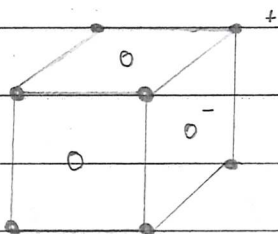


Magnons

1.

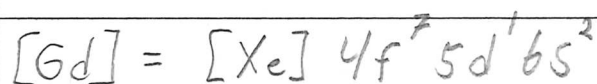
Imagine yourself inside a crystalline solid. (You must be tiny of course).

You would see the atomic nuclei regularly placed in a periodic arrangement, and the electrons will "circle" around them. However, you notice that in this particular solid some of the atoms have more protons than electrons and others have less. You are in an ionic crystal

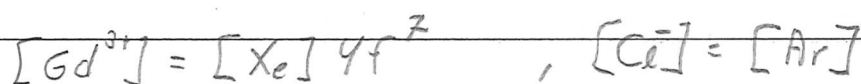


Although some of the electrons are displaced, they are nevertheless localized around their nuclei; the crystal is an insulator

Looking closer, you see that one of the types of atoms is Gadolinium, Gd (element 64, rare-earth 45 electrons). Its natural electron configuration is



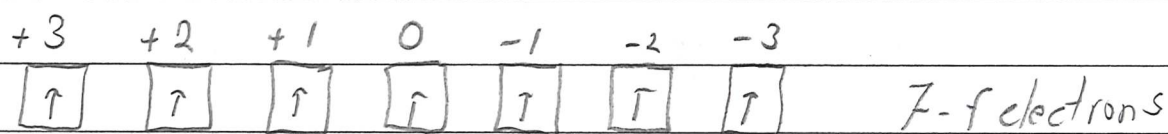
but in the ionic crystal the 5d and 6s electrons have moved over to the other type of atom, for instance Ce^{12} of which there are 3 for each Gd. So in the crystal



With a magnetometer you are also able to see that the Gd^{3+} ions have a magnetic dipole moment, while Ca have none. Also you notice that the dipole moments of nearby Gd^{3+} ions tend to align with each other.

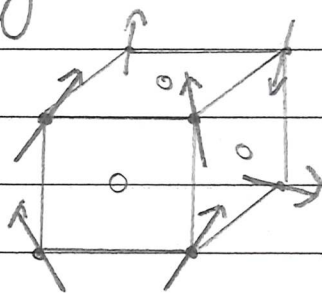
Then you wake up from your imaginary world and become your theoretical physicist self again!

Knowing your QM you realize that the dipole moments must come from the electrons in the partially filled f-shell. as filled shells contribute 0 to total orb. ang. mom L and spin S . The dipole moments from the nuclei are much smaller



$$S = 7/2, L = 0$$

$$S = 7/2$$



The dipole moments want to align with nearby moments

$$H = -J \sum \vec{S}_i \cdot \vec{S}_{i'}$$

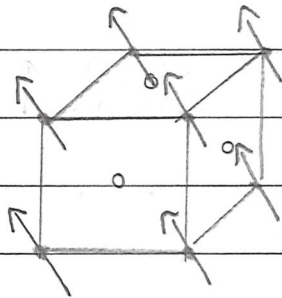
nearest
-neighbor

Spin- $1/2$ operator

Heisenberg model
caused by virtual
hopping of electrons.

The Heisenberg model has a Global $O(3)$ symmetry. You can rotate all the spins the same amount, and H is unchanged.

For $T < T_c$ (Curie temp.), the free energy (which should be minimized) is minimized by a state where all spins are aligned in the same direction.



so is invariant under $O(2)$ rot

so $O(3)$ has been broken to $O(2)$

for $T > T_c$ The dipole moments are largely uncorrelated and we have a paramagnetic insulator. (The moments will align with an external field).

let's see how we can investigate the broken symmetry phase.

