

UiO : **Department of Physics**
University of Oslo

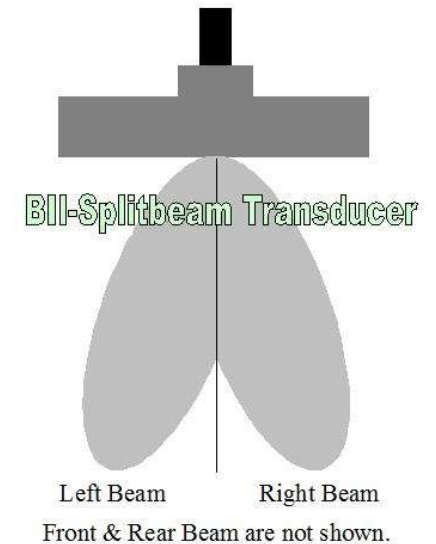
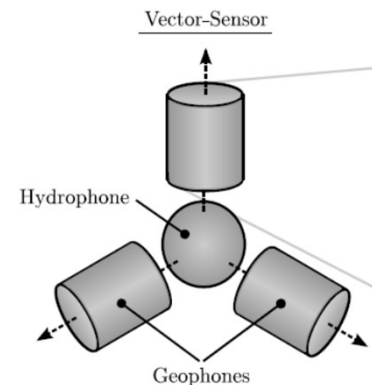
End-fire or differential arrays – from cardioid microphones to Yagi antennas

Sverre Holm

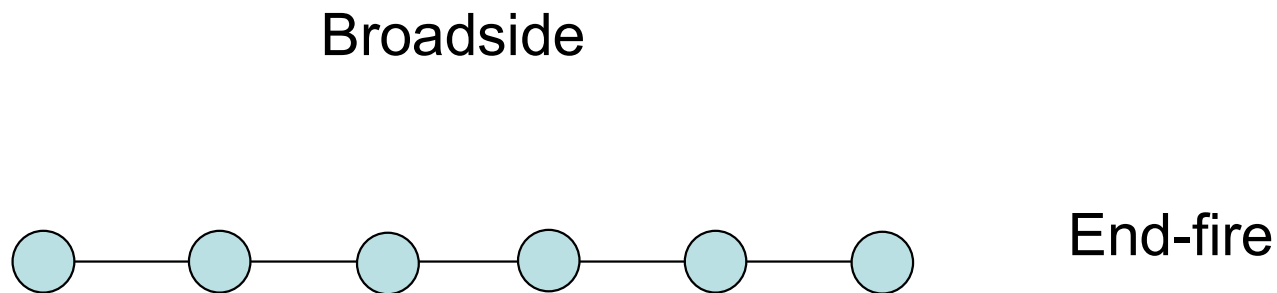


Seemingly dissimilar applications

- Microphones
- Yagi antennas
- Split beam echo sounder
- Vector sensors



End-fire arrays



Broadside:	$\sin \theta_1,$	θ_1 angle relative to broadside direction
End-fire:	$\cos \theta_2 = \sin \theta_1,$	as $\theta_2 = 90 - \theta_1$ angle rel to end-fire direction

Differential arrays – from cardioid microphones to Yagi antennas

- Part 1:
Directional microphones (2nd order arrays)
- Part 2:
Nth order arrays and Yagi-Uda antennas



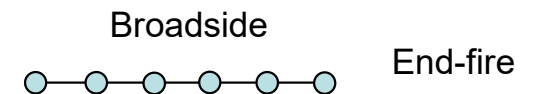
N-element end-fire vs broadside array

- Broad-side:

- Element distance: $\sim \lambda/2$

- Array gain: $\max N$

- Beamwidth: $\theta \propto \frac{\lambda}{D} = \frac{c}{Df}$ or $\frac{\lambda}{D} \Big|_{D=N\lambda/2} = \frac{2}{N}$



- End-fire:

- Element distance: $\ll \lambda/2$

- Array gain: N^2 (theoretical maximum)

- Super-directive or supergain

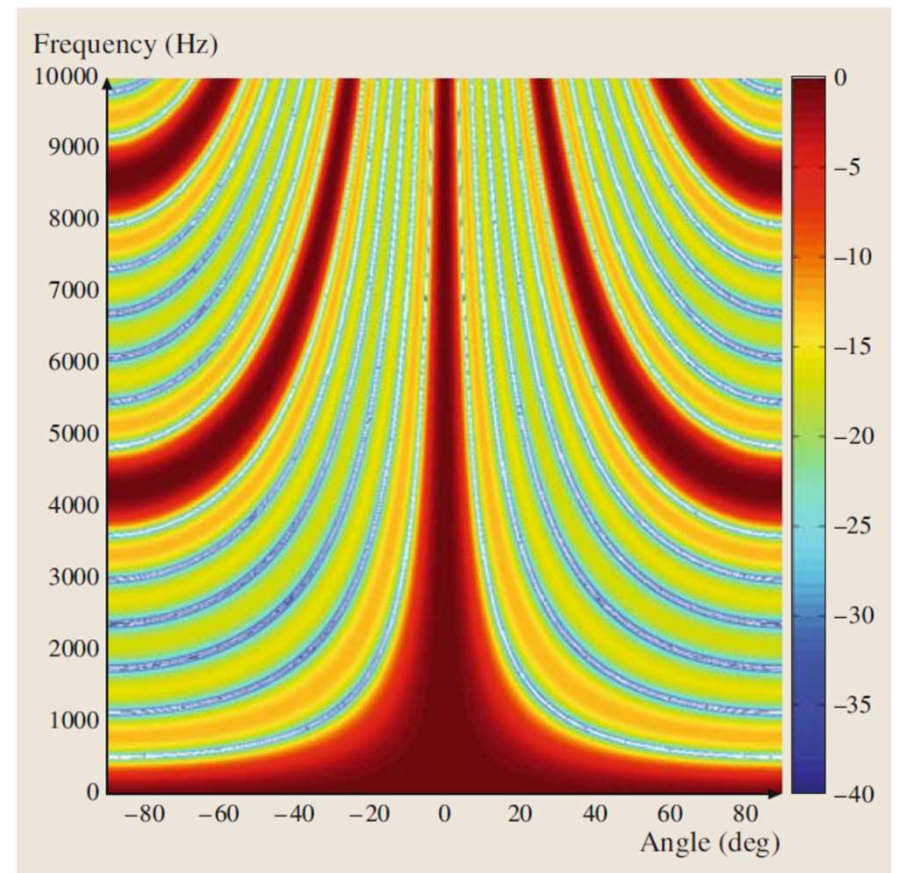
- Almost frequency-independent beam pattern

Large frequency variation for the Uniform Linear Array

$$\theta_{BW} \approx \lambda/D = \frac{c}{Df}$$

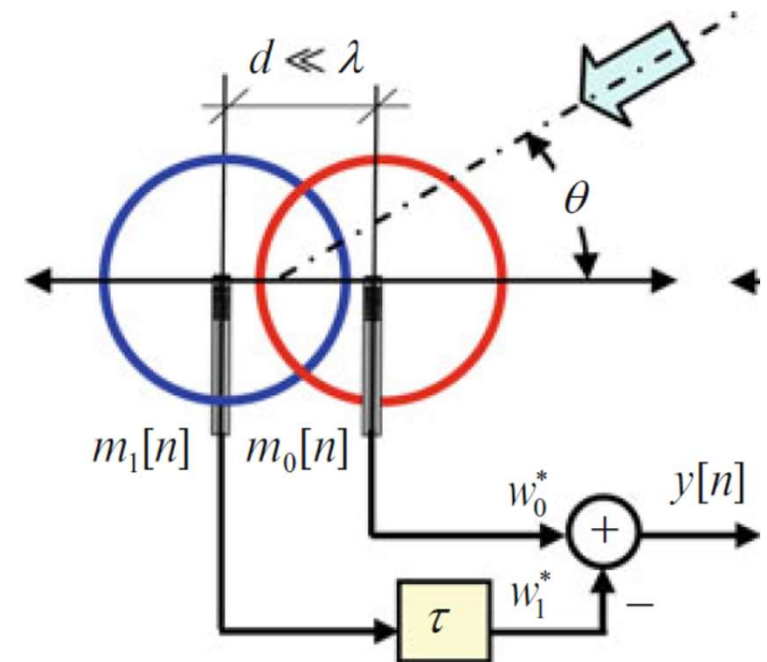
- 7-element uniformly spaced array, $d=8$ cm, unsteered
- $d = \lambda @ f=4250$ Hz
($c=340$ m/s)

Elko and Meyer. «Microphone arrays»
2008

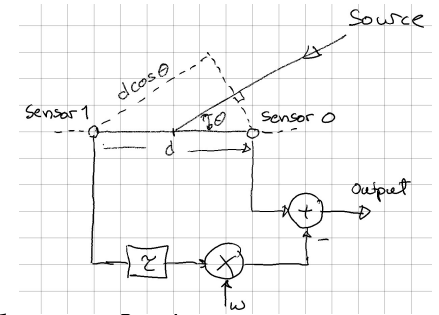
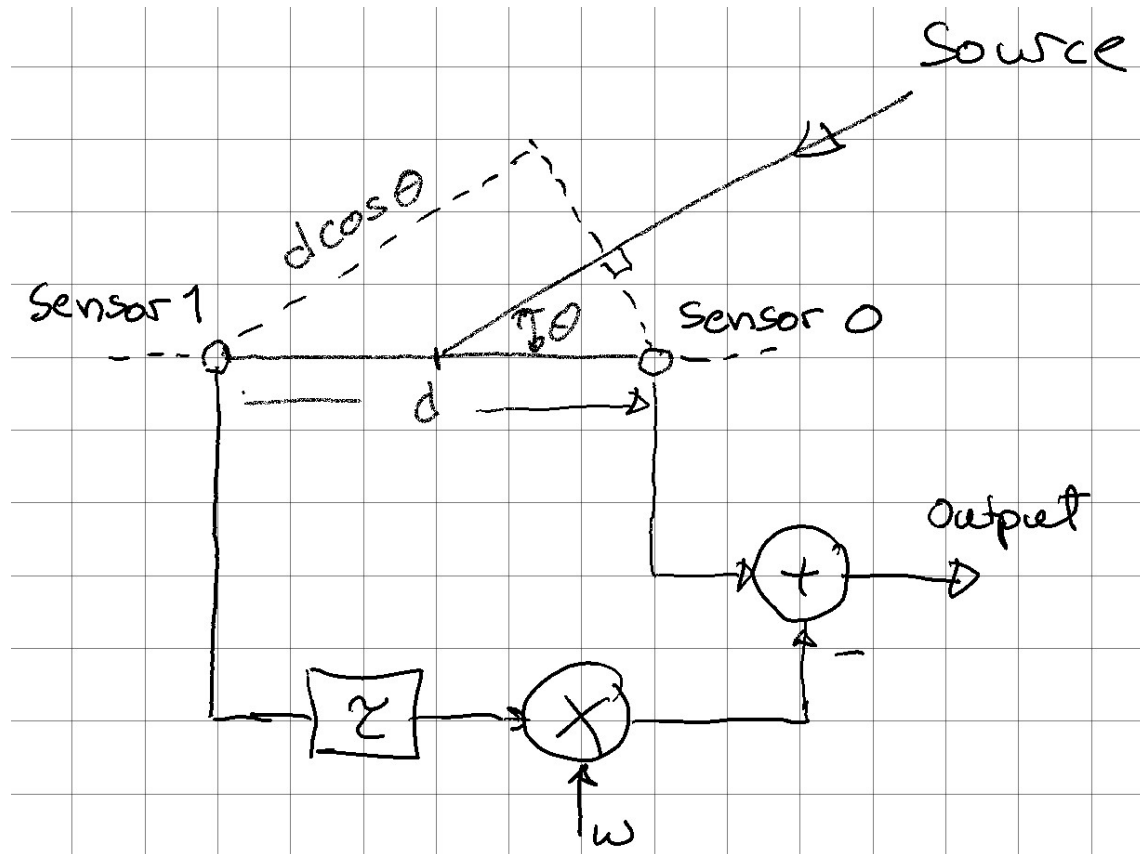


