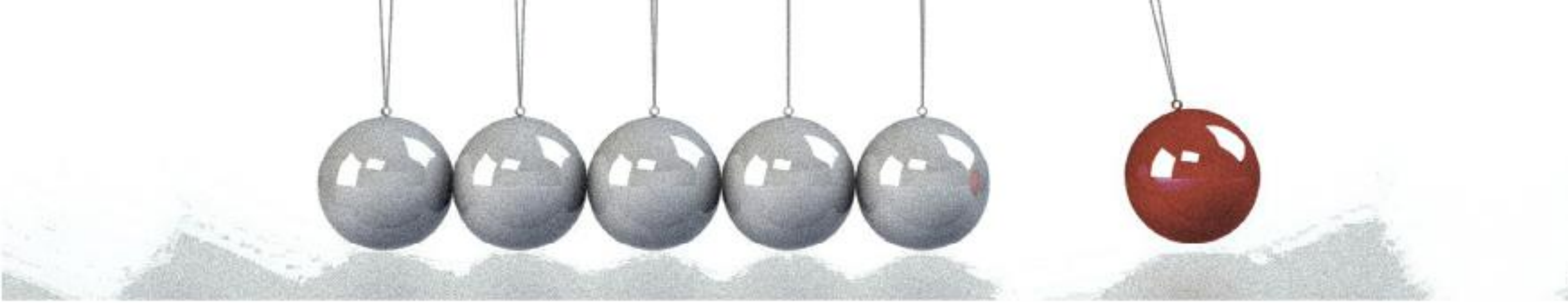


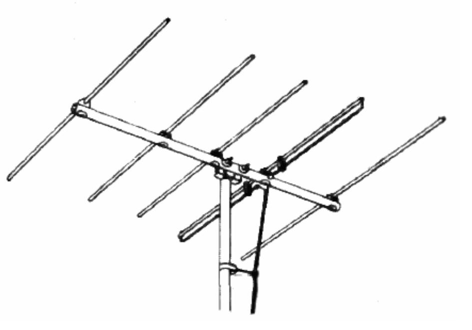
UiO : Department of Physics
University of Oslo

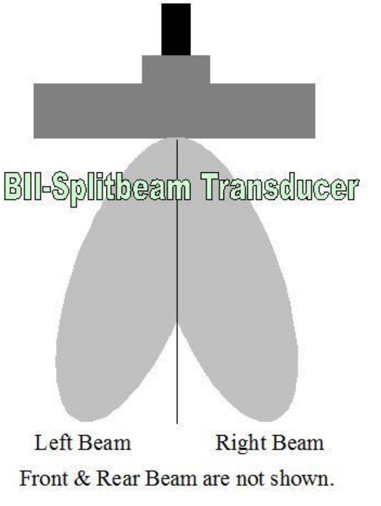
End-fire or differential arrays – from cardioid microphones to Yagi antennas

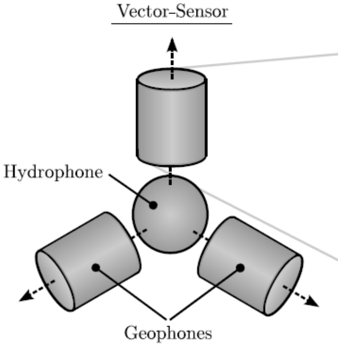
Sverre Holm

Seemingly dissimilar applications

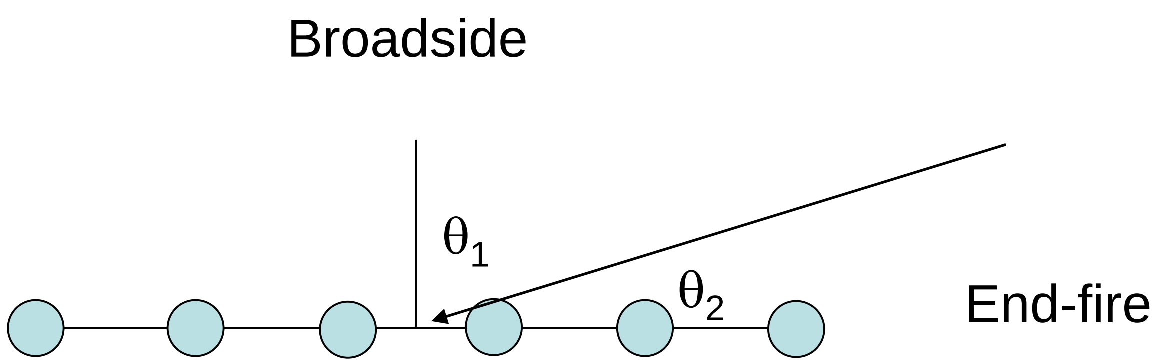
- Microphones
- Yagi antennas
- Split beam echo sounder
- Vector sensors







End-fire arrays



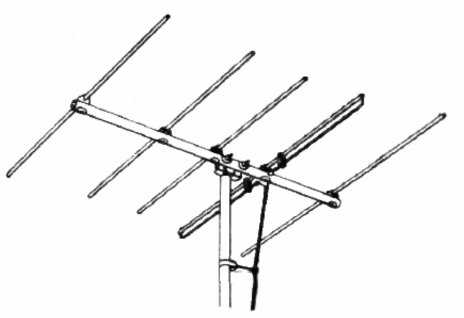
Broadside: sin θ_1, θ_1 angle relative to broadside direction

End-fire: cos θ_2 = sin θ_1, as $\theta_2 = 90-\theta_1$ angle rel to end-fire direction

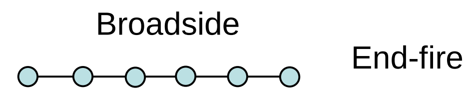
Differential arrays – from cardioid microphones to Yagi antennas

- Part 1:
  Directional microphones (2nd order arrays)

- Part 2:
  Nth order arrays and Yagi-Uda antennas



N-element end-fire vs broadside array

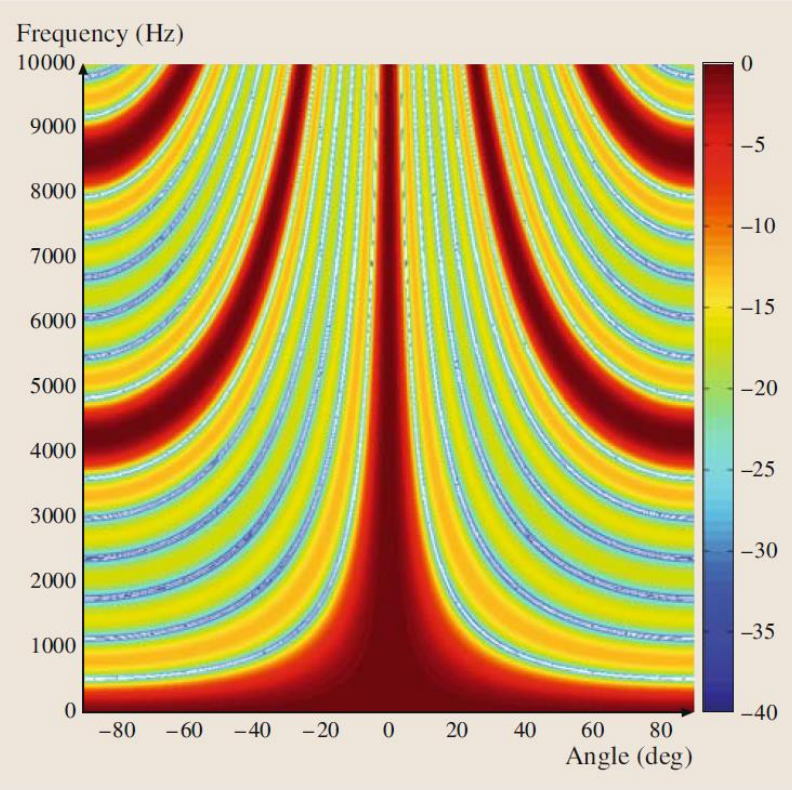


- Broad-side:
  - Element distance: ~ λ/2
  - Array gain: max N
  - Small angle beamwidth: $\theta \propto \frac{\lambda}{D} = \frac{c}{Df} \text{ or } \frac{\lambda}{D}\Big|_{D=N\lambda/2} = \frac{2}{N}$
- End-fire:
  - Element distance: << λ/2
  - Array gain: N^2 (theoretical maximum)
    - Super-directive or supergain
  - Almost frequency-independent beam pattern

The problem with wide bandwidth and the Uniform Linear Array

$$\theta_{BW} \approx \lambda/D = \frac{c}{Df}$$

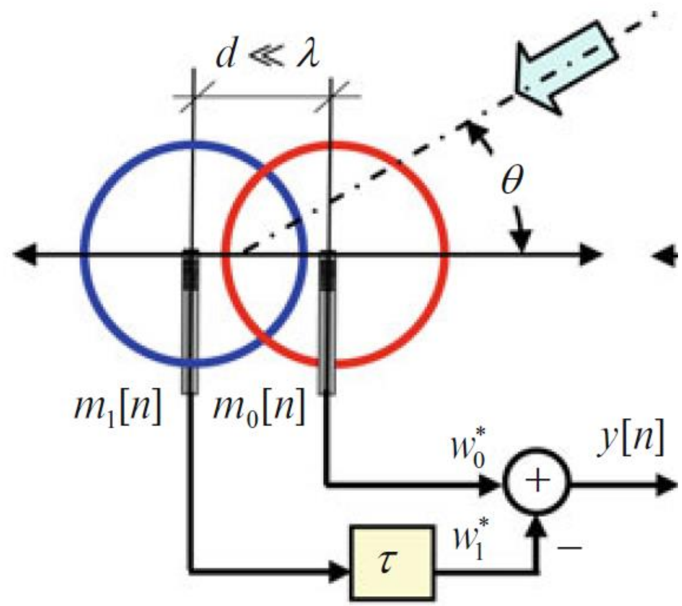
- 7-element uniformly spaced array, d=8 cm, unsteered
- $d = \lambda$@ f=4250 Hz (c=340 m/s)



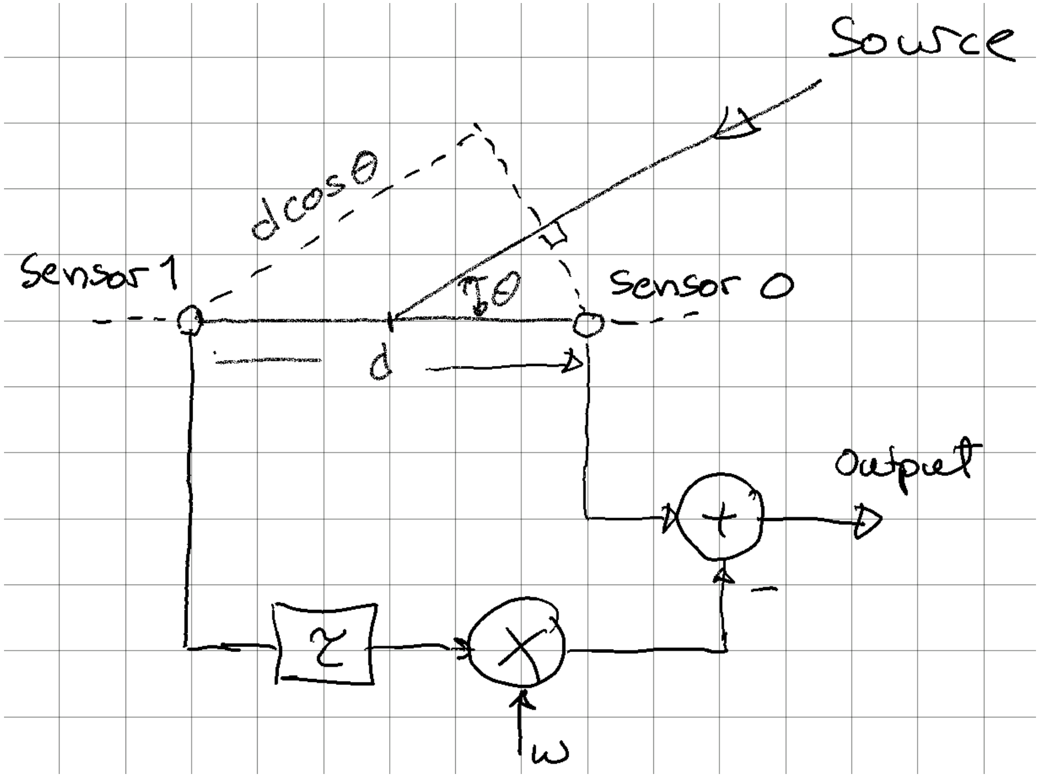
Elko and Meyer. «Microphone arrays» 2008

