Waves with Power-Law Attenuation: Corrections

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In a book with 731 equations and 117 figures, it is unfortunately almost inevitable that there will be some errors. If you find some, please send suggestions to me at this email address: sverre.holm (a) fys.uio.no.

These are corrections and additions for Holm (2019). **Bold text** needs to be added and replace ~~stricken out text~~.

1 Introduction

- Page 12, Fig 1.4: New figure and additions to caption (no effect on main text)
- Page 12, misprint: replace $\mathrm{d}^{-t/\tau_\sigma}$ by $\mathrm{e}^{-t/\tau_\sigma}$ in:

$$G(t) = E_e + E_e \left(\frac{\tau_\varepsilon}{\tau_\sigma} - 1 \right) \mathrm{e}^{-t/\tau_\sigma}, \tag{1.14}$$

3 Models of linear viscoelasticity

- Page 79, Fig 3.7: τ_σ in formula in upper figure should be τ

4 Absorption Mechanisms and Physical Constraints

- Page 113, Sect. 4.4 and the discussion of Buckingham 2005, 2008. As pointed out in (Buckingham 2020), these papers show that the viscous model is causal in the strong sense, not only in the weak sense.

5 Power-law wave equations from constitutive equations

- Page 126, Fig 5.4: New figure and additions to caption (no effect on main text)
- Page 133, Sect. 5.3.1: Missing minus after last equal sign:

$$\Delta c_{ph} \approx \frac{c_0}{2} \tau^{y-1} \sin \frac{\pi y}{2} \omega^{y-1} = -c_0^2 \alpha_0 \tan \frac{\pi y}{2} \omega^{y-1} \tag{5.35}$$

6 Phenomenological power-law wave equations

- Of the four references to Zhao & McGough (2016*a*), three of them should be to Zhao & McGough (2016*b*) instead (pages 163, 168, and 171)

- Page 166, line 1, Sect. 6.1.2.1:
  "... the phase velocity increases as a function of frequency, but then may start falling and eventually become ~~negative~~ **zero**."

- Page 168, Final equation on page should have ω^{y-1} not ω^y:

  $$c_{ph} = \frac{1}{\frac{1}{c_0} + \alpha_0 \tan{(\pi y/2)} \omega^{y-1}}. \tag{6.20}$$

  Thus c_{ph} will increase monotonically (except for $y = 1$) and this is consistent with the condition of (4.36). The lack of agreement with (4.35) for the attenuation for $y > 1$ seems to be consistent with the observation in Zhao & McGough (2016*b*) that this wave equation gives a non-causal, i.e. a non-passive, solution for $y = 1.139$, $y = 1.5$, and $y = 2$.

  ~~Thus c_{ph} will decrease monotonically when $\tan(\pi y/2) > 0$, i.e. when $y < 1$. Since that is not consistent with the condition of (4.36), it can be concluded that this model does not satisfy the criterion for passivity of Sect. (4.3) regardless of whether $y \leq 1$ or $y > 1$. This is consistent with the observation in Zhao & McGough (2016a) that this wave equation gives a non-causal, i.e. a non-passive, solution for $y = 1.139$, $y = 1.5$, and $y = 2$.~~

7 Justification for power laws and fractional models

- Page 216, misprint in fourth line below Eq. (7.105): change (5.2.2) to (5.22): "just like the half-order fractional Newton model of **(5.22)**"

- Page 201, change text under Eq. (7.59) from ~~where the order may be in the range $0 \leq \alpha \leq 1$.~~ to **resulting in $\tilde{E}(\omega) \approx E_0^{1-\alpha}(\mathrm{i}\omega\eta_0)^\alpha$ which extends (7.51) to the range $0 \leq \alpha \leq 1$.**

- Page 218, change the equation to

  $$T(y_0) = \frac{1}{\sqrt{2g}} \int_0^{y_0} \frac{1}{(y_0 - y)^{0.5}} \frac{\mathrm{d}s}{\mathrm{d}y}\, \mathrm{d}y, \tag{7.108}$$

  where **$s(y)$ gives the shape of the curve and** g is the gravity of the Earth. **It is proportional to the Caputo fractional derivative of order 0.5 of (A.44)**. Therefore this is considered to be the first physical problem that requires a fractional derivative.

