

Errata:
S. Holm, Waves with Power-Law Attenuation,
Springer, 2019

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12th September, 2023

In a book with 731 equations and 117 figures, it is unfortunately inevitable that there will be some errors.

A corrected version of the book¹ was published online early 2023. Unfortunately that introduced a new misprint which is in Part I of this document. Minor, mostly cosmetic corrections applicable to both the original and the corrected version are in Part II. The items which have been corrected in the 2023 version are in part III of this document and therefore only apply to the 2019 version.

The latest version of this document can be found at <http://folk.uio.no/sverre>. If you find more errors, please send them to me at 'sverre.holm (a) fys.uio.no'.

Bold text should be added and replace ~~stricken out~~ text.

Part I

Corrections to 2023 version

- Page 283, Fig. A.1, caption: "... increments of 0.9" is a misprint which should be "... increments of **0.1**" (4.3.2023). See also Fig. A.1 on page 5 of this document.

¹Holm, S. (2019) *Waves with Power-Law Attenuation*, Springer and ASA Press., Switzerland, <https://link.springer.com/book/10.1007/978-3-030-14927-7>.

Part II

Minor corrections

1 Introduction

- Page 19: It is the hope that this chapter will inspire ... (6.12.2022).

2 Classical wave equations

- Page 34, Eq. (2.23): Remove the term $\Re(\sqrt{\frac{\rho_0}{E}})$ from the equation (8.12.2022).
- Page 71: Biography of Duhem, line 5, misprint: Change 'on' to 'in': "the footnote **in** Sect. 1.2" (6.12.2022).
- Page 56, above Eq. (2.92), misprint: "...is given in Bass et al. (1995, 1995)" should be "...Bass et al. (1995, **1996**)" (22.04.2023).

3 Models of linear viscoelasticity

- Pages 75-78: The terminology of Sect. 3.2.2 "The Kelvin-Voigt model" is consistent with that of Sect. 2.4. However, changing E to E_e — the equilibrium value — would make it consistent with the other models and in particular Sect. 5.2.1. The change would apply to Figs. 3.4 and 3.5, as well as all equations with E in Sect. 3.2.2 (14.8.2023).
- Page 79, Fig. 3.7: τ_σ in formula in upper figure should be τ (6.8.2019).

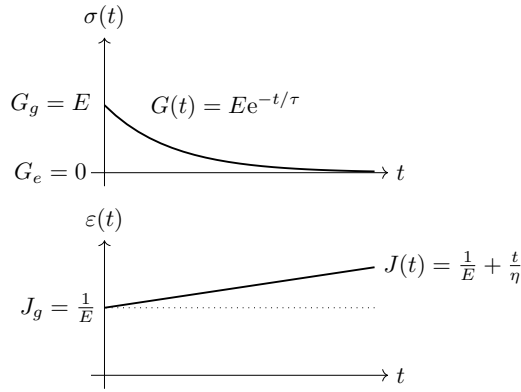


Fig. 3.7

9 Power laws and fractal scattering media

- Page 266, Eq. (9.32) should be (5.5.2023):

$$\ln Z = \ln(K\rho')^{1/2} = \frac{1}{2} [\ln \rho' - \ln(1/K)]. \quad (9.32)$$

- Page 267, Eq. (9.35), z is missing in two places (12.09.2023):

$$H(\omega, z) = \exp \left(-\frac{\gamma^{(e)}(\omega)\omega^2}{8c_0^2} z - i \frac{\gamma^{(s)}(\omega)\omega^2}{8c_0^2} z \right). \quad (9.35)$$

5 Power-Law Wave Equations Appendix B

- Page 124, Eq. (5.10): Parameter range should be $0 < \alpha, \beta \leq 1$ (19.06.2023).
- Page 153, just after Eq. (5.96): This model is similar to the **complex modulus** constitutive law of ... (6.12.2022).
- Page 153, two lines further down: also an approximation to the **elasticity** response of a tube ... (6.12.2022).

- Page 298, Eq. (B.44): Change \mathbf{u}_p to \mathbf{u}_c (5.12.2020).

7 Justification for power laws and fractional models

- Page 209: All instances of μ in in Eqs. (7.75)-(7.77) should be replaced by viscosity η (6.12.2022).
- Page 212, Eq. (7.86): Remove ' $= kR$ ' and change $\omega\tau_r$ below the equation to $\omega\tau$ (10.9.2021).

