End-fire or differential arrays – from cardioid microphones to Yagi antennas

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Seemingly dissimilar applications

• Microphones
• Yagi antennas
• Split beam echo sounder
• Vector sensors
End-fire arrays

Broadside: $\sin \theta_1$, $\theta_1$ angle relative to broadside direction

End-fire: $\cos \theta_2 = \sin \theta_1$, as $\theta_2 = 90 - \theta_1$ angle rel to end-fire direction
Differential arrays – from cardioid microphones to Yagi antennas

• Part 1:
  Directional microphones (2nd order arrays)

• Part 2:
  Nth order arrays and Yagi-Uda antennas
N-element end-fire vs broadside array

• Broad-side:
  – Element distance: $\sim \lambda/2$
  – Array gain: max N
  – Beamwidth: $\theta \propto \frac{\lambda}{D} = \frac{c}{D f}$ or $\frac{\lambda}{D \big|_{D=N\lambda/2}} = \frac{2}{N}$

• End-fire:
  – Element distance: $<< \lambda/2$
  – Array gain: $N^2$ (theoretical maximum)
    • Super-directive or supergain
  – Almost frequency-independent beam pattern
Large frequency variation for the Uniform Linear Array

\[ \theta_{BW} \approx \frac{\lambda}{D} = \frac{c}{Df} \]

- 7-element uniformly spaced array, \( d=8 \text{ cm} \), unsteered
- \( d = \lambda \) at \( f=4250 \text{ Hz} \) (\( c=340 \text{ m/s} \))

Elko and Meyer. «Microphone arrays» 2008
Two-element array: filter interpretation

- **Conventional:**
  - Sum: FIR Low-pass beamforming: **steers peak**

- **Differential, end-fire**
  - Simplest case: $\tau=0$, $w_0=w_1$
  - Difference: FIR High-pass beamforming: **steers null**
• Weight will later be used to model propagation effects due to spherical spreading, 1/r-effect, in the near-field
• Sensors 1,2 could be the ears of an owl with τ due to internal coupling

$$\tau_d \cos \theta = \frac{d \cos \theta}{c} = \text{acoustic delay}$$

$$\tau = \text{processing delay, often mechanical implementation of processing delay}$$

$$R(\omega, \theta) = 1 - we^{-j\omega(\tau + \tau_d \cos \theta)}$$

$$\tau_d = \frac{d}{c}$$
Beampattern, effect of processing, \( w=1, f=fc: \tau + d/c = \text{half a period at } fc \)

Dipole (figure eight), \( \tau / \tau_d = 0, \)

hypercardioid, \( \tau / \tau_d = 1/3, \)

cardioid, \( \tau / \tau_d = 1, \)

almost omni \( \tau / \tau_d = 3 \)

Sharp null, good for direction finding

Split beam echo sounder

Microphone / Yagi
- speaker mic -
- common mic for vocals -
Cardioid, $\tau = \tau_d$: 
Dependency of angle of incidence

Note, same angle dependency for all frequencies $< f_c$

Uncini, Fig 9.29a, $d=2.5$ cm. Max gain = $10\log 2^2 = 6$ dB

$$R(\omega, \theta) = 1 - we^{-j\omega(\tau + \tau_d \cos \theta)}, \quad \tau_d = d/c$$
Cardioid, $\tau = \tau_d$:
Dependence of element distance [cm]

Uncini, Fig 9.29b

$$R(\omega, \theta) = 1 - we^{-j\omega(\tau + \tau_d \cos \theta)}, \quad \tau_d = d/c$$
Dependency of different patterns

Uncini, Fig 9.29a, d=2.5 cm

\[ R(\omega, \theta) = 1 - \omega e^{-j\omega(\tau + \frac{d}{c} \cos \theta)}, \quad \tau = 0, \frac{0.33d}{c}, \frac{d}{c} \]

Figure-8, hypercardioid, cardioid
Cut-off frequency and angularity

- Zeroes of $R(\omega, \theta)$: $\omega(\tau + \tau_d \cos \theta) = 0, 2\pi$
- Maximum of $R(\omega, \theta)$ gives cut-off frequency:
  $$\omega_c(\tau + \tau_d \cos \theta) = \pi \implies \omega_c = \frac{\pi}{(\tau + \tau_d)} \quad [\theta = 0]$$