Two-element array: filter interpretation

- Conventional:
 - Sum: FIR Low-pass beamforming: **steers peak**
- Differential, end-fire
 - Simplest case: $\tau=0$, $w_0=w_1$
 - Difference: FIR High-pass beamforming: **steers null**



Uncini, 2015, Fig. 9.24

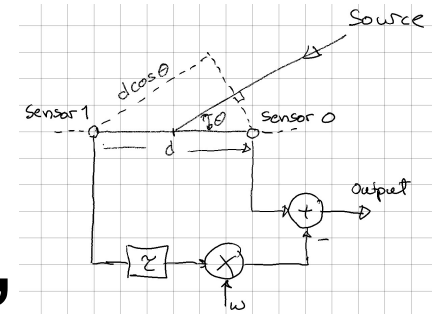


$\tau_d \cos \theta = d \cos \theta / c =$
acoustic delay

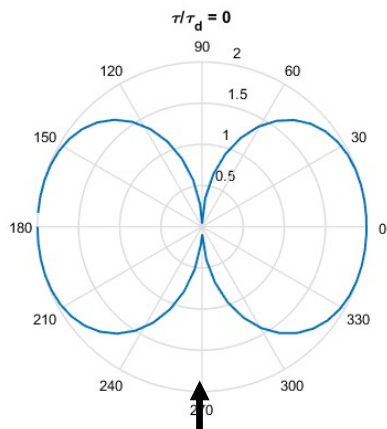
τ = processing delay,
often mechanical
implementation
of processing delay

$$R(\omega, \theta) = 1 - w e^{-j\omega(\tau + \tau_d \cos \theta)}, \quad \tau_d = d/c$$

- Weight will later be used to model propagation effects due to spherical spreading, $1/r$ -effect, in the near-field
- Sensors 1,2 could be the ears of an owl with τ due to internal coupling



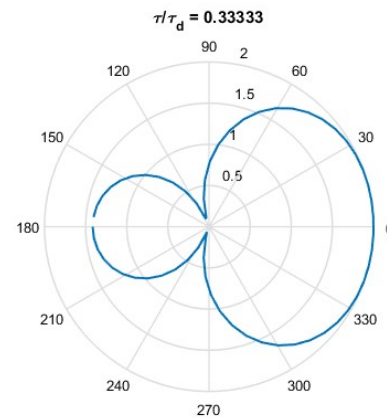
Beampattern, effect of processing, $w=1, f=f_c: \tau+d/c = \text{half a period at } f_c$



Sharp null, good
for direction finding

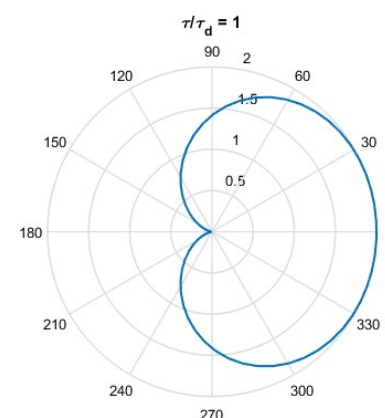
Dipole (figure eight),
 $\tau/\tau_d = 0,$

Split beam echo sounder



hypercardioid,
 $\tau/\tau_d = 1/3,$

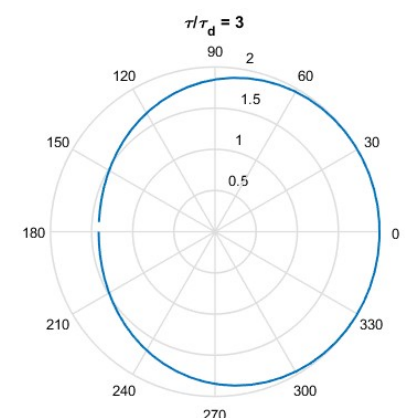
- speaker mic -



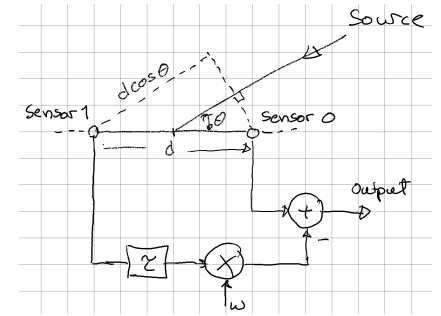
cardioid,
 $\tau/\tau_d = 1,$

Microphone / Yagi

- common mic for vocals -

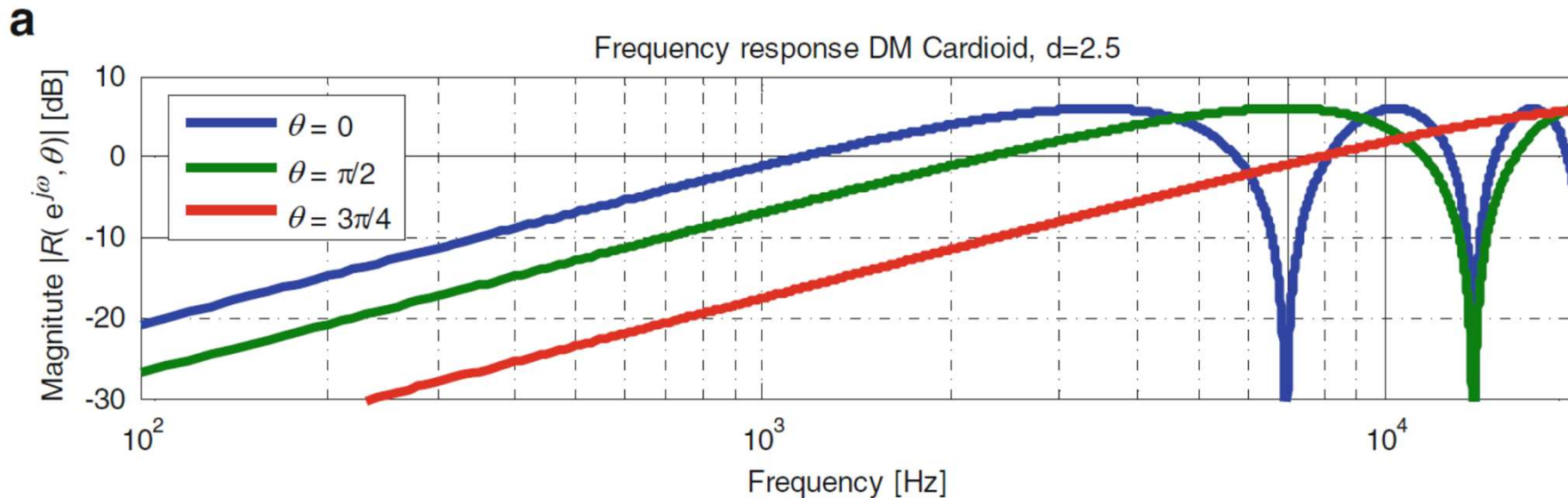


almost omni
 $\tau/\tau_d = 3$



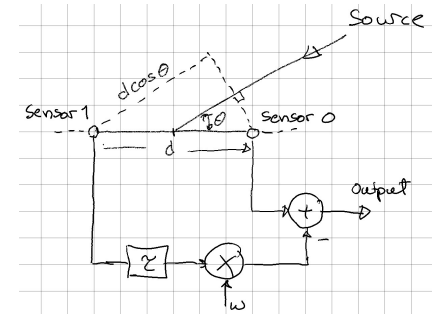
Cardioid, $\tau = \tau_d$: Dependency of angle of incidence

Note, same angle dependency for all frequencies $< f_c$

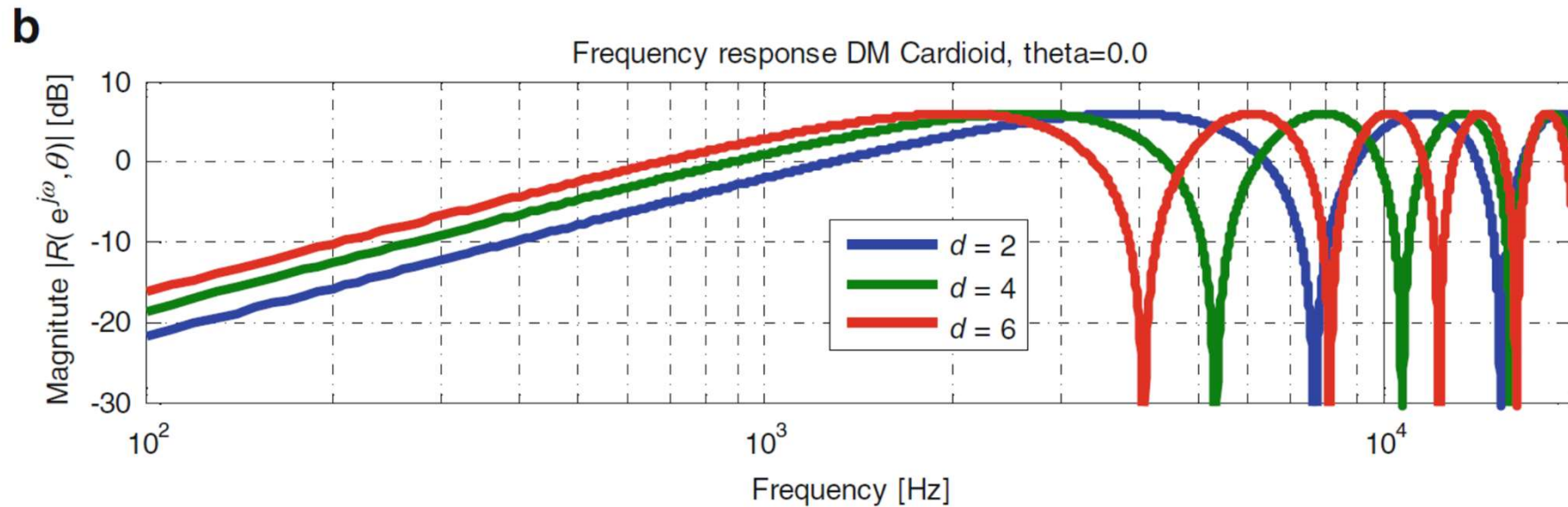


Uncini, Fig 9.29a, $d=2.5$ cm. Max gain = $10\log 2^2 = 6$ dB

$$R(\omega, \theta) = 1 - we^{-j\omega(\tau + \tau_d \cos \theta)}, \quad \tau_d = d/c$$



Cardioid, $\tau = \tau_d$: Dependency of element distance [cm]

