

Experimental Dissipative Quantum Chaos

Kristian Wold,
Lucas Sá,
Pedro Ribeiro,
Sergiy Denisov



Kristian Wold
Oslo Metropolitan University



Sergiy Denisov
Oslo Metropolitan University



Pedro Ribeiro
Instituto Superior Técnico



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University of Cambridge

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Roadmap

- Quantum Chaos and Random Matrix Theory (RMT)
 - Quantum Chaos is fuzzy, an Empirical Zoo!
 - Properties of matrices produced by different stochastic models
 - Connect with theoretical physical models and real experiments

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 - Retrieval protocol for obtaining the dynamics of quantum systems
 - Uses measurement data, Pauli expectation values, stemming from real hardware.
 - Lets us model quantum circuits as quantum channels.

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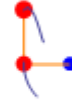
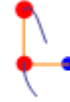
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 - Uses measurement data, Pauli expectation values, stemming from real hardware.
 - Lets us model quantum circuits as quantum channels.
- Results
 - Can determine the presence (or absence) quantum chaos in today's noisy quantum hardware
 - A class of parameterized quantum circuits can be described by a model from RMT called *diluted unitary*
 - Latent noise in quantum hardware is chaotic in nature, but non-chaotic dissipation can be engineered into the quantum circuits and detected in our setup.

Classical and Quantum Chaos

- Classical Chaos -
Initial perturbation ->
large evolved deviation

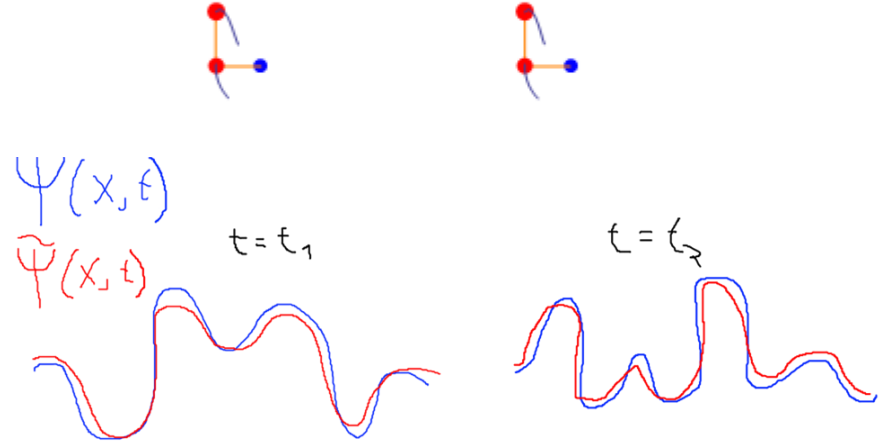
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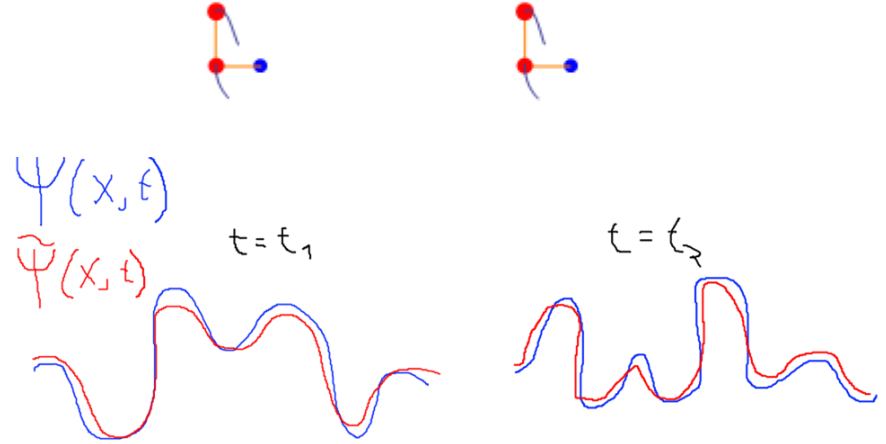
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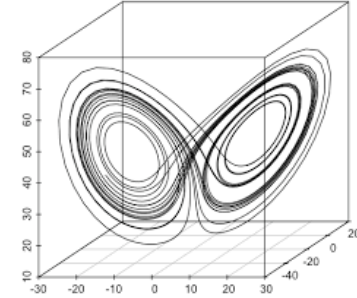
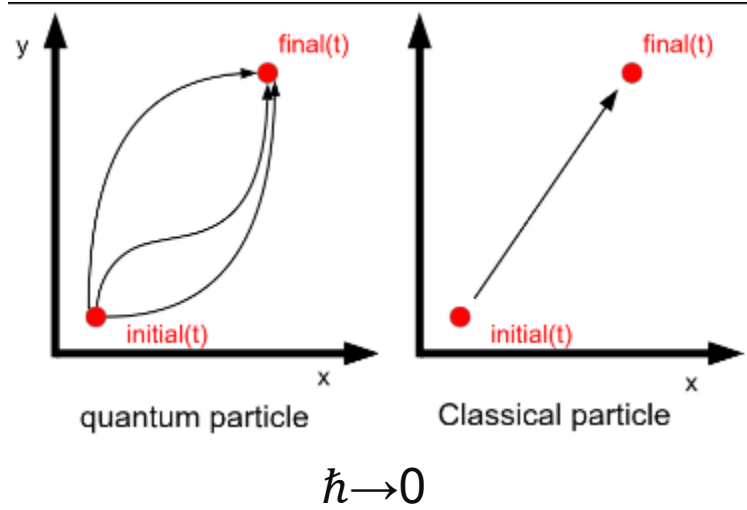
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- Classical phenomenon
emerge from quantum mechanics?
Chaos enters QM somewhere other
than on the levels of amplitudes.



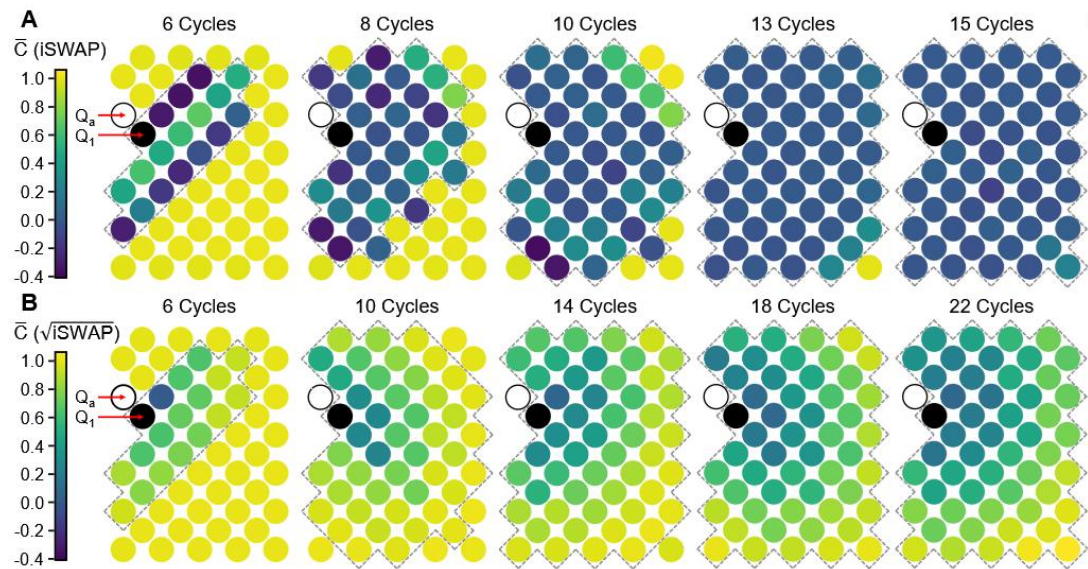
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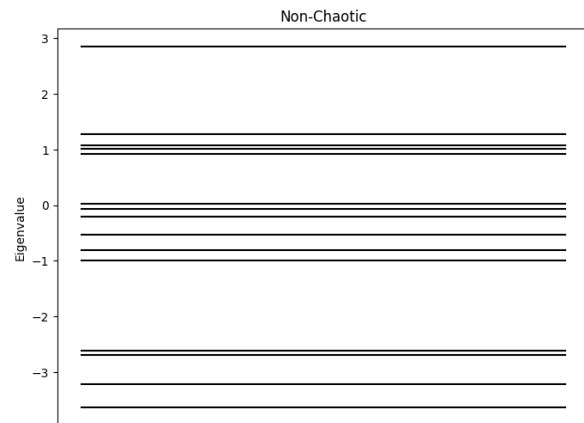
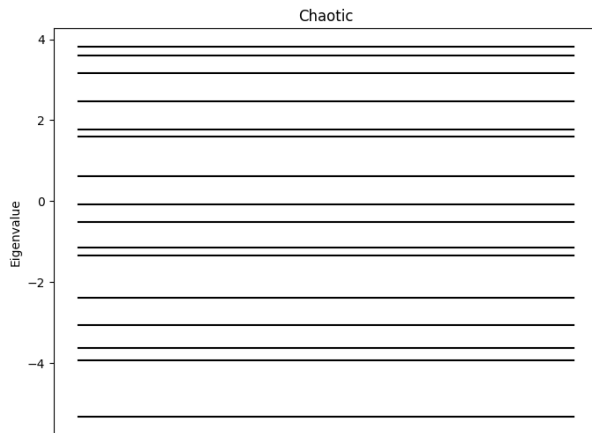
Yu Chen, Google, 2021, "Information Scrambling in Computationally Complex Quantum Circuits"