- Well below cut-off, phase is small:
  $$R(\omega, \theta) = 1 - e^{-j\omega(\tau + \tau_d \cos \theta)} \approx j\omega(\tau + \tau_d \cos \theta)$$
  - Gain proportional to frequency: may compensate
  - Same angle dependency for all frequencies
Cut-off frequency: delay=half a period

\[ f_c = \frac{\omega_c}{2\pi} = 0.5/(\tau + \tau_d) = 0.5/(\tau + \frac{d}{c}) \]

- Cardioid, \( \tau = d/c \): \( f_c = \frac{c}{4d} \)

- Uncini examples, \( d=2.5 \) cm: \( f_c = 3.4 \) kHz

- Next examples, \( d=8.5 \) mm: \( f_c = 10 \) kHz
Relatively insensitive to frequency: Cardioid below $f_c$, breaks up above $f_c$
Split beam echosounder

• 2-element transducer
  – Normal beam: add
  – Split-beam: subtract, no processing delay $\tau \rightarrow$ figure-of-8

• Assume a single broadside target
  – Normal beam: a peak
  – Split-beam: a null

• A null is a more precise indicator than a peak for when a target is exactly broadside
Pressure gradient microphone: compensated for 6 dB/octave

Compensated by the mass of the diaphragm

\[ d = 8.5\text{mm}, \quad \pi r_d = 1, \quad \theta = 0 \]
Difference array: Quite sensitive to parameter variations

- Broadband array needs an equalizer to boost low frequencies ⇔ large sensitivity to low-frequency self-noise
- Element distance, \( d << \lambda \)
  - But not too small, otherwise sensitivity to noise increases
- Higher order differential arrays are even more sensitive
If the mic diaphragm is open to the air on one side but closed at the other, it is considered to be pressure-operated: although it reacts to air pressure, it is not sensitive to direction, resulting in an omnidirectional mic pattern.

Where the diaphragm is open on both sides, as in this diagram, it responds to the pressure-gradient (the difference between the pressure at the front and the back of the diaphragm).

In this case, sound from the side results in even pressure on both sides of the diaphragm, which is why figure-of-eight mics reject sound from the side but are responsive to both the front and rear.

https://www.soundonsound.com/techniques/using-microphone-polar-patterns-effectively
Cardioid microphone

Most cardioid mics now incorporate a vented 'labyrinth' in a single-capsule design that manipulates the phase of sounds hitting the rear, to produce the desired cardioid pattern. The supercardioid and hypercardioid designs use the same principle to create a more focused pattern to the front, at the expense of reducing the rear rejection.

If you notice vents at the side of the mic head, the mic probably has a cardioid pattern (or a variation on it).

https://www.soundonsound.com/techniques/using-microphone-polar-patterns-effectively
Spherical spreading (from mouth)

http://resource.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial_files/Web-basics-pointsources.htm
Proximity effect of cardioid: bass boost when close-in

Difference due to $1/r$:

- **Distance $d=10$ cm:**
  - 1. element: 10 cm
  - 2. el.: 10+0.85 cm
  - Effective $w = 10/10.85 : 0.92$

- **Distance $d=2.5$ cm:**
  - $w = 2.5/(2.5+0.85) = 0.75$
Neumann U 47: First switchable pattern condenser microphone (1940’s)

Front and rear membranes: cardioid
- Sound coming from the front causes movement of the front membrane and reaches the inner side of the rear membrane through the perforations in the electrode.
- If only one membrane is connected, the microphone works as described above as a cardioid.

When connecting both cardioid halves in parallel, the capsule produces an omnidirectional pattern.

https://en-de.neumann.com/u-47
Shotgun microphone

Characteristics
• Low frequencies, supercardioid
• High: lobar
• Off axis, more sensitive to lower and less to higher frequencies: colored sound.

