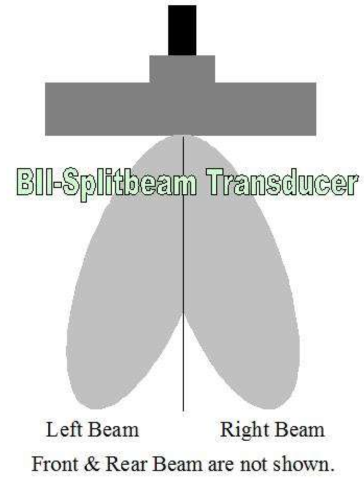
UiO : Department of Informatics
University of Oslo

# Differential arrays – from cardioid microphones to Yagi antennas

Sverre Holm

# Seemingly dissimilar applications

- End-fire arrays




- Echo sounder: Split beam for direction finding


- Microphones

- Yagi antennas

# N-element end-fire vs broadside array

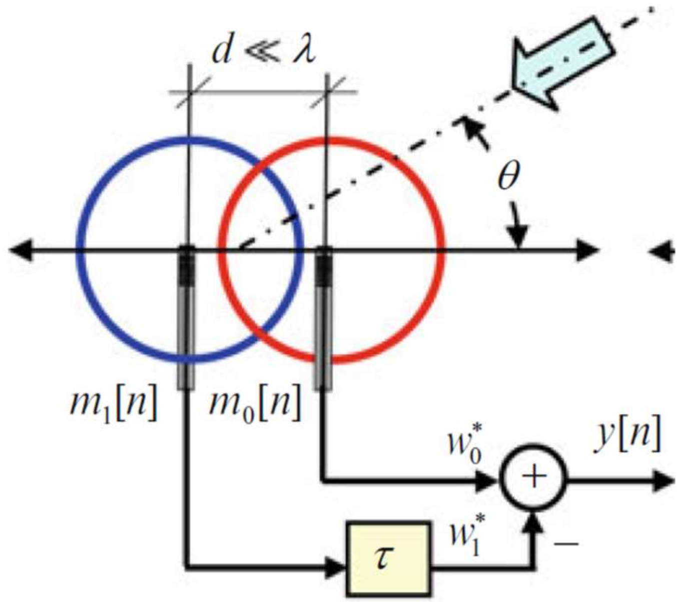
- Broad-side:
  - Element distance: ~ $\lambda/2$
  - Array gain: N
  - Beamwidth inverse proportional with frequency
- End-fire:
  - Element distance: << $\lambda/2$
  - Array gain: $N^2$ (theoretical maximum)
    - Super-directive or supergain
  - Almost frequency-independent beam pattern

# Literature

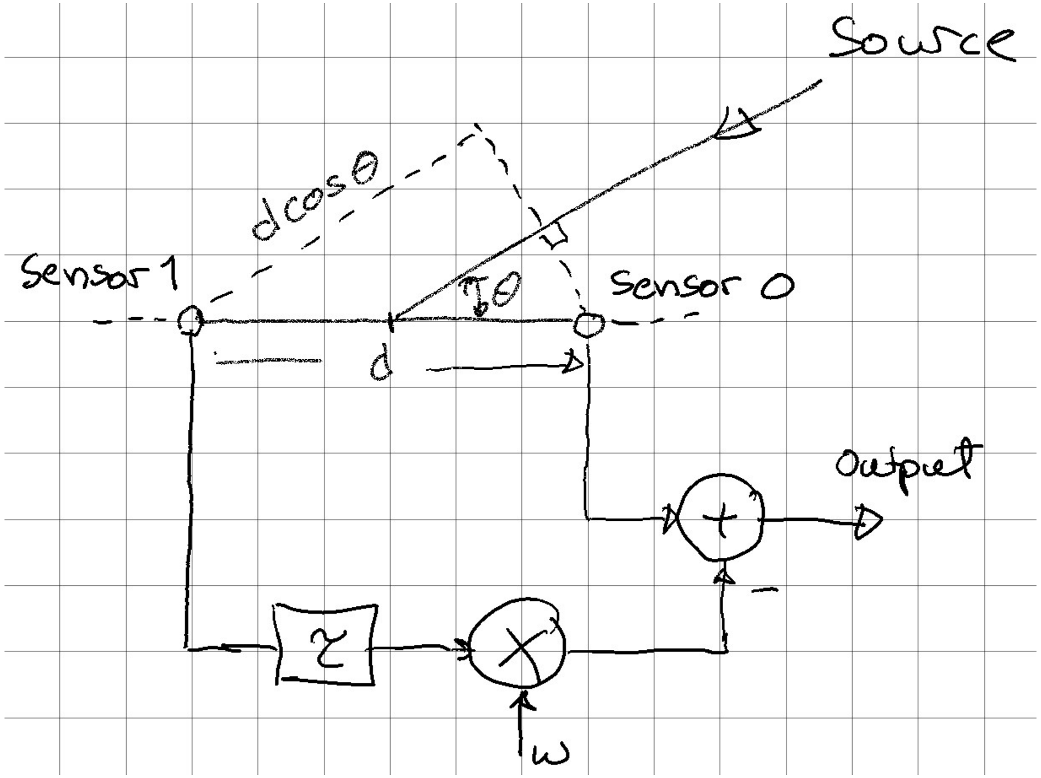

- Uncini, Aurelio. *Fundamentals of adaptive signal processing*. Springer International Publishing, 2015.
  - Sect. 9.4.2 Differential Sensor Array
- Elko, Gary W., and Jens Meyer. "Microphone arrays." *Springer handbook of speech processing*. Springer, Berlin, Heidelberg, 2008. 1021-1041.
- Elko, Gary W. "Differential microphone arrays." *Audio signal processing for next-generation multimedia communication systems*. Springer, Boston, MA, 2004. 11-65.
- Wikipedia: Yagi–Uda antenna

# Two-element array

- Conventional:
  - Sum: FIR Low-pass beamforming: **steers peak**
- Differential, end-fire
  - Simplest case: $\tau=0$, $w_0=w_1$
  - Difference: FIR High-pass beamforming: **steers null**

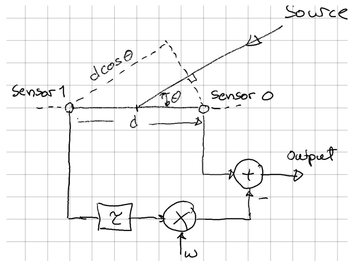



Uncini, 2015, Fig. 9.24

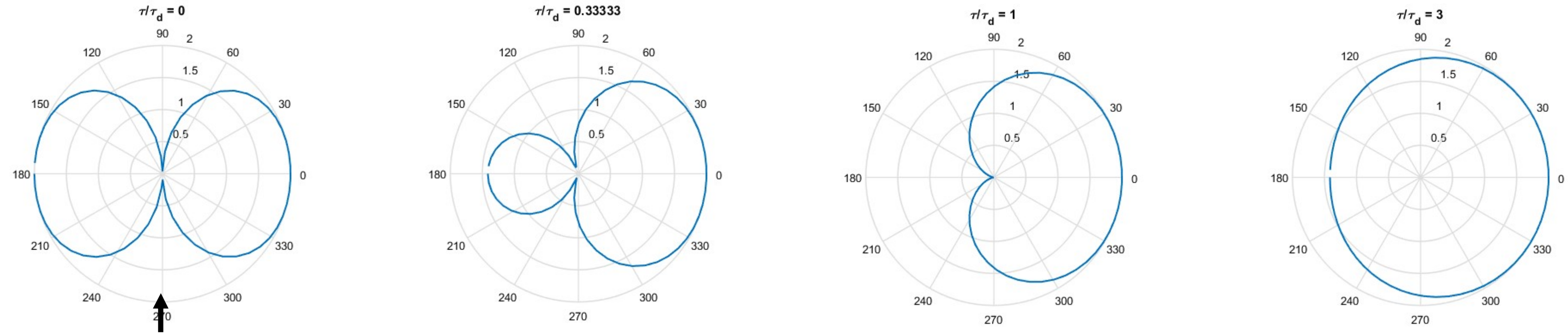



$$R(\omega, \theta) = 1 - w\mathrm{e}^{-\mathrm{j}\omega(\tau + \tau_d \cos\theta)}, \quad \tau_d = d/c$$