Ways to detect Quantum Chaos

- Classical Limit Quantum Chaos (Bohigas–Giannoni–Schmit)
- Information Scrambling
- Spectral Chaos - statistics on eigenvalues!

$$H(\vec{\theta}) \rightarrow \{E_1, E_2, \dots, E_n\}$$

$$s_n = E_{n+1} - E_n \sim p_{DW}(s) = \frac{\pi s}{4} e^{-\pi s^2/4}$$



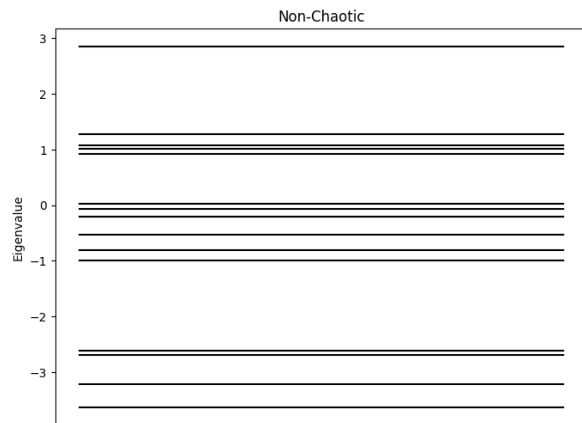
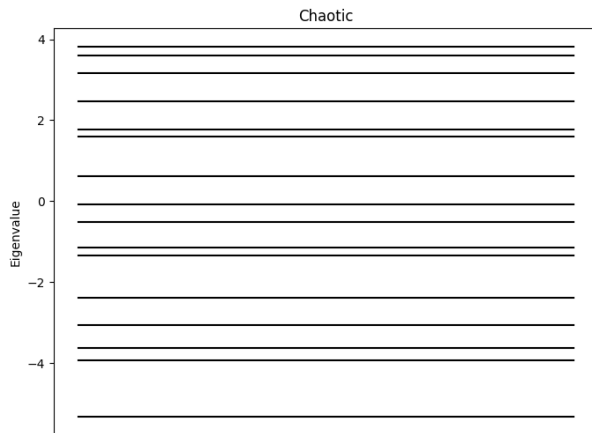
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- Information Scrambling
- Spectral Chaos - statistics on eigenvalues!
- Strength:
Can be used on systems without classical limit

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Random Matrix Theory

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Random Matrix Theory

- Random Matrix Theory investigate (spectral) properties of stochastically generated matrices.
- In general, a spectrum $\{\lambda_n\}$ of a matrix H relates to its elements in a very complicated way.
- Remarkably, can be computed analytically when the elements follow special stochastic models. In particular, the Gaussian Orthonormal Ensemble (GOE).

$$\det(H - \lambda I) = 0$$

$$G_{ij} \sim \mathcal{N}(0, \frac{1}{\sqrt{d}})$$

$$H = \frac{G + G^T}{\sqrt{2}}$$

$$P(\lambda_1, \dots, \lambda_N) = \frac{1}{Z_N} \prod_{i < j} |\lambda_i - \lambda_j| \prod_j e^{-\frac{1}{4}\lambda_j^2} d\lambda_j$$

Random Matrix Theory – Energy Spacing

- With a joint probability for the eigenvalues of a GOE system, the energy spacings can be computed

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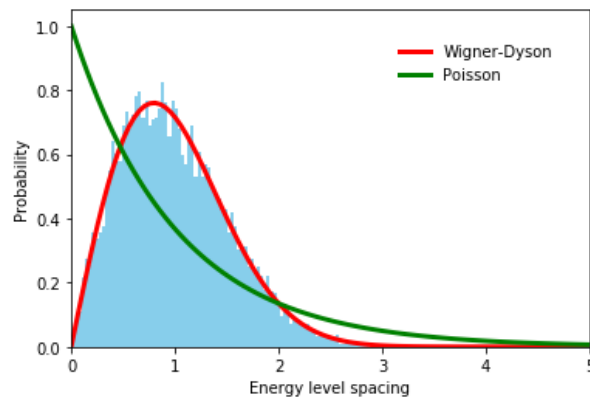
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Random Matrix Theory – Energy Spacing

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- Wigner-Dyson exhibit distinct eigenvalue repulsion, opposed to uncorrelated Poisson statistics

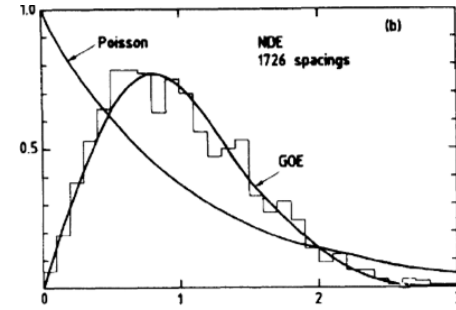
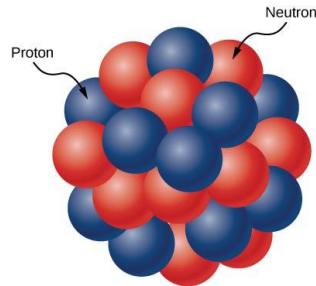
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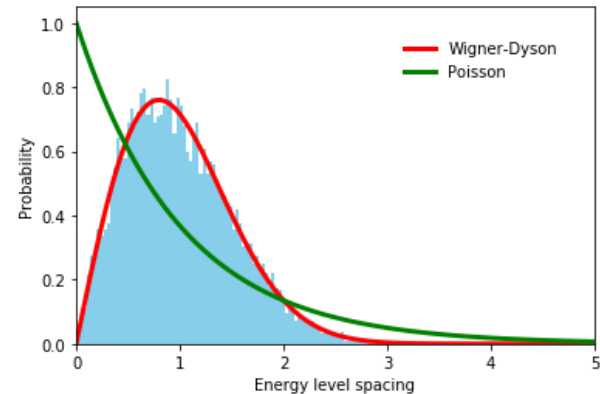
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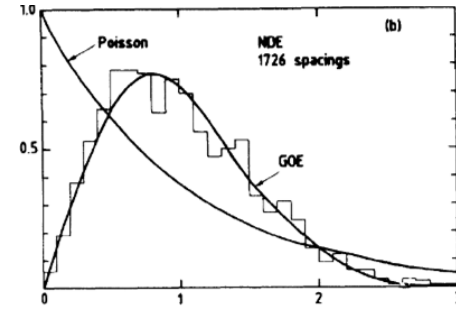
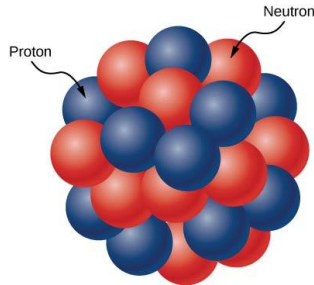
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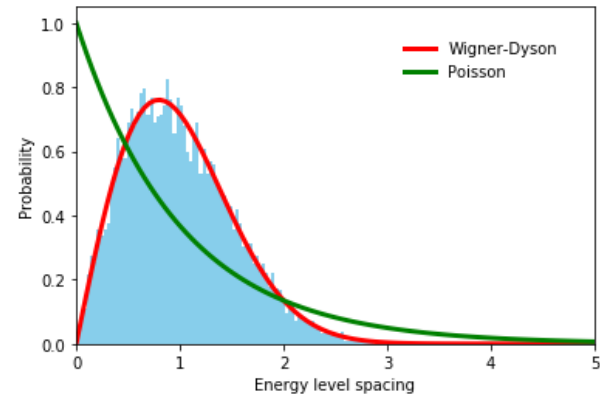
Random Matrix Theory – Energy Spacing

