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reading:

- H. Baer, E. -K. Park, X. Tata, "Collider, direct and indirect detection of supersymmetric dark matter," New J. Phys. **11**, 105024 (2009). [arXiv:0903.0555 [hep-ph]].

Kaluza-Klein Dark Matter *(cont.)*

The orbifold compactification also solves the problem that there are no chiral *fermions* in 5D (γ^5 is part of the Dirac algebra in that case, which means that Lorenz transformations mix between "left-handed" and "right-handed" states).[15] However, one may still decompose a 5D fermion as $f = \Pi_R f + \Pi_L f$, with the usual definition $\Pi_{R,L} = \frac{1}{2}(1 \pm \gamma^5)$, and note that the two parts do not mix if one is restricted to 4D Lorenz transformations. For every 5D fermion, there are thus *two* 4D chiral fermions – or, rather, two infinite KK towers of 4D chiral fermions. Therefore, one may assign different $P_{\mathbb{Z}_2}$ transformation properties to these states and thereby recover the SM situation at the zero mode level, where one has singlets ψ_s and doublets ψ_d of definite chirality:

$$\psi_d = \frac{1}{\sqrt{2\pi R}}\psi_{d_L}^{(0)} + \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\left(\psi_{d_L}^{(n)}\cos\frac{ny}{R} + \psi_{d_R}^{(n)}\sin\frac{ny}{R}\right), \qquad (144)$$

$$\psi_s = \frac{1}{\sqrt{2\pi R}}\psi_{s_R}^{(0)} + \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\left(\psi_{s_R}^{(n)}\cos\frac{ny}{R} + \psi_{s_L}^{(n)}\sin\frac{ny}{R}\right), \qquad (145)$$

where $f_{R,L}^{(n)} \equiv (\Pi_{R,L} f)^{(n)}$.

In general, 5D momentum conservation implies the conservation of the KK number n in 4D (when considering possible interaction terms) – but since the orbifold boundaries break translational invariance, one can no longer expect this to be the case. If the physics at the orbifold fixpoints is the same, however, there is still a $\mathbb{Z}_2$ mirror symmetry $y \leftrightarrow \pi R - y$ (which corresponds to translations by πR on $\mathbb{S}^1$, i.e. a remnant of the full translational invariance); the corresponding conserved quantity is *KK-parity* $(-1)^n$.[16] The lightest KK particle (LKP) is thus stable and provides in principle an interesting dark matter candidate.

[15] Recall that spinors transform as $\psi(x) \to \psi'(x') = S(\Lambda)\psi(x)$, where $S(\Lambda) = \mathbf{1} + \frac{1}{8}\Delta\omega_{\mu\nu}[\gamma^\mu, \gamma^\nu]$ for an infinitesimal Lorentz transformation $\Lambda^\mu_\nu = \delta^\mu_\nu + \Delta\omega^\mu_\nu$. In 4D, one thus has $\{S, \Pi_{R,L}\} = 0$, while this is no longer true in 5D.

[16] One may easily check this by observing that all fields can be expanded in terms of $\cos(ny/R)$ or $\sin(ny/R)$; in both cases, the replacement $y \to y - \pi R$ induces a total factor of $(-1)^n$ without changing the argument of these functions. In interactions, there appears the product of several of such factors – which has to equal unity since the theory should be invariant under these transformations, i.e. $(-1)^{\sum_i n_i} = 1$. For any j, it thus follows that $(-1)^{n_j} = (-1)^{\sum_{i\neq j} n_i}$, i.e. $(-1)^{n_j}$ is conserved.

Starting from massless 5D fields, all lowest-level KK modes would have the same mass $1/R$. In order to determine the LKP, one thus has to take into account both generic mass terms (generated, e.g., through electroweak symmetry breaking) as well as radiative corrections:

$$m_{(1)}^2 = R^{-2}\left[1 + R^2 m_{\rm EW}^2 + R^2 \delta m_{\rm rad}^2\right] \quad \text{(for Bosons)} \tag{146}$$

$$m_{(1)} = R^{-1}\left[1 + R\, m_{\rm EW} + R\, \delta m_{\rm rad}\right] \quad \text{(for Fermions)}\,. \tag{147}$$

Assuming $R^{-1} \sim 1\,\mathrm{TeV}$, the second term would be of the order of 10^{-1} for the top quark, but as small as 10^{-6} for electrons. The last term, on the other hand, is generically expected to be of the order of the respective gauge couplings α – which illustrates the importance of properly taking into account radiative corrections for the determination of the LKP. Some model-dependence is introduced here since these corrections depend on the choice of counterterms at the orbifold fixpoints that are necessary to renormalize otherwise infinite contributions.

