on the quantum statistical physics of dark matter freeze-out [1]

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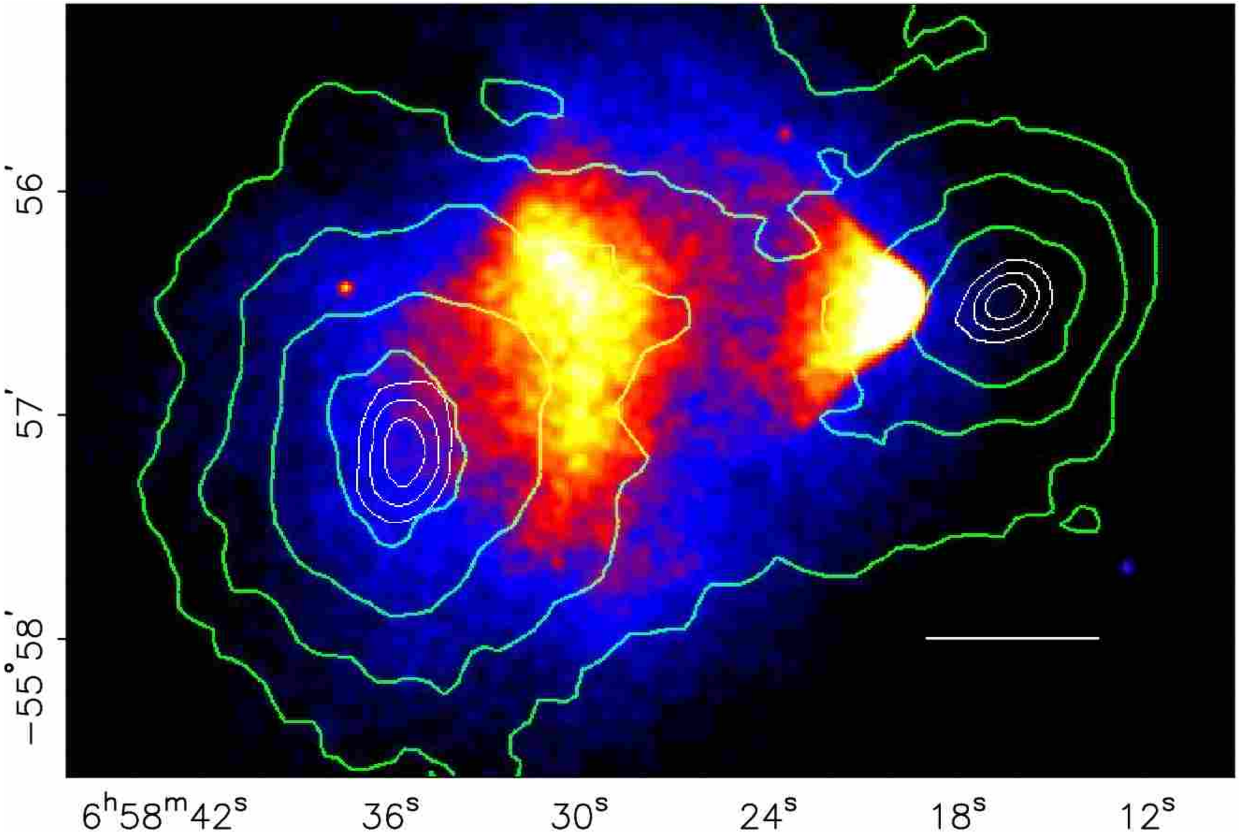
there is “dark matter”



[2] D. Clowe *et al*, *A direct empirical proof of the existence of DM*, astro-ph/0608407

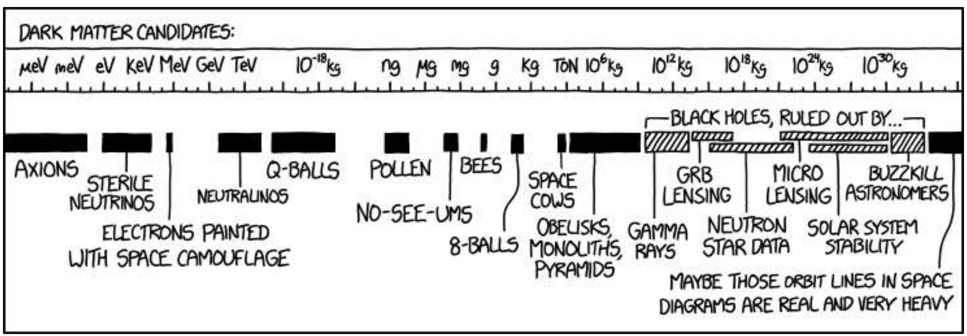
what is dark matter?

