UiO : Department of Informatics
University of Oslo

Fascinating wave phenomena: Where Newton and Hooke fall short

Sverre Holm, 2016

Centre for Innovative Ultrasound Solutions
For health care, maritime, and oil & gas

VOL. 84, NO. B9 JOURNAL OF GEOPHYSICAL RESEARCH AUGUST 10, 1979

Constant Q-Wave Propagation and Attenuation

EINAR KJARTANSSON

Geophys. J. R. astr. Soc. (1967) **13**, 529–539.

Linear Models of Dissipation whose Q is almost Frequency Independent—II

Michele Caputo*

REVIEWS OF GEOPHYSICS VOL. 2, NO. 4 NOVEMBER 1964

Q

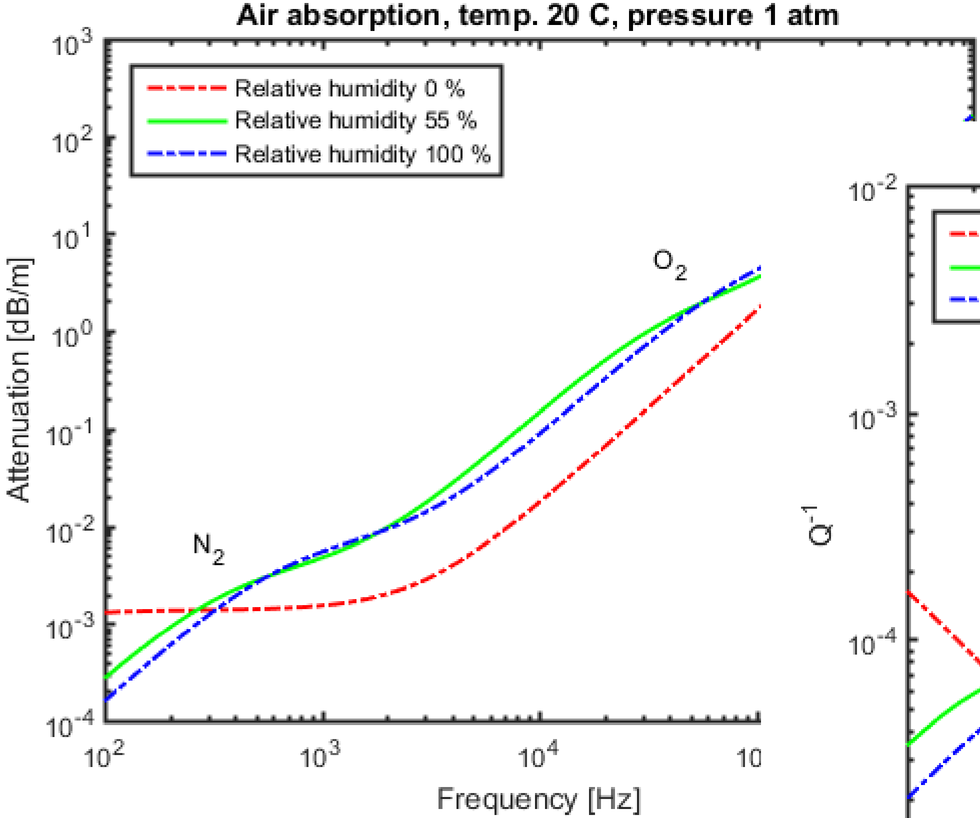
L. KNOPOFF

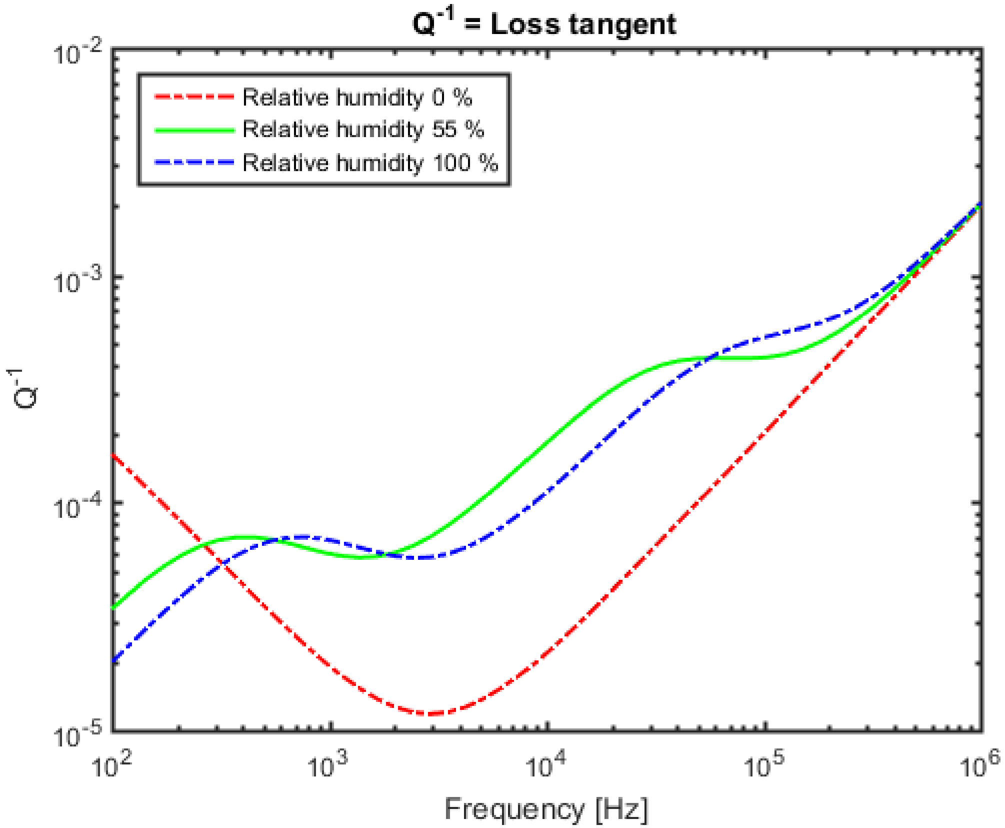
Department of Physics and Institute of Geophysics and Planetary Physics
University of California, Los Angeles

Abstract. Measurements of the specific attenuation factor $1/Q$ in homogeneous materials in the laboratory and in the field show overwhelmingly that $1/Q$ is substantially independent of frequency, whereas $1/Q$ varies as the first power of frequency