Holstein - Primakoff (1940)

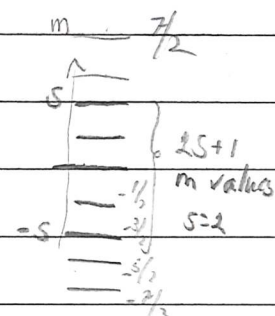
4

Spin-S raising & lowering ops: ($\hbar=1$)

$$S^- |S, m\rangle = \sqrt{(S+m)(S-m+1)} |S, m-1\rangle$$

$$S^+ |S, m\rangle = \sqrt{(S-m)(S+m+1)} |S, m+1\rangle$$

$$S^z |S, m\rangle = m |S, m\rangle$$



Set $n = S - m$: The deviation from the max value $m=S$
(or $m=S-n$) ($n \in [0, 2S]$)

$$S^- |S, S-n\rangle = \sqrt{(2S-n)(n+1)} |S, S-(n+1)\rangle$$

$$= \sqrt{2S} \sqrt{n+1} \sqrt{1 - \frac{n}{2S}} |S, S-(n+1)\rangle$$

$$S^+ |S, S-n\rangle = \sqrt{2S} \left(1 - \frac{n-1}{2S}\right)^{1/2} n^{1/2} |S, S-(n-1)\rangle$$

$$S^z |S, S-n\rangle = (S-n) |S, S-n\rangle$$

Set $|S, S\rangle = |0\rangle$ Boson ops: $a^+ |0\rangle = |1\rangle$ etc

$$|S, S-1\rangle = |1\rangle$$

$$a^+ a |n\rangle = n |n\rangle$$

$$|S, S-n\rangle = |n\rangle$$

Then

$$S^- = \sqrt{2S} a^+ \left(1 - \frac{a^+ a}{2S}\right)^{1/2}$$

$$S^+ = \sqrt{2S} \left(1 - \frac{a^+ a}{2S}\right)^{1/2} a$$

$$S^z = S - a^+ a$$

(Holstein - Primakoff transformation)

reproduces the spin algebra with bosons.
for instance

$$S^- |S, S-n\rangle = \sqrt{2S} a^+ \left(1 - \frac{a^+ a}{2S}\right)^{1/2} |n\rangle = \sqrt{2S} \sqrt{n+1} \left(1 - \frac{n}{2S}\right)^{1/2} |n+1\rangle$$

$$S^z |S, S-n\rangle = (S - a^+ a) |n\rangle = \underbrace{(S-n)}_{|S, S-n\rangle} \underbrace{|n\rangle}_{|S, S-(n+1)\rangle}$$

for $\langle a^\dagger a \rangle \ll 2S$ it is a good approx to expand $\sqrt{}$
To leading order one gets

$$\begin{aligned} S^- &= \sqrt{2S} a^- \\ S^+ &= \sqrt{2S} a^+ \\ S^z &= S - a^\dagger a \end{aligned}$$

The so-called linear spin wave approximation. (LSW)

FM Magnons (Square lattice)

$$H = -J \sum_{\langle \vec{r}, \vec{r}' \rangle} \vec{S}_{\vec{r}} \cdot \vec{S}_{\vec{r}'}$$

$\uparrow$ $\langle \vec{r}, \vec{r}' \rangle$ $\leftarrow$ nearest nbrs.

$\uparrow$
 $\uparrow$
 $\uparrow$
 $\uparrow$

$\uparrow$
 $\uparrow$
 $\uparrow$
 $\uparrow$

$\uparrow$
 $\uparrow$
 $\uparrow$
 $\uparrow$

FM ground state
 $\uparrow = |S, S\rangle$
 (Broken symmetry)

$$S^\pm = S^x \pm i S^y$$

$$= -J \sum_{\langle \vec{r}, \vec{r}' \rangle} \left\{ \frac{1}{2} (S_r^+ S_{r'}^- + S_r^- S_{r'}^+) + S_r^z S_{r'}^z \right\}$$

H.P.

$$= -J \sum_{\langle \vec{r}, \vec{r}' \rangle} \left\{ S (a_r a_{r'}^\dagger + a_r^\dagger a_{r'}) + (S - a_r^\dagger a_r) (S - a_{r'}^\dagger a_{r'}) \right\}$$

$$= -J \sum_{\langle \vec{r}, \vec{r}' \rangle} S^2$$

It doesn't need to be this simple. Other H 's will give more complicated expressions.

