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reading:

- V. Rubakov, A. Vlasov, "What do we learn from CMB observations", arXiv: 1008.1704.
- W. Hu, "Lecture Notes on CMB Theory: From Nucleosynthesis to Recombination", arXiv:0802.3688.
- M. Taoso, G. Bertone, A. Masiero, "Dark Matter Candidates: A Ten-Point Test," JCAP **0803**, 022 (2008) [arXiv:0711.4996 [astro-ph]].

5 Cosmic microwave background

The existence of the cosmic microwave background radiation (CMB), with its almost perfect isotropy and black-body spectrum of $T = 2.725\,\mathrm{K} = 2.3 \cdot 10^{-4}\,\mathrm{eV}$, is one of the most compelling evidences for big bang cosmology. It was released shortly after electrons and ions (re)combined to atoms, thus making the universe transparent for the first time; the temperature and redshift of last scattering (mainly Thompson scattering off free electrons) are given by $T_\mathrm{r} = 2970\,\mathrm{K} = 0.26\,\mathrm{eV}$ and $z_\mathrm{r} = 1090$ (these values are mainly related to the ionization energy of hydrogen and essentially independent of any cosmological parameters). Even more interesting is the existence of tiny temperature fluctuations in the CMB, of the order of $\delta T/T \sim 10^{-5}$, which correspond to tiny density fluctuations in the early universe. The CMB thus really is a snapshot of the universe at the time of last scattering, providing detailed information about the composition of the cosmological fluid.

The observed temperature fluctuations can conveniently be expanded in spherical harmonics:

$$\delta T(\phi,\theta)/T = \sum_{l,m} a_{lm} Y_{lm}(\phi,\theta)\,. \tag{74}$$

Observations indicate that the a_{lm} are Gaussian random variables, with

$$\langle a_{lm} a_{l'm'} \rangle \equiv C_l \delta_{ll'} \delta_{mm'}\,, \tag{75}$$

so the variance (or power) of the fluctuations becomes

$$\left\langle \left(\frac{\delta T}{T}\right)^2 \right\rangle = \sum_l \frac{2l+1}{4\pi} C_l \approx \int \frac{l(l+1)}{2\pi} C_l \, d(\ln l)\,. \tag{76}$$

Note that the relation between multipole number and angular scale is roughly $\theta \simeq \pi/l$; the appearance of the first peak in the power spectrum at $l \sim 200$ thus corresponds to an angular size of about 1°.

In conformal Newtonian gauge, with Φ and Ψ denoting the gravitational potentials, the temperature fluctuations observed today can be written as

$$\frac{\delta T}{T} = \frac{1}{4}\frac{\delta\rho_\gamma}{\rho_\gamma} + \Phi + v_\parallel + \int_{t_r}^{t_0} dt(\dot{\Phi} - \dot{\Psi})\,. \tag{77}$$

In total, there are thus four effects that change the photon temperature:

- Fluctuations in the energy density of photons ($\rho_\gamma \propto T^4$) at the time of last scattering.
- A redshift (blueshift) $\delta T/T = \Phi$ of photons that start off from a potential well (a potential hill). Together with the first effect, this is called the *Sachs-Wolfe effect*.
- A *Doppler shift* $\delta T/T = v_\parallel$, where $v_\parallel$ is the velocity component of the photon-baryon medium, at the time of last scattering, towards the direction of observation.
- The so-called *integrated Sachs-Wolfe effect* that arises from the fact that photons transverse time-dependent gravitational fields from the time of emission until today; for constant fields, they would gain(loose) the same amount of energy when entering a potential well(hill) as they would loose(gain) when leaving it again.

Numerically, the first two terms in Eq. (77) dominate; they receive their main contribution from the baryon-photon system (which to an excellent approximation is a single fluid prior to recombination) and dark matter.

Before recombination, density perturbations in the baryons and photons inside the horizon oscillate like in Eq. (71).

$$\delta_\gamma \simeq \delta_b \propto \cos kr_s\,. \tag{78}$$

Since the phase of these oscillations is uniquely determined, we expect peaks in the fluctuation spectrum at $k_n \equiv n\pi/r_s$. This corresponds to an angular size of $\theta_n \simeq (\pi/k_n)/\eta_0 = r_s/(n\eta_0)$ today, where η_0 is the conformal time since last scattering, i.e. the comoving distance to the last-scattering surface. Maximal values for C_l are thus expected at

$$l_{\max} \simeq \frac{\pi}{\theta_n} = n\pi\frac{\eta_0}{r_s}\,. \tag{79}$$

Because z_r is essentially independent of cosmological parameters, the sound horizon at last scattering, r_s, provides a cosmological standard ruler. η_0, on the other hand, depends on the expansion history of the universe since the release of the CMB:

$$\eta_0 = \int_{t_r}^{t_0} \frac{dt}{a} = \int_0^{z_r} \frac{dz}{a_0 H} = \frac{1}{a_0 H_0} \int_0^{z_r} \frac{dz}{\sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}} \,. \tag{80}$$

Furthermore, the angular scale for a given co-moving distance depends on the geometry. The position of the peaks thus provides a way to measure both curvature and late-time evolution of the universe (with a certain degeneracy).