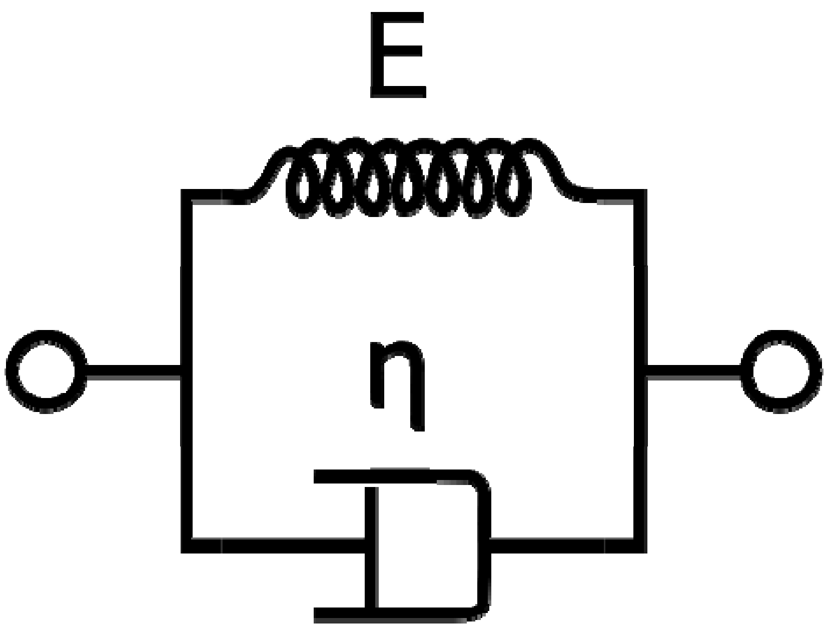
Air: approx quadratic attenuation

$$Q^{-1} \propto \text{attenuation} \cdot \text{velocity}/\text{frequency}$$





Hooke and Newton



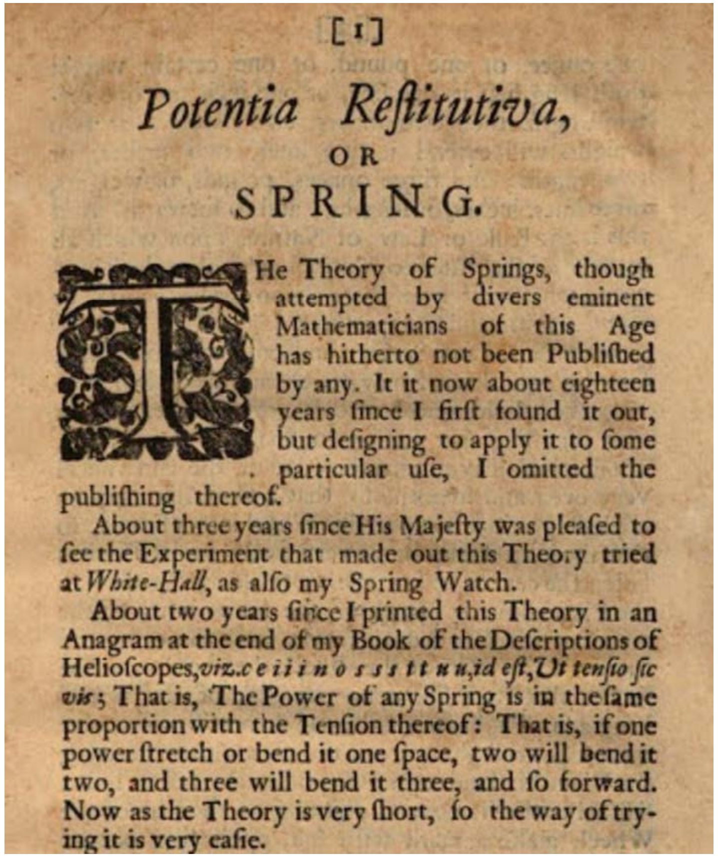
Natural philosophers:

- Hooke (1635-1703)
- Newton (1642-1726)

Non-Hookean

Here’s what Hooke postulated

- 1660: ceiiinosssttuv
- 1678: ut tensio, sic vis
- ‘as the extension, so the force’



[1]

Potentia Reſtitutiva,

OR

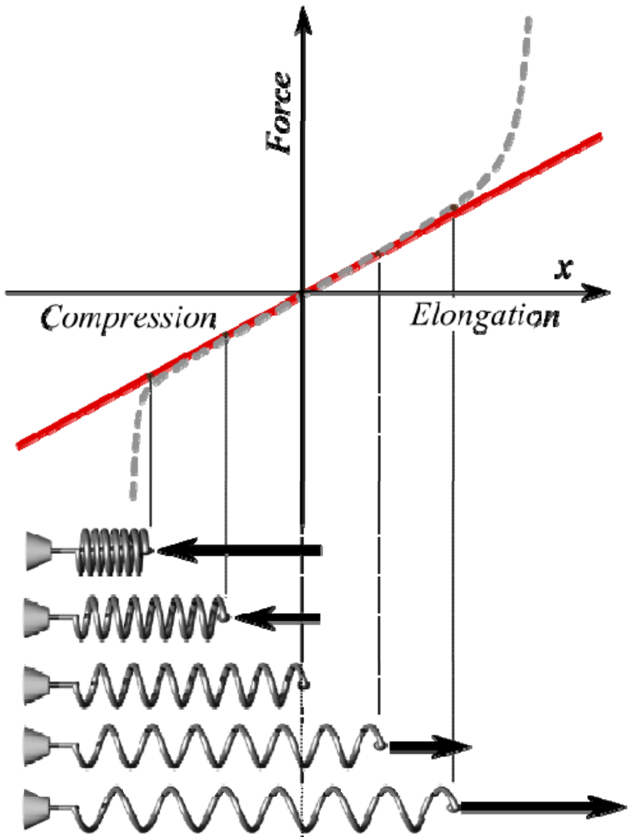
SPRING.

THe Theory of Springs, though attempted by divers eminent Mathematicians of this Age has hitherto not been Publiſhed by any. It it now about eighteen years ſince I firſt found it out, but deſigning to apply it to ſome particular uſe, I omitted the publiſhing thereof.