No linear terms.

$$-JS \sum_{\langle \vec{r}, \vec{r}' \rangle} (a_{r'}^\dagger a_r + a_r^\dagger a_{r'} - a_r^\dagger a_r - a_{r'}^\dagger a_{r'})$$

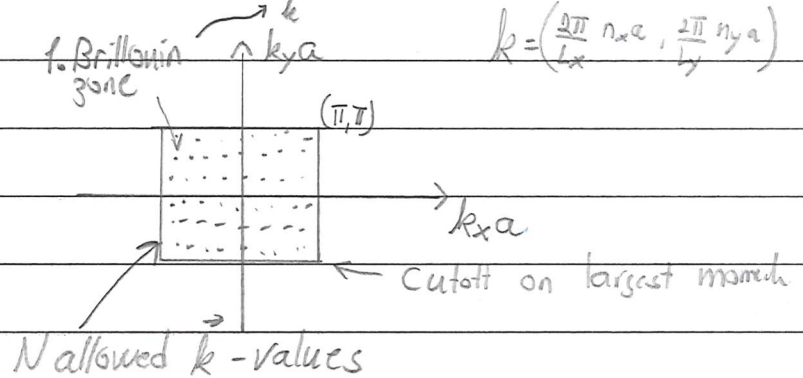
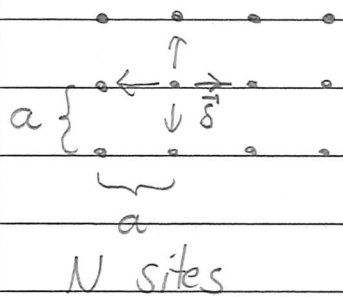
$$+ O(S^0)$$

($1/S$ - expansion)

Fourier transforms

$$a_{\vec{r}} = \frac{1}{N} \sum_{\vec{k}} a_{\vec{k}} e^{-i\vec{k} \cdot \vec{r}}$$

$$a_{\vec{r}}^{\dagger} = \frac{1}{N} \sum_{\vec{k}} a_{\vec{k}}^{\dagger} e^{i\vec{k} \cdot \vec{r}}$$



$$\sum_{\langle \vec{r} \vec{r}' \rangle} a_{\vec{r}}^{\dagger} a_{\vec{r}} = \sum_{\vec{r}} \frac{1}{2} \sum_{\vec{\delta}} \frac{1}{N} \sum_{\vec{k}, \vec{k}'} a_{\vec{k}'}^{\dagger} a_{\vec{k}} e^{i(\vec{k}' - \vec{k}) \cdot \vec{r}} e^{i\vec{k} \cdot \vec{\delta}}$$

$$\text{identity: } \sum_{\vec{r}} e^{i(\vec{k}' - \vec{k}) \cdot \vec{r}} = N \delta_{\vec{k}', \vec{k}}$$

We find

$$\sum_{\langle \vec{r} \vec{r}' \rangle} a_{\vec{r}}^{\dagger} a_{\vec{r}} = \sum_{\vec{k}} a_{\vec{k}}^{\dagger} a_{\vec{k}} \cdot \frac{1}{2} \sum_{\vec{\delta}} e^{i\vec{k} \cdot \vec{\delta}}$$