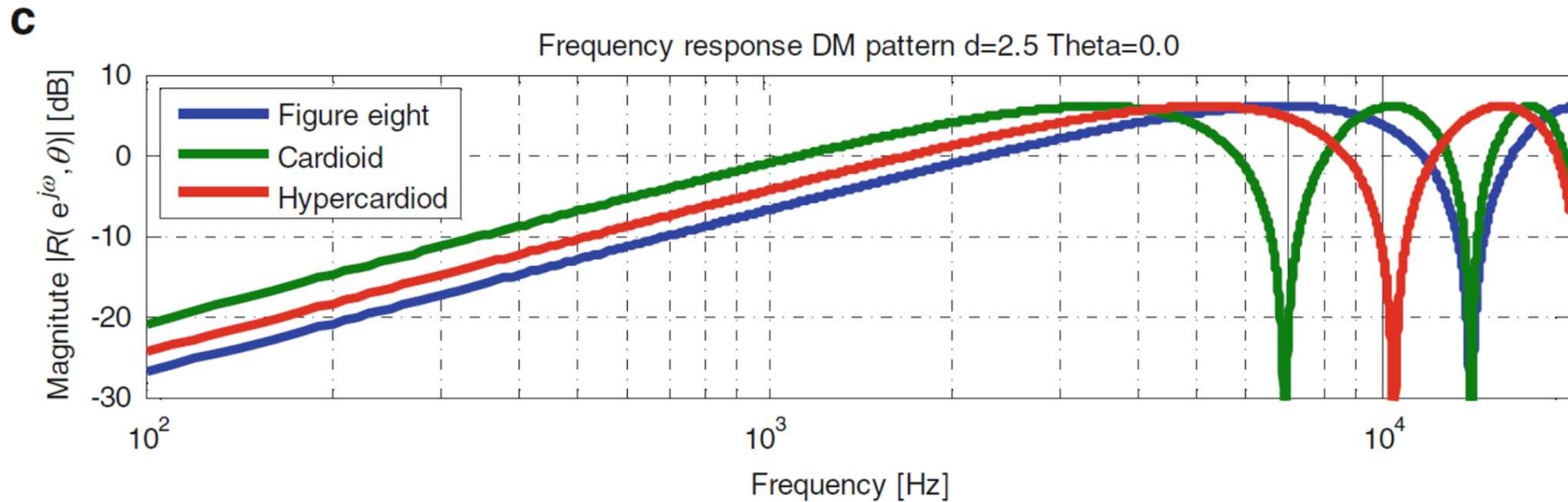


Uncini, Fig 9.29b

$$R(\omega, \theta) = 1 - we^{-j\omega(\tau + \tau_d \cos \theta)}, \quad \tau_d = d/c$$

Dependency of different patterns

Zeroes, maxima



Uncini, Fig 9.29a, $d=2.5$ cm

$$R(\omega, \theta) = 1 - w e^{-j\omega(\tau + \tau_d \cos \theta)}, \quad \tau = 0, \quad 0.33d/c, \quad d/c$$

Cut-off frequency and angularity

- Zeroes of $R(\omega, \theta)$: $\omega(\tau + \tau_d \cos \theta) = 0, 2\pi$
- Maximum of $R(\omega, \theta)$ gives cut-off frequency:

$$\omega_c(\tau + \tau_d \cos \theta) = \pi \quad \Rightarrow \quad \omega_c = \pi / (\tau + \tau_d) \quad [\theta = 0]$$

- Well below cut-off, phase is small:

$$R(\omega, \theta) = 1 - e^{-j\omega(\tau + \tau_d \cos \theta)} \approx j\omega(\tau + \tau_d \cos \theta)$$

- Gain proportional to frequency: may compensate
- **Same angle dependency for all frequencies**

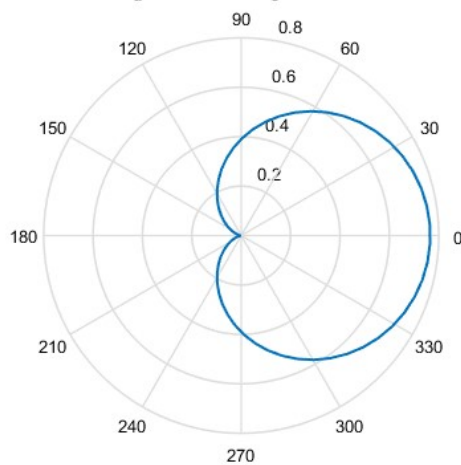
Cut-off frequency: delay=half a period

$$f_c = \frac{\omega_c}{2\pi} = 0.5/(\tau + \tau_d) = 0.5/(\tau + \frac{d}{c})$$

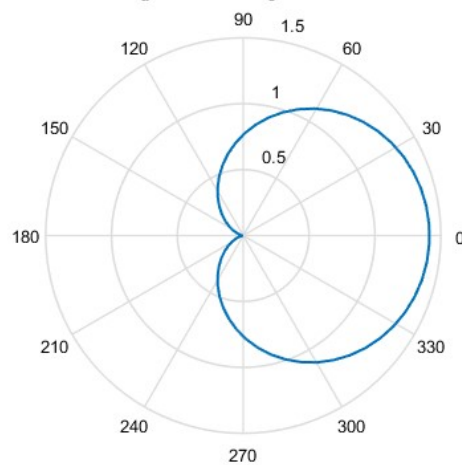
- Cardioid, $\tau = d/c$: $f_c = \frac{c}{4d}$
- Uncini examples, $d=2.5$ cm: $f_c = 3.4$ kHz
- Next examples, $d=8.5$ mm: $f_c = 10$ kHz

Relatively insensitive to frequency: Cardioid below f_c , breaks up above f_c

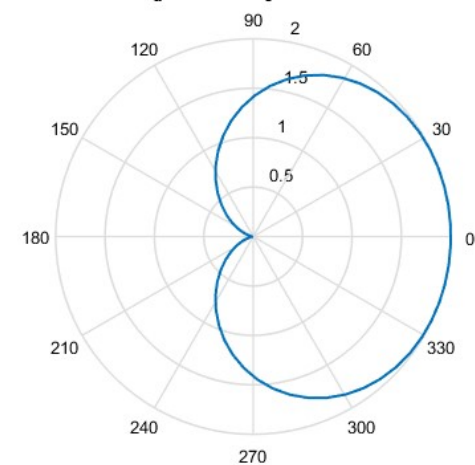
$\tau/\tau_d = 1, f = 0.25*f_c = 2500 \text{ Hz}$



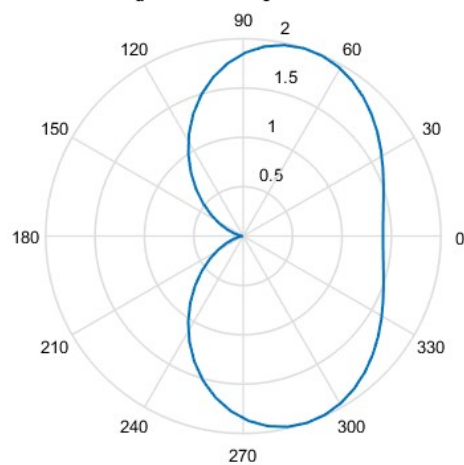
$\tau/\tau_d = 1, f = 0.5*f_c = 5000 \text{ Hz}$



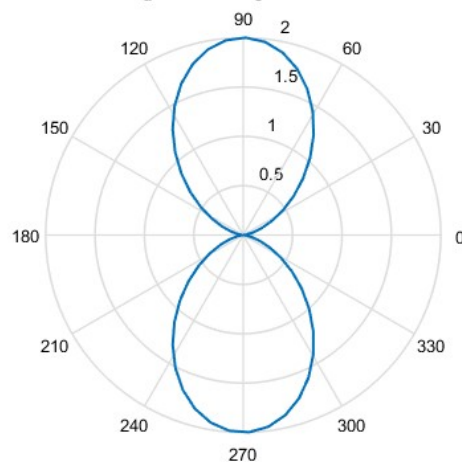
$\tau/\tau_d = 1, f = 1*f_c = 10000 \text{ Hz}$



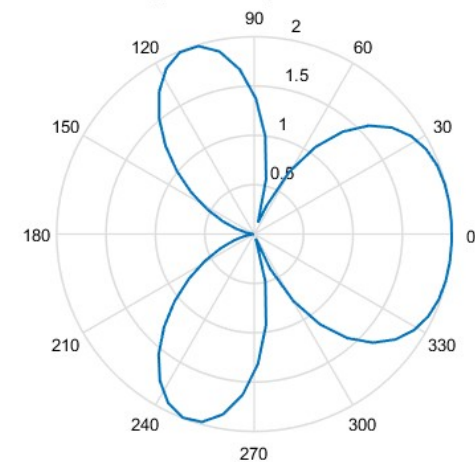
$\tau/\tau_d = 1, f = 1.5*f_c = 15000 \text{ Hz}$



$\tau/\tau_d = 1, f = 2*f_c = 20000 \text{ Hz}$

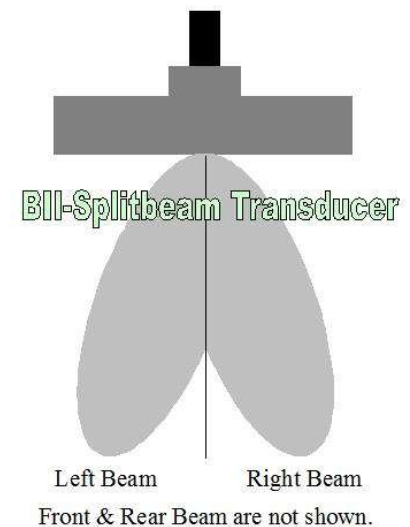


$\tau/\tau_d = 1, f = 3*f_c = 30000 \text{ Hz}$



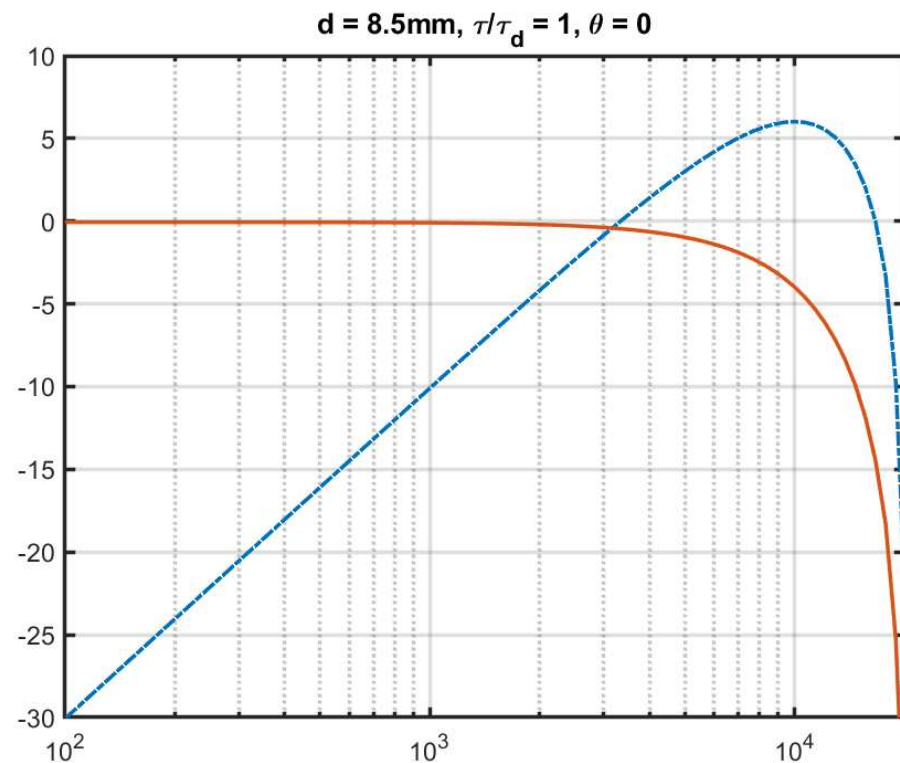
Split beam echosounder

- 2-element transducer
 - Normal beam: add
 - Split-beam: subtract, no processing delay $\tau \rightarrow$ figure-of-8
- Assume a single broadside target
 - Normal beam: a peak
 - Split-beam: a null
- A null is a more precise indicator than a peak for when a target is exactly broadside



Pressure gradient microphone: compensated for 6 dB/octave

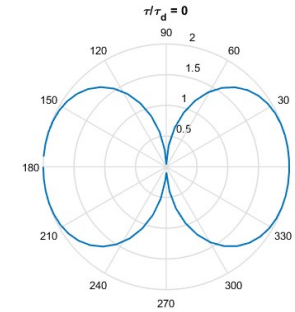
Compensated
by the mass of
the diaphragm



