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Why Non-Thermal?

- The most widely studied dark matters are the thermal WIMPS ⇒ relic satisfied by thermal freeze-out.
- Direct detection experiments however yielded null results ⇒ upper bound on dark matter nucleon cross section.

Non-observation of dark matter may indicate that these particles are more feebly interacting than we think.
Thermal Freeze-Out: A Brief Recap

- Particles which were in thermal equilibrium in early universe decouple or freezes out when their rate of interactions can not keep up with the expansion of the universe.

- Boltzmann Equation:

\[
\hat{L} f_a = \left( \frac{\partial}{\partial t} - H p_a \frac{\partial}{\partial p_a} \right) f = \mathcal{C}_{a+b \leftrightarrow i+j} [f_a] \tag{1}
\]

where,

\[
\mathcal{C} [f_a] \simeq \int \frac{d^3 p_b}{2E_b} \frac{d^3 p_i}{2E_i} \frac{d^3 p_j}{2E_j} \times (2\pi)^4 \delta^4 (p_a + p_b - p_i - p_j) \times \\
\left( |\mathcal{M}|^2_{a+b \rightarrow i+j} f_a f_b (1 \pm f_i) (1 \pm f_j) - \\
|\mathcal{M}|^2_{i+j \rightarrow a+b} f_i f_j (1 \pm f_a) (1 \pm f_b) \right) \tag{2}
\]
Eq. (2) can be simplified further using some approximations ...

- CP invariance is assumed i.e.

\[ |M|_i+j \rightarrow a+b|^2 = |M|_{a+b \rightarrow i+j}^2 \equiv |M|_i+j^2 \] (say)

\[ \Rightarrow \]

\[ C[f] \sim \int \frac{d^3 p_a}{2E_a} \frac{d^3 p_b}{2E_b} \frac{d^3 p_i}{2E_i} \frac{d^3 p_j}{2E_j} \times (2\pi)^4 \ \delta^4 (p_a + p_b - p_i - p_j) \times \]

\[ |M|^2 \left( f_a f_b (1 \pm f_i)(1 \pm f_j) - f_i f_j (1 \pm f_a)(1 \pm f_b) \right) \]
Pauli blocking and stimulated emission terms are neglected.

\[ \mathcal{C}[f] \approx \int \frac{d^3 p_a}{2E_a} \frac{d^3 p_b}{2E_b} \frac{d^3 p_i}{2E_i} \frac{d^3 p_j}{2E_j} \times (2\pi)^4 \delta^4(p_a + p_b - p_i - p_j) \times |\mathcal{M}|^2 \left( f_a f_b - f_i f_j \right) \]  

(3)
If particles $i$ and $j$ belong to the thermal soup, they are assumed to follow the classical Maxwell-Boltzmann distribution function i.e. $f \sim e^{-\frac{E}{T}}$

$$f_i f_j = e^{-\frac{E_i + E_j}{T}} = e^{-\frac{E_a + E_b}{T}} = f_a^{eq} f_b^{eq}$$

Eq. (3) simplifies to:

$$\mathcal{C}[f] \simeq \int \frac{d^3 p_a}{2E_a} \frac{d^3 p_b}{2E_b} \frac{d^3 p_i}{2E_i} \frac{d^3 p_j}{2E_j} \times (2\pi)^4 \delta^4(p_a + p_b - p_i - p_j) \times$$

$$\left| \mathcal{M} \right|^2 \left( f_a f_b - f_a^{eq} f_b^{eq} \right)$$  (4)
If \( a \) and \( b \) are both identical particles (say \( \chi \)), then integrating Eq. (4) on both sides by \( \int_0^\infty d^3 p \), and remembering that \( n_\chi \sim \int_0^\infty f(p) d^3 p \), we finally arrive at the conventional form:

\[
\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle \left( n_\chi^2 - n_\chi^{eq} \right)
\]

where, \( \sigma = \sigma_\chi \chi \to \text{all} \) and,

\[
\langle \sigma v \rangle = \frac{1}{8T M_\chi^4 K_2^2 \left( \frac{M_\chi}{T} \right)} \int_4^{\infty} \sigma \left( s - 4M_\chi^2 \right) \sqrt{s} K_1 \left( \frac{\sqrt{s}}{T} \right) ds
\]