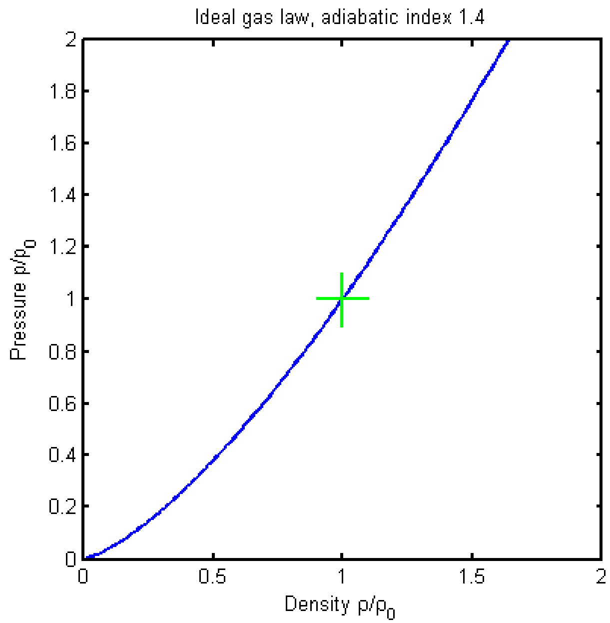
About three years ſince His Majeſty was pleaſed to ſee the Experiment that made out this Theory tried at *White-Hall*, as alſo my Spring Watch.

About two years ſince I printed this Theory in an Anagram at the end of my Book of the Deſcriptions of Helioſcopes, *viz.* *c e i i i n o s s s t t u u, id eſt, Ut tenſio ſic vis*; That is, The Power of any Spring is in the ſame proportion with the Tenſion thereof: That is, if one power ſtretch or bend it one ſpace, two will bend it two, and three will bend it three, and ſo forward. Now as the Theory is very ſhort, ſo the way of trying it is very eaſie.

Nonlinear spring, nonlinearity in gas

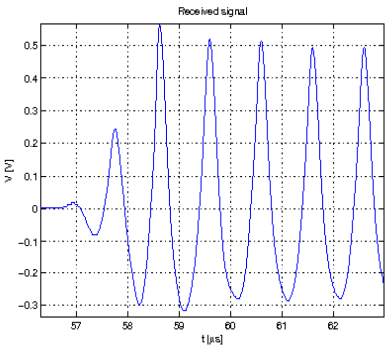


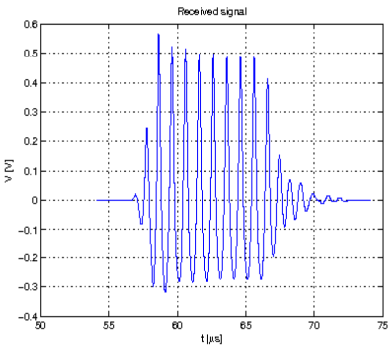
$$\frac{p}{p_0} = \frac{\rho}{\rho_0}^{\gamma}$$

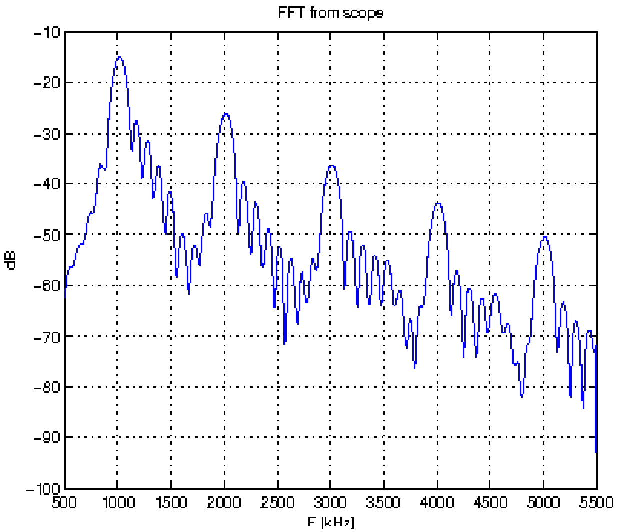


The Great Wave off Kanagawa. Katsushika Hokusai (1760–1849)