Two-element array: filter interpretation

- Conventional:
  - Sum: FIR Low-pass beamforming: **steers peak**
- Differential, end-fire
  - Simplest case: $\tau=0$, $w_0=w_1$
  - Difference: FIR High-pass beamforming: **steers null**



Uncini, 2015, Fig. 9.24

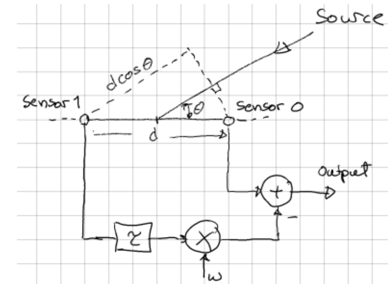


Acoustic delay:

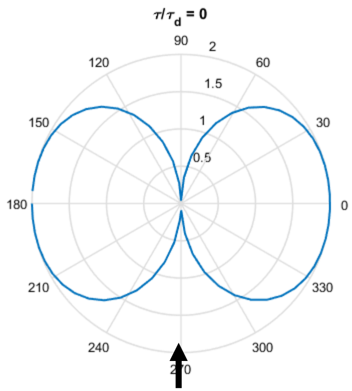
$\tau_d \cos\theta = d \cos\theta / c$

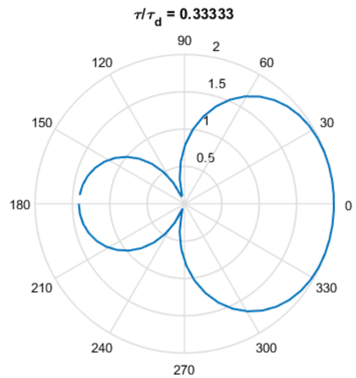
Processing delay: τ, often mechanical implementation

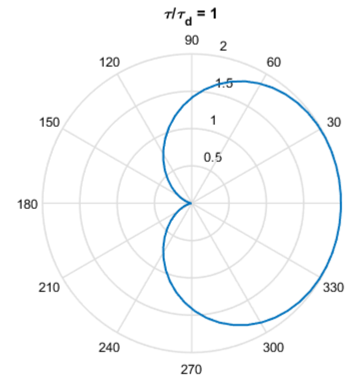
- Response: $R(\omega, \theta) = 1 - w e^{-j\omega(\tau + \tau_d \cos\theta)}, \quad \tau_d = d/c$
- Weight *w* will be used to model propagation effects due to spherical spreading, 1/r-effect, in the near-field
- Ears of owl, lizard, … : τ , *w* due to internal coupling

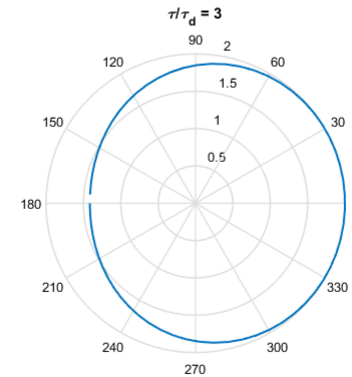


Beampattern, effect of processing @ f=fc: τ+d/c = half a period at fc, w=1









Sharp null, good for direction finding

Dipole (figure eight), $\tau/\tau_d = 0$,

hypercardioid, $\tau/\tau_d = 1/3$,

cardioid, $\tau/\tau_d = 1$,

almost omni $\tau/\tau_d = 3$

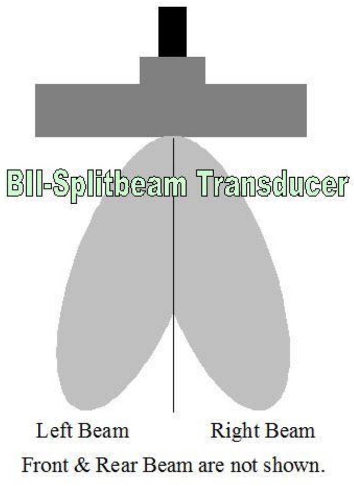
Split beam echo sounder

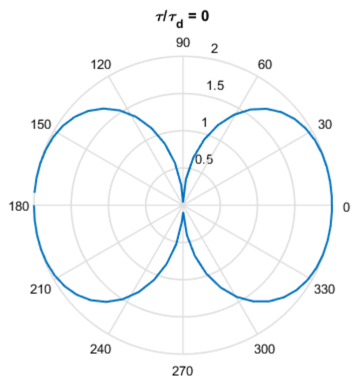
Microphone / Yagi

- speaker mic -

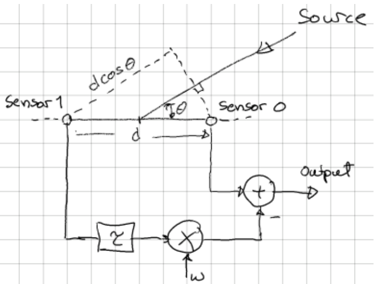
- common mic for vocals -

Split beam echosounder



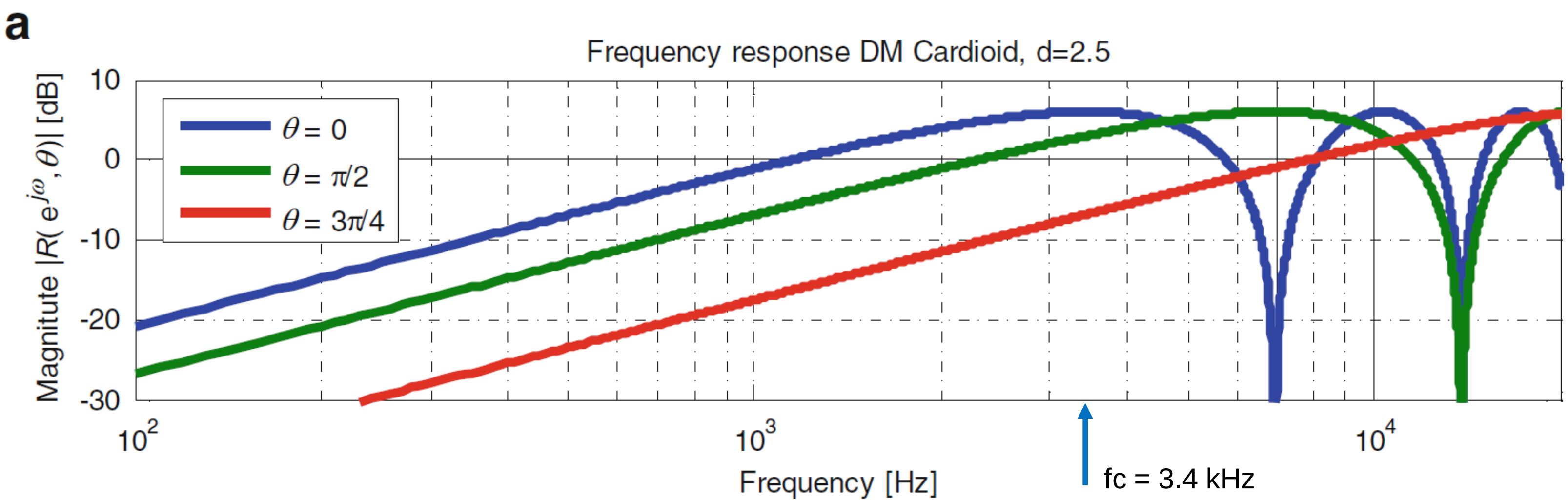


- 2-element transducer
  - Normal beam: add
  - Split-beam: subtraction only delay τ → figure-of-8
- If single broadside target
  - Normal beam: a peak
  - Split-beam: a null
- A null is a **more precise** indicator than a peak for when a target is exactly broadside



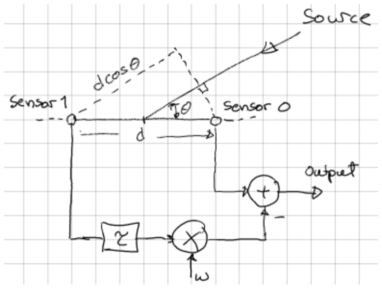
Cardioid, $\tau = \tau_d$:
Dependency of angle of incidence

Note, same angle dependency for all frequencies < fc



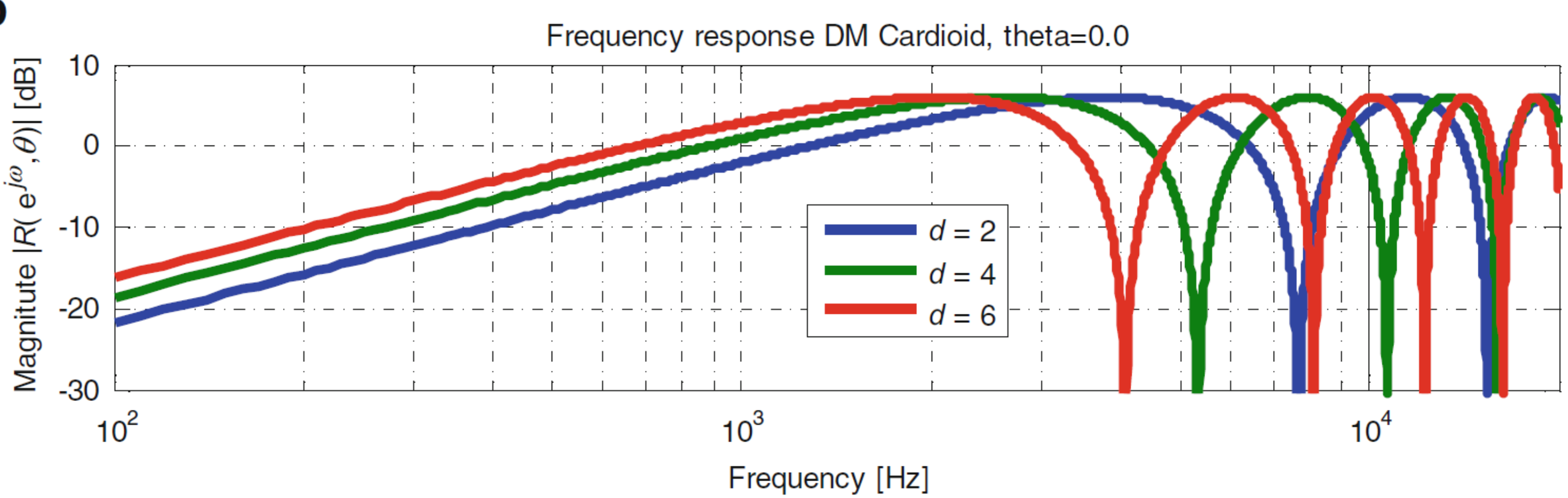
Uncini, Fig 9.29a, d=2.5 cm. Max gain = $10\log 2^2$ = 6 dB

$$R(\omega,\theta) = 1 - w\mathrm{e}^{-\mathrm{j}\omega(\tau+\tau_d\cos\theta)}, \qquad \tau_d = d/c$$



