



THE NORWEGIAN ASTRONOMY AND ASTROPHYSICS
OLYMPIAD COMMITTEE

IOAA NOTES

Part 5: Stellar Astronomy

Author:
Đorđe Veljković

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1 Stellar Physics Basics

This handout will talk about stellar properties that are necessary to understand how stars behave, how they've come about, how they end and everything in between. We assume that you have understanding of light as a wave, as it is central in the field we call "spectroscopy". Spectroscopy is the analysis of spectra (in other words, emitted and/or absorbed radiation) with purposes of gaining understanding of e.g. chemical composition and movement.

Throughout the handout, and in many problems you'll encounter later on, the term *blackbody* will come up. We regard stars as blackbodies in the sense that they don't reflect or scatter radiation, but only absorb and completely reemit it. This is a theoretical approximation, but often a valid and useful one for the purposes of the Olympiad. Blackbody radiation produces continuous spectra, which is one of the three types of spectra defined by Kirchhoff. You can see all types of spectra in Figure 1. The emission spectrum is observed with hot gases at low density and low pressure, while the absorption spectrum is observed when light from a continuous spectrum passes through a cold low-density gas.

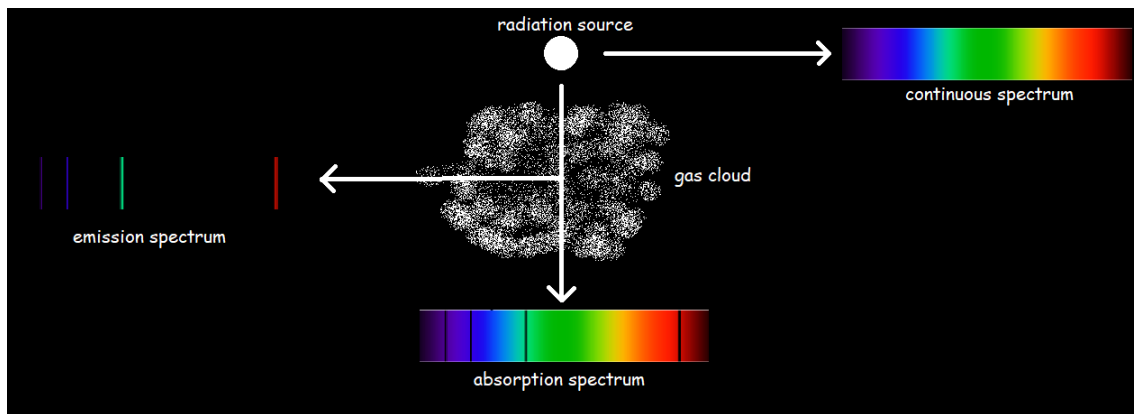


Figure 1: Three different types of spectra used in spectroscopy. The emission spectrum is only observed with hot, thin gases at low pressure.

Another important observation serving as the basis for Stefan-Boltzmann's Law and Wien's Displacement Law is the fact that the intensity of blackbody radiation only depends on the wavelength observed, and the temperature of the blackbody. This property is described using Planck's Law, which defines the intensity as follows:

$$B(\lambda; T) = B_\lambda(T) = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \quad (1)$$

You don't need to know this formula, only realise that everything except λ and T is a constant. In astronomy, we use the usual units of nm when we talk about wavelengths, but you might also come about Ångström, denoted by Å, where $1 \text{ nm} = 10 \text{ Å}$, named after the Swedish physicist Anders J. Ångström who is regarded as the father of spectroscopy.

1.1 Stefan-Boltzmann's Law

Starting from this assumption, we can derive Stefan-Boltzmann's Law; the derivation is far outside of the scope of IOAA. It is done by integrating (1) from 0 to ∞ , which involves an extremely complex integral, so we are satisfied with simply presenting the equation describing the law:

$$F = \sigma T^4, \quad \sigma = 5.67 \cdot 10^{-8} \frac{\text{W}}{\text{m}^2\text{K}^4} \quad (2)$$

Although simple at first sight, two things need further underlining. First of all, the F in the formula denotes what we call the radiation flux density, or in other words, how much power is

radiated per unit of area. One thing to keep an eye out for is that the constant σ is given in a unit such that the temperature that you plug in *must* be in Kelvin.

The luminosity of a blackbody, L , is then simply the flux multiplied by the surface area of the radiating object. In other words, flux doesn't depend on the shape, but luminosity does! In our case, the area is usually approximated as the area of a sphere, or $4\pi R^2$, so that:

$$L = 4\pi R^2 \cdot F = 4\pi R^2 \cdot \sigma T^4 \quad (3)$$

The temperature in both of these formulae is what we call the star's *effective temperature*. Now, the further we move away from the star, the lower the flux will be. If we are at a distance r from a blackbody, the flux at that distance would be:

$$F' = \frac{L}{4\pi r^2} = \frac{4\pi R^2 \cdot F}{4\pi r^2} = \left(\frac{R}{r}\right)^2 F \quad (4)$$

Now, from observational astronomy, you might remember that the angular diameter of a star observed in the sky can be expressed as $\alpha = 2 \cdot \frac{R}{r}$, so $\frac{R}{r} = \frac{\alpha}{2}$. This means that if we measure the flux (which we can do!) and know the angular diameter of the star (which we can observe!), we can find the effective temperature!