Weight will be used to model propagation effects due to spherical spreading, 1/r-effect, in the near-field




# Beampattern, effect of delay at f=fc, w=1




Sharp null, good
for direction finding

Dipole (figure eight), $\tau/\tau_d = 0$,

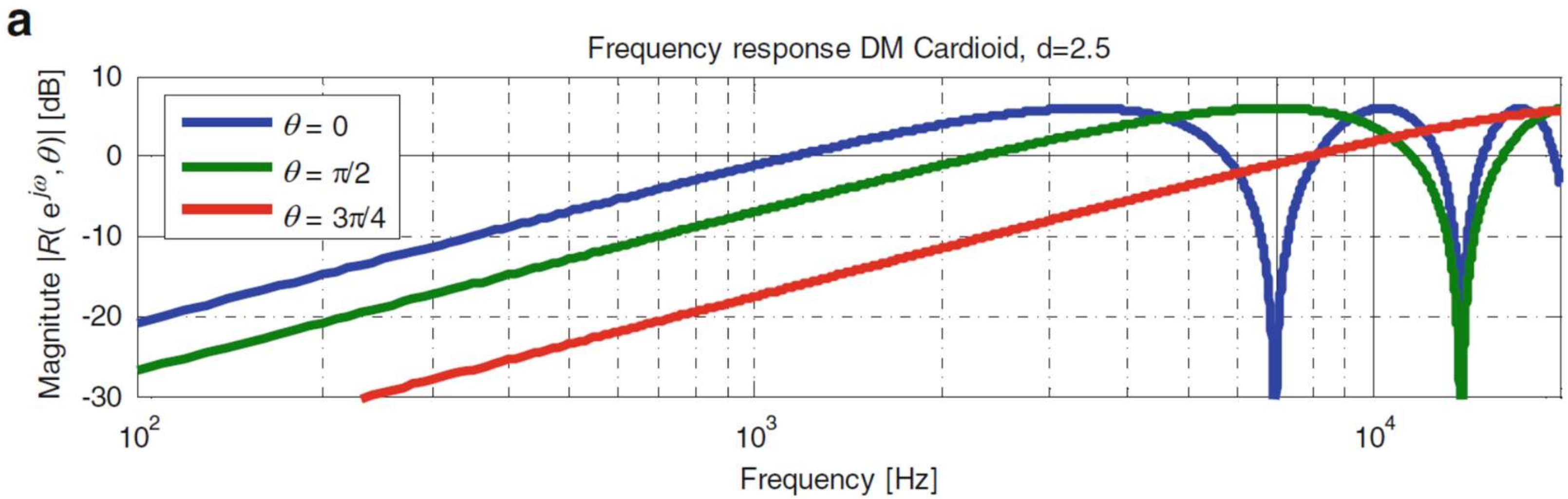
hypercardioid, $\tau/\tau_d = 1/3$,

cardioid, $\tau/\tau_d = 1$,
[Most common mic]

almost omni $\tau/\tau_d = 3$

Split beam echo sounder --- Microphone / Yagi antenna ---

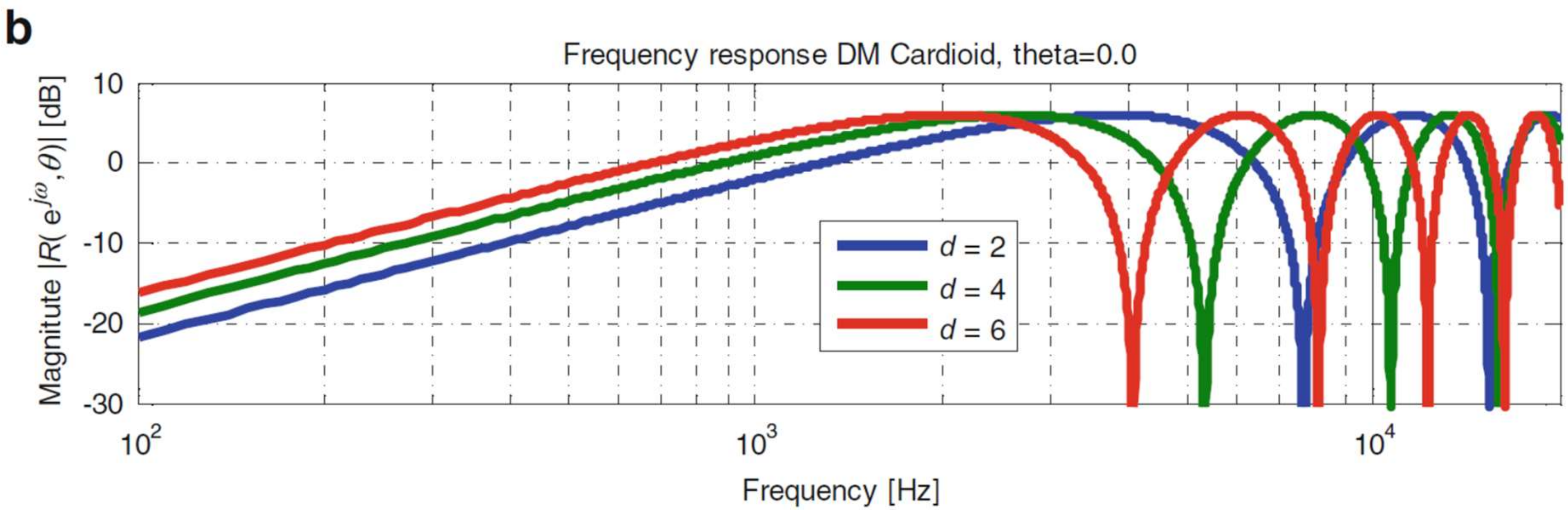
# Cardioid, $\tau = \tau_d$ : Dependency of angle of incidence




Uncini, Fig 9.29a, d=2.5 cm. Max gain = $10\log 2^2$ = 6 dB

$$R(\omega, \theta) = 1 - w\mathrm{e}^{-\mathrm{j}\omega(\tau + \tau_d \cos\theta)}, \quad \tau_d = d/c$$

# Cardioid, $\tau = \tau_d$ : Dependency of element distance

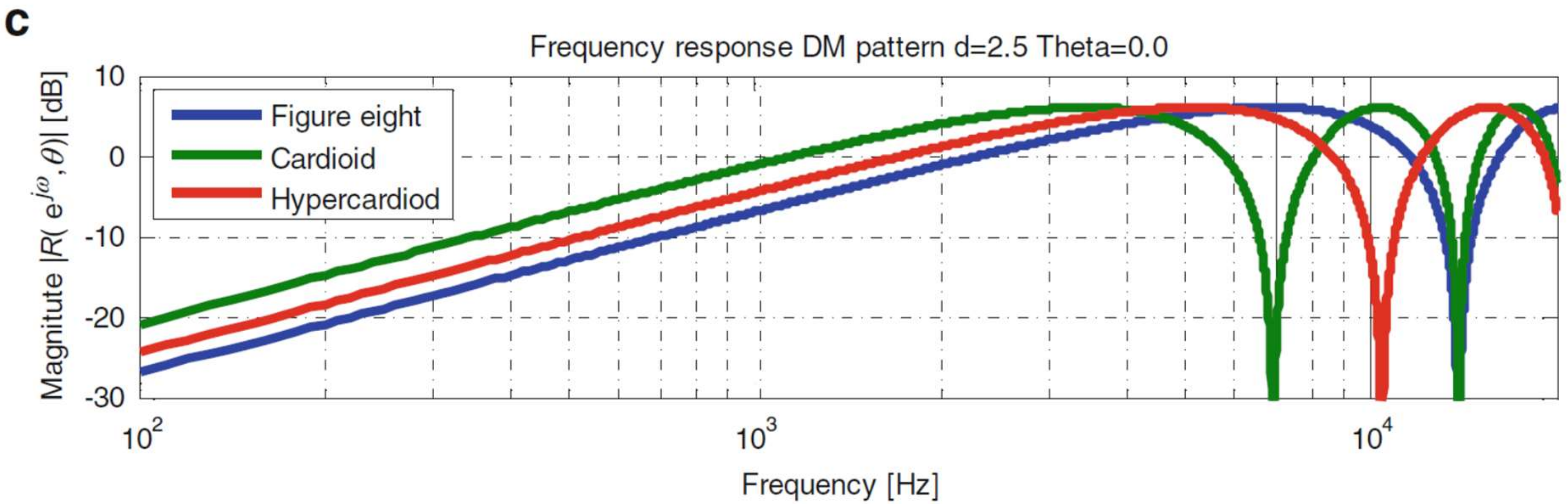



Uncini, Fig 9.29b

$$R(\omega,\theta) = 1 - w\mathrm{e}^{-\mathrm{j}\omega(\tau+\tau_d\cos\theta)}, \quad \tau_d = d/c$$