yet to be discovered particles? basic requirements:

- not visible $\Rightarrow$ electrically neutral
- around long ago & still today $\Rightarrow$ stable or very long-lived
- correct structure formation long ago $\Rightarrow$ rather heavy

known particles fail to satisfy these requirements

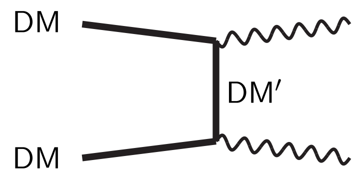
candidates from `https://xkcd.com/2035/`:



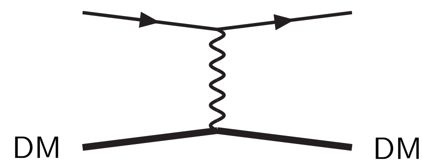
in this talk: “wimp paradigm”

postulate the existence of weakly interacting massive particles (“heavy neutrinos”) which cannot decay and are thus stable

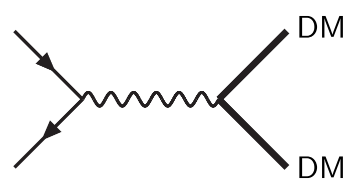
“indirect detection” from galactic center:



“direct detection” by nuclear recoil:



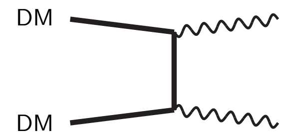
“collider search” through missing energy:



text-book wimp is in trouble

lee-weinberg equation[3] ($n =$ number density, $H =$ Hubble rate)

$$(\partial_t + 3H)n = -\langle \sigma v_{\text{rel}} \rangle \left(n^2 - n_{\text{eq}}^2\right)$$



linearize around equilibrium:

$$n = n_{\text{eq}} + \delta n\ , \quad n^2 - n_{\text{eq}}^2 \approx 2 n_{\text{eq}} \delta n$$

parametrize cross section:

$$\langle \sigma v_{\text{rel}} \rangle \equiv \frac{\alpha^2}{M^2}\ , \quad M \equiv M_{\text{DM}}$$

[3] B.W. Lee and S. Weinberg, *Cosmological Lower Bound...*, Phys. Rev. Lett. 39 (77) 165

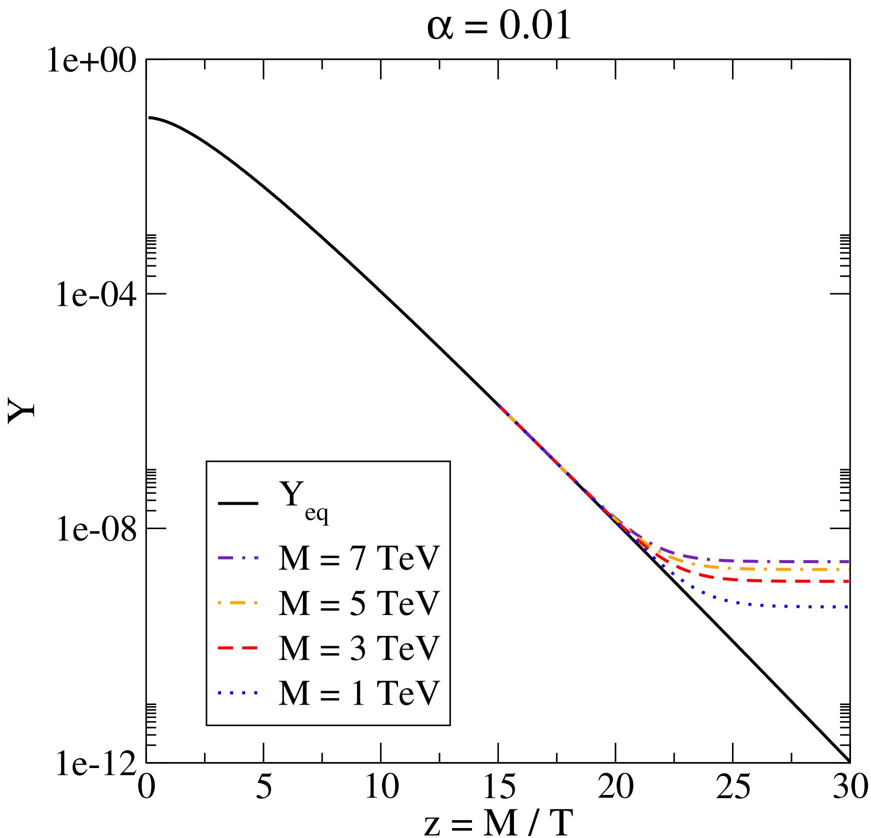
$$\Rightarrow \boxed{(\partial_t + 3H)n \approx -\frac{2\alpha^2 n_{\text{eq}}}{M^2}\,\delta n}$$