1.2 Wien's Displacement Law

If we differentiate (1) with respect to λ and setting this equal to zero, we find the wavelength at which a blackbody with a certain temperature radiates with the strongest intensity. This is also known as Wien's Displacement Law and is described by this formula:

$$b = \lambda_{\max} T, \quad b \approx 2.898 \cdot 10^{-3} \text{ m} \cdot \text{K} \quad (5)$$

Notice that b is a constant value! The displacement part of the full name of the law comes from the fact that as temperature changes, the peak of the radiation intensity curve moves! Check out how it does at <https://www.geogebra.org/classic/dwpqj3jy>. The total intensity radiated from the body in question is equal to the area under the curve, but what we're looking at is the peak, or where it's strongest!

2 Spectroscopy

Spectroscopy is the science of analysing spectra, looking at how spectral lines behave under different conditions and interpreting what that tells us about whatever we're observing. It's not only used in astronomy, and is an essential building block in chemistry, medicine and many other things.

In common for all fields of science utilising spectroscopy is that it's used to tell us about the chemical composition of things. If you remember Figure 1, when we pass light of a continuous spectrum through a cold gas, on the other side, we will observe lines. You can view these lines as a sort of unique ID of the composition of the gas, as each and every chemical element will have its own unique effect on the spectrum. This aspect of spectroscopy helps us see exactly what different stars are made out of.

On astronomical scales, we observe the Doppler effect, which we assume you are familiar with. A consequence of this effect is shifting of spectral lines observed from a far away star. The shift is either red if the star is moving away from us, or blue if it is moving closer to us. The shift in wavelength is given by the following formula:

$$\Delta\lambda = \lambda - \lambda_0 = \frac{v}{c}\lambda_0, \quad (6)$$

where v is the velocity of the star along our line of sight. A negative velocity obtained by this formula would indicate that the star is moving towards us, and a positive velocity indicates the opposite.

With this, a natural question arises: if we see an absorption line at a certain wavelength, how can I know whether it's there because of a Doppler shift (and how much has it shifted?) or because that's exactly where the rest wavelength is? Lab experiments have been performed on Earth on very many elements and their combinations, meaning we know how their spectral lines relate to each other, so we have a resting reference on Earth. Using this resting reference, if we see the same pattern in a spectrum coming from a far-away star, we can exactly quantify the Doppler shift and thereby the velocity of the star!

Another thing we might observe with spectra due to movement is so-called spectral line broadening. Since stars are very big in diameter, there is a difference in the light we observe from the part of the star rotating towards us and the part rotating away. The part rotating towards us will shift the light towards the bluer end of the spectrum, and the part rotating away from us will shift it towards the redder end of the spectrum. When we observe light coming from a star, we will get light from both of these sides, and the spectral line will appear wider!

Another phenomenon you should be aware of that our Sun exhibits quite a lot is spectral line *splitting*. This occurs as a consequence of strong magnetic fields which affect the energy levels of atoms. A static magnetic field will split a spectral line into three, and the two new "copies" will be shifted symmetrically from the zero point both towards the blue and red ends. This is useful for measuring magnetic field strengths from astronomical objects, and we can notice it quite a lot with our own Sun when there is lots of sunspot activity.

Although we usually present spectra to you as a block of colour going from violet on the left to red on the right, the reality is that we look at the graph of flux intensity against wavelength, i.e. the same graph that might be presented to you in a problem involving Wien's Law. Figure 2 shows the difference between the idealised graphs you usually get presented and an actual spectrum we see from a star. All the dips in the plot correspond to an absorption line, while any peak would correspond to an emission line.