With all these preparations, we can now introduce the model of *universal extra dimensions* (UED) which is essentially the standard model in 5D, compactified on an $\mathbb{S}^1/\mathbb{Z}_2$ orbifold. Up to the first KK level, the spectrum of states looks very similar to the case of supersymmetry: one heavy partner (of the same spin, however!) for each standard model particle – two, in fact, for the fermions – and five physical Higgs states (three of which arise as linear combinations from KK states of the Goldstone modes associated to the standard Higgs doublet and the Goldstone-like $W^5_{(1)}$ modes of the $SU(2)$ gauge fields). Note that the standard model in 5D is not renormalizable and can thus only be treated as an effective 'low'-energy theory up to some cutoff scale Λ (which is usually taken to be $\Lambda R \sim 20-40$ in order to ensure that perturbation theory still works). In the *minimal* version of this model (mUED), one assumes that the above mentioned boundary terms give negligible contributions to the KK masses at energies close to Λ – which means that the mUED model can be fully described by only two new parameters, the compactification scale R and the cutoff scale Λ. The stability of the electroweak vacuum, in fact, requires $\Lambda \sim 5$ in this case such that the only free parameter is given by the compactification scale.

A full calculation of the mUED mass spectrum shows that the LKP is given by the first KK excitation of the photon, which is well approximated by the first KK excitation $B^{(1)}$ of the weak hypercharge gauge boson because the Weinberg angle at $n > 0$ is very effectively driven to zero. For $R^{-1} \gtrsim 800\,\mathrm{GeV}$, radiative corrections actually lead to $m_{B^{(1)}} < R^{-1}$ so the $B^{(1)}$ is also lighter than the KK graviton (for which $m_{G^{(1)}} \simeq R^{-1}$). The $B^{(1)}$ thus satisfies all requirements for a nice WIMP dark matter candidate; it acquires the correct relic density for compactification scales of $R^{-1} \sim 1.3\,\text{TeV}$. In arriving at this result, it turns out that co-annihilations with KK-leptons are important, in particular the s-channel mediated process $B^{(1)}e^{(1)} \to e^{(2)}\gamma$. The $B^{(1)}$ itself annihilates dominantly into leptons ($\sim 60\,\%$) and quarks ($\sim 35\,\%$), which is quite different from the neutralino case.

In a phenomenological approach, one may also consider non-minimal versions of the UED model and take the radiative mass corrections of the KK particles to be more or less free parameters. Since all KK modes are rather close in mass anyway, co-annihilations are generically important and this procedure can significantly change the relic density. What is more, one can also tune the masses such that the LKP is no longer the $B^{(1)}$ but the $H^{(1)}$ or $Z^{(1)}$ (the $\nu^{(1)}$ is excluded for the same reason as the $\tilde{\nu}$ – but introducing right-handed neutrinos with a Dirac mass term provides a work-around in exactly the same way). Finally, people have investigated the option to consider more than one UED. This brings about new WIMP candidates, the 'spinless KK photon or Z boson', which are linear combinations of the extra-dimensional vector field components (a larger number of extra dimensions, however, also makes the problem of non-renormalizability more severe, which implies that one has to lower the cutoff-scale considerably).

8.2 SUSY candidates

The basic idea of supersymmetry (SUSY) is to extend the standard model such that every bosonic degree of freedom gets a new fermionic degree of freedom and vice versa. A full review of supersymmetry is beyond the scope of this lecture, but let us mention that in the minimal supersymmetric standard model (MSSM), this is done with the minimal possible field content (that takes into account all Yukawa couplings). This means that every SM gauge boson gets a spin $\frac{1}{2}$ 'gaugino' $\tilde{\lambda}$ as a partner (note that these need to be Majorana particles in order to keep the number of degrees of freedom for each state equal to two) and there is one scalar partner for each SM fermion helicity state – right- and left-handed 'sfermions' $\tilde{f}_R$ and $\tilde{f}_L$. Two Higgs doublets[17] are needed in order to cancel triangle anomalies; 3 degrees of freedom are Goldstone modes that give masses to the gauge bosons just as in the SM while the remaining 5 are physical degrees of freedom and given by the standard Higgs boson h, a heavy version H, a pseudoscalar A and

[17]Both Higgs fields acquire vacuum expectation values, $\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$ and $\langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$. The relation to the gauge boson mass is determined by $m_W^2 = \frac{1}{2}g^2(v_1^2 + v_2^2)$ and the ratio of the vacuum expectation values is usually denoted by $\tan\beta \equiv \frac{v_2}{v_1}$ (note that the SM Higgs boson h is contained in H_2).

two charged Higgs fields $H^{\pm}$.

