7 galaxy is the most eccentric ellipse. The closer we get to the centre of an elliptical galaxy, the denser the distribution of stars is. They contain little to no cold gas from which new stars might be able to form; on the contrary, X-ray observations of the interstellar medium revealed gas at temperatures of several million degrees. The stars in elliptical galaxies are usually older smaller red stars.

1.4.2 Lenticular Galaxies

These galaxies are denoted with S0, and represent a sort of a middle ground between elliptical and spiral galaxies. They don't have a lot of material, but do possess a central flat disk made up of stars, which you will also see with spiral galaxies. The Tully-Fisher relation that we will explore soon holds for observations of lenticular galaxies as well.

1.4.3 Spiral Galaxies

Finally, spiral galaxies are, like the name suggests, galaxies that have spiral arms. In the centre of spiral galaxies lie bulges that have similar structure to elliptical galaxies. Spiral galaxies have a further subdivision into normal spirals and barred spirals, just like you can see in Figure 1.

Normal spirals are denoted with S followed by either a, b or c, depending on the size of the central region, the thickness of the spiral arms, and how "open" the spirals are.

Barred spirals are denoted with SB, but otherwise follow the same a, b, c sequence as normal spirals. The difference is that the central part of barred galaxies is shaped like a bar.

The cores of these galaxies contain older stars and globular clusters, and are where stars are born, but the younger stars that are bluer usually populate the spiral arms.

By experimental observations of spiral galaxies, we have developed the so-called Tully-Fisher relation which connects the rotational velocity of a galaxy with its luminosity (and thereby its mass). This relation shows us that the velocity of the components in a galaxy doesn't depend on how far out from the galaxy's centre they are, and is an indicator of the existence of black matter. The relation is:

$$L \propto v^\alpha \tag{5}$$

where α is a number that varies between galaxies, but the most common value is around 3.

Another property that holds for spiral galaxies is the mass-luminosity relation, just like for main sequence stars where $L \propto M^{3.5}$.

1.4.4 Non-Morphological Classification of Galaxies

Finally, it is not only the shape that helps us classify galaxies. Any galaxy can also be said to be either normal or active, based on their brightnesses.

Normal galaxies are galaxies with constant brightness originating solely from their stars. This radiation is almost purely thermal, and we observe absorption spectra from these galaxies. Such normal galaxies are the vast majority of the galaxies known to us today.

Most galaxies have a black hole at their centre. It is hypothesised that when gas accretes (accumulates, collects) around the supermassive black hole at the centre, the resulting accretion disk emits radiation of immense energies. This gas is also shot out and goes along magnetic field lines in immense velocities, which can be noticed on radio observations. These galaxies with high energy emittance, brightness variability and where we observe emission spectra are called active galaxies and have active galactic nuclei (AGN).

Active galaxies are further classified as either Seyfert galaxies (rapid changes in brightness, assumed to have very small AGNs), radio-galaxies (elliptical galaxies with incredible radio-energies coming from shooting out high-velocity charged particle jets), blazars (radio-galaxies that shoot their jets directly towards us and are bright across the entire spectrum), and quasars (the brightest objects we know of in the universe).

2 Cosmic Distance Ladder

Another essential aspect of cosmology is the so-called cosmic distance ladder. How *do* we measure vast distances in space? The cosmic distance ladder is the name given to a set of methods used to measure cosmic distances. The longer the distances get, the more different methods we want to use. You will see a lot of diagrams like Figure 2 with more or fewer measurement methods, but these are the most important ones for you to know.

