

fermions? $z_{10} = \sum_{n=0}^{\infty} \langle n | e^{-\beta \hat{H}} | n \rangle = e^{-\frac{1}{2}\beta\omega} + e^{-\frac{3}{2}\beta\omega}$

$$\Rightarrow \mathcal{F} = \left[-\frac{3}{8} - \beta^{-1} \right] \frac{d^3 h}{(2\pi)^3} \ln(1 + e^{-\beta\omega_x}) \equiv \left[-\frac{3}{8} \right] + T^4 \mathcal{J}_F\left(\frac{m}{T}\right)$$

asymptotics? • $\mathcal{J}_F\left(\frac{m}{T} \gg 1\right) = \mathcal{J}_B\left(\frac{m}{T} \gg 1\right)$

• $\mathcal{J}_F\left(\frac{m}{T} \ll 1\right) = \frac{7}{8} \cdot \left(-\frac{\pi^2}{90}\right) + 2 \cdot \frac{1}{24} \left(\frac{m}{T}\right)^2 + \mathcal{O}\left(\frac{m^4}{T^4}\right)$

NB: no non-analytic terms $\sim \left(\frac{m^2}{T^2}\right)^{\frac{3}{2}}$

Example: ϕ^4 theory with N_f "free" fermions and N_b "free" bosons
(fermionic d.o.f.) (bosonic d.o.f.)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}m_\phi^2 \phi^2 - \frac{\lambda}{4!} \phi^4 + \dots$$

- $V_0(\phi)$: $T=0$ potential

(+) assume $m_{f,b} \propto \phi$ (like in SM: Higgs mechanism)

$$\Rightarrow \mathcal{F} = V_0(\phi) + T^4 \left\{ \sum_{b=1}^{N_b} \mathcal{J}_B\left(\frac{m_b}{T}\right) + \sum_{f=1}^{N_f} \mathcal{J}_F\left(\frac{m_f}{T}\right) \right\} \equiv V_T(\phi) + \phi\text{-indep. terms}$$

(- $\text{eff } \frac{\pi^2}{90} T^4$ for $T \gg m_{f,b}$)

high T : $V_T(\phi) = V_0(\phi) + \frac{T^2}{24} \left\{ \sum m_b^2(\phi) + 2 \sum m_f^2(\phi) \right\}$

↑ includes $\psi, \bar{\psi}$!
↑ count 3 d.o.f. for vectors!

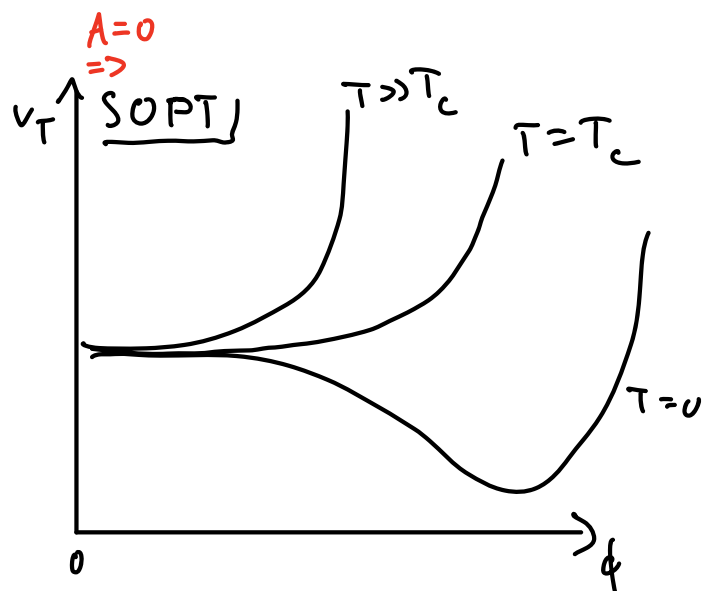
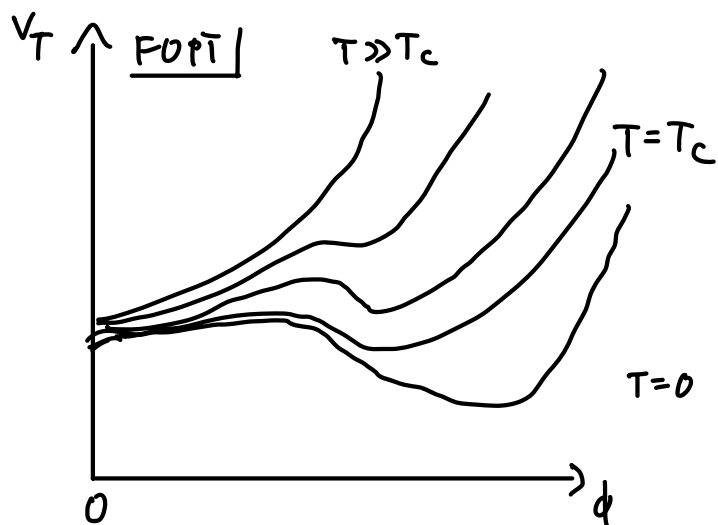
$$- \frac{T}{12\pi} \sum (m_b^2)^{\frac{3}{2}}$$