The Lagrangian for this theory (conveniently derived in the superfield formalism which, however, is also beyond the scope of this lecture) shows a perfect symmetry between SM and SUSY fields; in particular, all couplings, masses and other quantum numbers in the SUSY sector are fixed and (except for the spin) the same as for the corresponding SM partners. Achievements of the MSSM include, in particular, a solution to the fine-tuning problem (since all radiative contributions to the Higgs mass that are quadratic in the cutoff scale cancel) and a unification of all coupling constants at the scale of grand unified theories (GUT), i.e. at around 10^{16} GeV; SUSY also helps to bring electroweak precision measurements in even better agreement with theoretical expectations.

In general, one would expect interaction terms in the MSSM that generically lead to a rather fast decay of the proton. In order to avoid such a dangerous behavior, one often postulates the conservation of *R-parity* which is defined as $R \equiv (-1)^{3B+L+2s}$; it is positive for all SM particles and negative for all SUSY particles. An important consequence of the conservation of R-parity, to which we will return shortly, is that the lightest supersymmetric particle (LSP) is stable.

Obviously, the world is not perfectly supersymmetric so SUSY has to be broken. Usually, one only considers *soft SUSY* breaking in order not to spoil the successes of the MSSM mentioned above. This means that one only allows for SUSY-breaking terms in the Lagrangian that preserve the separate conservation of baryon B and lepton number L, and do not break the SM gauge symmetries nor give rise to quadratic divergences. The most general form of $\mathcal{L}_{\rm soft}$ that satisfies all these criteria can then be written as

$$\begin{aligned}\mathcal{L}_{\rm soft} \;=\; & \underbrace{-\frac{1}{2}M^a\tilde{\lambda}^a\tilde{\lambda}^a}_{\text{gaugino masses}} + \underbrace{\tilde{\mathbf{q}}_L^{i^*}\mathbf{M}^2_{q_L}\tilde{\mathbf{q}}^i_L + \left\{\tilde{\mathbf{q}}^i_L \to \tilde{\mathbf{l}}^i_L, \tilde{\mathbf{u}}_R, \tilde{\mathbf{d}}_R, \tilde{\mathbf{e}}_R\right\}}_{\text{squark and slepton masses}} \\ & + \underbrace{m_1^2 H_1^{i^*}H_1^i + m_2^2 H_2^{i^*}H_2^i - \epsilon_{ij}B\mu H_1 H_2^j + h.c.}_{\text{Higgs masses}} \\ & + \underbrace{\epsilon_{ij}\left(\tilde{\mathbf{e}}_R^{i^*}\mathbf{A}_E\mathbf{Y}_E\tilde{\mathbf{l}}^i_L H_1^i + \tilde{\mathbf{d}}_R^{i^*}\mathbf{A}_D\mathbf{Y}_D\tilde{\mathbf{q}}^i_L H_1^i - \tilde{\mathbf{u}}_R^{i^*}\mathbf{A}_U\mathbf{Y}_U\tilde{\mathbf{q}}^i_L H_2^i\right)}_{\text{'trilinear terms'} \sim A\tilde{f}_R\tilde{f}_L H} .\end{aligned} \tag{148}$$

Here, the soft trilinear couplings $\mathbf{A}$, the Yukawa couplings $\mathbf{Y}$ and the soft sfermion masses $\mathbf{M}$ are 3×3 matrices in generation space. In full generality, $\mathcal{L}_{\rm soft}$ contains 105 free parameters. Together with 18 SM parameters and the μ parameter (which appears as an analogue to the Higgs mass in the superpotential), this theory is therefore sometimes referred to as 'MSSM-124'.

There are several simplifying assumptions that are often made about the form of $\mathcal{L}_{\rm soft}$ (which eventually of course should follow from the so far unknown mechanism of soft SUSY breaking). The one that is most often adopted is the so-call *GUT condition* where one demands that all gauge couplings unify at the GUT scale and that all gaugino masses are the same at this scale. At low energies, this implies

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2}\,, \tag{149}$$

where the subscripts refer to the $U(1)$, $SU(2)$ and $SU(3)$ gauge groups, respectively. Numerically, the above relation implies $M_1 \approx 0.5M_2$ and $M_2 \approx 0.3M_3$.

