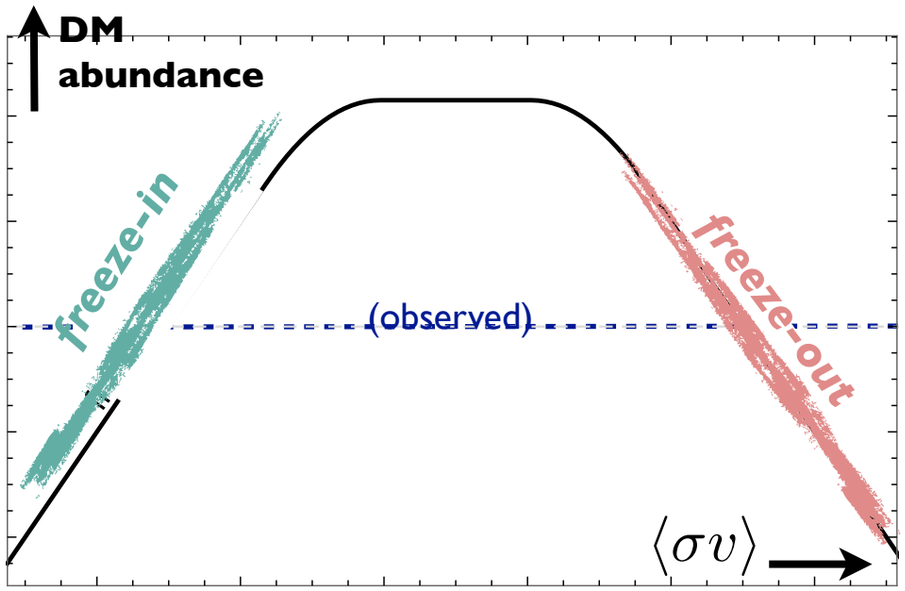
Orsay, 2 November 2022

AstroParticle Symposium
Université Paris-Saclay

Dark Matter from the Primordial Plasma

From freeze-in to pandemic production

Torsten Bringmann



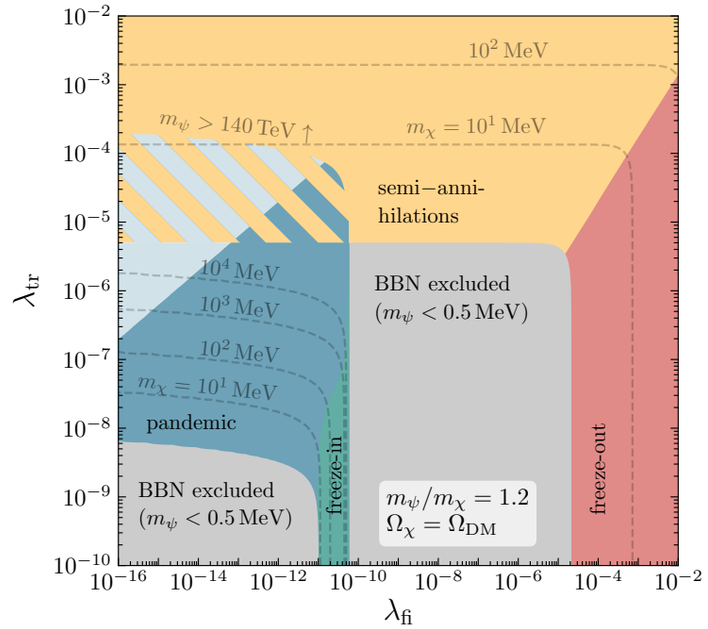
standard production mechanisms

Based on

TB, Heeba, Kahlhoefer & Vangsnes, JHEP '22

TB, Depta, Hufnagel, Ruderman & Schmidt-Hoberg, PRL '21

TB, Depta, Hufnagel, Kersten, Ruderman & Schmidt-Hoberg, arXiv:2206.10630



'pandemic' production

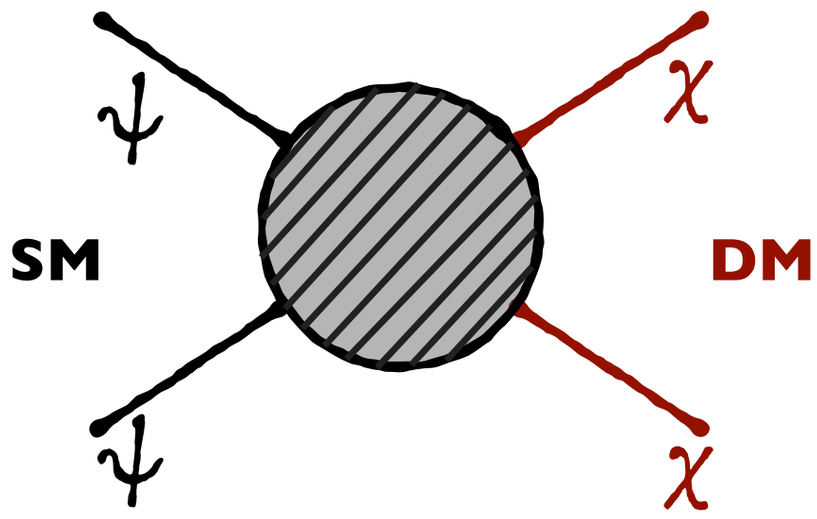
UiO : University of Oslo

The origin of dark matter

- Existence of (particle) DM = evidence for BSM physics
- Guiding principle: any convincing model for dark matter must include a production mechanism

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- Typical interaction with the primordial heat bath:



FIMPs or WIMPs ?

- Smooth transition between two regimes:

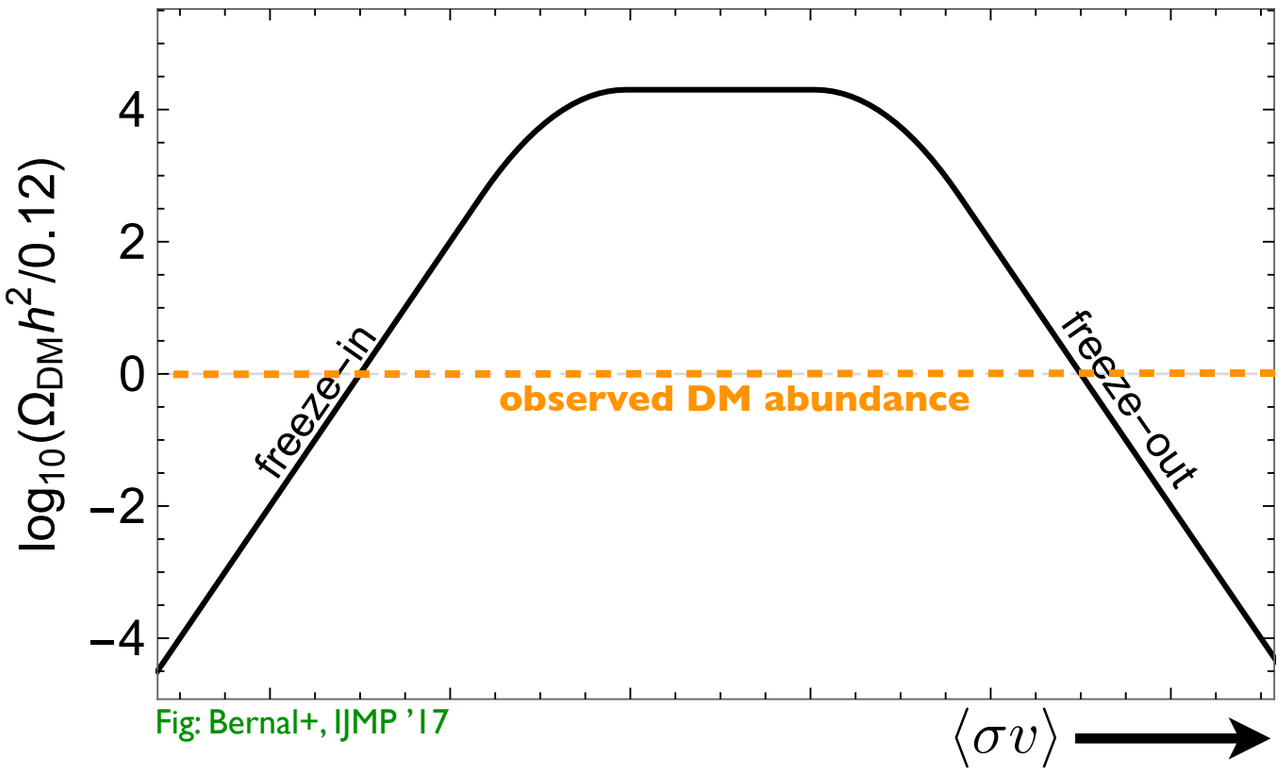


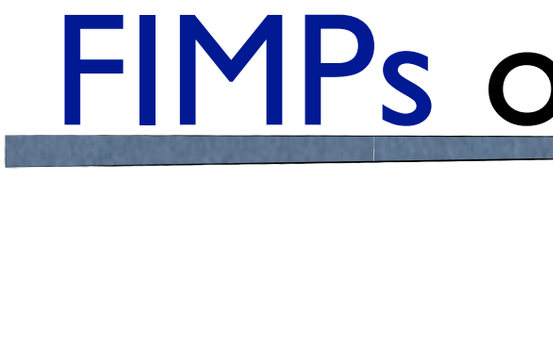
Fig: Bernal+, IJMP '17

FIMPs or WIMPs ?

- Smooth transition between two regimes:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = \langle\sigma v\rangle n^2_{\chi,\mathrm{eq}}$$

freeze-in



depends on initial conditions

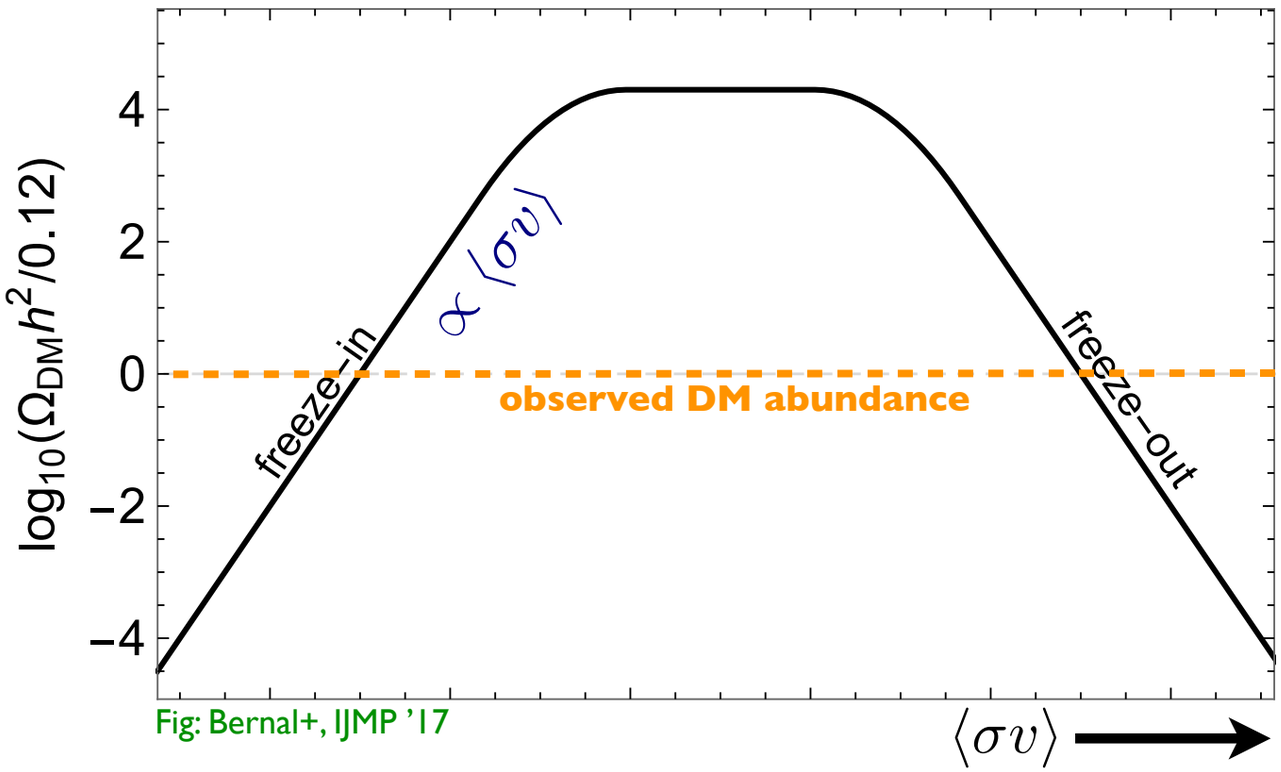


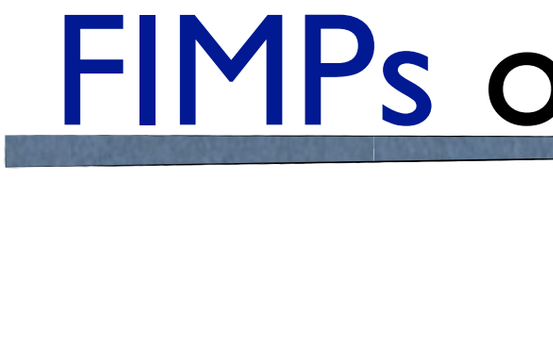
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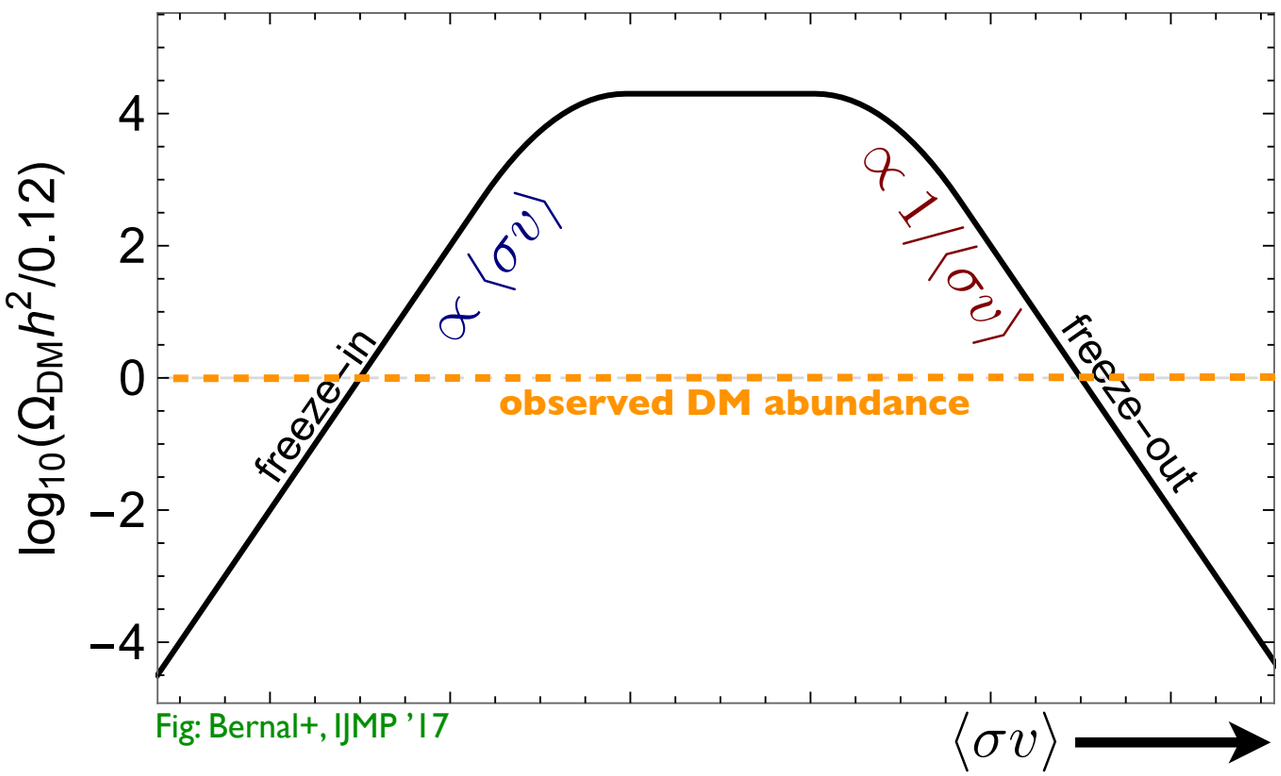
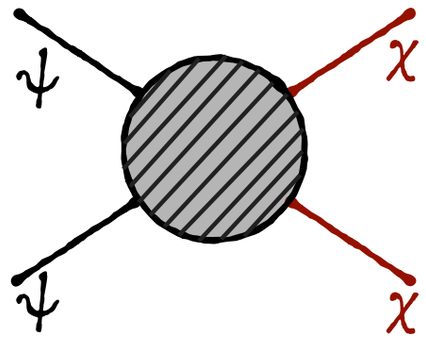