So far, we have neglected the effect of dark matter (apart from its contribution to the total curvature, as discussed above). Let us first estimate the direct effect, namely its contribution to the gravitational potential. From the Poisson equation, we get

$$\nabla^2 \Phi_\chi = -\frac{k^2}{a^2}\Phi_\chi = 4\pi G \delta\rho_\chi \,. \tag{81}$$

Since $\delta\rho_\chi = \delta_\chi \cdot \rho_\chi \propto a^{-2}$, it follows that $\partial_t \Phi_\chi = 0$ and $\Phi_\chi \propto k^{-2}$. For a relativistic medium (with $\Phi \ll 1$), the Newtonian equation of hydrodynamic equilibrium (40) is generalized to

$$\nabla p = -(p+\rho)\nabla\Phi \,. \tag{82}$$

With $p = p_\gamma = \rho_\gamma/3$ and $\rho = \rho_b + \rho_\gamma$, this can be written as $\frac{1}{3}\delta\rho_\gamma = -(\frac{4}{3}\rho_\gamma + \rho_b)\Phi$ in Fourier space. Using $\Phi \approx \Phi_\chi$, the first two terms in Eq. (77) thus combine to

$$\frac{\delta T}{T} \approx -\frac{3}{4}\frac{\rho_b}{\rho_\gamma}\Phi_\chi \,. \tag{83}$$

Note that this is proportional to ρ_b – without baryons, there would thus be no effect of Φ on the CMB. The scaling $\Phi_\chi \propto k^{-2}$ largely explains the observed overall suppression of power on small scales.

There is another, more indirect effect from the interplay of baryons and DM. Consider, on top of the contributions in Eq. (83) the contribution from baryon oscillations to the gravitational potential, which adds a term $A\cos(kr_s)$. For the observed *square* of the fluctuations, interference terms become important:

$$\left(\frac{\delta T}{T}\right)^2 \supset A^2\cos^2(kr_s) - \frac{3}{2}A\frac{\rho_b}{\rho_\gamma}\Phi_\chi\cos(kr_s) + \left(\frac{3}{4}\frac{\rho_b}{\rho_\gamma}\Phi_\chi\right)^2 \,. \tag{84}$$

Peaks with odd n ($\cos[kr_s] = -1$) are thus enhanced and peaks with even n ($\cos[kr_s] = 1$) are suppressed – an effect that should be most pronounced at small l (due to $\Phi_\chi \propto k^{-2}$).

At small scales, the CMB power is rather strongly suppressed (on top of the expected k^{-4} scaling) by two effects: CMB photon scattering on high-energy free electrons in the plasma (Sunyaev-Zel'dovich effect) and the free streaming of CMB photons after their mean free path increases drastically already quite some time before recombination (Silk damping). There are various other mechanisms that result in (usually less important) changes of the CMB spectrum that we will not discuss here. After 7 years of data, the observations of the WMAP satellite can all nicely be described by a rather small set of cosmological parameters that, on top of that, can be determined to a high precision. Again, one finds overwhelming evidence for a DM component that is about five times as big as the baryonic component (the latter of which being in agreement with BBN data).

6 Summary – DM properties

The preceding lectures can be summarized as follows: There is overwhelming evidence for a sizable dark matter component in the universe, coming from observations at distance scales that range from the size of small galaxies, to clusters of galaxies, to cosmological scales close to the present size of the horizon. Its cosmological abundance can be determined rather precisely as

$$\Omega_\chi h^2 = 0.1186 \pm 0.0031\,, \tag{85}$$

which is much more than the abundance of ordinary ("baryonic") matter, $\Omega_B h^2 = 0.02217 \pm 0.00033$. (Here, $h = 67.9 \pm 1.5$ is the present Hubble expansion rate in units of 100 km/s/Mpc). Note that the ratio of dark to baryonic matter locally can differ drastically from the cosmological value of $\Omega_\chi/\Omega_B \approx 5$: while, e.g., the Milky Way within the solar radius is largely dominated by baryons, it might be as high as ~ 1000 for Dwarf galaxies.

Even though the *amount* of dark matter is known very well, so far basically nothing is known about its *nature*. Since all evidence is obtained somehow indirectly, i.e. through its gravitational effects, also the list of inferred properties looks more like a list of what dark matter is *not*: In particular, dark matter must

- be *non-baryonic* in order to be consistent with BBN and CMB,
- be *electrically neutral* because it does not seem to emit electromagnetic radiation (it is dark...),
- be *color-neutral*, because such heavy partons (confined inside color-neutral hadrons) would interact too strongly with baryons (with consequences for BBN, the CMB and even the stability of galactic disks),

- have a *small coupling* to bosons charged under $SU(2)$ in order to evade constraints from direct detection (see chapter 13),
- be *cold* (highly non-relativistic), because only if matter is dominantly dissipation-less and has negligible free-streaming effects observations of large scale structures agree with the results from numerical N-body simulations of structure formation,
- be *collision-less* in order to explain observations like the bullet cluster. (Again excluding baryonic dark matter, but not putting very stringent constrains on the self-interaction of typical particle DM candidates).