- Leap of faith: Check energy spacing of some real physical system! Surprisingly good correspondence.
- Not a priori obvious. A nuclear Hamiltonian is something very specific. Why should it share characteristics of something very generic?



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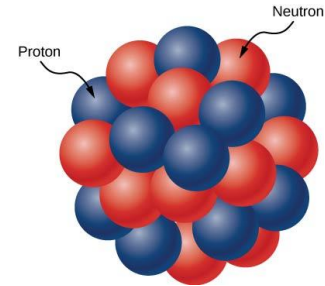
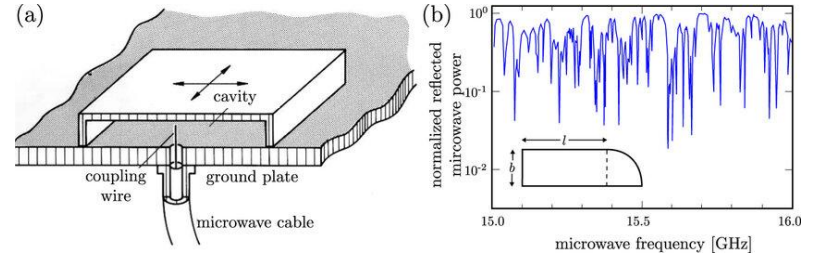


Experimental Quantum Chaos

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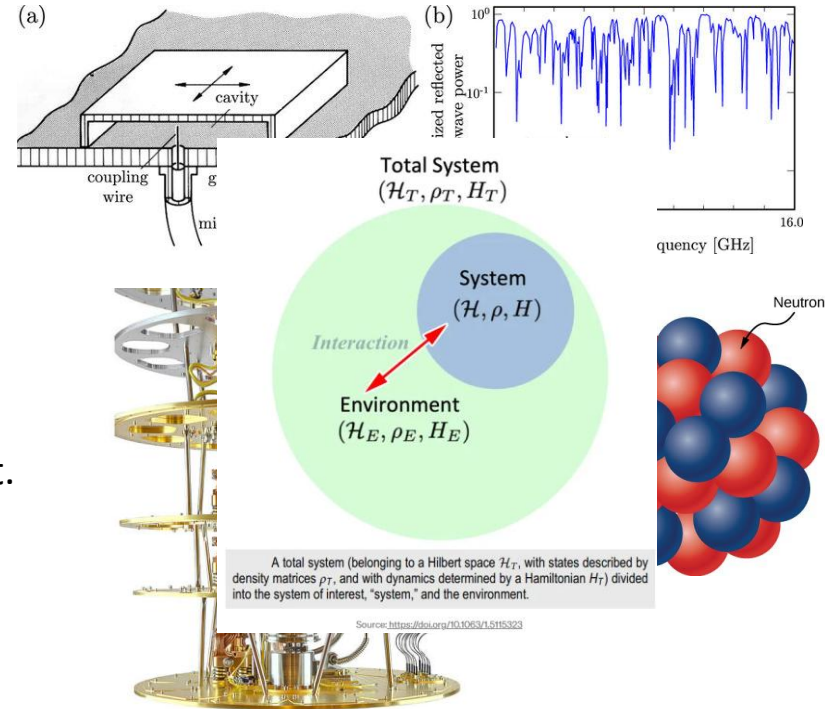
Natural: nuclei

Engineered: microwave Billiards and quantum computers.



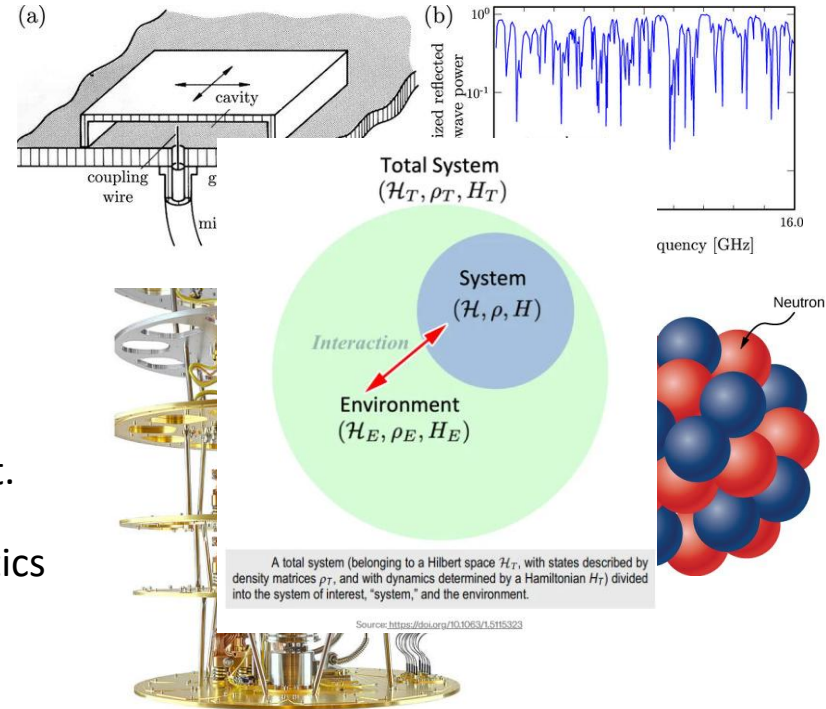
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- What systems to investigate? What characteristics to look for?



Tempting Idea – Quantum Computers!



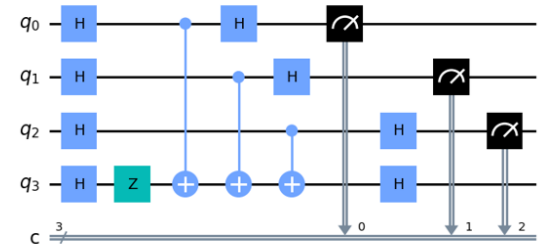
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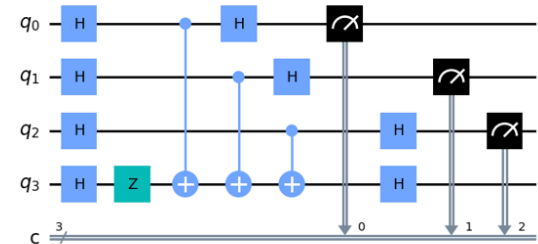
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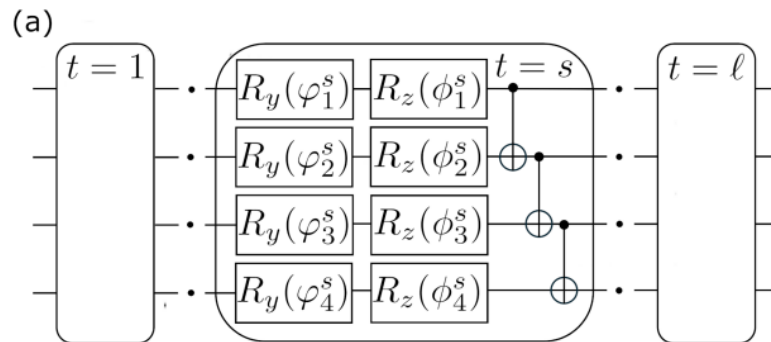
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- Noisy! A curse for some. A blessing for us.