↑ count 1 d.o.f. for vectors - only longitudinal d.o.f.!

+ higher orders in T/m (next term: $m^4 \log \frac{T}{m}$)

$$\equiv \frac{D}{2} \left(T^2 - \frac{M^2}{0} \right) \phi^2 - \frac{A}{3} T \phi^3 + \frac{\lambda_T}{4!} \phi^4 + \dots$$

↑ "T"
↑ only from bosons!
↑ needed for FORT!
 $\lambda_T = \lambda + B \cdot \log \frac{T}{m}$



b) strong couplings?

→ need full thermal FT:

• expand $z = \text{Tr} [e^{-\beta(\hat{H}_0 + \hat{H}_{int})}]$ in $\epsilon = \frac{g^2 T}{\omega_R}$ *exercise ii*

→ IR divergence for $g^2 T \gg \omega_R$

• thermal masses $m_B^2(T) = m_0^2 + \underline{c g^2 T^2}$

↗ not for transverse / "magnetic" vector components

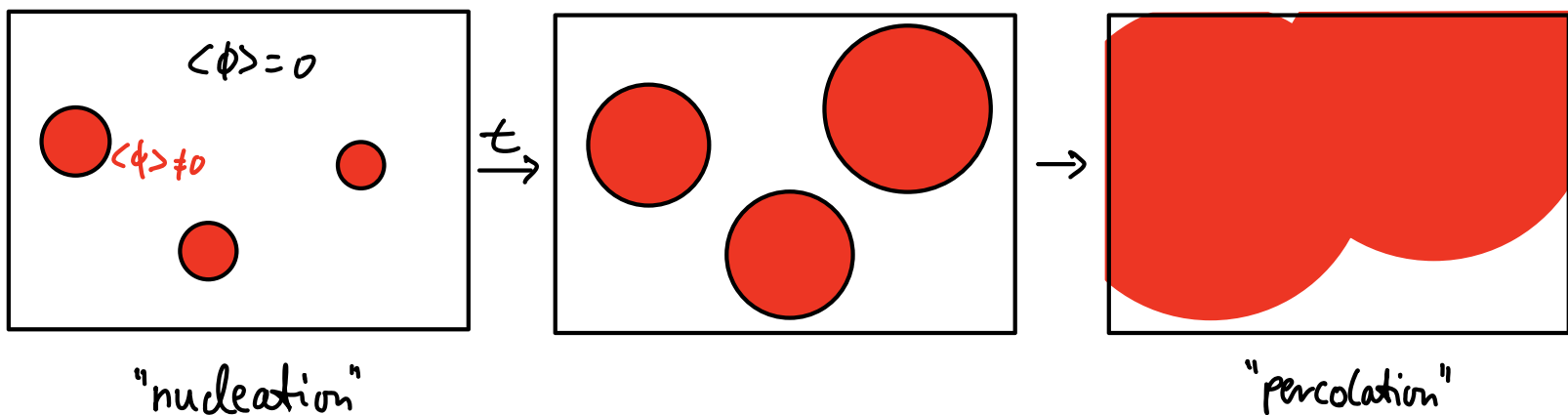
→ expansion always works for $g \ll 1$!

