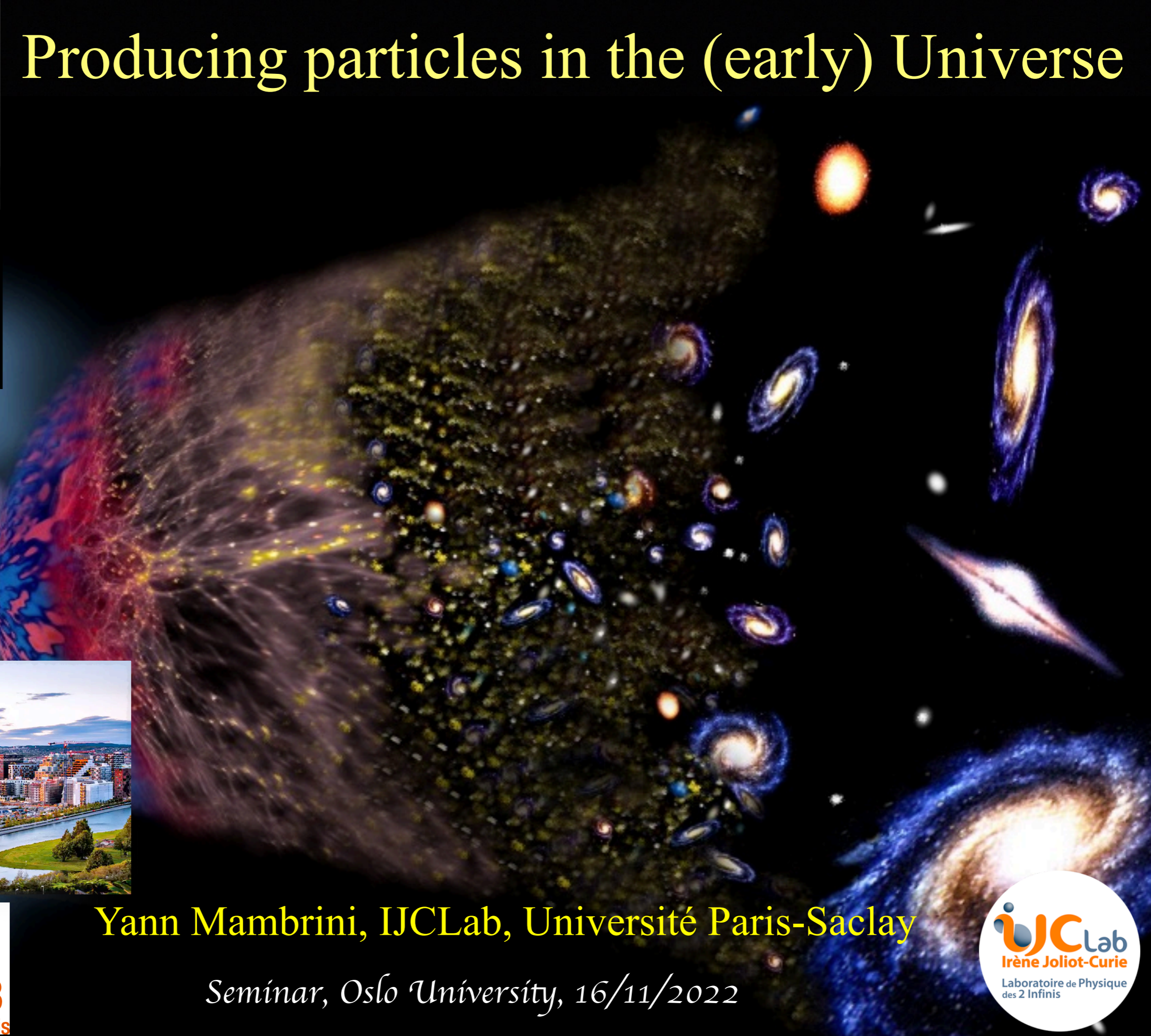




*« Available energy is the main object at stake in the struggle for existence and the evolution of the world. »*

**L. Boltzmann**

# Producing particles in the (early) Universe



Yann Mambrini, IJCLab, Université Paris-Saclay

Seminar, Oslo University, 16/11/2022



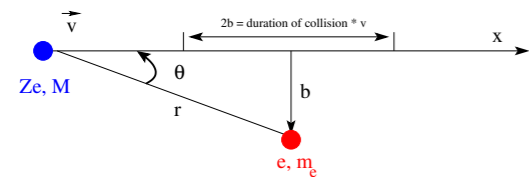
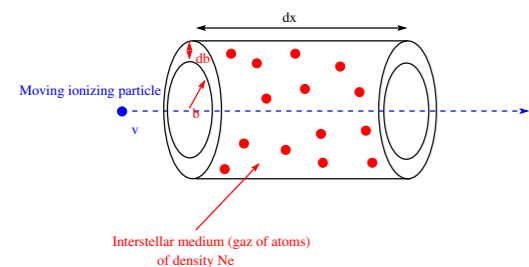
# PhD students :



Simon Clery (IJCLab, Paris-Saclay)



Sarunas Verner (Minneapolis U.)