• If \( \chi \) can decay then an extra term depletion term will be added to the "Rate Equation"..

\[
\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle \left( n_\chi^2 - n_\chi^{2\text{eq}} \right) - \langle \Gamma_\chi \rangle (n_\chi - n_\chi^{\text{eq}})
\]  

(5)

where, \( \Gamma_\chi \) is the relevant width of \( \chi \),

\[
\langle \Gamma_\chi \rangle = \frac{K_1(T)}{K_2(T)} \Gamma_\chi
\]

\( K_1 \) and \( K_2 \) are modified Bessel function of order 1 and 2.
In terms of $Y$, 

$$\frac{dY_\chi}{dx} = - \frac{s\langle \sigma v \rangle}{H x} \left( Y_\chi^2 - Y_\chi^{2_{eq}} \right)$$

A formal solution of this equation is given by:

The Early Universe, Kolb & Turner
Non-Thermal Dark Matter : Basic Concepts

- Condition for thermalisation of a system of particles in the background of an expanding universe:

\[ \frac{\Gamma}{H} \gg 1 \]

where,

\[ \Gamma = \text{Interaction Rate} = n_{eq} \langle \sigma v \rangle, \quad H = \text{Expansion Rate}. \]

- Freeze-out: \( \frac{\Gamma}{H} \ll 1 \) (\( T < T_{fo} \))

- Freeze-in: \( \frac{\Gamma}{H} \ll 1 \) (at all epochs)
• Number density of non-thermal dark matter is very small ⇒ initial abundance is almost negligible.

• Non-thermal dark matters hence needed to be produced ⇒ comoving number density gradually rises and finally saturates to satisfy relic density.

• Usually the most dominant production channels are from decay of heavier particles.

• The decaying particle may be in thermal equilibrium or can itself be out-of-equilibrium.
• If the decaying particle is in thermal equilibrium, then the usual form of rate equation in terms of $n$ (or $Y$) can be used.

• For example, we can use: $\langle \Gamma \rangle = \frac{K_1(T)}{K_2(T)} \Gamma$ etc ...

• But if the mother particle is not in equilibrium, then such an equation cannot be used.

• Now, $\langle \Gamma \rangle = \frac{\int \Gamma f_{DM}(p) d^3p}{\int f_{DM}(p) d^3p}$ (from definition only...)

$\Rightarrow$ Knowledge of the non-equilibrium distribution function, $f_{\text{mother}}$ is necessary to solve for $f_{DM}$.

$\Rightarrow$ Need for coupled set of Boltzmann equations...
The difference between the freeze-in and freeze-out scenario is best illustrated by the following plots:

Hall et al., arXiv:0911.1120
A new $U(1)_{B-L}$ model

- **Gauge Group**: $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$

- **The Lagrangian**:

  $$\mathcal{L}_{BL} = i \bar{\eta}_L \gamma_\mu D^\mu_\eta \eta_L + i \bar{\xi}_L \gamma_\mu D^\mu_\xi \xi_L + i \sum_{i=1}^2 \bar{\chi}_{iR} \gamma_\mu D^\mu_{\chi_i} \chi_{iR}$$

  $$- \frac{1}{4} F_{ZBL}^{\mu \nu} F_{ZBL, \mu \nu} + \sum_{i=1}^2 (D^u_{\phi_i \phi_i})^\dagger (D_{\phi_i \mu \phi_i})$$

  $$- \sum_{i=1}^2 \left( y_{\xi_i} \bar{\xi}_L \chi_{iR} \phi_2 + y_{\eta_i} \bar{\eta}_L \chi_{iR} \phi_1 + h.c. \right) - V(H, \phi_1, \phi_2) + \mathcal{L}_\Delta$$

Sudhanwa Patra et.al., arXiv:1607.04029
where,

\[
V(H, \phi_1, \phi_2) = \mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \mu_1 \phi_1^\dagger \phi_1 + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \mu_2 \phi_2^\dagger \phi_2 \\
+ \lambda_2 (\phi_2^\dagger \phi_2)^2 + \rho_1 (H^\dagger H)(\phi_1^\dagger \phi_1) + \rho_2 (H^\dagger H)(\phi_2^\dagger \phi_2) + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) \\
+ \mu \left( \phi_2 \phi_1^\dagger + \phi_2^\dagger \phi_1 \right)
\]