The “Cosmic Distance Ladder”

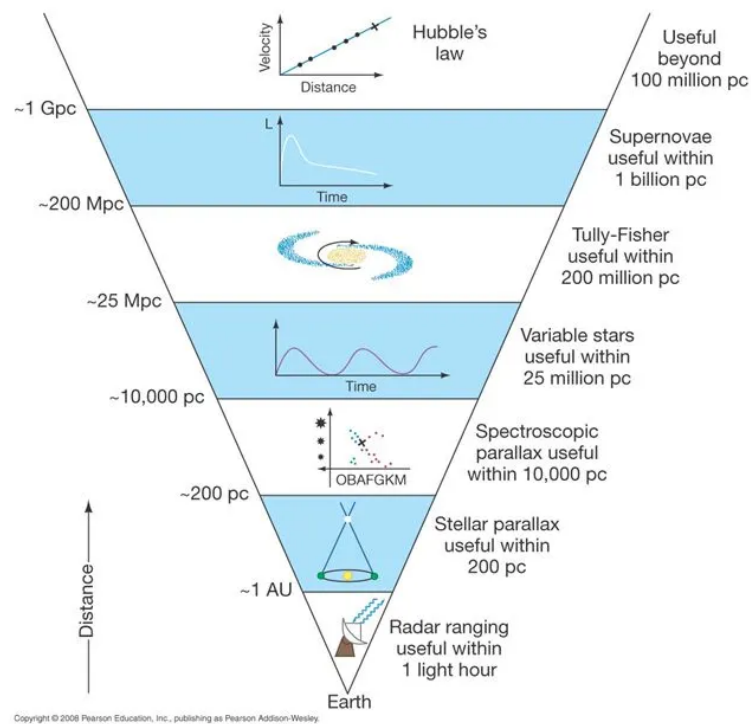


Figure 2: The cosmic distance ladder.

The first step (or rung, as the steps on the ladder are actually called!) is radar measurement. The range for this is approximately 1 light hour, which is on the order of magnitude of the distance between Earth and Saturn. All further distance measurements follow the principle that they need to be calibrated using what we know from shorter measuring, e.g. stellar parallax needs to be calibrated using radar ranging before we can safely use it for longer distances.

2.1 Parallax

Already at the second step, we're seeing a unit of measurement we haven't used previously, namely pc, which stands for parsec. Don't believe Star Wars when they tell you that the Millennium Falcon did the Kessel Run in less than 12 parsecs - parsecs are a measure of distance, not time, despite the "secs" in the name.

The name parsec is actually short for parallax arcsecond, which is a good segue into the second method of distance measurement, namely parallax. You can observe the parallax effect yourself right now, albeit not astronomically. If you stretch out your arm and put a thumb up (and give yourself a thumbs-up for having come this far), and alternate between closing your left and your right eyes, it will look as if your thumb is moving, even though you're keeping it in place!

This exact effect can be used to determine the distance to close stars. It takes six months for Earth to go from one side of the Sun to the opposite side. If we observe the position of a star in the sky at one point in time, and then six months later, we might observe a small shift, just like seen in Figure 3.

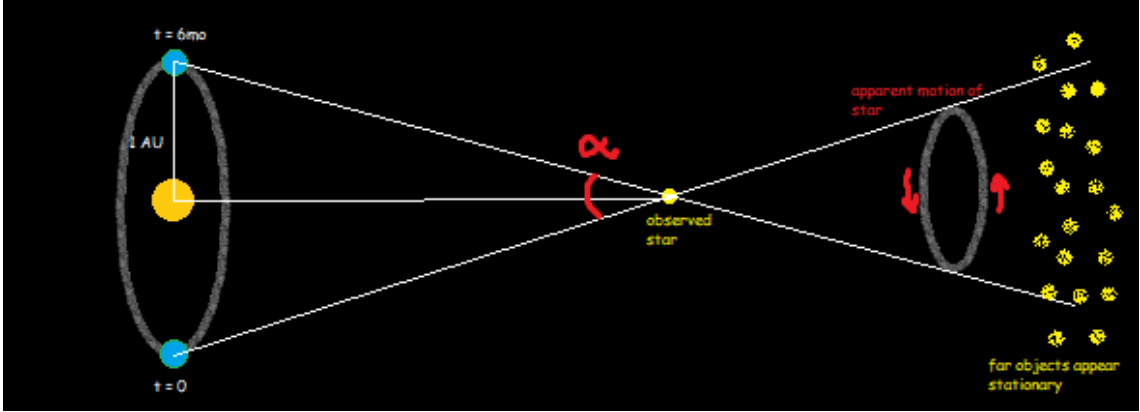


Figure 3: The parallax effect and definition of a parsec.