In purely phenomenological (or bottom-up) approaches, one makes a number of simplifying assumptions that lead to various versions of phenomenological MSSMs (pMSSMs) that are defined at the electroweak scale, i.e. directly at the scale where observations should be affected. One example is the MSSM-7 where one chooses

$$\mathbf{A}_U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_t \end{pmatrix}, \quad \mathbf{A}_D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_b \end{pmatrix}, \quad \mathbf{A}_E = \mathbf{0} \tag{150}$$

and $\mathbf{M}_{q_L} = \mathbf{M}_{l_L} = \mathbf{M}_u = \mathbf{M}_U = \mathbf{M}_D = \mathbf{M}_E = m_0\mathbf{1}$. An obviously appealing aspect is the universality, but this ansatz automatically also guarantees the separate conservation of all lepton numbers (L_e, L_μ and L_τ) as well as the non-existence of both CP-violations (beyond those of the SM) and flavor changing neutral currents (since all matrices are diagonal). In general, in fact, all these points are of considerable concern since there are tight observational bounds that, consequently, exclude large parts of the full MSSM-124. In the MSSM-7, the free parameters that are specified at the electroweak energy scale are usually μ, M_2, $\tan\beta$, m_A, m_0, A_b and A_t.

Top-down approaches, on the other hand, make a number of simplifying assumptions at the GUT scale; by solving the renormalization group equations, one can then obtain the values for masses and couplings at the (observable) low-energy scale. Often, these approaches are motivated by concrete mechanisms for SUSY breaking. In the *constrained MSSM* (cMSSM), e.g., one adopts the GUT condition and specifies a common gaugino mass $m_{1/2}$, a universal scalar mass m_0 (for both sfermions and the Higgs bosons) and a single trilinear coupling $A_0\mathbf{1} = \mathbf{A}_U = \mathbf{A}_D = \mathbf{A}_E$. In order for electroweak symmetry breaking to be successful (i.e. for the Higgs potential to develop a minimum), one needs to specify also $\tan\beta$ and the sign of μ. In total, these are 4 parameters + one sign at the GUT scale that define the cMSSM. While the cMSSM is sometimes also referred to as mSUGRA, models of *minimal supergravity* are strictly speaking even more constrained by the relation $B = A_0 - m_0$ (which is why B instead of A_0 is often used to specify the model) and the fact that the gravitino (which is completely unconstrained in the cMSSM) has the same mass as the scalars, $m_{3/2} = m_0$.

8.2.1 Neutralino DM

In the MSSM, there appear 4 Majorana interaction eigenstates – two neutral gauginos $\tilde{B}$ and $\tilde{W}_3$ as well as two Higgsinos $\tilde{H}_1^0$ and $\tilde{H}_2^0$. The mass term for these states as it appears in the Lagrangian is given by

$$\left(\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0\right) \begin{pmatrix} M_1 & 0 & -\frac{g' v_1}{\sqrt{2}} & \frac{g' v_2}{\sqrt{2}} \\ 0 & M_2 & \frac{g v_1}{\sqrt{2}} & \frac{g v_2}{\sqrt{2}} \\ -\frac{g' v_1}{\sqrt{2}} & \frac{g v_1}{\sqrt{2}} & 0 & -\mu \\ \frac{g' v_2}{\sqrt{2}} & \frac{g v_2}{\sqrt{2}} & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{W}_3 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix} . \qquad (151)$$

The corresponding mass eigenstates are obtained by diagonalizing the above expression and called *neutralinos*:

$$\tilde{\chi}_i^0 = N_{i1}\tilde{B} + N_{i2}\tilde{W}_3 + N_{i3}\tilde{H}_1^0 + N_{i4}\tilde{H}_2^0 \, . \qquad (152)$$

The lightest of these states, often simply referred to as *the* neutralino $\chi \equiv \tilde{\chi}_1^0$, makes an excellent WIMP DM candidate if it is the LKP. For phenomenological discussions, one is often interested in its *Gaugino fraction* $Z_g \equiv |N_{11}|^2 + |N_{12}|^2$ and *Higgsino fraction* $Z_h \equiv |N_{13}|^2 + |N_{14}|^2 = 1 - Z_g$. Note that if the GUT condition is employed, it follows from Eqs. (149) and (151) that a gaugino fraction close to 1 means that the neutralino is essentially a Bino.