- The model is well motivated since:
  - It is anomaly free.
  - It can give rise to neutrino mass.
  - It can accommodate a thermal dark matter.
- The charge assignment is given in the following table:

<table>
<thead>
<tr>
<th>Field</th>
<th>SU(2)_L charge</th>
<th>U(1)_Y charge</th>
<th>U(1)_{B-L} charge</th>
<th>VEV</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM Fermions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( l_L \equiv (v_L, e_L)^T )</td>
<td>2</td>
<td>(-\frac{1}{2})</td>
<td>(-1)</td>
<td></td>
</tr>
<tr>
<td>( Q_L \equiv (u_L, d_L)^T )</td>
<td>2</td>
<td>(\frac{1}{6})</td>
<td>(\frac{1}{3})</td>
<td>0</td>
</tr>
<tr>
<td>( e_R )</td>
<td>1</td>
<td>(-1)</td>
<td>(-1)</td>
<td></td>
</tr>
<tr>
<td>( u_R )</td>
<td>1</td>
<td>(\frac{2}{3})</td>
<td>(\frac{1}{3})</td>
<td></td>
</tr>
<tr>
<td>( d_R )</td>
<td>1</td>
<td>(-\frac{1}{3})</td>
<td>(\frac{1}{3})</td>
<td></td>
</tr>
<tr>
<td>BSM Fermions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \xi_L )</td>
<td>1</td>
<td>0</td>
<td>(\frac{4}{3})</td>
<td>0</td>
</tr>
<tr>
<td>( \eta_L )</td>
<td>1</td>
<td>0</td>
<td>(\frac{1}{3})</td>
<td></td>
</tr>
<tr>
<td>( \chi_{1R} )</td>
<td>1</td>
<td>0</td>
<td>(\frac{2}{3})</td>
<td></td>
</tr>
<tr>
<td>( \chi_{2R} )</td>
<td>1</td>
<td>0</td>
<td>(\frac{2}{3})</td>
<td></td>
</tr>
<tr>
<td>Scalars</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( H )</td>
<td>2</td>
<td>(\frac{1}{2})</td>
<td>0</td>
<td>(\nu)</td>
</tr>
<tr>
<td>( \phi_1 )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>(\nu_1)</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>(\nu_2)</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>3</td>
<td>1</td>
<td>(-2)</td>
<td>(\nu_{\Delta})</td>
</tr>
</tbody>
</table>
Points to highlight regarding the model:

- The scalars acquire vev after symmetry breaking:

\[
H^0 = \frac{1}{\sqrt{2}}(v + \tilde{h}) + \frac{i}{\sqrt{2}} \tilde{G},
\]

\[
\phi_1 = \frac{1}{\sqrt{2}}(v_1 + \tilde{h}_1) + \frac{i}{\sqrt{2}} \tilde{A}_1,
\]

\[
\phi_2 = \frac{1}{\sqrt{2}}(v_2 + \tilde{h}_2) + \frac{i}{\sqrt{2}} \tilde{A}_2,
\]

- The fields \((\tilde{h}_1, \tilde{h}_2, \tilde{h}_3)\) mix with each other
  \[\Rightarrow 3 \times 3\] PMNS-like mixing matrix parametrised by \(\theta_{12}, \theta_{13}, \theta_{23}\).

- \(\tilde{A}_1\) and \(\tilde{A}_2\) also mix together to give rise to a massive physical pseudoscalar with mass \(M_A\).
• For the Fermions:

\[ \mathcal{L}_{\text{fermion-mass}} = \left( \begin{array}{cc} \bar{\xi}_L & \bar{\eta}_L \end{array} \right) \mathcal{M}_{\text{fermion}} \left( \begin{array}{c} \chi^1_R \\ \chi^2_R \end{array} \right) \]

where,

\[ \mathcal{M}_{\text{fermion}} = \left( \begin{array}{cc} y_{\xi} v_2 & y_{\xi} v_2 \\ y_{\eta} v_1 & y_{\eta} v_1 \end{array} \right) \]