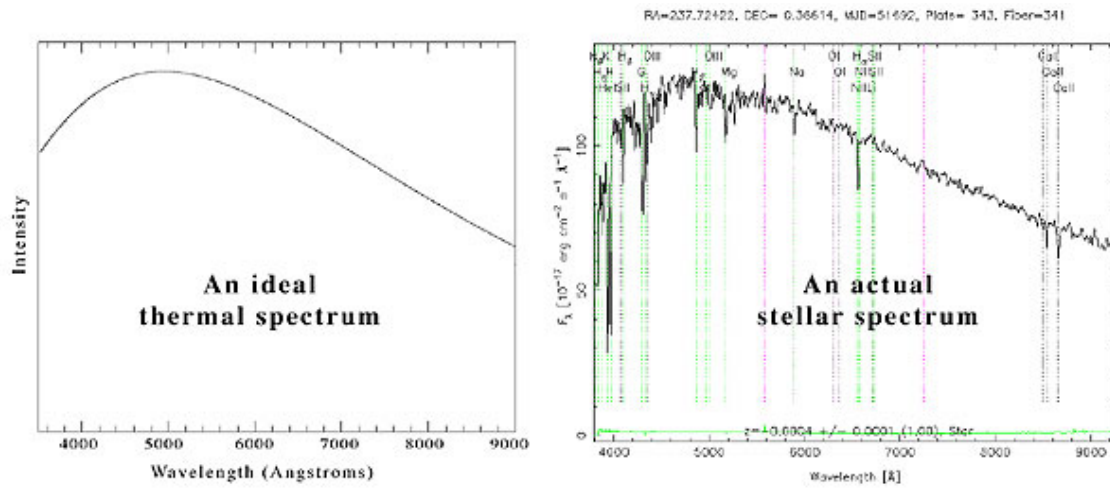


Figure 2: An ideal spectrum next to an actual spectrum from a star. The dips in the plot correspond to absorption lines.

The dips in the spectrum aren't always the same. Some stars that have the same elements in their composition will have stronger dips than others, which corresponds to greater absorption, maybe indicating more of that exact element in the composition. Even though the stellar spectrum is obviously not ideal, it is still continuous enough to allow us to apply Wien's Law!

3 Magnitudes

3.1 Apparent vs. Absolute

And we finally get to those dreaded magnitudes that everyone mixes up so often. The observational handout focused on applying apparent magnitudes to observing stars, but here, we will see mathematically what lies behind these and put some meaning behind the numbers.

In astronomy, we differ between apparent magnitude and absolute magnitude. Apparent magnitude is, as suggested in the previous handout, used more for a subjective measure of how bright something is in the sky. It is relative to us as observers on Earth, but we do regard it as a constant number, no matter where on Earth you are, assuming clear observing conditions. The definition of magnitudes comes from flux which we have looked at previously.

For apparent magnitude, the following relationship holds for the difference in magnitude between two stars:

$$m_1 - m_2 = -2.5 \log_{10} \left(\frac{F_1}{F_2} \right) \quad (7)$$

The apparent magnitude of any single star is found using a reference star. For apparent magnitudes, the reference star is Vega and it is defined to have magnitude $m = 0$. Rearranging the above to solve for the flux ratio gives:

$$\frac{F_2}{F_1} = 100^{\frac{m_1 - m_2}{5}} \quad (8)$$

which tells us that a magnitude difference of 5 means that one of the stars is 100 times brighter than the other. Note that *lower* magnitude means that the object is *brighter*.

Absolute magnitude is a more standardised way of measuring how bright a celestial object is, and doesn't have an observer on Earth as reference. Rather, absolute magnitude is a measure of how bright something would be at a distance of 10 parsec. Since the distance is standardised, if we use the flux-luminosity relationship given by (4), we can write:

$$M_1 - M_2 = -2.5 \log_{10} \left(\frac{L_1}{L_2} \right)$$

which is the same form as the apparent magnitude relation, only here we use the luminosities that are completely independent on how far we are from the object in question. In problems, you will often be asked to express the luminosity of a star in solar luminosities, so you would usually set $M_2 = 4.83$ and $L_2 = L_{\odot}$.

3.2 Colour Index

You might or might not have heard the word "bolometric" being used in the context of magnitudes. If something is given as "bolometric", it means it was derived from a measurement across all wavelengths. In reality, we don't always measure things across the entire spectrum, so we have implemented the so-called UBV system. In this system, absolute magnitudes are measured through ultraviolet, blue, and visual filters. Differences between pairs of these magnitudes are referred to as "colour index", and can be used to classify stars. Newer systems also use red and infrared filters, but we won't go into these here.

The U-B colour index is the difference between the ultraviolet magnitude and the blue magnitude, and the B-V colour index between blue and visual, and these are the ones used most often. If we plot these indices against each other, we get a so-called colour-colour diagram. Colour-colour diagrams allow us to compare stellar properties across clusters, even if we don't know the distance to them, and can be very useful for calibrating instruments on what we already know.

Now, how do we get the bolometric magnitudes if we only measure with a couple of filters? The answer to that is bolometric correction, which is a factor we add to the visual magnitude to get the bolometric magnitude. This number is most often negative and very large, and there is a

problem with the fact that there have been disagreements on how exactly this value should be standardised and what the zero point is. As of 2015, the standard is to use an arbitrarily-set bolometric magnitude for the Sun and base the rest on that. If you need to take bolometric correction into account, you will be given the necessary factors.

