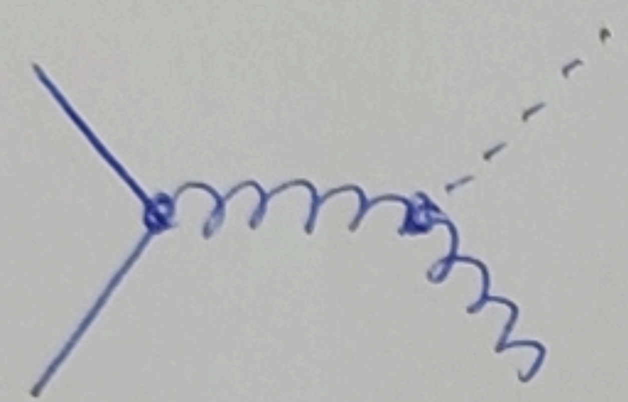
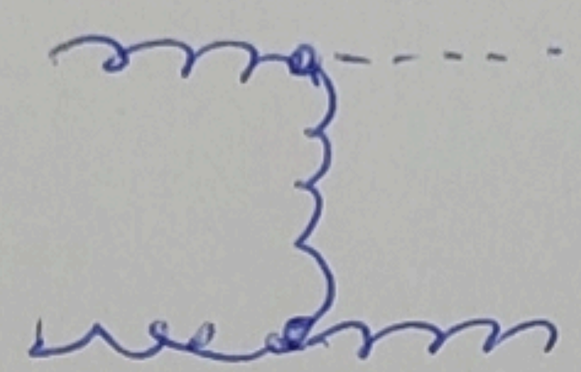
Axion cosmology

Different possible production mechanisms for axions in the early universe:

- thermal production $\mathcal{L} \sim \frac{\alpha_s}{8\pi} \frac{a}{f_a} G\tilde{G}$





$\sigma \sim \frac{\alpha_s^3}{f_a^2}$

thermalisation if $\Gamma \sim n\sigma v \sim H$

$n_q \sim n_g \sim T^3$, $H \sim \frac{T^2}{M_P}$ $\longrightarrow$ $\frac{n\sigma v}{H} \sim \left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)^2 \frac{T}{10^{12}\,\mathrm{GeV}}$

→ only produced if T_R very high
for consistency of this estimate need $f_a > T_R$

→ would correspond to hot DM if major contribution ↯

- misalignment mechanism

consider background field (small mass → large occupation number)

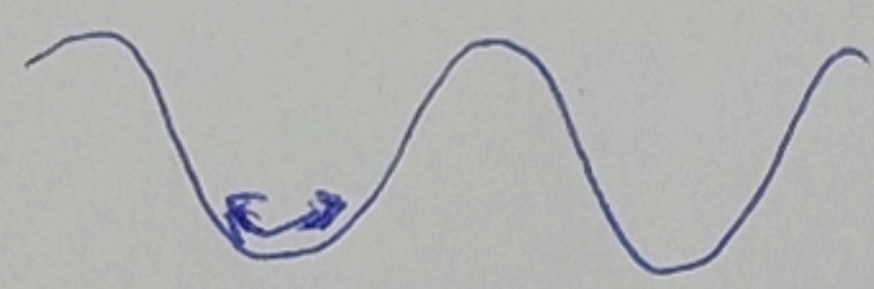
assume homogeneous field $\phi(x,t) = \phi(t)$

(this is the case if PQ is broken before inflation)