Note that the angles and sizes are absolutely not to scale in Figure 3 and are exaggerated for the purposes of illustration. The distance as represented by the horizontal line connecting our Sun to the observed star is given as:

$$d = \frac{1}{p} \quad (6)$$

and p in the above figure would be half of the angle α . If you express p in arcsec, then you will get d in parsecs, where

$$1 \text{ arcsec} = \frac{1}{60} \text{ arcmin} = \frac{1}{3600}^\circ \text{ and } 1 \text{ parsec} = \frac{1 \text{ AU}}{1 \text{ arcsec}} = 3.08 \cdot 10^{16} \text{ m} \quad (7)$$

Sometimes, you might also come about the unit mas, which is simply a milliarcsecond, or 10^{-3} arcsec. This type of parallax is what we call stellar parallax, or trigonometric parallax, since it's a direct calculation using angles and distances.

The other type of parallax you see on the ladder is the so-called spectroscopic parallax. This is a method that combines parallax with observations of magnitudes. We can find distances using the following formula:

$$m - M = 5 \log(d) - 5 = 5 \log\left(\frac{d}{10 \text{ pc}}\right) \quad (8)$$

which can essentially be read as "objects with the same magnitude are the same distance away".

Now, a little caveat here is that this is not exactly the complete formula. There is a phenomenon called interstellar extinction where stellar dust and gas absorb and scatter the light coming from stars to us, and as such affect the apparent magnitude. As such, you might sometimes hear that the magnitude needs to be "corrected for extinction". The correction for extinction can be represented as a function like so:

$$m - M = 5 \log\left(\frac{d}{10 \text{ pc}}\right) + A(d) \quad (9)$$

where $A(d)$ is the extinction as a function of distance. If you need to take extinction into account, you will be told so, but unless it's specified, you're free to assume that the correction has been performed. Nonetheless, you should be aware of this phenomenon.

2.2 Standard Candles

Standard candles are objects of known luminosity - from the previous handout, if we know the luminosity and measure the flux, we can determine the distance.

2.2.1 Cepheids

How does this work with variable stars, you might wonder. Even though these stars are variable and pulsate, there are variable stars called cepheids that pulsate extremely regularly. By measuring this period, we can determine the absolute magnitude of the cepheid by the following formula:

$$M = -2.78 \cdot (\log(T) - 1) - 4 \quad (10)$$

You don't need to remember this formula, but you might need to find it yourself in a data analysis question sometime, so just be aware that there is a relationship. Figure 4 shows a typical cepheid curve, and we measure the period by looking at the distance between the peaks, and then plug into the formula above.

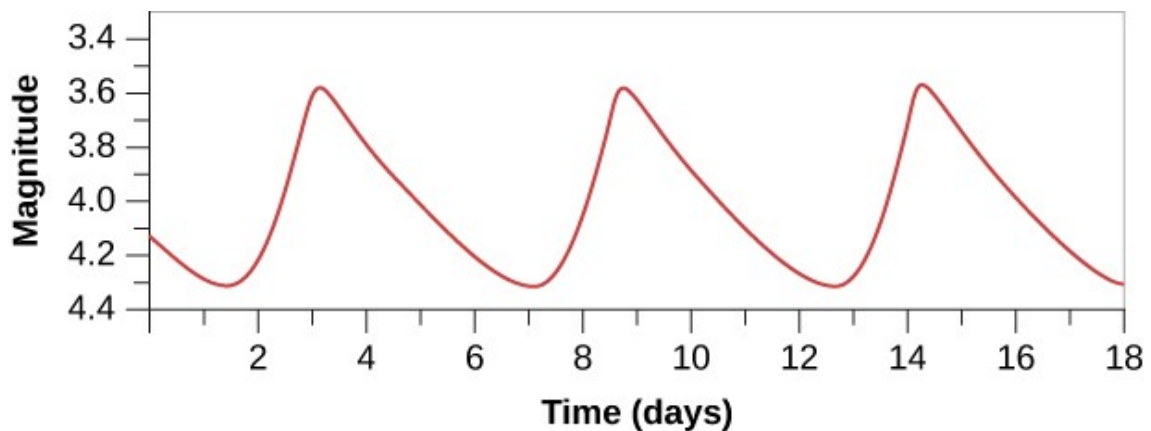


Figure 4: A typical cepheid light curve.