Debye screening removes IR divergence

• ...

4. Bubble nucleation (+ gravitational waves)

FOPTs in real space:



"Thin wall approximation":

Energy of single bubble: $E(R) = - \frac{4\pi}{3} R^3 \Delta V_T + 4\pi R^2 \sigma$

$\uparrow$ free energy difference $\uparrow$ surface tension (calculated from "bounce action")

$\leadsto$ need $\frac{dE}{dR} < 0$ for expansion

$$\Rightarrow R > R_c = \frac{2\sigma}{\Delta V_T}$$

$$\leadsto E_c \equiv E(R_c) = \frac{16\pi}{3} \frac{\sigma^3}{\Delta V_T^2}$$

$$\propto \frac{1}{T_c^3} (T_c - T)^2$$

NB: "critical" here $\neq$ "critical" in T_c !

$\leadsto$ formally diverges as $T \rightarrow T_c$
 "easier to form bubbles as $T \rightarrow 0$ "

nucleation rate: $\frac{\Gamma}{V} \sim T_c^4 e^{-E_c/T_c}$

$\uparrow$ [...]

cosmology: $H^2(T) \sim M_{Pl}^{-2} T^4$ (1st Friedmann equation)

$\leadsto$ first bubble nucleates when (~ 1 / Hubble volume)

$$M_{Pl}^{-4} T^8 \sim T_c^4 e^{-E_c/T_c} \quad \left| \text{for } T < T_c, \frac{E_c}{T} \text{ decreases rapidly from } \infty \right.$$

$\leadsto$ need $T_c = \text{few } \times T$

$$\Rightarrow \frac{E_c}{T} \sim \ln(T_c^4 / M_{Pl}^4) \quad | M_{Pl} = 1.2 \cdot 10^{19} \text{ GeV}$$

$$\sim 100 + 4 \ln \frac{T_c}{100 \text{ GeV}}$$

- transition rate $\beta \equiv \left. \frac{d}{dt} \log T_c \frac{\Gamma}{V} \right|_{t=t_p}$
 $\leadsto$ fractional volume in metastable phase $\approx e^{-\beta(t-t_p)}$

- transition strength $\alpha \equiv \frac{\Delta(\text{latent heat})}{S_{sm}} = \Delta(g_{\mu\nu} T^{\mu\nu})$