# Dependency of different patterns

Zeroes, maxima




Uncini, Fig 9.29a, d=2.5 cm

$$R(\omega, \theta) = 1 - w\mathrm{e}^{-\mathrm{j}\omega(\tau+\tau_d \cos\theta)}, \quad \tau = 0, \ \ 0.33d/c, \ \ d/c$$

Figure-8, hypercardioid, cardioid

# Cut-off frequency and angularity

- Zeroes of $R(\omega,\theta)$: $\omega(\tau + \tau_d \cos\theta) = 0,\ 2\pi$
- Maximum of $R(\omega,\theta)$ gives cut-off frequency:

$$\omega_c(\tau + \tau_d \cos\theta) = \pi \quad \Rightarrow \quad \omega_c = \pi/(\tau + \tau_d) \quad [\theta = 0]$$

- Well below cut-off, phase is small:

$$R(\omega,\theta) = 1 - e^{-j\omega(\tau + \tau_d \cos\theta)} \approx j\omega(\tau + \tau_d \cos\theta)$$

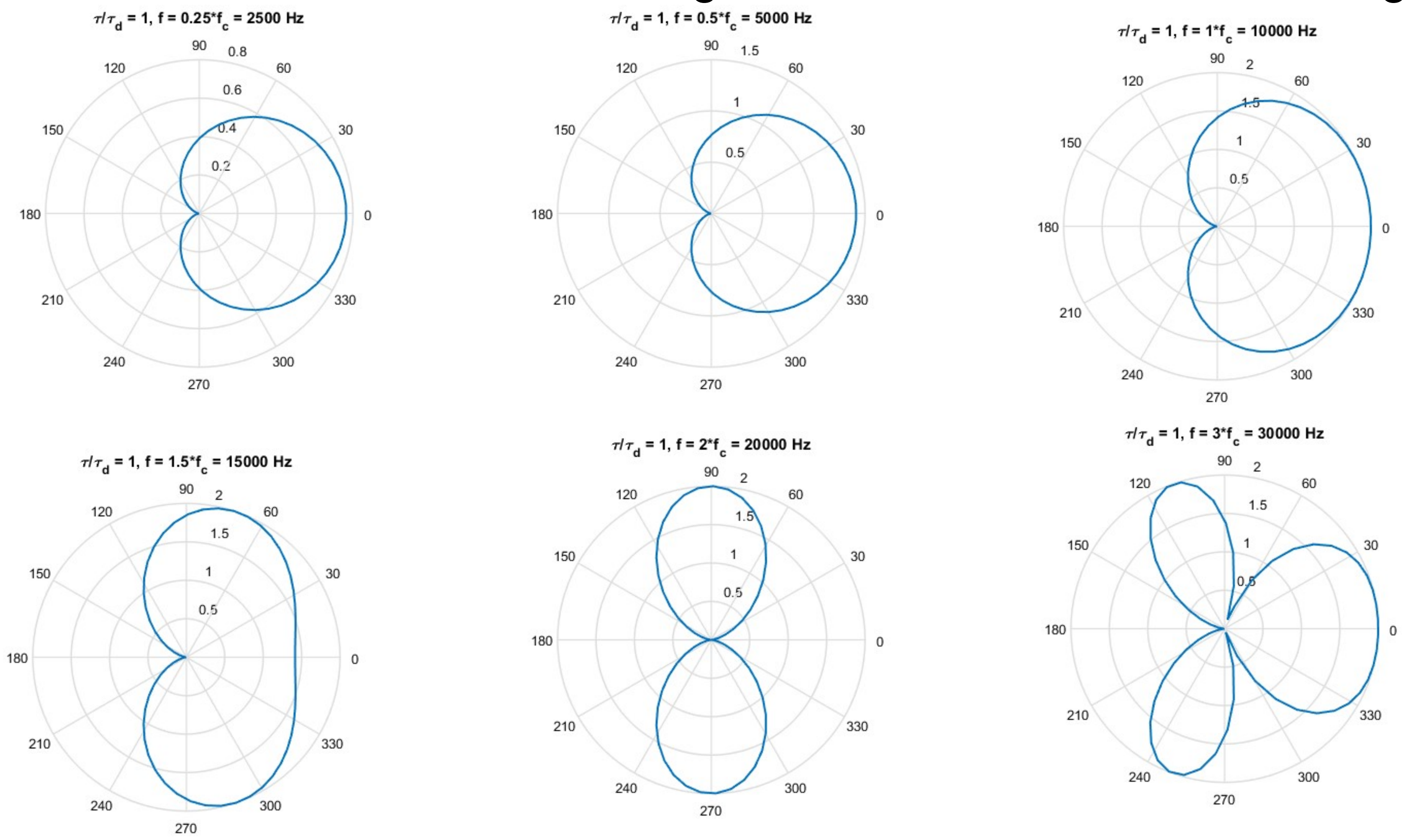
  - Gain proportional with frequency: compensated
  - **Same angle dependency for all frequencies**

# Cut-off frequency

$$f_c = \frac{\omega_c}{2\pi} = 0.5/(\tau + \tau_d) = 0.5/(\tau + \frac{d}{c})$$

- Cardioid: $f_c = \frac{c}{4d}$
- Uncini examples, d=2.5 cm: $f_c$ = 3.4 kHz
- Next examples, d=8.5 mm: $f_c$ = 10 kHz

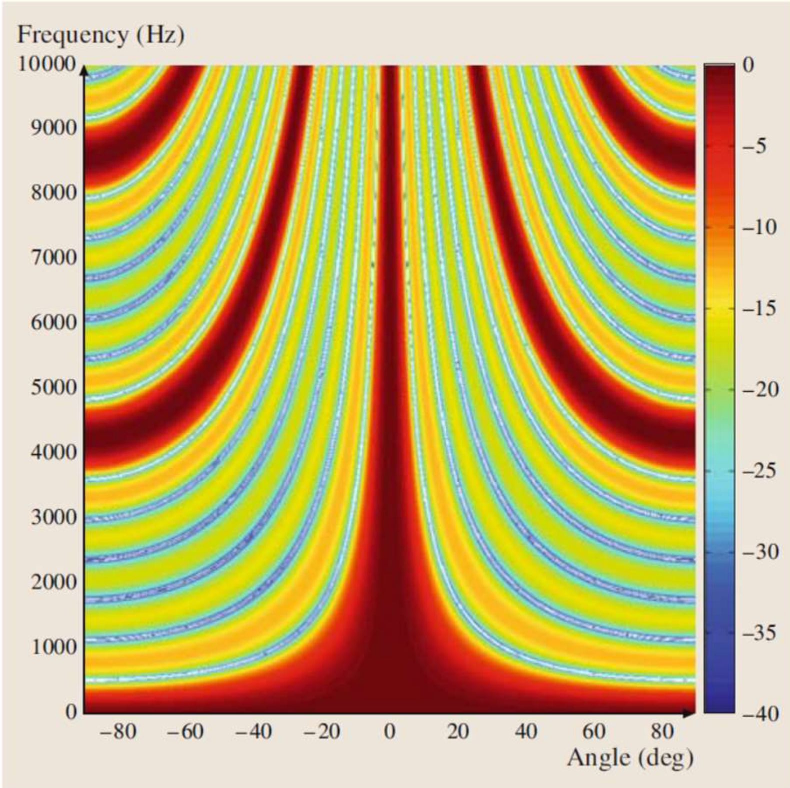
# Relatively insensitive to frequency: Cardioid below $f_c$, breaks up above $f_c$