Nonlinear pulse shape in water

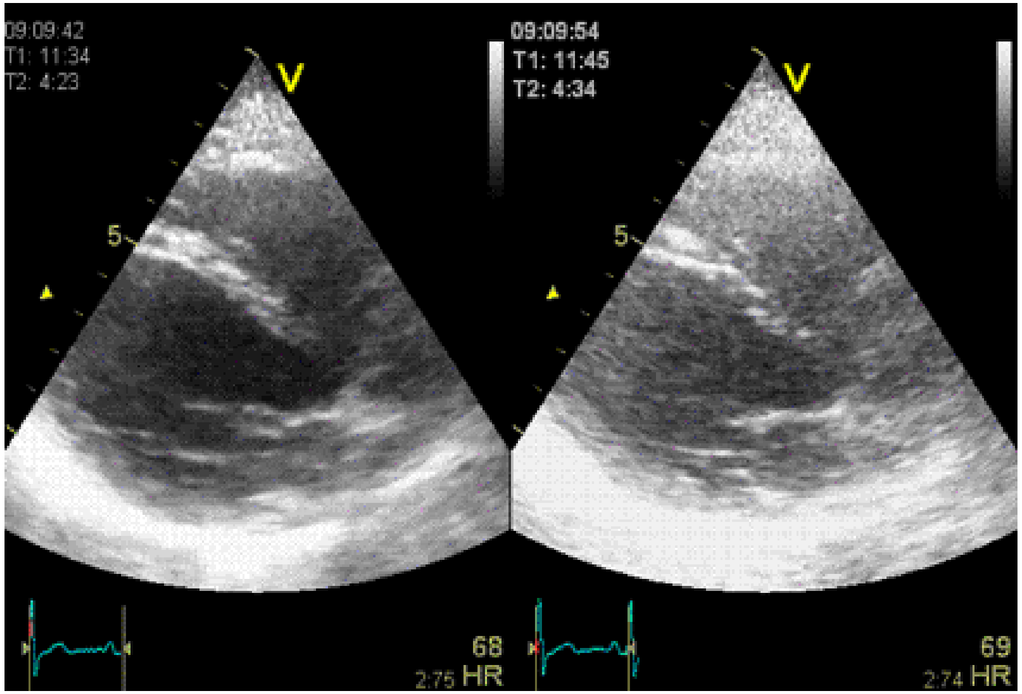


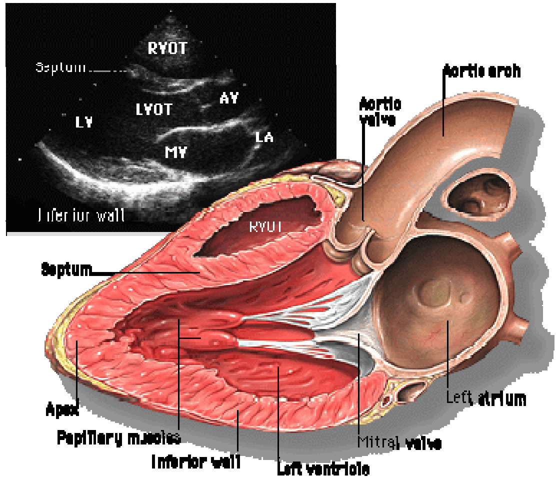




Fabrice Prieur, Sept. 2009

Tissue Harmonic Medical Imaging

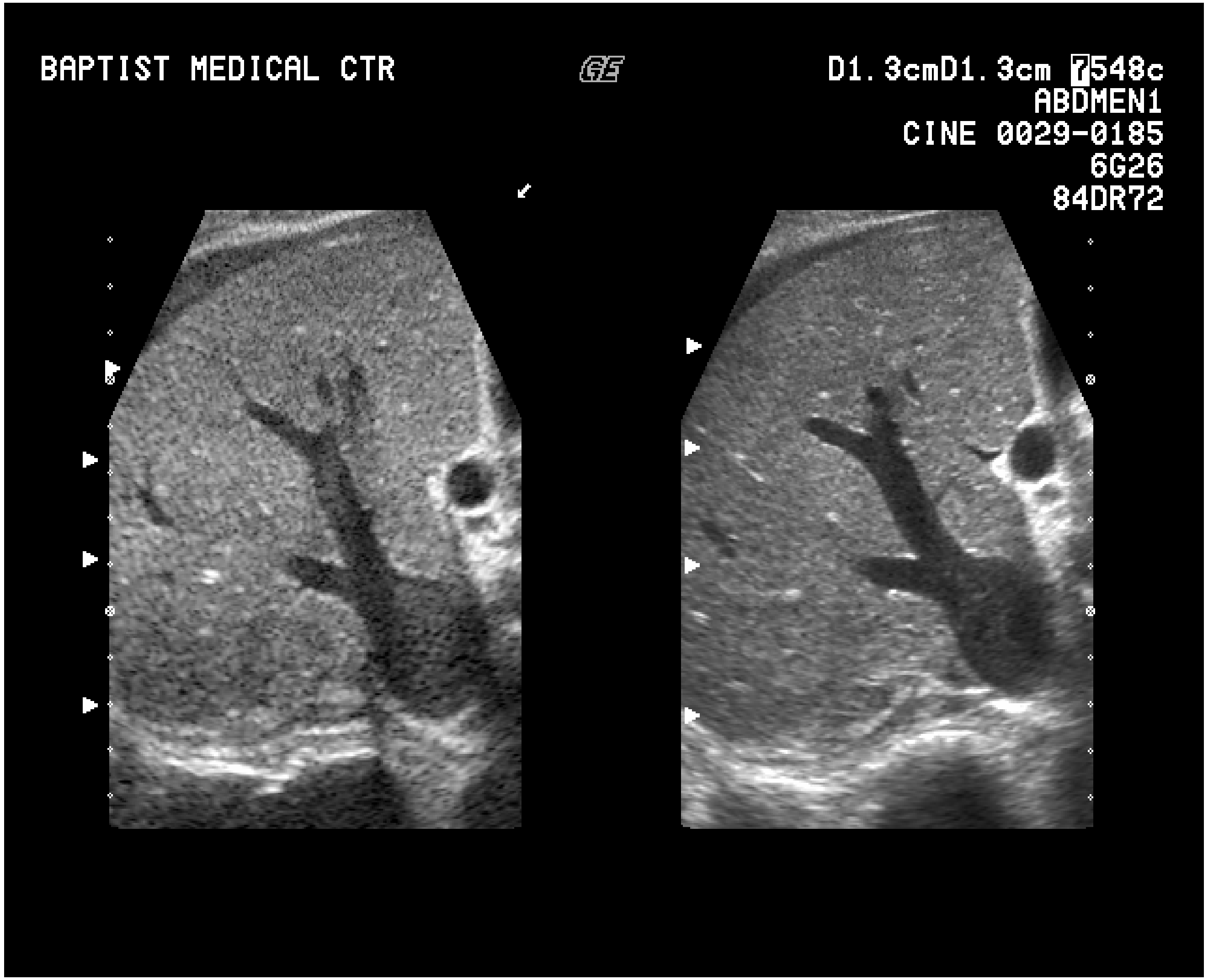




- Ultrasound image of a heart (parasternal view) using second harmonic (left) and fundamental (right) signals.
- Courtesy of Asbjørn Støylen, NTNU, Trondheim, Norway