2.2.2 Type Ia Supernovae

Type Ia Supernovae, as you know from the previous handout, happen when White Dwarfs reach the Chandrasekhar Limit of about $1.4M_{\odot}$. Since the explosion happens at essentially the same mass, all type Ia supernovae have approximately the same peak absolute magnitude of -19.3 ± 0.03 , and then follow approximately the same curve once they dim. This in conjunction with (9) can tell us the distance to the supernova in question based on the apparent magnitude we observe.

2.3 Tully-Fisher

As touched upon before, Tully-Fisher gives us the luminosity, and knowing luminosity immediately lets us know the distance via magnitudes!

2.4 The Hubble-Lemaître Law (for distance measurement)

And finally we get to the Hubble-Lemaître Law, mostly known as just Hubble's Law, which for us serves as a stepping stone into cosmological theories. Here, we will just present the formula, and we'll go into further detail about how exactly this beautifully simple expression has come about later.

We will explore later the concept of our universe constantly expanding, and as such, objects are moving away, or *receding* from us. This recessional velocity, v can be found by observing spectra from far away objects and using Doppler's famous formulae based on the redshift observed. Once v is found, the distance to the object can be found as:

$$d = \frac{v}{H_0}, \tag{11}$$

where H_0 is a constant approximately equal to $73 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$, with $1 \text{ Mpc} = 10^6 \text{ pc}$. Later, we will examine exactly what the origin of this constant is, and define it a little more precisely.

3 Cosmological Concepts and Theories

The most important cosmological concept you need to know about and that lays the foundation for all modern astronomy and astrophysics is that we live in an expanding universe.

3.1 ... then nearly fourteen billion years ago

An expanding universe implies that there was once a point where everything had to expand from. This singularity (reminder, a singularity is something we are currently unable to describe mathematically) where the entire universe was condensed into a point of infinitely-small volume exploded in what we today refer to as the Big Bang.

The earliest moments after the Big Bang, most commonly referred to as Planck Time, or the first $5 \cdot 10^{-44}$ s after the Big Bang are also not able to be described physically or mathematically with what we know today. After this time, we know that the universe experienced an extremely rapid expansion in the order of magnitude of 10^{50} and cooling, which are still going on today.

A tangible proof of the Big Bang is that we can still observe residual radiation from the event even today. We see it in the form of Cosmic Microwave Background Radiation (or just CMB for short). It comes from all directions in space, and behaves as black body radiation with a temperature of approximately 2.73 K. Figure 5 shows this background radiation as published in 2013 after 15 months of observation. The irregularities and asymmetries in this temperature map can tell us a lot about our universe, and have helped us confirm the validity of our current cosmological models.