In the literature, there have literally been zillions of proposals to explain the nature of dark matter. Roughly, they can be divided into two classes: modifications of gravity or (new) elementary particles that make up the dark matter. While attempts of the former type often very nicely explain isolated phenomena like the flattening of rotation curves, they usually lack a sound theoretical motivation and generally fail to give a consistent description of *all* observed phenomena.

For the latter option, the first interesting point to note is that the standard model of particle physics does not contain any particle with the above properties – so the very existence of dark matter provides strong evidence for *physics beyond the standard model*. Many proposed particle dark matter candidates, however, are disfavored for similar reasons as given above for modifications of gravity. A *good dark matter candidate* should thus be consistent with all observations *and* have an independent (strong) motivation from particle physics. As it turns out, there are a couple of proposals that do, indeed, satisfy all these criteria.

7 Thermal relics

7.1 Weakly interacting massive particles

Let us consider the case of some massive ('cold') particle χ in the early universe that interacts with standard model particles f through annihilations $\bar{\chi}\chi \leftrightarrow \bar{f}f$. The evolution of its number density is given by the Boltzmann equation (or, rather, its first moment, see Section 7.2):

$$\partial_t \left(a^3 n_\chi\right) = \dot{n}_\chi + 3Hn_\chi = -\langle\sigma v\rangle \left(n_\chi^2 - n_\chi^{\mathrm{eq}2}\right) , \tag{86}$$

where $\langle\sigma v\rangle$ is the effective, thermally averaged annihilation cross section times the relative velocity of the two annihilating χ particles and n_χ^{eq} their number density in thermal equilibrium.

The limiting behavior of this ODE can be easily understood. At early times, or very high temperature, the Hubble expansion term can be neglected because the annihilation term on the rhs dominates ($n_\chi \propto a^{-3} \propto T^3$ vs. $H \propto t^{-1} \propto a^{-2} \propto T^2$): as a result, n_χ is very efficiently forced to follow the equilibrium solution, almost independent of the initial conditions. When the particles become non-relativistic, the interaction rate $\Gamma \sim \langle \sigma v \rangle n_\chi$ will start to fall behind the Hubble expansion rate H due to the Boltzmann suppression of $n_\chi \propto \exp[-m_\chi/T]$. Eventually, the rhs can be completely neglected and the co-moving χ number density stays constant – until today, where it will contribute to the measured value of Ω. The process of leaving the thermal distribution, at some temperture $T_{\rm cd}$, is known as (chemical) decoupling or *freeze-out*. Numerically, one finds (assuming dominantly s-wave annihilation)

$$\Omega_\chi h^2 \sim 0.1 \times \frac{3 \cdot 10^{-26} \mathrm{cm}^3/\mathrm{s}}{\langle \sigma v \rangle|_{T_{\rm cd}}} \,. \tag{87}$$

The fact that this agrees with the observed value for $\Omega_\chi h^2 \sim 0.1$, see Eq. (85), for particles with masses and coupling strenths at the electroweak scale $[\langle \sigma v \rangle \sim \alpha^2_{SU(2)} c/(100\,\mathrm{GeV})^2 \sim 10^{-3} (10^{10}\mathrm{cm/s})(10^{-16}\mathrm{cm})^2]$ is sometimes referred to as the *WIMP miracle*. Such weakly interacting massive particles are predicted in many extensions of the standard model of elementary particle that were introduced to solve its shortcomings at energies above the electroweak scale ($\sim 100\,\mathrm{GeV}$), in particular the fine-tuning problem connected to the fact that the scalar sector is not protected against radiative corrections (the sum of loop contributions to the Higgs mass, in particular, are quartic in the assumed cutoff scale of the theory). Examples for such extensions include supersymmetry (SUSY), Little Higgs models and theories with extra dimensions and will be discussed in more detail later.

The lightest of the new particles predicted in these theories is often neutral; if stable on cosmological time scales (which is usually guaranteed by some internal symmetry), it provides an excellent DM candidate – not only because it will be thermally produced in the early universe, with a relic density that is generically of the right order of magnitude, but also because it provides, in principle, ways to test the DM particle hypothesis by means other than gravitational: Since the interaction with standard model particles are more or less of the standard $SU(2)$ type (i.e. weak but not unreasonably weak), one may hope to i) produce WIMPs at colliders, ii) observe their recoil off heavy nuclei in large detectors or iii) detect the results of galactic DM pair annihilation in the spectrum of cosmic rays of various kinds. All these detection strategies will later be discussed in some detail.