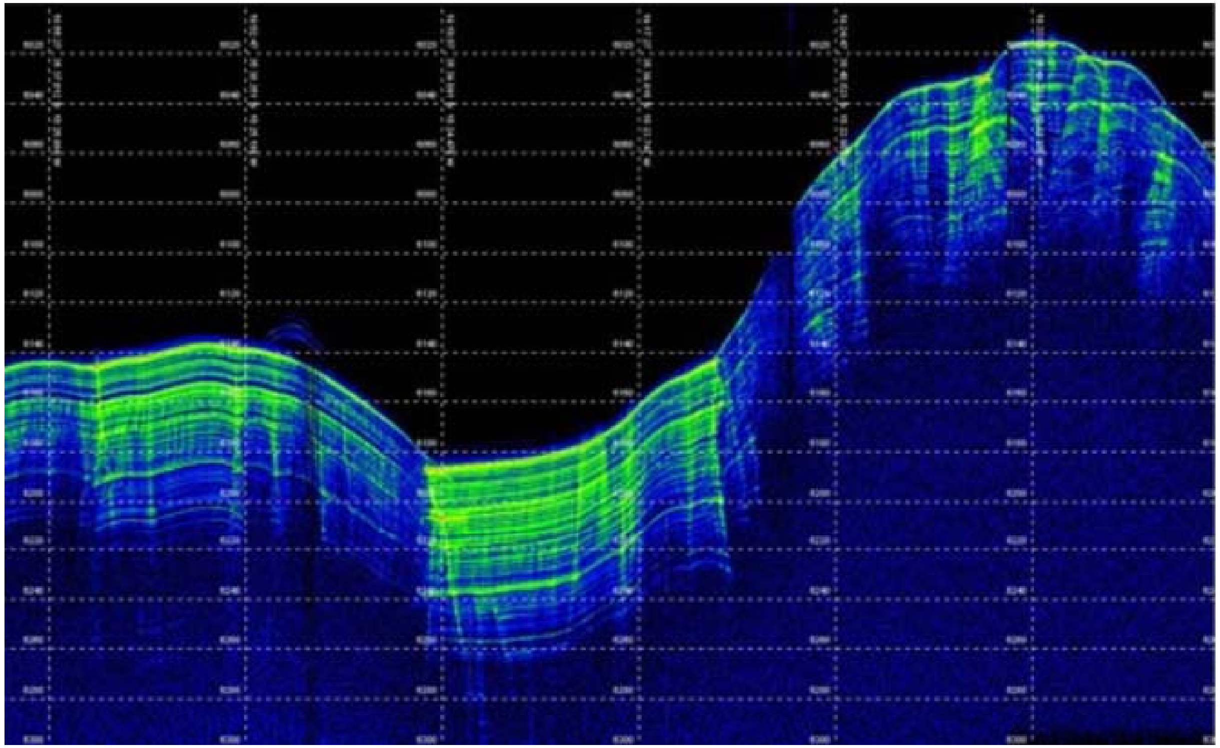
Liver

- Harmonic or octave imaging
- Default mode in most ultrasound scanners



Fundamental 2. harmonic

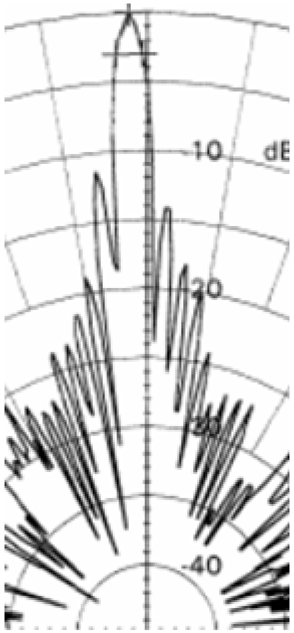
Parametric sub-bottom profiler



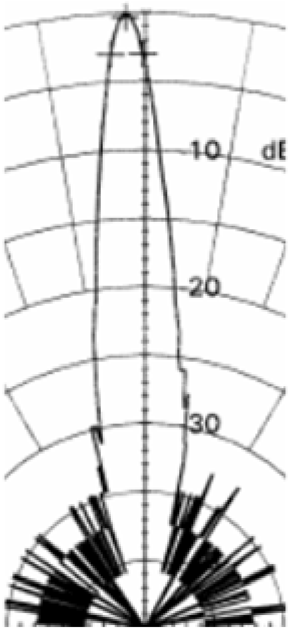
TOPAS parametric sonar.
Water depth >4600 m, penetration depth 60-90 m.
Courtesy of Kongsberg Maritime, Norway

Primary and secondary beamprofiles

Primary: 40 kHz



Secondary: 4 kHz

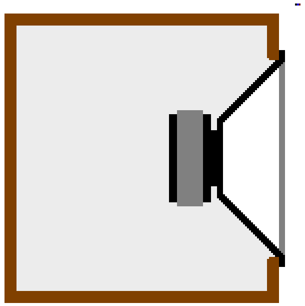


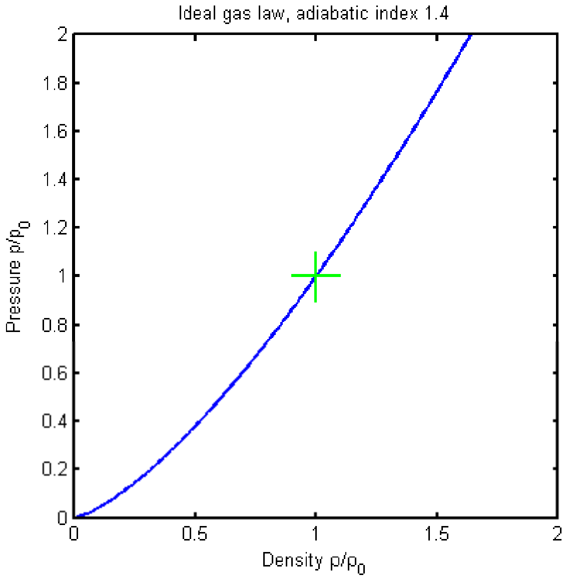
Conventional sonar would have required 5-10 times larger aperture

Parametric audio sound source

- Non-linear interaction
- Holosonics: Audio Spotlight
  - http://www.holosonics.com/

Even your subwoofer is nonlinear!



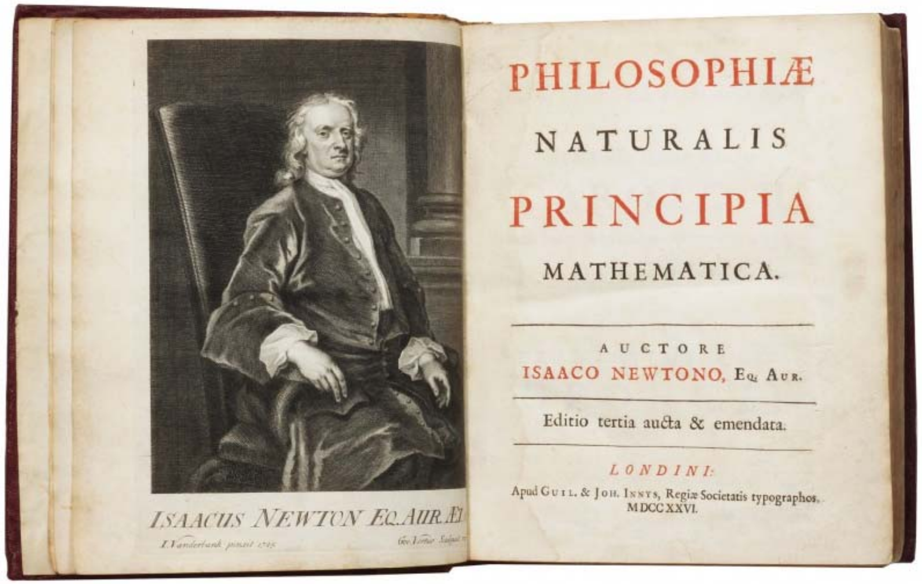


Non-Newtonian

- Aftenposten Viten, 3 april 2015: “I år er påsken på riktig dato, og det var det Newton som fant ut”
- Science 2.0: “This Year Easter Falls On The Correct Date According To Newton”

Here's what Newton postulated in 1687

"The resistance arising from the want of lubricity in the parts of a fluid is, cæteris paribus (other factors being equal), proportional to the velocity with which the parts of the fluid are separated from each other."