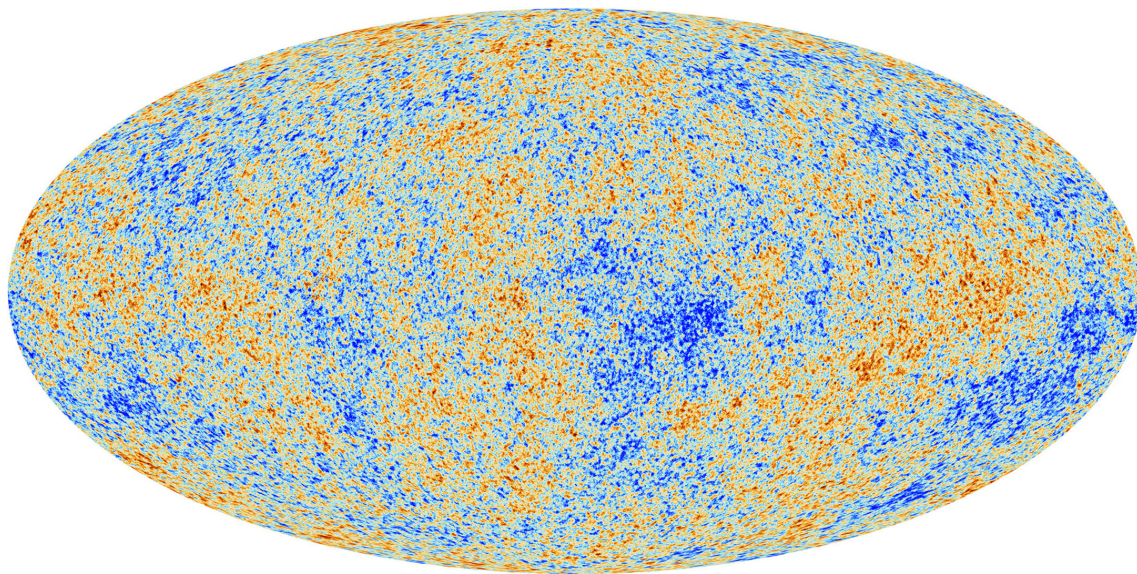


Figure 5: The Cosmic Microwave Background Radiation as imaged by ESA's Planck Space Telescope. The difference between the red and blue parts is on the order of a couple of millikelvin.

3.2 Properties of the Universe

Today, we assume that our universe is homogeneous and isotropic at large scales. We call this the "cosmological principle", and it is important to note that it holds for large scales, so irregularities on a smaller scale still fit the model. Isotropy means that whatever direction we look in, the universe looks the same. Homogeneity means that the universe looks the same in every point.

Both of these support what we call the Copernican principle, namely the fact that our Solar System isn't special at all when it comes to its position, meaning that what we observe from here can be said to be representative for an observation from any other point in the universe.

There is one strange thing about the CMB that we have dubbed the "axis of evil" that contradicts this principle a little. We won't go into further detail here, but we have measured a significant (albeit small) difference in the CMB when observed through one half of our solar system as compared to another. This is strange and as of yet an unanswered phenomenon, since the Copernican principle suggests that everything should be the same no matter what direction we look in.

3.3 The Geometry of the Universe

A consequence of the homogeneity and isotropy assumptions is that the universe has no centre and no edge. Beyond that, there are three different possibilities for the geometry of the universe: an elliptic (closed) universe, a hyperbolic (open) universe, and a parabolic (flat, Euclidian) universe. Most cosmologists are of the belief that our universe is flat.

Let us explore what these different geometries mean and where they come from. To do so, we must introduce a parameter called the scale factor. This parameter, denoted by the letter a (you might sometimes see R used as well, but it doesn't really matter), is a time-dependent quantity that helps us say something about how much the universe is expanding. The value(s) of the scale factor are not very important for you, but you should realise how it behaves. If the universe expands to double its size, then the scale factor will be doubled as well. The convention is that the scale factor today is $a_0 = 1$.

The scale factor is made so that the length of any line in space will expand proportionally to a , and any volume will expand proportionally to a^3 . When we say that any length expansion is proportional to a , we can also include wavelengths here! As such, we can write:

$$\frac{\lambda_0}{\lambda} = \frac{a_0}{a} \quad (12)$$

where λ_0 and a_0 are the current wavelength and scale factor, while λ and a were the wavelength and scale factor at time of emission. Since the scale factor has been growing with time, we can see that $\frac{a_0}{a} > 1$, meaning that $\lambda_0 > \lambda$ and the wavelength is being redshifted.

From the Doppler effect, we know that the redshift can be quantified as:

$$z = \frac{\lambda_0 - \lambda}{\lambda} \quad (13)$$

Rearranging this redshift expression and inserting into (12) gives a relationship between the observed redshift and the scaling factor as:

$$z + 1 = \frac{a_0}{a} \quad (14)$$

More on the scaling factor: since we now know that wavelength is proportional to a , it means that the energy of a photon is proportional to a^{-1} . In the background radiation, the CMB, we can show (beyond the scope of this) that the number of photons is conserved. This means that as the universe expands, the density of photons will be proportional to a^{-3} because of volume expansion. This means that the energy density of the CMB is proportional to a^{-4} . Finally, by Stefan-Boltzmann, we know that the energy density is proportional to T^4 . This means that the temperature of the CMB is proportional to a^{-1} . In other words, when the universe was twice as small as it is now, it was twice as hot!