Applications:
• Film industry: dialog pickup on the shooting set
• Sport events
• Birds at great distances

Polar Response of the Shure VP89L Shotgun Mic.
Shotgun microphone

N^{th} order differential array

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Split beam echo sounder
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Exact differential array

• Plane wave (far-field) pressure field:
  \[ p(r, t) = A_0 e^{i(\omega_0 t - k r \cos \theta)} \]

• Spatial derivative (drop time) = pressure gradient:
  \[ \left| \frac{d}{dr} p(k, r) \right| = j k \cos \theta A_0 e^{-j k r \cos \theta} \]

• Beam pattern shape
  \[ \propto k \cos \theta = \frac{\omega}{c} \cos \theta \]
  – Similar to previous derivation:
  \[ R(\omega, \theta) = 1 - e^{-j \omega (\tau + \tau_d \cos \theta)} \bigg|_{\tau=0} \approx j \omega (\tau_d \cos \theta) \]
  – \sim\text{no-delay difference array} (\tau=0), \text{figure-of-eight}
Differential array (cont)

- \( jk \cos \theta = \frac{j \omega}{c} \cos \theta \)
  - high-pass 6 dB/octave

- Pressure gradient from conservation of linear momentum (Euler):
  \[
  \rho_0 \frac{\partial \mathbf{v}}{\partial t} = - \nabla p \Rightarrow \nabla p \propto j \omega |v|
  \]

- Therefore called pressure gradient or velocity microphone

- If 3-D: vector sensor
Vector sensors
– underwater acoustics: low f.

It is shown that the multichannel receiver using a single vector sensor can offer significant size reduction for coherent acoustic communication at the carrier frequency of 12 kHz, compared with a pressure sensor line array.


This paper proposes a mode domain beamforming method for a 3 x 3 uniform rectangular array of two-dimensional (2D) acoustic vector sensors with inter-sensor spacing much smaller than the wavelengths


Masking from industrial noise can hamper the ability to detect marine mammal sounds near industrial operations, whenever conventional (pressure sensor) hydrophones are used for passive acoustic monitoring. ... Improvements in signal-to-noise ratio of up to 15 dB are demonstrated on bowhead whale calls, which were otherwise undetectable using conventional hydrophones.

Far-field: n’th order differential array

\[ \frac{d^n}{dr^n} p(k, r) = A_0 (jk \cos \theta)^n e^{-jkr \cos \theta} \]

- Beampattern \( \propto \cos^n \theta \)
- Frequency response: \( \propto \omega^n \): 6n dB/octave
Near-field: n’th order differential array

Pressure:

\[ p(r, t) = A_0 e^{i(\omega_0 t)} \frac{e^{-jk_0 r \cos \theta}}{r} \]

\[
\frac{d^n}{dr^n} p(k, r, \theta) = A_0 \frac{n!}{r^{n+1}} e^{-jkr \cos \theta} (-1)^n \sum_{m=0}^{n} \frac{(jkr \cos \theta)^m}{m!}
\]

– Sum of dipole-like terms of type \( \cos^m \theta \)

– May optimize coefficients for desirable properties

– Differential array, n=1, i.e. 2 terms in sum:

\[ R(\omega, \theta) \approx j\omega (\tau + \tau_d \cos \theta) = j\omega (a_0 + a_1 \cos \theta) \]
Differential array in practice

• Approx. by finite differences, $d \ll \lambda$
• Response: $\propto \omega^n \left( a_0 + a_1 \cos \theta + a_2 \cos^2 \theta + \ldots + a_n \cos^n \theta \right)$
• $n=1$: Figure-of-8 $a_0=0, a_1=1$
  Hypercardioid $a_0=1/4, a_1=3/4$
  Cardioid $a_0=1/2, a_1=1/2$

$$R(\omega, \theta) \approx j\omega (\tau + \tau_d \cos \theta), \quad \tau = 0, \tau_d/3, \tau_d$$
First, second order differential array

Uncini, 2015, Fig. 9.24
Beam pattern, $\cos^n$

\[ \propto \omega^n \left( a_0 + a_1 \cos \theta + a_2 \cos^2 \theta + \ldots + a_n \cos^n \theta \right) \]
Maximum directional gain
1st – 4th order

\[ \propto \omega^n \left( a_0 + a_1 \cos \theta + a_2 \cos^2 \theta + \ldots + a_n \cos^n \theta \right) \]