8 Power laws and porous media

- Page 229, there should be $\approx$ rather than the final $=$ in Eq. (8.5):

$$G_{GS,s}(t) = \frac{\gamma_s t_s^{1-\alpha}}{\Gamma(1-\alpha)} h_s(t) \approx \frac{\gamma_s}{\Gamma(1-\alpha)} \cdot t^{-\alpha}, \quad t \geq 0. \tag{8.5}$$

- Page 243, Eqs. (8.41-8.42), k_s should be k.

9 Power laws and fractal scattering media

- Page 264, change the equation in the center of the expression to

$$\underbrace{Z = \rho' c = \sqrt{\rho' K}}_{\text{Acoustic impedance}} \tag{9.17}$$

Addendum

- Page 267, heading of ex. A.1: “Fractional derivative of order **0...1**”, rather than “0.1”.
- Eq. (B.44): Change $\mathbf{u}_p$ to $\mathbf{u}_c$

Acknowledgement

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References

Buckingham, M. J. (2020), ‘The causal properties of the compressional wave in an unconsolidated marine sediment’, *J. Theor. Comput. Acoust.* **28**(01), 2050003.

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Zhao, X. & McGough, R. J. (2016*a*), ‘Numerical evaluation of time-domain Green’s functions for space-fractional wave equations’, *J. Acoust. Soc. Am.* **140**(4), 3187–3187.

Zhao, X. & McGough, R. J. (2016*b*), ‘Time-domain comparisons of power law attenuation in causal and noncausal time-fractional wave equations’, *J. Acoust. Soc. Am.* **139**(5), 3021–3031.

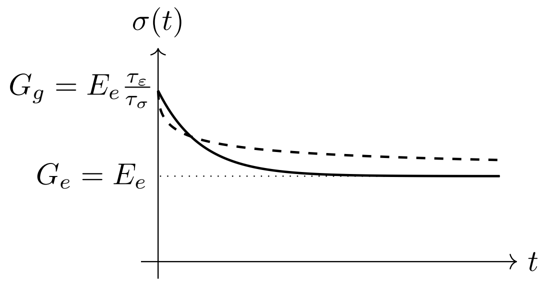

Fig. 1.4 Relaxation moduli of Zener model with an exponential time response, (1.13) (solid line) **with $\boldsymbol{E = 1}$, $\boldsymbol{\eta = 0.5}$, and $\boldsymbol{\tau_\varepsilon = 1}$**, and for the fractional Zener model (dashed line) for $\alpha = 0.5$, $\boldsymbol{\tau_\varepsilon = 2}$, which asymptotically approaches a power law function, (1.30). The asymptotic values are the glass modulus, $G_g = G(0^+)$ and the equilibrium modulus, G_e for infinite time

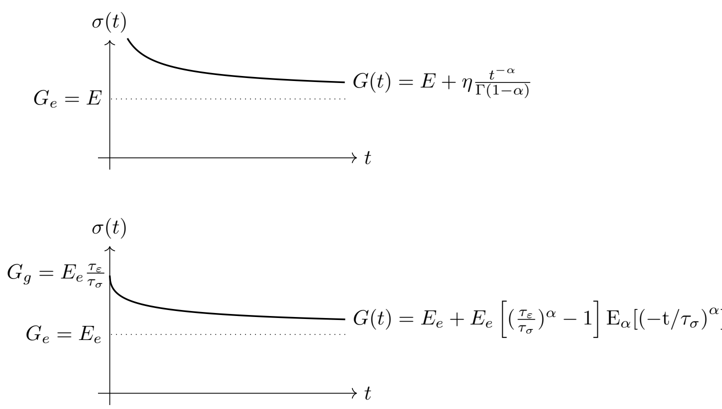

Fig. 5.4 Relaxation moduli of fractional Kelvin-Voigt model (upper) **with $\boldsymbol{E = 1}$, $\boldsymbol{\eta = 1}$** and fractional Zener model (lower) with $\alpha = 0.5$, $\boldsymbol{\tau_\varepsilon = 4}$