The geometries we mentioned before can be quantified by the so-called curvature value, or κ . A $\kappa = -1$ value indicates an open universe, $\kappa = 0$ is flat, and $\kappa = 1$ is a closed universe. An open universe will continue expanding forever, eventually ripping its own atoms apart from the expansion. A closed universe will eventually cease expanding and collapse upon itself at the end.

Based on the different values of curvature, we can draw graphs of the scaling factor and how it develops in time with the different geometries. Such a sketch is given in Figure 6.

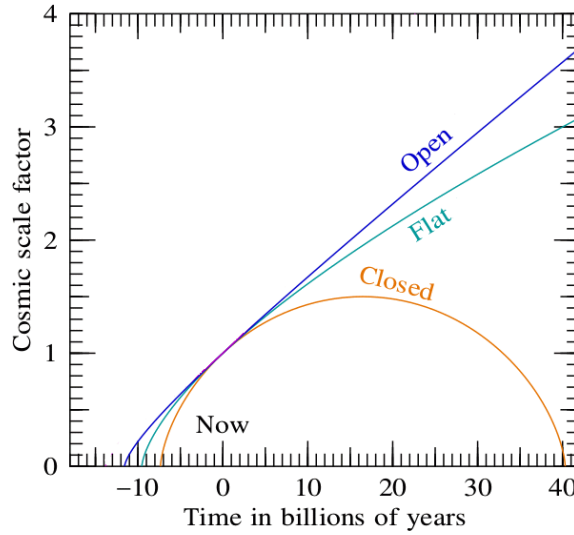


Figure 6: The three possibilities of development of the scale factor, depending on the curvature.

For an open universe, the scale factor is approximately linear in time, or $a \propto t$. For a flat universe, it has been found that $a \propto t^{\frac{2}{3}}$ is the function the scale factor will tend to. Currently, no matter the type of universe, the scaling factor is approximately directly proportional to time, i.e. $a \propto t$.

3.4 The Hubble-Lemaître Law

Now we have laid the groundwork necessary to write out the actual full form of the Hubble-Lemaître Law. Using the scaling factor described above, we can define the so-called proper distance in space. This distance, which we'll denote with d is the distance you would find by taking one time instant and measuring with a ruler. Since it depends on the scaling factor, it changes with time. The second distance of importance is the so-called comoving distance, which is a distance that has compensated for universe expansion and is constant. We denote it with r .

The formula relating the two is:

$$d(t) = a(t) \cdot r \implies r = \frac{d(t)}{a(t)} \quad (15)$$

where a is the scaling factor as described previously. By differentiating both sides of this equation with respect to time, we get the formula:

$$\dot{d} = \dot{a} \cdot r \quad (16)$$

The derivative of the distance is the velocity, just like we usually have in physics! Using the expression for the comoving distance from (15), we can write

$$v = \frac{\dot{a}}{a} \cdot d \quad (17)$$

where v in the expression above denotes the recession velocity. Notice the dot above the scaling factor - this is just shorthand for the time derivative. Any quantity that is differentiated with respect to time can be written with a little dot above the letter (or two dots if you differentiate twice). As an example, acceleration is the derivative of velocity, which is the derivative of displacement, or in other words:

$$a = \dot{v} = \ddot{d} \quad (18)$$

Notice how the equation above has the form of $v = H \cdot d$ which you have seen many times previously. In essence, we have that the Hubble constant is not actually constant! To be more precise, we define H as the Hubble parameter, and realise that it is actually time dependent as well!

The value of $H_0 = 73 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$ is actually the value of the Hubble parameter calculated for today. In cosmology, we often use the subscript 0 to denote something relating to the present day! The age of our universe can actually be approximated by the reciprocal of the Hubble parameter for today, i.e.

$$\text{age of universe} \approx \frac{1}{H_0} \quad (19)$$

That finally leads us to the form of the law you have been taught so far:

$$v = H(t) \cdot d \quad (20)$$

$$t = t_{\text{now}} = t_0 \implies v = H_0 \cdot d \quad (21)$$

There is actually another form of this law, which I myself prefer, and it involves using the cosmological redshift. The above equation actually comes from a hidden assumption that $v \ll c$, so that $cz \approx v$. If we swap this back into Hubble's law, we get a form that also works for relativistic velocities.

$$d = \frac{cz}{H_0} \quad (22)$$

This allows us to determine immense distances in space simply by knowing the redshift!