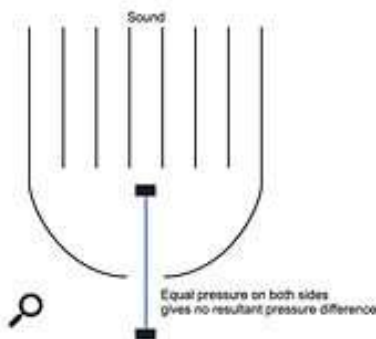
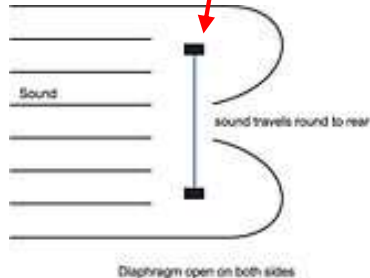
Difference array: Quite sensitive to parameter variations

- Broadband array needs an equalizer to boost low frequencies \Leftrightarrow large sensitivity to low-frequency self-noise
- Element distance, $d \ll \lambda$
 - But not too small, otherwise sensitivity to noise increases
- Higher order differential arrays are even more sensitive

Figure-of-eight microphone



Membrane
Front



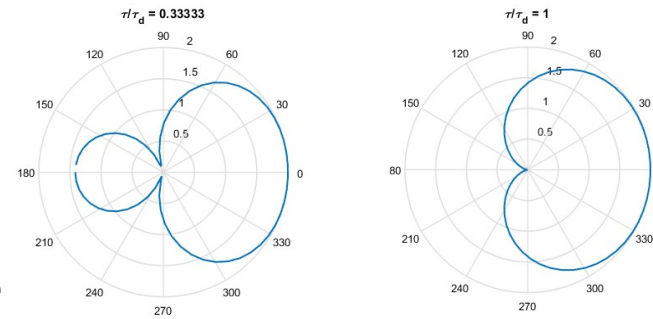
If the mic diaphragm is open to the air on **one side** but closed at the other, it is considered to be pressure-operated: although it reacts to air pressure, it is not sensitive to direction, resulting in an **omnidirectional** mic pattern.

Where the diaphragm is **open on both sides**, as in this diagram, it responds to the pressure-gradient (the difference between the pressure at the front and the back of the diaphragm).

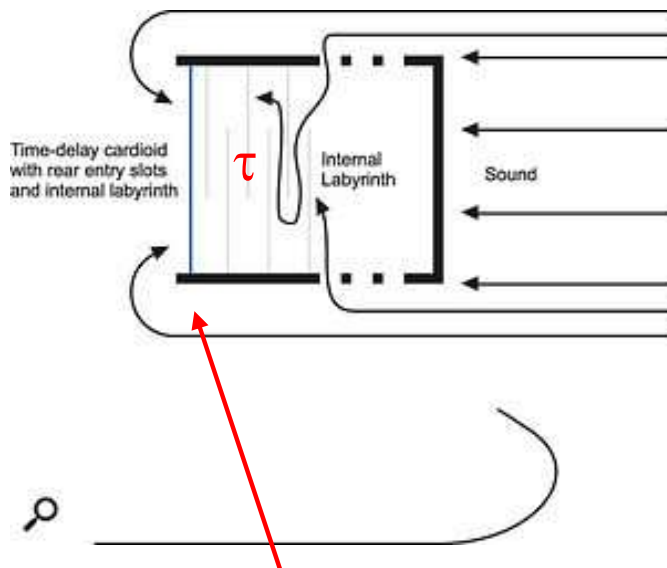
In this case, sound from the side results in even pressure on both sides of the diaphragm, which is why **figure-of-eight** mics reject sound from the side but are responsive to both the front and rear.

<https://www.soundonsound.com/techniques/using-microphone-polar-patterns-effectively>

Cardioid microphone



Front d



Membrane



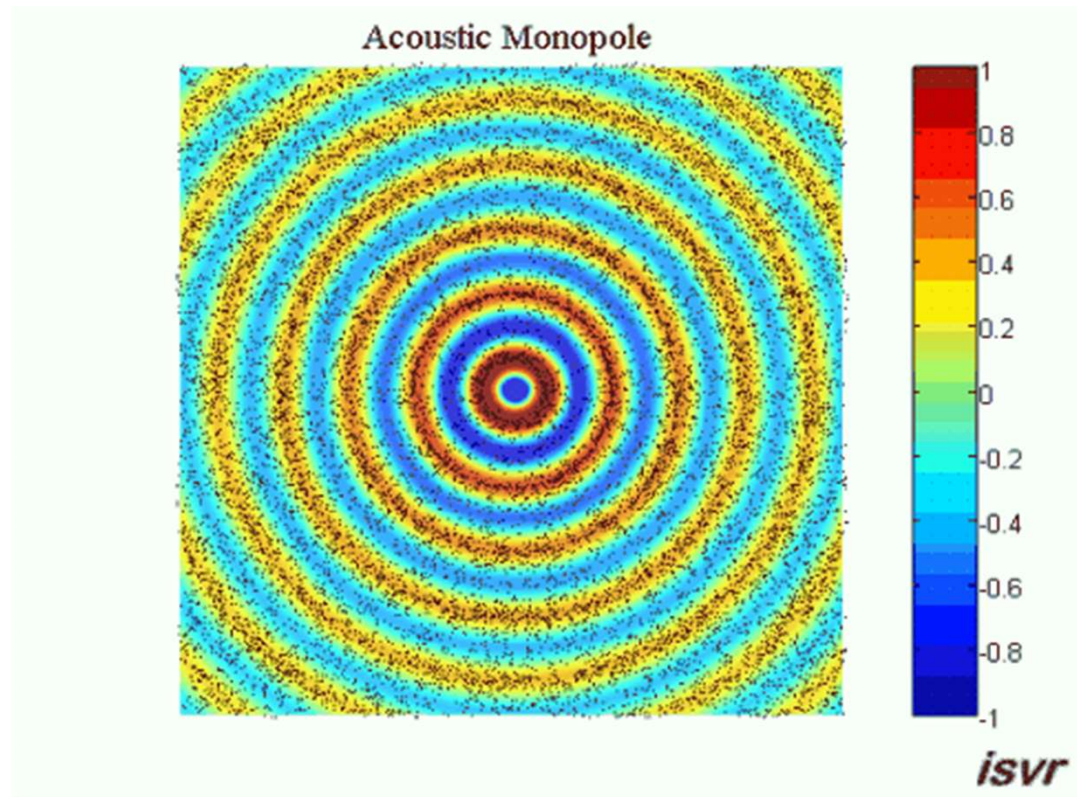
Most cardioid mics now incorporate a vented 'labyrinth' in a single-capsule design that manipulates the phase of sounds hitting the rear, to produce the desired cardioid pattern.

The supercardioid and hypercardioid designs use the same principle to create a more focused pattern to the front, at the expense of reducing the rear rejection.

If you notice vents at the side of the mic head, the mic probably has a cardioid pattern (or a variation on it).

<https://www.soundonsound.com/techniques/using-microphone-polar-patterns-effectively>

Spherical spreading (from mouth)

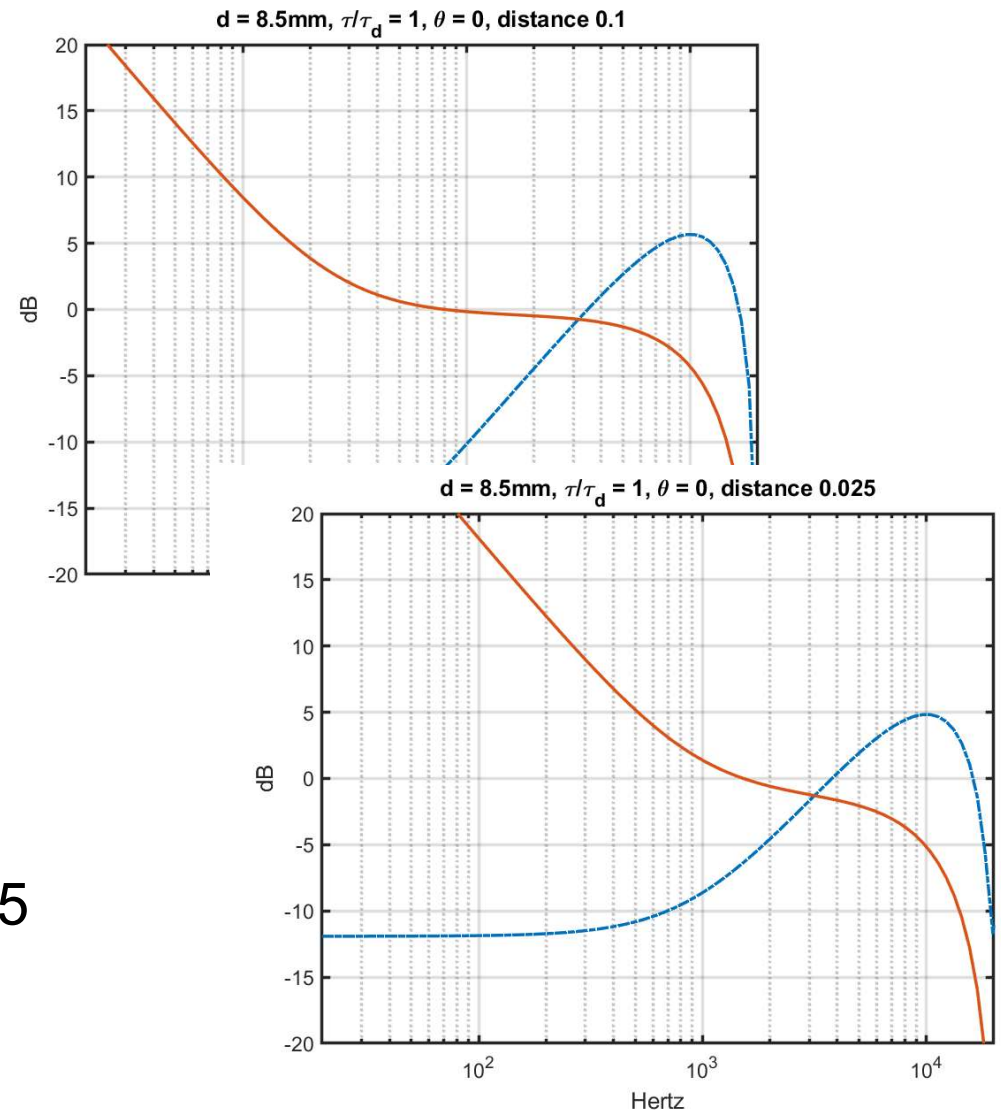


http://resource.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial_files/Web-basics-pointsources.htm

Proximity effect of cardioid: bass boost when close-in

Difference due to $1/r$:

- Distance $d=10$ cm:
 - 1. element: 10 cm
 - 2. el.: 10+0.85 cm
 - Effective $w = 10/10.85 : 0.92$
- Distance $d=2.5$ cm:
 - $w = 2.5/(2.5+0.85) = 0.75$



Neumann U 47: First switchable pattern condenser microphone (1940's)

Front and rear membranes: cardioid

- Sound coming from the front causes movement of the front membrane and reaches the inner side of the rear membrane through the perforations in the electrode.
- If only one membrane is connected, the microphone works as described above as a cardioid.

When connecting both cardioid halves in parallel, the capsule produces an omnidirectional pattern.