$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$ + FRW metric

$$\Rightarrow \ddot{\phi} + 3H\dot{\phi} - \underbrace{a^{-2}\nabla^2\phi}_{\approx 0} + \frac{\partial V}{\partial \phi} = 0$$

$$V \simeq f_a^2\, \tilde{m}_a^2(T)\left[1 - \cos\frac{a}{f_a}\right] \simeq \frac{1}{2}\tilde{m}_a^2(T)\, a^2$$



warm up: <u>static universe</u>

$$\ddot{\phi} + m_a^2\phi = 0 \qquad \text{harmonic oscillator}$$

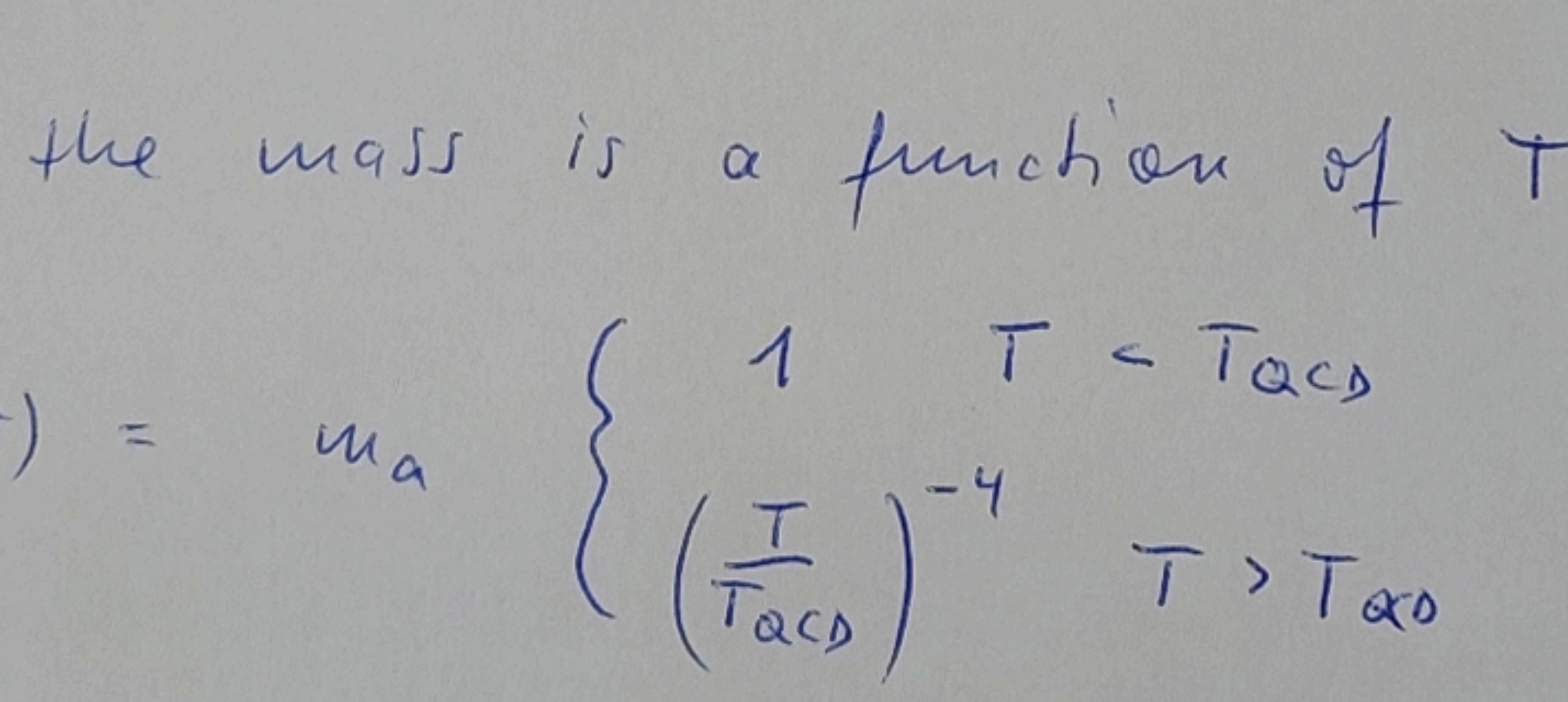
$$\Rightarrow \phi(t) = \phi_0 \cos(mt)$$