3.5 The First Friedmann Equation and Density

Alexander Friedmann developed two equations to describe a homogenous, isotropic universe, both following from relativity and being far outside the scope of this handout to derive. However, some consequences and properties of the first equation are still something you should be aware of. The first Friedmann equation has the following form, and is there to help us solve for the scaling factor:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \cdot \rho(t) - \frac{\kappa c^2}{a^2} \quad (23)$$

Notice how the curvature parameter appears here as well, but also one more new, but extremely important one - the time dependent space density. On large scales, the density is uniform, but still dependent on time. For a flat universe with $\kappa = 0$, we have what's called the critical density. Remembering that $H_0 = \frac{\dot{a}}{a}$, this density can be expressed as:

$$\rho_{0,c} = \frac{3H_0^2}{8\pi G} \quad (24)$$

This means that it is exactly the density that decides the geometry of the universe! If the density is exactly the critical density, the universe is flat. If the density is bigger than the critical density, we have a closed universe, and if it's smaller, we have an open universe.

We have a way of quantifying this as well, using what we call the density parameter, such that:

$$\Omega = \frac{\rho}{\rho_c} \quad (25)$$

or, in words: the density parameter is the ratio of the observed density to the critical density.

Equation (23) is actually the less common form of the equation. More commonly, you will come about the following:

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{0,R}a^{-4} + \Omega_{0,M}a^{-3} + \Omega_{0,k}a^{-2} + \Omega_{\Lambda} \quad (26)$$

where all the different omegas are different density parameters. The densities in question are:

- R stands for radiation - this is the energy density of photons, just like we discussed when we first introduced the scale factor
- M stands for mass - this is what we refer to as baryonic density
- k stands for curvature, and the corresponding density is the spatial curvature density - explaining this is a bit beyond our paygrade, so we're happy with just knowing that it exists
- Λ denotes dark energy, and is a perfect segue into the next section for this handout!

3.6 Dark Energy

Everything we have mentioned thus far has relied on an assumption of a matter-dominated universe. You might have heard pop-scientific articles and modern media mention dark energy quite a lot, and it is a topic of constant discussion and research in today's times.

Observations from the last century have shown that the universe is expanding in an accelerating fashion. This meant that there had to be *something* working repulsively, or opposite to gravity. This something is postulated to be *dark energy* and is a building block of the popular Λ_{CDM} model. CDM here stands for Cold Dark Matter, and we think that our known matter makes up only 5% of the universe, while dark matter and dark energy make up 25% and 70%, respectively.

Based on this model involving dark matter and energy, the universe has gone through three phases in its lifecycle so far:

- radiation-dominated era where electromagnetic radiation was the most important form of energy and when more complex matter started getting produced - Friedmann's equations suggest that the scale factor developed according to $a \propto t^{\frac{1}{2}}$, and while matter was made, radiation energy density fell according to a^{-4} like we saw before.
- matter-dominated era during which galaxies, stars and everything we know of matter today formed
- cosmological-dominated era in which we are now, where dark energy and dark matter dominate

There are also two phases before the radiation-dominated phase, but these ended approximately 1 s before the Big Bang. These eras are the hadron era, which lasted for about 10^{-4} s and during which protons and neutrons were made. Right after that followed the lepton era which lasted up to 1 s and during which electrons were made.

Look at (26) again - there is no scale factor term next to the density parameter of dark energy! This means that ever since the beginning of the universe, the dark energy density has remained exactly the same. Since we know that our universe is expanding, for dark energy density to remain the same, somehow more and more dark energy needs to be created as the universe expands.

Just to underline the difference between the two, dark energy is what drives the accelerated expansion of the universe, while dark matter is what makes up most of the mass of galaxies.