<https://en-de.neumann.com/u-47>

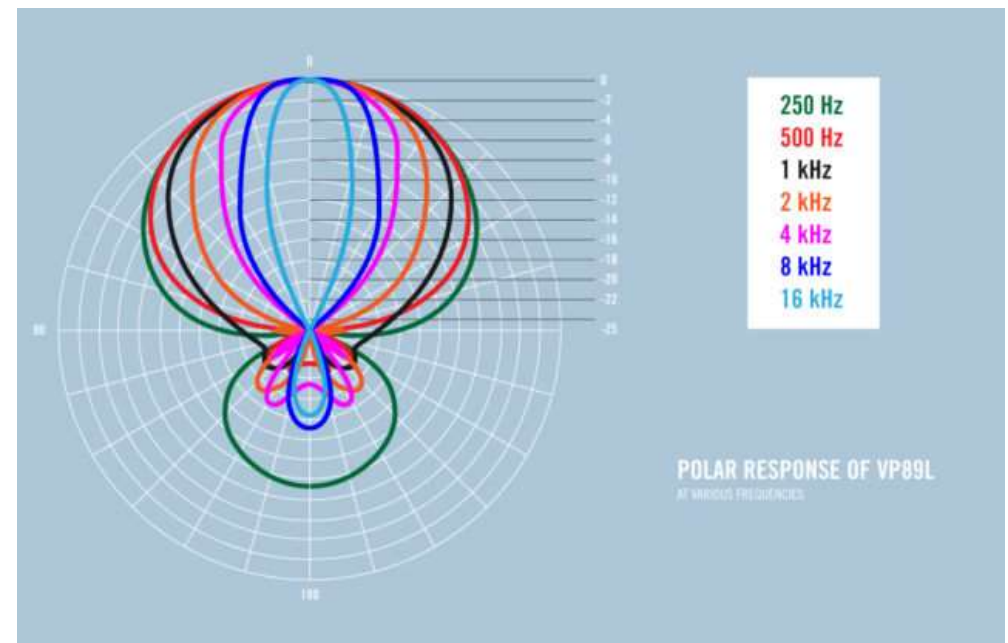
Shotgun microphone

Characteristics

- Low frequencies, supercardioid
- High: lobar
- Off axis, more sensitive to lower and less to higher frequencies: colored sound.

Applications:

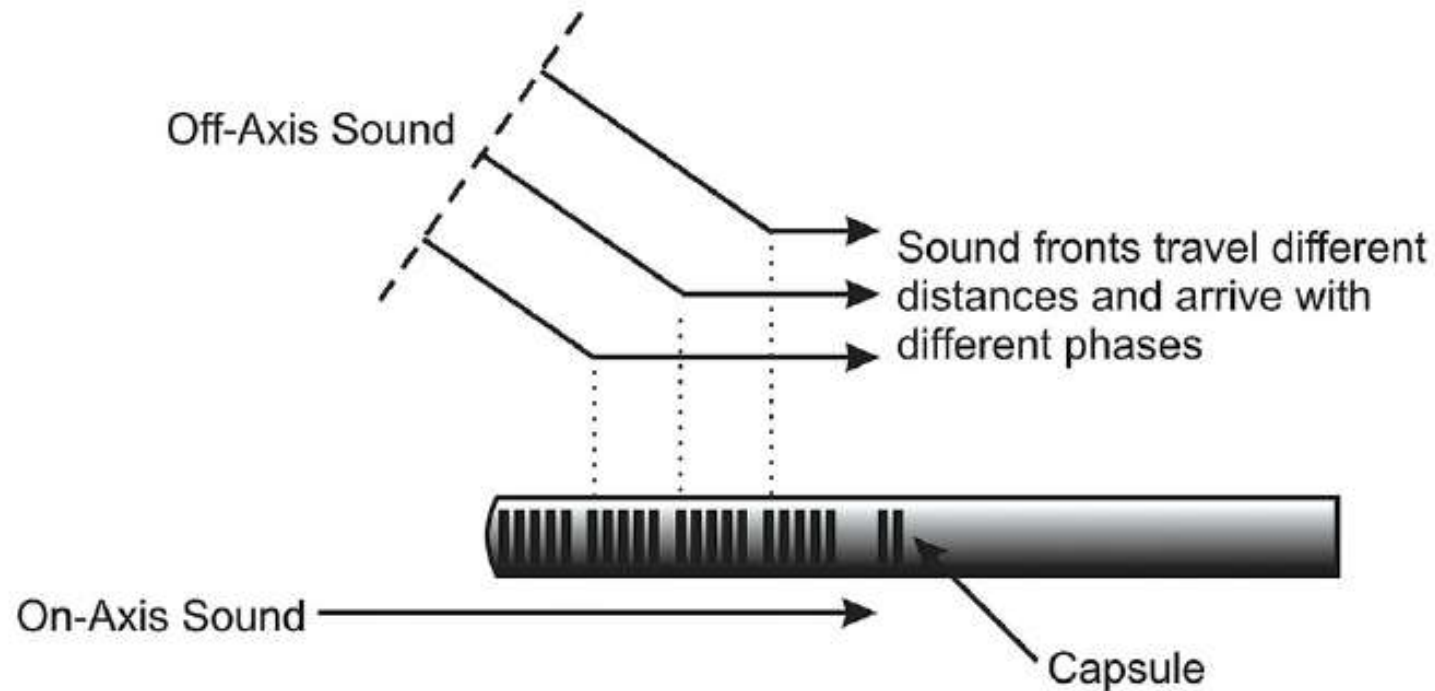
- Film industry: dialog pickup on the shooting set
- Sport events
- Birds at great distances



Polar Response of the Shure VP89L Shotgun Mic.

Shotgun microphone

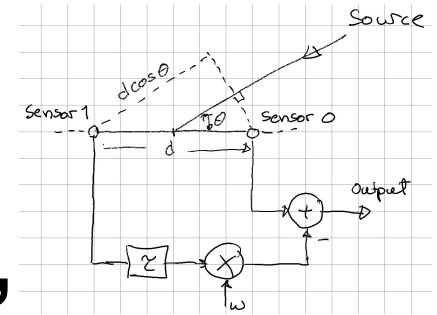
N^{th} order differential array



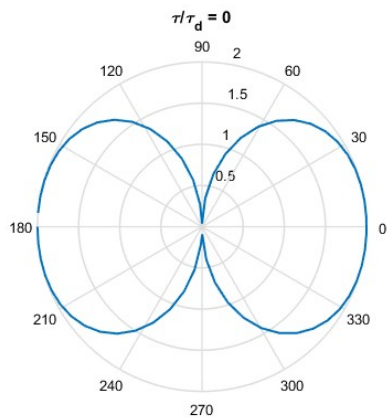
<https://www.speechrecsolutions.com/guides/Shotgun%20Mic%20Tutorial.pdf>

Differential arrays – from cardioid microphones to Yagi antennas

- Part 1:
Directional microphones (2nd order arrays)
- Part 2:
Nth order arrays and Yagi-Uda antennas

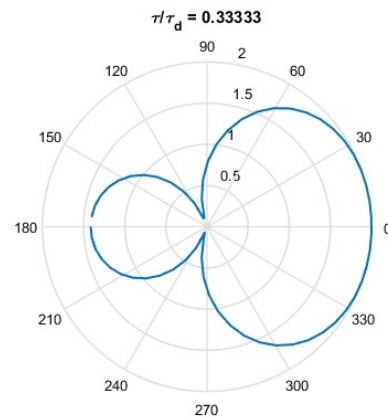


Beampattern, effect of processing, $w=1$, $f=f_c$



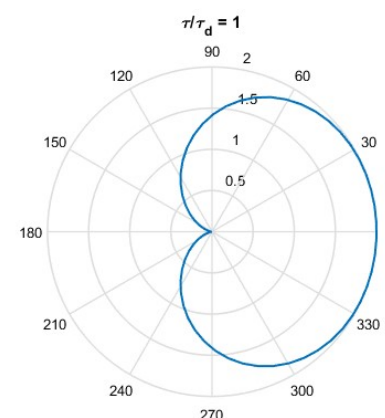
Dipole (figure eight),
 $\tau/\tau_d = 0$,

Split beam echo sounder



hypercardioid,
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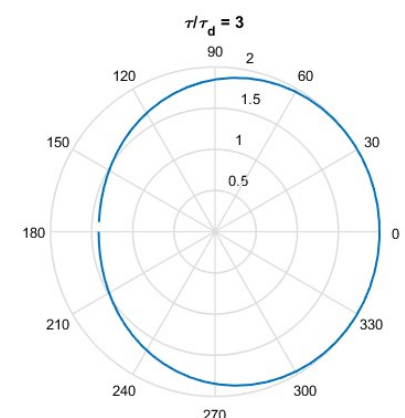
- speaker mic -



cardioid,
 $\tau/\tau_d = 1$,

Microphone / Yagi

- common mic for vocals -



almost omni
 $\tau/\tau_d = 3$

Exact differential array

- Plane wave (far-field) pressure field:

$$p(r, t) = A_0 e^{j(\omega_0 t - kr \cos \theta)}$$

- Spatial derivative (drop time) = pressure gradient:

$$\left| \frac{d}{dr} p(k, r) \right| = jk \cos \theta A_0 e^{-jkr \cos \theta}$$

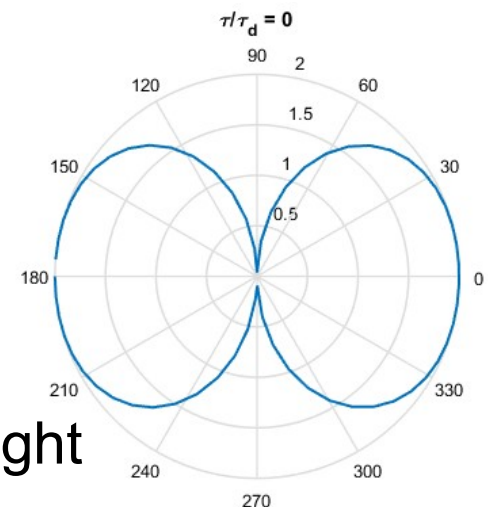
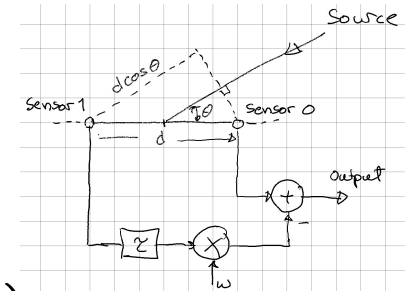
- Beam pattern shape

$$\propto k \cos \theta = \frac{\omega}{c} \cos \theta$$

- Similar to previous derivation:

$$R(\omega, \theta) = 1 - e^{-j\omega(\tau + \tau_d \cos \theta)} \big|_{\tau=0} \approx j\omega(\tau_d \cos \theta)$$

- ~no-delay difference array ($\tau=0$), figure-of-eight

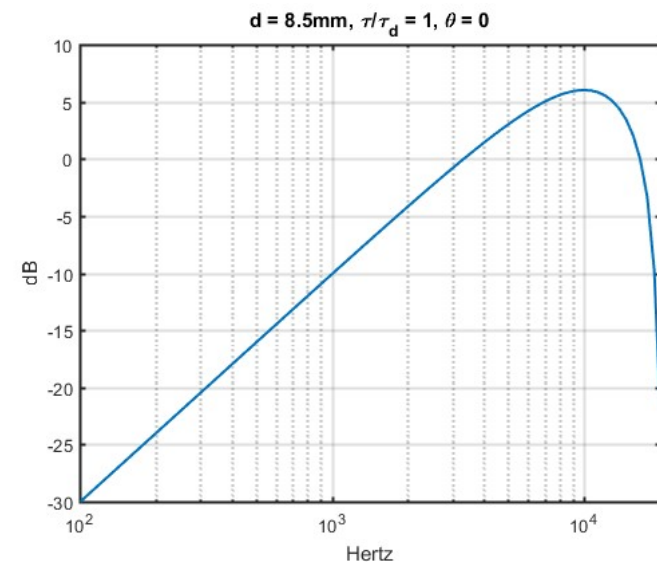


Differential array (cont)

- $j k \cos \theta = \frac{j \omega}{c} \cos \theta$
– high-pass 6 dB/octave
- Pressure gradient from conservation of linear momentum (Euler):

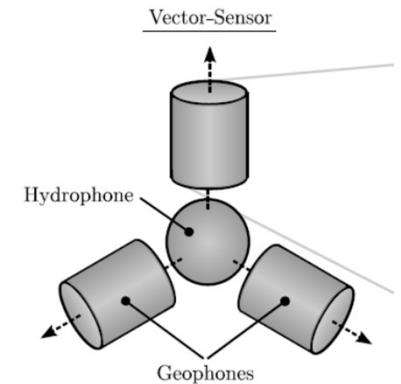
$$\rho_0 \frac{\partial v}{\partial t} = -\nabla p \Rightarrow \nabla p \propto j \omega |v|$$

- Therefore called **pressure gradient** or **velocity** microphone
- If 3-D: vector sensor



Vector sensors

– underwater acoustics: low f.