8 Power laws and porous media

- Page 231: Replace $t_p^{-\alpha_p}$ with $t^{-\alpha_p}$ in Eq. (8.11) (8.2.2023).
- Page 243, Eqs. (8.41-8.42): k_s should be k (5.12.2020).

Part III

Corrections to 2019 version

These are corrections which have all been fixed in the 2023 version of the book.

1 Introduction

- Page 12, Fig 1.4: New figure and additions to caption, with no effect on main text (6.8.2019).

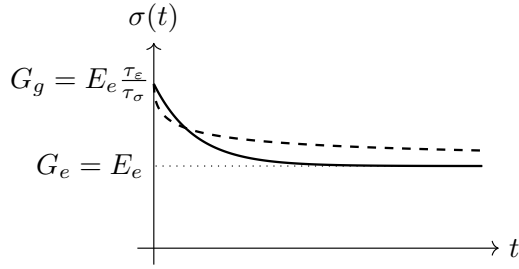


Fig. 1.4 Relaxation moduli of Zener model with an exponential time response, (1.13) (solid line) **with $E = 1$, $\eta = 0.5$, and $\tau_\epsilon/\tau_\sigma = 1$** , and for the fractional Zener model (dashed line) for $\alpha = 0.5$, $\tau_\epsilon/\tau_\sigma = 2$, which asymptotically approaches a power law function, (1.30). The asymptotic values are the glass modulus, $G_g = G(0^+)$ and the equilibrium modulus, G_e for infinite time

- Page 12: Misprint, replace d^{-t/τ_σ} by e^{-t/τ_σ} in (6.8.2019):

$$G(t) = E_e + E_e \left(\frac{\tau_\epsilon}{\tau_\sigma} - 1 \right) e^{-t/\tau_\sigma}, \quad (1.14)$$

2 Classical wave equations

- Page 48, Eq. 2.69: Move $2^{1/2}$ from numerator to denominator (27.7.2022):

$$\alpha_k \approx \frac{1}{c_0(2\tau_\epsilon)^{1/2}} \left(1 - \frac{\tau_\sigma}{4\tau_\epsilon} \right) \omega^{1/2}. \quad (2.69)$$

5 Power-law wave equations from constitutive equations

- Page 120, above Eq. (5.1): “...and $\alpha_0 = 0.5/(2\pi \cdot 8.686)$ Np/radian frequency-cm” should be changed to: “...and $\alpha_0 = 0.5/[(2\pi)^y \cdot 8.686]$ Np/radian frequency [MHz]^y/cm” (17.5.2022)
- Page 120, below Eq. (5.1): “...with 0.14 dB/cm/MHz and $y = 1.21$ to breast with 0.75 dB/cm/MHz and $y = 1.5$ ” should be changed to “...with 0.14 dB/cm/MHz^{1.21} and $y = 1.21$ to breast with 0.75 dB/cm/MHz^{1.5} and $y = 1.5$ ” (17.5.2022)
- Page 126:

- Eq. (5.16): The limits should be $0 < \alpha \leq 1$. (22.9.2021)
- Eq. (5.17): The relaxation modulus of the Fractional Maxwell model has incorrect scaling factor and should be (19.3.2022):

$$G(t) = \frac{\eta}{\tau^\alpha} E_\alpha [-(t/\tau_\sigma)^\alpha], \quad (5.17)$$

- Eq. (5.19): The creep compliance has incorrect exponent and scaling factor (19.3.2022):

$$J(t) = \frac{\tau^\alpha}{\eta} + \frac{1}{\eta} \frac{t^\alpha}{\Gamma(1+\alpha)}. \quad (5.19)$$

- Fig 5.4: New figure and additions to caption with no effect on main text (6.8.2019).