Non-Newtonian: Thixotropic

Viscosity decreases with time, e.g. paint:

- Starts with high viscosity
- Agitated: Low viscosity - easy to smear out
- At rest: High viscosity - doesn't drip

Honey, coal-water slurries, waxy crude oil, cytoplasm of cells

Non-Newtonian: Rheopectic

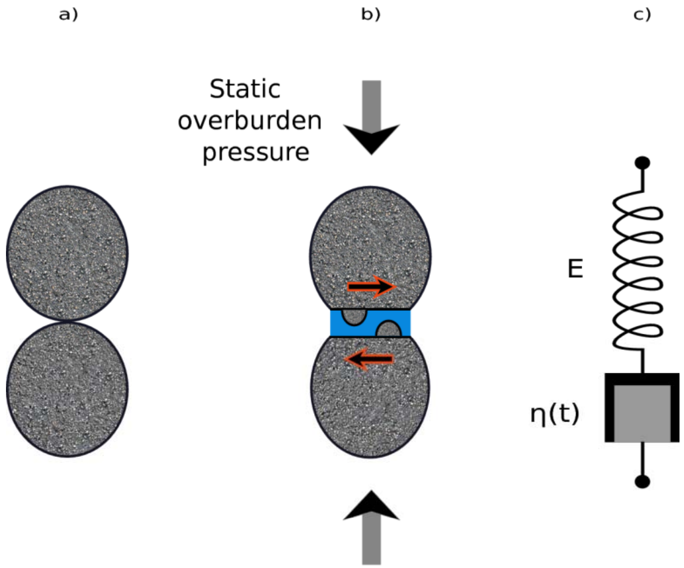
Viscosity increases with time

DIY Slime

- Borax powder
- White glue
- Distilled water
- Talcum powder (opt.)

Grain-to-grain shearing

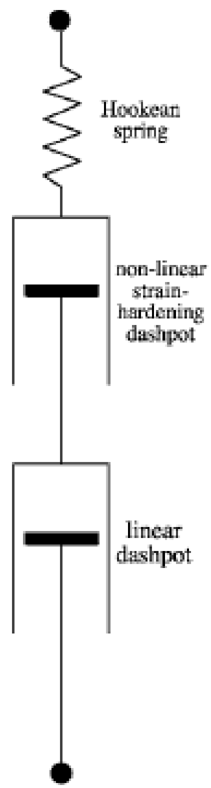
- Elastic behavior from stick-slip, intergranular micro-asperities.
- Viscous dissipation in pore fluid film.



- Buckingham, «Wave propagation, stress relaxation, and grain-to-grain shearing in saturated, unconsolidated marine sediments,» J. Acoust Soc Am, 2000

Rheopecty in unconsolidated material

- After being triggered, shearing becomes progressively more difficult to sustain, viscosity: $\eta(t) = \eta_0 + \theta_s t$
- Time-varying Maxwell model = Rheopectic model
- Power law memory: $h_s(t) \approx t_s^{-1}(t/t_s)^{-\gamma}$
- Pandey, Holm, "Connecting the grain-shearing mechanism of wave propagation in marine sediments to fractional order wave equations," J. Acoust. Soc. Am. 2016



(b)

Power law ⇔ non-integer derivative

- 1921: Nutting: Power law stress response of a fluid to a constant strain load
- 1951: Scott Blair: Nutting's law is the response of a system described by a non-integer derivative

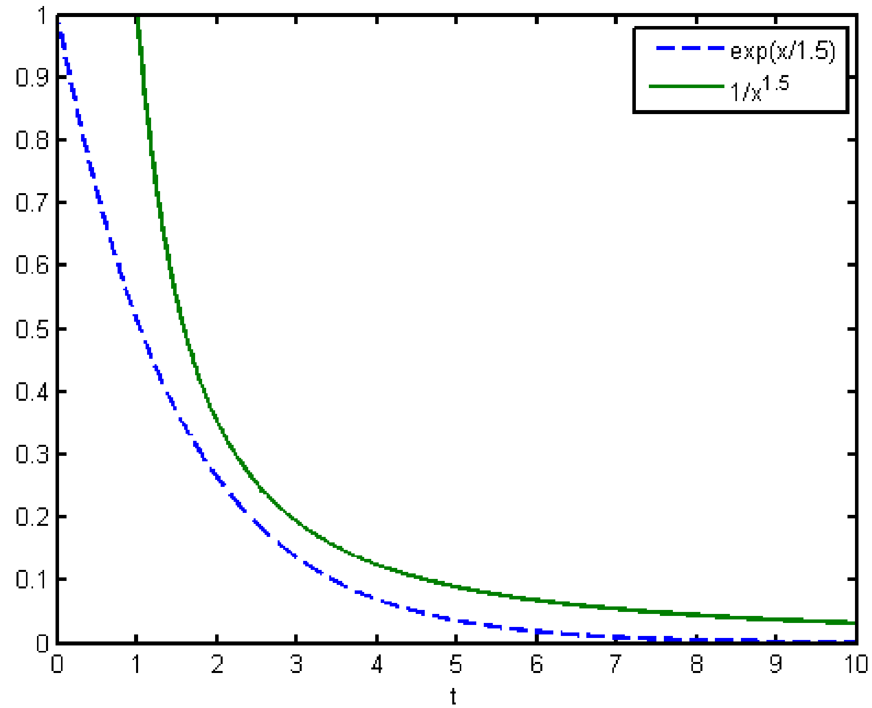
A different form of memory function

Exponential: $e^{-t/T}$

- Relaxation function
- Spring-damper system

Power law: t^{-a}

- Long-tail memory
- Scale-invariant, $(bt)^{-a} = b^{-a}\, t^{-a}$




This is a Romanesque Cauliflower, which is a natural approximation of a fractal.