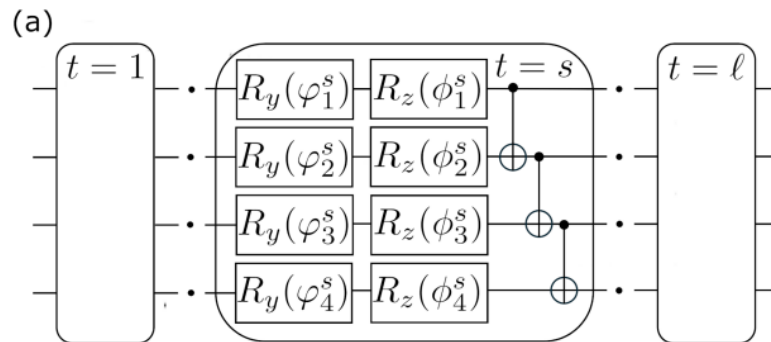
A first experiment – Parameterized Quantum Circuits

- Quantum circuit consisting of layers of random rotations and “ladders” of CNOTs



A first experiment – Parameterized Quantum Circuits

- Quantum circuit consisting of layers of random rotations and “ladders” of CNOTs
- Universal approximator, when noiseless. Converges quickly, consistent with experimental resolution for $l=10$.



Retrieving the Circuit

- Set up parametric model of a general quantum channel ($r > 1$ means non-unitary!)

$$\rho_t = T(\rho_0, \boldsymbol{\theta}) = \sum_{i=1}^r K_i(\boldsymbol{\theta}) \rho_0 K_i(\boldsymbol{\theta})^\dagger$$

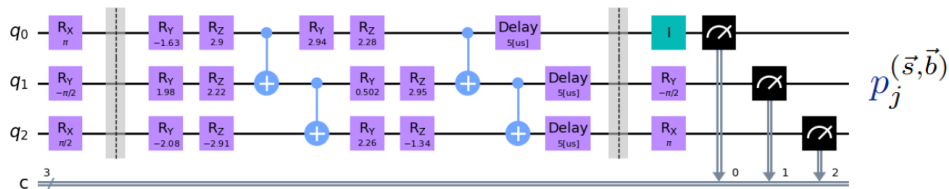
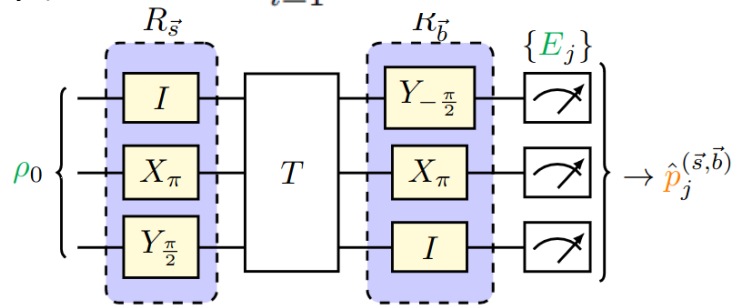
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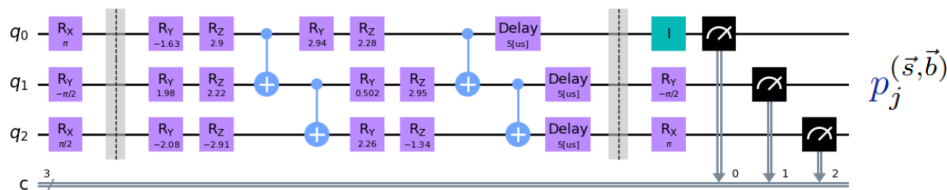
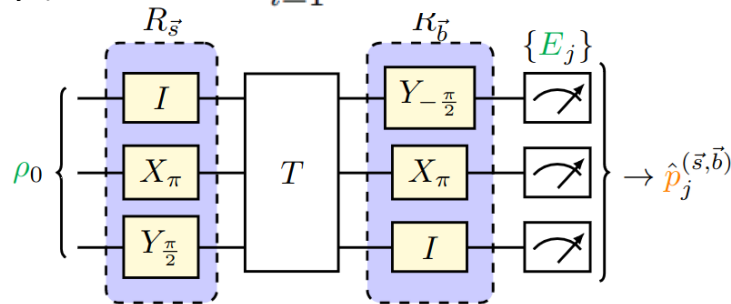


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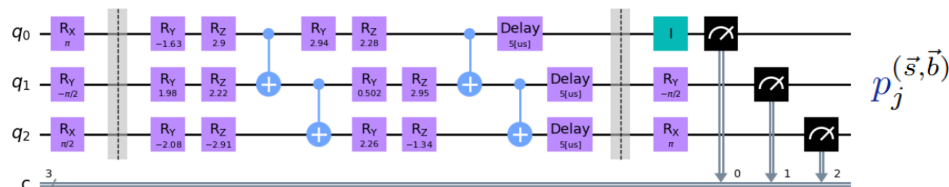
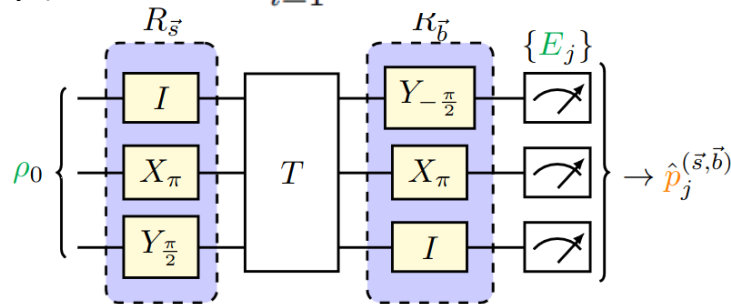
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- Retrieve the Pauli-string measurements, both in-silico and on real hardware
- Minimize a loss function over measurements by optimizing parameters of the model T
- Diagonalize T . obtaining the spectrum

$$\Phi_T ||\psi_R^{(i)}\rangle\rangle = \lambda_i ||\psi_R^{(i)}\rangle\rangle$$

$$\{\bar{\lambda}_i\}_i = \{\lambda_i\}_i \quad |\lambda_i| \leq 1$$

$$||\psi_R^{(0)}\rangle\rangle = ||\rho_{ss}\rangle\rangle$$



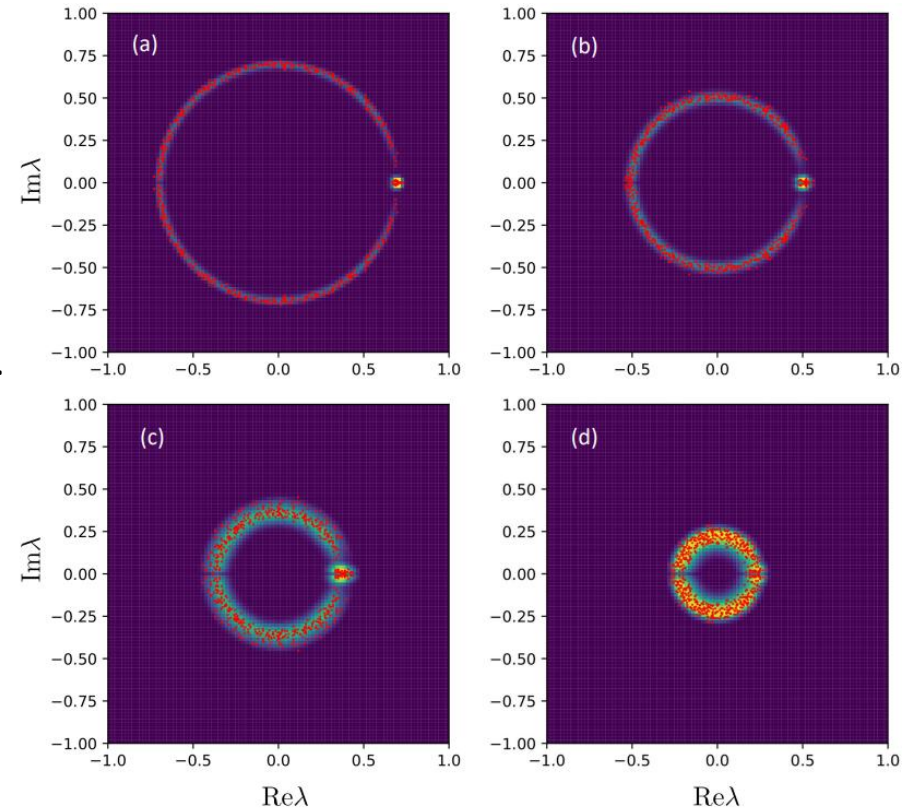
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Spectrum of Retrieved Circuit