equilibrium number density is a known function of T, M:

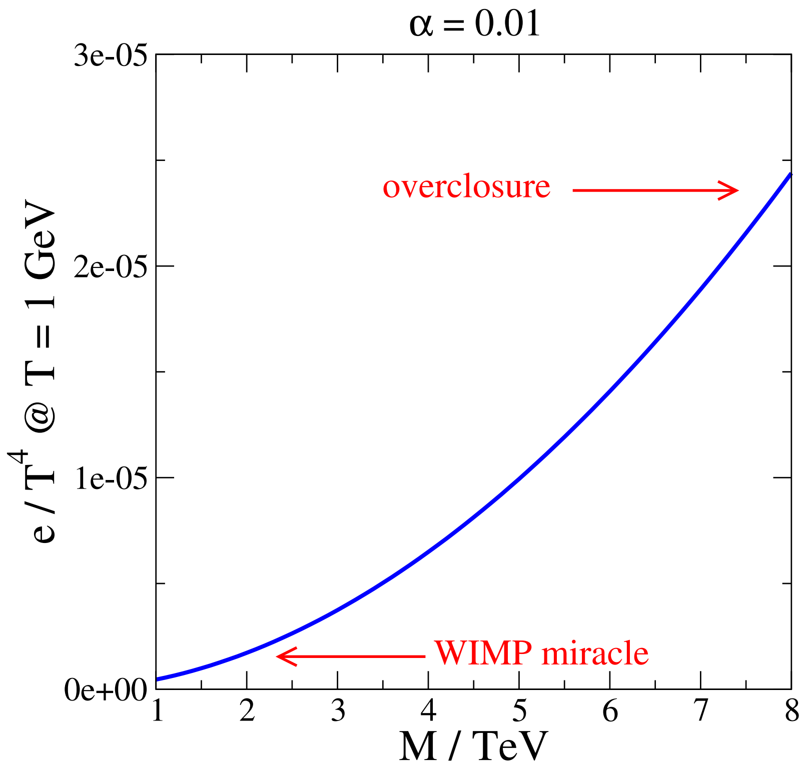
$$n_{\text{eq}} \propto \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{1}{e^{\sqrt{p^2+M^2}/T} \pm 1} \approx \left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T}$$

the right-hand side becomes very small if $\alpha^2 n_{\text{eq}}/M^2 \ll H$

indeed a numerical solution shows a "freeze-out" ($Y \equiv n/s$):



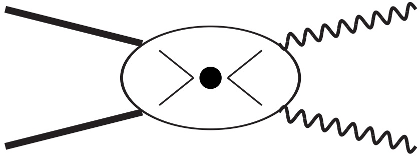
$\Rightarrow$ final energy density ($e \equiv Mn$) grows faster than $\sim M$:



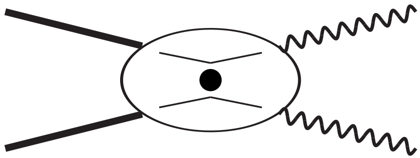
null searches at LHC push up M, so danger of "overclosure"

could increased $\langle \sigma v_{\text{rel}} \rangle$ help?

large cross sections could originate via “resonant” effects

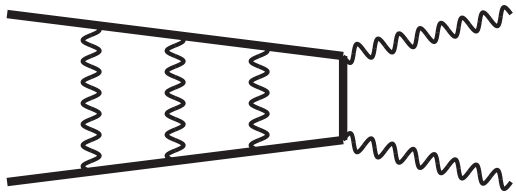


s-channel



t-channel $\Rightarrow$ this talk

simplest t-channel enhancement:[4] "sommerfeld effect"[5]



$$\langle \sigma v_{\text{rel}} \rangle \longrightarrow \langle \sigma_{\text{tree}} v_{\text{rel}} S(v_{\text{rel}}) \rangle$$

for attractive Coulomb-like interaction:

$$S(v_{\text{rel}}) \sim \frac{\alpha}{v_{\text{rel}}} \quad \text{for} \quad v_{\text{rel}} \lesssim \alpha$$