# Nothing like the large frequency variation for the Uniform Linear Array

$$\theta_{BW} \approx \lambda/D = \frac{c}{Df}$$

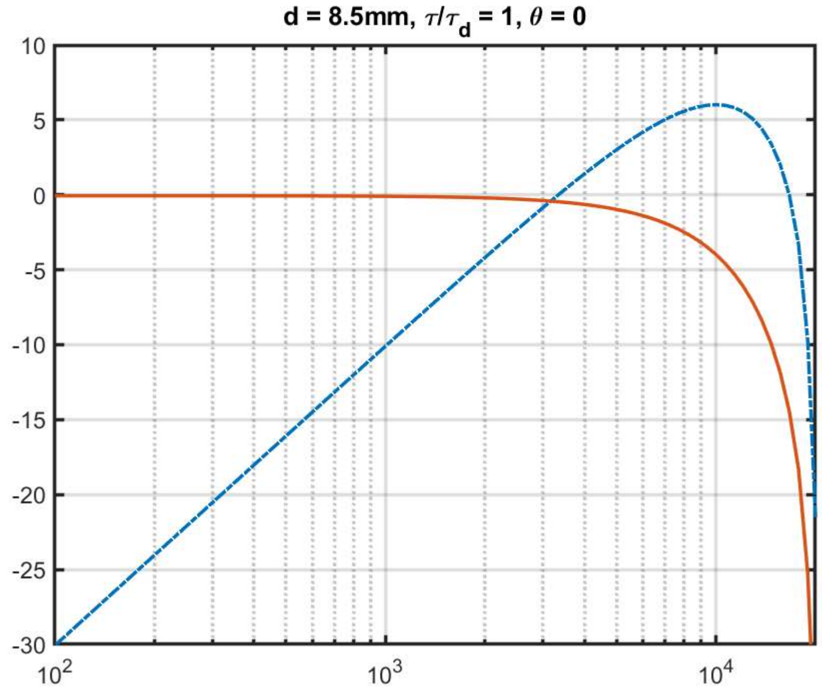
- Seven-element uniformly spaced array, d=8, unsteered
- $d = \lambda$ @ f=4250 Hz (c=340 m/s)




Elko and Meyer. «Microphone arrays» 2008

# Pressure gradient microphone: compensated for 6 dB/octave

Compensated by the mass of the diaphragm



# Difference array: Quite sensitive to parameter variations

- Broadband array needs an equalizer to boost low frequencies ⇔ large sensitivity to low-frequency self-noise
- Element distance, d << $\lambda$
  - But not too small, otherwise sensitivity to noise increases
- Higher order differential arrays are even more sensitive

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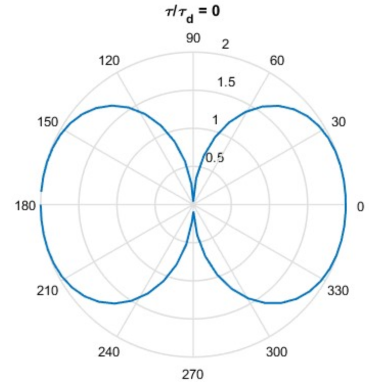
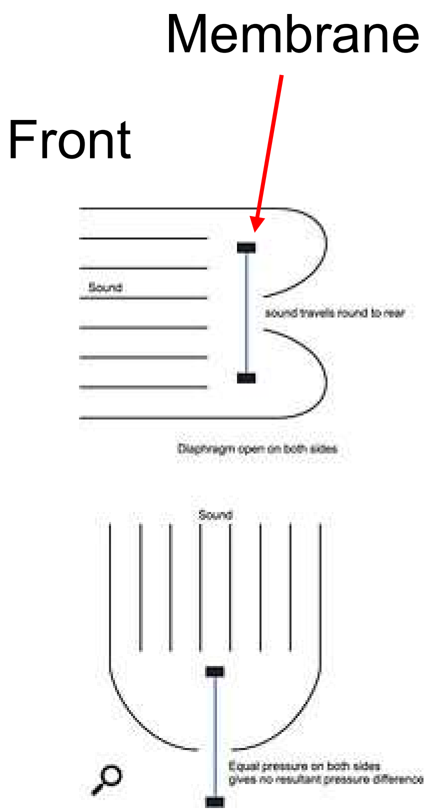



# Figure-of-eight microphone

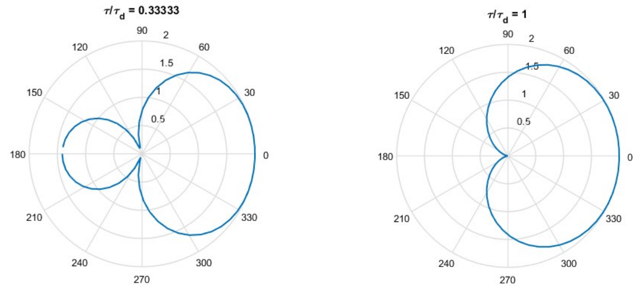



If the mic diaphragm is open to the air on one side but closed at the other, it is considered to be pressure-operated: although it reacts to air pressure, it is not sensitive to direction, resulting in an **omnidirectional** mic pattern.

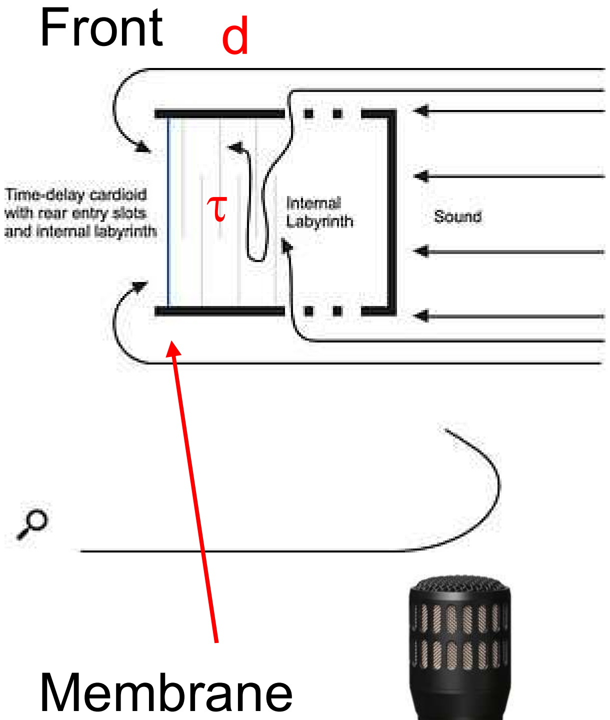
Where the diaphragm is **open on both sides**, as in this diagram, it responds to the pressure-gradient (the difference between the pressure at the front and the back of the diaphragm).

In this case, sound from the side results in even pressure on both sides of the diaphragm, which is why **figure-of-eight** mics reject sound from the side, but are responsive to both the front and rear.

https://www.soundonsound.com/techniques/using-microphone-polar-patterns-effectively




# Cardioid microphone





Most cardioid mics now incorporate a vented 'labyrinth' in a single-capsule design that manipulates the phase of sounds hitting the rear, to produce the desired cardioid pattern.

The supercardioid and hypercardioid designs use the same principle to create a more focused pattern to the front, at the expense of reducing the rear rejection.