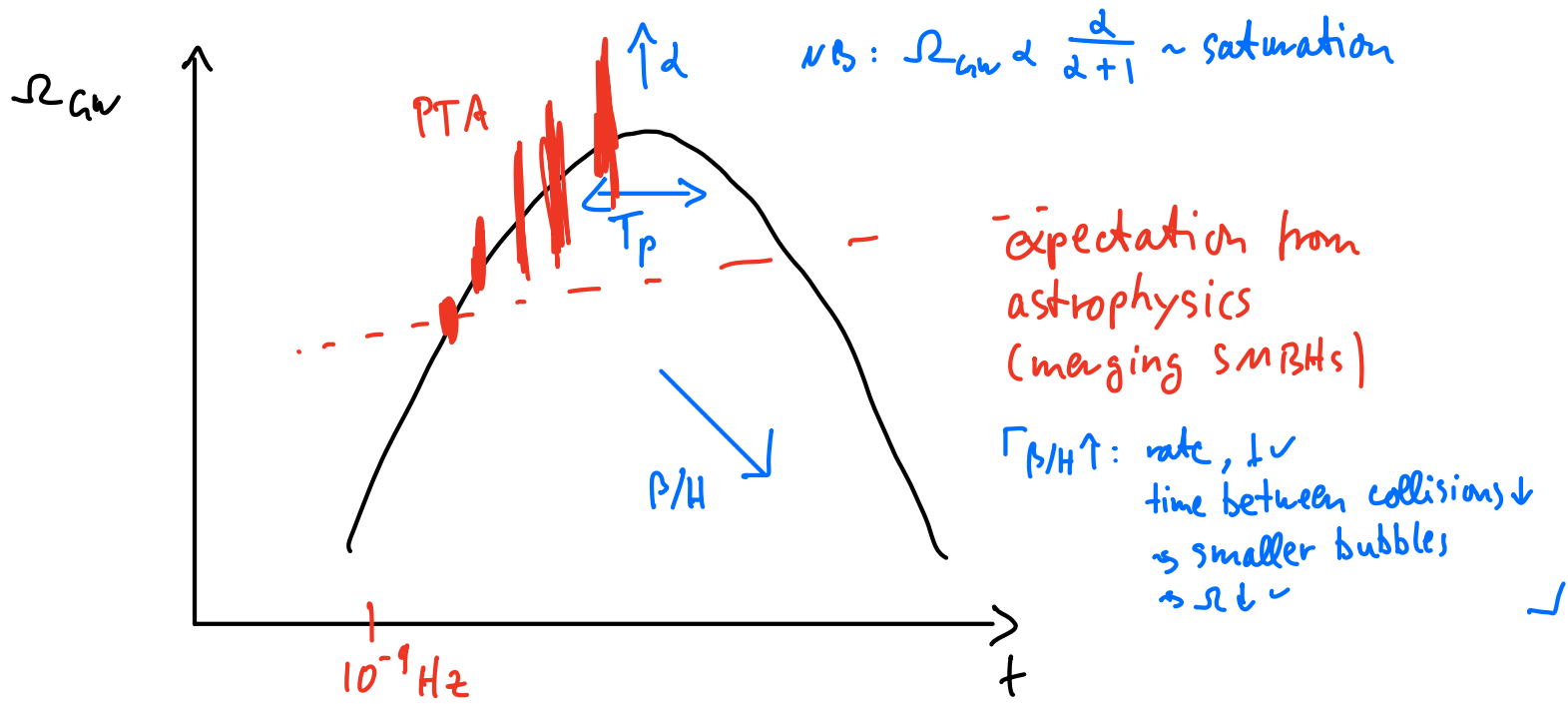
gravitational waves: $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + h_{\mu\nu} dx^\mu dx^\nu$

- "ripples" in spacetime / perturbations of local ("dust") rulers
- propagate at speed of light
- need $\ddot{Q}$ to generate

$\rightarrow$ generated by bubbles due to

1. collisions for $t \lesssim t_p$
 $\leadsto$ relatively short period, sub-dominant contribution to Ω_{GW}
2. bulk fluid motion in plasma ("sound waves") for $t \gtrsim t_p$
 colliding sound shells after percolation, each of those being $\sim$ spherical
 $\leadsto$ dominant contribution
3. turbulences
 $\leadsto$ non-linearity of fluid eqns.; more difficult to study, typically sub-dominant

$\leadsto$ need simulations...

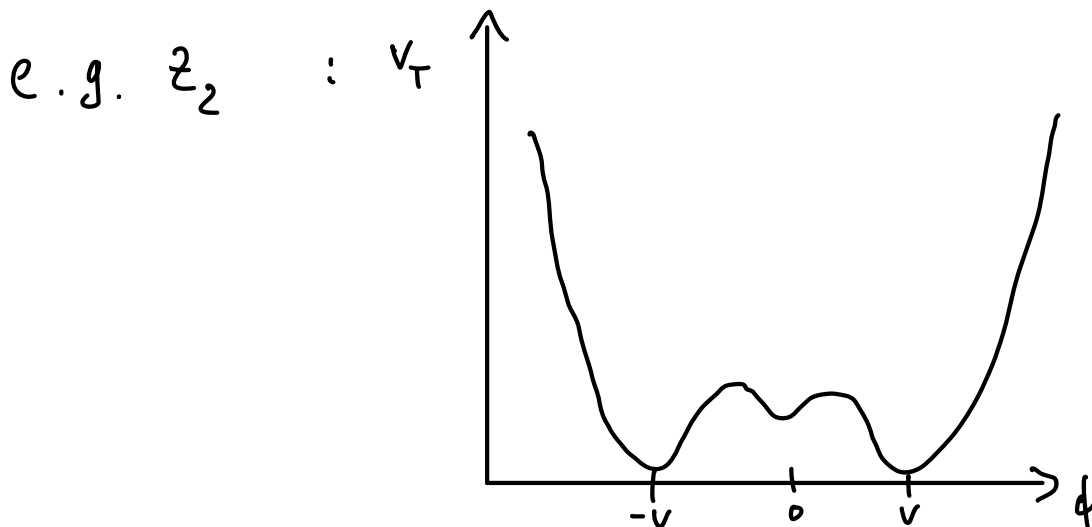


5. Symmetries (+ topological defects)

PTs are typically associated with breaking of symmetries

$\leadsto$ "false vacuum" $\Leftrightarrow$ "symmetric phase"

