Extra Dimensions

1. Basic idea + consequences
2. Vector fields [introduce "orbifolds"]
3. Fermions [surprise: same idea works here!]
4. Historical notes
   [original KK idea + a bit on follow-up]
5. The SM in 5D
   ["bosonic supersymmetry", radiative corrections, ... if time allows]

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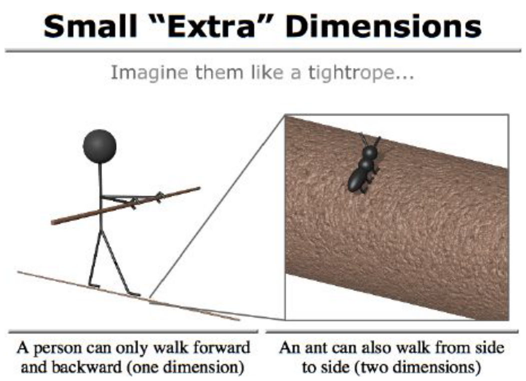
1. Basic idea



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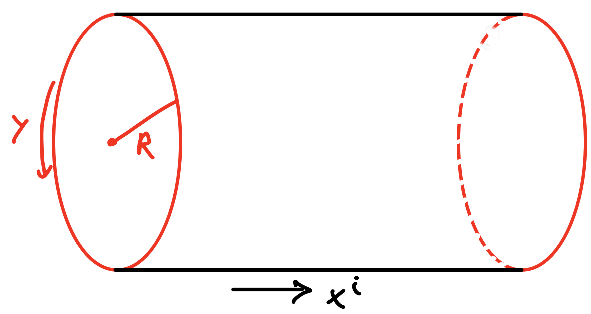


$A = \{0, \dots, d-1\}$

$d = 3+n$

$$X^A = \begin{pmatrix} [c]t \\ x^1 \\ x^2 \\ x^3 \\ y^1 \\ \vdots \\ y^n \end{pmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{ordinary 3D} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} n \text{ compactified extra dimensions} \end{matrix}$$

$\rightsquigarrow$ invisible for distances $\gg R$

$\qquad\qquad = \qquad\quad =$ energies $\ll R^{-1}$

Q: Why no extra time dimensions?

A: Lorentz symmetry!

$SO(1,3)$ $\rightarrow$ $SO(1,4)$ = picture above
$\searrow$ $SO(2,3)$: allows rotations between time components
=> closed time-like curves (CTCs) possible
=> causality ⚡

- generic consequences

A) Modification of Newton's law $\Phi_N \propto r^{-1}$

Poisson eq. : $\nabla^2 \Phi_N = 0$ gravitational potential in vacuum

flat space (in Euclidian coordinates) : $\nabla^2 = \left(\frac{\partial}{\partial X^1}\right)^2 + \ldots + \left(\frac{\partial}{\partial X^{d-1}}\right)^2$

polar coordinates...

$\Rightarrow \Phi_N \propto r^{-(1+n)}$ => gravitational force falls of faster / "gravity is more diluted"

current limit on deviations from $\Phi_N \propto r^{-1}$:

cf. PDG

$R \lesssim 10^{-2}\,\text{cm}$ (torsion balance detectors)

$\Leftrightarrow R^{-1} \gtrsim 10^{-3}\,\text{eV}$ NB: this is gravity alone! Much stronger constraints otherwise

B) Variation of constants

$$S = \frac{1}{16\pi \hat{G}} \int d^{4+n} X \sqrt{|g|}\, R$$

higher-dim. grav. constant

| e.g. $g_{AB}\, dX^A dX^B \overset{\text{spacetime is separable}}{=} g_{\mu\nu}(x^\delta)\, dx^\mu dx^\nu + \tilde{g}_{pq}(x^\delta; y^\gamma)\, dy^p dy^q$

$$= \frac{1}{16\pi\hat{G}} \underbrace{\int d^4x \sqrt{|g|}\, R^{(4)}}_{\text{GR in 4D}} \underbrace{\int d^n y \sqrt{|g|}}_{\equiv V \sim R^n} + \dots$$

$R^{(4)}$: 4D Ricci scalar

V: "volume" of internal space

$\dots$ $\hat{=}$ additional d.o.f., TBD below ...