Fig: Bernal+, IJMP '17

$$\frac{dn_\chi}{dt} + 3Hn_\chi = \langle\sigma v\rangle \left(n^2_{\chi,\mathrm{eq}} - n^2_\chi\right)$$

freeze-out



insensitive to initial conditions

Freeze-in: Case study

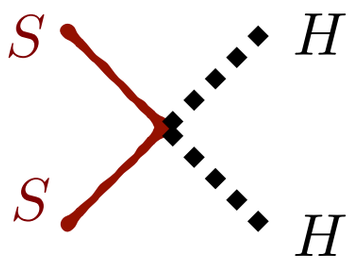
TB, Heeba, Kahlhoefer & Vangsnes, JHEP '22

- Scalar Singlet DM

$$\mathcal{L} = \frac{1}{2}\partial_\mu S \partial^\mu S + \frac{1}{2}\mu_S^2 S^2 + \frac{1}{2}\lambda_{hs} S^2 |H|^2 + \frac{1}{4}\lambda_s S^4$$

Silveira & Zee, PLB '85

  - before EWSB:



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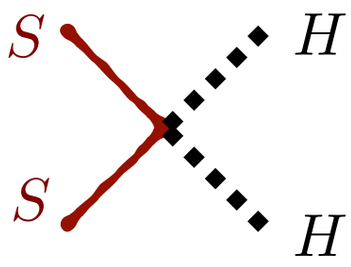
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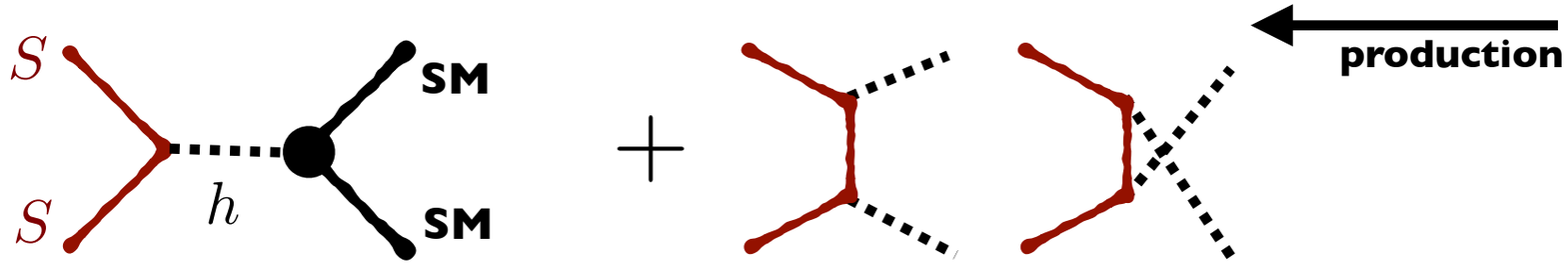
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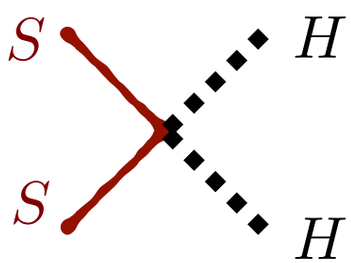
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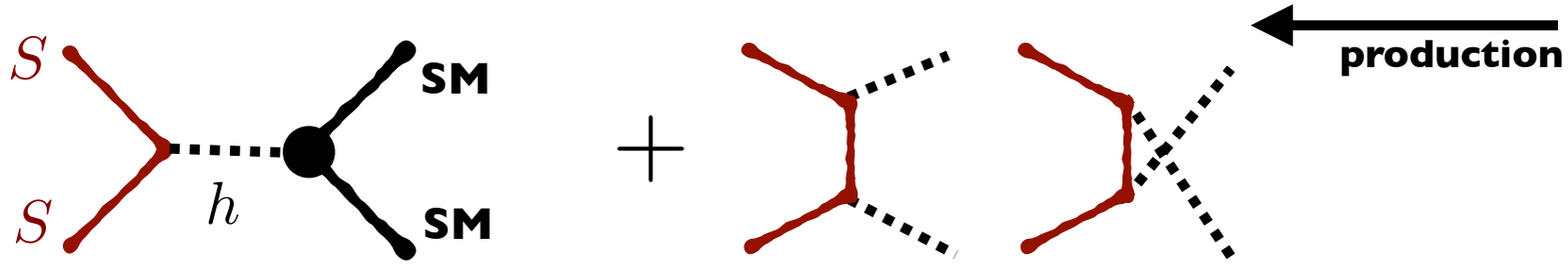
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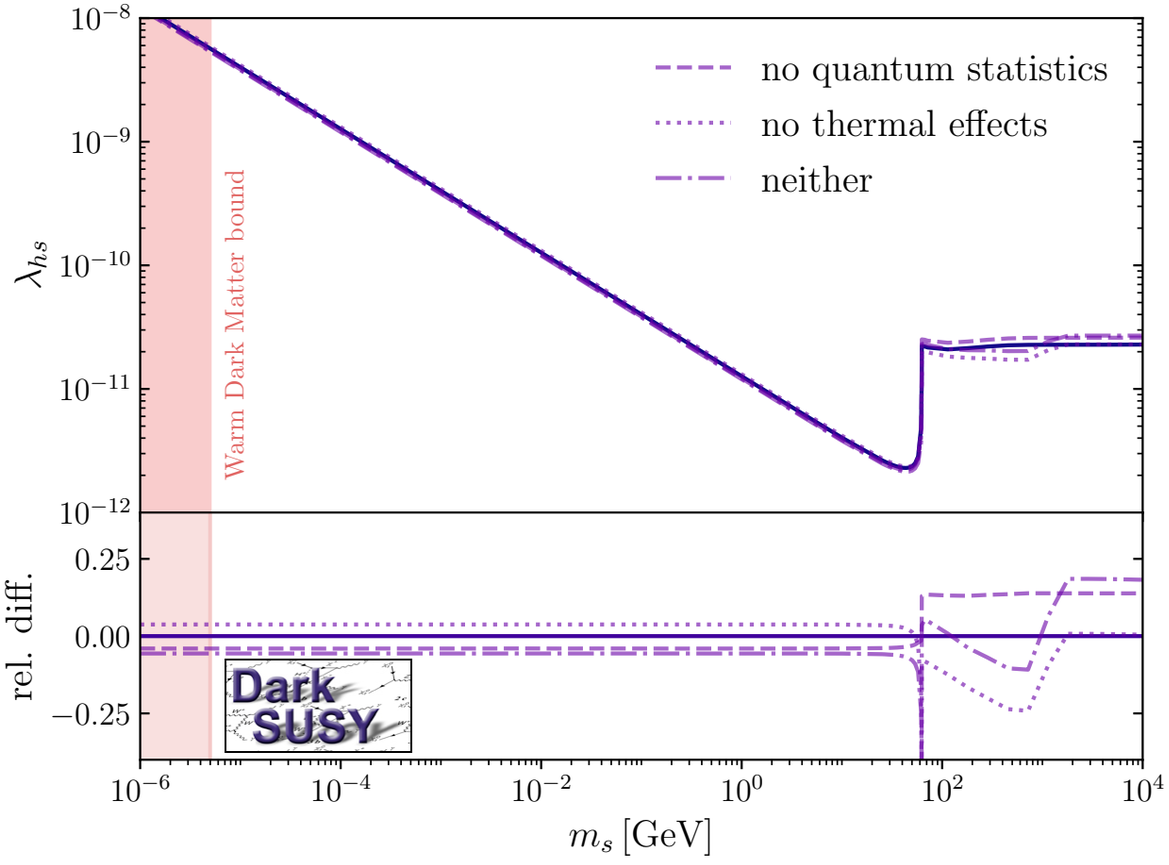
Silveira & Zee, PLB '85

  - before EWSB:



  - after EWSB:





- High reheating temperature

Freeze-in: Case study

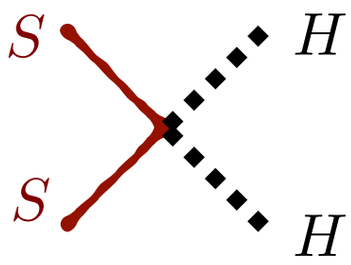
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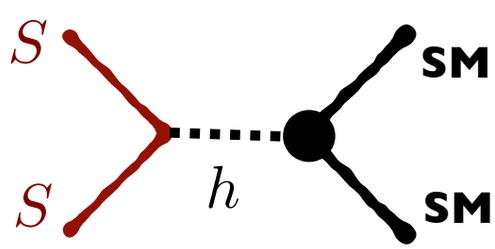
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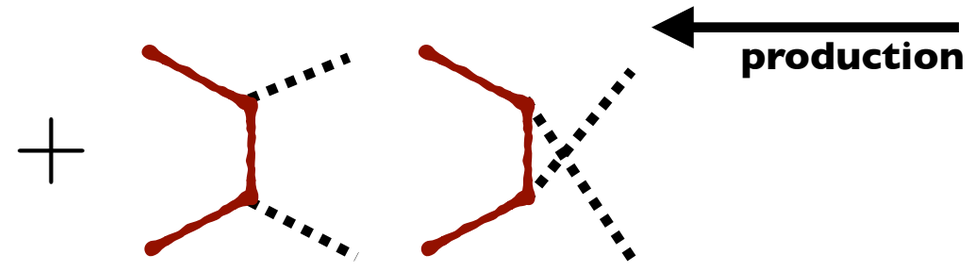
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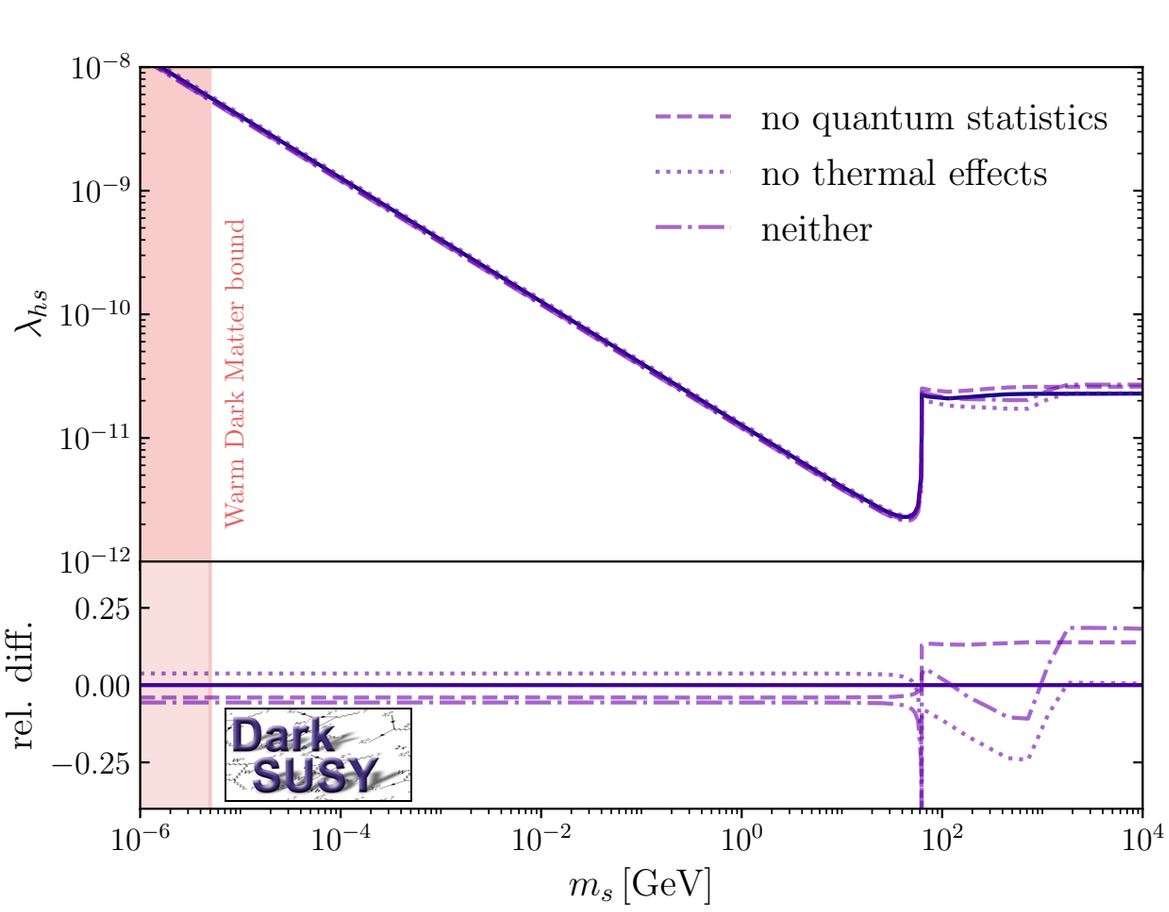
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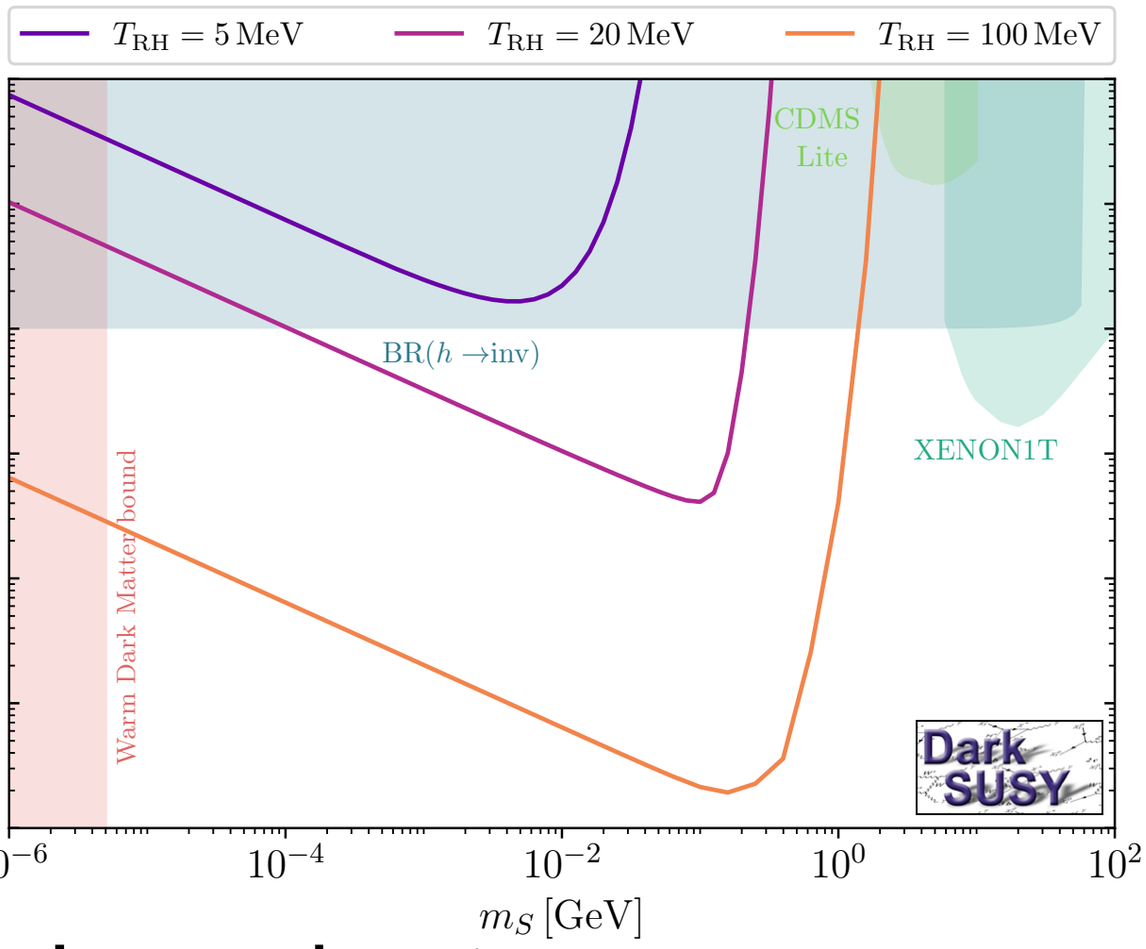
- after EWSB:







- High reheating temperature



- Low reheating temperature

A new production mechanism

- In the past, many variants of freeze-out scenario have been explored:

ELDER DARK MATTER

HIDDEN SECTOR FREEZE-OUT

KINDER DARK MATTER

CANNIBAL DARK MATTER

FORBIDDEN DARK MATTER

ZOMBIE DARK MATTER

SIMP DARK MATTER

...