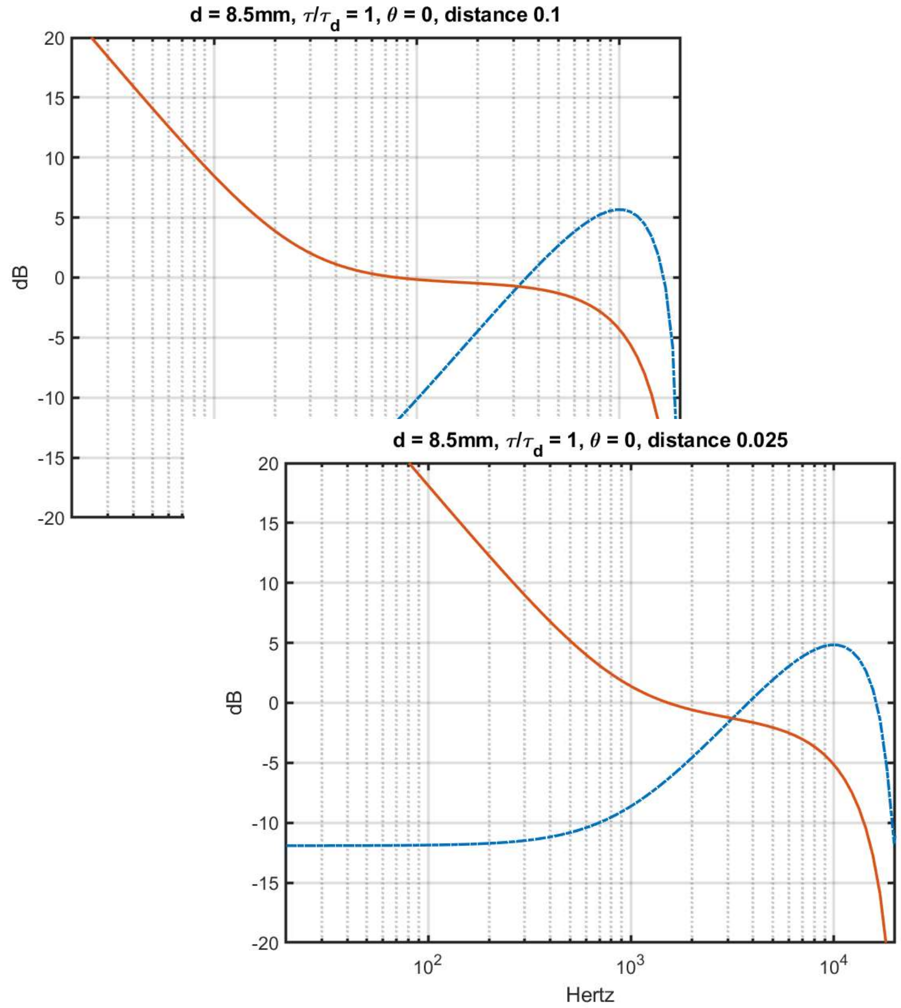
**If you notice vents at the side of the mic head, the mic probably has a cardioid pattern (or a variation on it).**

https://www.soundonsound.com/techniques/using-microphone-polar-patterns-effectively

# Proximity effect of cardioid: bass boost when close-in

Difference due to 1/r:

- d=10 cm:
  - 1. element: 10 cm
  - 2. el.: 10+0.85 cm
  - Effective w = 10/10.85 = 0.92
- d=2.5 cm:
  - w= 10/12.5 = 0.8



# Exact differential array

- Plane wave (far-field) pressure field:

$$p(r,t) = A_0 e^{\mathrm{j}(\omega_0 t - kr\cos\theta)}$$

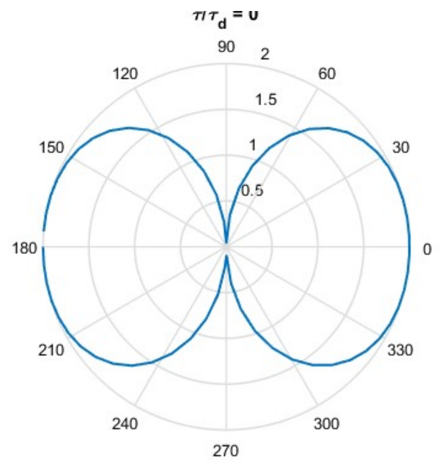
- Spatial derivative (dropping time):

$$\left|\frac{\mathrm{d}}{\mathrm{d}r} p(k,r)\right| = \mathrm{j}k\cos\theta A_0 e^{-\mathrm{j}kr\cos\theta}$$

- Beam pattern shape

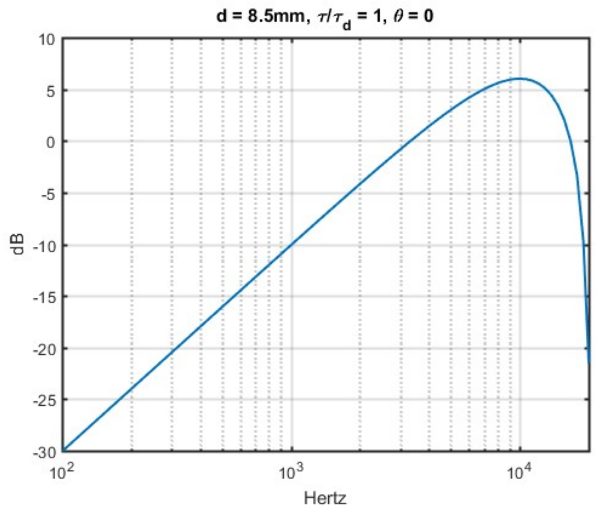
$$\propto k\cos\theta = \frac{\omega}{c}\cos\theta$$

  – ~no-delay difference array (τ=0)



# Differential array (cont)

- $\mathrm{j}k\cos\theta = \frac{\mathrm{j}\omega}{c}\cos\theta$
  - high-pass 6 dB/octave
- Pressure gradient from conservation of linear momentum (Euler):




$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla p \Rightarrow \nabla p \propto \mathrm{j}\omega |v|$$

- Therefore called pressure gradient or velocity microphone

# Far-field: n'th order differential array

$$\frac{\mathrm{d}^n}{\mathrm{d}r^n} p(k,r) = A_0 (\mathrm{j}k\cos\theta)^n e^{-\mathrm{j}kr\cos\theta}$$

- Beampattern $\propto \cos^n\theta$
- Frequency response: $\propto \omega^n$: 6n dB/octave

# Near-field

Pressure: $p(r,t) = A_0 e^{\mathrm{j}(\omega_0 t)} \frac{e^{-\mathrm{j}k_0 r \cos\theta}}{r}$

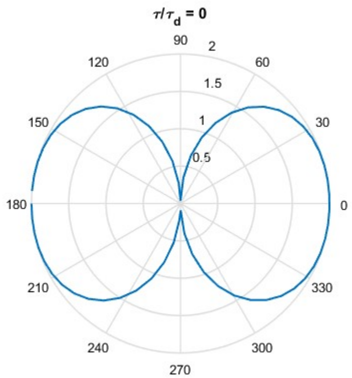
$$\frac{\mathrm{d}^n}{\mathrm{d}r^n} p(k,r,\theta) = A_0 \frac{n!}{r^{n+1}} \mathrm{e}^{-\mathrm{j}kr\cos\theta} (-1)^n \sum_{m=0}^{n} \frac{(\mathrm{j}kr\cos\theta)^m}{m!}$$

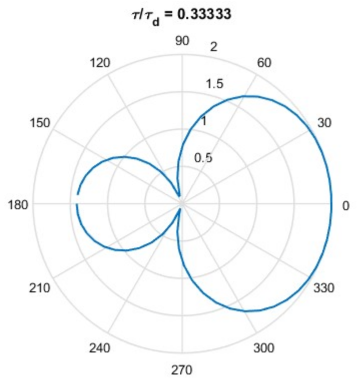
- Sum of dipole-like terms of type $\cos^m\theta$
- May optimize coefficients for desirable properties
- Compare to differential array, n=1:

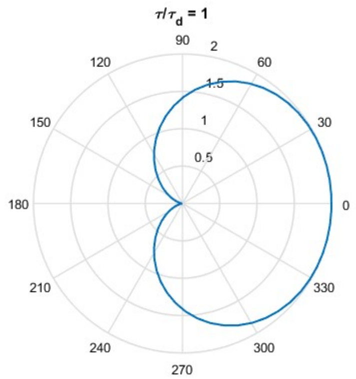
$$R(\omega,\theta) = 1 - \mathrm{e}^{-\mathrm{j}\omega(\tau+\tau_d\cos\theta)} \approx \mathrm{j}\omega(\tau+\tau_d\cos\theta)$$

# Differential array in practice

- Approx. by finite differences, d << λ.
- Response: $\propto \omega^n \left(a_0 + a_1 \cos\theta + a_2 \cos^2\theta + \ldots + a_n \cos^n\theta\right)$
- n=1: Figure-of-8 $a_0$=0, $a_1$=1
  Hypercardioid $a_0$=1/4 $a_1$=3/4
  Cardioid $a_0$=1/2 $a_1$=1/2