4 Stellar Life

Buckle in for a story about the journey of a star, from how it's born to how it can eventually end up. This understanding is necessary when we delve into HR diagrams later. Figure 3 shows how a star's life might go on depending on several factors, most notably their size. This is a simplified view on stellar evolution which is an extremely complex process, but it should give you a more than good enough starting point for understanding how stars develop. Some of the phases are called branches, and you will realise why once we explain it through HR-diagrams. For now, just gloss over the name, and we'll come back to it in the next section.

Stars start from enormous and dense gaseous molecular clouds that become unstable and collapse under their own gravity. These clouds are not homogenous and have regions that are more dense, which are candidates for forming stars. The collapse is not instantaneous, rather taking a couple of millions of years, during which gas and dust can accumulate around these denser regions in a process we call accretion and form protostars (proto is Ancient Greek for "first"). Jets of material are also shot away from protostars, and this is how we actually detect them, since the winds are very characteristic for such stars. This is the first stage in a star's development, and protostars are not yet capable of having fusion processes going on in their cores.

From a protostar, there are four different pathways that can be taken. The most interesting one to begin with is how a protostar ends up on the so-called main sequence. With more gas and dust accumulation, mass grows, and therefore the gravitational force which shrinks the protostar. Shrinkage leads to increase in density and core temperature, and finally hydrogen begins to fuse. The gravitational force is balanced by the radiation pressure from the fusion reactions and the star has reached a hydrostatic equilibrium. If the star is large enough, above $1.3M_{\odot}$, hydrogen fuses directly into helium in a process we call the CNO-cycle. Below this mass, we observe the proton-proton chain, still ending up with helium, but having deuterium as a middle step.

How long a star spends on the main sequence is directly related to the mass of the star. The more massive the star, the shorter its life will be. We can actually describe this with an equation that we choose not to derive, but present here for completeness' sake:

$$\frac{t_A}{t_B} = \left(\frac{M_B}{M_A} \right)^{2.5} \quad (9)$$

As a reference value, we often use the stellar lifetime of our Sun at about 10 billion years, with a mass of about $2 \cdot 10^{30}$ kg. The intuitive reason behind bigger stars living shorter is that they need more radiation pressure to counteract the inward gravitational force, and as such must consume fuel faster to generate more radiation.

Once the main sequence star has burnt up all of the hydrogen in its core, it moves to the subgiant branch, where hydrogen in the outer shell becomes the main source of fusion. Because of this, the star starts to expand and cool, and the core is left to be idle and not perform more fusion.

The red giant stage is very similar to the subgiant stage, only that the star has been completely depleted of hydrogen, so there is no more fusion, and it starts to collapse under its own gravity. As it collapses, the pressure on the core will increase, and with it the temperature, which will enable fusion of helium at one point. Once helium fusion has started, there will be a sudden increase in radiation pressure, and the star will expand. Smaller stars will exhibit a helium flash wherein they will release pressure built up from the helium fusion in a span of a couple of minutes. That's a lot of energy!

The next stage is the horizontal branch, where the star is stable again and fuses helium using the so-called triple-alpha process. In this process, helium fuses into beryllium, which fuses with another helium atom to create carbon!

Soon after, the star moves to the asymptotic giant branch where the core dies down again, but this time it has carbon and oxygen. Just like hydrogen came from the shell in the subgiant branch, helium and hydrogen come from the shell during this stage. Once they get to the core, they cause temporary energy increases which we refer to as thermal pulses.

Now, the only thing left is carbon, which many stars aren't massive enough to come close to the temperatures needed to fuse it. Such stars start contracting in an unstable manner, blowing hot gas away and creating a planetary nebula. Don't be fooled by the name - these nebulae have nothing to do with planets, and were named based on their planet-like shapes when observed through telescopes!

After a while, the contractions stop, and the remainder of the core is a white dwarf that is unable to fuse any elements. The equilibrium to gravity in white dwarfs comes from electrons between close atoms exerting immense pressure on one another. It is hypothesised that this process can end when no more energy is left and the white dwarf turns into a black dwarf.

The process we have described here is what will happen to our Sun and how it will end its lifetime. It's still got more than 5 billion years to go, so that's a problem for another day (or eon)!

Now, let's look at some other branches! A white dwarf can under certain conditions, the most important of which is crossing the Chandrasekar limit of $1.4M_{\odot}$, undergo a supernova and turn into an incredibly dense and compact neutron star! The neutron stars receive lots of energy from the supernova that created it, which can cause it to rotate at a frequency of up to 700 times a second! Such rotating neutron stars are called pulsars, and the pulsing we see is due to magnetic fields created by the rotation. The reason for the great density is that they form with incredible gravitational pressure that combines protons and electrons into neutrons. Neutrons can be more tightly packed as they don't create any empty space, like the proton-electron interaction in a normal atom does.

If the core is larger than three solar masses, it will have passed the Tolman-Oppenheimer-Volkoff limit (or just TOV for short), and the core will continue to collapse into a black hole! All of the above happens with more massive stars that can go up to $40M_{\oplus}$ and are able to fuse beyond carbon, all the way up to iron.