- Can we get a qualitatively different freeze-in scenario?



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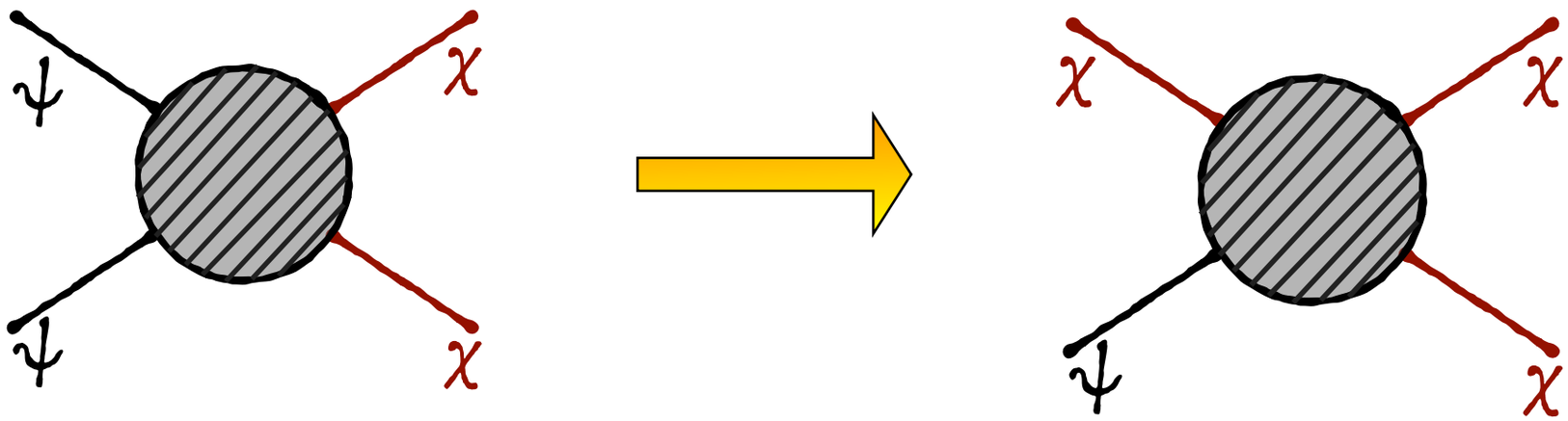
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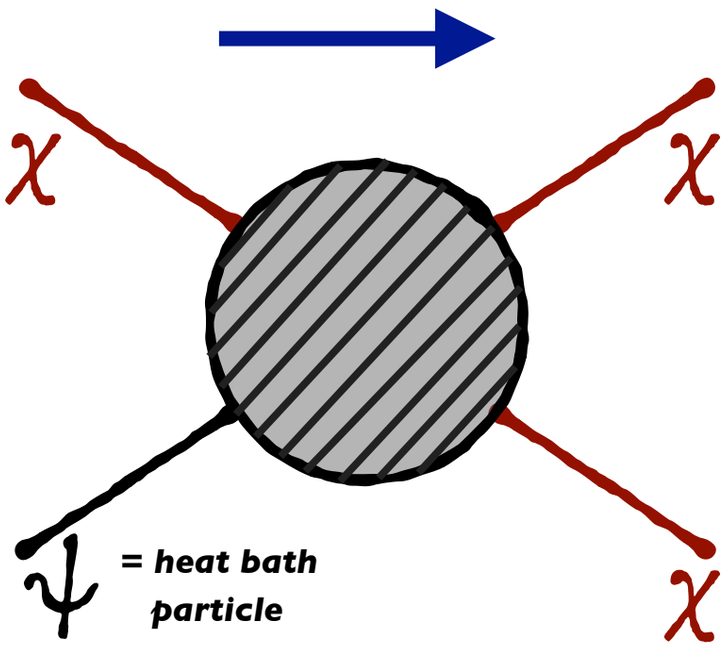
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A new production mechanism



TB, Depta, Hufnagel, Rudermann & Schmidt-Hoberg, 2103.16572

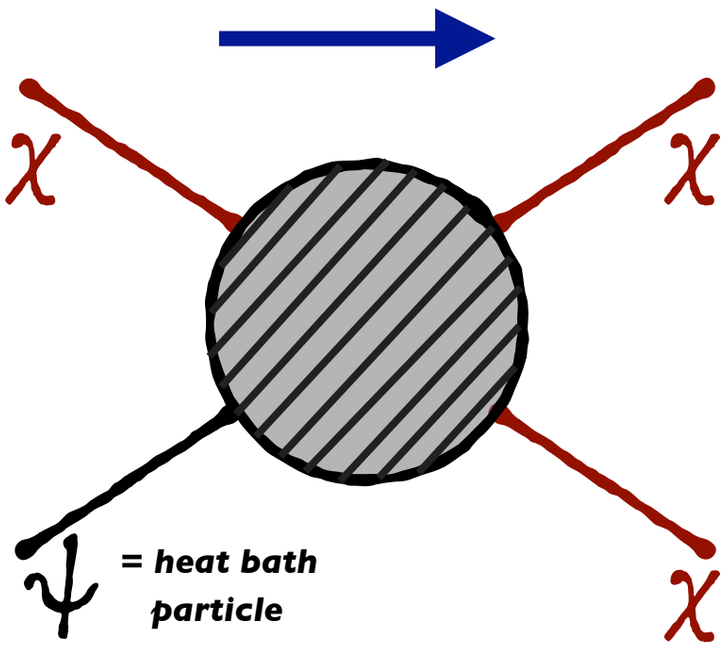
Hryczuk & Laletin, 2104.05684

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[for $n_\chi \ll n_\psi^{\rm eq}$]

A new production mechanism

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A Contribution to the Mathematical Theory of Epidemics.

By W. O. Kermack and A. G. McKendrick.

(Communicated by Sir Gilbert Walker, F.R.S.—Received May 13, 1927.)

S ***# susceptible individuals***

I ***# infected individuals***

recovered $(R = \mathrm{tot} - S - I)$

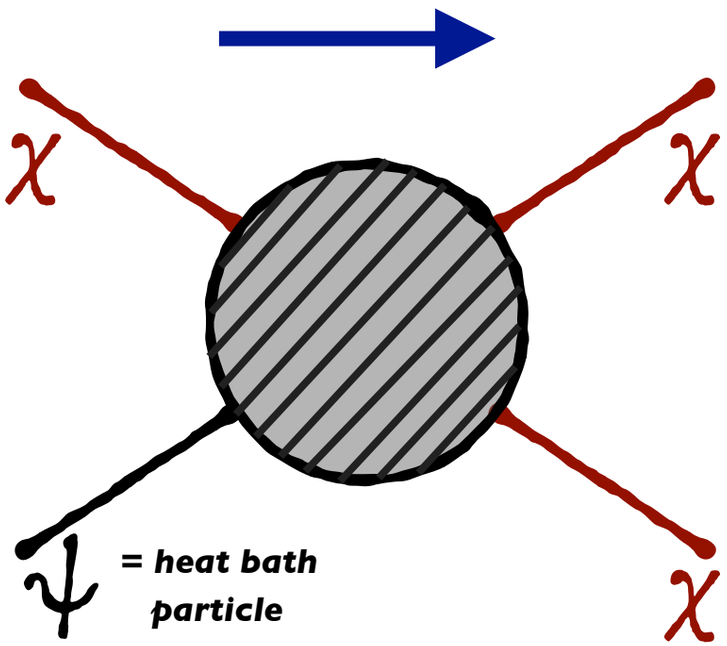
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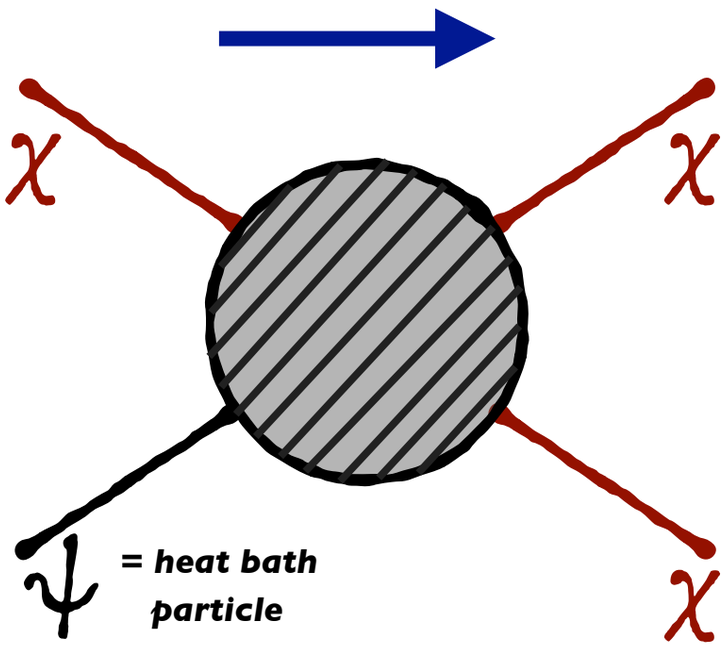
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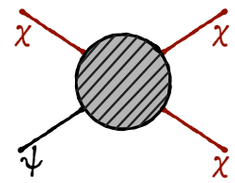
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➡ *reproduction number, or 'R-value':* $R \equiv \frac{\beta S}{\gamma} = \frac{n_\psi^{\rm eq}\langle\sigma v\rangle}{3H}$

Exponential DM production

$\mathcal{L} \supset (\lambda_{\rm tr}/3!)\psi\chi^3$

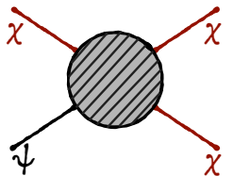


$= -i\,\lambda_{\rm tr}$

toy model

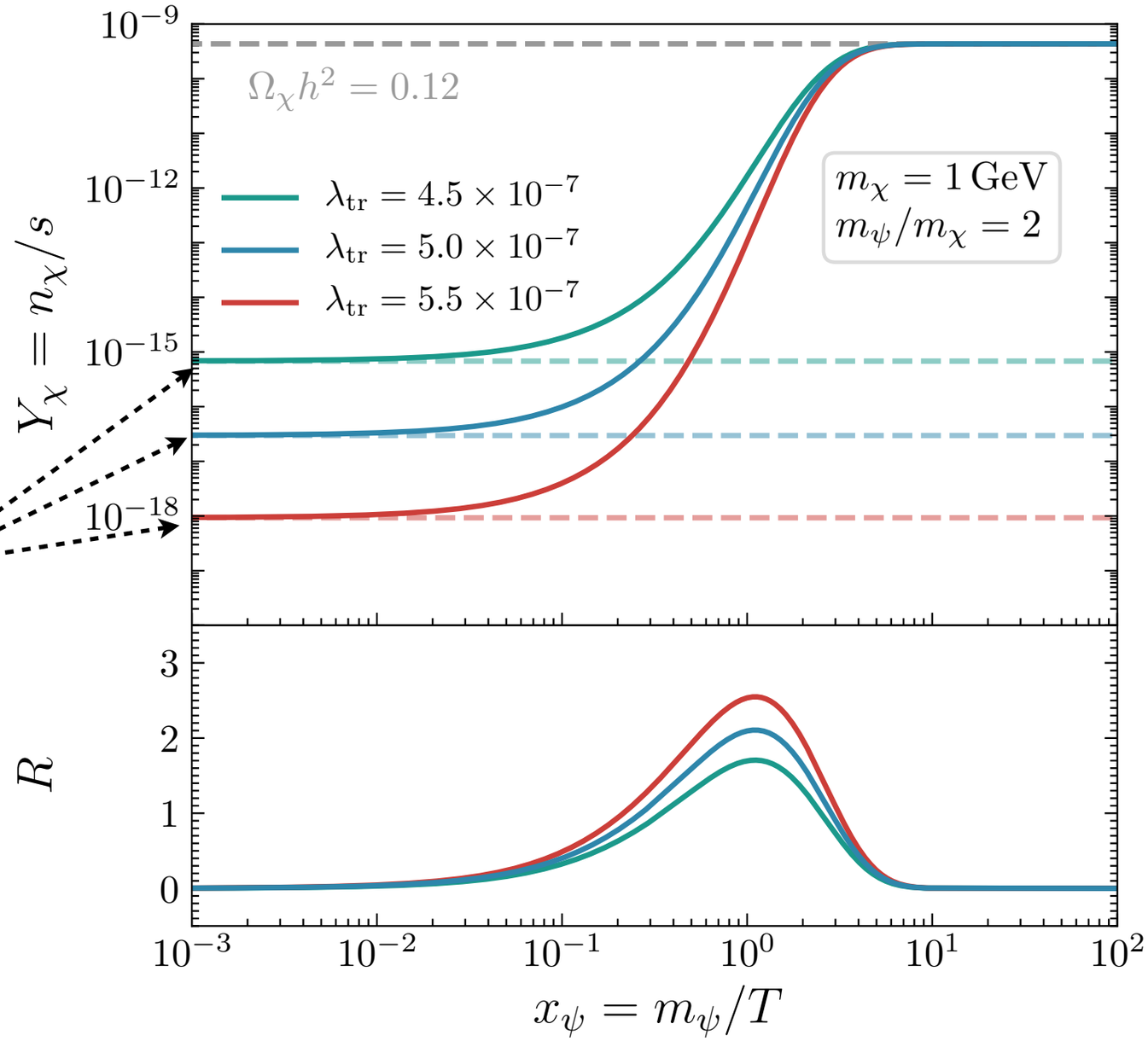
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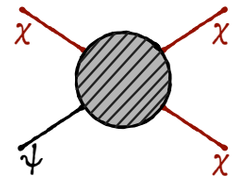


Tiny initial DM abundance

$$R \equiv \frac{\beta S}{\gamma} = \frac{n_\psi^{\rm eq}\langle\sigma v\rangle}{3H}$$

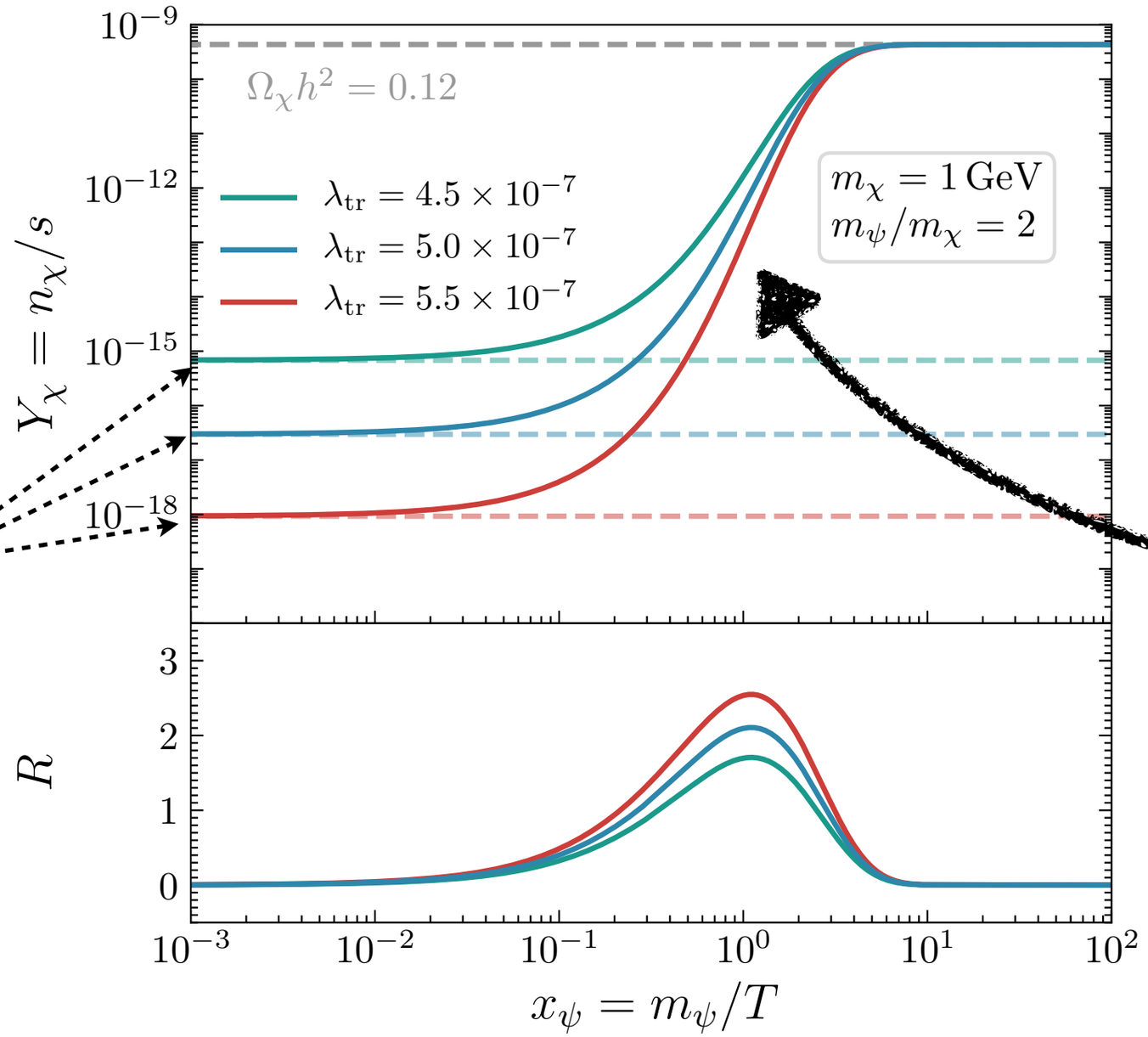
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toy model