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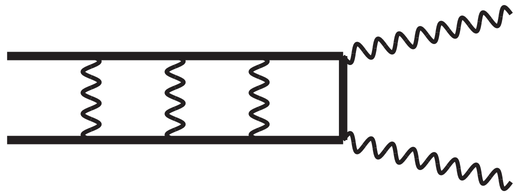
[4] e.g. J. Hisano *et al*, *Non-perturbative effect on ... dark matter*, hep-ph/0610249

[5] e.g. L.D. Landau and E.M. Lifshitz, *Quantum Mechanics*, Third Edition, §136

in similar spirit but more recent:[6] bound states



$$M_{\text{bound}} = 2M - \Delta E \;\Rightarrow\; e^{-M_{\text{bound}}/T} > e^{-2M/T}$$

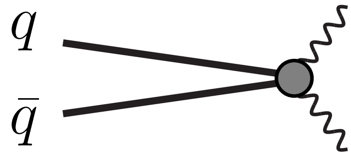
this is quantum mechanics in a statistical background

(typically the dark sector contains several species, DM and DM',
and perhaps only one of them forms bound states)

[6] e.g. B. von Harling and K. Petraki, *Bound-state formation for ...*, 1407.7874

some quantum statistical physics

physical picture of the decay



energy released in the inelastic reaction is $2M \gg T \Rightarrow$ the "hard" annihilation process is effectively local[7,8]

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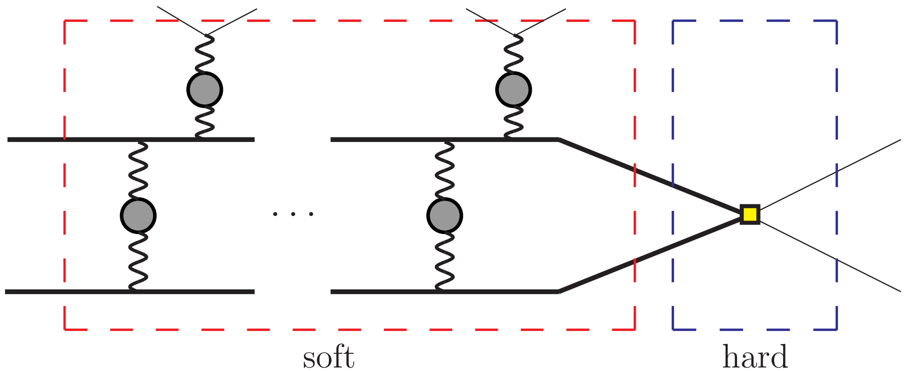
7 G.T. Bodwin *et al*, *Rigorous QCD analysis of ... annihilation ...*, hep-ph/9407339

8 e.g. L.S. Brown and R.F. Sawyer, *Nuclear reaction rates in a plasma*, astro-ph/9610256

but there are also soft effects

before annihilation there's plenty of time for interactions:[9]



⇒ "debye screening", "landau damping", "salpeter correction"

[9] plot from S. Biondini and ML, *Re-derived overclosure bound...*, 1706.01894

definition of thermally averaged sommerfeld factor

if θ, η annihilate DM and $\bar{\text{DM}}$, then the simplest annihilation operator is the "s-wave" or "singlet-channel" one:

$$\frac{1}{M^2}\frac{1}{\mathcal{Z}}\sum_{m\neq 0} e^{-E_m/T} \underbrace{\langle m|\theta^\dagger\eta^\dagger\,\eta\theta|m\rangle}_{\sum_n \langle m|\theta^\dagger\eta^\dagger|n\rangle\langle n|\eta\theta|m\rangle} \equiv \langle \mathcal{O}_s\rangle_T$$

denote by $\bar{S}_i$ enhancement factor over tree-level

$$\bar{S}_i \equiv \frac{\langle \mathcal{O}_i\rangle_T / \langle \mathcal{O}_i\rangle_{T,\text{tree}}}{n^2_{\text{eq}}/(n^2_{\text{eq}})_{\text{tree}}}$$

perturbative evaluation

the 2-body problem can be reduced to a 1-body problem:

$$E_m \quad =: \quad E' + \underbrace{\left[2M_{\text{rest}} + \frac{k^2}{4M_{\text{kin}}} \right]}_{\text{center-of-mass energy}} \quad .$$

converting $\sum_m$ into integrals over E' and k and carrying out the integral over k we are left with

$$\langle \theta^\dagger \eta^\dagger \, \eta \theta \rangle_T = e^{-2M_{\text{rest}}/T} \left(\frac{M_{\text{kin}} T}{\pi} \right)^{3/2} \int_{-\Lambda}^{\infty} \frac{\mathrm{d}E'}{\pi} e^{-E'/T} \rho_s(E')$$

how to obtain the "spectral function" $\rho_s(E')$?