The supernovae mentioned come in a few different flavours as well, most notably type I and type II, with further subdivisions. The difference between the types is that type I don't have hydrogen and we don't see hydrogen absorption lines, while we do with type II. Type Ia supernovae occur as a result of fusion on dwarf stars' surfaces, while all the others are due to core collapse, and also exhibit silicone absorption lines. Type Ib and Ic do not, and they are caused by very strong stellar winds that blow away the outer shell. With Ic, the winds are so strong that helium is blown away, so helium lines aren't present in these, while they are in Ib.

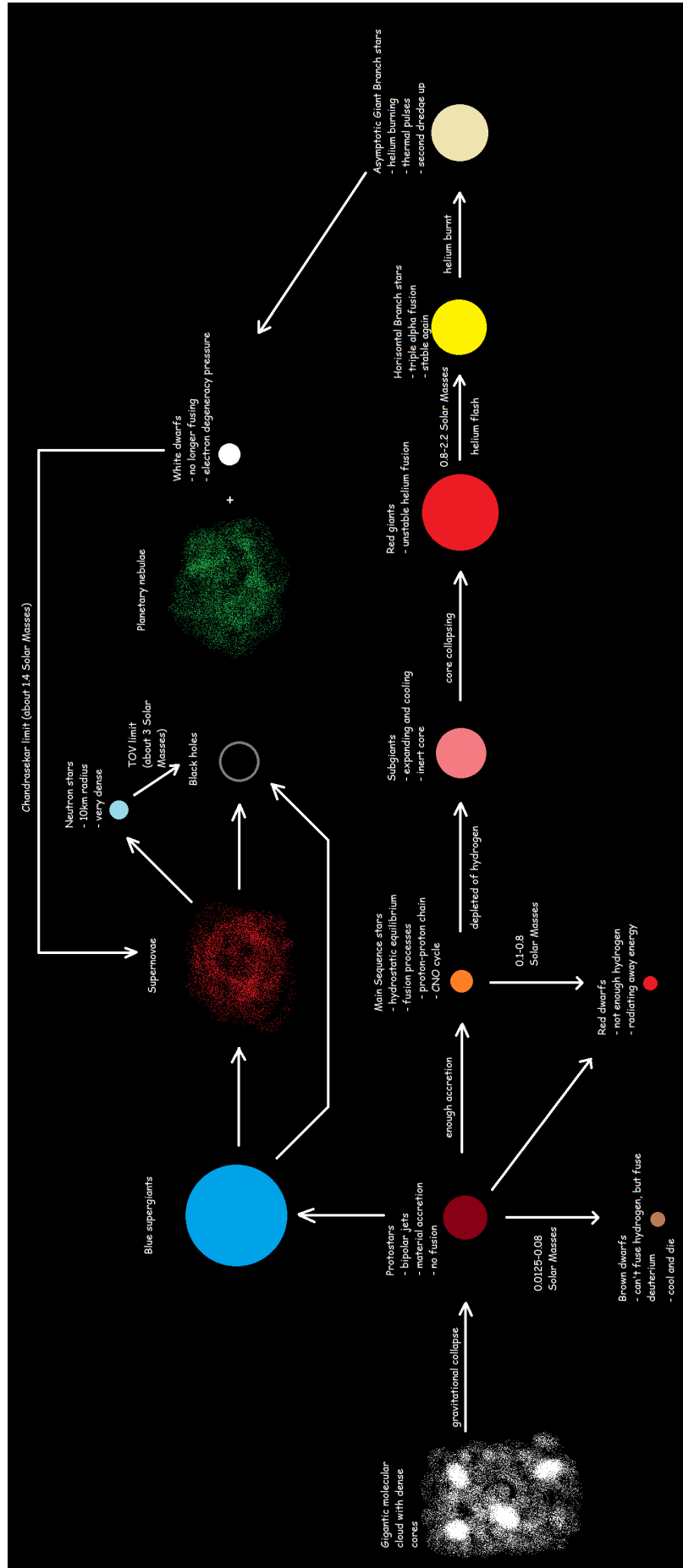


Figure 3: A graphical representation of possible pathways through stellar evolution.

5 Hertzsprung-Russel Diagrams

Figure 4 shows the HR diagram we most commonly present in problems. More descriptive HR diagrams can be found online, but we are keeping it simple here. The diagram reveals several well-defined regions when stars are plotted as points according to some of their properties. Also notice that temperature decreases along the x-axis.