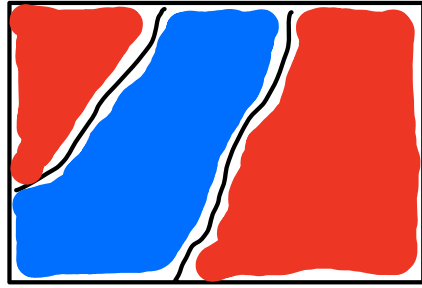
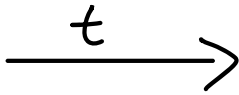
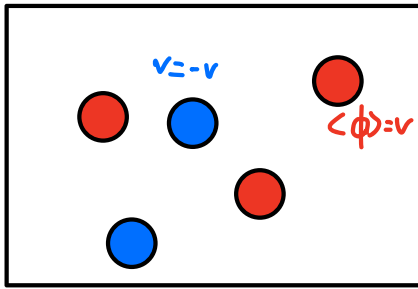
"true vacuum" $\Leftrightarrow$ "broken phase"



$\leadsto$ theory / $\mathcal{L}(\phi)$ symmetric under $\phi \leftrightarrow -\phi$,
but no such symmetry for ground state!

e.g. $\phi \equiv v + \varphi \leadsto \mathcal{L}(\varphi)$ not symmetric
 $\equiv -v + \tilde{\varphi} \quad \mathcal{L}(\tilde{\varphi}) = \quad =$

in real space?



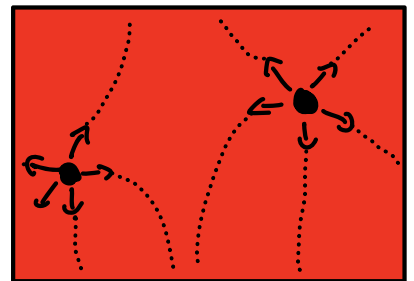
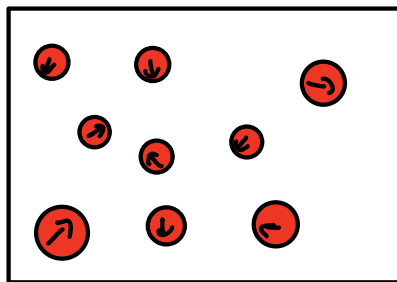
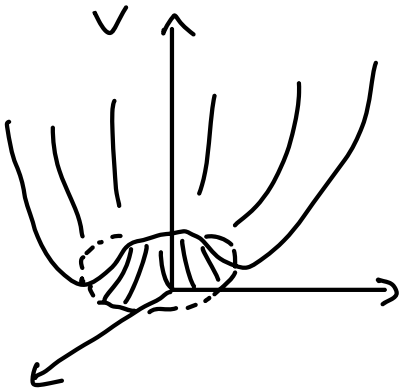
$\uparrow$ "domain walls" (2D)

$-v < \phi < v$ (ϕ must be continuous!)

$\Rightarrow$ width given by $\delta E \stackrel{!}{=} 0$, with

$$E = \int d^3x \, v > 0 \neq \text{vacuum}$$

e.g. $U(1) : v = v(|\phi|)$ for $\phi(x) = (h(x) + v) e^{i\varphi(x)} \leadsto \langle \phi \rangle = v \cdot e^{i\beta}$



"cosmic strings" (1D)

again: finite energy
density from $\delta E \stackrel{!}{=} 0$

$SO(3) \rightarrow U(1) : \text{"monopoles" (0D) point-like}$

Bottom line: cosmo-constraints on top. defects $\Rightarrow$ constraints on fund. phys.!