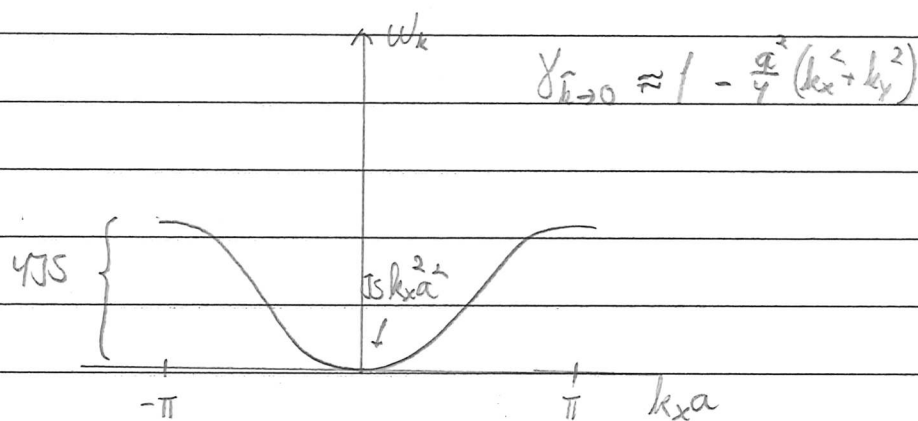
Similarly

$$\sum_{\langle \vec{r} \vec{r}' \rangle} a_{\vec{r}} a_{\vec{r}'} = \sum_{\vec{k}} a_{\vec{k}} a_{\vec{k}} \cdot \left(\frac{1}{2} \sum_{\vec{\delta}} 1 \right)$$

$$H = -JS^2 \cdot 2N + \sum_{\vec{k}} a_{\vec{k}}^{\dagger} a_{\vec{k}} JS \left(4 - \sum_{\vec{\delta}} e^{i\vec{k} \cdot \vec{\delta}} \right) + O(S^0)$$

$$\gamma_{\vec{k}} = \frac{1}{2} (\cos k_x a + \cos k_y a) \quad \frac{4JS(1 - \gamma_{\vec{k}})}{2dJS(1 - \gamma_{\vec{k}})} \quad \gamma_{\vec{k}} = \frac{1}{d} \sum_{i=1}^d \cos k_i a$$

$a_{\vec{k}}^+$ is then recognized as a Magnon creation operator that creates a magnon with momentum $\vec{k}$ and energy $\omega_{\vec{k}} = 4JS(1 - \gamma_{\vec{k}}) = 4JS(1 - \frac{1}{2}(\cos k_x a + \cos k_y a))$



Goldstone theorem:

$SO(3)$ symmetry of H : 3 generators

- FM state is rot inv about its magnetization: 1 Generator

of broken generators: 2 (Expect 2 Goldstone bosons!)

But Only 1 Goldstone boson here

Reason: In non-rel. QFT Goldstone bosons with dispersion k^z , z even count double (!)

see. H.B. Nielsen, S. Chadha, NPB 445 (1976) "On how to count Goldstone modes"

Finite temperature

Tot. Magn.

$$M = \sum_r \langle S_r^z \rangle = \sum_r (S - \langle a_r^\dagger a_r \rangle) = NS - \sum_k \langle a_k^\dagger a_k \rangle$$

$$\sum_k \langle a_k^\dagger a_k \rangle = \sum_k \frac{1}{e^{\beta u_k} - 1} \approx N \int \frac{d^d(ka)}{(2\pi)^d} \frac{1}{\underbrace{[(JS)^{1/2} ka]^2}_p}$$

$\uparrow$ Base occ. factor $\uparrow$ expands about $k=0$

$u_k = JS(\bar{k}a)^2$

$$= N \left(\frac{T}{JS} \right)^{d/2} \underbrace{\int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2}}_{A_d}$$

($k_0=1$)

So $M = N \left(S - \left(\frac{T}{JS} \right)^{d/2} A_d \right)$

But... for $d \leq 2$ $A_d \rightarrow \infty$ (small p divergence).
 So our assumption of a FM ground state breaks down.
 This is an example of the

Mermin-Wagner theorem

: A system with a continuous global symmetry (and short-range interactions) cannot have long-range order at finite T in $d \leq 2$.

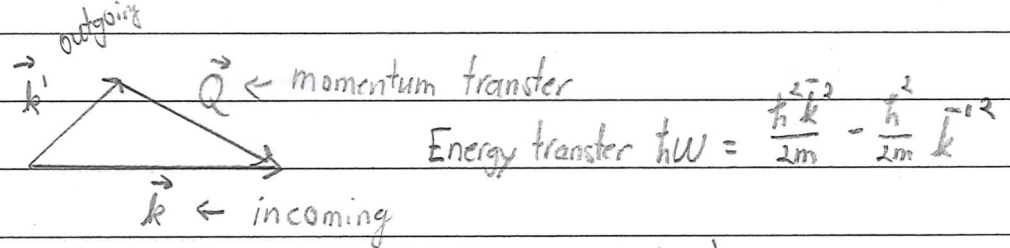
Neutron Scattering (and Magnons)

9.

$$g_p = 5.59$$

$$g_n = -3.82$$

The Neutron has no net electric charge, but a magnetic dipole moment. So it penetrates materials easily, and interacts with the spins in the material.