it comes from a Schrödinger equation for a Green's function:

$$
\begin{aligned}
[H_T - i\,\Gamma_T(r) - E']G(E';\mathbf{r},\mathbf{r}') &= \delta^{(3)}(\mathbf{r}-\mathbf{r}')\;, \\
\lim_{\mathbf{r},\mathbf{r}'\to\mathbf{0}} \operatorname{Im} G(E';\mathbf{r},\mathbf{r}') &= \rho(E')
\end{aligned}
$$

here the Hamiltonian has a standard from

$$
H_T = -\frac{\nabla_{\mathbf{r}}^2}{M} + V_T(r)\;, \quad r = |\mathbf{r}|\;,
$$

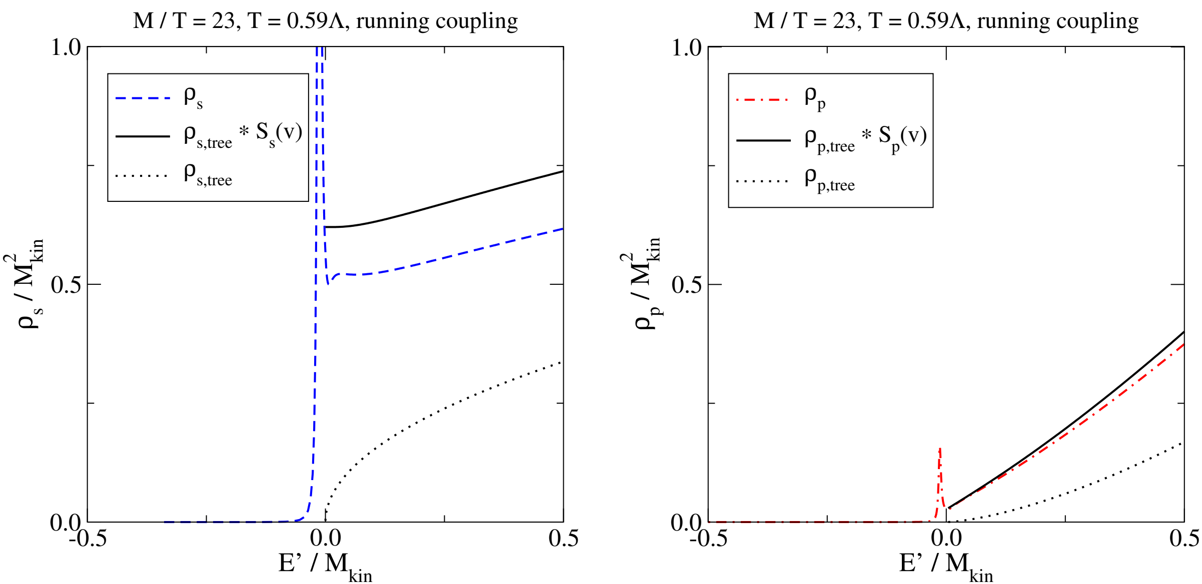
whereas $-i\Gamma_T(r)$ accounts for real scatterings with the plasma

structures at leading non-trivial order

$$V_T(r) = -\underbrace{\frac{g_s^2 C_F}{4\pi}\Big[m_D}_{\text{"salpeter correction"}} + \underbrace{\frac{\exp(-m_D r)}{r}}_{\text{"debye screening"}}\Big]$$

$$\Gamma_T(r) = \underbrace{\frac{g_s^2 C_F T}{2\pi}\int_0^\infty \frac{dz\, z}{(z^2+1)^2}\Big[1 - \frac{\sin(z m_D r)}{z m_D r}\Big]}_{\text{"landau damping"}}$$