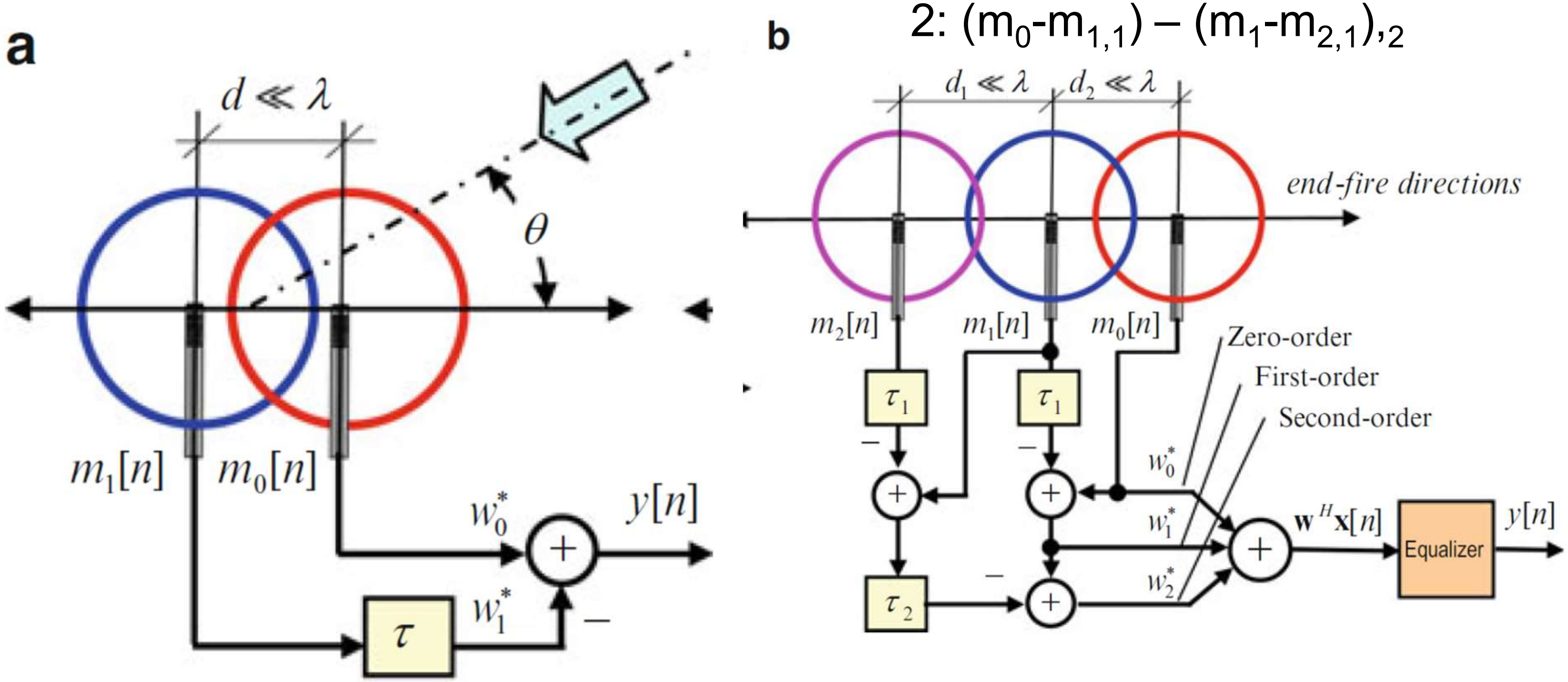


# First, second order differential array

0: $m_0$
1: $m_0 - m_{1,1}$
2: $(m_0-m_{1,1}) - (m_1-m_{2,1}),_2$




Uncini, 2015, Fig. 9.24

# Maximum directional gain, n=1…4

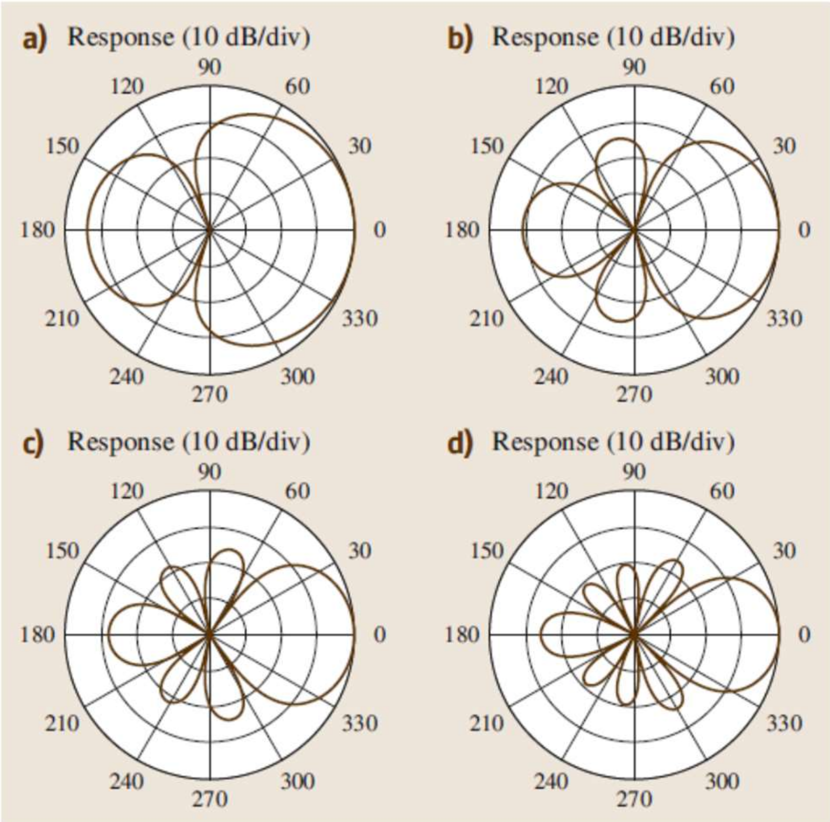


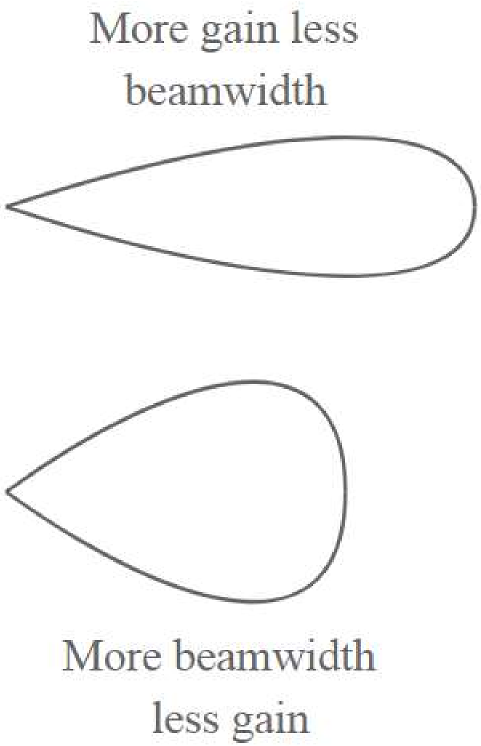

**Fig. 50.12a–d** Directional patterns that have maximum directional gain for differential microphone arrays for up to forth-order arrays

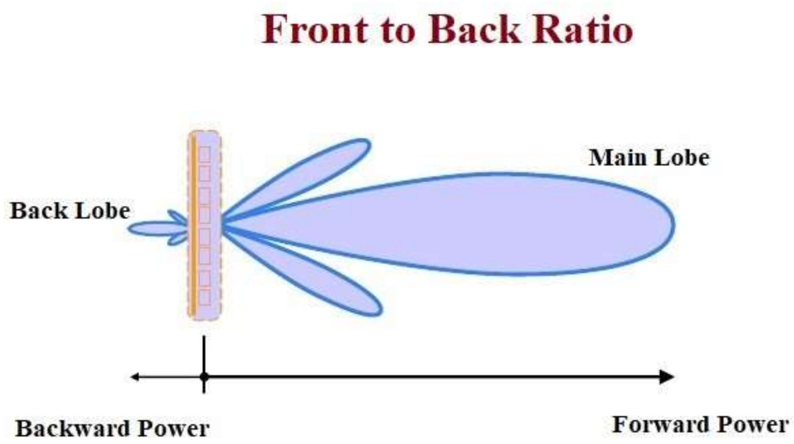
n=1: hypercardioid

n=2: Narrower beam than that of $\cos^2$

Elko and Meyer. «Microphone arrays» 2008

# Performance metrics: Beamwidth, Gain, Front/Back




https://www.electronics-notes.com/articles/antennas-propagation/yagi-uda-antenna-aerial/gain-directivity.php
https://www.everythingrf.com/community/what-is-front-to-back-ratio-in-an-antenna

