Magnetic behavior of thin superconducting disks with $B$-dependent $J_c$

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The ac susceptibility, $\chi = \chi' + i\chi''$, of a superconducting thin disk in a perpendicular magnetic field is calculated in the critical-state model assuming a field-dependent critical current, $J_c(B)$. We find both analytically and numerically that the asymptotic behavior at large ac-field amplitudes $B_{am}$ changes from $\chi' \propto B_{am}^{-3/2}$ and $\chi'' \propto B_{am}^{-1}$ for the Bean model, to $\chi' \propto B_{am}^{-3}$ and $\chi'' \propto B_{am}^{-2}$ for $J_c$ decreasing with $|B|$ as $|B|^{-1}$ or faster. In the parametric $\chi''(\chi')$ plot the peak of $\chi''$ increases in magnitude and shifts toward $\chi' = 0$. This allows an easy experimental discrimination between a Bean model behavior, one with $J_c(B)$, and one where flux creep is an ingredient. Account of the $B$-dependence of $J_c$ improves substantially agreement with available experimental $\chi''(\chi')$ data.

During the last years much attention has been paid to the critical state model (CSM) analysis of thin samples in perpendicular magnetic fields. Within the Bean model, the magnetic behaviour of a long thin strip [1] and a thin circular disk [2] has already been thoroughly investigated. However, an analysis of thin samples with a field-dependent critical current, $J_c(B)$, has so far been restricted to calculation of flux distributions and magnetization loops [3,4]. In this work we calculate complex ac susceptibility for a thin disk with $B$-dependent $J_c$. The results are compared with similar results for long samples in a parallel field [5], with results of flux creep simulations [6] and with experimental data [7].

We consider a thin superconducting disk of radius $R$ and thickness $d$, where $d \ll R$. A set of integral equations derived in [4] is solved numerically to obtain the field and current density distributions for a given applied field $B_a$. Repeating this procedure for different $B_a$ we calculate the magnetization hysteresis loops $M(B_a)$. The ac susceptibility $\chi = \chi' + i\chi''$ is then determined by the first coefficient of the Fourier series of $M(t) = M[B_a(t)]$ for oscillating applied field $B_a(t) = B_{am} \cos \omega t$.

The most dramatic change of the $\chi(B_{am})$ behavior due to a $B$-dependence of $J_c$ is found at large $B_{am}$. For the real part, $\chi'$, we find a power-law behavior with different exponents for different $J_c(B)$-dependences ranging from -3/2 (for the Bean model) to -3, see Fig. 1. This asymptotic power-law behavior can also be obtained analytically. We find that if $J_c \propto |B|^{-\alpha}$ at large $|B|$, then

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**Figure 1.** High-field behavior of the real part of the susceptibility normalized to the Meissner state value $\chi_0 = 8R/3\pi d$: (1) the Bean model; (2) $J_c = J_c0/(1 + b^{1/2})$; (3) $J_c = J_c0/(1 + b)$; (4) $J_c = J_c0 e^{-b}$, where $b = 2|B|/\mu_0 J_c0$. “NH Physica C 341–348 (2000) 2045–2046” Elsevier Science B.V. All rights reserved.

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Figure 2. Parametric $\chi''(\chi')$ plot for the Kim model, $J_c = J_{c0}/(1 + 2|B|/3\mu_0 J_{c0})$ (full line) and for the Bean model (dashed line). Symbols show experimental susceptibility data from [7].

$$\chi'' \propto \chi'^{2/3} \begin{cases} B_{am}^{-1-s}, & s < 1 \\ B_{am}^{-2}, & s > 1 \end{cases}$$

(1)

It should be emphasized that this result holds true not only for a thin disk but also for any geometry of the superconductor.

A very convenient way to analyze ac susceptibility data is to consider a $\chi'(\chi'')$ parametric plot, see Fig. 2. For the Bean model the $\chi''$ has a maximum value of 0.24 at $\chi' = -0.38$, while for a $B$-dependent $J_c$ the $\chi''$ peak increases in magnitude and shifts towards $\chi' = 0$. Also, the high-field part of the plot is strongly modified. Meanwhile, the asymptotic behavior at low fields, $\chi''/\chi_0 = 32/15\pi(1 + \chi'/\chi_0)$ (dotted line) remains unchanged for any $J_c(B)$. These results are qualitatively similar to those for long samples in parallel field [5]. It can also be seen from Fig. 2 that the curve calculated for a $B$-dependent $J_c$ provides a much better agreement with available experimental data [7].

It is interesting to compare our $\chi''(\chi')$ plots to the ones obtained by calculations based on a non-linear current-voltage curve, $J \propto E^{1/n}$, $n < \infty$. Shown in Fig. 3 together with the CSM results is a $\chi''(\chi')$-curve (dotted line) drawn in accordance to typical graphs presented in [5]. In contrast to the effect of having a $B$-dependent $J_c$, the $\chi''$ peak shifts here towards $\chi' = -1$ and the slope at $\chi' \to -1$ becomes significantly steeper. Thus, an analysis of the $\chi''(\chi')$ plot allows one to discriminate between a Bean model behavior, one with $J_c(B)$, and one where flux creep is an ingredient.

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