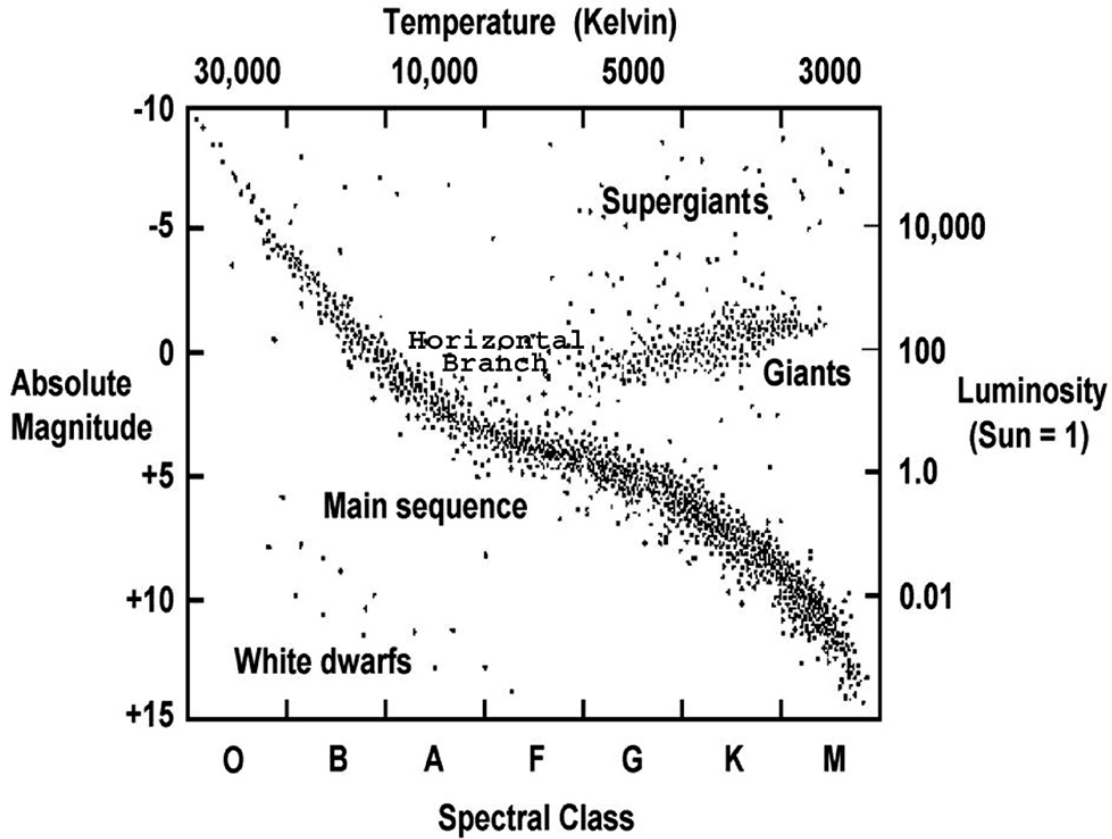


Figure 4: The HR-diagram for Milky Way stars.

The shape stretching across the diagonal of the plot is what we call the main sequence, and is where stars are able to fuse hydrogen. Above the main sequence are stars that can fuse more elements, and below, we have stars that are towards the end of their lifetimes.

In the development, you can see the horizontal branch as moving from the main sequence to the giants. The asymptotic giant branch is not as visible on this diagram, so we provide Figure 5 to visualise the paths easier. The asymptotic giant branch is called like that as it approaches the quality of the red giant branch the further it stretches.

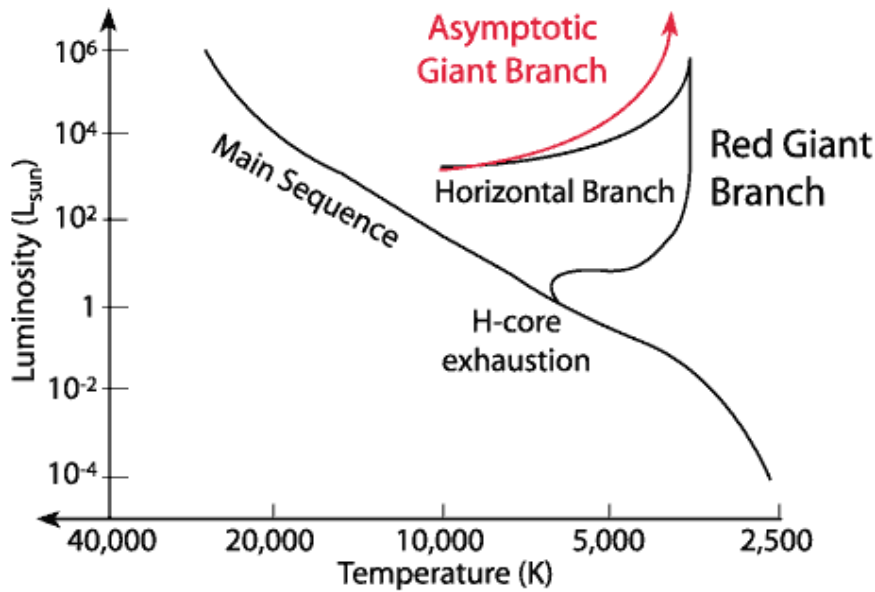


Figure 5: A simplified sketch showing evolution of a star from the main sequence towards supergiants.