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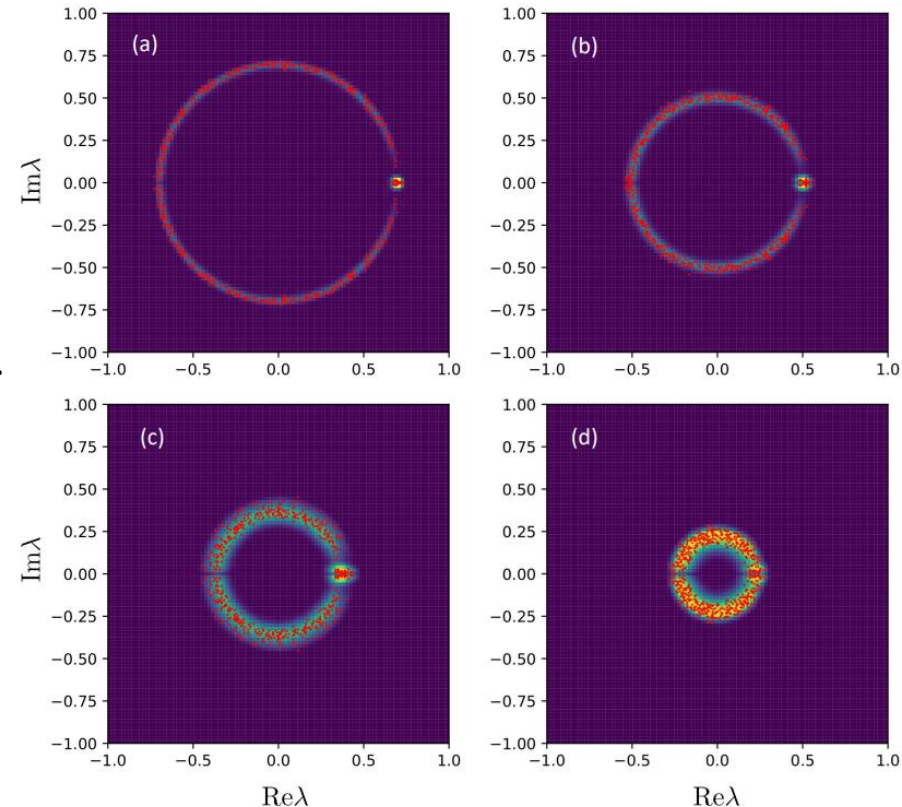
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The eigenvalues distribute in an annulus.
Sharp boundary of an inner and outer radius.
Annulus collapse and broaden as a function of L .



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The eigenvalues distribute in an annulus.
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Annulus collapse and broaden as a function of L .
- Very particular form! Do we know of any theoretical models capturing this behaviour?



Diluted Unitaries

Sá. et. al. *Spectral transitions and universal steady states in random Kraus maps and circuits*. Phys. Rev. B 102, 134310 (2020)

- A two-parameter stochastic ensemble of quantum channels.

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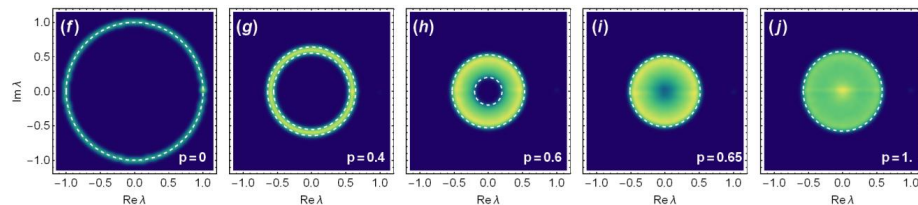
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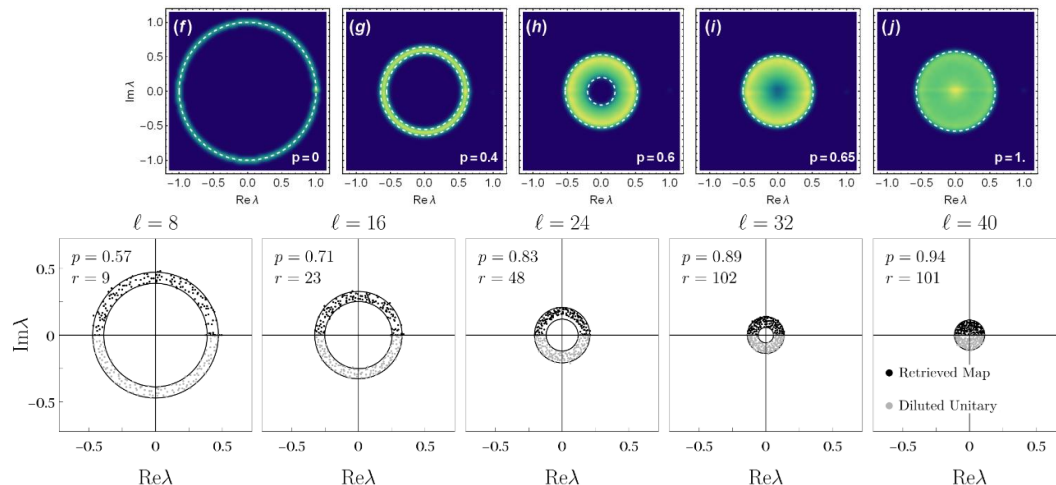
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- Inner and outer radius is related to $\mathbf{p}$ and $\mathbf{r}$