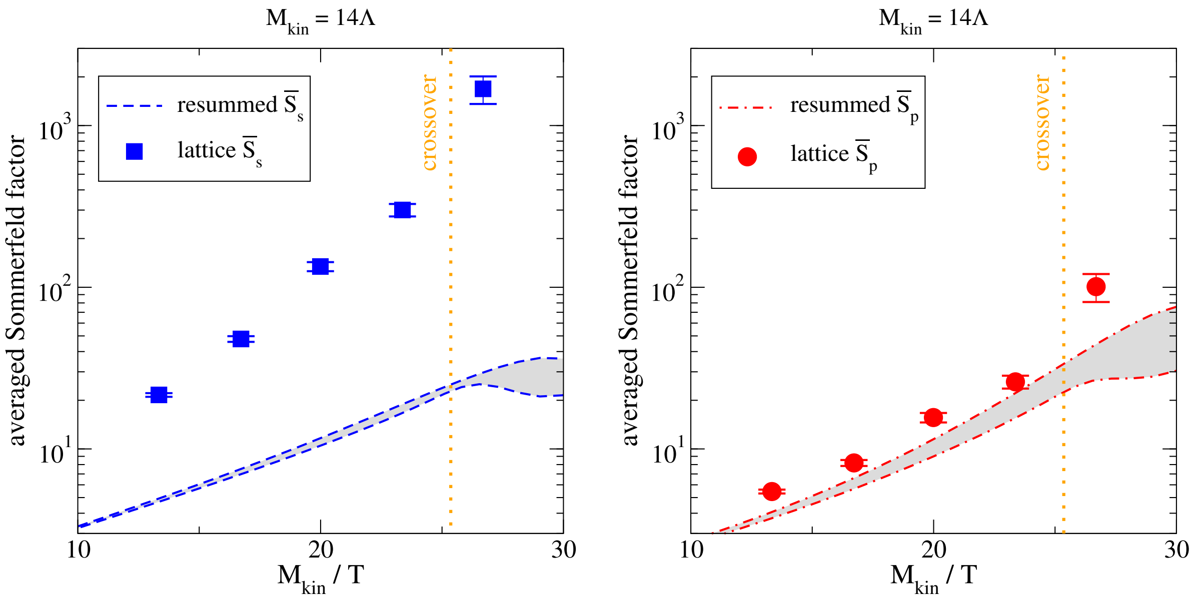
examples of perturbative evaluations of $\rho_{s,p}(E')$[10]



s-wave $\qquad$ p-wave

[10] S. Kim and ML, … *thermally averaged p-wave Sommerfeld factor*, 1904.07882

comparisons with non-perturbative data[10]



s-wave p-wave

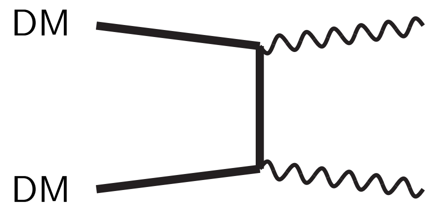
time $\longrightarrow$

relation to observational constraints

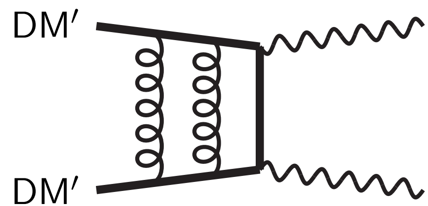
why is cosmology different from the present day?

long ago: $t \sim 10^{-12}$ s, $T \sim 100$ GeV

DM annihilation:



DM' annihilation:

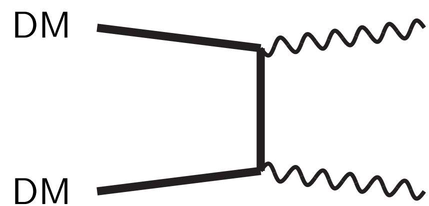


DM $\leftrightarrow$ DM' is in thermal equilibrium $\Rightarrow$ annihilation can proceed through the heavier DM' channel if this is more efficient

today the DM' channel should not be active any more

present universe: $t \sim 10^{17}$ s, $T \ll$ eV

DM annihilation is active in galactic centers, but with a small $\langle \sigma v_{\text{rel}} \rangle$ (e.g. p-wave suppressed)