Cardioid, $\tau = \tau_d$:
Dependency of element distance [cm]

b

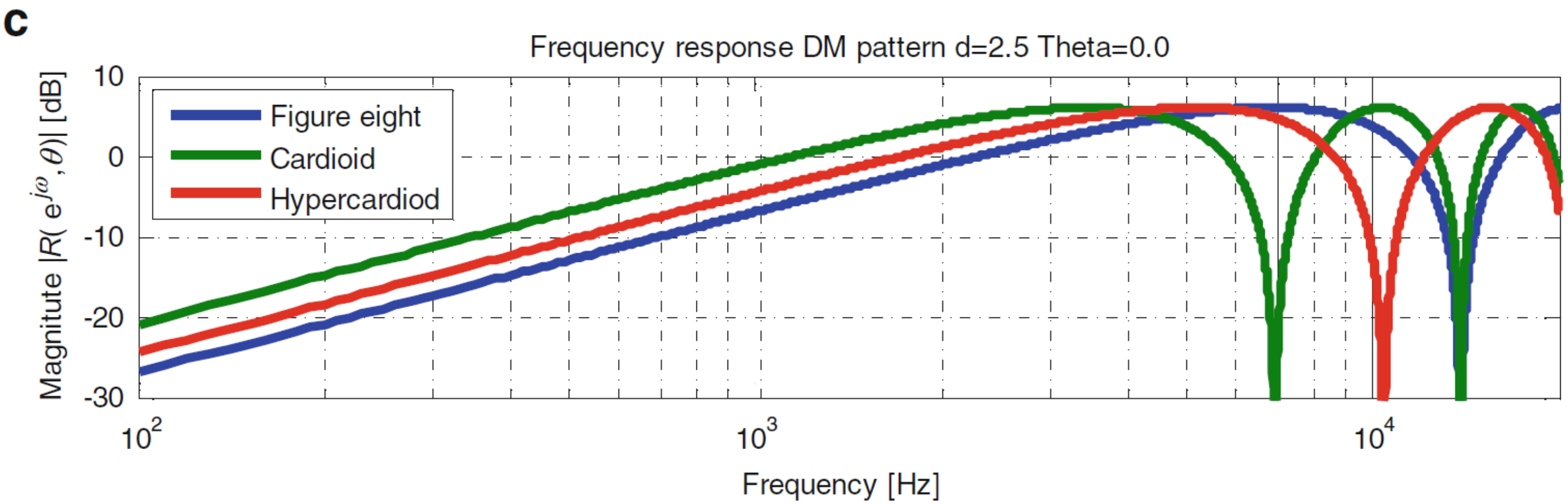


Uncini, Fig 9.29b

$$R(\omega, \theta) = 1 - w\mathrm{e}^{-\mathrm{j}\omega(\tau + \tau_d \cos\theta)}, \qquad \tau_d = d/c$$

Dependency of different patterns

Zeroes, maxima



Uncini, Fig 9.29a, d=2.5 cm

$$R(\omega,\theta) = 1 - w e^{-j\omega(\tau + \tau_d \cos\theta)}, \quad \tau = 0, \ 0.33d/c, \ d/c$$

Figure-8, hypercardioid, cardioid

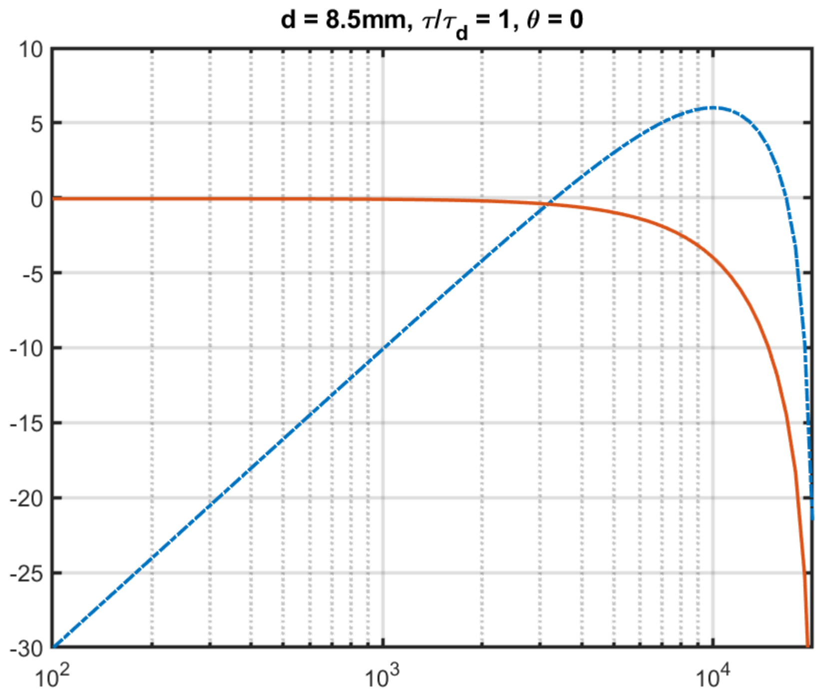
Cut-off frequency and angularity

$$R(\omega,\theta) = 1 - \mathrm{e}^{-\mathrm{j}\omega(\tau+\tau_d\cos\theta)}, \quad \tau_d = d/c$$

- Zeroes: $\omega(\tau + \tau_d \cos\theta) = 0,\ 2\pi$
- Maximum of *R(ω,θ)* gives cut-off frequency:
  $\omega_c(\tau + \tau_d \cos\theta) = \pi \quad \Rightarrow \quad \omega_c|_{\theta=0} = \pi/(\tau + \tau_d)$
- Well below cut-off, phase is small:
  $R(\omega,\theta) \approx \mathrm{j}\omega(\tau + \tau_d \cos\theta)$
  - **Same angle dependency for all frequencies**
  - Gain proportional to frequency

Pressure gradient microphone: compensated for 6 dB/octave

Compensated by the mass of the diaphragm

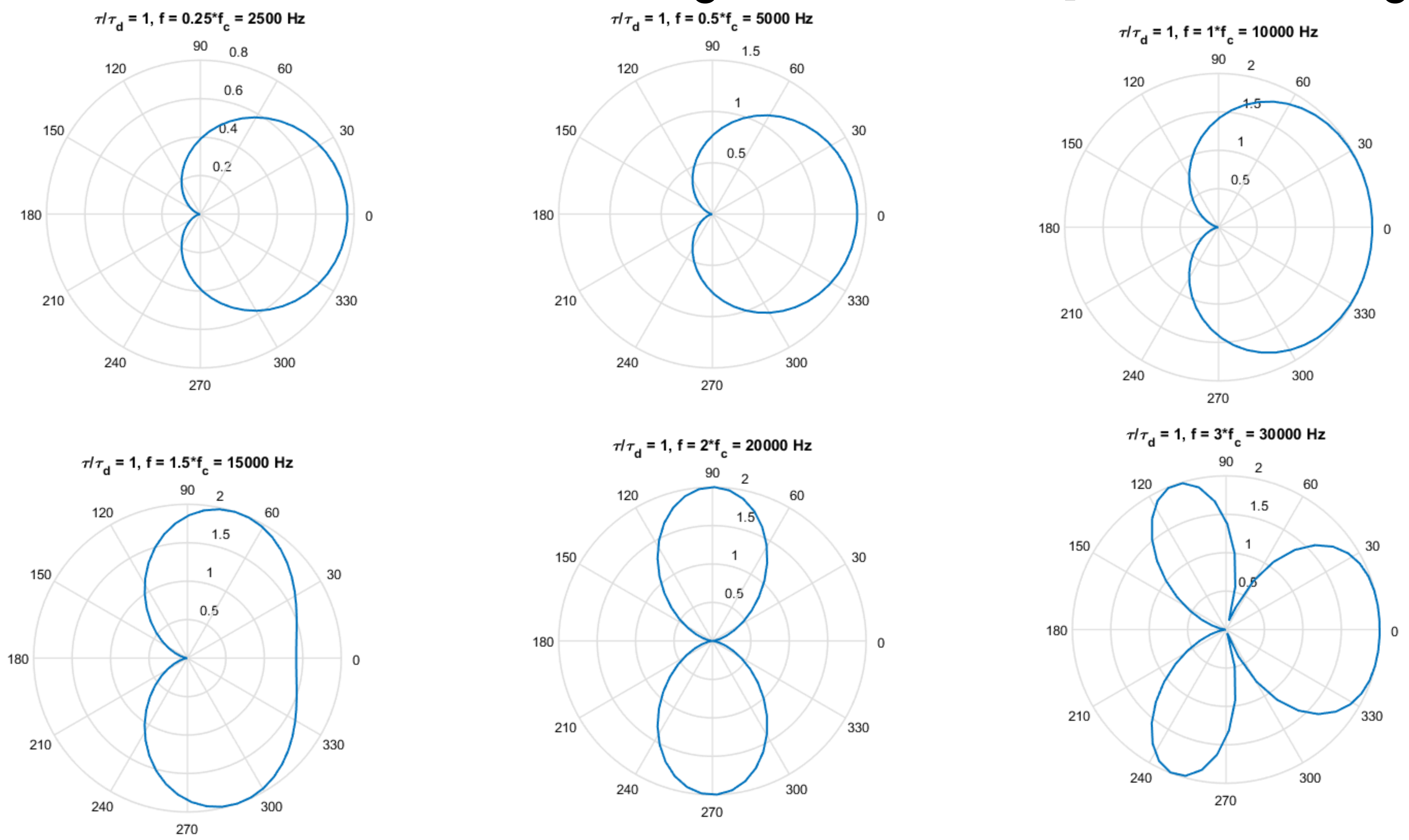


Cut-off frequency: delay=half a period

$$f_c = \frac{\omega_c}{2\pi} = 0.5/(\tau + \tau_d) = 0.5/(\tau + \frac{d}{c})$$

- Cardioid, τ = *d/c*: $f_c = \frac{c}{4d}$
- Uncini examples, d=2.5 cm: f_c = 3.4 kHz
- Next examples, d=8.5 mm: f_c = 10 kHz

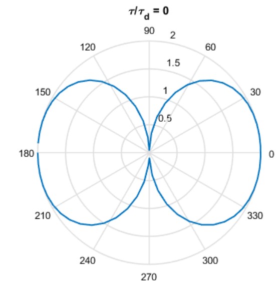
Relatively insensitive to frequency: Cardioid below f_c, breaks up above f_c



Difference array: Quite sensitive to parameter variations

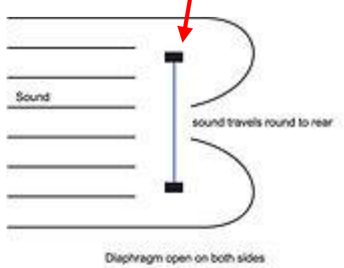
- Broadband array needs an equalizer to boost low frequencies ⇔ large sensitivity to low-frequency self-noise
- Element distance, d << λ
  - But not too small, otherwise sensitivity to noise increases
- Higher order differential arrays are even more sensitive

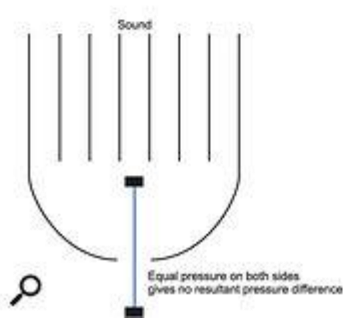
Figure-of-eight microphone



Membrane

Front





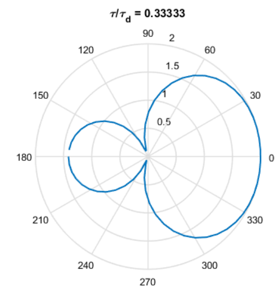
If the mic diaphragm is open to the air on one side but closed at the other, it is considered to be pressure-operated: although it reacts to air pressure, it is not sensitive to direction, resulting in an **omnidirectional** mic pattern.

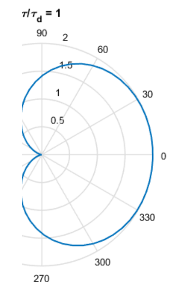
Where the diaphragm is **open on both sides**, as in this diagram, it responds to the pressure-gradient (the difference between the pressure at the front and the back of the diaphragm).