Tiny initial DM abundance



exponential growth $R \gtrsim 1$

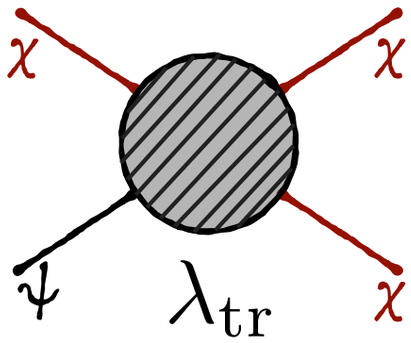
large H

small $n_\psi^{\rm eq}$

$$R \equiv \frac{\beta S}{\gamma} = \frac{n_\psi^{\rm eq}\langle\sigma v\rangle}{3H}$$

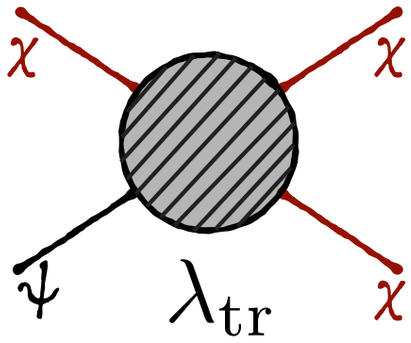
Adding freeze-in production

‘transmission’



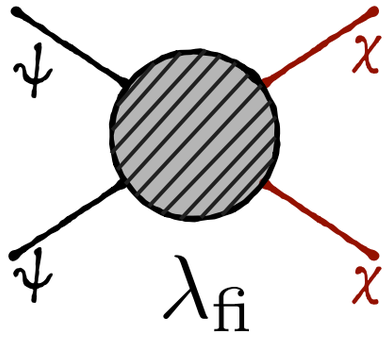
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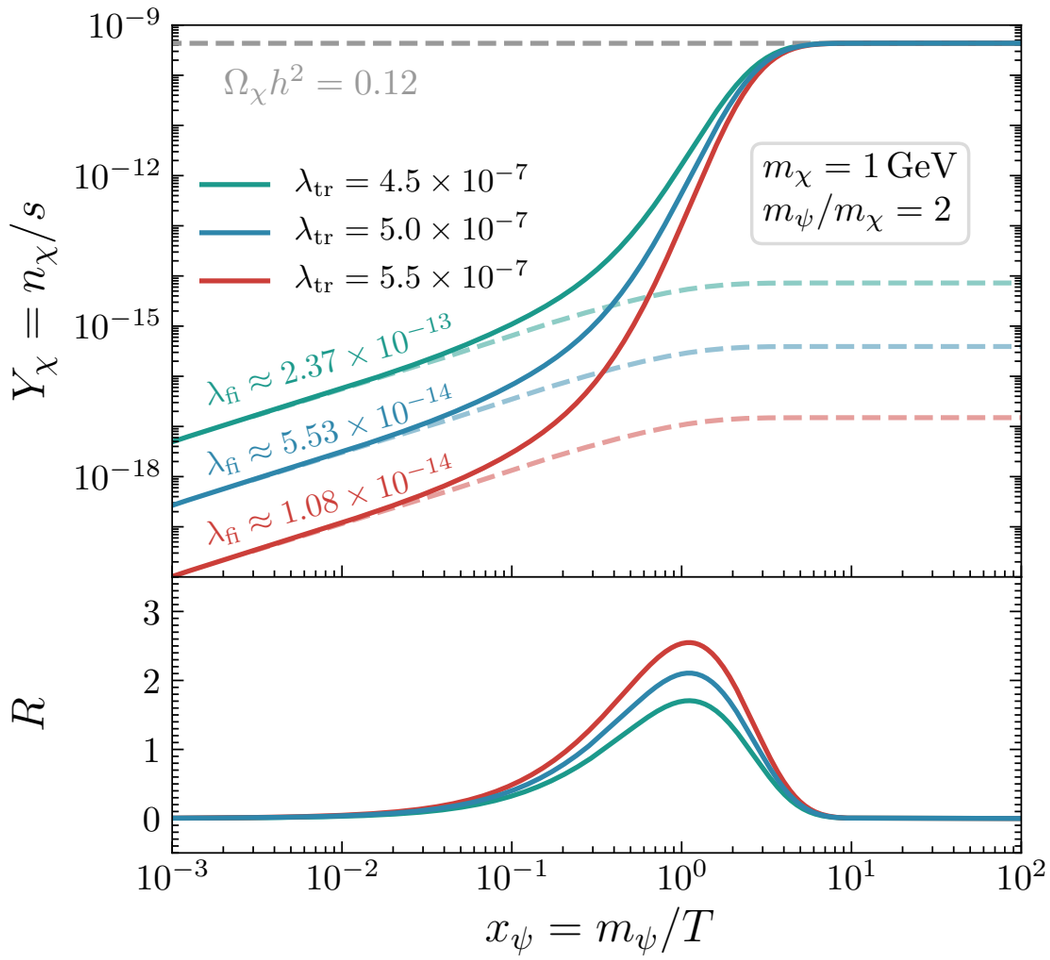
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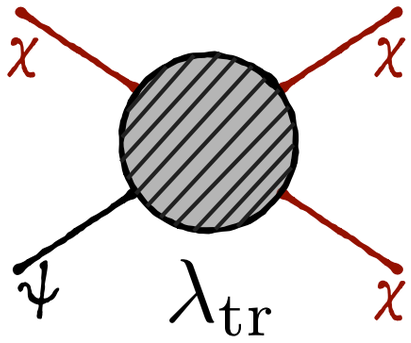
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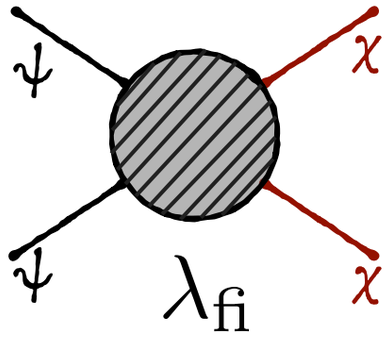
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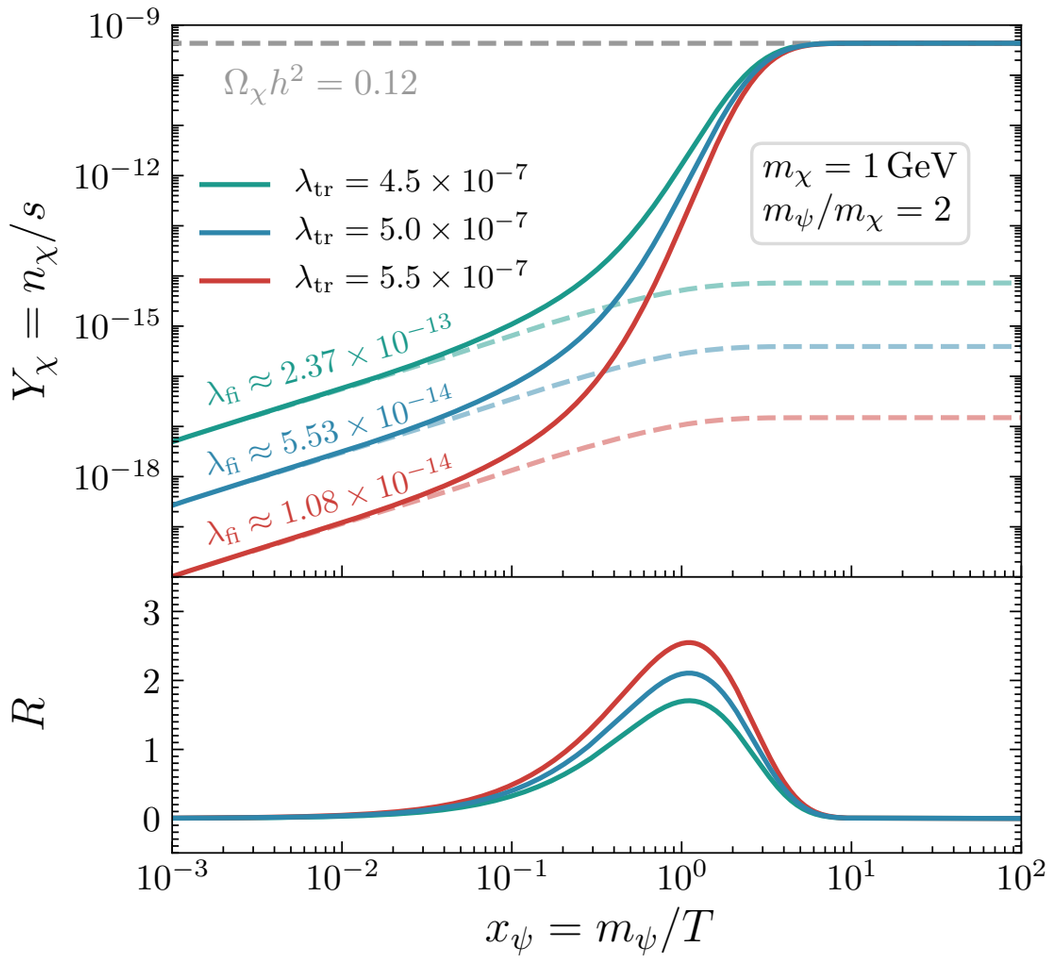
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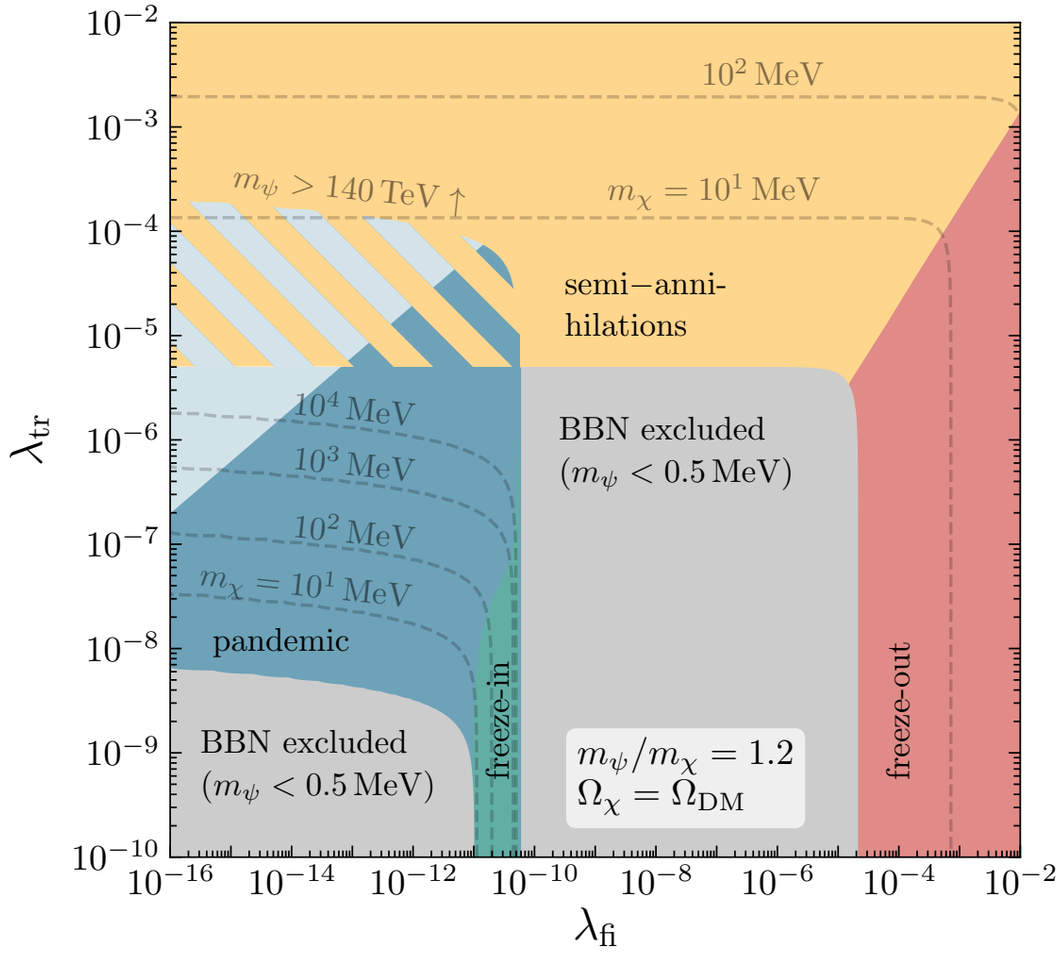


+

‘freeze-in’