Now comes the question of spectral classes. Even though the HR-diagram often has four different axes drawn on it, it is essentially looking at two stellar properties: temperature and luminosity. Luminosity and absolute magnitude are closely intertwined, as are the spectral class and temperature. A popular mnemonic to remember the order of spectral classes is "Oh, Be A Fine Girl/Guy, Kiss Me".

Spectral classes are tightly connected to temperature, but the classification division is done based on the stars' spectral characteristics. O-star spectra have characteristically strong hydrogen and helium lines. This makes sense, as O-stars are the largest and have most of these elements available for fusion. The hydrogen and helium lines get weaker as we progress through classes, and more elements are added, until we finally reach M-stars that have molecular absorption lines!

There are further subdivisions inside on the spectral classes, ranging from 0 to 9 based on temperature. A G0 star would therefore be hotter than a G9 star. Finally, we want to discriminate stars based on luminosity, so we add a roman numeral indicating the luminosity class. From I to V, we get supergiants, bright giants, giants, subgiants and main sequence stars. White dwarfs get a D added on. With such a system, our Sun would be classified as a G2 V star. There are further subdivisions for these luminosity classes, but those are beyond what is needed for the olympiad.

The HR diagram you often see is not unique. We can draw a specific HR diagram for any star cluster or galaxy, and the different ones will exhibit vastly different properties. The distribution of stars can actually tell us something about the age of a cluster. Knowing from earlier in this text that massive stars live shorter lifespans, an absence of O-class and B-class stars in the HR-diagram can indicate that the star cluster is older. We can quantify this using what we call turn-off points.