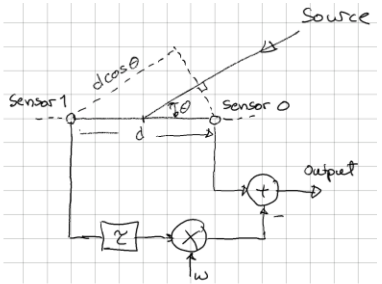
In this case, sound from the side results in even pressure on both sides of the diaphragm, which is why **figure-of-eight** mics reject sound from the side but are responsive to both the front and rear.

https://www.soundonsound.com/techniques/using-microphone-polar-patterns-effectively

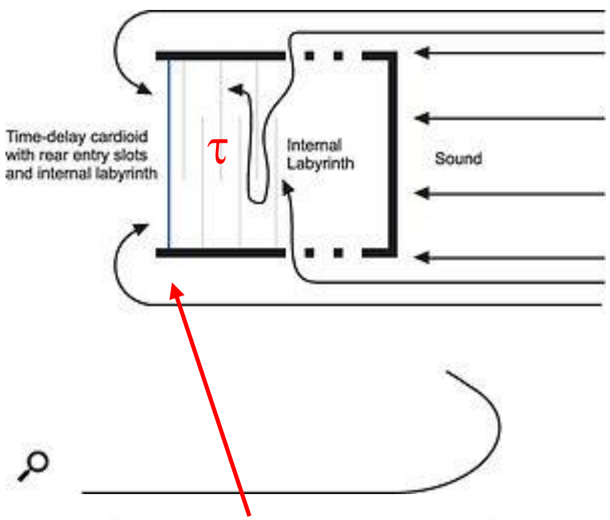
Cardioid mic







Front d



Membrane

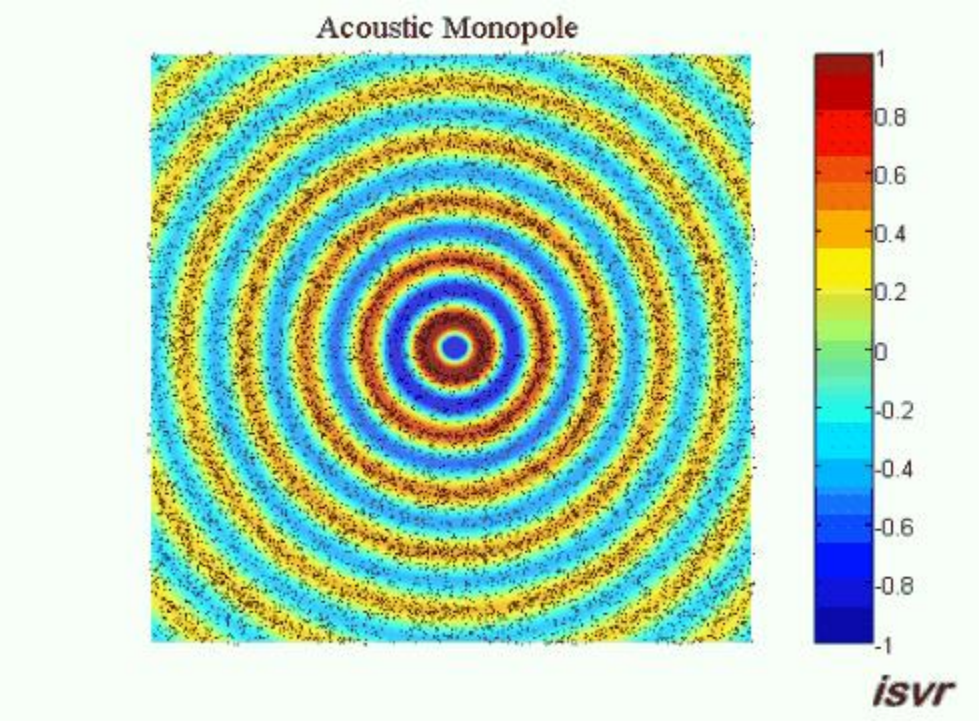
Most cardioid mics now incorporate a vented 'labyrinth' in a single-capsule design that manipulates the phase of sounds hitting the rear, to produce the desired cardioid pattern.

The supercardioid and hypercardioid designs use the same principle to create a more focused pattern to the front, at the expense of reducing the rear rejection.

If you notice vents at the side of the mic head, the mic probably has a cardioid pattern (or a variation on it).

https://www.soundonsound.com/techniques/using-microphone-polar-patterns-effectively

Spherical spreading (from mouth)

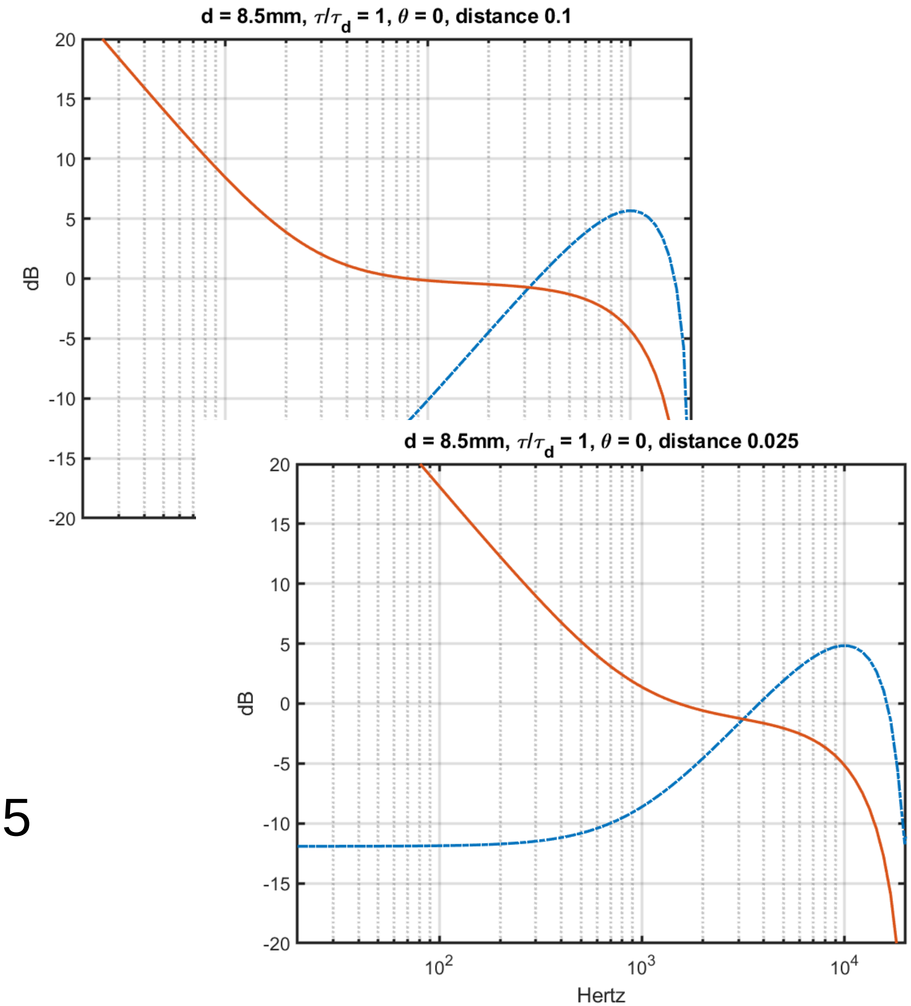


http://resource.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial_files/Web-basics-pointsources.htm

Proximity effect of cardioid: bass boost when close-in

Difference due to 1/r:

- Distance d=10 cm:
  - 1. element: 10 cm
  - 2. el.: 10+0.85 cm
  - Effective w = 10/10.85 = 0.92
- Distance d=2.5 cm:
  - w = 2.5/(2.5+0.85) = 0.75



Neumann U 47: First switchable pattern condenser microphone (1940's)

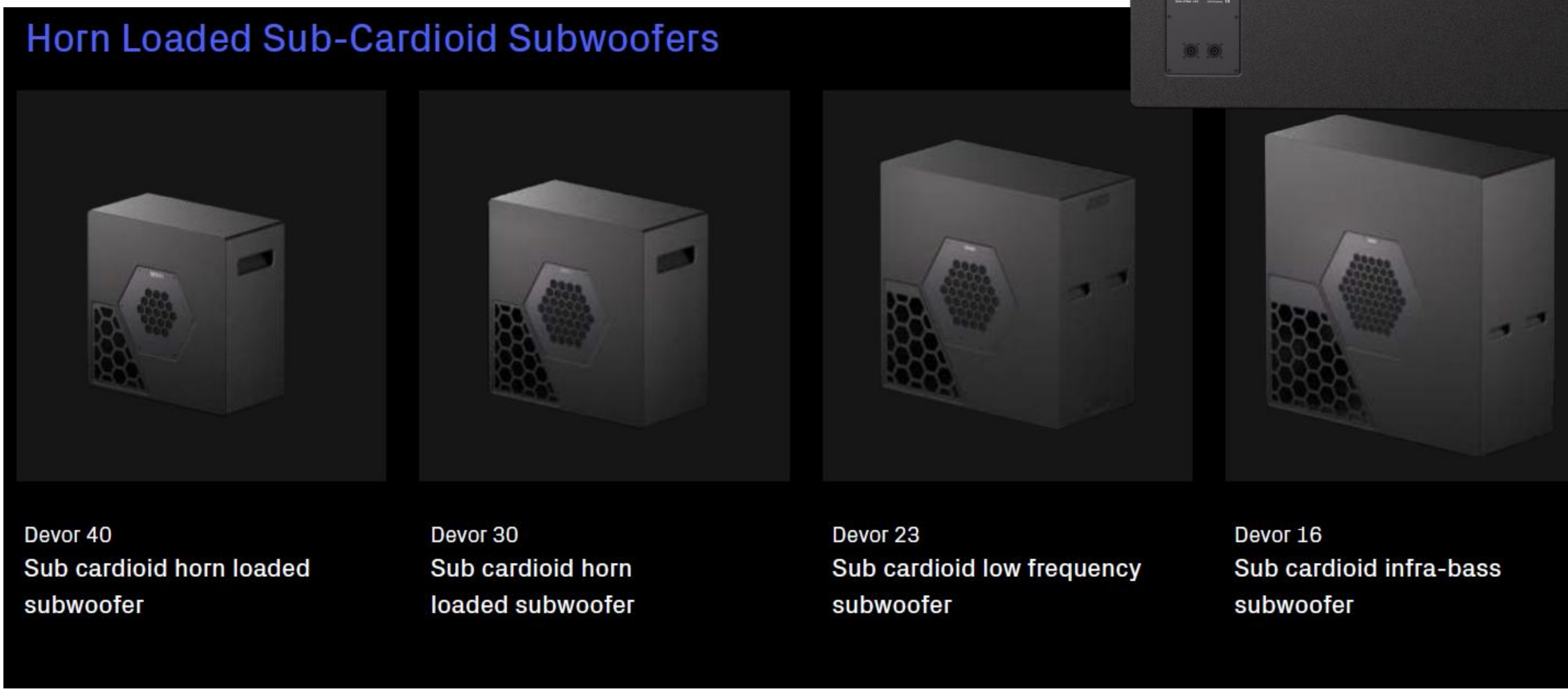
Front and rear membranes: cardioid

- Sound coming from the front causes movement of the front membrane and reaches the inner side of the rear membrane through the perforations in the electrode.
- If only one membrane is connected, the microphone works as described above as a cardioid.

When connecting both cardioid halves in parallel, the capsule produces an omnidirectional pattern.