<u>Expanding universe</u> → Hubble friction

→ damped harmonic oscillator

also the mass is a function of T

$$\tilde{m}_a(T) = m_a \begin{cases} 1 & T < T_{QCD} \\ \left(\frac{T}{T_{QCD}}\right)^{-4} & T > T_{QCD} \end{cases} \qquad m_a \simeq 6\,\text{meV}\left(\frac{10^9\,\text{GeV}}{f_a}\right)$$



For high T / early times

1) $T > T_c$

$3H > \tilde{m}_a$ overdamped

$\ddot{\phi} + 3H\dot{\phi} \simeq 0 \Rightarrow \dot{\phi} \propto a^{-3} \quad \left(H = \frac{\dot{a}}{a}\right)$

$\Rightarrow \dot{\phi} \simeq 0 \Rightarrow \phi \simeq \phi_0$ field is frozen

2) $T < T_c$

$3H < \tilde{m}_a$ underdamped $\Rightarrow$ oscillations

$$\ddot{\phi} + 3H\dot{\phi} + \tilde{m}(T)\,\phi \simeq 0$$

slow change compared to oscillations in ϕ

$\rightarrow$ WKB ansatz $\quad \phi = \underbrace{A(t)}_{\text{slow}}\, e^{i\overbrace{\theta(t)}^{\text{fast}}}$

Plugging this ansatz into EOM $\rightarrow$ 2 equations

$\underline{\text{Re}}$ $\quad \underbrace{\frac{\ddot{A}}{A} + 3H\frac{\dot{A}}{A}}_{\ll \dot{\theta}^2} - \dot{\theta}^2 + m^2 \simeq 0 \quad \Rightarrow \dot{\theta} = \pm m$

$\theta = \pm \int dt\, m$

$\underline{\text{Im}}$ $\quad \dot{A} + \frac{A}{2}\left(3H + \frac{\ddot{\theta}}{\dot{\theta}}\right) \overset{\dot{\theta} \simeq m}{=} \dot{A} + A\left(\frac{3}{2}\frac{\dot{a}}{a} + \frac{1}{2}\frac{\dot{m}}{m}\right)$

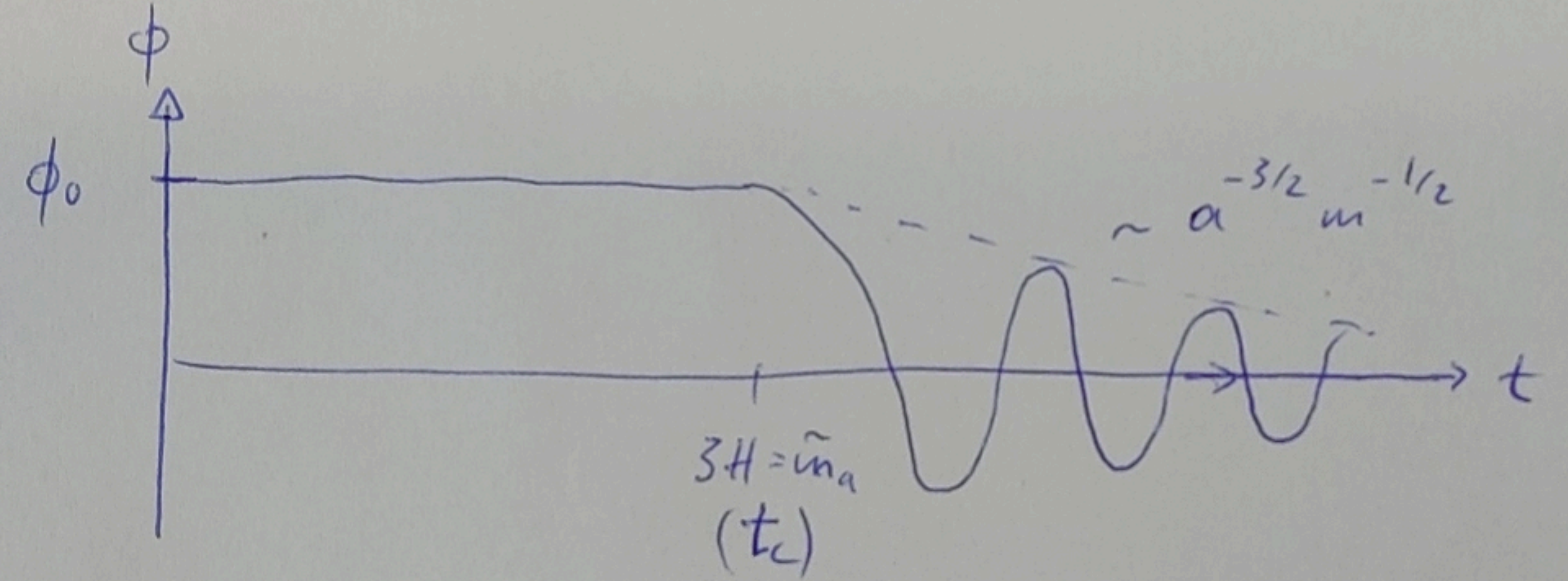
$\Rightarrow A(t) \propto a^{-3/2}\, m^{-1/2}$

