Waves with Power-Law Attenuation: Corrections
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These are corrections and additions for [Holm (2019)]. Bold text needs to be added and replace stricken out text.

In a book with 731 equations and 117 figures, it is unfortunately almost inevitable that there will be some errors. If you find some, please send suggestions to me at this email address: sverre.holm (a) fys.uio.no.

1 Introduction

- Page 12, Fig 1.4: New figure and additions to caption (no effect on main text)
- Page 12, misprint: replace $d^{-t/\tau_\sigma}$ by $e^{-t/\tau_\sigma}$ in:

$$G(t) = E_e + E_e \left( \frac{\tau_e}{\tau_\sigma} - 1 \right) e^{-t/\tau_\sigma}, \quad (1.14)$$

3 Models of linear viscoelasticity

- Page 79, Fig 3.7: $\tau_\sigma$ in formula in upper figure should be $\tau$

5 Power-law wave equations from constitutive equations

- Page 126, Fig 5.4: New figure and additions to caption (no effect on main text)
- Page 133, Sect. 5.3.1: Missing minus after last equal sign:

$$\Delta c_{ph} \approx C_0 \gamma^{-1} \sin \frac{\pi y}{2} \omega^{y-1} = -C_0^2 \alpha_0 \tan \frac{\pi y}{2} \omega^{y-1} \quad (5.35)$$

6 Phenomenological power-law wave equations

- Of the four references to Zhao & McGough (2016a), three of them should be to Zhao & McGough (2016b) instead (pages 163, 168, and 171)
Page 166, line 1, Sect. 6.1.2.1:
“... the phase velocity increases as a function of frequency, but then may start falling and eventually become negative zero.”

Page 168, Final equation on page should have $\omega y^{-1}$ not $\omega y$:
\[ c_{ph} = \frac{1}{\sigma_0 + \alpha_0 \tan (\pi y/2)} \omega y^{-1}. \] (6.20)

Thus $c_{ph}$ will increase monotonically (except for $y = 1$) and this is consistent with the condition of (4.36). The lack of agreement with (4.35) for the attenuation for $y > 1$ seems to be consistent with the observation in Zhao & McGough (2016b) that this wave equation gives a non-causal, i.e. a non-passive, solution for $y = 1.139, y = 1.5,$ and $y = 2$.

Since that is not consistent with the condition of (1.30), it can be concluded that this model does not satisfy the criterion for passivity of Sect. (4.3) regardless of whether $y \leq 1$ or $y > 1$. This is consistent with the observation in Zhao & McGough (2016a) that this wave equation gives a non-causal, i.e. a non-passive, solution for $y = 1.139, y = 1.5,$ and $y = 2$.

7 Justification for power laws and fractional models

Page 216, misprint in fourth line below Eq. (7.105): change (5.2.2) to (5.22): “just like the half-order fractional Newton model of (5.22)”

Page 201, change text under Eq. (7.59) from where the order may be in the range $0 \leq \alpha \leq 1$ to resulting in $\tilde{E}(\omega) \approx E_0^{1-\alpha} (i\omega \eta_0)^\alpha$ which extends (7.51) to the range $0 \leq \alpha \leq 1$.

Page 218, change the equation to
\[ T(y_0) = \frac{1}{\sqrt{2g}} \int_{0}^{y_0} \frac{1}{(y_0 - y)^{0.5}} ds \ dy, \] (7.108)
where $s(y)$ gives the shape of the curve and $g$ is the gravity of the Earth. It is proportional to the Caputo fractional derivative of order 0.5 of (A.44). Therefore this is considered to be the first physical problem that requires a fractional derivative.

8 Power laws and porous media

Page 229, there should be $\approx$ rather than the final $=$ in Eq. (8.5):
\[ G_{GS,s}(t) = \frac{\gamma_s^{1-\alpha}}{\Gamma(1-\alpha)} h_s(t) \approx \frac{\gamma_s}{\Gamma(1-\alpha)} \cdot t^{-\alpha}, \quad t \geq 0. \] (8.5)

Page 243, Eqs. (8.41-8.42), $k_s$ should be $k$. 

2
9 Power laws and fractal scattering media

- Page 264, change the equation in the center of the expression to

\[ Z = \frac{\gamma c}{\sqrt{\rho K}} \]  

(9.17)

Addendum

- Page 267, heading of ex. A.1: “Fractional derivative of order 0...1”, rather than “0.1”.
- Eq. (B.44): Change \( u_p \) to \( u_c \)

Acknowledgement

Thanks to professor James B. Spicer for valuable input.

References


Fig. 1.4 Relaxation moduli of Zener model with an exponential time response, (1.13) (solid line) with $E = 1$, $\eta = 0.5$, and $\tau_\varepsilon = 1$, and for the fractional Zener model (dashed line) for $\alpha = 0.5$, $\tau_\varepsilon = 2$, which asymptotically approaches a power law function, (1.30). The asymptotic values are the glass modulus, $G_g = G(0^+)$ and the equilibrium modulus, $G_e$ for infinite time.

Fig. 5.4 Relaxation moduli of fractional Kelvin-Voigt model (upper) with $E = 1$, $\eta = 1$ and fractional Zener model (lower) with $\alpha = 0.5$, $\tau_\varepsilon = 4$. 

\[ G_g = E_e \frac{\tau_\varepsilon}{\tau_\sigma} \]
\[ G_e = E_e \]
\[ G(t) = E + \eta \frac{t^{-\alpha}}{\Gamma(1-\alpha)} \]
\[ G(t) = E_e + E_e \left[ \left( \frac{t}{\tau_\sigma} \right)^\alpha - 1 \right] E_\alpha \left[ -t/\tau_\sigma \right]^\alpha \]