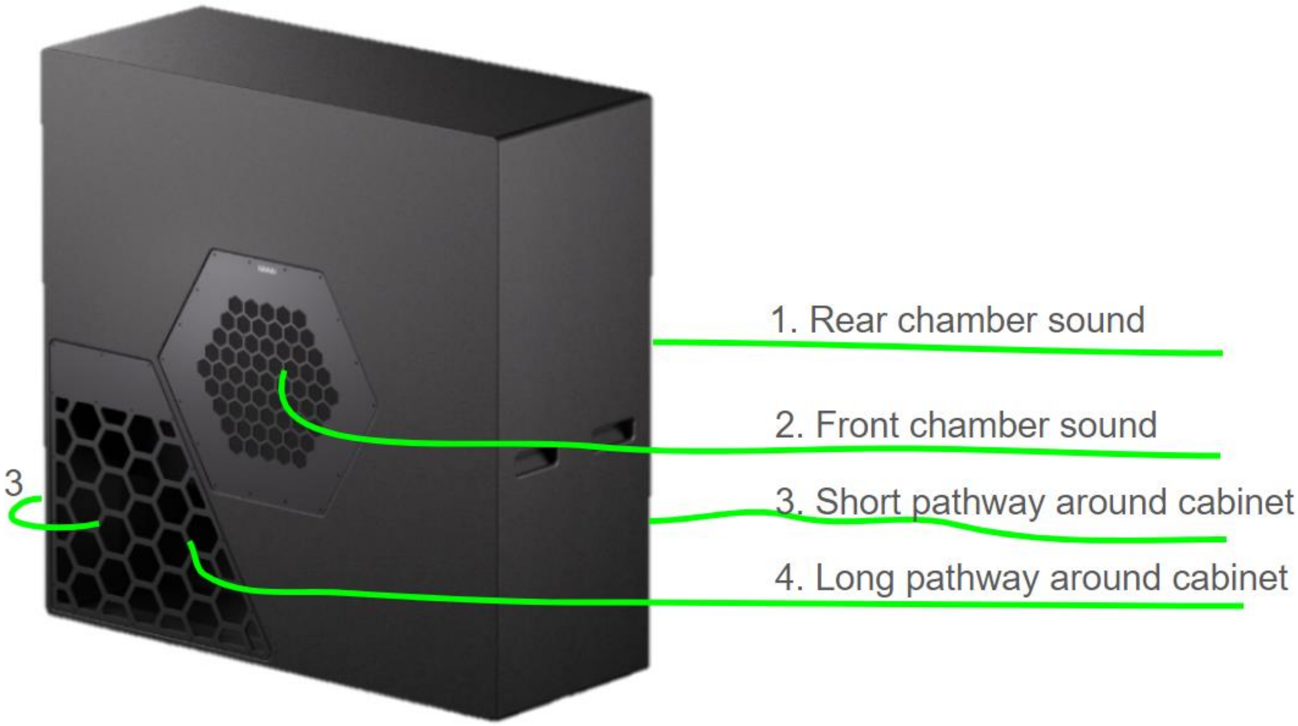
https://en-de.neumann.com/u-47

End-fire subwoofers



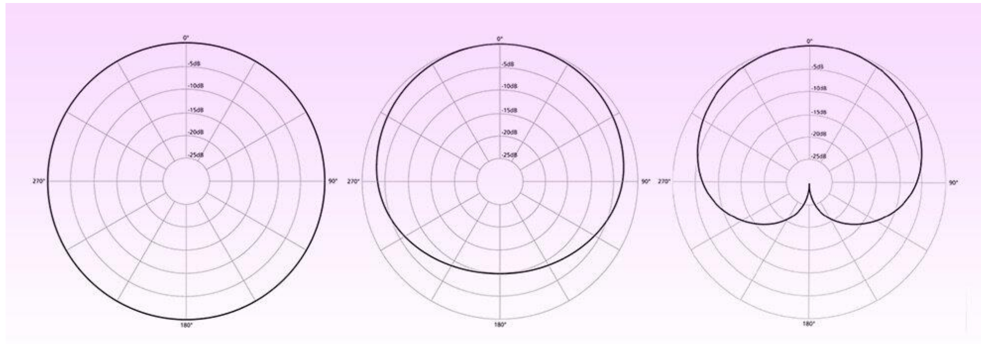
www.nnnn.no

NNNN Devor: N=4-element end-fire



Devor 23: 8-10 dB attenuation behind speaker (23 Hz speaker)

Sub cardioid



(12) **United States Patent**
Skramstad

(10) **Patent No.: US 11,882,400 B2**
(45) **Date of Patent: Jan. 23, 2024**

(54) **DIRECTIONAL LOUDSPEAKER**

(71) Applicant: **NNNN AS**, Årnes (NO)

(72) Inventor: **Rune Skramstad**, Drammen (NO)

(73) Assignee: **NNNN AS**, Årnes (NO)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 232 days.

(21) Appl. No.: **17/623,309**

(22) PCT Filed: **Jun. 24, 2020**

(56) **References Cited**

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181/152
2,751,997 A * 6/1956 Gately, Jr. H04R 1/2865
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EP 3 018 915 B1 6/2018

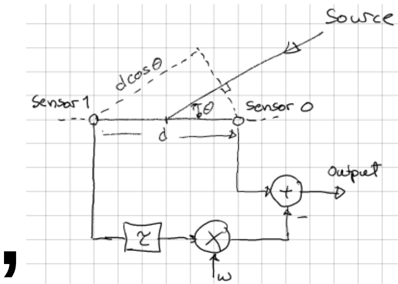
(Continued)

https://www.production-expert.com/production-expert-1/why-you-should-be-using-subcardioid-mics

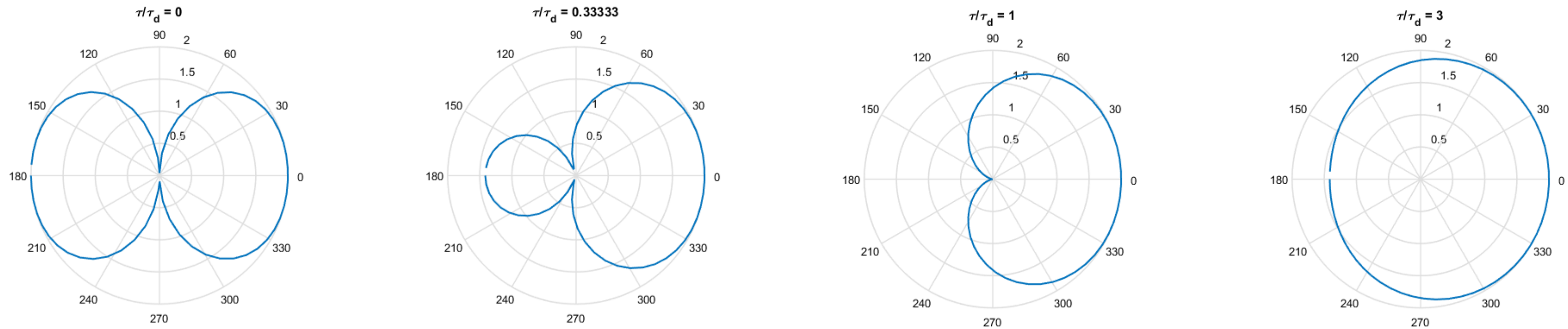
Differential arrays – from cardioid microphones to Yagi antennas

- Part 1:
  Directional microphones (2nd order arrays)

- Part 2:
  Nth order arrays and Yagi-Uda antennas



Beampattern, effect of processing, w=1, f=fc



Dipole (figure eight), $\tau/\tau_d = 0$,

hypercardioid, $\tau/\tau_d = 1/3$,

cardioid, $\tau/\tau_d = 1$,

almost omni $\tau/\tau_d = 3$

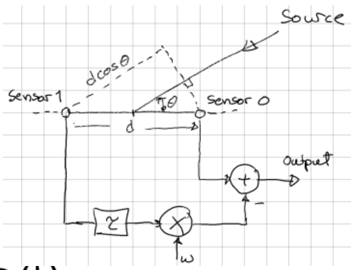
Split beam echo sounder

Microphone / Yagi

- speaker mic -

- common mic for vocals -

Exact differential array



- Plane wave (far-field) pressure field:

$$p(r,k,t) = A_0 e^{\mathrm{j}(\omega_0 t - kr\cos\theta)}$$

- Spatial derivative (drop time) = pressure gradient:

$$\left|\frac{\mathrm{d}}{\mathrm{d}r}p(k,r)\right| = \mathrm{j}k\cos\theta\; p(k,r)$$

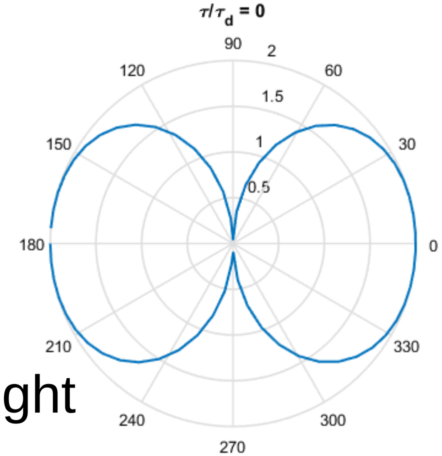
- Beam pattern shape

$$\propto k\cos\theta = \frac{\omega}{c}\cos\theta$$

  - Similar to previous derivation:

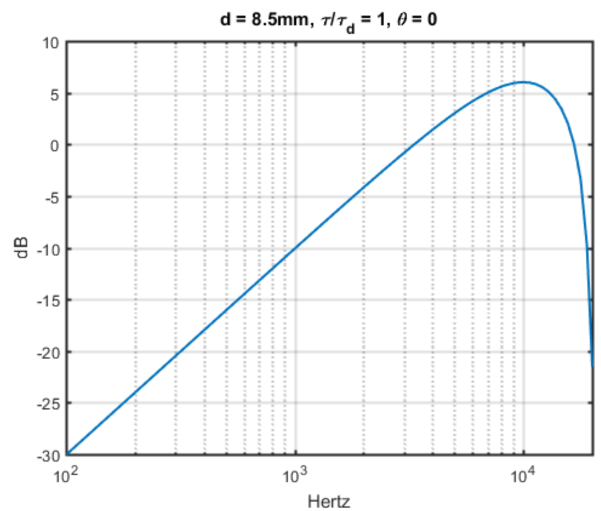
$$R(\omega,\theta) = 1 - \mathrm{e}^{-\mathrm{j}\omega(\tau+\tau_d\cos\theta)}|_{\tau=0} \approx \mathrm{j}\omega(\tau_d\cos\theta)$$

  - ~no-delay difference array (τ=0), figure-of-eight



Differential array (cont)

- $\mathrm{j}k\cos\theta = \frac{\mathrm{j}\omega}{c}\cos\theta$
  - high-pass 6 dB/octave

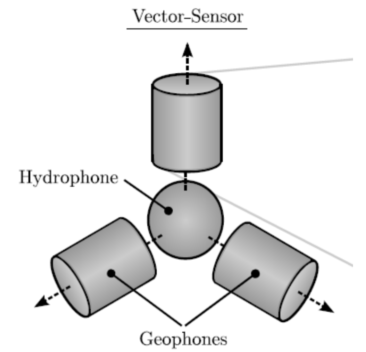


- Pressure gradient from conservation of linear momentum (Euler):

$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla p \ \Rightarrow \ \nabla p \propto \mathrm{j}\omega |v|$$

- Therefore called pressure gradient or velocity microphone
- If 3-D: vector sensor

Vector sensors – underwater acoustics: low f.



It is shown that the multichannel receiver using a single vector sensor can offer significant size reduction for coherent acoustic communication at the carrier frequency of 12 kHz, compared with a pressure sensor line array.

- Song, Abdi, Badiey, Hursky, (2011). Experimental demonstration of underwater acoustic communication by vector sensors. IEEE J Ocean Eng,

This paper proposes a mode domain beamforming method for a 3 x 3 uniform rectangular array of two-dimensional (2D) acoustic vector sensors with inter-sensor spacing much smaller than the wavelengths

- Guo, Yang, Miron, (2015). Low-frequency beamforming for a miniaturized aperture three-by-three uniform rectangular array of acoustic vector sensors. J Acoust Soc Am..

Grønlandshval: 25-900 Hz

Masking from industrial noise can hamper the ability to detect marine mammal sounds near industrial operations, whenever conventional (pressure sensor) hydrophones are used for passive acoustic monitoring. … Improvements in signal-to-noise ratio of up to 15 dB are demonstrated on bowhead whale calls, which were otherwise undetectable using conventional hydrophones.

- Thode,Kim, Norman, Blackwell, Greene (2016). Acoustic vector sensor beamforming reduces masking from underwater industrial noise during passive monitoring. J Acoust Soc Am.