$\Rightarrow$ (effective) 4D coupling $G = V^{-1}\hat{G}$

- similar for other interactions
- strong constraints on variations (x^μ-dependence) of fund. couplings

} extra dimensions must be "stabilized"!

[= not an easy task at all, in general ...]

C) The "Kaluza-Klein tower"

Consider e.g. a massive scalar field in 5D (= 1+3+1)

[+ flat spacetime, i.e. Minkowski]

$$0 = \left(\Box^{(5)} + m^2\right) \phi(x^\mu, y) \quad \text{"Klein-Gordon equation"}$$

$$\hat{=} \left(\partial_t^2 - \nabla^2 - \partial_y^2 + m^2\right)\phi$$

∇^2: standard 3D Laplacian

compactification: $y \sim y + 2\pi R$ [i.e. $f(y) = f(y+2\pi R)\ \forall f$]

$$\Rightarrow \phi(x^\mu, y) = \sum_{n=0}^{\infty} \phi^{(n)}(x^\mu)\, e^{-i\frac{n}{R}y} \qquad \Rightarrow \phi(x^\mu, y) = \phi(x^\mu, y+2\pi R)$$

$$\Rightarrow \partial_y^2 \phi = -\sum \left(\frac{n}{R}\right)^2 \phi^{(n)}(x^\mu)\, e^{-i\frac{n}{R}y}$$

$\Rightarrow$ one eq. for each Fourier component:

$$\Rightarrow \quad 0 = \Big(\partial_t^2 - \nabla^2 + \underbrace{\frac{n^2}{R^2} + m^2}_{\equiv m_n^2}\Big)\,\phi^{(n)}(x^\mu) \quad = \text{KG in 4D!}$$

$\Rightarrow$ In 4D, such a theory is equivalent to an infinite "tower" of states / scalar fields with mass m_n!

interpretation?

- Kinetic energy in y-direction $\hat{=}$ rest mass in x^i-direction (conservation of energy!)
- small R $\hat{=}$ heavy $m_{n\geq 1}$ $\hat{=}$ invisible at low energies ✓

NB:
- KK tower is a generic prediction
- exact structure depends on
  - internal geometry (here just S^1)
  - types of fields / particles that can propagate in the EDs

    (NB: only gravity <u>always</u> feels the EDs, the rest is model-dependent)

2. Vector fields

e.g. a single vector in 5D:

$$A^M = (A^0, A^1, A^2, A^3, A^5) \quad = \text{vector under 5D Lorentz trafos}$$

$$A^M \to A'^M = \Lambda^M{}_N A^N$$

$\Rightarrow$ • A^μ = vector under 4D Lorentz trafos

$$A^\mu \to A^{\mu\prime} = \Lambda^\mu{}_\nu A^\nu \qquad \left[\text{or } \Lambda^M{}_N = \begin{pmatrix} \Lambda^\mu{}_\nu & \vec{0} \\ \vec{0}^T & 1 \end{pmatrix} \right]$$

• A^5 = scalar under 4D Lorentz trafos

$$A^5 \to A^{5\prime} = A^5$$

$\Rightarrow$ in the effective, 4D theory expect:

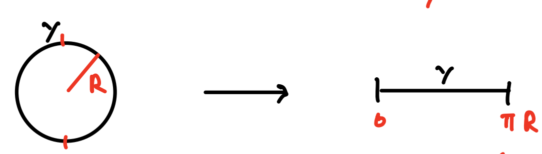
- a light ($m \ll \frac{1}{R}$) vector field + KK tower
- a light ($m \ll \frac{1}{R}$) scalar " " " "

not observed!

solution: compactify on "orbifold" $S^1/\mathbb{Z}_2$ instead of circle S^1!