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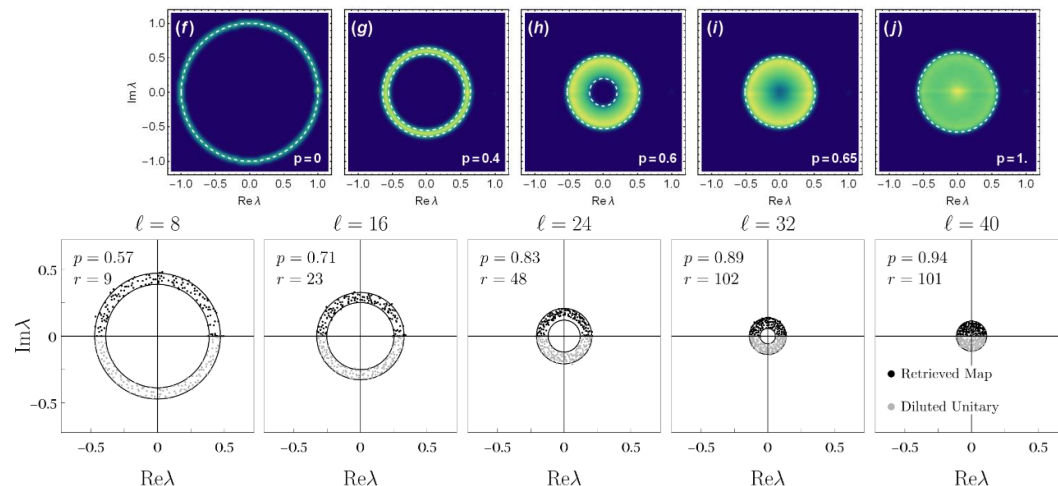
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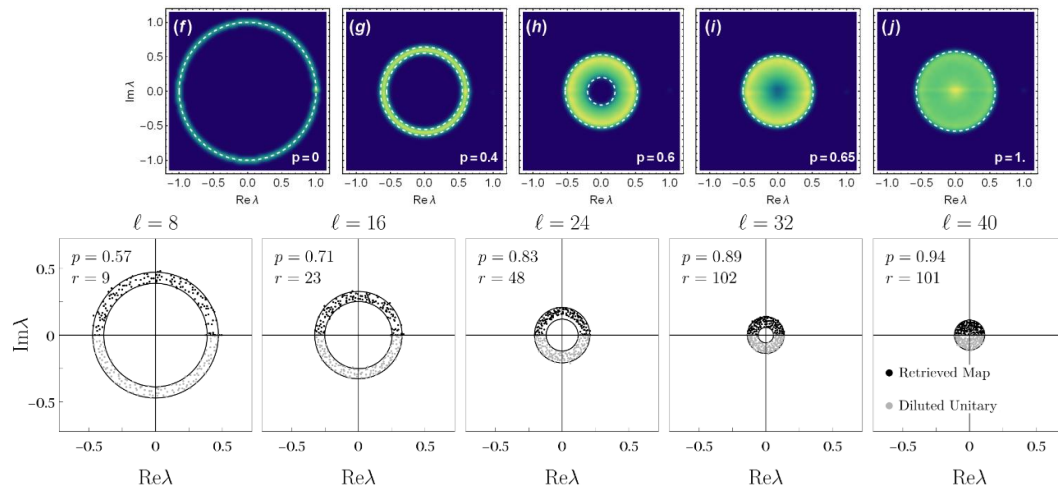
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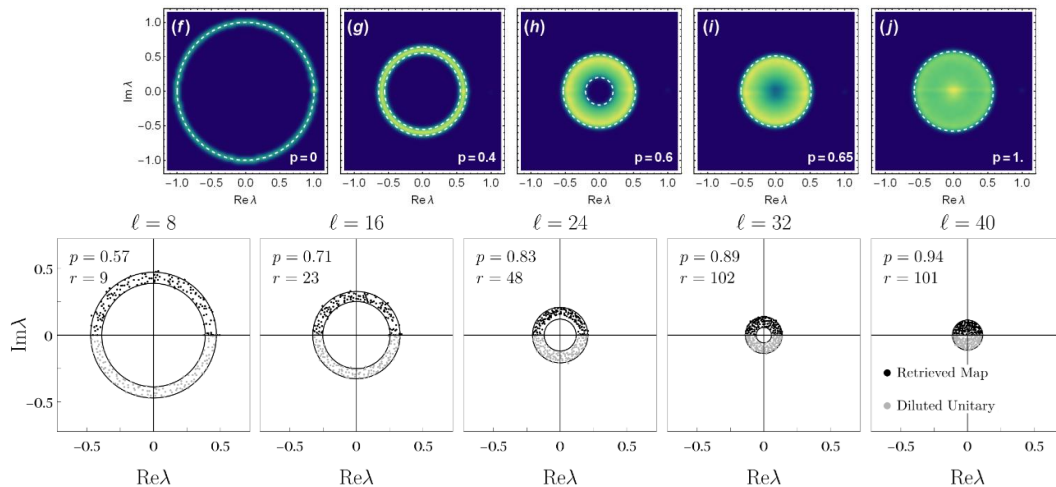
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- Potentially exclude typical reductionistic noise models. E.g. amplitude and phase damping.



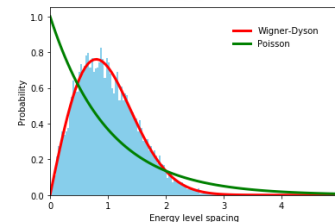
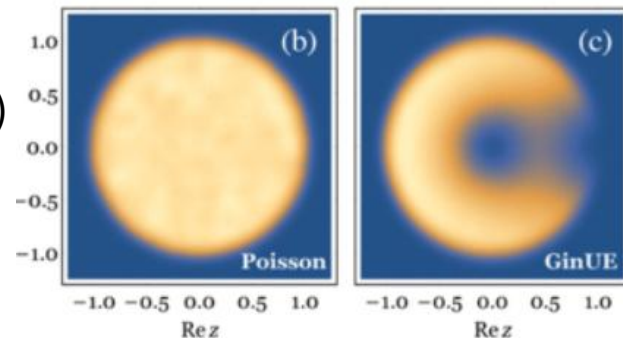
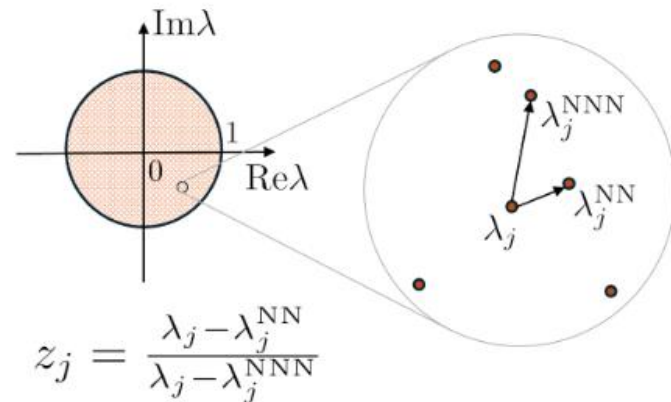
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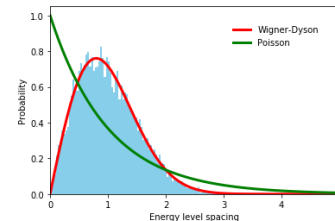
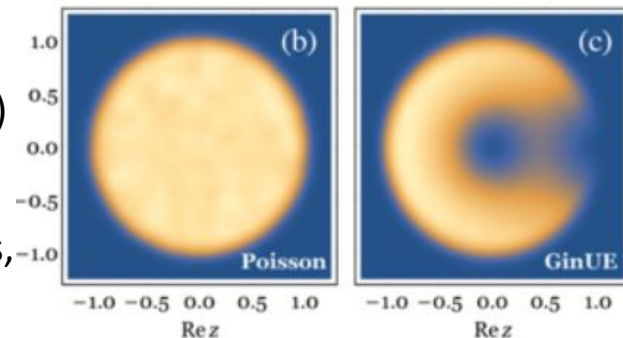
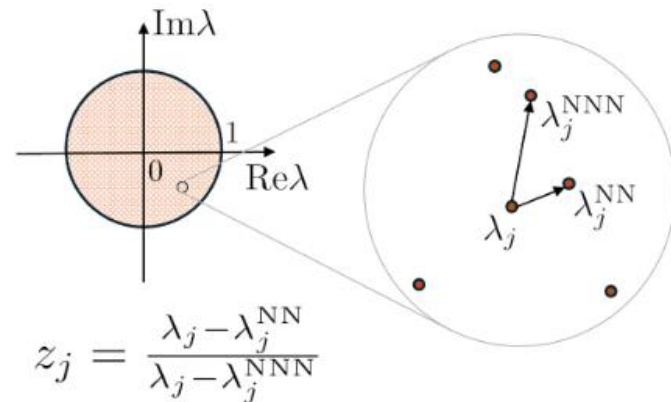
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What about dissipative effects that are very regular?
- An appropriate measure is complex spacing ratio (CSR).
Repulsion between eigenvalues (chaos) induce a bitten-donut shape, while decorrelation (non-chaotic) induce a disk.