# Optimization, first order

## Maximum gain

- n=1: hypercardioid

$$E_{HC_1}(\theta) = \frac{1 + 3\cos\theta}{4}.$$

- Array gain: 20log(n+1)= 10log$N^2$
  - N is no of elements

## Best front-back ratio

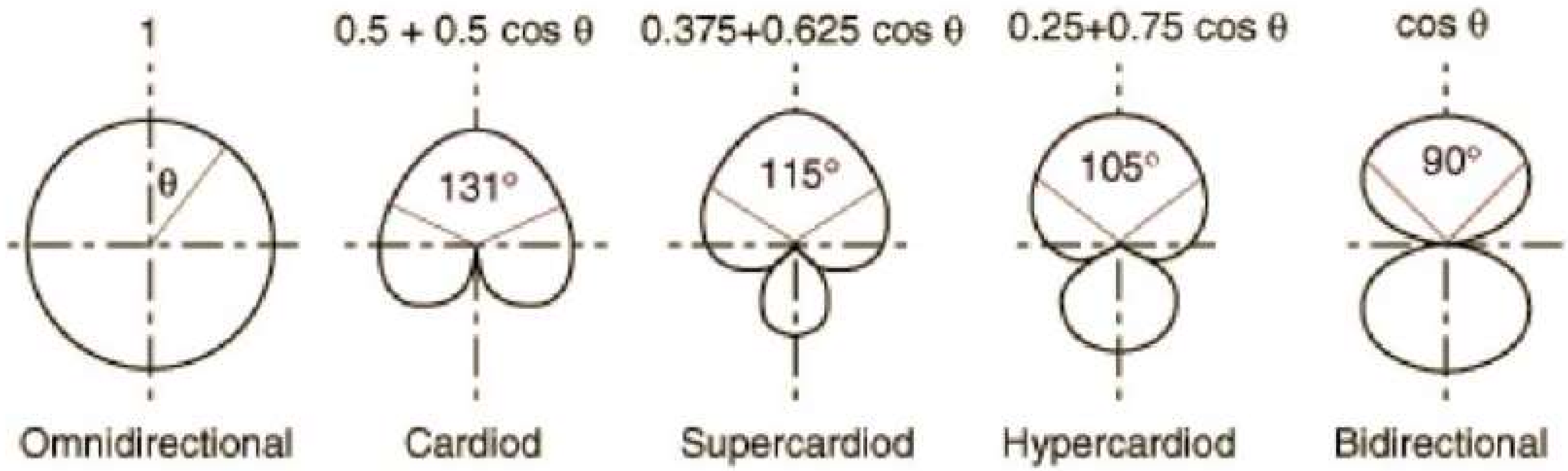
- n=1: supercardioid

$$E_{SC_1}(\theta) = \frac{\sqrt{3} - 1 + (3 - \sqrt{3})\cos\theta}{2}.$$

Elko, "Differential microphone arrays", 2004

# Hyper- vs super-cardioid
# Max gain vs best front/back-ratio

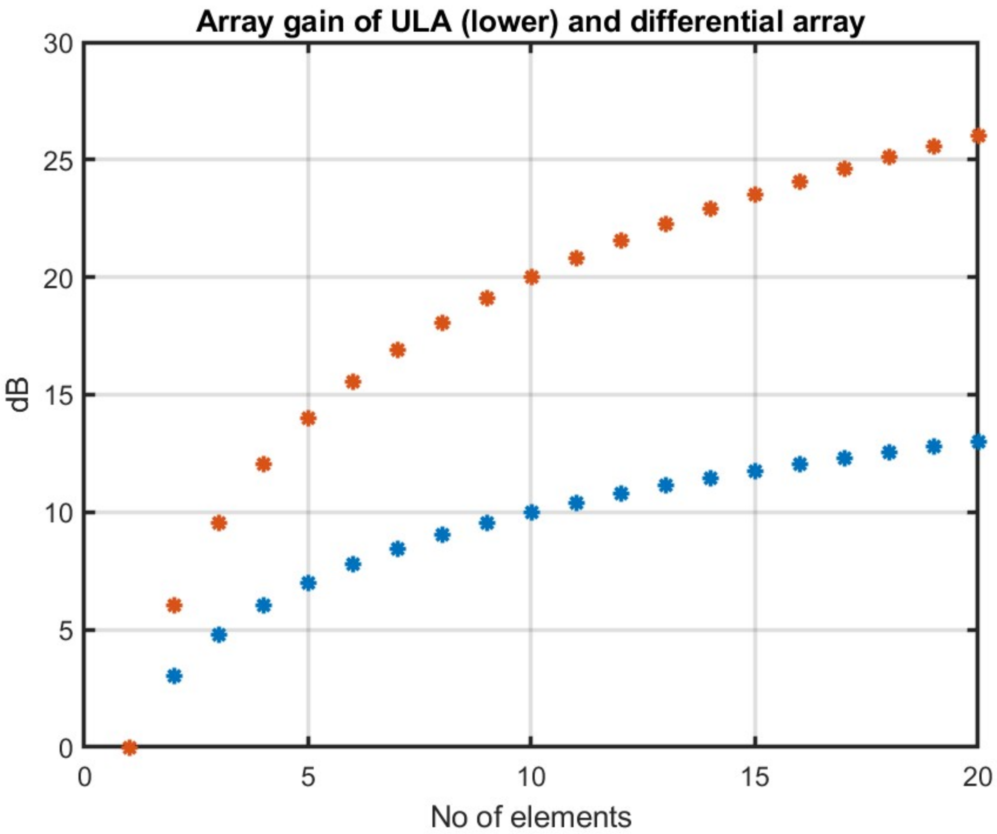
Theory: 0.366, 0.634




Best front/back-ratio, maximum gain

Quaranta, Dimino, D'altrui, General guidelines for acoustic antenna designed for beamforming noise source localization, 2007