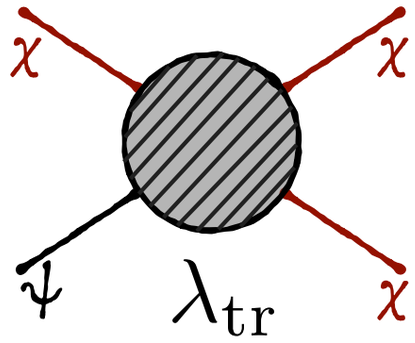






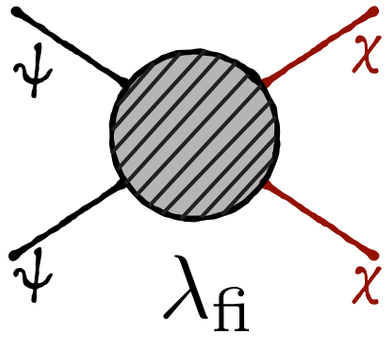
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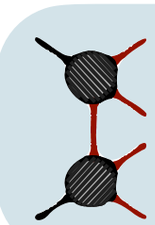
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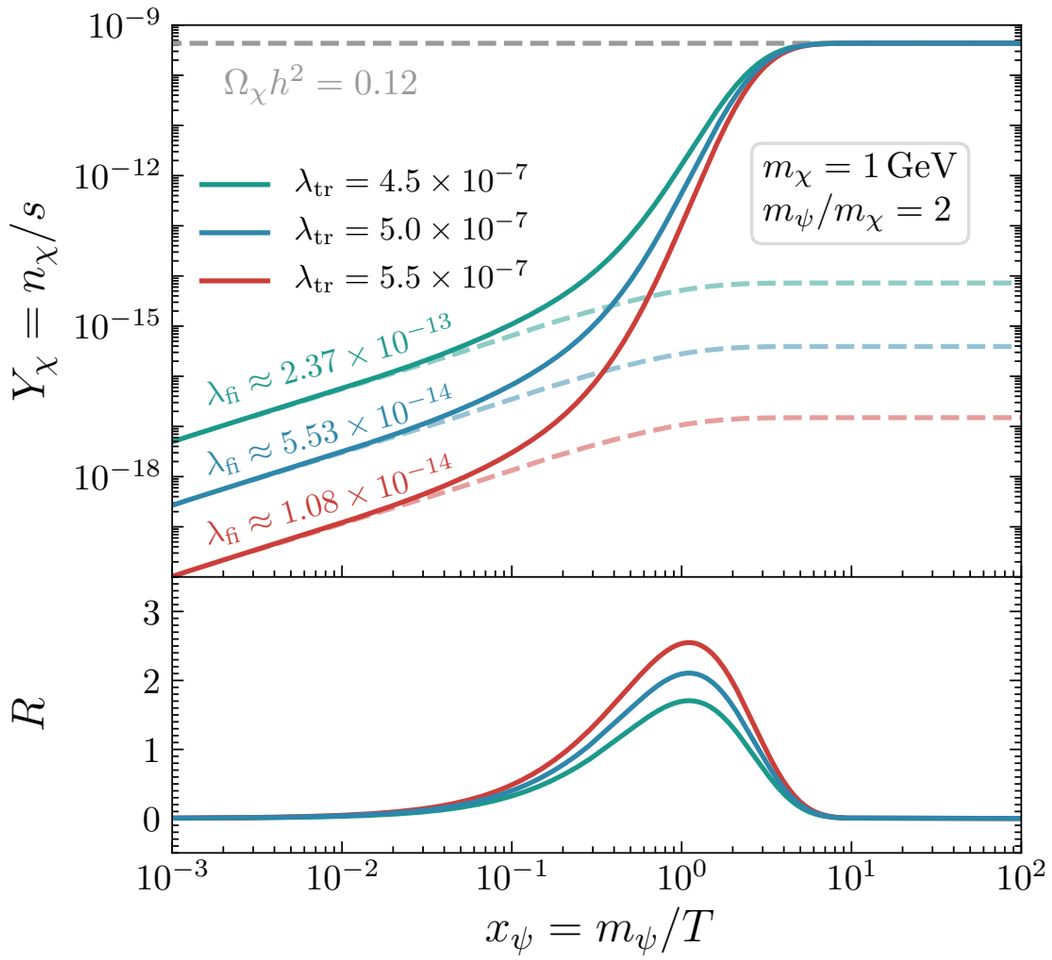
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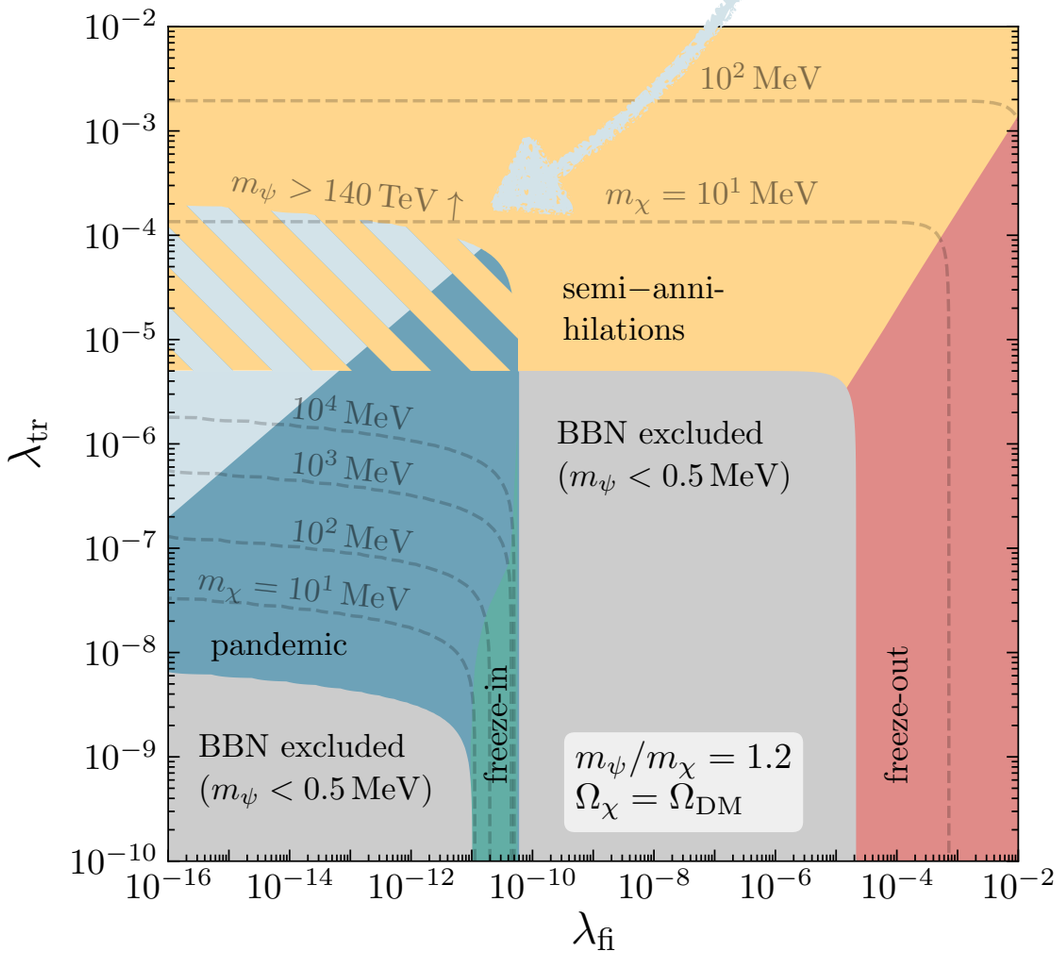
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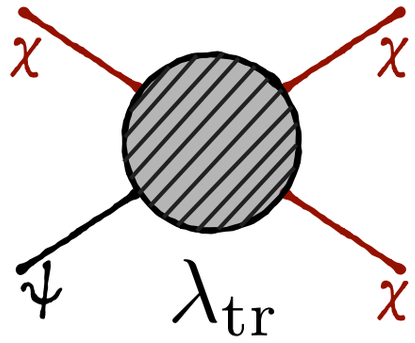
2→4 freeze-in dominates





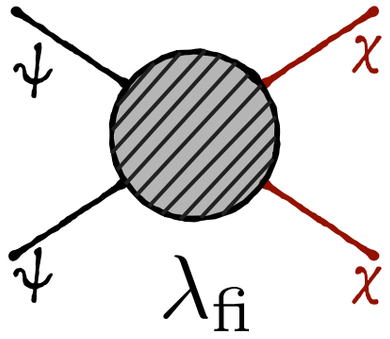
Adding freeze-in production

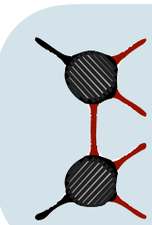
‘transmission’



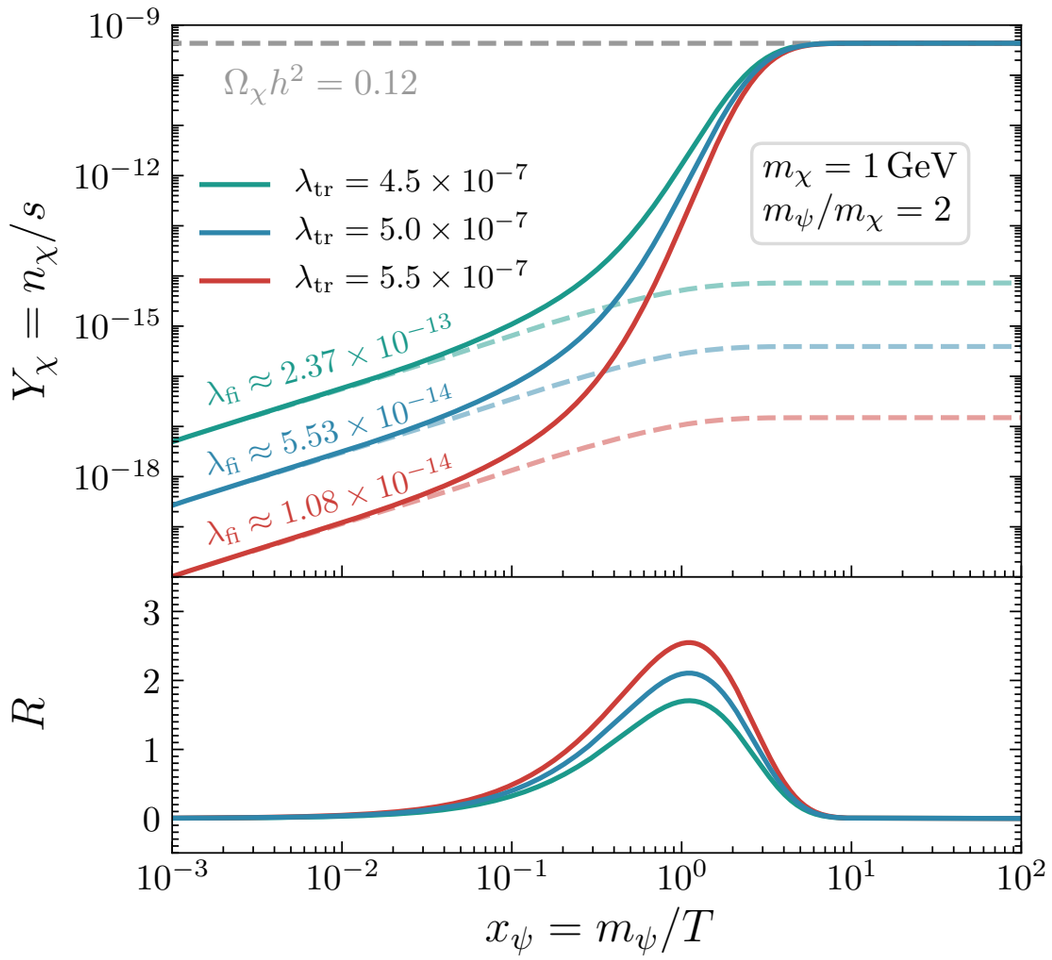
+

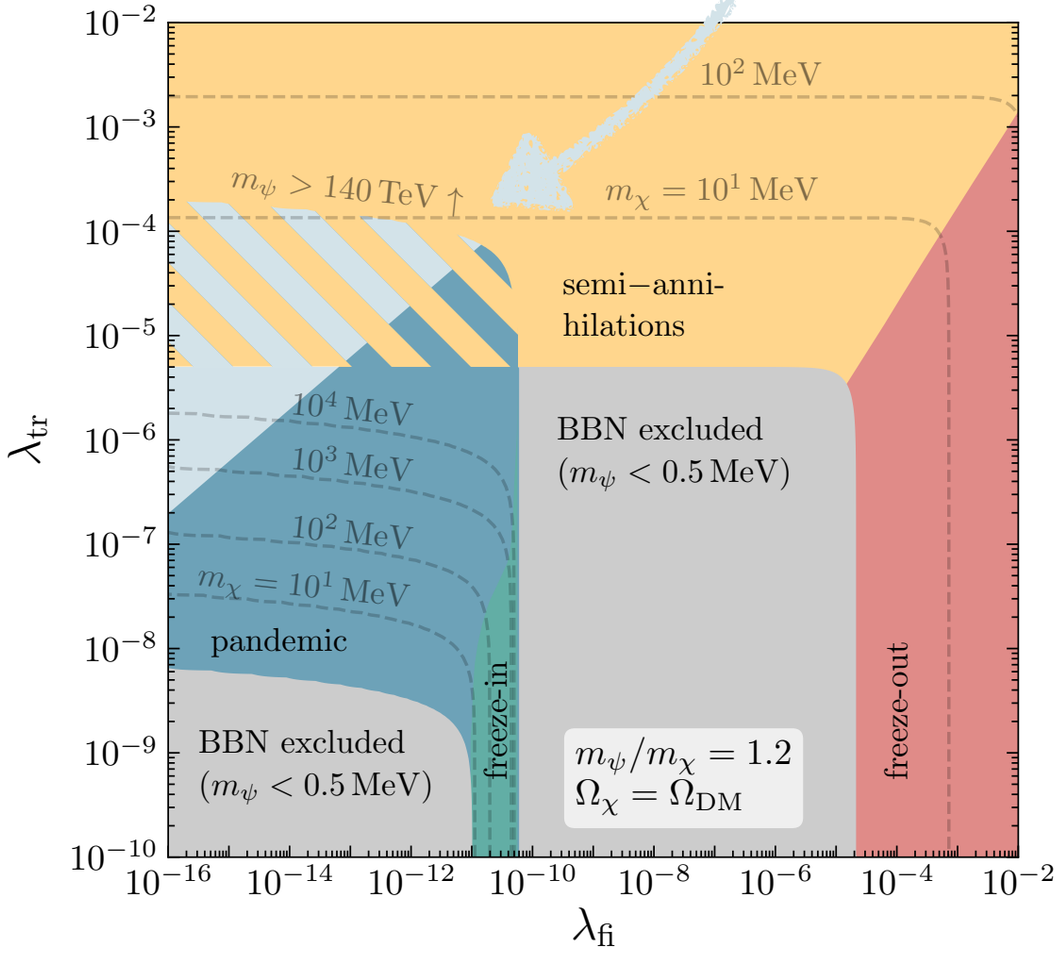
‘freeze-in’





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➡ *‘Pandemic’ production turns out to be a rather generic mechanism for the genesis of DM!*

Signals ?

- Necessarily model-dependent
  - 'Pandemic DM' describes a class of models, just like 'WIMP' does

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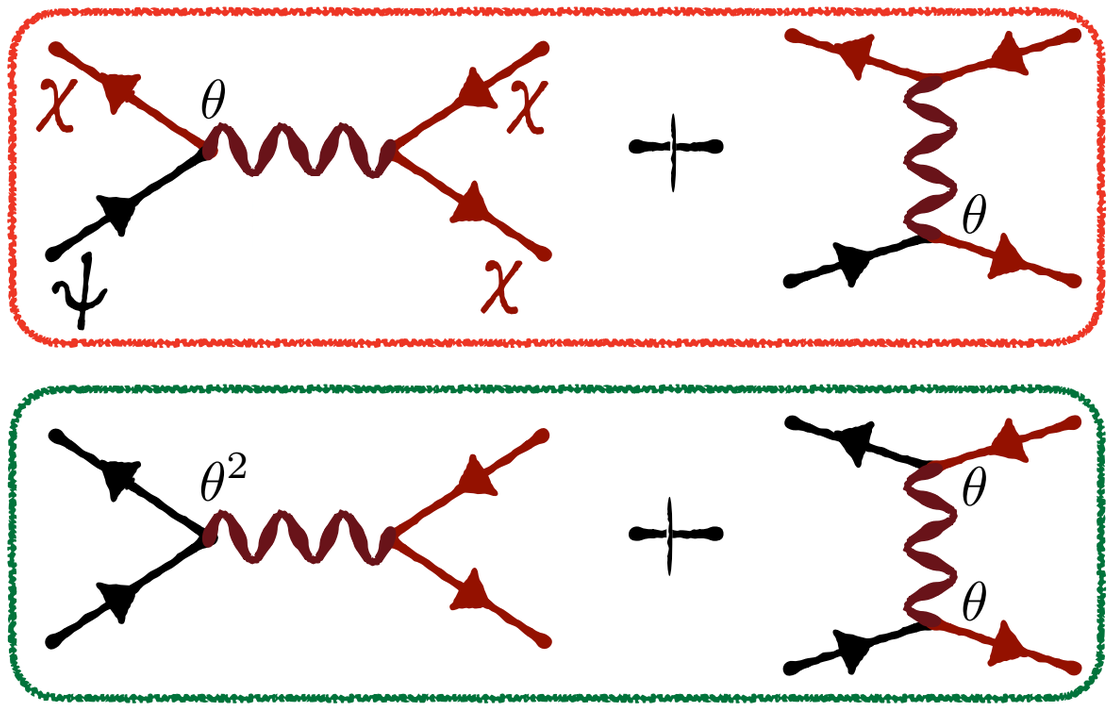
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transmission $\propto \theta$

freeze-in $\propto \theta^2$

Sterile neutrinos



- An excellent, well-motivated dark matter candidate
- Production by SM processes: oscillations with active neutrinos, combined with CC and NC scatterings