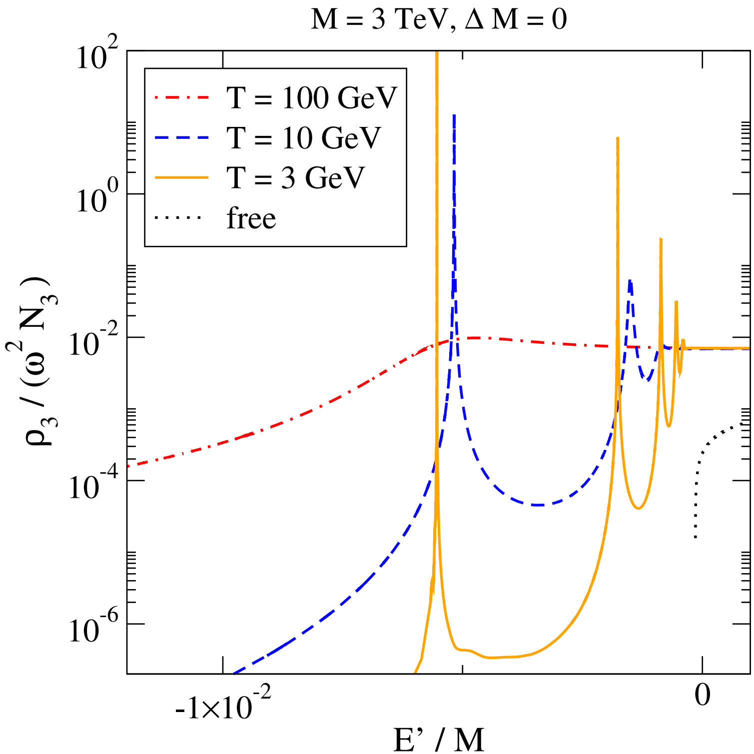


the heavier DM' decayed long ago, and plays no practical role in cosmology (however it can be searched for at the LHC)

$$\Rightarrow \; 0 < \frac{M_{\text{DM'}} - M}{M} \ll 1 \text{ leads to interesting effects}$$

relation to heavy ion collisions

example of T-dependence of a DM' spectral function



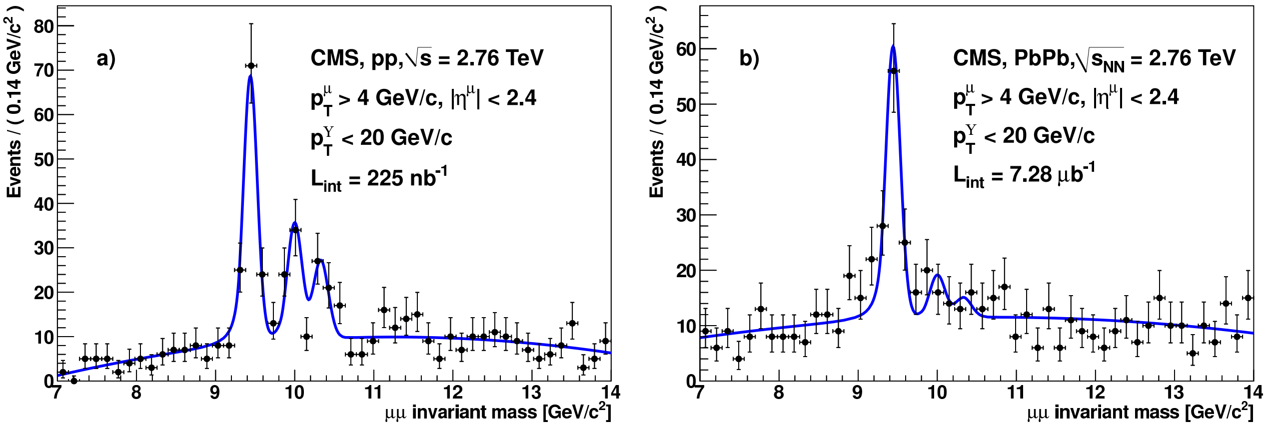
what are we talking about on the QCD side?