# Maximum theoretical array gain, end-fire vs uniform linear array

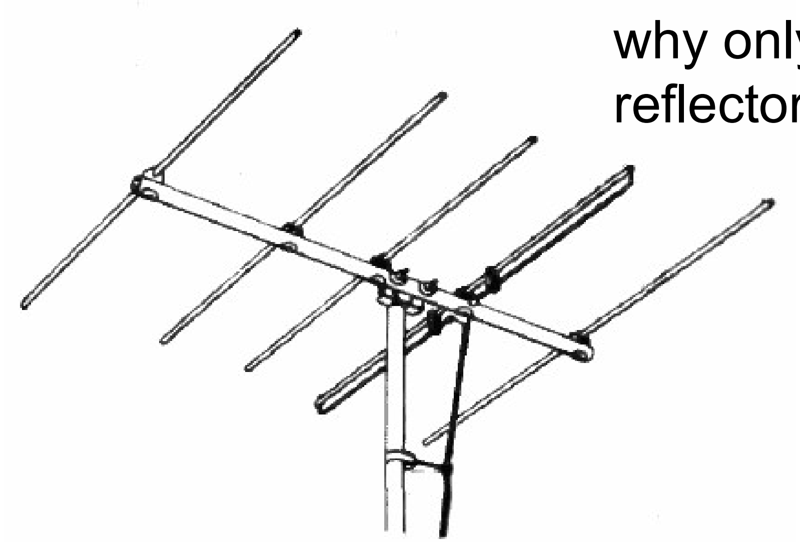



$$AG = 10 \log N^2, \quad BW \propto \frac{1}{\sqrt{N}}$$

$$AG = 10 \log N, \quad BW \propto \frac{\lambda}{D} = \frac{2}{N}$$

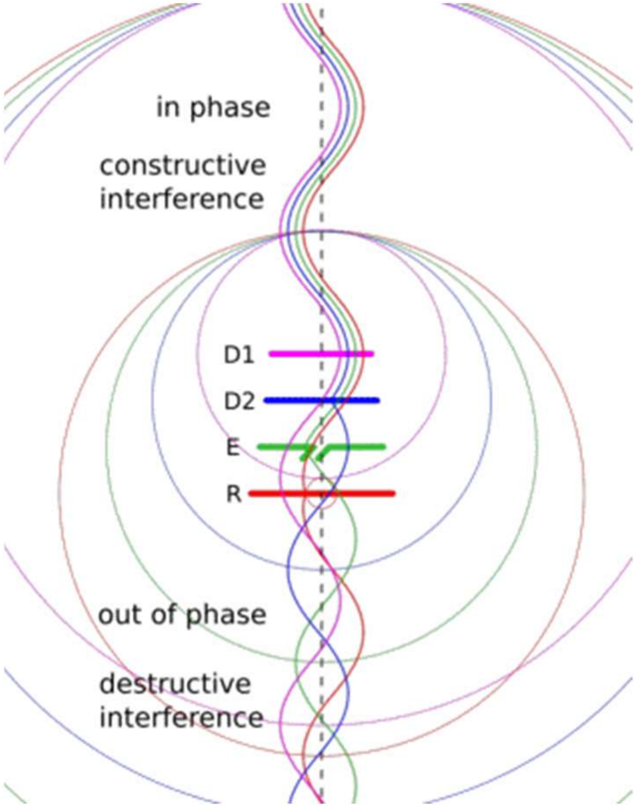
# Yagi-Uda antenna: Parasitic elements

- Single active element
  - Length $\lambda/2$ => narrowband
- Passive elements:
  - Reflector
    - typ 5% longer: Capacitive reactance - voltage phase lags that of the current
  - Directors:
    - typ 5% shorter: Inductive reactance - current phase lags phase of the voltage
    - Ex. 3 or 17 directors



why only 1 reflector?

# Phasing animation (from Wikipedia)



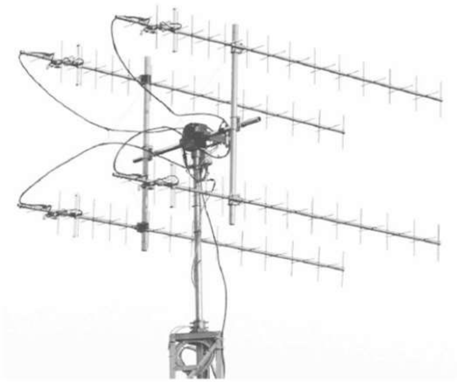

- Time delays due to element distance
- Reradiation from passive elements (parasitic)
  - Field behind first reflector is nearly canceled
- Inherently narrow-band

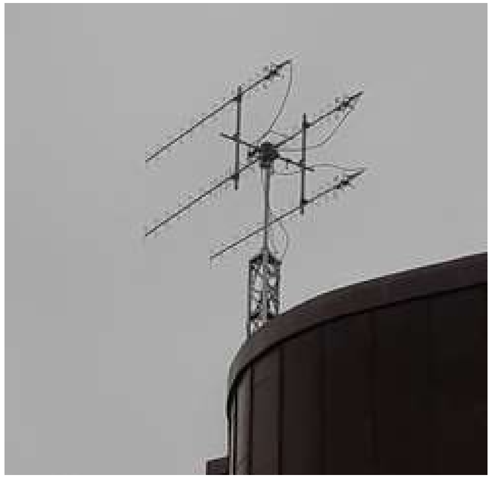
*Uda, S., 1925, "On the Wireless Beam of Short Electric Waves". Journ. Institute of Electrical Engineers of Japan*

*Yagi, Hidetsu; Uda, Shintaro, 1926,"Projector of the Sharpest Beam of Electric Waves" Proc. of the Imperial Academy of Japan.*

# Physics department, UiO

- CubeSTAR:
  - 437.465 MHz, $\lambda$=0.686 m
  - 432-438 MHz amateur band
- Circularly polarized
  - 4 x 436CP30, each with:
  - 2 x (13 directors+1 reflector+1 driven element) = 30 elements
- 4 stacked together
  - 1.143 m=1.67 $\lambda$, gain=20.5 dB, -3 dB beamwidth=16 deg





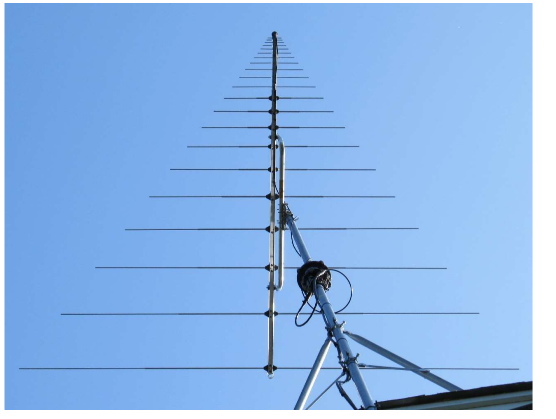
Eirik Vikan, UiO Satellite Ground Station: Simulation, Implementation and Verification, MSc, 2011

https://www.duo.uio.no/bitstream/handle/10852/11067/Eirik_Vikan_UiO_Satellite_Ground_Station_Simulation_Implementation_and_Verification.pdf

https://www.m2inc.com/FG436CP30

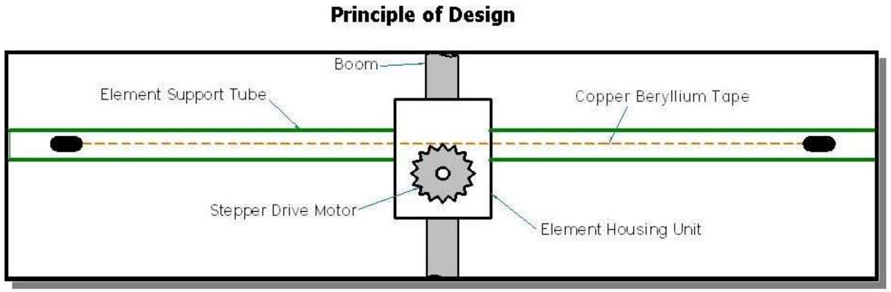
# Broadband? Log periodic array

- VHF/UHF, 50-1300 MHz
- 21 elements, 2 m boom
- Forward gain: 10 to 12 dBi (rel isotropic)
  - Like a 4-5-element Yagi
- Complex as several or all elements are active
- Create CLP-5130-1N



# Broadband? Adjustable element lengths

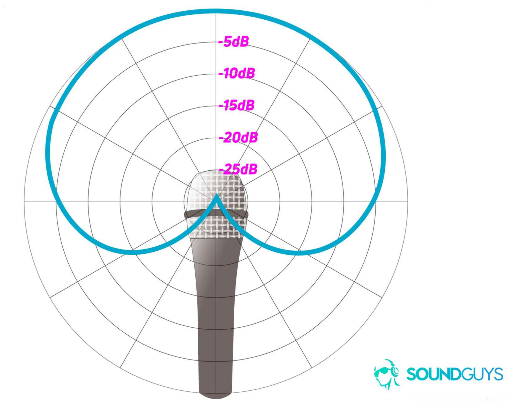
Multi-frequency

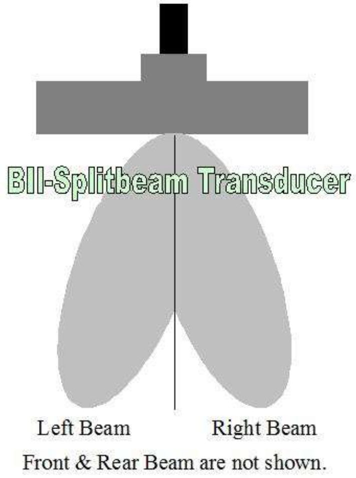




3-element adjustable Yagi,
Russian Antarctica Base:
RI1ANR 14-52 MHz

# Differential arrays

- Array gain up to $N^2$ vs N for ULA
- Frequency-independent beampattern
- Sensitive designs
  - commonly N=2




- Microphone: proximity effect
- Yagi antenna: narrowband