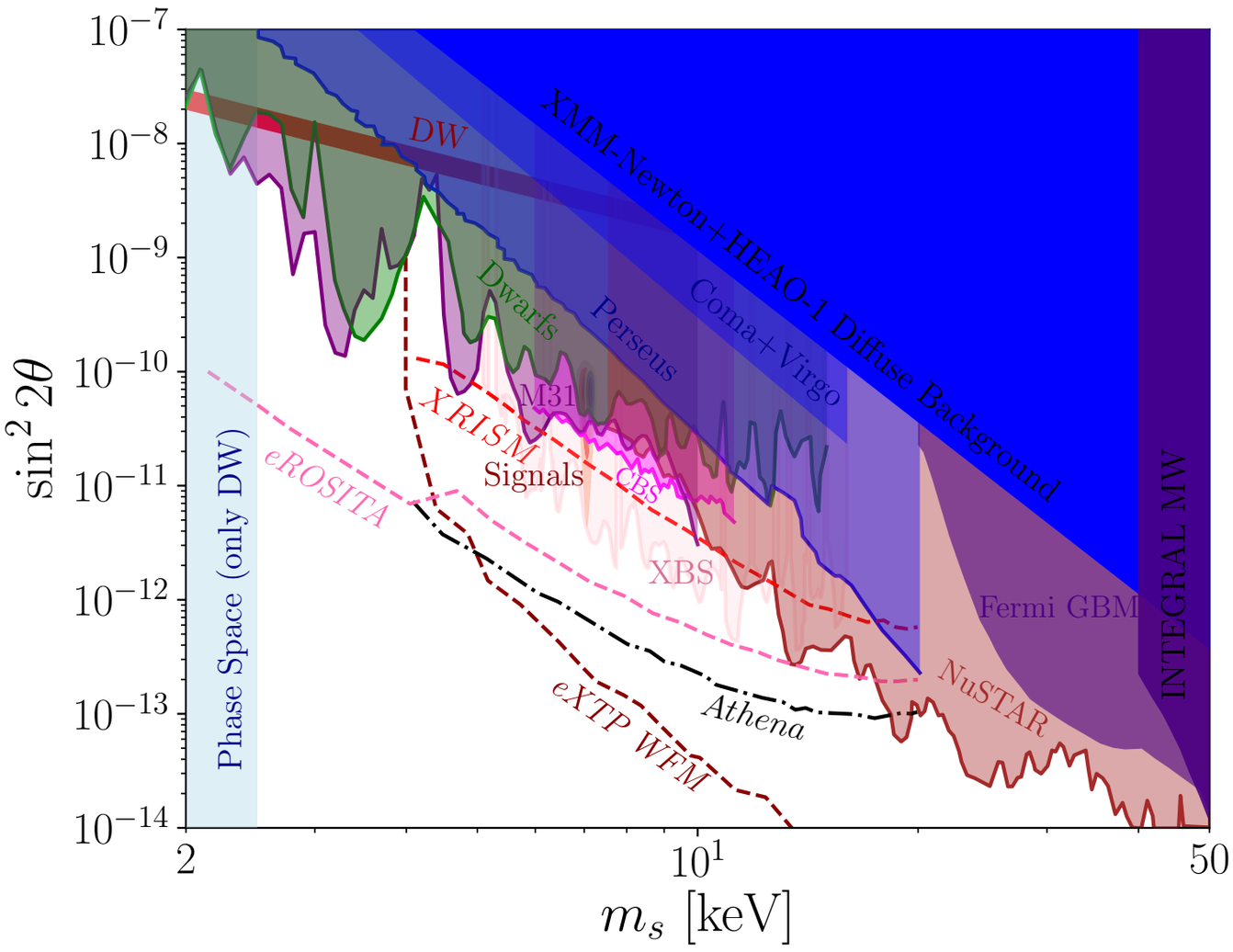
Dodelson & Widrow, PRL '94

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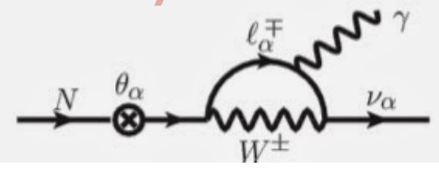
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Dodelson & Widrow, PRL '94



Abazajian+, 2203.7377

X-ray lines



- Unfortunately, this scenario is ruled out by observations…

Interacting sterile neutrinos

TB, Depta, Hufnagel, Kersten, Ruderman & Schmidt-Hoberg, arXiv:2206.10630

- Let's add a scalar ϕ that only couples to the sterile neutrinos

$$\mathscr{L} \supset \frac{y}{2}\phi\bar{\nu}_s\nu_s \quad \Rightarrow \quad \frac{y}{2}\phi[\sin^2\theta\bar{\nu}_\alpha\nu_\alpha - \sin\theta\cos\theta(\bar{\nu}_\alpha\nu_s + \bar{\nu}_s\nu_\alpha) + \cos^2\theta\bar{\nu}_s\nu_s]$$

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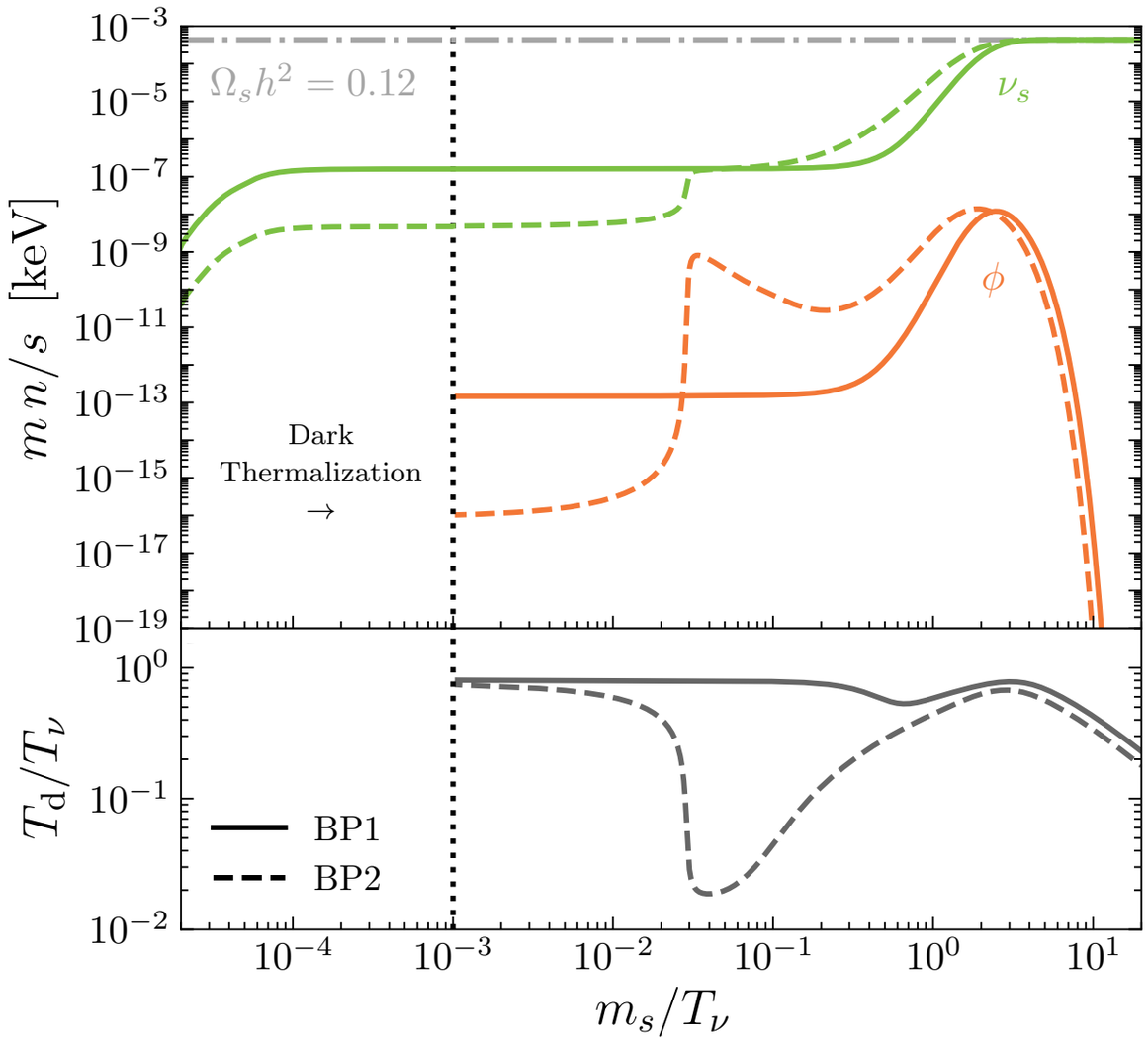
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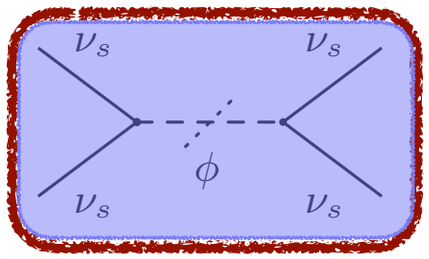
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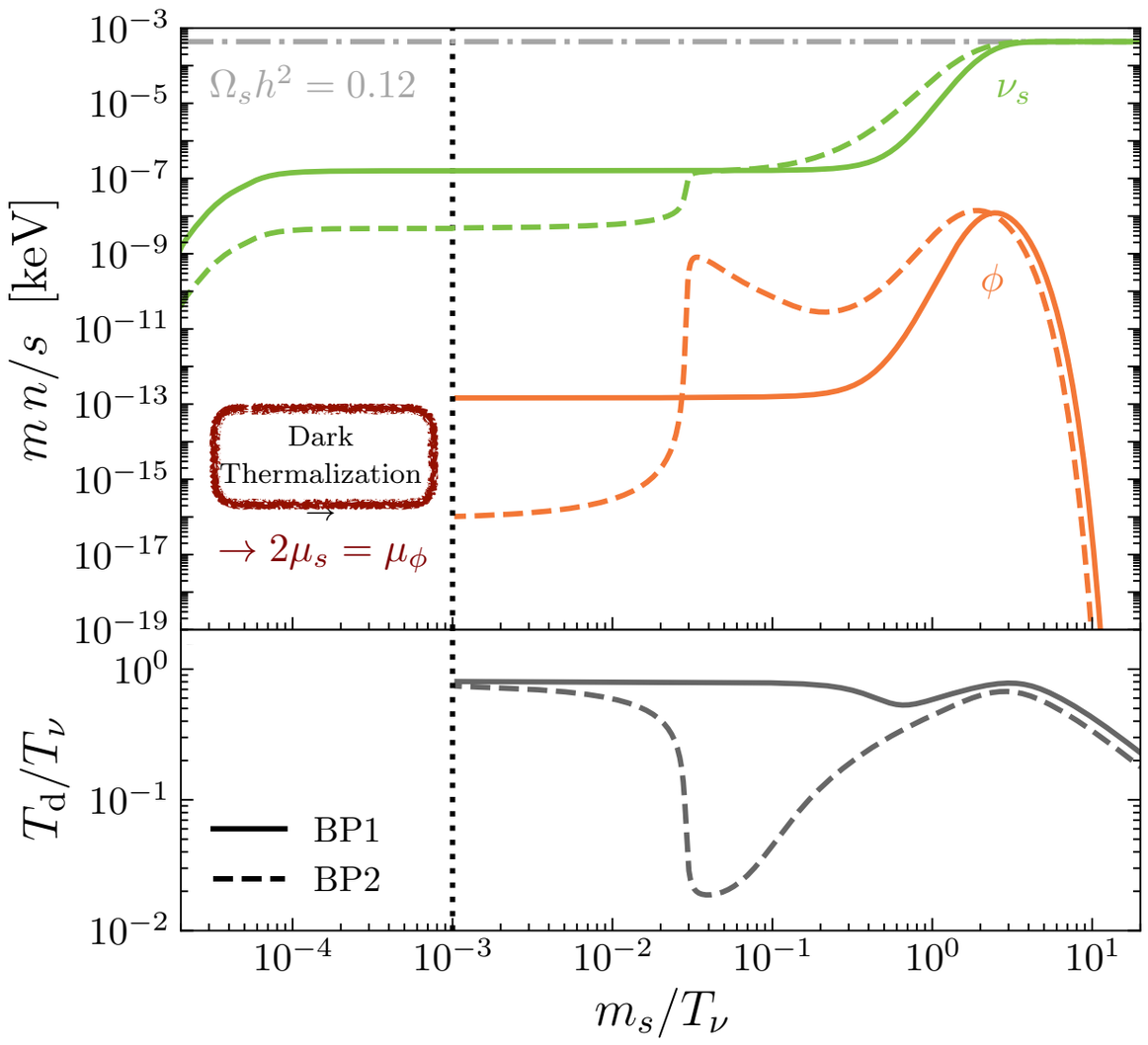
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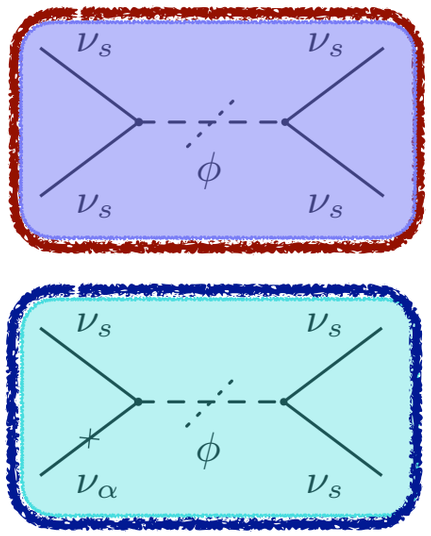
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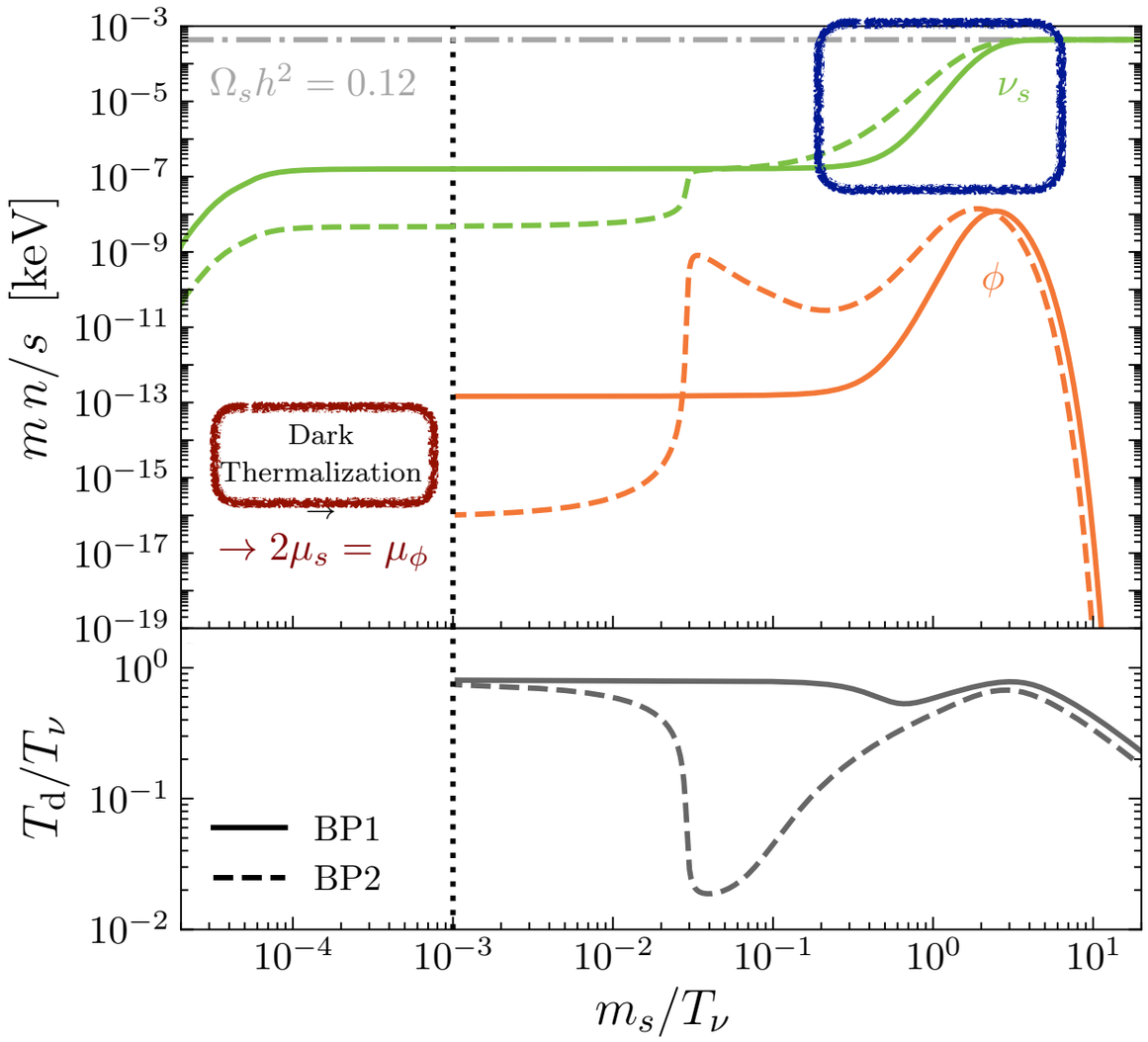
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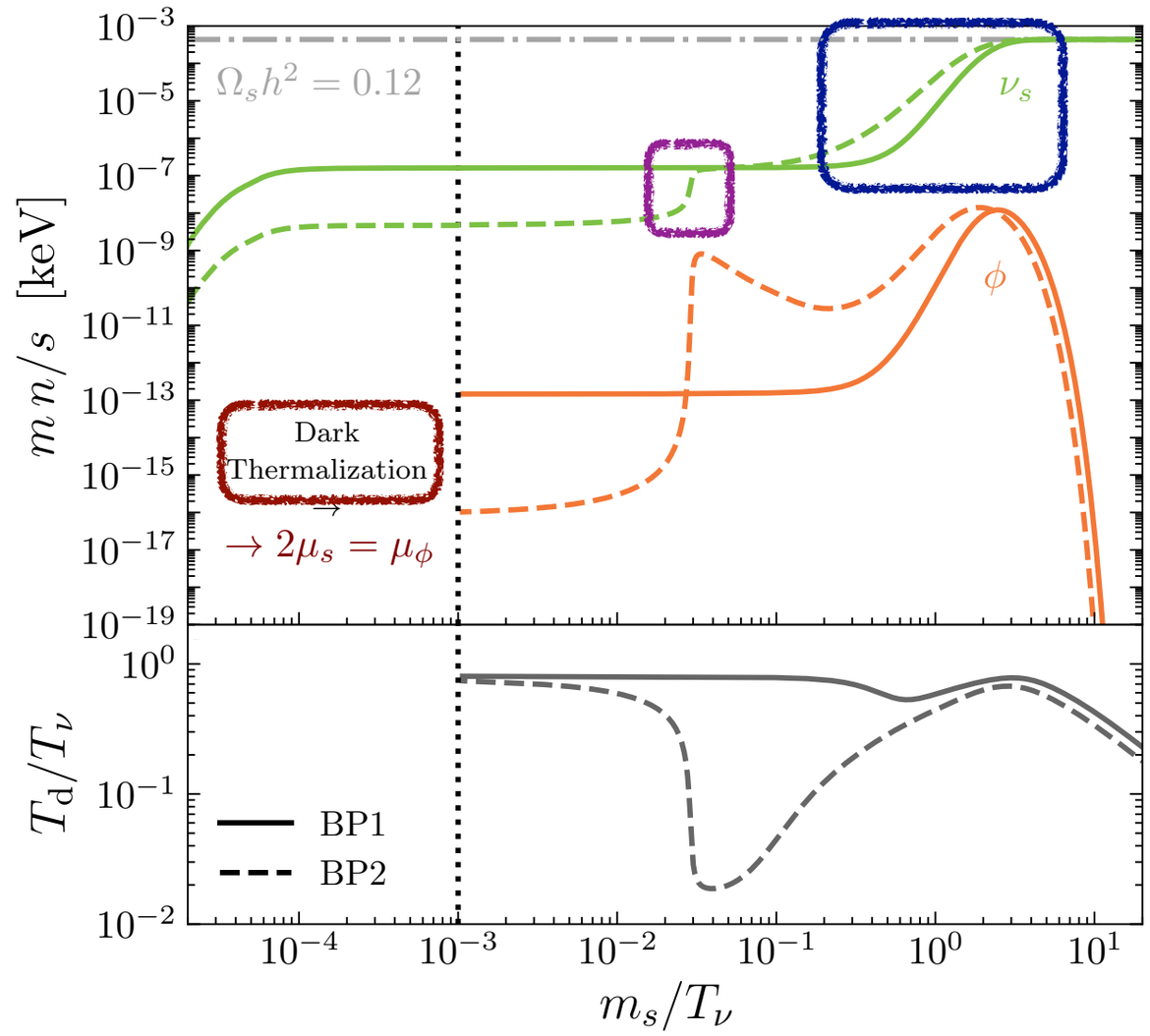
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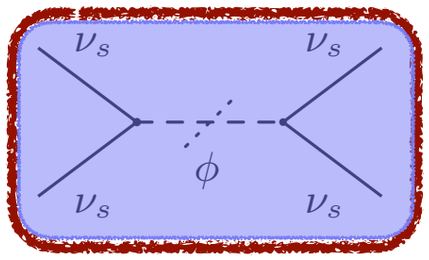
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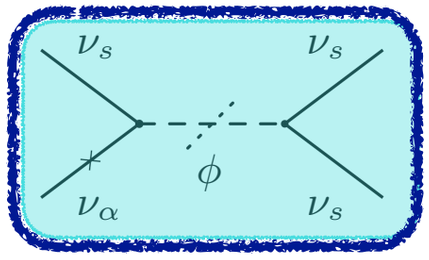
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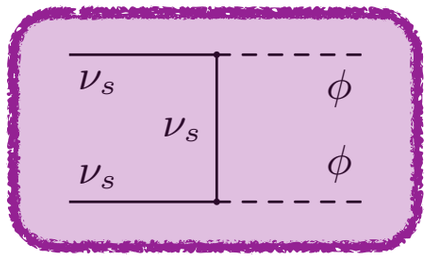
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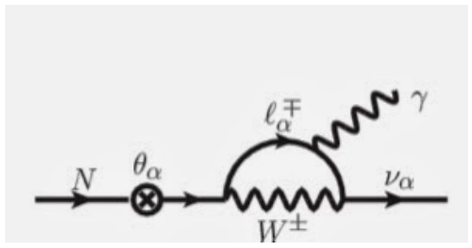
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Sterile neutrinos… revived !