It is shown that the multichannel receiver using a single vector sensor can offer significant size reduction for coherent acoustic communication at the carrier frequency of **12 kHz**, compared with a pressure sensor line array.

- Song, Abdi, Badiey, Hursky, (2011). Experimental demonstration of underwater acoustic communication by vector sensors. IEEE J Ocean Eng,

This paper proposes a mode domain beamforming method for a 3 x 3 uniform rectangular array of two-dimensional (2D) acoustic vector sensors with inter-sensor spacing much smaller than the wavelengths

- Guo, Yang, Miron, (2015). **Low-frequency** beamforming for a miniaturized aperture three-by-three uniform rectangular array of acoustic vector sensors. J Acoust Soc Am..

Grønlandshval: 25-900 Hz

Masking from industrial noise can hamper the ability to detect marine mammal sounds near industrial operations, whenever conventional (pressure sensor) hydrophones are used for passive acoustic monitoring. ... Improvements in signal-to-noise ratio of up to 15 dB are demonstrated on bowhead whale calls, which were otherwise undetectable using conventional hydrophones.

- 1.3.2023– Thode, Kim, Norman, Blackwell, Greene (2016). Acoustic vector sensor beamforming reduces masking from underwater industrial noise during passive monitoring. J Acoust Soc Am.

Far-field: n'th order differential array

$$\frac{d^n}{dr^n} p(k, r) = A_0 (jk \cos \theta)^n e^{-jkr \cos \theta}$$

- Beampattern $\propto \cos^n \theta$
- Frequency response: $\propto \omega^n$: 6n dB/octave

Near-field: n'th order differential array

Pressure: $p(r, t) = A_0 e^{j(\omega_0 t)} \frac{e^{-jk_0 r \cos \theta}}{r}$

$$\frac{d^n}{dr^n} p(k, r, \theta) = A_0 \frac{n!}{r^{n+1}} e^{-jkr \cos \theta} (-1)^n \sum_{m=0}^n \frac{(jkr \cos \theta)^m}{m!}$$

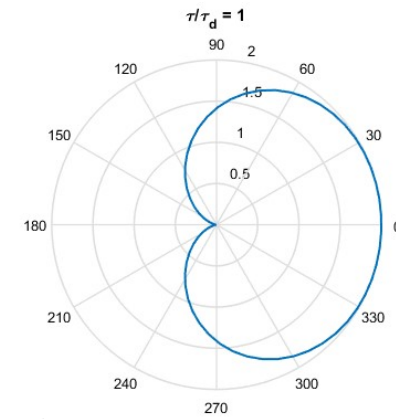
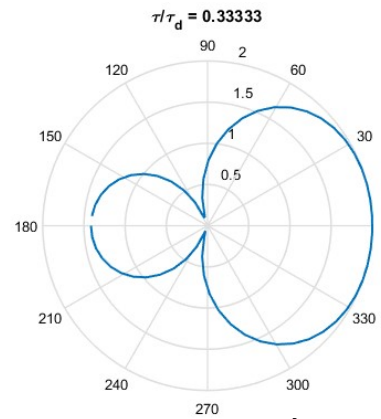
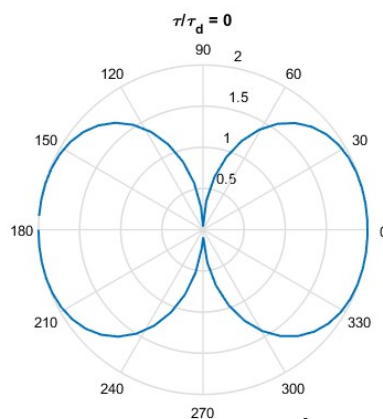
- Sum of dipole-like terms of type $\cos^m \theta$
- May optimize coefficients for desirable properties
- Differential array, $n=1$, i.e. 2 terms in sum:

$$R(\omega, \theta) \approx j\omega(\tau + \tau_d \cos \theta) = j\omega(a_0 + a_1 \cos \theta)$$

Differential array in practice

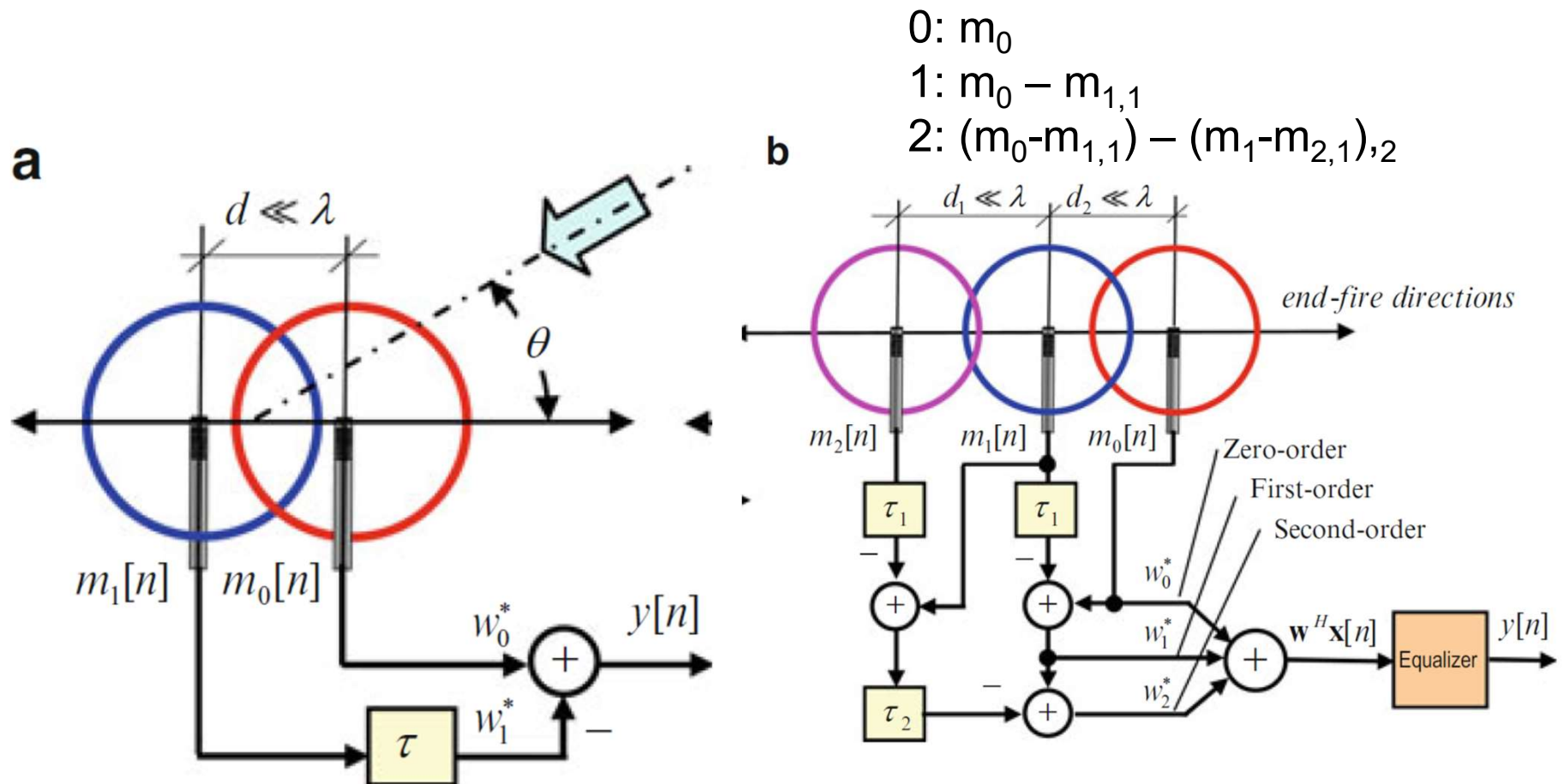
- Approx. by finite differences, $d \ll \lambda$
- Response: $\propto \omega^n (a_0 + a_1 \cos \theta + a_2 \cos^2 \theta + \dots + a_n \cos^n \theta)$
- $n=1$:

Figure-of-8	$a_0=0, \quad a_1=1$
Hypercardioid	$a_0=1/4 \quad a_1=3/4$
Cardioid	$a_0=1/2 \quad a_1=1/2$



$$R(\omega, \theta) \approx j\omega(\tau + \tau_d \cos \theta), \quad \tau = 0, \tau_d/3, \tau_d$$

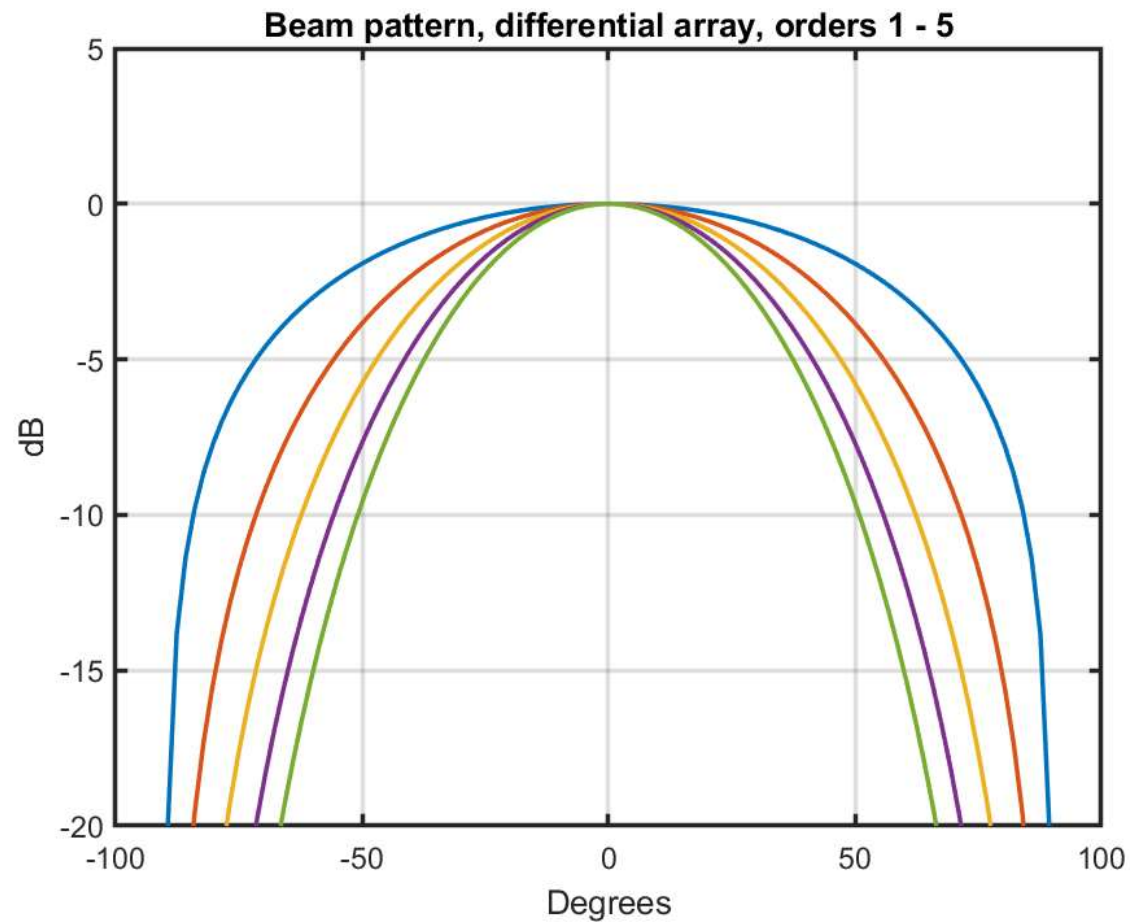
First, second order differential array



Uncini, 2015, Fig. 9.24

$$\propto \omega^n \left(a_0 + a_1 \cos \theta + a_2 \cos^2 \theta + \dots + a_n \cos^n \theta \right)$$