Sá. et. al. *Complex Spacing Ratios: A Signature of Dissipative Quantum Chaos*. Phys. Rev. X 10, 021019 – Published 27 April, 2020

Second experiment – Engineered non-chaotic Noise

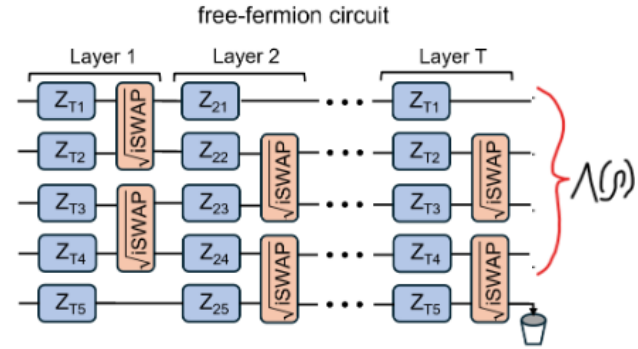
- Latent noise seem very chaotic.
What about dissipative effects that are very regular?
- An appropriate measure is complex spacing ratio (CSR).
Repulsion between eigenvalues (chaos) induce a bitten-donut shape, while decorrelation (non-chaotic) induce a disk.
- What systems gives statistics that are not bitten donuts, and can we even detect it accurately?



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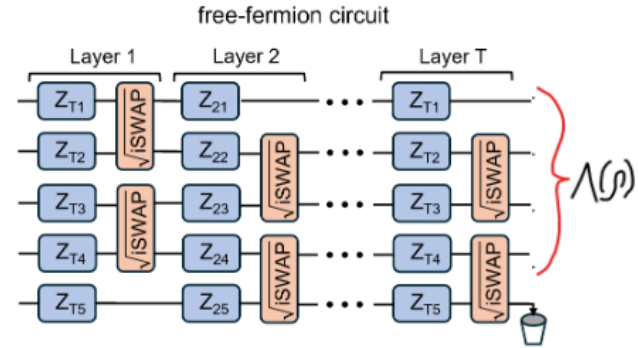
Trace-out Free-Fermion (FF) Circuits

- Inspiration from circuits simulating fermionic systems. Very constrained gate-set. Lots of symmetry.



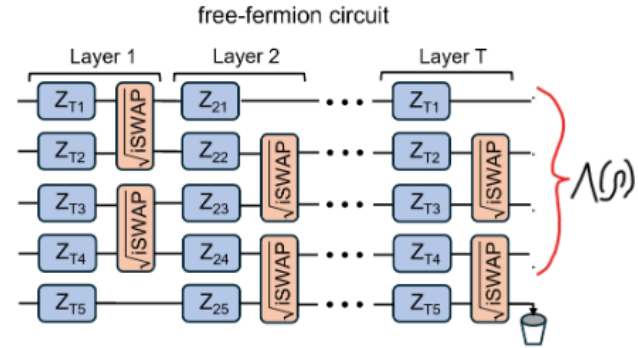
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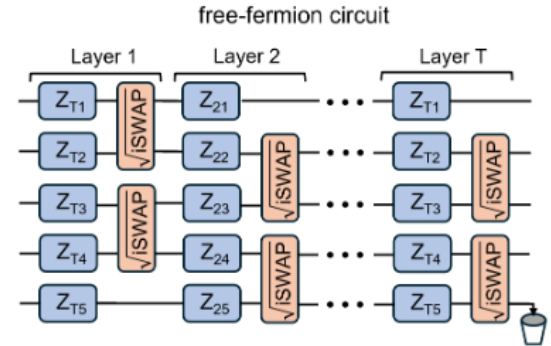
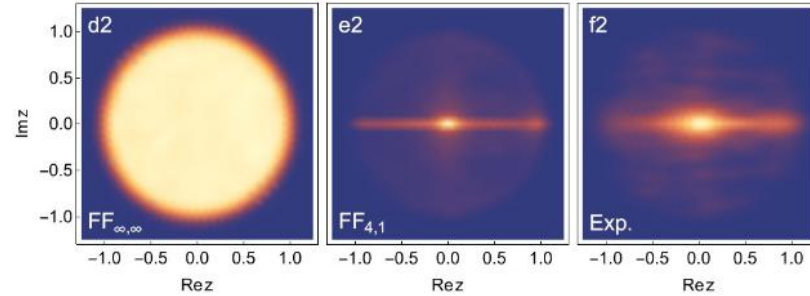
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- Inspiration from circuits simulating fermionic systems. Very constrained gate-set. Lots of symmetry.
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- For a high-fidelity computer and shallow circuits, the hope is that the engineered openness is dominating over the latent one, and is detectable.



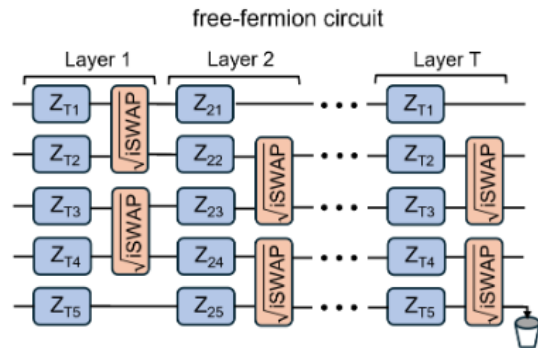
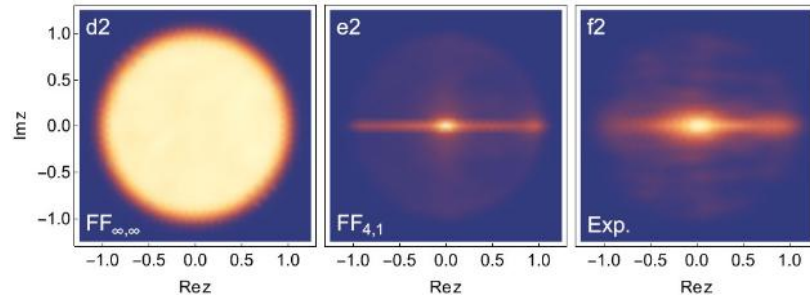
Results and Comparisons

- For the infinite size FF, we get the expected disk. For 4+1 qubits, we get strong finite size effects. But at any rate, it is very different from the bitten donut.



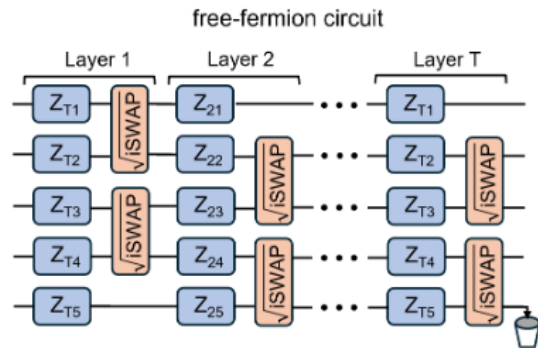
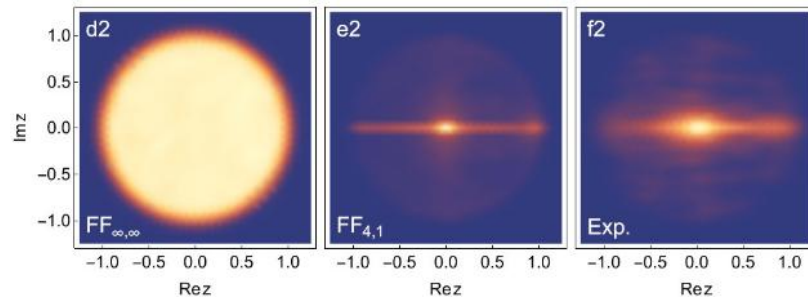
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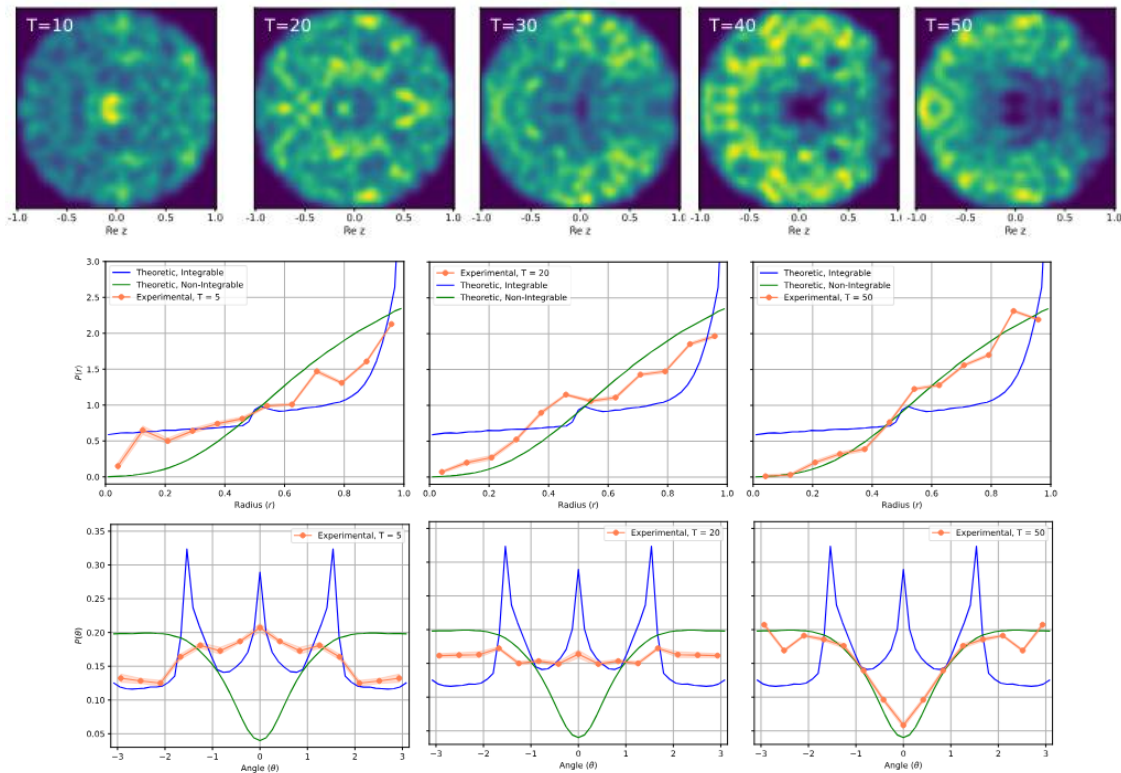
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- Previous was for $l=5$. How does the CSR change as you increase the number of layers?