Plugging A and θ back into the ansatz and keeping the real part:

$$\phi(t) = \underbrace{C \cdot a(t)^{-3/2}\, m(t)^{-1/2}}_{\text{slowly varying amplitude}} \underbrace{\cos \int_{t_c}^{t} dt'\, \tilde{m}_a(t')}_{\text{oscillating part}}$$

Match amplitude at $t = t_c$

$$\phi(t) = \phi_0 \left(\frac{a(t_c)}{a(t)}\right)^{3/2} \left(\frac{m(t_c)}{m(t)}\right)^{1/2} \cos \int_{t_c}^{t} dt'\, \tilde{m}_a(t')$$



<u>Energy density</u>

$$\rho \sim \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} m^2 \phi^2$$

$T > T_c$ $\quad \rho \overset{\dot{\phi}=0}{=} \frac{1}{2}\tilde{m}_a^2(T)\,\phi_0^2$ $\quad$ not constant because $m(T)$

$$\rho \sim m_a^2(T) \sim \frac{1}{T^8} \sim a^8 \sim t^4 \quad \text{in RD}$$

$$\phi_0 = \theta_i f_a$$

$T < T_c$ Plug in $\phi(t)$

$$\frac{1}{2}\left(m^2\phi^2 + \dot{\phi}^2\right) \simeq \frac{1}{2}\tilde{m}_a^2(t)\,\phi_0^2\left(\frac{a(t_c)}{a(t)}\right)^3 \frac{\tilde{m}_a(t_c)}{\tilde{m}_a(t)}\overbrace{\cos^2\int + \sin^2\int}^{1}$$