Far-field: n’th order differential array

$$\left|\frac{\mathrm{d}^n}{\mathrm{d}r^n} p(k,r)\right| = (\mathrm{j}k\cos\theta)^n \; p(k,r)$$

- Beampattern $\propto \cos^n \theta$
- Frequency response: $\propto \omega^n$: 6n dB/octave

Near-field: n'th order differential array

Pressure: $p(r,t) = A_0 e^{\mathrm{j}(\omega_0 t)} \frac{e^{-\mathrm{j}k_0 r \cos\theta}}{r}$

$$\frac{\mathrm{d}^n}{\mathrm{d}r^n} p(k,r,\theta) = A_0 \frac{n!}{r^{n+1}} \mathrm{e}^{-\mathrm{j}kr\cos\theta} (-1)^n \sum_{m=0}^{n} \frac{(\mathrm{j}kr\cos\theta)^m}{m!}$$

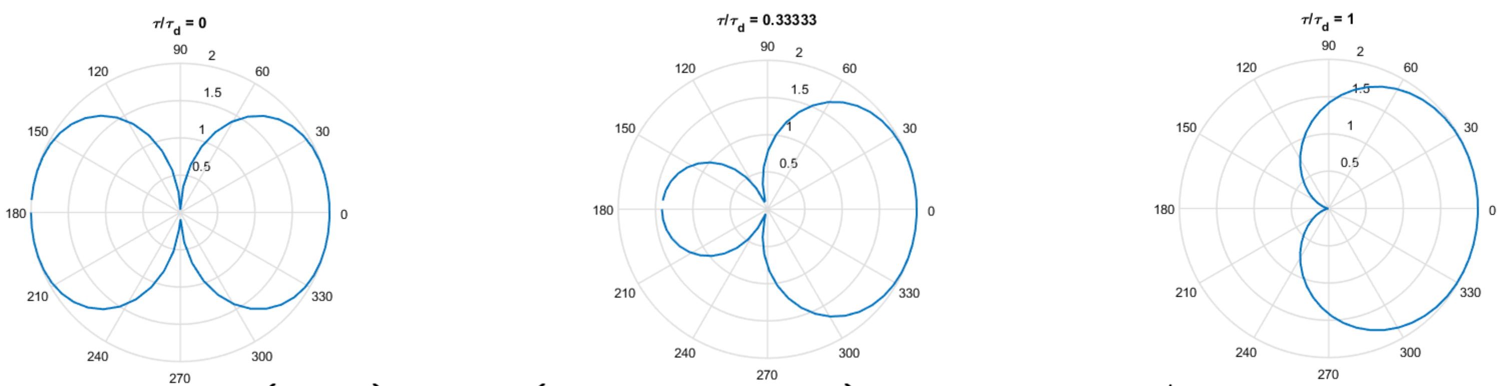
- Sum of dipole-like terms of type $\cos^m \theta$
- May optimize coefficients for desirable properties
- Differential array, n=1, i.e. 2 terms in sum:

$$R(\omega,\theta) \approx \mathrm{j}\omega(\tau + \tau_d \cos\theta) = \mathrm{j}\omega(a_0 + a_1 \cos\theta)$$

Differential array in practice

- Approx. by finite differences, d << λ
- Response: $\propto \omega^n \left(a_0 + a_1 \cos\theta + a_2 \cos^2\theta + \ldots + a_n \cos^n\theta\right)$
- n=1:

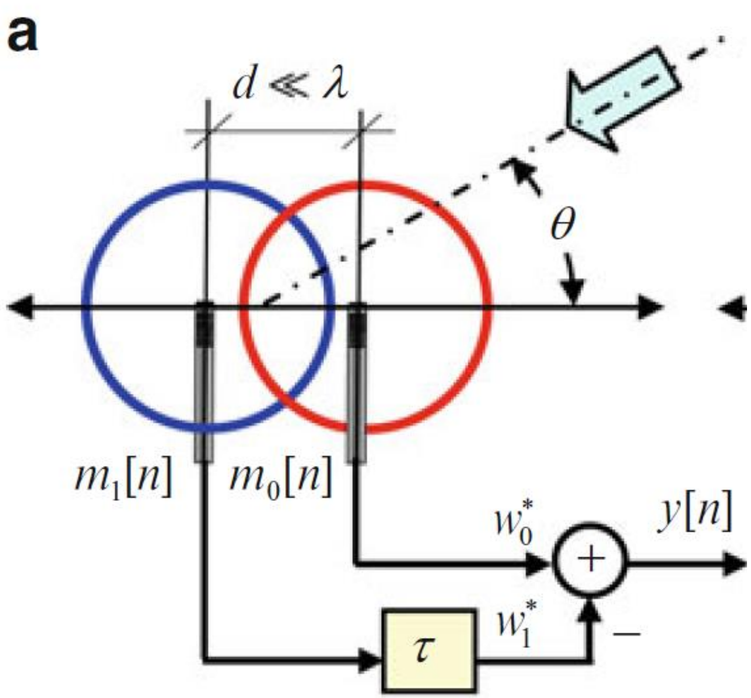
n=1:	Figure-of-8	a_0=0,	a_1=1
	Hypercardioid	a_0=1/4	a_1=3/4
	Cardioid	a_0=1/2	a_1=1/2



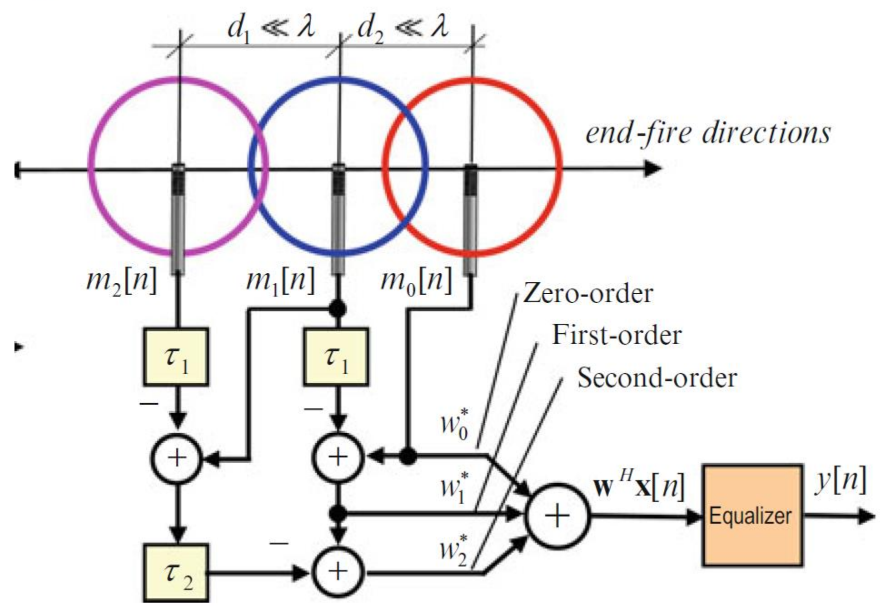
$$R(\omega,\theta) \approx \mathrm{j}\omega(\tau + \tau_d \cos\theta), \quad \tau = 0, \tau_d/3, \tau_d$$

First, second order differential array

0: m_0
1: $m_0 – m_{1,1}$
2: $(m_0 - m_{1,1}) – (m_1 - m_{2,1})_{,2}$



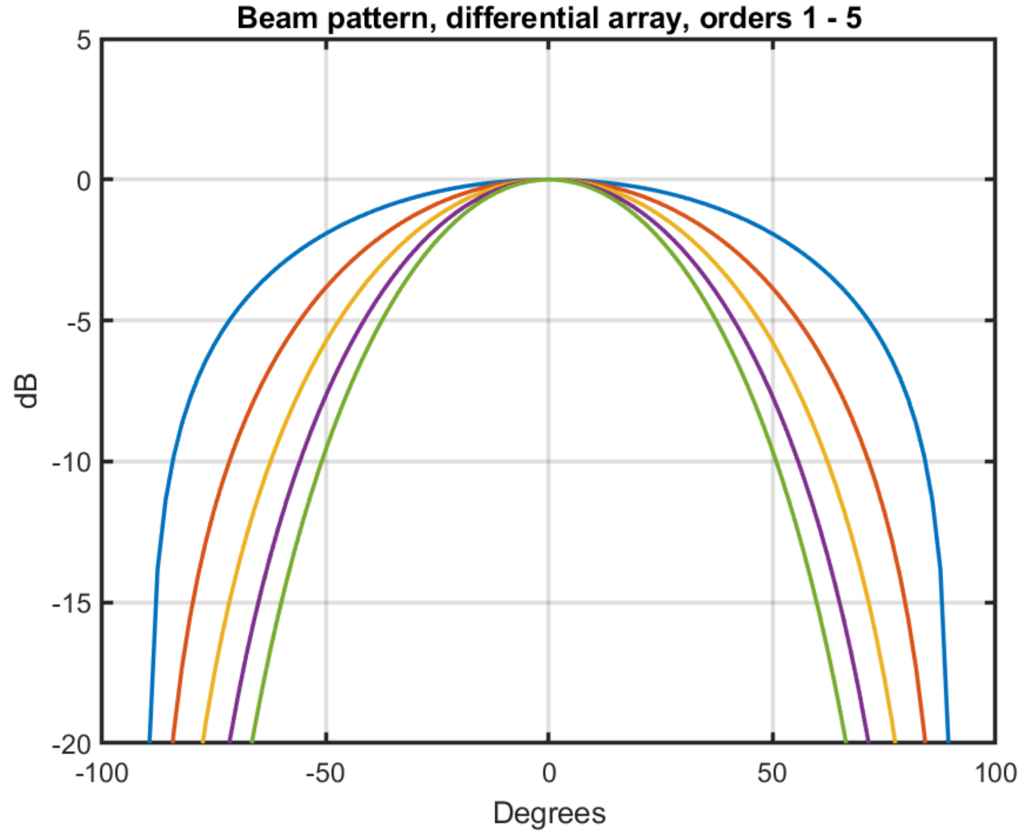
b



Uncini, 2015, Fig. 9.24

$$\propto \omega^n \left(a_0 + a_1 \cos\theta + a_2 \cos^2\theta + \ldots + a_n \cos^n\theta\right)$$

Beam pattern, $\cos^n$



$$\propto \omega^n \left(a_0 + a_1 \cos\theta + a_2 \cos^2\theta + \ldots + a_n \cos^n\theta\right)$$

Maximum directional gain 1st – 4th order

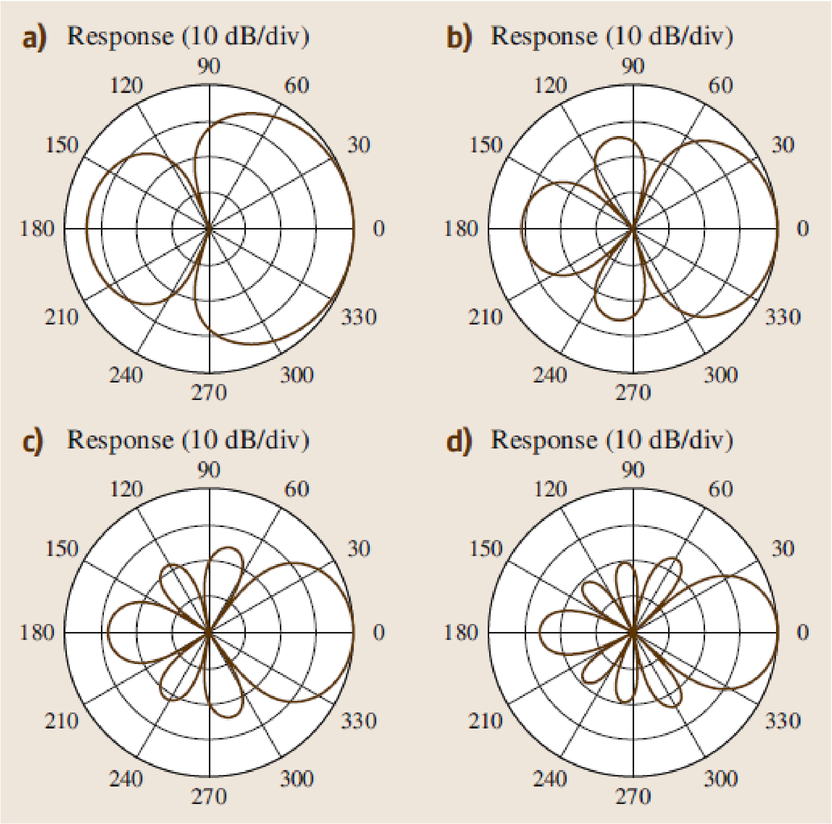


Fig. 50.12a–d Directional patterns that have maximum directional gain for differential microphone arrays for up to forth-order arrays