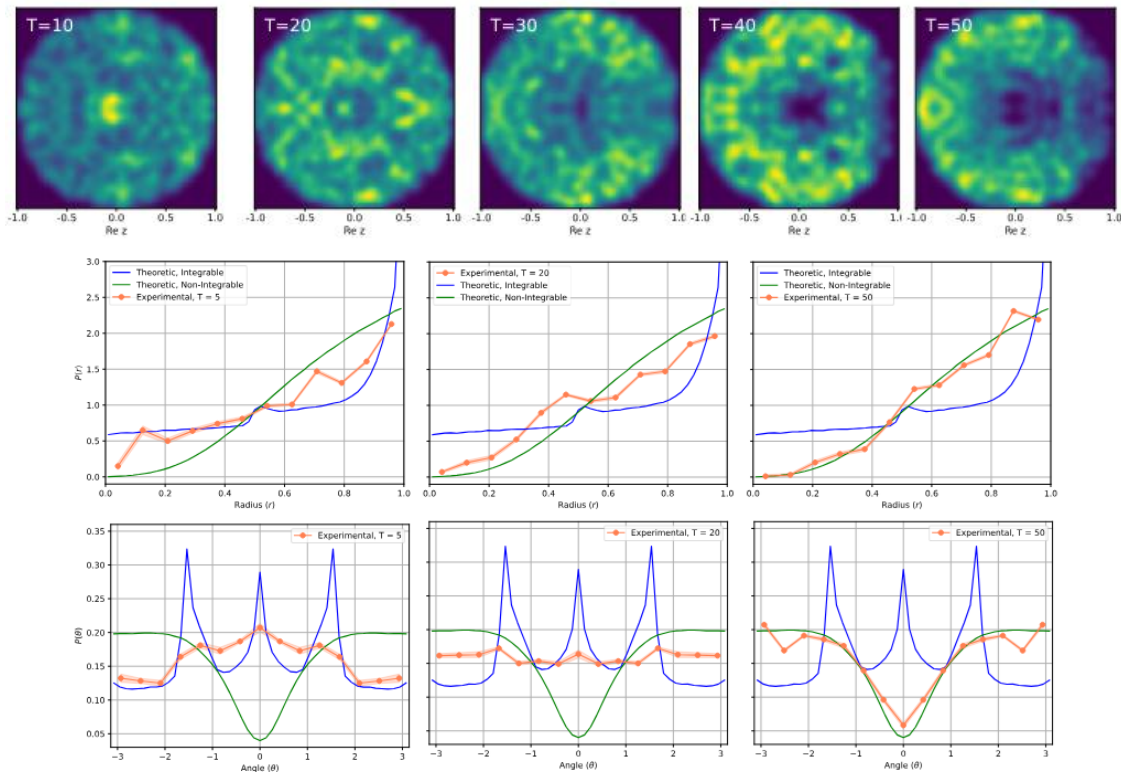
Disk to Donut Transition

- As the depth increase, latent noise become more dominant.



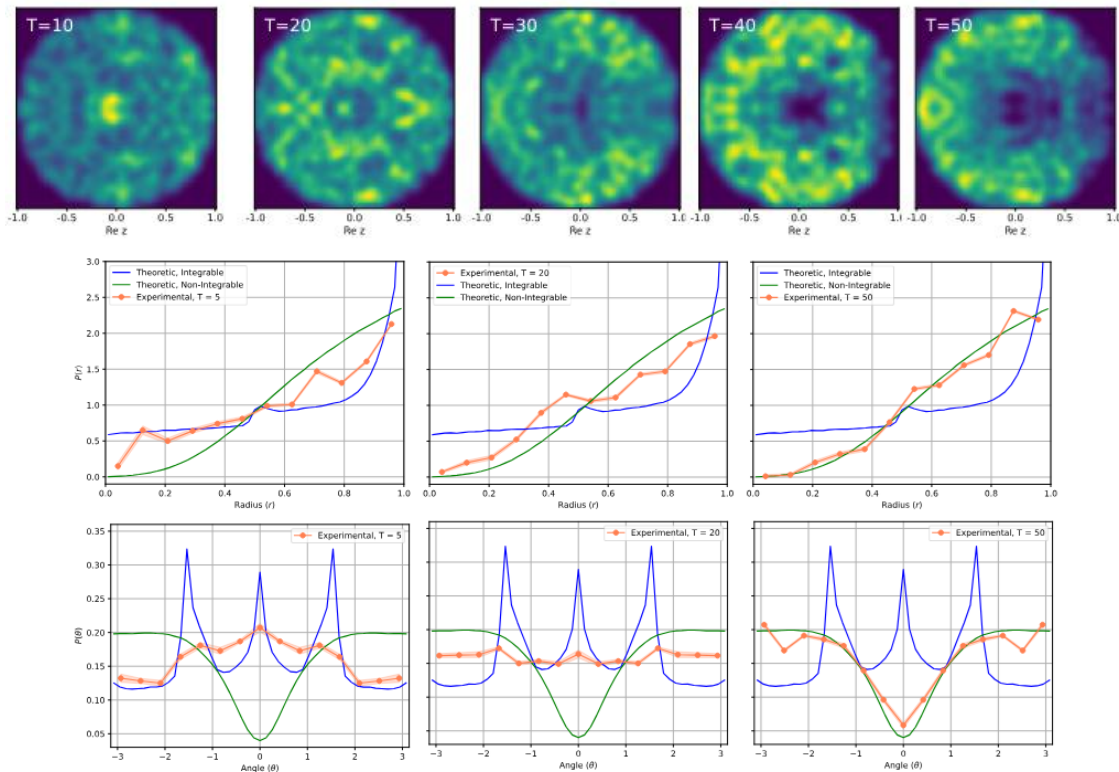
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- We see observe that the latent noise induce a disk to donut transition, showing that it is chaotic in nature.
- Further observations that hopefully inspire how to think about quantum noise.



Quantum Process Tomography

- Other presentation!