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reading:

- D. Hooper, S. Profumo, "Dark matter and collider phenomenology of universal extra dimensions", Phys. Rept. **453**, 29-115 (2007). [hep-ph/0701197].

7.4 The smallest gravitationally bound WIMP halos

For the case of chemical decoupling, we could derive the resulting DM relic density today. Let us now investigate whether there are also observational consequences from the kinetic decoupling of WIMPs. At $T \gg T_{\rm kd}$, WIMPs are tightly coupled to the heat bath, so we actually have $\delta_\chi = \delta_b = \frac{4}{3}\delta_\gamma$ (for adiabatic fluctuations; note that such high temperatures were not considered in the discussion of Section 4). Afterwards, DM density perturbations can freely grow (logarithmically) under the influence of gravitation; close to kinetic decoupling, however, first a remaining viscous coupling between the DM and radiation fluids and then the free-streaming of the WIMPs generate an exponential cutoff in the power spectrum. The mass contained within a region with the size of the corresponding cutoff-scale is given by

$$M_{\rm fs} \approx 2.9\times 10^{-6} \left(\frac{1+\ln\left(g_{\rm eff}^{1/4} T_{\rm kd}/50\ \mathrm{MeV}\right)/19.1}{\left(m_\chi/100\ \mathrm{GeV}\right)^{1/2} g_{\rm eff}^{1/4} \left(T_{\rm kd}/50\ \mathrm{MeV}\right)^{1/2}} \right)^3 M_\odot\,. \quad (129)$$

Acoustic oscillations in the WIMP component lead to a similar exponential cutoff in the power spectrum and the characteristic damping mass in this case is

$$M_{\rm ao} = \frac{4\pi}{3}\,\frac{\rho_\chi}{H^3}\Big|_{T=T_{\rm kd}} \approx 3.4\times 10^{-6} \left(\frac{T_{\rm kd}\, g_{\rm eff}^{1/4}}{50\,\mathrm{MeV}}\right)^{-3} M_\odot\,. \quad (130)$$

Which of these two damping mechanisms is more efficient depends on the specific combination of m_χ and $T_{\rm kd}$. The actual cutoff in the power spectrum is thus given by

$$M_{\rm cut} = \max\left[M_{\rm fs}, M_{\rm ao}\right]\,, \quad (131)$$

corresponding to the mass of the smallest gravitationally bound objects that will form much later, during the matter-dominated era, when the primordial density fluctuations enter the non-linear regime. For the neutralino, the mass of these smallest protohalos can be anything between 10^{-11} and almost 10^{-3} solar masses, depending on the supersymmetric model parameters. The lightest Kaluza-Klein particle (LKP) in theories with universal extra dimensions is another DM candidate which we will discuss later; in this case, the range of the smallest protohalo masses would be rather small, with $M_{\rm cut} \sim 10^{-6} M_\odot$. Direct probes of $M_{\rm cut}$ would thus provide very interesting information about the particle nature of dark matter.

Even though the smallest clumps should in principle appear as gamma-ray point sources (see Section 10), however, these are unlikely to be resolved. It may therefore be much more promising to focus on anisotropy probes like the one-point probability function of the diffuse gamma-ray flux. Gravitational lensing seems to be extremely challenging due to the small size of these objects and the fact that their virial radius is much larger than the Einstein radius. However, it has been proposed that multiple images of time-varying sources in strong lensing systems may probe also very small objects. Even sub-solar objects could create observable strong gravitational lensing events and be used to detect small subhalos close to the cutoff mass.

For all these considerations, one should keep in mind that numerical N-body simulations presently cannot resolve the small scales we are interested in here, so relatively little is known about what happens to the smallest protohalos during structure formation and until today. The most likely scenario, however, seems to be that even though they lose some of their material due to tidal interactions in merger processes or encounters with stars, most of the mass resides in a dense and compact central core that remains intact.

7.5 Hot thermal relics

Before moving on to the discussion of possible WIMP candidates, let us briefly look at the opposite extreme of thermal production in the early universe, where the relics leave thermal equilibrium while still highly relativistic. In this case, their number density at and before freeze-out is given by

$$n_\chi = g_\chi \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{E/T} \pm 1} = \frac{\zeta(3)}{\pi^2} g_\chi T^3 \times \left\{ \begin{array}{ll} 1 & \text{for Bosons } (-) \\ \frac{3}{4} & \text{for Fermions } (+) \end{array} \right. , \quad (132)$$

where the Riemann zeta function, $\zeta(x) \equiv \sum_{k=1}^{\infty} k^{-x}$, evaluates to $\zeta(3) \approx 1.2021$. After decoupling, the number density dilutes with the expanding space as