n=1: hypercardioid

n=2: Narrower beam than that of $\cos^2$

~ shotgun (lobar) microphone

Elko and Meyer. «Microphone arrays» 2008

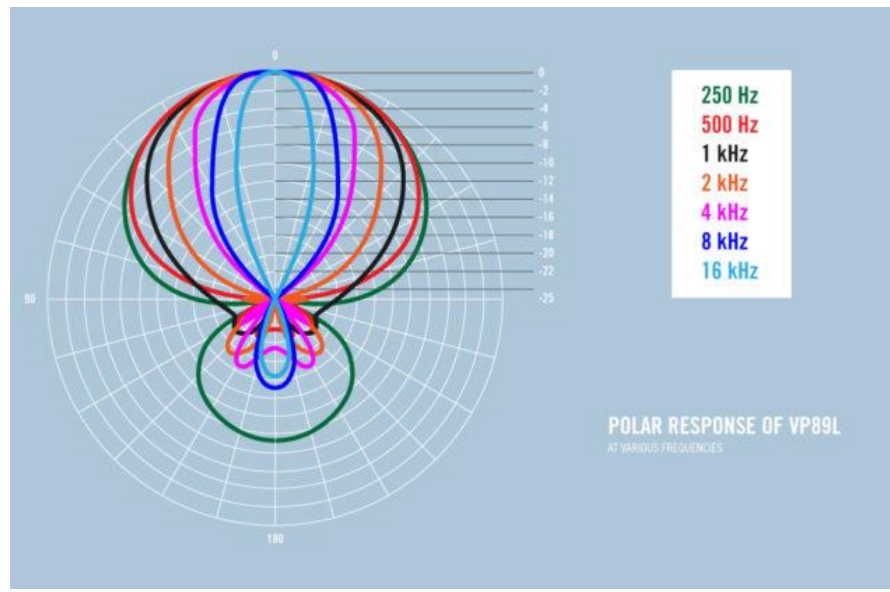
Shotgun microphone

Characteristics

- Low frequencies, supercardioid
- High: lobar
- Off axis, more sensitive to lower and less to higher frequencies: colored sound.

Applications:

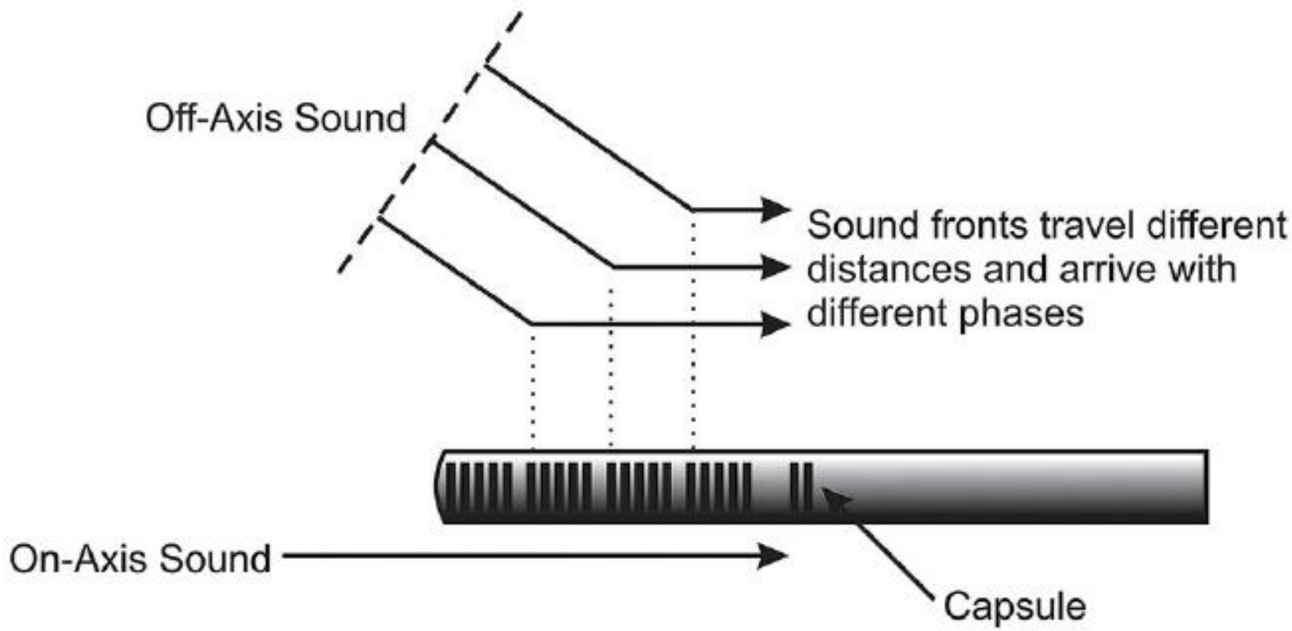
- Film industry: dialog pickup on the shooting set
- Sport events
- Birds at great distances



Polar Response of the Shure VP89L Shotgun Mic.

Shotgun microphone

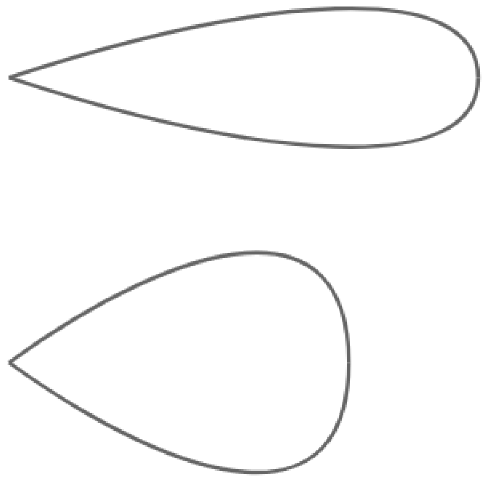
Nth order differential array



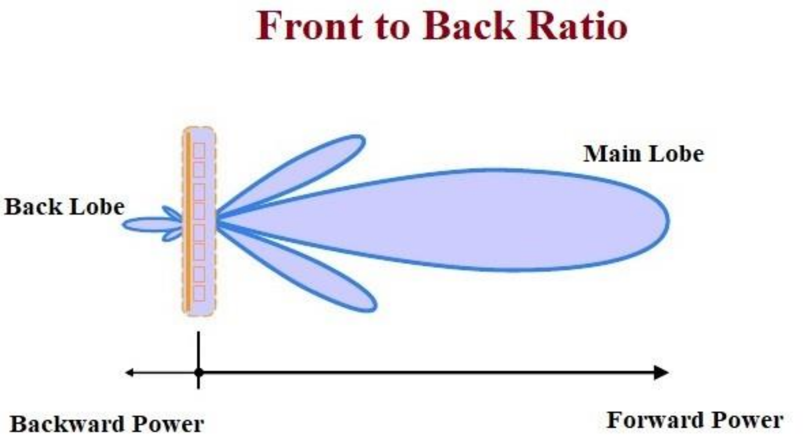
https://www.speechrecsolutions.com/guides/Shotgun%20Mic%20Tutorial.pdf

Performance metrics: Beamwidth, Gain, Front/Back

More gain less beamwidth



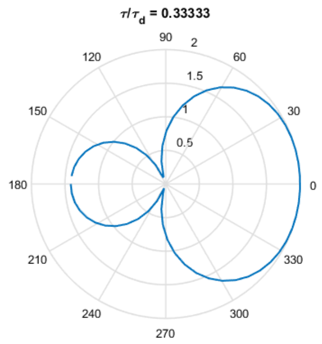
More beamwidth less gain



https://www.electronics-notes.com/articles/antennas-propagation/yagi-uda-antenna-aerial/gain-directivity.php
https://www.everythingrf.com/community/what-is-front-to-back-ratio-in-an-antenna

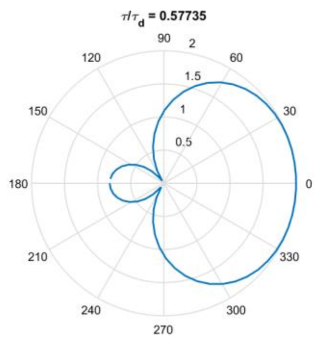
Optimization, first order

$$E(\theta,\omega) \propto \omega\,(a_0 + a_1 \cos\theta)$$



Maximum gain

- n=1: hypercardioid $E_{HC_1}(\theta) = \dfrac{1 + 3\cos\theta}{4}.$
- Array gain: 20log(n+1)= 10logN^2
  - N is no of elements



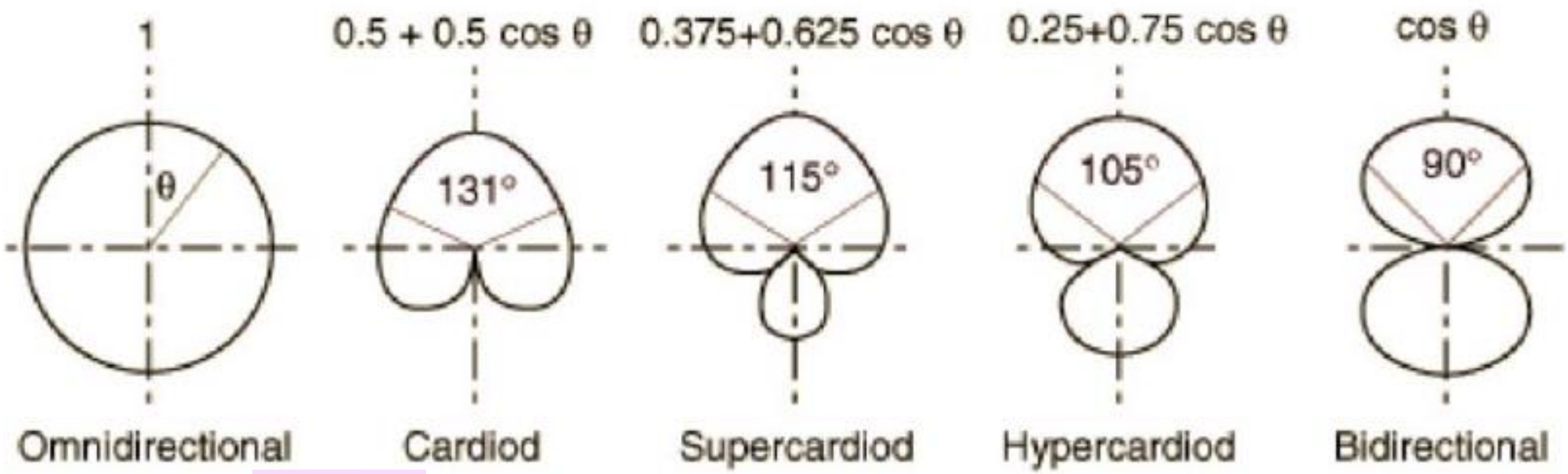
Best front-back ratio

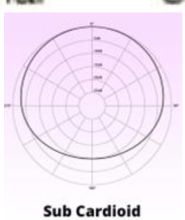
- n=1: supercardioid $E_{SC_1}(\theta) = \dfrac{\sqrt{3} - 1 + (3 - \sqrt{3})\cos\theta}{2}.$

Elko, "Differential microphone arrays”, 2004

Hyper- vs super-cardioid
Max gain vs best front/back-ratio

(Theory: 0.366, 0.634)