only ∂_t wrt oscillating part

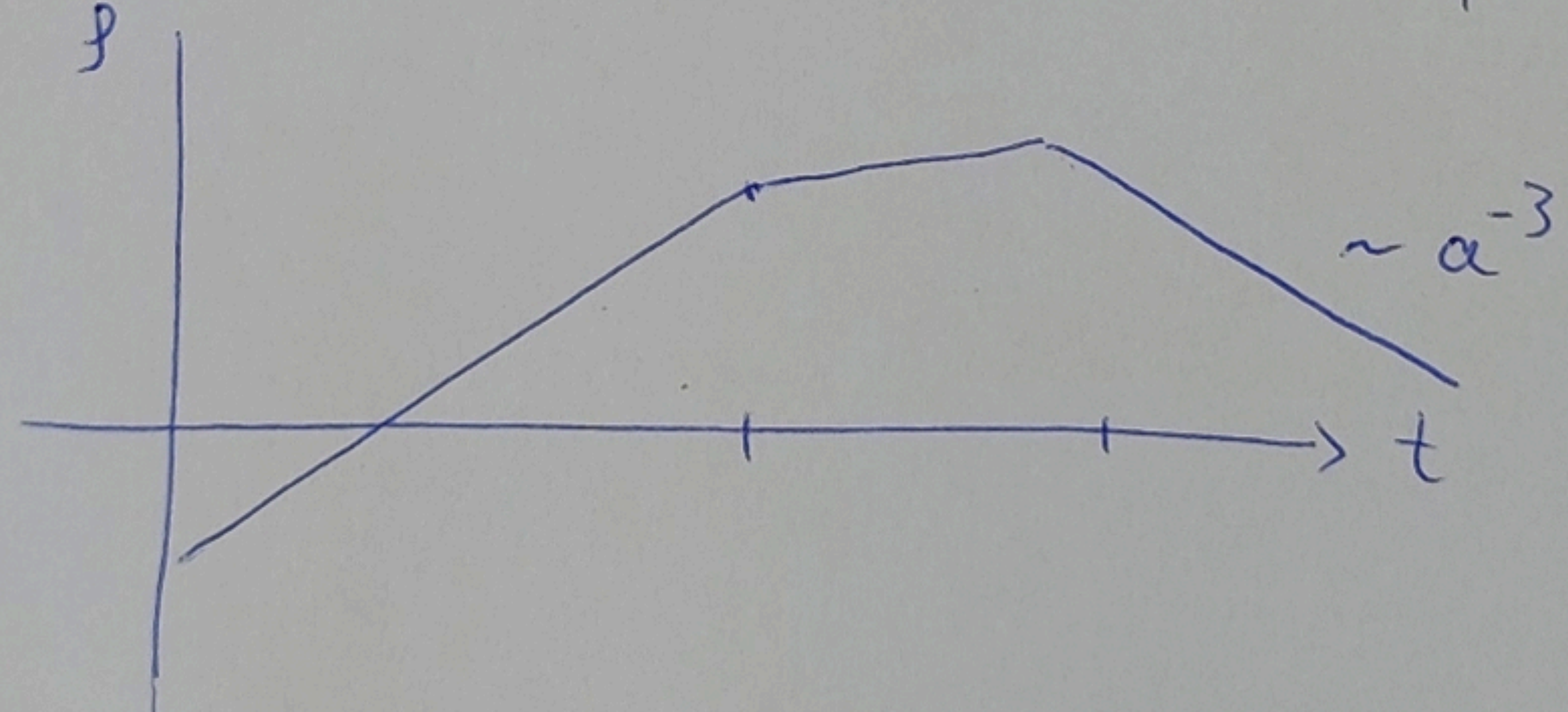
$$\Rightarrow \rho \simeq \frac{1}{2}\tilde{m}_a(t)\,\tilde{m}_a(t_c)\left(\frac{a(t_c)}{a(t)}\right)^3 \phi_0^2$$