- Page 128, Eq. (5.23): Change to positive sign in the exponent for the creep compliance of the Fractional Newton (spring-pot) model (19.3.2022):

$$J(t) = \frac{1}{\eta} \frac{t^\alpha}{\Gamma(1+\alpha)}. \quad (5.23)$$

- Page 133, Sect. 5.3.1: Missing minus after last equal sign (6.8.2019):

$$\begin{aligned} \Delta c_{ph} &\approx \frac{c_0}{2} \tau^{y-1} \sin \frac{\pi y}{2} \omega^{y-1} \\ &= -c_0^2 \alpha_0 \tan \frac{\pi y}{2} \omega^{y-1} \end{aligned} \quad (5.35)$$

- Page 140, Eq. (5.67b): The first τ has lost the subscript. Due to a misprint, the frequency limits

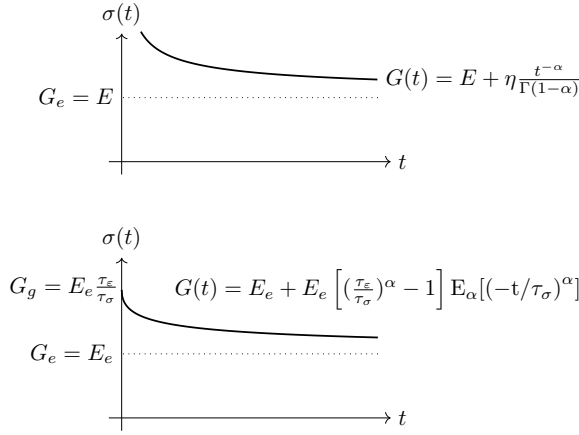


Fig. 5.4 Relaxation moduli of fractional Kelvin-Voigt model (upper) with $E = 1$, $\eta = 1$ and fractional Zener model (lower) with $\alpha = 0.5$, $\tau_\varepsilon/\tau_\sigma = 4$

have also been reversed compared to Eq. (5.66b) (20.1.2022):

$$c_{ph} = c_0 \frac{\tau_\varepsilon^{\alpha/2}}{\cos \frac{\pi\alpha}{4}} \left[1 - \frac{1}{4} \left[\frac{\tau_\sigma}{\tau_\varepsilon} \right]^\alpha \right]^{-1} \omega^{\alpha/2}, \quad (5.67)$$

$$(\omega\tau_\sigma)^\alpha \ll 1 \ll (\omega\tau_\varepsilon)^\alpha$$

6 Phenomenological power-law wave equations

- There are four references to "Zhao and McGough (2016)" in this chapter (5.12.2020):
 - Page 166: This one is correct
 - Pages 163, 168, and 171: Change them to Zhao and McGough (2016b)
- Page 166, top line (6.8.2019): "... the phase velocity increases as a function of frequency, but then may start falling and eventually become **negative zero**."
- Page 168: Final equation on page should have ω^{y-1} not ω^y (5.12.2020):

$$c_{ph} = \frac{1}{\frac{1}{c_0} + \alpha_0 \tan(\pi y/2) \omega^{y-1}}. \quad (6.20)$$

Thus c_{ph} will increase monotonically (except for $y = 1$) and this is consistent with the condition of (4.36). The lack of agreement with

(4.35) for the attenuation for $y > 1$ seems to be consistent with the observation in Zhao and McGough (2016b) that this wave equation gives a non-causal, i.e. a non-passive, solution for $y = 1.139$, $y = 1.5$, and $y = 2$.

Thus c_{ph} will decrease monotonically when $\tan(\pi y/2) > 0$, i.e. when $y < 1$. Since that is not consistent with the condition of (4.36), it can be concluded that this model does not satisfy the criterion for passivity of Sect. (4.3) regardless of whether $y \leq 1$ or $y > 1$. This is consistent with the observation in Zhao and McGough (2016) that this wave equation gives a non-causal, i.e. a non-passive, solution for $y = 1.139$, $y = 1.5$, and $y = 2$.

- Page 172: Add this new reference as "Zhao and McGough (2016b)" at the end of the list (5.12.2020):

X. Zhao, R.J. McGough, Time-domain comparisons of power law attenuation in causal and non-causal time-fractional wave equations. J. Acoust. Soc. Am. 139(5), 3021–3031 (2016b).