TB, Depta, Hufnagel, Kersten, Ruderman & Schmidt-Hoberg, arXiv:2206.10630

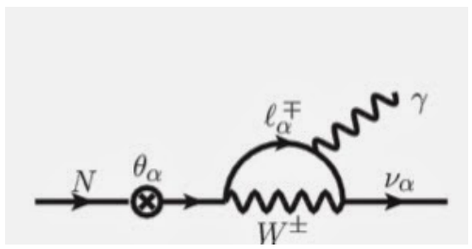
- Observational constraints
  - (Standard) X-ray lines



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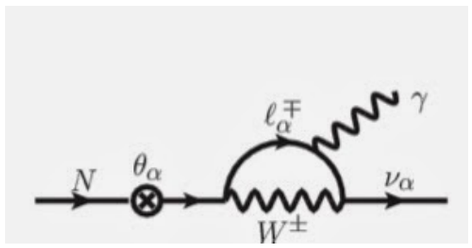
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Garzilli+, MNRAS '21

$$\lambda_{\mathrm{FS}} < 0.24\,\mathrm{Mpc}$$
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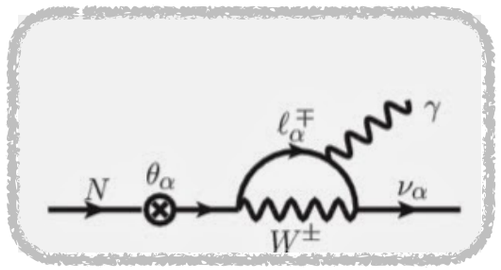
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Palanque-Delabrouille+, JCAP '20

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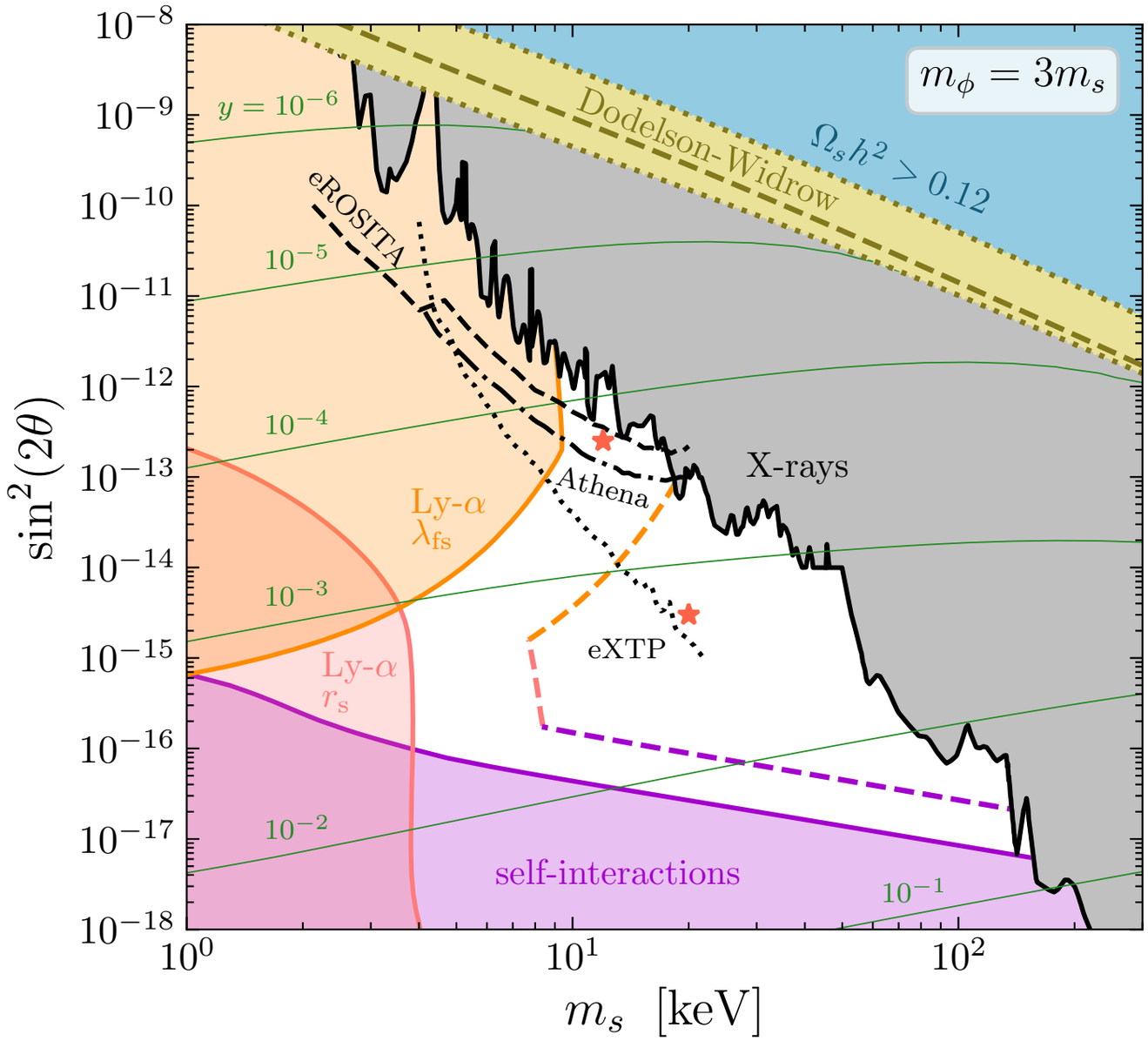
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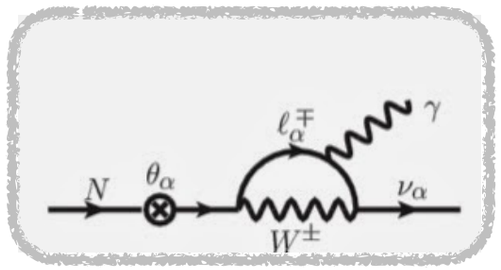


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  - Bounded from above *and* below
  - Significant parts in observational reach

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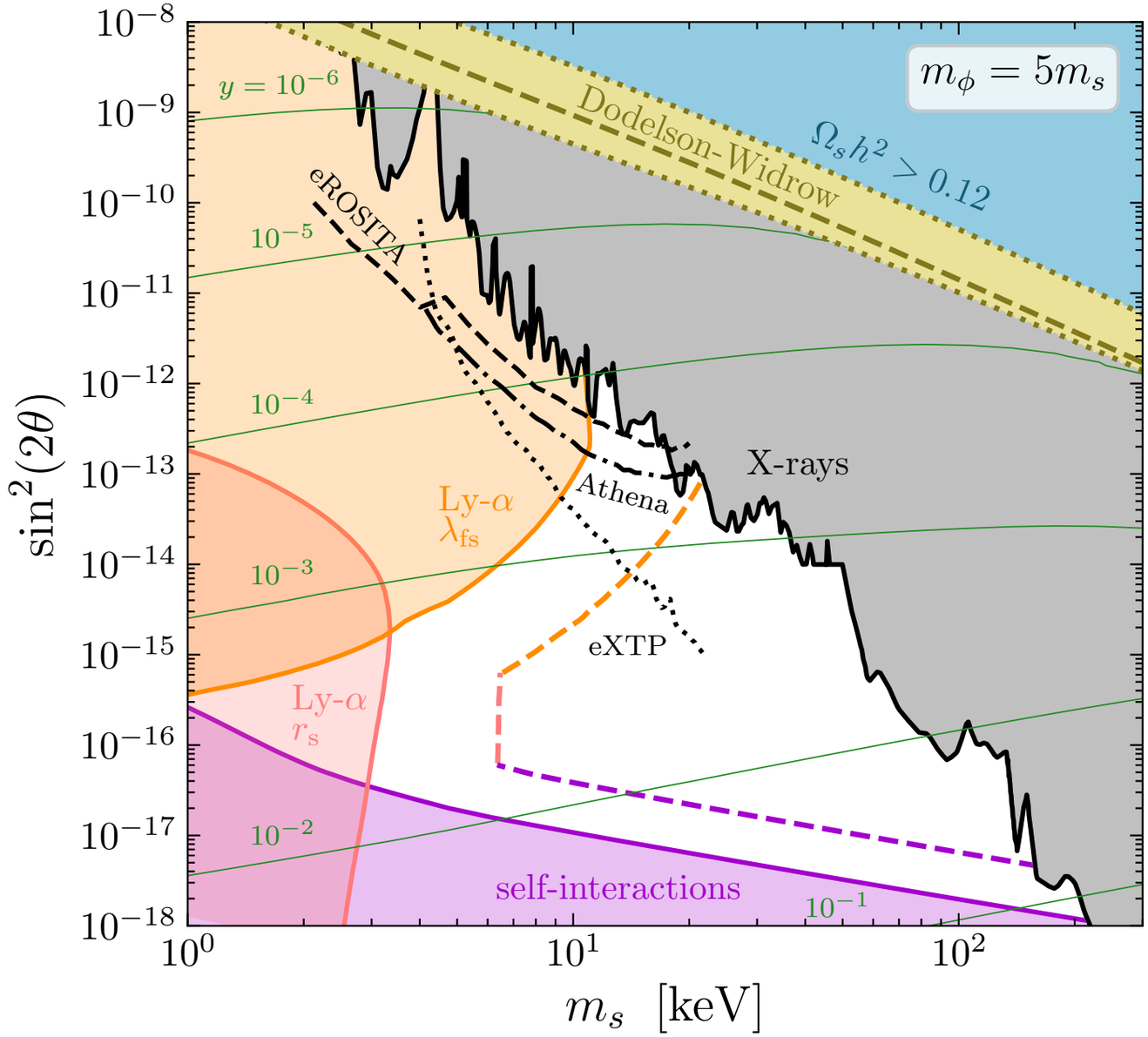
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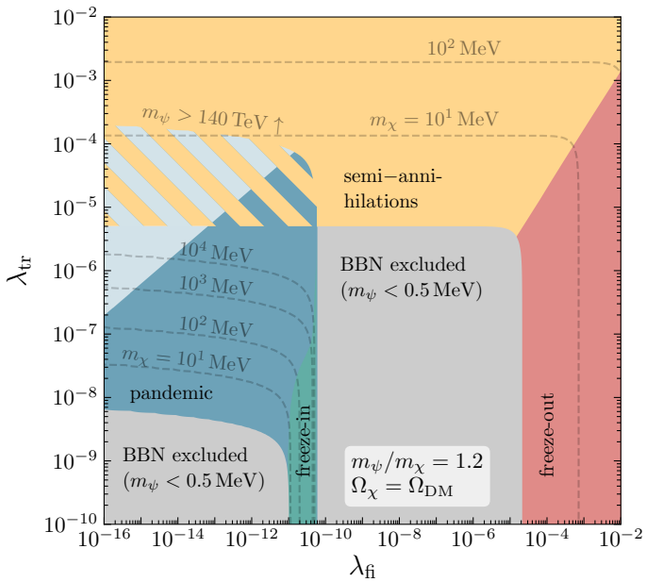
Palanque-Delabrouille+, JCAP '20



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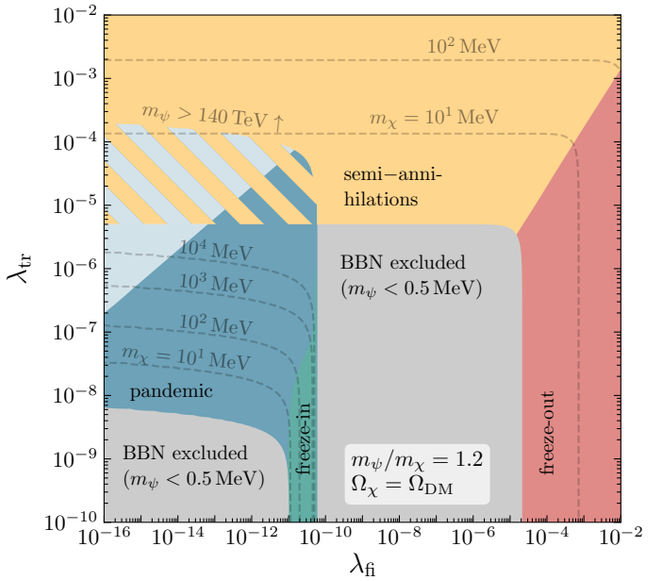
Conclusions