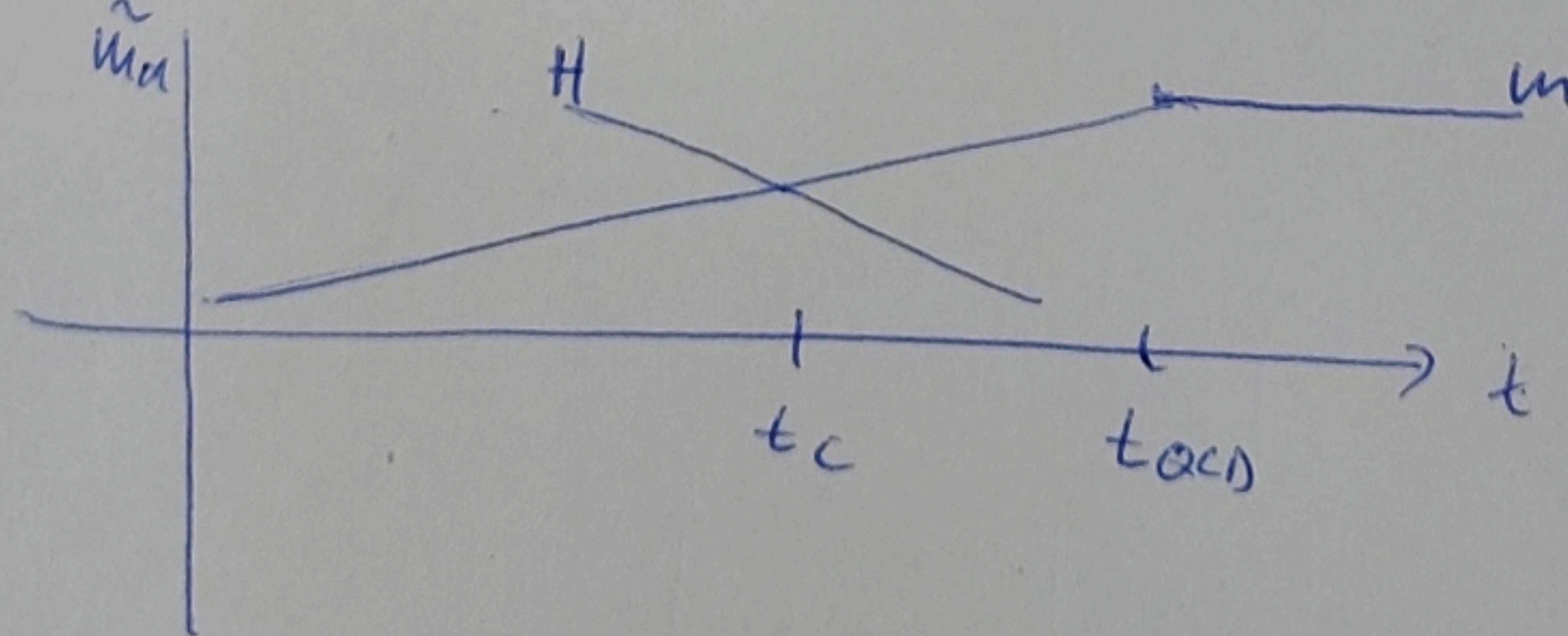
Scaling $\quad T > T_c \quad \sim t^4$

Assume $\quad T_c > T > T_{QCD} \quad \sim \frac{m_a}{a^3} \sim \frac{t^2}{t^{3/2}} \sim t^{1/2}$

$\quad T_c > T_{QCD} > T \quad \sim \frac{1}{a^3} \sim t^{-3/2}$

scales like matter ! after QCD PT





Can show pressure and velocity dispersion ≈ 0.

$\Rightarrow$ CDM

what is the ordering of T_c and T_{QCD}?

$3H(T_c) = \tilde{m}_a(T_c)$ defines T_c

$\underline{T_c < T_{QCD}}$: $\quad 3\frac{T_c^2}{M_p} \simeq m_a \quad \Rightarrow \quad T_c \simeq \left(\frac{m_a M_p}{3}\right)^{1/2}$

$\underline{T_c > T_{QCD}}$ $\quad 3\frac{T_c^2}{M_p} \simeq m_a \left(\frac{T_c}{T_{QCD}}\right)^{-4} \quad \Rightarrow \quad T_c \simeq \left(\frac{m_a M_p T_{QCD}^4}{3}\right)^{1/6}$

Turnover $\quad 3\frac{T_{QCD}^2}{M_p} \simeq m_a \approx 6\,\text{meV}\left(\frac{10^9\,\text{GeV}}{f_a}\right)$