PHYSICAL REVIEW E 94, 032606 (2016)

Linking the fractional derivative and the Lomnitz creep law to non-Newtonian time-varying viscosity

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(Received 13 May 2016; published 23 September 2016)

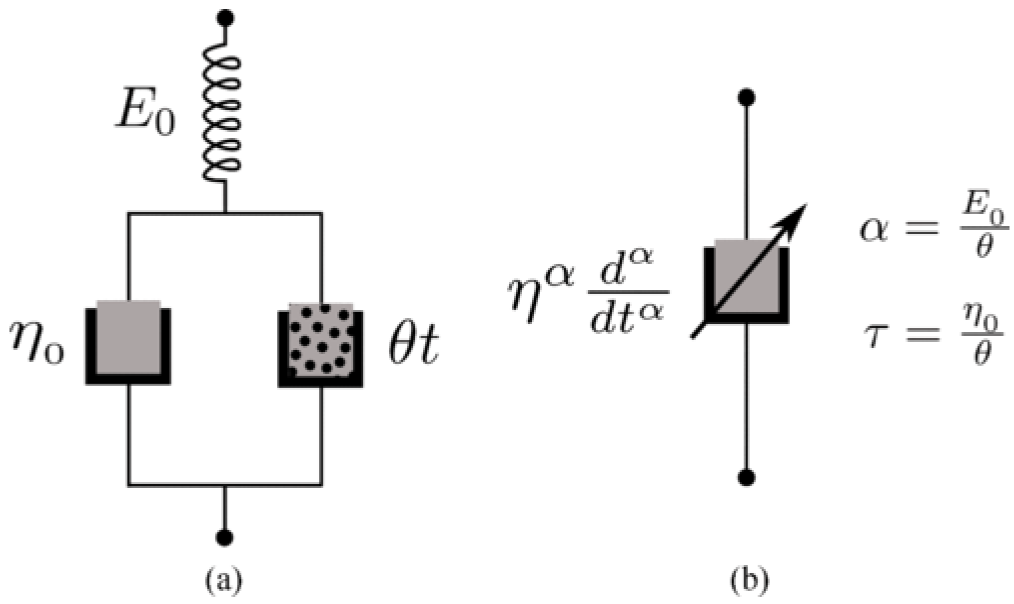
Many of the most interesting complex media are non-Newtonian and exhibit time-dependent behavior of thixotropy and rheopecty. They may also have temporal responses described by power laws. The material behavior is represented by the relaxation modulus and the creep compliance. On the one hand, it is shown that in the special case of a Maxwell model characterized by a linearly time-varying viscosity, the medium's relaxation modulus is a power law which is similar to that of a fractional derivative element often called a springpot. On the other hand, the creep compliance of the time-varying Maxwell model is identified as Lomnitz's logarithmic creep law, making this possibly its first direct derivation. In this way both fractional derivatives and Lomnitz's creep law are linked to time-varying viscosity. A mechanism which yields fractional viscoelasticity and logarithmic creep behavior has therefore been found. Further, as a result of this linking, the curve-fitting parameters involved in the fractional viscoelastic modeling, and the Lomnitz law gain physical interpretation.