• The mass and gauge basis states are related by:

\[ \left( \begin{array}{c} \xi_L \\ \eta_L \end{array} \right) = \mathcal{U}_L \left( \begin{array}{c} \psi^2_L \\ \psi^1_L \end{array} \right), \quad \left( \begin{array}{c} \chi^1_R \\ \chi^2_R \end{array} \right) = \mathcal{U}_R \left( \begin{array}{c} \psi^2_R \\ \psi^1_R \end{array} \right). \]

where,

\[ \mathcal{U}_{L,R} = \left( \begin{array}{cc} \cos \theta_{L,R} & \sin \theta_{L,R} \\ -\sin \theta_{L,R} & \cos \theta_{L,R} \end{array} \right) \]

• The physical states are \( \psi_1 = \psi^1_L + \psi^1_R \) and \( \psi_2 = \psi^2_L + \psi^2_R \).
• The breaking of $U(1)_{B-L}$ gauge group also gives the extra gauge boson its mass.

$$M^2_{Z_{BL}} = \left( \frac{g_{BL} v_2}{\beta} \right)^2 (1 + 4\beta^2), \text{ where } \beta = \frac{v_2}{v_1}$$

• The set of independent parameters relevant for our analysis are as follows:
  $\theta_{12}, \theta_{13}, \theta_{23}, \theta_L, \theta_R, M_{h_2}, M_{h_3}, M_A, M_{\psi_1}, M_{\psi_2}, M_{Z_{BL}}, g_{BL}$ and $\beta$. 

Non-thermal Dark Matter

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• $\psi_1$ is our dark matter candidate.

• The dominant production modes of this dark matter are:

  $h_1 \rightarrow \psi_1 \psi_1$, $h_2 \rightarrow \psi_1 \psi_1$, $Z_{BL} \rightarrow \psi_1 \psi_1$.

• The decay widths strongly depend on $g_{BL}$ and for non-thermality $g_{BL} \sim 10^{-10}$.

• Very small $g_{BL} \Rightarrow Z_{BL}$ is also not in equilibrium.

  $Z_{BL}$ production : $h_2 \rightarrow Z_{BL}Z_{BL}$.

  $Z_{BL}$ depletion : $Z_{BL} \rightarrow f \bar{f}$ and $Z_{BL} \rightarrow \psi_1 \bar{\psi}_1$. 
• $h_2$ can however equilibrate through their mixing with SM Higgs ($h_1$).

• We can not solve for $Y_{\psi_1}$ directly, rather we need to find $f_{Z_{BL}}$ and consequently solve for $f_{\psi_1}$ coupled Boltzmann equations.

$$
\begin{align*}
\hat{L} f_{Z_{BL}} &= C_{h_2 \rightarrow Z_{BL} Z_{BL}} + C_{Z_{BL} \rightarrow all} \\
\hat{L} f_{\psi_1} &= \sum_{s=h_1, h_2} C_{S \rightarrow \overline{\psi}_1 \psi_1} + C_{Z_{BL} \rightarrow \overline{\psi}_1 \psi_1}
\end{align*}
$$

• SM Higgs ($h_1$) gets its mass after EWPT. So, when $T > T_{EWPT}$, $h_1$ decay is not allowed.
Simplifying the Liouville Operator

- For isotropic homogeneous universe, FRW metric gives:

\[ \hat{L} = \left( \frac{\partial}{\partial t} - H p \frac{\partial}{\partial p} \right) \]

- Using conservation of entropy:

\[ \frac{dT}{dt} = -HT \left( 1 + \frac{T g_s'(T)}{3g_s(T)} \right)^{-1} \]

- Change of coordinates:

\[ r = \frac{m_0}{T}, \]

\[ \xi_p = \left( \frac{g_s(T_0)}{g_s(T)} \right)^{1/3} \frac{p}{T}, \]
• The Liouville operator simplifies to:

\[ \hat{L} = rH \left(1 + \frac{Tg'_s}{3g_s} \right)^{-1} \frac{\partial}{\partial r}, \]

• Chosen Benchmark (\(\beta = 10^{-3}\)):