Cross section (Magnetic scattering only)

$$\frac{d^2\sigma}{d\Omega d\omega} = f(\vec{Q}) \sum_{\alpha\beta} \left(\delta_{\alpha\beta} - \frac{Q_\alpha Q_\beta}{Q^2} \right) S^{\alpha\beta}(\vec{Q}, \omega)$$

$\sim 10^{-24} \text{ cm}^2$ (Nuclear c.s.) falls off with $1/Q$
 $\uparrow$ Neutrons couple to moments $\perp \vec{Q}$
 $\uparrow$ Dynamical structure factor (Theory focus on this)

$$S^{\alpha\beta}(\vec{Q}, \omega) = \int_{-\infty}^{\infty} dt \langle S_{\vec{Q}}^\alpha(t) S_{-\vec{Q}}^\beta(0) \rangle e^{i\omega t}, \quad S_{\vec{Q}}^\alpha = \sum_{\vec{r}} S_{\vec{r}}^\alpha e^{-i\vec{Q} \cdot \vec{r}}$$

$$Z = \sum_n e^{-\beta E_n} \quad \text{energy eigenstates}$$

$$\langle S_{\vec{Q}}^\alpha(t) S_{-\vec{Q}}^\beta(0) \rangle e^{i\omega t} = \frac{1}{Z} \sum_{n,m} \langle n | e^{iHt} S_{\vec{Q}}^\alpha e^{-iHt} | m \rangle \langle m | S_{-\vec{Q}}^\beta | n \rangle e^{i\omega t}$$

$$= \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} e^{i(\omega - (E_m - E_n))t} \langle n | S_{\vec{Q}}^\alpha | m \rangle \langle m | S_{-\vec{Q}}^\beta | n \rangle$$

Integrate over t :

$$S^{\alpha\beta}(\vec{Q}, \omega) = \frac{2\pi}{\sum_n e^{-\beta E_n}} \sum_{n,m} \delta(\omega - (E_m - E_n)) e^{-\beta E_n} \langle n | S_{\vec{Q}}^\alpha | m \rangle \langle m | S_{-\vec{Q}}^\beta | n \rangle$$

for $T \rightarrow 0$ ($\beta \rightarrow \infty$) Ground state dominates sum $Z = \sum_n e^{-\beta E_n} \sim e^{-\beta E_0}$ 10.

$$S^{\alpha\beta}(\bar{Q}, \omega) \stackrel{T=0}{=} 2\pi \sum_m \delta(\omega - E_m) \langle 0 | S_{\bar{Q}}^{\alpha} | m \rangle \langle m | S_{\bar{Q}}^{\beta} | 0 \rangle$$

- Peaks when $\omega = \text{any } E_m$, Energy levels of system
- Spectral weight : $\langle 0 | S_{\bar{Q}}^{\alpha} | m \rangle \langle m | S_{\bar{Q}}^{\beta} | 0 \rangle$

Linear spin waves : $S_{\bar{Q}}^+ = \sqrt{2SN} a_{\bar{Q}}^+$, $S_{\bar{Q}}^- = \sqrt{2SN} a_{\bar{Q}}$

$$S^{+-}(\bar{Q}, \omega) = 2\pi \sum_m \delta(\omega - E_m) (2SN) \langle 0 | a_{\bar{Q}} | m \rangle \langle m | a_{\bar{Q}}^+ | 0 \rangle$$

Now assume that $|m\rangle$ are states composed of non-interacting magnons
 $|1434\rangle = |1: \bar{Q}_1, 4: \bar{Q}_2, 3: \bar{Q}_3 \text{ etc.} \rangle$

Then we only get a contribution from states with 1 magnon having momentum $\bar{Q}$:

$$S^{+-}(\bar{Q}, \omega) = 2\pi 2SN \delta(\omega - \omega_{\bar{Q}})$$