Beam pattern, \cos^n



$$\propto \omega^n (a_0 + a_1 \cos \theta + a_2 \cos^2 \theta + \dots + a_n \cos^n \theta)$$

Maximum directional gain

1st – 4th order

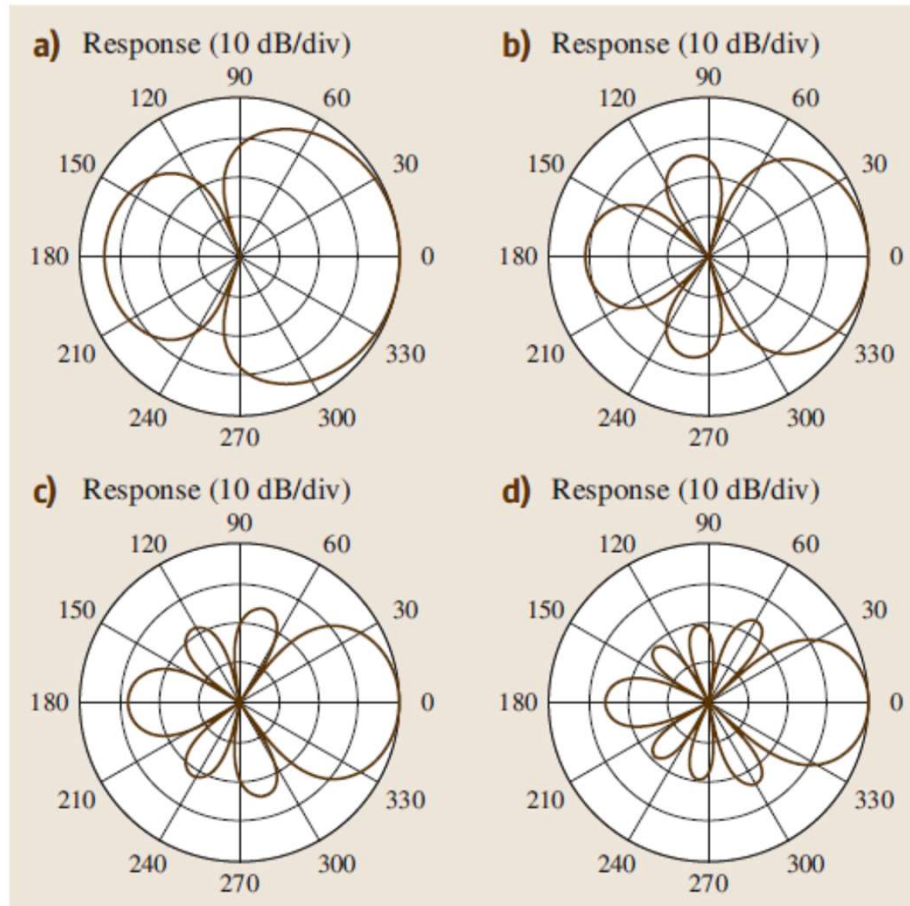


Fig. 50.12a–d Directional patterns that have maximum directional gain for differential microphone arrays for up to fourth-order arrays

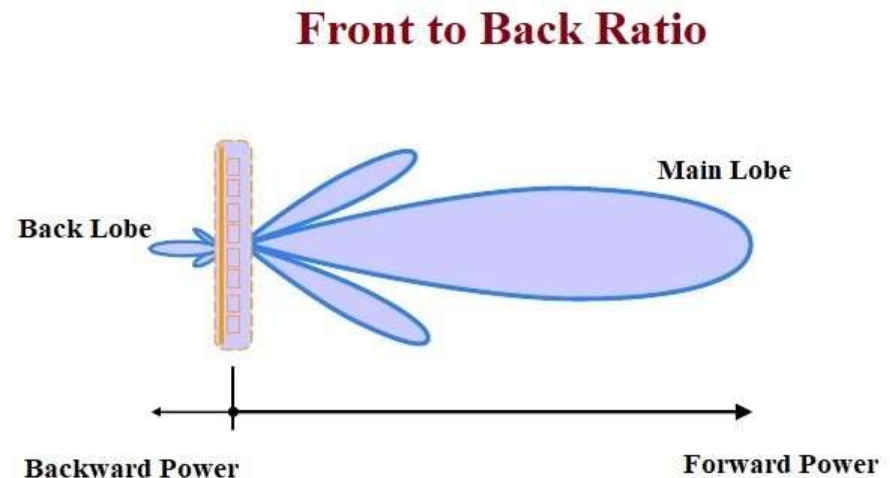
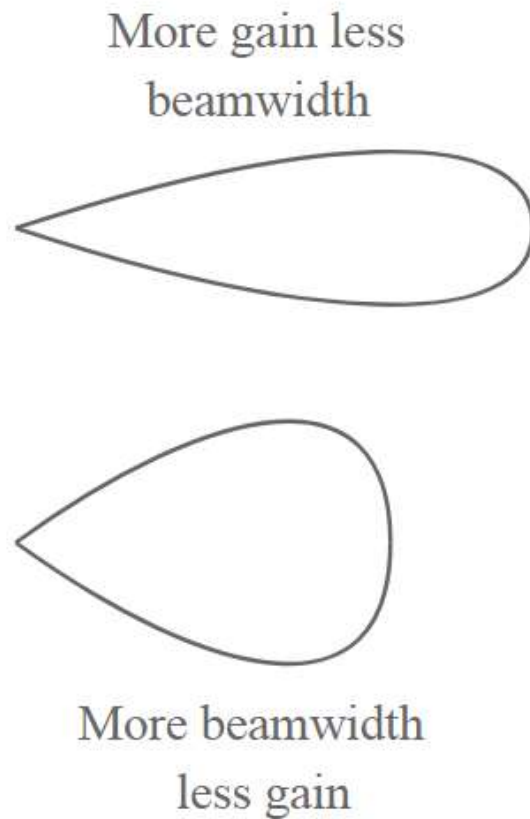
$n=1$: hypercardioid

$n=2$: Narrower beam than that of \cos^2

~ shotgun (lobar) microphone

Elko and Meyer.
«Microphone arrays» 2008

Performance metrics: Beamwidth, Gain, Front/Back



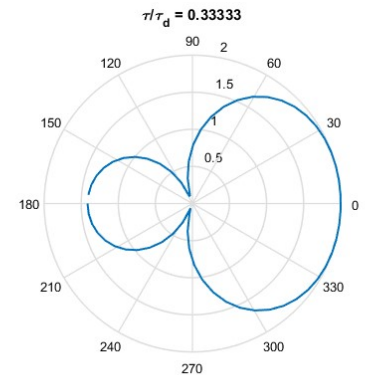
<https://www.electronics-notes.com/articles/antennas-propagation/yagi-uda-antenna-aerial/gain-directivity.php>
<https://www.everythingrf.com/community/what-is-front-to-back-ratio-in-an-antenna>

Optimization, first order

$$E(\theta, \omega) \propto \omega (a_0 + a_1 \cos \theta)$$

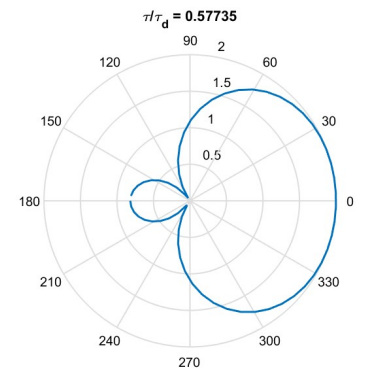
Maximum gain

- $n=1$: hypercardioid $E_{HC_1}(\theta) = \frac{1 + 3 \cos \theta}{4}$.
- Array gain: $20\log(n+1) = 10\log N^2$
 - N is no of elements



Best front-back ratio

- $n=1$: supercardioid $E_{SC_1}(\theta) = \frac{\sqrt{3} - 1 + (3 - \sqrt{3}) \cos \theta}{2}$.