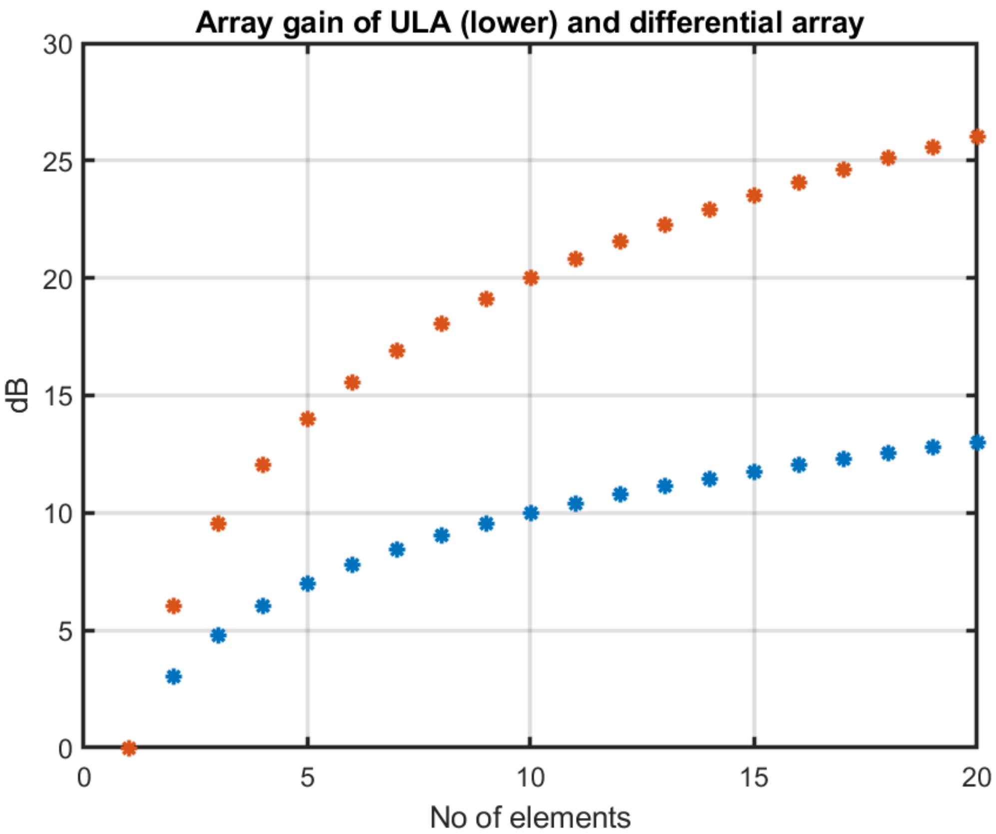




Best front/back-ratio, maximum gain

Quaranta, Dimino, D'altrui, General guidelines for acoustic antenna designed for beamforming noise source localization, 2007

Maximum theoretical array gain, end-fire vs uniform linear array

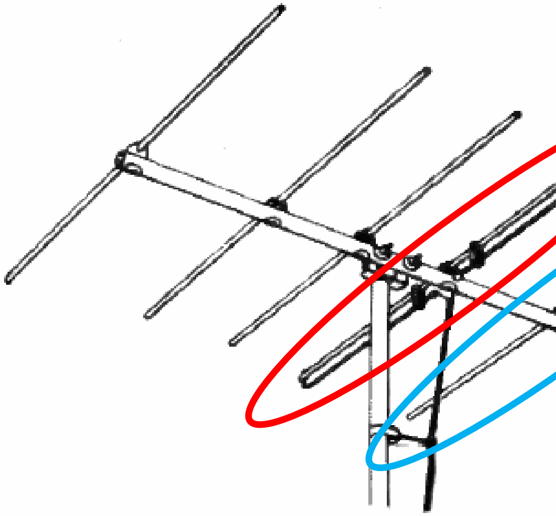


$$AG = 10\log N^2, \qquad BW \propto \frac{1}{\sqrt{N}}$$

$$AG = 10\log N, \qquad BW \propto \frac{\lambda}{D} = \frac{2}{N}$$

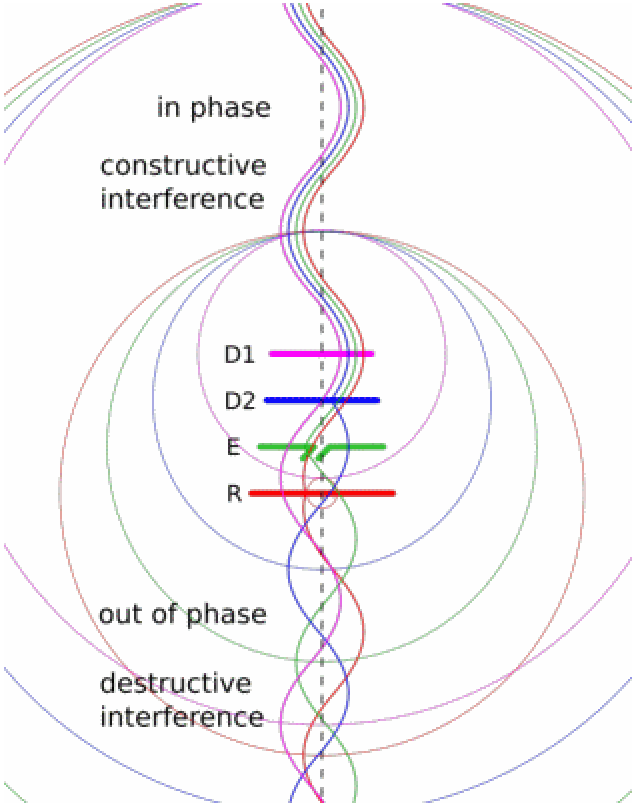
Yagi-Uda antenna: n-1 parasitic elements

- Single active element
  - Length $\lambda/2$ => narrowband
- Passive elements:
  - Reflector
    - typ 5% longer: Capacitive reactance - voltage phase lags that of the current
  - Directors (ex: 3 or 17):
    - typ 5% shorter: Inductive reactance - current phase lags phase of the voltage
- Delays:
  - τ_d – element distance
  - τ – element length, i.e. frequency dependent
  -



Why never
> 1 reflector?

Phasing animation (from Wikipedia)



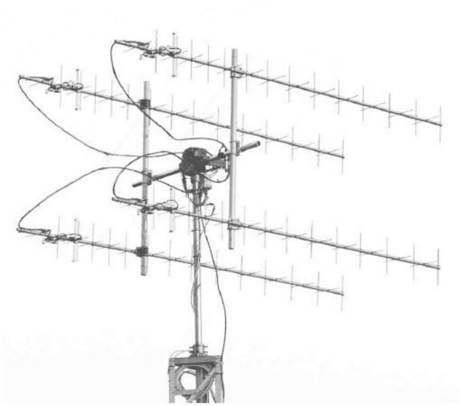
- Time delays due to element distance
  - Phasing in element due to length vs wavelength
- Reradiation from passive elements (parasitic)
  - Field behind first reflector is ≈ 0
- Inherently narrow-band

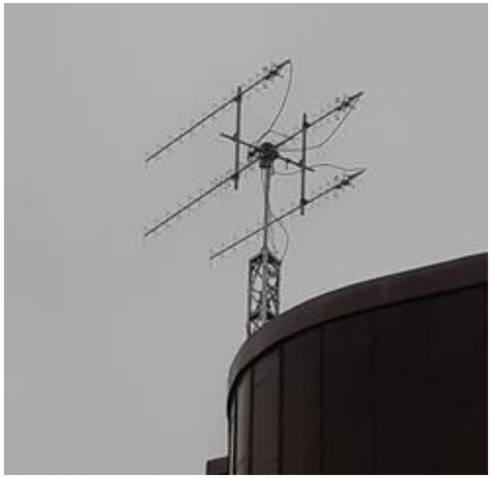
Uda, S., 1925, "On the Wireless Beam of Short Electric Waves". Journ. Institute of Electrical Engineers of Japan

Yagi, Hidetsu; Uda, Shintaro, 1926,"Projector of the Sharpest Beam of Electric Waves" Proc. of the Imperial Academy of Japan.

Physics department, UiO

- CubeSTAR downlink:
  - 437.465 MHz, λ=0.686 m
  - 432-438 MHz radio amateur band
- Circularly polarized
  - 4 x 436CP30, each with:
  - 2 x (13 directors+1 reflector+1 driven element) = 30 elements
  - Horizontal and vertical polarization
- 4 stacked together (~ broadside array)
  - 1.143 m=1.67 λ, gain=20.5 dB, -3 dB beamwidth=16 deg





Eirik Vikan, UiO Satellite Ground Station: Simulation, Implementation and Verification, MSc, 2011

https://www.duo.uio.no/bitstream/handle/10852/11067/Eirik_Vikan_UiO_Satellite_Ground_Station_Simulation_Implementation_and_Verification.pdf

https://www.m2inc.com/FG436CP30

Broadband? Log periodic array

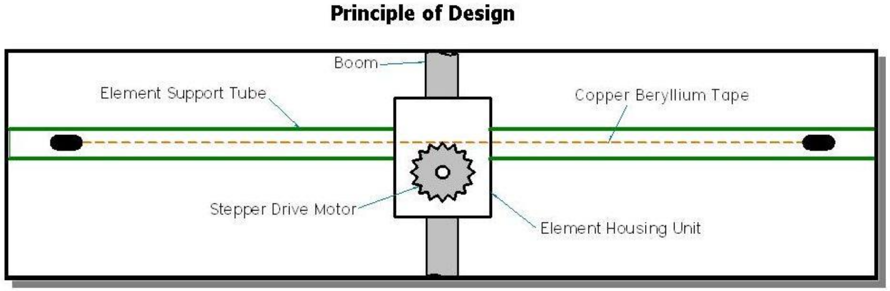
- VHF/UHF, 50-1300 MHz
- 21 elements, 2 m boom
- Forward gain: 10 to 12 dBi (rel isotropic)
  - Like a 4-5-element Yagi
- Complex as several or all elements are active

- Create CLP-5130-1N

Broadband? Adjustable element lengths

Multi-frequency

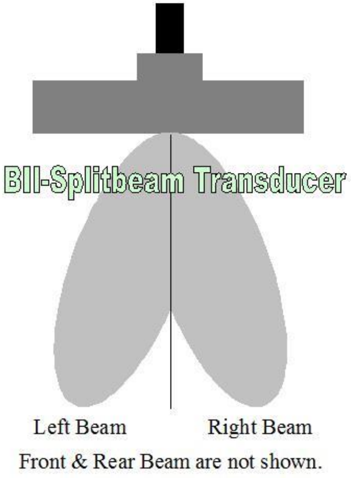
Bi-directional mode

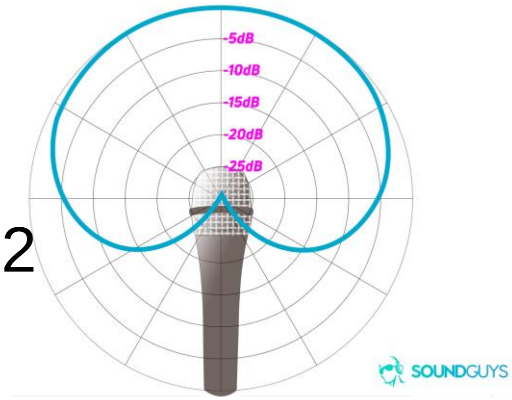


3-element adjustable Yagi, Russian Antarctica Base: RI1ANR 14-52 MHz

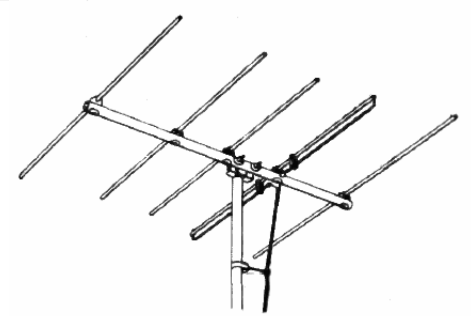
Differential arrays

- Array gain up to N^2 vs N for ULA
- Frequency-independent beampattern
- Sensitive designs
  - Most common mic N=2





- Microphone: proximity effect
- Yagi antenna: narrowband



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- Uncini, Aurelio. *Fundamentals of adaptive signal processing*. Springer International Publishing, 2015.
  - Sect. 9.4.2 Differential Sensor Array
- Elko, Gary W., and Jens Meyer. "Microphone arrays." *Springer handbook of speech processing*. Springer, Berlin, Heidelberg, 2008. 1021-1041.
- Elko, Gary W. "Differential microphone arrays." *Audio signal processing for next-generation multimedia communication systems*. Springer, Boston, MA, 2004. 11-65.
- Wikipedia: Yagi–Uda antenna