In the cMSSM/mSUGRA, the neutralino is the LSP in a large part of parameter space: only for $m_{1/2} \gg m_0$ the LSP is charged, typically the $\tilde{\tau}$, and thus not a suitable DM candidate; the region for very large $m_0 \gg m_{1/2}$, on the other hand, is excluded because electroweak symmetry breaking does not take place. One can single out four regions in the $m_0 - m_{1/2}$ plane where the neutralino acquires the correct relic density to account for all dark matter observed today:

- The *bulk region* at relatively low values of both m_0 and $m_{1/2}$ where the annihilation into leptons – mediated by relatively light sleptons in the t-channel – are most important to set the relic density for neutralino masses of $m_\chi \sim 100\,\text{GeV}$.[18] This region is by now completely excluded by the LHC. Higher values of m_0 and/or $m_{1/2}$, on the other hand, generically imply smaller annihilation cross sections – so in order not to overproduce dark matter, the other cMSSM regions are defined by three possible enhancement mechanisms for the effective annihilation rate:
- The *co-annihilation region* corresponds to a thin strip at large $m_{1/2}$, close to the region where the LSP is charged. Here, the effective annihilation rate is enhanced because the neutralino is even closer in mass with the $\tilde{\tau}$ (or, at very large values of $m_{1/2}$ and only if $|A_0| \gg 0$, the $\tilde{t}$) and co-annihilations become important. Correspondingly, the mass of the neutralino can be pushed up to several 100 GeV (for co-annihilations with staus) or even beyond 1 TeV (for co-annihilations with stops).

[18]Note that today, at very small relative velocities, the annihilation of neutralinos into light fermions is strongly helicity suppressed by a factor of m_f^2/m_χ^2. This is because the Pauli principle in this limit enforces the system of two incoming neutralinos to be a Spin-0 state. The outgoing fermion and antifermion, on the other hand, have opposite helicities in the limit of $m_f \to 0$ and thus form a Spin-1 system.

- In the *funnel region* that appears, only for large $\tan\beta$, at large values of m_0 *and* $m_{1/2}$, the mass of the neutralino is tuned such that resonant annihilation through an s-channel pseudoscalar Higgs boson is the main process to set the relic density, i.e. $s(T \simeq T_{\rm cd}) \simeq 4M_\chi^2(1+v_{\rm cd}^2) \simeq m_A^2$. The mass of the neutralino is a few hundred GeV in this case.
- The *focus point* or *hyperbolic branch region* is characterized by a thin strip close to the region where electroweak symmetry breaking does no longer occur, i.e. with large values of m_0. This is the only region where the neutralino is not a more or less pure Bino but acquires a considerable Higgsino fraction – which allows for the very efficient annihilation into $SU(2)$ gauge bosons. For $Z_g \ll 1$, the neutralino is rather heavy and almost degenerate in mass with charginos, $m_\chi^+ \simeq m_\chi \gtrsim 1\,\text{TeV}$; as expected, annihilation is mostly to Z and $W^\pm$ bosons (through t-channel exchange of charginos). The latter two regions do not exist in mSUGRA models.

Various mechanisms have been proposed for the breaking of supersymmetry. For our purpose the most interesting aspect is whether the gravitino (to be discussed later) is even lighter than the neutralino and thus provides the true LSP. This is the case in gauge-mediated SUSY breaking, where the GUT condition is automatically satisfied and the gravitino gets a mass of the order of eV (though it can be as heavy as 1 GeV). In gravity-mediated SUSY breaking models, on the other hand, also the gravitino acquires a mass at the electroweak scale and it is much easier to have a neutralino LSP. Finally, one should also mention so-called anomaly mediation models (AMSB), where the gaugino masses vanish at tree level and the induced relation between the masses at the low-energy scale does no longer follow Eq. (149). In particular, one finds $M_1 \simeq 2.8 M_2$ which allows for *Wino* dark matter. The relic density requirement is in that case met for $m_\chi \gtrsim 2\,\text{TeV}$ and the neutralino $\chi \simeq \tilde{W}^0$ is part of an almost mass-degenerate triplet $(\tilde{W}^+, \tilde{W}^0, \tilde{W}^-)$. The gravitino mass in this scenario is $m_{3/2} \gg 1\,\text{TeV}$ and thus does not pose any problems.