$\Rightarrow f_a \sim 10^{17}$ GeV

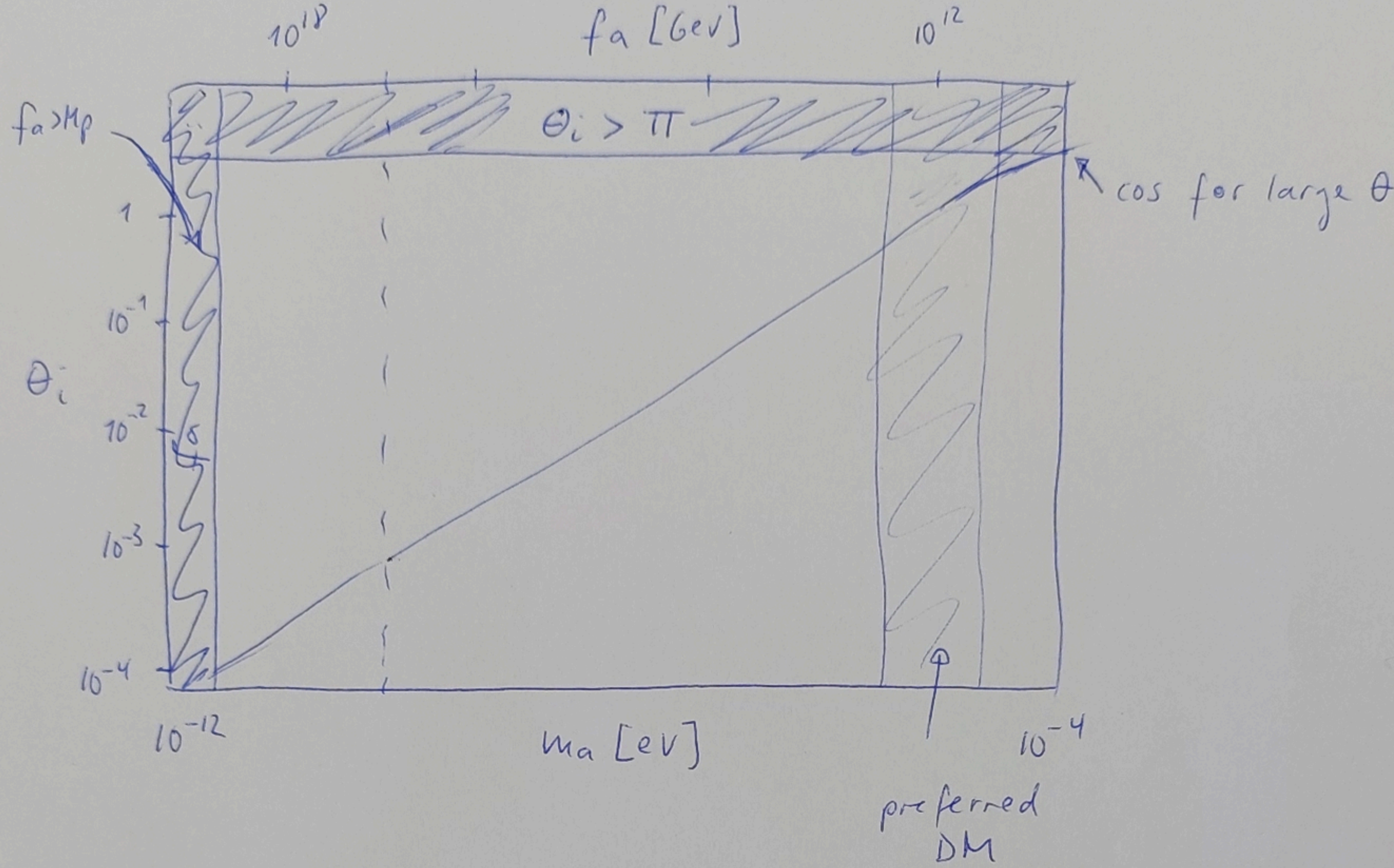
Now use $\quad H_c \sim \frac{T_c^2}{M_p} \sim \frac{1}{a_c^2}$ and plug into ρ to obtain Ωh^2