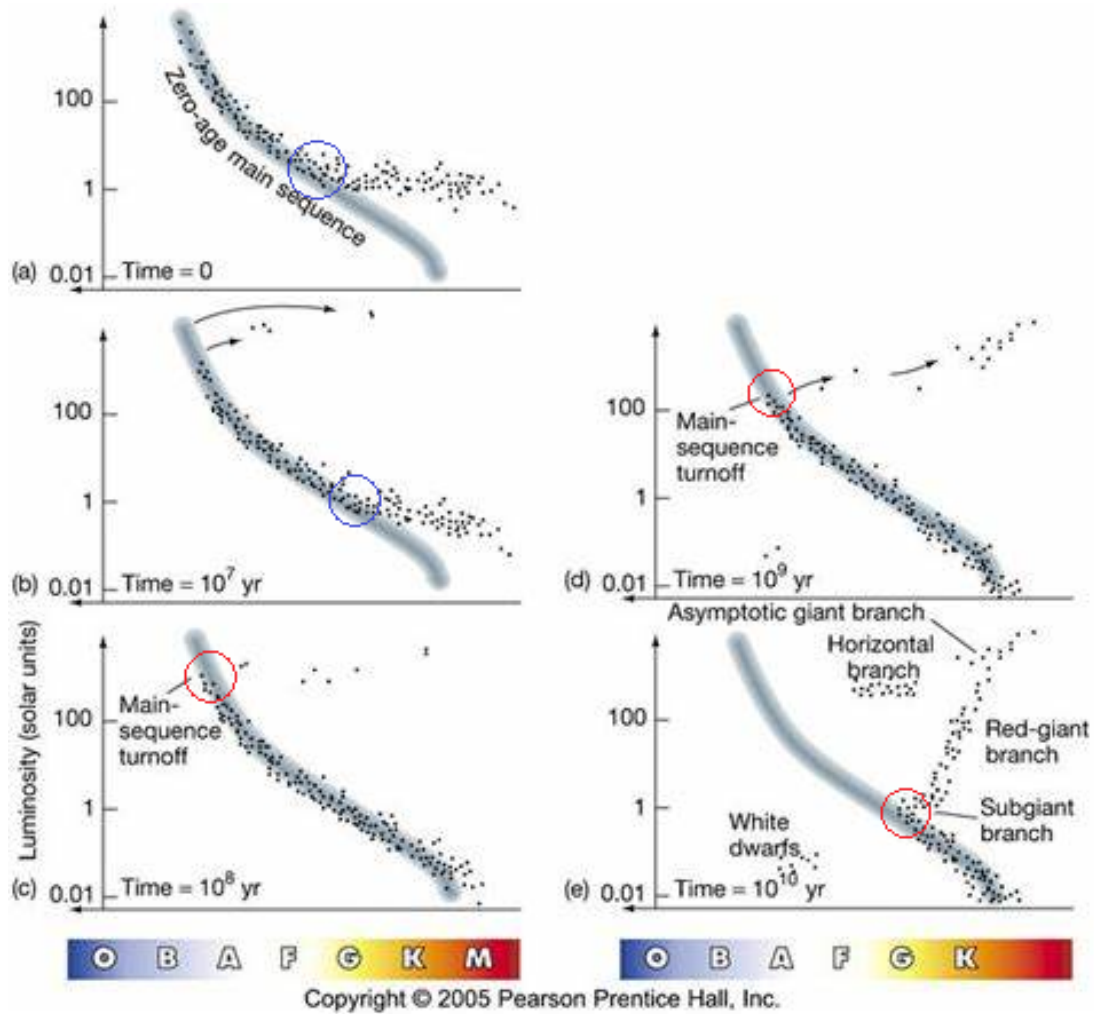


Figure 6: Multiple HR diagrams showing different qualities for different ages. The regions circled in red are the so-called turn-off points; the ones in blue are not.

Figure 6 shows different HR diagrams for star clusters of different ages. From the paragraph above, if we look at the diagrams (c), (d), and (e), we see that O- and B-stars slowly start to disappear and the main sequence branches off at the turn-off point. The turn-off point will be at a specific temperature. From the temperature, we can find the spectral class, and therefore the mass of the star. We can plug the mass into (9), use what we know about our Sun as reference, and find an approximate value of the age of a given cluster using the turn-off point.

Now, be wary of HR diagrams that look like (a) and (b) - although these have clear spots where they branch off of the main sequence, they aren't turn-off points. These clusters are actually so young that most of the big stars are populating the upper left part of the main sequence, but not enough time has passed for stars to get to the lower right part of the main sequence.

6 Variable Stars

Variable stars also exist, but these are just a part of the bigger picture - no star will spend enough time as a variable to be completely defined by its variability. These are difficult to put on HR-diagrams because of the rapid changes in their luminosity, but they do occupy a very specific region we refer to as the instability strip. This strip is marked clearly in Figure 7.

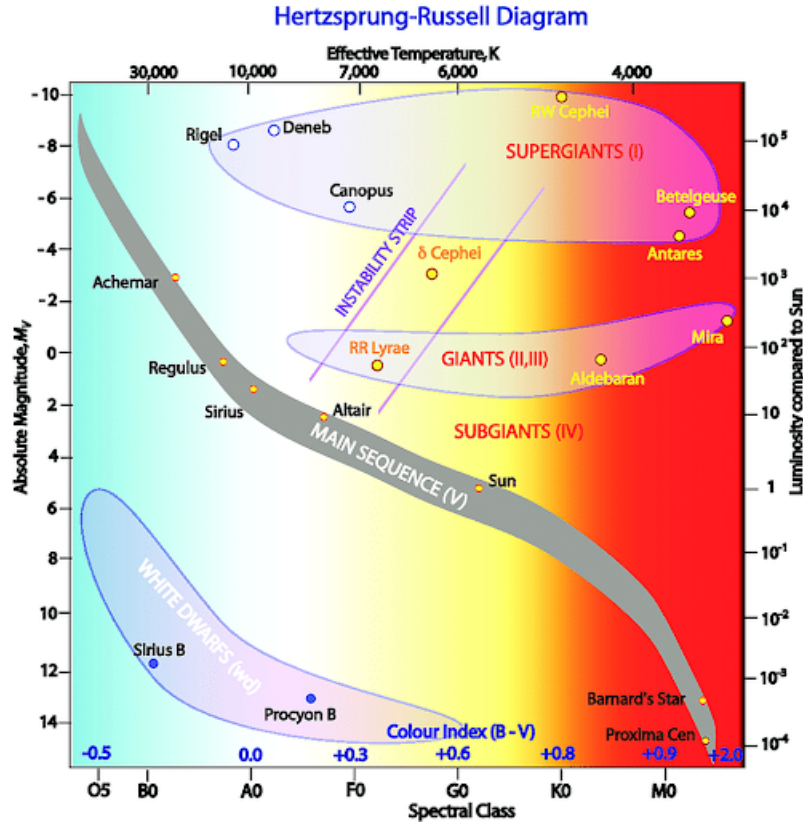


Figure 7: An HR-diagram with the instability strip drawn clearly in.

There are multiple different types of variable stars, the two most interesting ones being:

- cepheids are variable stars with extremely regular changes in brightness that can be described mathematically - we use them as a standard candle when measuring distance, and you can read more about that in the next handout.
- RR Lyrae are very much alike cepheids, but a lot dimmer. They tend to be in globular clusters, meaning they're found at any galactic coordinate and not just around the galactic plane.

These stars are intrinsic variables, meaning that they actually experience surface changes that affect how bright they appear to us. Intrinsic variables can be further classified into pulsating variables which we've described above as having very regular cycles, or eruptive variables, which are mostly protostars and other stars that gain/lose matter and have irregular bursts of brightness.

Extrinsic variables also exist, but the variations in brightness come from other sources, for example being eclipsed. Some binaries can be classified as extrinsic variables, and you can read more about them in the next handout.