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Corresponding values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_{Z_{BL}})</td>
<td>1 TeV</td>
</tr>
<tr>
<td>(M_{h_2})</td>
<td>5 TeV</td>
</tr>
<tr>
<td>(M_{\psi_1})</td>
<td>10 GeV</td>
</tr>
<tr>
<td>(g_{BL})</td>
<td>(1.75 \times 10^{-11})</td>
</tr>
<tr>
<td>(\theta_{12})</td>
<td>0.1000 rad</td>
</tr>
<tr>
<td>(\theta_{13})</td>
<td>0.01 rad</td>
</tr>
<tr>
<td>(\theta_{23})</td>
<td>0.06 rad</td>
</tr>
<tr>
<td>(\theta_L = \theta_R)</td>
<td>(\pi/4) rad</td>
</tr>
</tbody>
</table>
Calculation of Collision Terms

- $Z_{BL}(p) \rightarrow f(p') \bar{f}(q')$

$$\mathcal{C}_{Z_{BL} \rightarrow ff}[f_{Z_{BL}}(p)] = \frac{1}{2E_p} \int \frac{g_f d^3 p'}{(2\pi)^3 2E_{p'}} \frac{g_f d^3 q'}{(2\pi)^3 2E_{q'}} \times$$

$$(2\pi)^4 \delta^4(\tilde{p} - \tilde{p'} - \tilde{q'}) \times |\mathcal{M}|^2 \times [-f_{Z_{BL}}(p)]$$

$$= -f_{Z_{BL}}(p) \times \frac{1}{2E_p} \int \frac{g_f d^3 p'}{(2\pi)^3 2E_{p'}} \frac{g_f d^3 q'}{(2\pi)^3 2E_{q'}} (2\pi)^4 \delta^4(\tilde{p} - \tilde{p'} - \tilde{q'}) \times |\mathcal{M}|^2$$

- In a general frame moving with momentum 'p' :

$$\Gamma'_{Z_{BL} \rightarrow ff} = \frac{1}{2E_p} \int \frac{g_f d^3 p'}{(2\pi)^3 2E_{p'}} \frac{g_f d^3 q'}{(2\pi)^3 2E_{q'}} (2\pi)^4 \delta^4(\tilde{p} - \tilde{p'} - \tilde{q'}) \times |\mathcal{M}|^2$$
\[ \mathcal{C}_{Z_{BL} \rightarrow f \bar{f}}[f_{Z_{BL}}(\xi_p)] = -f_{Z_{BL}}(\xi_p) \times \left[ \Gamma_{Z_{BL} \rightarrow f \bar{f}}^{\text{rest}} \times \frac{M_{Z_{BL}}}{E_{Z_{BL}}} \right] \]

\[ = -f_{Z_{BL}}(\xi_p) \times \Gamma_{Z_{BL} \rightarrow f \bar{f}}^{\text{rest}} \times \frac{r_{Z_{BL}}}{\sqrt{\xi_p^2 B(r)^2 + r_{Z_{BL}}^2}} \]

- Hence,

\[ \mathcal{C}_{Z_{BL} \rightarrow all} = -f_{Z_{BL}}(\xi_p) \times \Gamma_{Z_{BL} \rightarrow all} \times \frac{r_{Z_{BL}}}{\sqrt{\xi_p^2 B(r)^2 + r_{Z_{BL}}^2}} \]

where,

\[ \left( \frac{g_s(T)}{g_s(T_0)} \right)^{1/3} = \left( \frac{g_s(M_{sc}/r)}{g_s(M_{sc}/r_0)} \right)^{1/3} \equiv B(r) \]
\[ h_2(k) \rightarrow Z_{BL}(p) Z_{BL}(q') \]

\[
\mathcal{C}_{h_2 \rightarrow Z_{BL}Z_{BL}}[f_{Z_{BL}}(p)] = 2 \times \frac{1}{2 E_p} \int \frac{g_{h_2} d^3 k}{(2\pi)^3} \frac{g_{Z_{BL}} d^3 q'}{(2\pi)^3} \times \\
\frac{2}{2 E_k} \frac{2}{2 E_{q'}} \times \\
(2\pi)^4 \delta^4(\vec{k} - \vec{p} - \vec{q'}) \times |\mathcal{M}|^2 \\
|_{h_2 \rightarrow Z_{BL}Z_{BL}} \\
[f_{h_2}(1 \pm f_{Z_{BL}})(1 \pm f_{Z_{BL}}) - f_{Z_{BL}} f_{Z_{BL}} (1 \pm f_{h_2})] \\
= 2 \times \frac{1}{2 E_p} \int \frac{g_{h_2} d^3 k}{(2\pi)^3} \frac{g_{Z_{BL}} d^3 q'}{(2\pi)^3} (2\pi)^4 \delta^4(\vec{k} - \vec{p} - \vec{q'}) \times |\mathcal{M}|^2 \times [f_{h_2}(k)].
\]