$$\Omega h^2 = \begin{cases} 0{,}15 \; \theta_i^2 \left(\frac{f_a}{10^{12}\,\text{GeV}}\right)^{7/6} & T_c > T_{QCD} \\ 2\cdot 10^5 \; \theta_i^2 \left(\frac{f_a}{10^{17}\,\text{GeV}}\right)^{3/2} & T_c < T_{QCD} \end{cases}$$

θ_i initial misalignment angle $\quad \theta_i \in [-\pi, \pi]$

$\rightarrow$ free parameter (naturally $\mathcal{O}(1)$)

Note smaller $m_a \longleftrightarrow$ larger $f_a \longleftrightarrow$ larger $\Omega_a h^2$



what if $T_R > f_a$? → symmetric phase restored

→ PQ breaking after inflation

→ different values of θ_i for different causal patches

*) $\theta_i^2 \rightarrow \langle \theta_i^2 \rangle \sim 1 \left(\frac{\pi^2}{3} \right)$ prediction for average

*) not homogeneous anymore → $\phi(x,t)$

→ non-zero momentum modes in axion field

→ similar contribution to Ωh^2

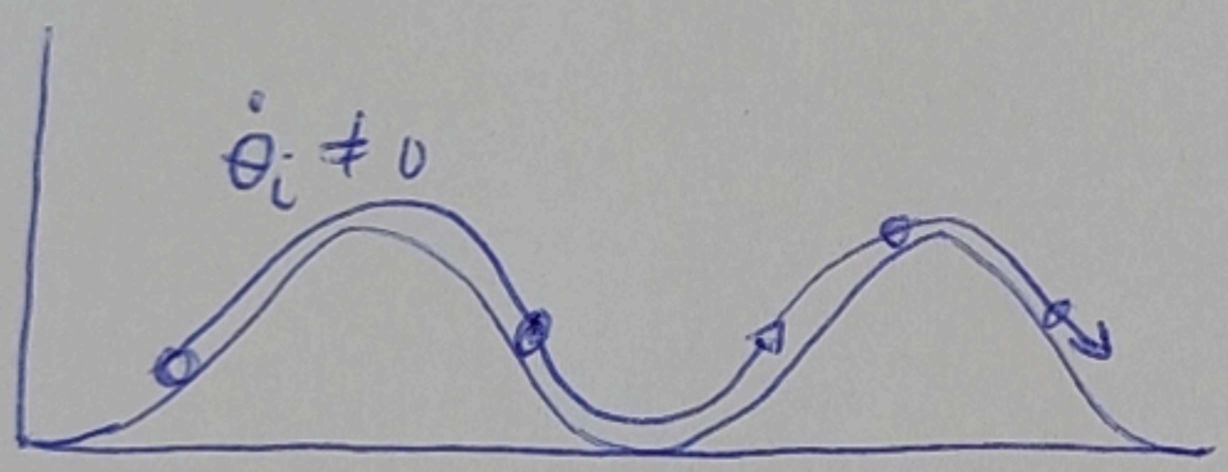
inhomogeneities may also lead to axion miniclusters!

*) topological defects due to complex vacuum structure (domain walls and cosmic strings)
usually unstable and decays to axions.

$$\sum \qquad \Omega_a h^2 \sim \left(\frac{f_a}{10^{12}\,\mathrm{GeV}}\right)^{7/6} \qquad \begin{array}{ll} 0{,}15\ \theta_i^2 & f_a > T_R \\ 0{,}7 & f_a < T_R \end{array}$$

There are of course variants which lead to different predictions.

E.g. <u>kinetic misalignment</u>



$\rightarrow$ Ωh^2 for smaller f_a