- n=1: hypercardioid
- n=2: Narrower beam than that of \( \cos^2 \)
- \( \approx \) shotgun (lobar) microphone

Elko and Meyer. «Microphone arrays» 2008
Performance metrics:
Beamwidth, Gain, Front/Back

More gain less beamwidth

More beamwidth less gain

Front to Back Ratio

https://www.everythingrf.com/community/what-is-front-to-back-ratio-in-an-antenna
Optimization, first order

\[ E(\theta, \omega) \propto \omega (a_0 + a_1 \cos \theta) \]

**Maximum gain**

- \( n=1: \) hypercardioid  \[
E_{HC1}(\theta) = \frac{1 + 3 \cos \theta}{4}.
\]
- Array gain: 20\log(n+1) = 10\log N^2
  - \( N \) is no of elements

**Best front-back ratio**

- \( n=1: \) supercardioid  \[
E_{SC1}(\theta) = \frac{\sqrt{3} - 1 + (3 - \sqrt{3}) \cos \theta}{2}.
\]

Elko, "Differential microphone arrays", 2004
Hyper- vs super-cardioid
Max gain vs best front/back-ratio

Theory: 0.366, 0.634

Best front/back-ratio, maximum gain

Quaranta, Dimino, D’altrui, General guidelines for acoustic antenna designed for beamforming noise source localization, 2007
Maximum theoretical array gain, end-fire vs uniform linear array

\[ AG = 10 \log N^2, \quad BW \propto \frac{1}{\sqrt{N}} \]

\[ AG = 10 \log N, \quad BW \propto \frac{\lambda}{D} = \frac{2}{N} \]
Yagi-Uda antenna: n-1 parasitic elements

- Single active element
  - Length $\lambda/2$ => narrowband

- Passive elements:
  - Reflector
    - typ 5% longer: Capacitive reactance - voltage phase lags that of the current
  - Directors:
    - typ 5% shorter: Inductive reactance - current phase lags phase of the voltage
    - Ex. here: 3 or 17 directors

Why only 1 reflector?
Phasing animation (from Wikipedia)

• Time delays due to element distance
  – Phasing in element due to length vs wavelength

• Reradiation from passive elements (parasitic)
  – Field behind first reflector is nearly canceled

• Inherently narrow-band

Uda, S., 1925, "On the Wireless Beam of Short Electric Waves". Journ. Institute of Electrical Engineers of Japan

Physics department, UiO

- **CubeSTAR:**
  - 437.465 MHz, $\lambda=0.686$ m
  - 432-438 MHz radio amateur band

- **Circularly polarized**
  - 4 x 436CP30, each with:
    - 2 x (13 directors+1 reflector+1 driven element) = 30 elements

- **4 stacked together (~ broadside array**
  - 1.143 m=$1.67 \lambda$, gain=20.5 dB, -3 dB beamwidth=16 deg

Eirik Vikan, UiO Satellite Ground Station: Simulation, Implementation and Verification, MSc, 2011

https://www.duo.uio.no/bitstream/handle/10852/11067/Eirik_Vikan.UiO_Satellite_Ground_Station_Simulation_Implementation_and_Verification.pdf

https://www.m2inc.com/FG436CP30
Broadband? Log periodic array

- VHF/UHF, 50-1300 MHz
- 21 elements, 2 m boom
- Forward gain: 10 to 12 dBi (rel isotropic)
  - Like a 4-5-element Yagi
- Complex as several or all elements are active

- Create CLP-5130-1N
Broadband? Adjustable element lengths

Multi-frequency
Bi-directional mode

3-element adjustable Yagi,
Russian Antarctica Base:
RI1ANR 14-52 MHz
Differential arrays

- Array gain up to $N^2$ vs $N$ for ULA
- Frequency-independent beampattern
- Sensitive designs
  - Most common mic $N=2$

- Microphone: proximity effect
- Yagi antenna: narrowband
Literature

  – Sect. 9.4.2 Differential Sensor Array