- There are a handful of generic ways of DM genesis from the primordial heat bath



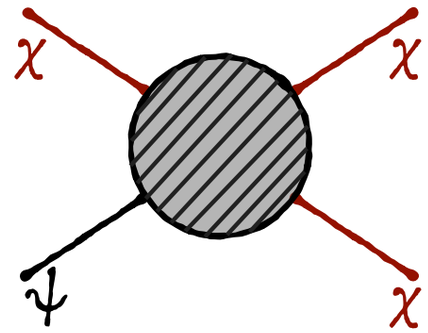
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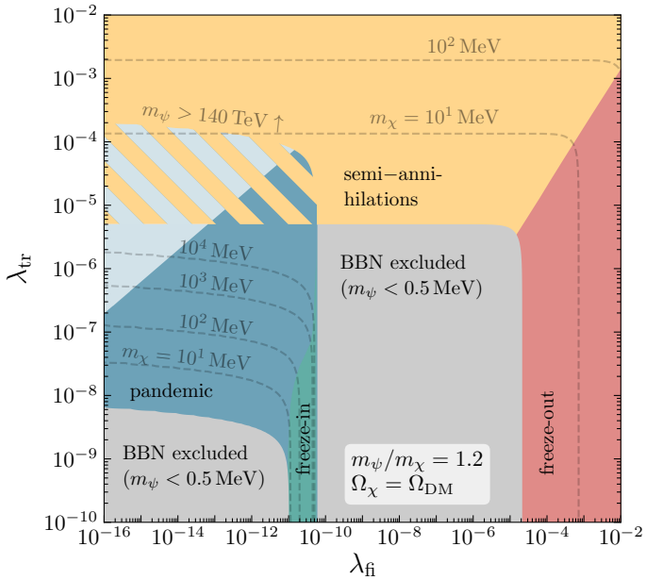
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  Mathematical analogy to spread of diseases works almost scarily well

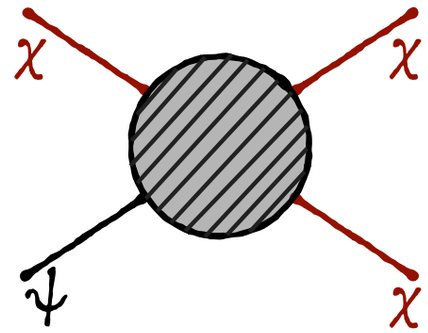


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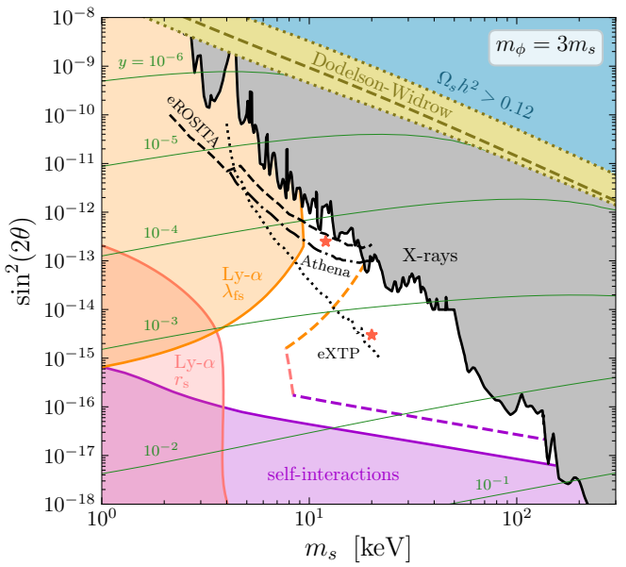
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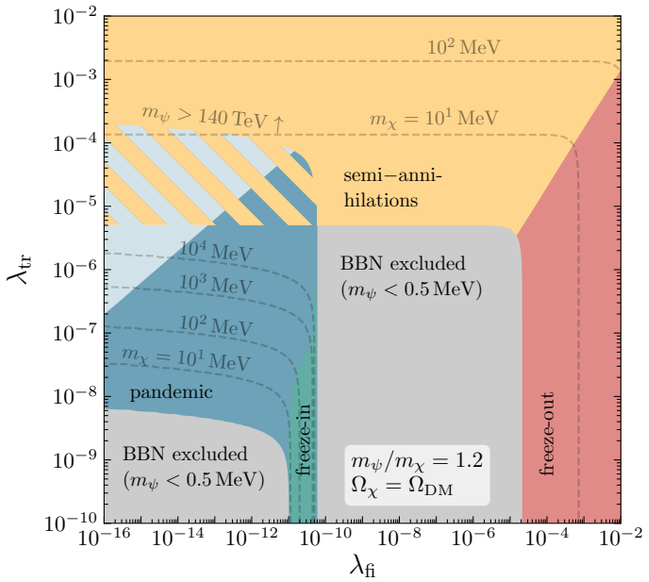


- A new *minimal* scenario for sterile neutrino DM ?!

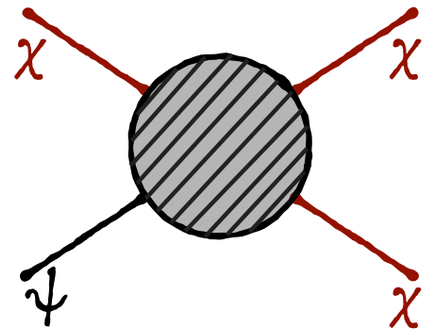


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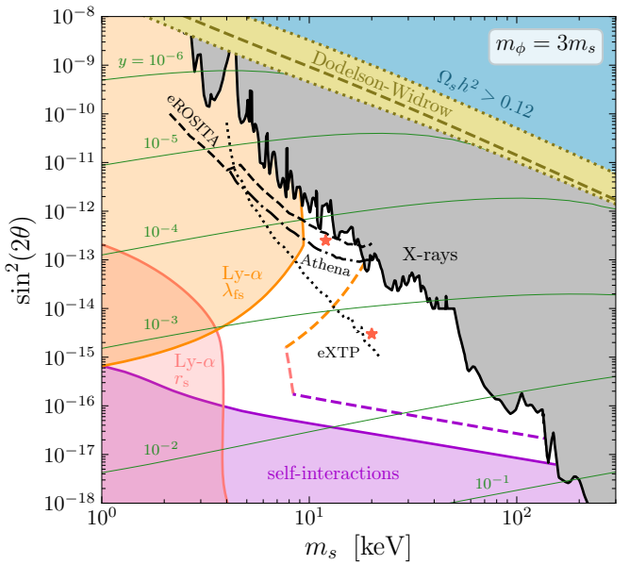
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Thanks for your attention!

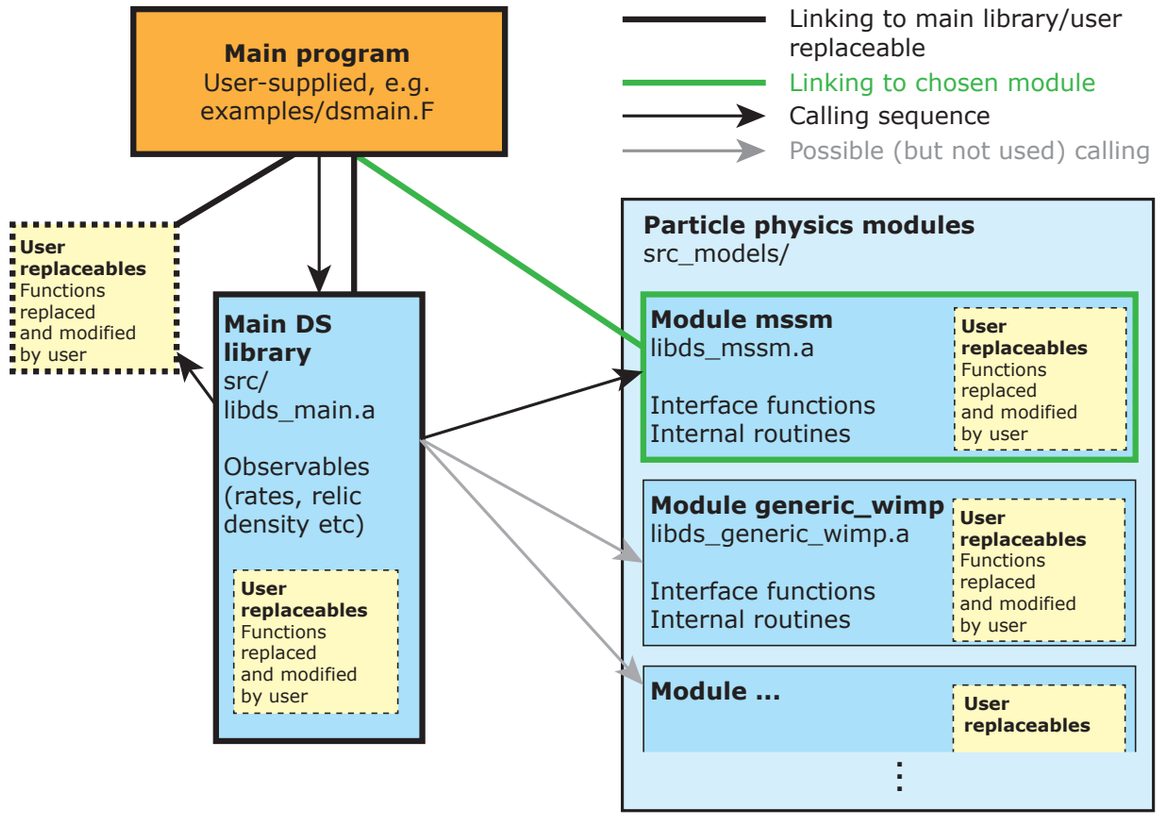
DarkSUSY



TB, Edsjö, Gondolo, Ullio & Bergström, JCAP '18

http://darksusy.hepforge.org

- Numerical package to calculate 'all' DM related quantities:
  - *relic density + kinetic decoupling*
  - *generic SUSY models + laboratory constraints implemented*
  - *cosmic ray propagation*
  - particle yields for generic DM annihilation or decay
  - *indirect detection rates: gammas, positrons, antiprotons, neutrinos*
  - *direct detection rates*
  - …



DarkSUSY

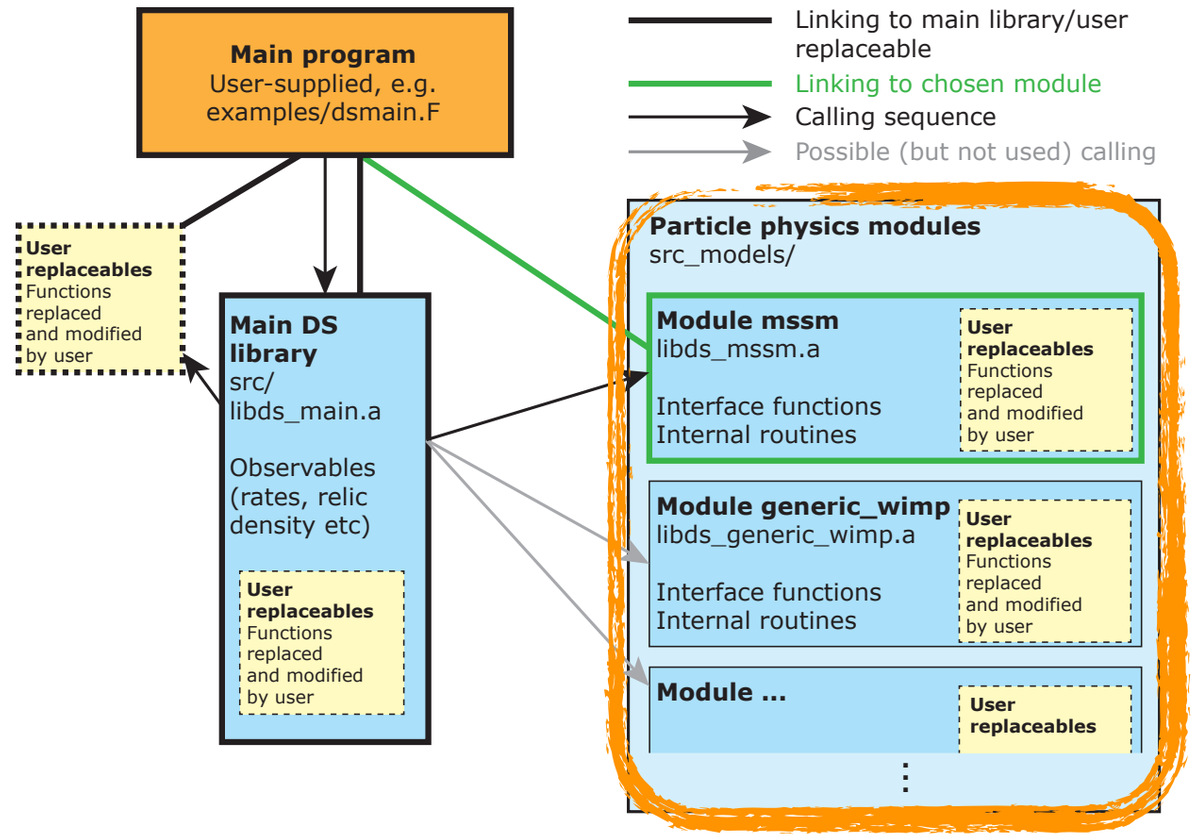


TB, Edsjö, Gondolo, Ullio & Bergström, JCAP '18

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Since version 6: no longer restricted to supersymmetric DM !

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  - *relic density + kinetic decoupling* **(also for $T_{\rm dark} \neq T_{\rm photon}$)**
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  - …



since 6.1: DM self-interactions

since 6.2: 'reverse' direct detection
(incl. full Q^2 -dependent scattering!)

since 6.3: freeze-in

Collision term for FIMPs

$$C[f_\chi] = \frac{1}{2g_\chi}\int\frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{E}}\int\frac{d^3k}{(2\pi)^3 2\omega}\int\frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}}(2\pi)^4\delta^{(4)}(\tilde{p}+p-\tilde{k}-k)$$

$$\times\left[|\mathcal{M}|^2_{\bar{\chi}\chi\leftarrow\bar{\psi}\psi}\, f_\psi(\omega)f_\psi(\tilde{\omega}) - |\mathcal{M}|^2_{\bar{\chi}\chi\rightarrow\bar{\psi}\psi}\, f_\chi(E)f_\chi(\tilde{E})\{1\pm f_\psi(\omega)\}\{1\pm f_\psi(\tilde{\omega})\}\right]$$

'production' 'annihilation'

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'production' 'annihilation'

$$\Rightarrow \boxed{C[f_\chi] = \langle\sigma v\rangle_{\chi\chi\to\psi\psi}\left(n_\chi^{\rm MB}\right)^2}$$

annihilation of ***would-be*** MB population

- Only 2 integrals after exploiting spherical symmetry:

(γ : Lorenz boost to CMS)

$$\boxed{\langle\sigma v\rangle_{\chi\chi\to\psi\psi} = \frac{8x^2}{{K_2}^2(x)}\int_1^\infty d\tilde{s}\,\tilde{s}\,(\tilde{s}-1)\int_1^\infty d\gamma\,\sqrt{\gamma^2-1}e^{-2\sqrt{\tilde{s}}x\gamma}\sigma_{\chi\chi\to\psi\psi}(s,\gamma)}$$

TB, Heeba, Kahlhoefer & Vangsnes, JHEP '22
(see also Lebedev & Toma, PLB '19
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$$\sigma_{\chi\chi\to\psi\psi}(p,\tilde{p}) = \frac{(2\pi)^4}{4N_\psi E\tilde{E}\,v_{\rm M\o l}}\int\frac{d^3k}{(2\pi)^3 2\omega}\int\frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}}\,\delta^{(4)}(\tilde{p}+p-\tilde{k}-k)\left|\overline{\mathcal{M}}\right|^2\{1\pm f_\psi(\omega)\}\{1\pm f_\psi(\tilde{\omega})\}$$

Collision term for FIMPs

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→ $K_1(2\sqrt{\tilde{s}}x)/(2\sqrt{\tilde{s}}x)$ ✔

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→ 1

- In *this* formulation, direct analogy with WIMP case!

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→ 1

- In *this* formulation, direct analogy with WIMP case!
  - Can recycle sophisticated numerical tools for thermal averages
  - Easier to estimate higher-order corrections