8.2.2 (Mixed) sneutrino DM

The only other possible WIMP DM candidate in the MSSM is a sneutrino LSP. The correct relic density is obtained for $600\,\text{GeV} \lesssim m_{\tilde{\nu}} \lesssim 700\,\text{GeV}$, mostly set by Z-boson exchange in the s-channel (in principle, there is another solution with sneutrino masses smaller than a few GeV – but this window is excluded by LEP bounds on the decay width of the Z boson which imply $m_{\tilde{\nu}} \gtrsim 44\,\text{GeV}$). However, the relatively strong coupling to the Z also mediates large interaction rates with nuclei in direct DM searches (through a t-channel Z exchange). The resulting spin-independent scattering cross-section per nucleon is $\sigma_n^{\rm SI} \sim 10^{-3}\,\text{pb}$ – about 4 orders of magnitude larger than current limits on this quantity from the CDMS experiment. The $\tilde{\nu}$ is thus not a suitable candidate to make up the (dominant part of the) observed DM today.

On the other hand, the fact that neutrinos are not massless implies that the standard model needs to be extended and the simplest phenomenological way to do so is by introducing right-handed neutrinos and a Dirac mass term. As a consequence, also the MSSM has to be extended correspondingly, essentially treating (s)neutrinos and (s)leptons on equal footings. For soft SUSY breaking, this means that there appear two new terms in the Lagrangian:

$$\mathcal{L}_{\rm soft} = \mathcal{L}_{\rm soft}^{\rm MSSM} + \tilde{\nu}_R^* \mathbf{M}_{\nu_R}^2 \tilde{\nu}_R - \epsilon_{ij} \tilde{\nu}_R^* \mathbf{A}_{\tilde{\nu}} \mathbf{Y}_{\tilde{\nu}} \tilde{\mathbf{l}}_L^i H_2^i \,. \tag{153}$$

In the $(\tilde{\nu}_L, \tilde{\nu}_R)$ basis, the sneutrino mass matrix for one generation is then given by

$$M_{\tilde{\nu}}^2 = \begin{pmatrix} M_{\tilde{l}}^2 + \frac{1}{2} m_Z^2 \cos\beta & \frac{1}{\sqrt{2}} A_{\tilde{\nu}} v \sin\beta \\ \frac{1}{\sqrt{2}} A_{\tilde{\nu}} v \sin\beta & M_{\tilde{\nu}_R}^2 \end{pmatrix}, \tag{154}$$

where $v \equiv \sqrt{v_1^2 + v_2^2}$ and the Dirac neutrino mass has been neglected; note that large mixings between the neutrino interaction eigenstates are possible for large values of $A_{\tilde{\nu}}$.The mass eigenstates are obtained by diagonalization as

$$\begin{array}{rcl} \tilde{\nu}_1 & = & \cos\theta_{\tilde{\nu}} - \sin\theta_{\tilde{\nu}} \\ \tilde{\nu}_2 & = & \sin\theta_{\tilde{\nu}} + \cos\theta_{\tilde{\nu}} \end{array}, \quad \text{with} \quad \sin 2\theta_{\tilde{\nu}} = \frac{\sqrt{2} A_{\tilde{\nu}} v \sin\beta}{M_{\tilde{\nu}_1}^2 - M_{\tilde{\nu}_2}^2}. \tag{155}$$

We thus have three additional free parameters compared to the MSSM that can be chosen as, e.g., $\theta_{\tilde{\nu}}$, $A_{\tilde{\nu}}$ and $M_{\tilde{\nu}_1}$. In particular, it is possible to choose them in such a way that $\tilde{\nu}_1$ is the LSP, with $m_{\tilde{\nu}_1}$ much smaller than all other sparticle masses. Compared to the MSSM $\tilde{\nu}$, couplings to the Z boson are suppressed by a factor of $\sin^2\theta_{\tilde{\nu}}$ which, at least naively, means that even relatively large mixing angles of $\sin\theta_{\tilde{\nu}} \sim 0.1$ can avoid current direct detection constraints. In fact, even the constraints from non-standard contributions to the width of the Z boson can be greatly mitigated. As a result, the $\tilde{\nu}_1$ can be a viable DM candidate for almost any mass $m_{\tilde{\nu}_1} \gtrsim 10\,\mathrm{GeV}$ (for much of the parameter space, one also has to require $\sin\theta_{\tilde{\nu}} \sim 0.01$).