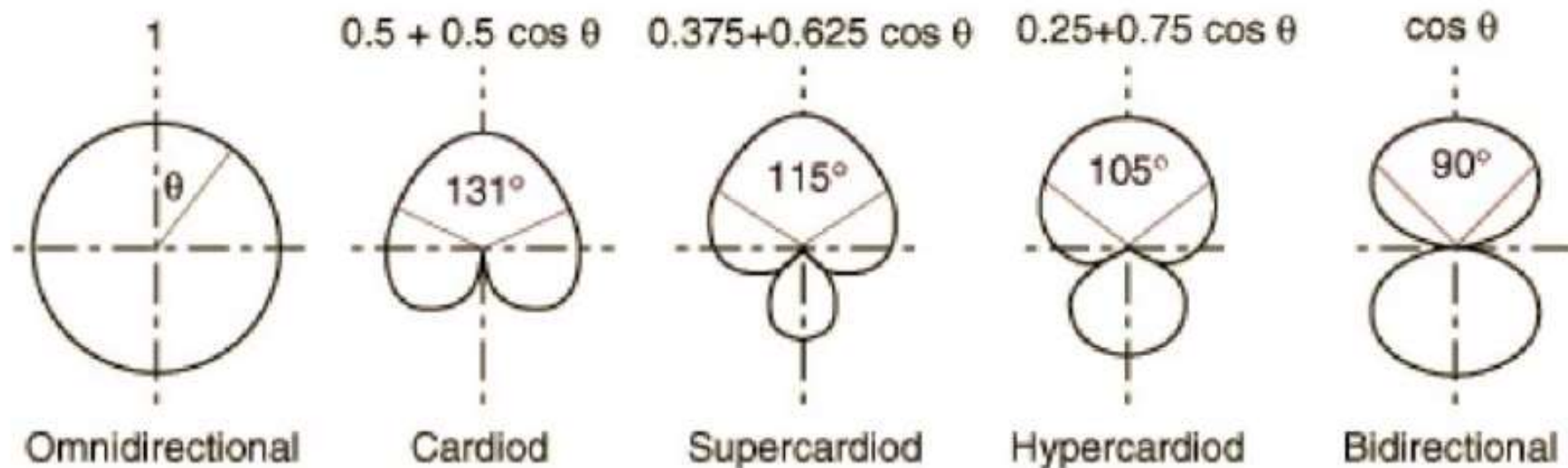


Elko, "Differential microphone arrays", 2004

Hyper- vs super-cardioid

Max gain vs best front/back-ratio

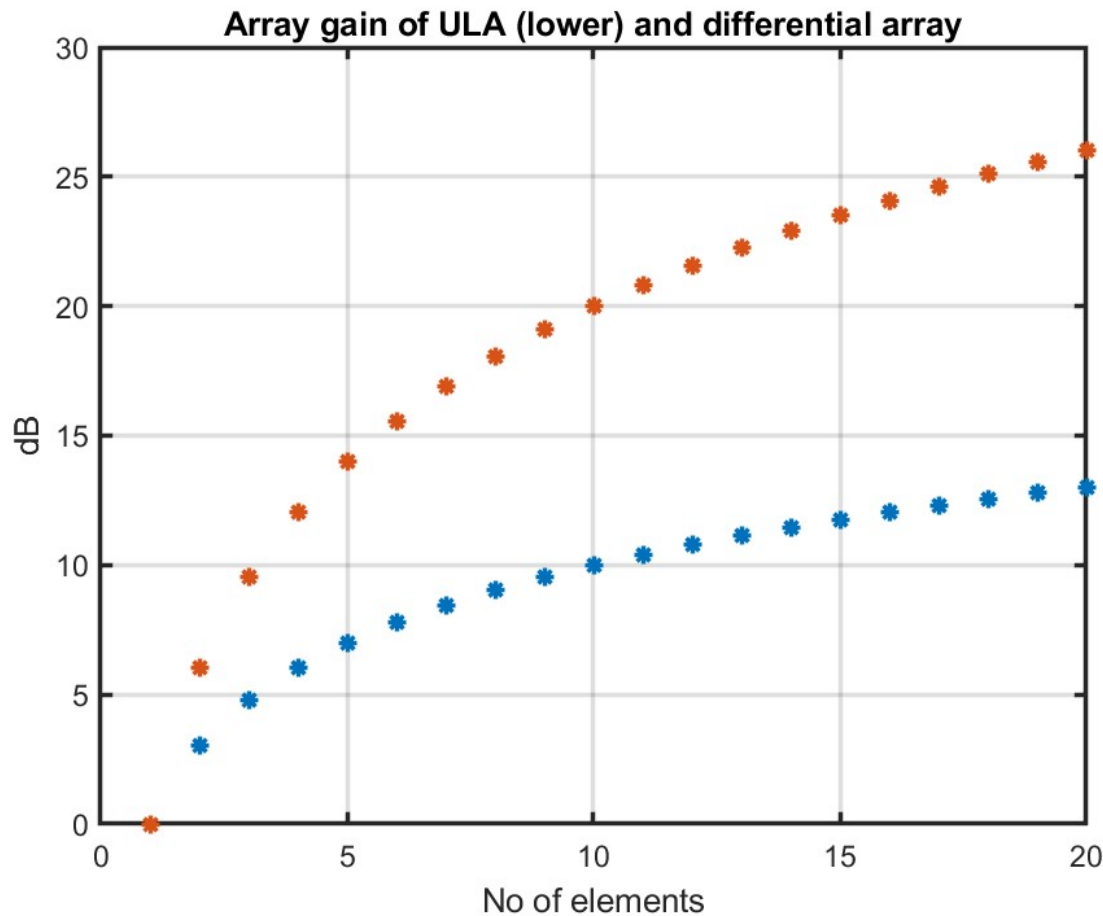
Theory: 0.366, 0.634



Best front/back-ratio, maximum gain

Quaranta, Dimino, D'altrui, General guidelines for acoustic antenna designed for beamforming noise source localization, 2007

Maximum theoretical array gain, end-fire vs uniform linear array



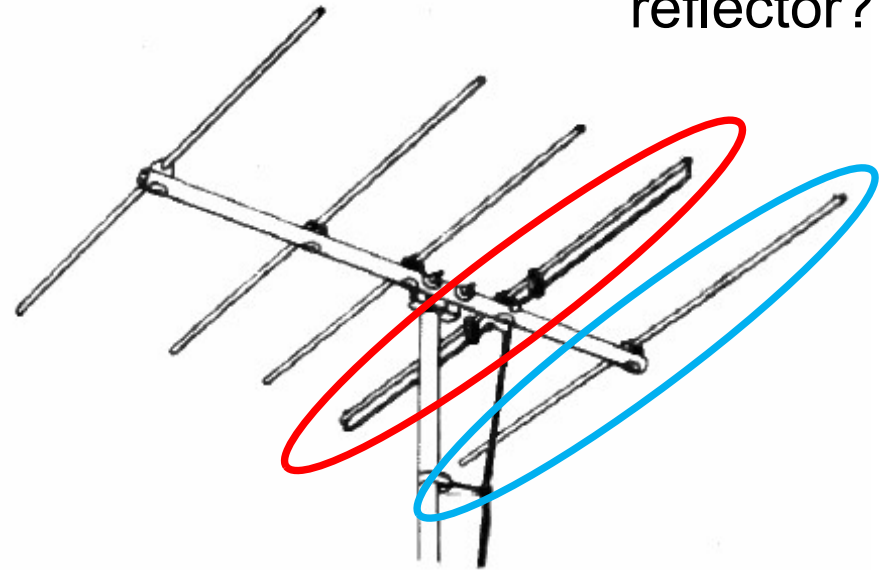
$$AG = 10 \log N^2, \quad BW \propto \frac{1}{\sqrt{N}}$$

$$AG = 10 \log N, \quad BW \propto \frac{\lambda}{D} = \frac{2}{N}$$

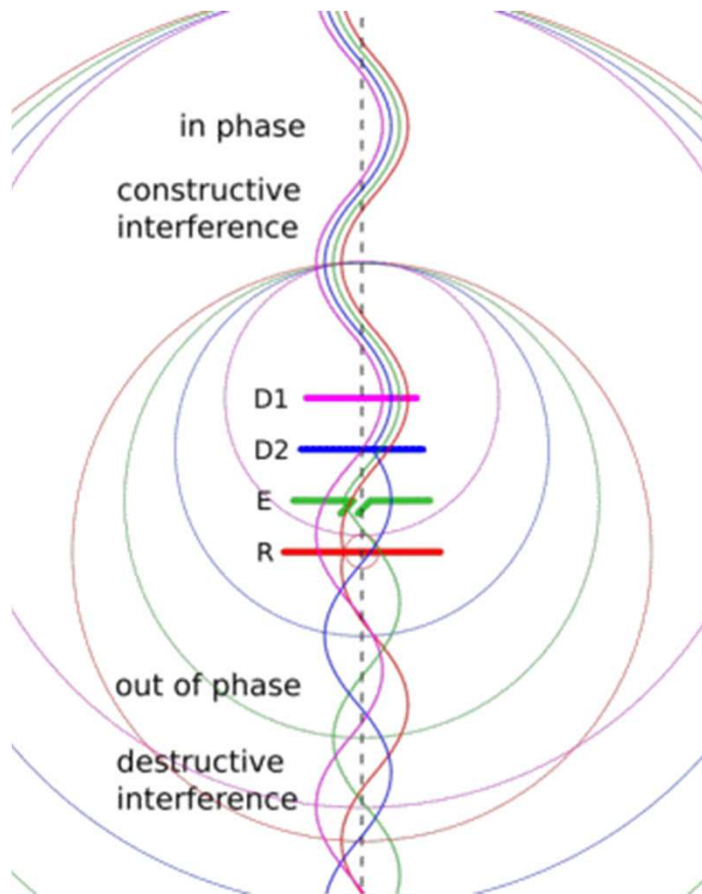
Yagi-Uda antenna: n-1 parasitic elements

- Single **active** element
 - Length $\lambda/2 \Rightarrow$ narrowband
- Passive elements:
 - **Reflector**
 - typ 5% longer: Capacitive reactance - voltage phase lags that of the current
 - Directors:
 - typ 5% shorter: Inductive reactance - current phase lags phase of the voltage
 - Ex. here: 3 or 17 directors

Why only 1 reflector?



Phasing animation ([from Wikipedia](#))



- Time delays due to element distance
 - Phasing in element due to length vs wavelength
- Reradiation from passive elements (parasitic)
 - Field behind first reflector is nearly canceled
- Inherently narrow-band

Uda, S., 1925, "[On the Wireless Beam of Short Electric Waves](#)". Journ. Institute of Electrical Engineers of Japan

Yagi, Hidetsu; Uda, Shintaro, 1926, "[Projector of the Sharpest Beam of Electric Waves](#)" Proc. of the Imperial Academy of Japan.

Physics department, UiO

- CubeSTAR:
 - 437.465 MHz, $\lambda=0.686$ m
 - 432-438 MHz radio amateur band
- Circularly polarized
 - 4 x 436CP30, each with:
 - 2 x (13 directors+1 reflector+1 driven element) = 30 elements
- 4 stacked together (~ broadside array)
 - 1.143 m = 1.67λ , gain=20.5 dB, -3 dB beamwidth=16 deg

Eirik Vikan, UiO Satellite Ground Station: Simulation, Implementation and Verification, MSc, 2011

https://www.duo.uio.no/bitstream/handle/10852/11067/Eirik_Vikan_UiO_Satellite_Ground_Station_Simulation_Implementation_and_Verification.pdf

<https://www.m2inc.com/FG436CP30>



Broadband? Log periodic array

- VHF/UHF, 50-1300 MHz
- 21 elements, 2 m boom
- Forward gain: 10 to 12 dBi (rel isotropic)
 - Like a 4-5-element Yagi
- Complex as several or all elements are active
- Create CLP-5130-1N



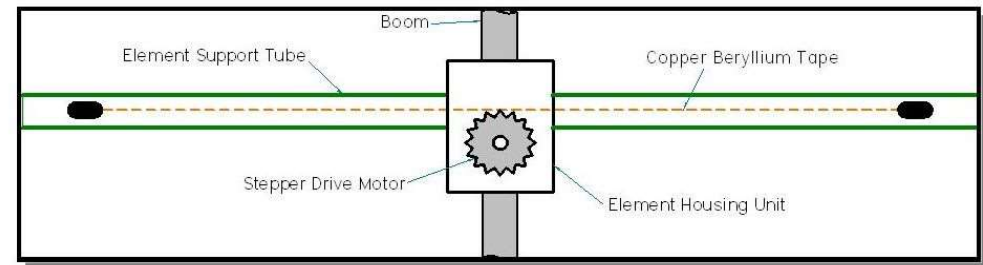
Broadband? Adjustable element lengths

Multi-frequency
Bi-directional mode



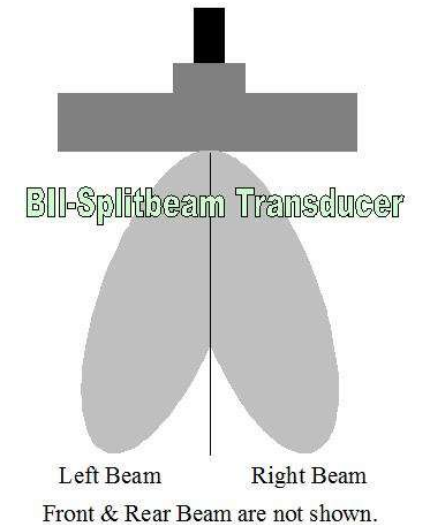
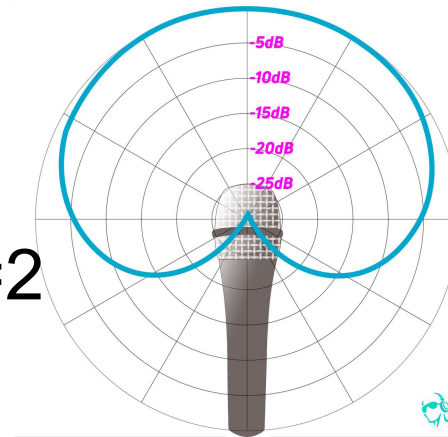
3-element adjustable Yagi,
Russian Antarctica Base:
RI1ANR 14-52 MHz

Principle of Design



Differential arrays

- Array gain up to N^2 vs N for ULA
- Frequency-independent beampattern
- Sensitive designs
 - Most common mic $N=2$
- Microphone: proximity effect
- Yagi antenna: narrowband



Literature

- Uncini, Aurelio. *Fundamentals of adaptive signal processing*. Springer International Publishing, 2015.
 - Sect. 9.4.2 Differential Sensor Array
- Elko, Gary W., and Jens Meyer. "Microphone arrays." *Springer handbook of speech processing*. Springer, Berlin, Heidelberg, 2008. 1021-1041.
- Elko, Gary W. "Differential microphone arrays." *Audio signal processing for next-generation multimedia communication systems*. Springer, Boston, MA, 2004. 11-65.
- Wikipedia: [Yagi–Uda antenna](#)