**Here,**

\[
|\mathcal{M}|^2 |_{h_2 \rightarrow Z_{BL}Z_{BL}} = \frac{g_{h_2}^2 Z_{BL} Z_{BL}}{2 \times 9} \left( 2 + \frac{(E_p E_{q'} - \vec{p} \cdot \vec{q'})^2}{M_{Z_{BL}}^4} \right)
\]
\[ \mathcal{C}^{h_2\rightarrow Z_{BL}Z_{BL}} [f_{Z_{BL}}(p)] = \frac{g_{h_2Z_{BL}Z_{BL}}^2}{6 \left(4\pi\right)^2} \frac{1}{E_p} \int \frac{k^2 dk \ d(\cos \theta)}{E_k E_q'(p, k, \cos \theta)} \times \]

\[ \delta(E_k - E_p - E_q'(p,k,\theta)) \times \left( 2 + \frac{(E_p E_q'(k, p, \theta) + p^2 - pk \cos \theta)^2}{M_{Z_{BL}}^4} \right) \times [f_{h_2}(k)] \]

where, \( E_q' = \sqrt{k^2 + p^2 + M_{Z_{BL}}^2 - 2pk \cos \theta} \)

- In terms of the chosen coordinates,

\[ E_q' = T \sqrt{\xi_k^2 \mathcal{B}(r)^2 + \xi_p^2 \mathcal{B}(r)^2 + r_{Z_{BL}}^2 - 2 \mathcal{B}(r)^2 \xi_k \xi_p \cos \theta} \]
\[ C_{h_2 \to Z_{BL} Z_{BL}} = \frac{r}{8 \pi M_{sc}} \frac{B^{-1}(r)}{\xi_p \sqrt{\xi_p^2 B(r)^2 + \left( \frac{M_{Z_{BL}} r}{M_{sc}} \right)^2}} \times \]

\[ \frac{g_{h_2 Z_{BL} Z_{BL}}^2}{6} \left( 2 + \frac{(M_{Z_{BL}}^2 - 2M_{h_2}^2 Z_{BL})^2}{4M_{Z_{BL}}^4} \right) \times \]

\[ \left( -e^{-\sqrt{\left( \xi_k_{\min}(\xi_p,r) \right)^2 B(r)^2 + \left( \frac{M_{h_2} r}{M_{sc}} \right)^2}} - e^{-\sqrt{\left( \xi_k_{\max}(\xi_p,r) \right)^2 B(r)^2 + \left( \frac{M_{h_2} r}{M_{sc}} \right)^2}} \right) \]
The distribution functions:

\[ \beta = 0.001 \]

\[ f_Z(B L)(\xi_p, r) \]

\[ f_{\nu_1}(\xi_p, t) \]
• Knowing the distribution function, we are finally able to calculate the comoving number density

\[ Y \equiv \frac{n}{s}. \]
Comparing with the approximate solution

- What if we had assumed that the system is close to equilibrium and used rate equations in terms of $Y$. 

\[ Y (\equiv \frac{n}{s}) \]

\[ r (\equiv \frac{M_{h1}}{T}) \]

\[ \Gamma_{ZBL} \rightarrow \text{all} > N_{Th} \]

\[ \Gamma_{ZBL} \rightarrow \bar{\psi}_1 \psi_1 > N_{Th} \]

\[ \Gamma_{ZBL} \rightarrow \text{all} > \text{Th} \]

\[ \Gamma_{ZBL} \rightarrow \bar{\psi}_1 \psi_1 > \text{Th} \]

\[ M_{\psi_1} = 10 \text{ GeV}, \ M_{Z_{BL}} = 1 \text{ TeV}, \ M_{h2} = 5 \text{ TeV}, \ g_{BL} = 1.75 \times 10^{-11} \]
THANK YOU