DOI: 10.1103/PhysRevE.94.032606

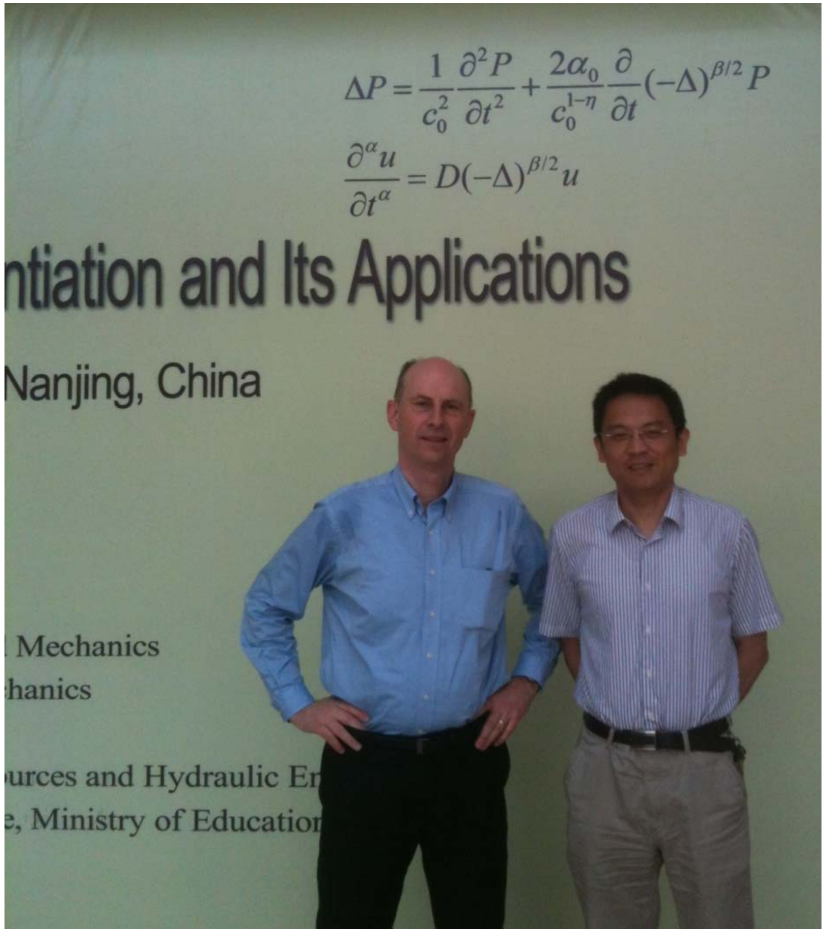
Surprising result

A linearly increasing viscosity (with time) + a spring

Relaxation response = a non-integer derivative damper

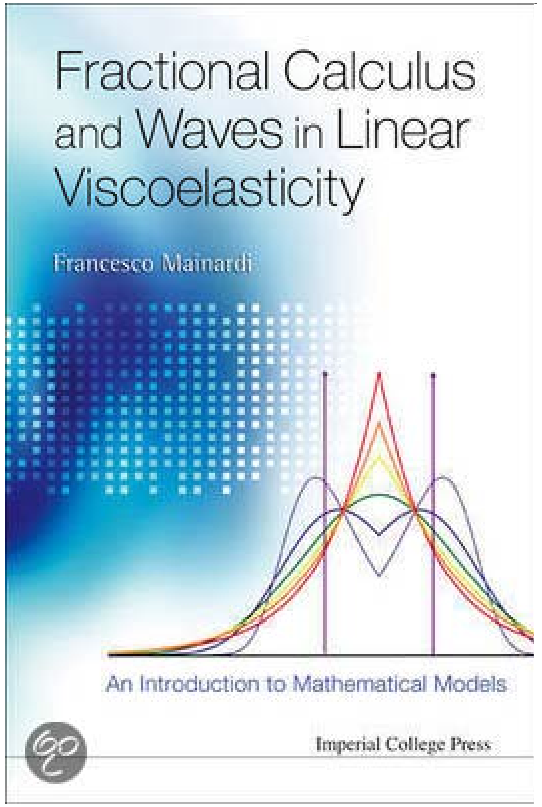


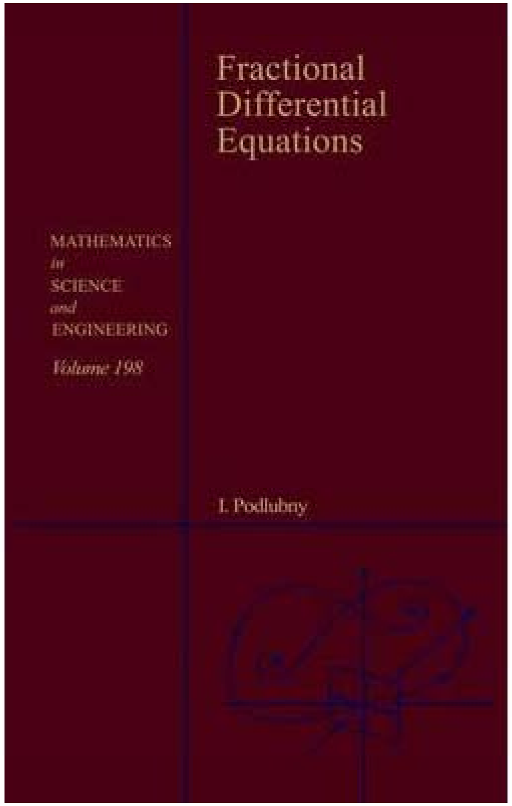
Fractional derivatives have been around for a while …



- Prof. Wen Chen, Hohai University, Nanjing

- Chen and Holm, "Fractional Laplacian time-space models for linear and nonlinear lossy media exhibiting arbitrary frequency dependency," J. Acoust. Soc. Amer., 2004.





Progress in Fractional Differentiation and Applications
An International Journal

Fractional Calculus & Applied Analysis
An International Journal for Theory and Applications

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(Electronic) ISSN 1314-2224

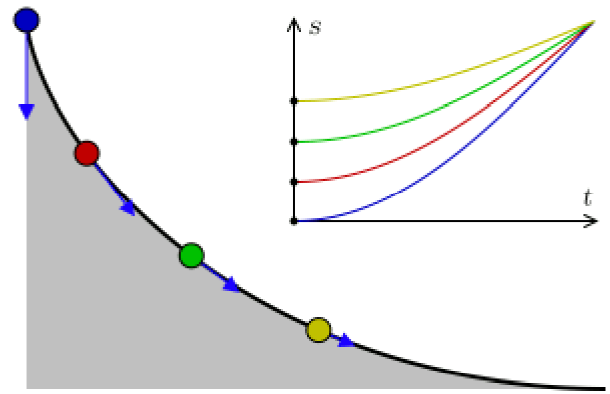
SURVEY PAPER

ON A FRACTIONAL ZENER ELASTIC WAVE EQUATION

Sven Peter Näsholm [1], Sverre Holm [2]

Abel’s Integral Equation - $D^{0.5}$

- First mechanical problem with a half order derivative
- Tautochrone curve, total time for the particle to fall:

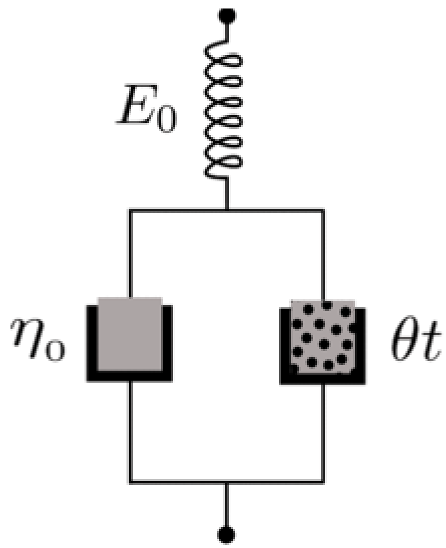


$$T(y_0) = \frac{1}{\sqrt{2g}} \int_0^{y_0} \frac{1}{(y_0 - y)^{0.5}} dy$$

  – Abel, Auflösung einer mechanischen Aufgabe, J. Reine u. Angew. Math, 1826,

Surprising result

- A linearly increasing viscosity (with time) + a spring

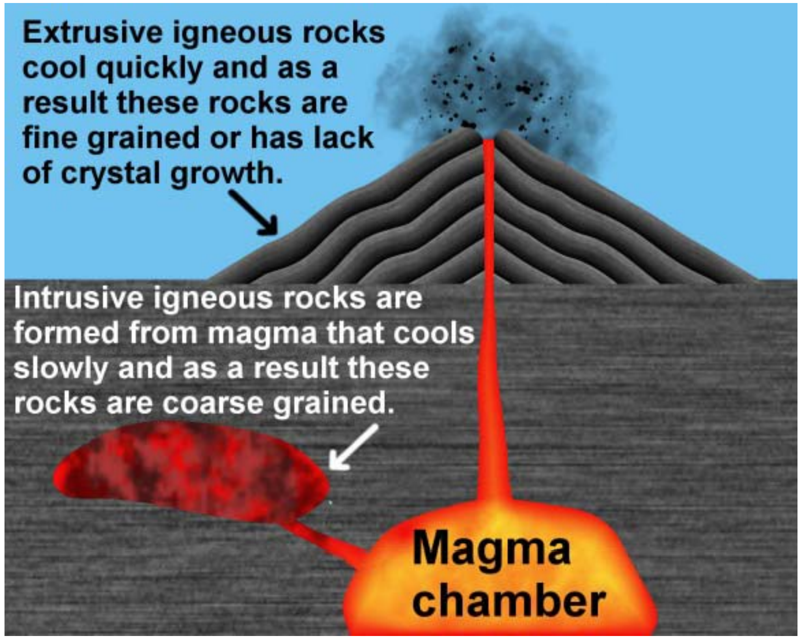


- Creep response = Lomnitz law, 1956 (Dietrich 1978):

$$\epsilon(t) \propto 1 + q \ln(1 + at)$$

- Creep behavior of igneous rocks
- Connection to time-varying viscosity

Igneous rocks



Complex media

Non-Hookean

Non-Newtonian

UiO : Department of Informatics
University of Oslo

Waves in Complex Media.
A Constitutive Equation Approach

version 0.42

06 December 2016

Sverre Holm
University of Oslo

Complex media and non-Hookean and non-Newtonian models

- Non-Hookean behavior: mature field of nonlinear acoustics
- Time-dependent non-Newtonian behavior has been much less explored
- May be one of the sources of power law and constant Q characteristics of complex media