$$n_\chi = \frac{a_{\rm cd}^3}{a^3} \, n_\chi|_{T_{\rm cd}} = \frac{\zeta(3)}{\pi^2} g_\chi \frac{g_{\rm eff}^s(T_{\rm cd})}{g_{\rm eff}^s(T_\gamma)} T_\gamma^3 \times \left\{ \begin{array}{ll} 1 & \text{for Bosons } (-) \\ \frac{3}{4} & \text{for Fermions } (+) \end{array} \right. , \quad (133)$$

where the last step follows from the conservation of entropy, Eq. (15), and T_γ now refers to the temperature of the photons; the decoupling temperature can as usual be estimated by comparing the annihilation rate Γ with the Hubble expansion rate H. As long as the relics are relativistic, they still keep their thermal distribution. Their temperature, $T_\chi = T_\gamma (g_{\rm eff}^s(T_{\rm cd})/g_{\rm eff}^s(T_\gamma))^{1/3}$, is lower than that of the photons because the latter are heated whenever a particle falls out of thermal equilibrium. Once the relic becomes non-relativistic, its energy density becomes $\rho_\chi = m_\chi n_\chi$, with n_χ given by the above equation.

Let us now discuss in some more detail the situation for standard model neutrinos. Those decouple around 4 MeV, a temperature at which the photons are still in equilibrium with positrons and electrons, such that

$$g^s_{\text{eff}}(T_{\text{cd}}) = 2 + 2 \cdot 2 \cdot \frac{7}{8} = \frac{11}{2}\,. \tag{134}$$

Only slightly later, around $T \sim 1\,\text{MeV}$, positrons and electrons annihilate such that the heat bath only contains the 2 photon degrees of freedom. This heats the photons with respect to the neutrinos such that

$$T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \approx 0.714\, T_\gamma\,. \tag{135}$$

From the observation of neutrino oscillations, $\Delta m^2_{\text{solar}} \simeq 7.5 \cdot 10^{-5}\,\text{eV}^2$ and $\Delta m^2_{\text{atmos}} \simeq 2.3 \cdot 10^{-3}\,\text{eV}^2$, we expect that $m_\nu \gg T_0 = 2.3 \cdot 10^{-4}\,\text{eV}$. The neutrinos should thus be non-relativistic today and therefore contribute to the total density as

$$\Omega_\nu = \frac{\sum_i m_{\nu_i}}{\rho_c} \times \frac{\zeta(3)}{\pi^2} 2 \frac{4}{11} \frac{3}{4} T_0^3 = \frac{\sum_i m_{\nu_i}}{93\,\text{eV}\,h^2}\,. \tag{136}$$

A very conservative bound on neutrino masses from cosmology can thus be deduced by requiring that the neutrinos do not contribute more than *all* of the observed dark matter, $\Omega_\nu < \Omega_\chi$, leading to $\sum_i m_{\nu_i} \lesssim 11$ eV. However, neutrinos would constitute *hot* dark matter, which is ruled out due to the associated large free-streaming effects. Still, neutrinos may contribute some small fraction to the total dark matter density. Taking constraints from CMB and large-scale structure observations into account, the combined cosmological bound on the sum of neutrino masses is then roughly given by $\sum_i m_{\nu_i} \lesssim 0.3$ eV. Oscillation data, on the other hand, imply a *lower* and therefore complementary bound of $\sum_i m_{\nu_i} \gtrsim 0.06$ eV. Standard model neutrinos thus contribute at the per cent-level to the total observed dark matter density.

8 WIMP candidates

Supersymmetry provides the most discussed, and arguably best motivated, WIMP candidate(s). Due to popular demand, however, I will first address candidates that appear in extra-dimensional scenarios.

8.1 Kaluza-Klein dark matter

At the beginning of the 20th century Nordström, and in particular Kaluza and Klein (KK) realized that there may exist more than the commonly conceived three spatial dimensions – as long as they are compactified on such a small scale that we have not yet been able to resolve them: at large distances, we should thus recover the familiar low-energy physics we are used to. The kinetic energy connected to the movement of a particle along the extra-dimensional directions would then appear as an additional mass contribution to the 4D observer. In a 5D space time, e.g., Lorenz invariance implies $m^2 = p^2 = p_\mu p^\mu - p_5^2$; after the re-arrangement $m_{\rm KK} \equiv m^2 + p_5^2 = p_\mu p^\mu$, this looks exactly like the corresponding relation in 4D, albeit with a different mass.