7 Justification for power laws and fractional models

- Page 201: Change text under Eq. (7.59) from where the order may be in the range $0 \leq \alpha \leq 1$, to **resulting in $\tilde{E}(\omega) \approx E_0^{1-\alpha} (i\omega\eta_0)^\alpha$ which extends (7.51) to the range $0 \leq \alpha \leq 1$** (6.8.2019).
- Page 212: Eq (7.90) has a minus too much. The correct expression is (16.6.2021):

$$\tilde{F}(\omega) = \frac{z^2}{8} \left[\frac{1}{1 - \frac{2J_1(iz)}{izJ_0(iz)}} - 1 \right] \quad (7.90)$$

$$= \frac{1}{4} \frac{z^2 J_1(iz)}{izJ_0(iz) - 2J_1(iz)}$$

- Page 216: Misprint in fourth line below Eq. (7.105): "just like the half-order fractional Newton model of (5.22) ~~(5.2.2)~~" (5.12.2020).
- Page 218: Change the equation to (5.12.2020):

$$T(y_0) = \frac{1}{\sqrt{2g}} \int_0^{y_0} \frac{1}{(y_0 - y)^{0.5}} dy, \quad (7.108)$$

where $s(y)$ gives the shape of the curve and g is the gravity of the Earth. It is proportional to the Caputo fractional derivative of order 0.5 of (A.44). Therefore this is considered to be the first physical problem that requires a fractional derivative.

8 Power laws and porous media

- Page 229, Eq. (8.5): Replace the final = with \approx (5.12.2020):

$$G_{GS,s}(t) = \frac{\gamma_s t_s^{1-\alpha}}{\Gamma(1-\alpha)} h_s(t) \approx \frac{\gamma_s}{\Gamma(1-\alpha)} \cdot t^{-\alpha}, \quad (8.5)$$

$$t \geq 0.$$

- Page 231: Eq. (8.13) needs k^2 in the third and fourth terms (19.3.2022):

$$k^2 - \frac{\rho_0}{K} \omega^2 + \frac{\gamma_p}{K} (i\omega)^{\alpha_p} k^2 + \frac{4}{3} \frac{\gamma_s}{K} (i\omega)^{\alpha_s} k^2 = 0. \quad (8.13)$$

- Page 245: Eq. (8.53) was missing A_2 (12.6.2021):

$$\nabla^2 u - \frac{1}{c_0^2} \frac{A_0}{A_2} \frac{\partial^2 u}{\partial t^2} - \frac{1}{c_0^2} \frac{\omega_c}{A_2} \frac{\partial u}{\partial t} = 0. \quad (8.53)$$

- Page 249: Eq. (8.58) is found by replacing ω_c by $\eta \frac{\sqrt{\tau}}{4\rho_0 B} \partial^{1/2} / \partial t^{1/2}$ in (8.53) (12.6.2021):

$$\nabla^2 u - \frac{1}{c_0^2} \frac{A_0}{A_2} \frac{\partial^2 u}{\partial t^2} - \frac{1}{c_0^2} \frac{\eta \sqrt{\tau}}{4\rho_0 B A_2} \frac{\partial^{3/2} u}{\partial t^{3/2}} = 0. \quad (8.58)$$

9 Power laws and fractal scattering media

- Page 263: Change the equation in the center of the expression to (5.12.2020):

$$\underbrace{Z = \rho' c = \sqrt{\rho' K}}_{\text{Acoustic impedance}} \quad (9.17)$$

Appendix A

- Page 281: Misprint in heading of ex. A.1: “... of order 0...1” \rightarrow 0.1 (5.12.2020).

- Page 283, Fig. A.1: The labels as well as the caption have incorrect values of α . The upper one should be “ $\alpha = 0$: Infinite memory” and the lower one should say “ $\alpha = 0.9$ ” (16.8.2022).

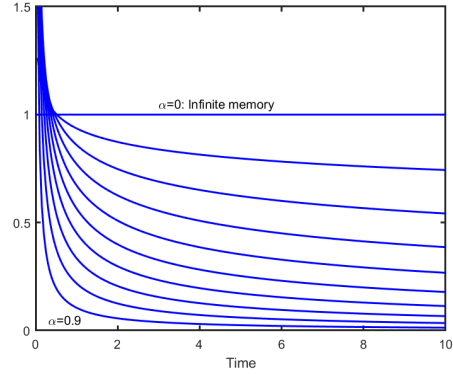


Fig. A.1 Power-law memory kernel in the convolution function of (A.48). The curves illustrate values of α from 0.1 in the upper curve to 0.9 in the lower curve in increments of 0.1 (inspired by Treeby & Cox (2010))

Appendix B

- Page 301: “ \mathbf{J}_f is the displacement current” should be “ \mathbf{J}_f is the free current” (23.3.2022).

Acknowledgement

Thanks to Sri Nivas Chandrasekaran, professor James B. Spicer, and Samuel Bryant for valuable input.