$\Rightarrow$ charm and bottom quarks at $T \sim 150...450$ MeV

$\Rightarrow$ non-equilibrium: gluons and light quarks are thermalized, charm and bottom quarks are "probes" in this background

$\Rightarrow$ bottom quark is non-relativistic ($m_b \sim 10...30\ T$), charm quark is a borderline case ($m_c \sim 2...6\ T$)

“melting” of bottomonium resonances can be observed! [11]



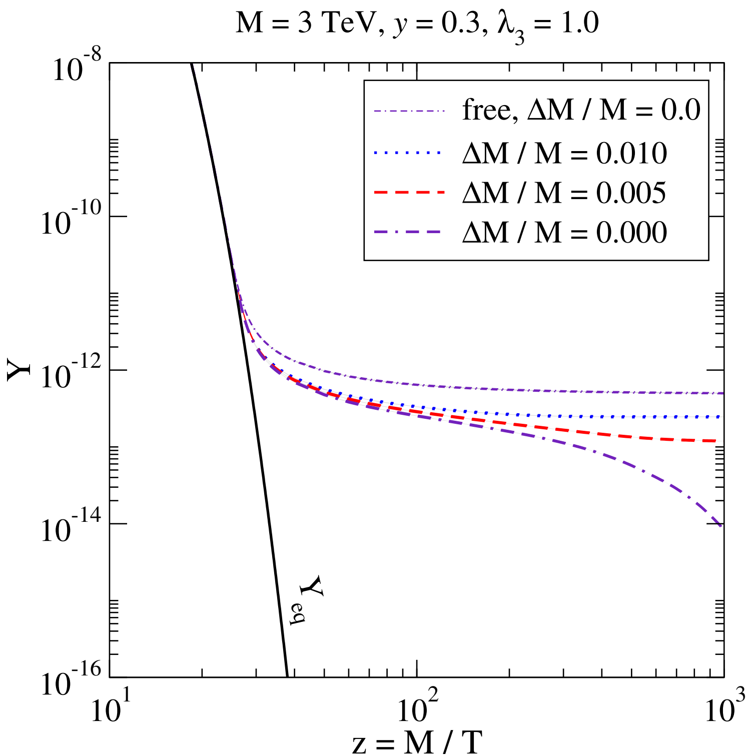
this follows a general pattern predicted theoretically [12]

[11] CMS Collaboration, *Suppression of excited* Υ *states* ... , 1105.4894

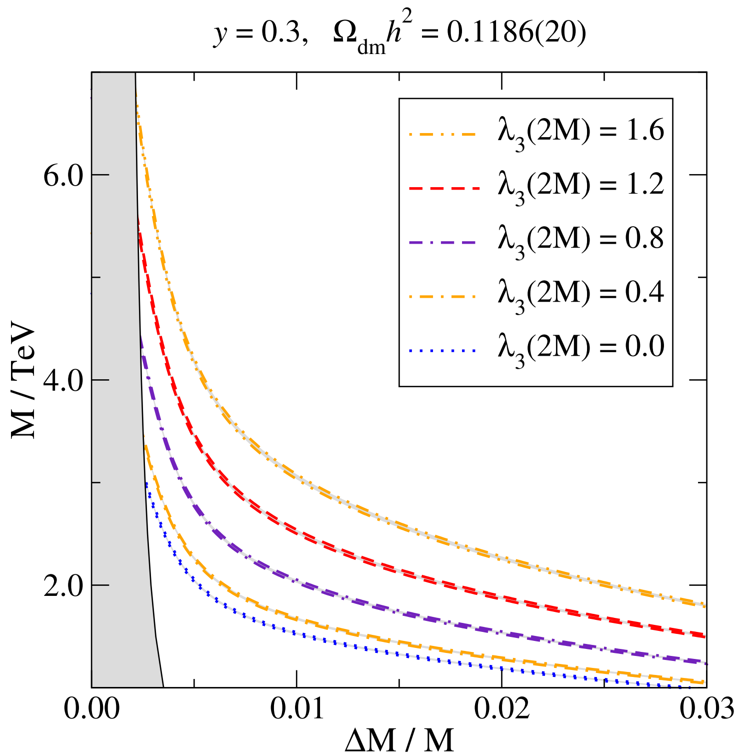
[12] e.g. F. Karsch, D. Kharzeev and H. Satz, ... *charmonium dissociation*, hep-ph/0512239

recap: what did we learn for cosmology?

if $\Delta M \equiv M_{\text{DM'}} - M$ is too small, late times problematic



keeping $\Delta M/M$ non-zero, large M is indeed possible



summary

$\Rightarrow$ apart from model uncertainties, generic dark matter studies contain theoretical ones

$\Rightarrow$ both quantum-mechanical effects (bound states, multiple interactions) and statistical physics phenomena (debye screening, $2 \to 2$ scatterings on plasma particles) may play a role

$\Rightarrow$ a strongly interacting DM' may increase $\langle \sigma v_{\text{rel}} \rangle$

$\Rightarrow$ model-specific studies are needed for definite conclusions