"add mirror symmetry" $\quad y \sim y + 2\pi R$

$$y \sim -y \qquad \text{"}P_{\mathbb{Z}_2}\text{"}$$

$\Leftrightarrow$ 

"branes" / "orbifold fixed points"

$\Rightarrow$ fields can be <u>even</u> under orbifold projections: $P_{\mathbb{Z}_2}\,\phi(x^\mu,y) = \phi(x^\mu,-y)$

or <u>odd</u> " " " : $P_{\mathbb{Z}_2}\,\phi(x^\mu,y) = -\phi(x^\mu,-y)$

$$\Rightarrow \bullet\ \phi_{even}(x^\mu,y) = \underbrace{\frac{1}{\sqrt{2\pi R}}}\phi^{(0)}_{even} + \underbrace{\frac{1}{\sqrt{\pi R}}}\sum_{n=1}^{\infty}\phi^{(n)}_{even}\cos\frac{ny}{R}$$

~ as before, w/ $e^{i\ldots} \to \cos\ldots$

convention to ensure canonical kinetic terms in 4D

$$\bullet\ \phi_{odd}(x^\mu,y) = \frac{1}{\sqrt{\pi R}}\sum_{n=1}\phi^{(n)}_{odd}\sin\frac{ny}{R}$$

$\leadsto$ odd fields have no zero (light) mode!

$\Rightarrow$ "trick": assign 5D A^μ to be even, A^5 to be odd!

[OK from 4D perspective because different fields

" " 5D " " orbifold breaks Lorentz invariance

(no longer translational invariance in y direction)]

Notes: • this is consistent with / required by gauge invariance:

$$A_N \to A_N + \partial_N\alpha(x^\mu,y)$$

[$N=\mu \Rightarrow \alpha$ must be even (if A_μ has zero modes $\Leftrightarrow$ A_μ even)

$\Rightarrow \partial_5\alpha$ is odd $\Rightarrow A_5$ must be odd, too]

• In this case, even the KK modes of A^5 are not physical – they are "Goldstone modes" that give mass to the massive A^μ modes.

$$S = -\frac{1}{4}\int d^5x \; F^{MN} F_{MN} \qquad ; \qquad F_{MN} = \partial_M A_N - \partial_N A_M$$

← necessary convention for above normalization

$$\Rightarrow \mathcal{L}^{4D} = -\frac{1}{4}\int_0^{2\pi R} dy \left(\partial_M A_N - \partial_N A_M\right)\left(\partial^M A^N - \partial^N A^M\right)$$

insert expansions for even, odd from above

$$\int_0^{2\pi} dy \; \sin(ny)\sin(my) = \delta_{mn}$$
$$\cos \quad \cos \quad = \delta_{mn}$$
$$\cos \quad \sin \quad = 0$$

⇒ momentum conservation!

$$\ldots$$

$$= -\frac{1}{4}\left(\partial_\mu A_\nu^{(0)} - \partial_\nu A_\mu^{(0)}\right)\left(\partial^\mu A^{\nu(0)} - \partial^\nu A^{\mu(0)}\right)$$

← standard $F_{\mu\nu}F^{\mu\nu}$ for zero modes (= kinetic terms)

$$-\frac{1}{4}\sum_{n=1}^{\infty}\left(\partial_\mu A_\nu^{(n)} - \partial_\nu A_\mu^{(n)}\right)\left(\partial^\mu A^{\nu(n)} - \partial^\nu A^{\mu(n)}\right)$$

← same, for $n \geq 1$

$$+\frac{1}{2}\sum_{n=1}^{\infty}\left(\partial_\mu A_5^{(n)} + \frac{n}{R} A_\mu^{(n)}\right)\left(\partial^\mu A_5^{(n)} + \frac{n}{R} A^{(n)\mu}\right)$$

→ A_5 can be transformed away by $\alpha = -\frac{R}{n} A_5^{(n)}$ "unitary gauge".