In order to make this statement more precise, let us consider the case of a 5D *scalar* field $\Phi(x^\mu, y)$, where y denotes the coordinate in the extra dimension. We assume compactification on a circle with radius R, i.e. we identify $y \sim y + 2\pi R$. This implies that we can write the y-dependence of Φ as a Fourier series,

$$\Phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}}\phi_{(0)}(x^\mu) + \frac{1}{2\sqrt{\pi R}}\sum_{|n|=1}^{\infty}\phi_{(n)}(x^\mu)e^{i\frac{n}{R}y}\,, \tag{137}$$

where $\Phi = \Phi^*$ implies $\phi_{(n)} = \phi^*_{(-n)}$. With $M \equiv (\mu, y)$, the kinetic part of the action for Φ then takes the form

$$\begin{aligned} S &= \int d^5x \left[(\partial_M\Phi)\left(\partial^M\Phi\right) - m^2\Phi^2\right] &(138)\\ &= \int d^4x \int_0^{2\pi R} dy \left[(\partial_\mu\Phi)(\partial^\mu\Phi) - (\partial_y\Phi)^2 - m^2\Phi^2\right] &(139)\\ &= \sum_{n=0}^{\infty}\int d^4x \left[\left(\partial_\mu\phi_{(n)}\right)\left(\partial^\mu\phi^*_{(n)}\right) - \left(m^2 + \frac{n^2}{R^2}\right)\left|\phi_{(n)}\right|^2\right]\,. &(140)\end{aligned}$$

From the 4D point of view, the scalar field is thus accompanied by an infinite *KK tower* of states with masses $m^2_{(n)} \equiv m^2 + n^2/R^2$.

For (gauged) *vector* fields, the situation is slightly more complicated. Starting with a massless 5D vector field, the full spectrum in the 4D theory is given by

$$A^M \longrightarrow \begin{cases} A^\mu_{(0)} & \text{(one massless 4D vector)} \\ A^5_{(0)} & \text{(one massless 4D scalar)} \\ A^\mu_{(n\geq 1)} & \text{(tower of massive 4D vectors with } m = n/R)\,. \end{cases} \tag{141}$$

Note that there are no massive scalar modes $A^5_{(n)}$; these play the role of Goldstone bosons that give a mass to the vector modes and get "eaten" in the process. To see this explicitly, one can integrate out the extra dimension in the 5D action in the same way as we did for the scalar field; one finds that the heavy scalar modes enter in the 4D action only in the combination $\left(\partial_\mu B^5_{(n)} + \frac{n}{R} B^\mu_{(n)}\right)$ – which means that they can be eliminated by a suitable gauge transformation of the $B^\mu_{(n)}$. Yet another way of understanding this result is to count the number of degrees of freedom: a massless 5D vector has 3 – which is the same as for massive 4D vectors; massless 4D vectors, on the other hand, have only 2 and thus need an additional scalar degree of freedom.

The above result, however, poses a problem: a massless scalar mode in the 4D theory, which in the case of gauge fields would couple with full gauge strength, is not observed and strongly constrained by observations. A possible solution is to compactify the extra dimension not on a circle $\mathbb{S}^1$ but on an *orbifold* $\mathbb{S}^1/\mathbb{Z}_2$ – i.e. to introduce an additional mirror symmetry between points that are mapped onto each other under the orbifold projection $y \to -y$. With $y \sim 2\pi R - y$, compactification thus effectively takes place on a line segment $[0, \pi R]$. Under such an orbifold projection $P_{\mathbb{Z}_2}$ any field ϕ transforms even, $P_{\mathbb{Z}_2}\phi(x^\mu, y) = \phi(x^\mu, -y)$, or odd, $P_{\mathbb{Z}_2}\phi(x^\mu, y) = -\phi(x^\mu, -y)$. Obviously, odd fields do not have zero modes, and in that way the above mentioned problem can be avoided by assigning suitable transformation properties under $P_{\mathbb{Z}_2}$. As a matter of fact, it is straightforward to show that gauge invariance already implies that A^5 has to transform odd if A^μ transforms even (which is a necessary property if one wants to recover the gauge field observed in 4D).[14]

In passing, let us note that this also halves the degrees of freedom for KK modes with $|n| \geq 1$ in Eq. (137), which now reads either of the following:

$$\Phi_{\text{even}}(x^\mu, y) = \frac{1}{\sqrt{2\pi R}}\phi_{(0)}(x^\mu) + \frac{1}{\sqrt{\pi R}} \sum_{|n|=1}^{\infty} \phi_{(n)}(x^\mu) \cos\frac{ny}{R} \tag{142}$$

$$\Phi_{\text{odd}}(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{|n|=1}^{\infty} \phi_{(n)}(x^\mu) \sin\frac{ny}{R}\,, \tag{143}$$

with *all* $\phi_{(n)}$ being real. Eq. (140) is unchanged, except for the fact that the sum does not include $n = 0$ for an odd field.

[14]Consider a gauge transformation $A_M(x^\mu, y) \to A_M(x^\mu, y) + D_M\theta(x^\mu, y)$. As discussed above, the 4D part of the vector should transform even under the orbifold projection, i.e. $A_\nu(x^\mu, y) \sim A_\nu(x^\mu, -y)$, which in turn implies that θ must transform even as well, $\theta(x^\mu, y) \sim \theta(x^\mu, -y)$. This, however, implies that $\partial_y\theta$, and thus A_5, must transform odd.