Fig. 5.9 Interaction of a high energy particle of charge  $Ze$  with an electron at rest.Fig. 5.10 Moving particle in an interstellar medium of density  $N_e$ .

distance at which the influence of the traveling particle on the electron is negligible. It corresponds roughly to the time when the orbital period is lower than the typical interaction time. In other words, if the electron takes more time to move around the nucleus than to interact with the moving particle, the electromagnetic influence of the later becomes weak. If one write  $\tau$  the interacting time and  $v_0$  the frequency of the rotating electron in the atom ( $v_0 = \omega_0/2\pi$ ), it corresponds to

$$\tau \approx \frac{2b}{v} < \frac{1}{v_0} \Rightarrow b < \frac{v}{2v_0} = b_{max} \quad (5.37)$$

The lower limit  $b_{min}$  can be obtained if we suppose, by a quantum treatment and the application of the uncertainty principle, that the maximum energy transfer is  $\Delta p_{max} = 2m_e v$  (because as we discussed earlier, the maximum velocity transferred to the electron is  $2v$ ) from  $\Delta p \Delta x \geq \hbar$  (Heisenberg principle) we have  $\Delta x \geq \hbar/2m_e v$ . We can then write

The two parts of the Lagrangian one needs to compute the scalar annihilation of Dark Matter  $SS \rightarrow h \rightarrow \bar{f}f$  are (see B.235)<sup>9</sup>

$$\begin{aligned} \mathcal{L}_{HSS} &= -\lambda_{HS} \frac{M_W}{2g} hSS \rightarrow C_{HSS} = -i \frac{\lambda_{HS} M_W}{g} \\ \text{and } \mathcal{L}_{Hff} &= -\frac{gm_f}{2M_W} h\bar{f}f \rightarrow C_{Hff} = -i \frac{gm_f}{2M_W} \end{aligned} \quad (B.145)$$

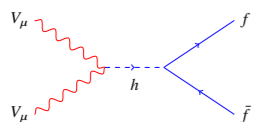
which gives

$$|\mathcal{M}|^2 = \frac{\lambda_{HS}^2 m_f^2 (s/2 - 2m_f^2)}{(s - M_H^2)^2 + \Gamma_H^2 M_H^2} \quad (B.146)$$

$\Gamma_H$  being the width of the Higgs boson (including its own decay into  $SS$ , see next section). When one implements this value of  $|\mathcal{M}|^2$  into Eq.(B.111) one obtains after simplification

$$\langle \sigma v \rangle_{f\bar{f}}^S = \frac{|\mathcal{M}|^2}{8\pi s} \sqrt{1 - \frac{m_f^2}{M_S^2}} = \frac{\lambda_{HS}^2 (M_S^2 - m_f^2) m_f^2}{16\pi M_S^2 (4M_S^2 - M_H^2)^2} \sqrt{1 - \frac{m_f^2}{M_S^2}}. \quad (B.147)$$

#### B.4.4.11 Annihilation in the case of vectorial Dark Matter to pairs of fermions



One can compute this annihilation cross section by the normal procedure or noticing that a neutral vectorial dark matter of spin 1 corresponds to 3 degrees of freedom. After averaging on the spin one can then write  $\langle \sigma v \rangle^V = \frac{3}{3 \times 3} \langle \sigma v \rangle^S = \frac{1}{3} \langle \sigma v \rangle^S$ . The academical computation for  $V_\mu(p_1) V_\mu(p_2) \rightarrow f\bar{f}$  gives

Yann Mambrini

# Particles in the Dark Universe

## A Student's Guide to Particle Physics and Cosmology

Springer

$$\ddot{S} + 3H\dot{S} + \left[ \frac{k^2}{a^2} + \mu\Phi_0 \cos(m_\Phi t) \right] S = 0 \quad (2.170)$$

Supposing  $a \approx \text{constant}$ , we can neglect  $H$ . This equation is one form of the Mathieu equation, which is the equation for an oscillator with a time dependant frequency  $\omega^2(t) = \frac{k^2}{a^2} + \mu\Phi_0 \cos(m_\Phi t)$ , and is present in a lot of classic phenomena involving periodical force. It can be shown that for

$$\frac{m_\Phi}{2} - \frac{\mu\Phi_0}{2m_\Phi} < \frac{k}{a} < \frac{m_\Phi}{2} + \frac{\mu\Phi_0}{2m_\Phi}, \quad (2.171)$$

we enter in a regime where the solution grows exponentially with time<sup>23</sup>. We can understand it easily, from the shape of the Mathieu equation, where, periodically, the coefficient  $\cos(m_\Phi t)$  becomes negative and drives the evolution of  $S$  toward an exponential solution, periodically. The evolution of  $S$  is shown in Fig.(2.8). A more refined treatment necessitate to compute the Bogoliubov coefficient to extract the occupation number [10], but we give in the following section a more intuitive view of the phenomena, solving the equation for the density of the  $\phi$  decay products. For the analytical solution of the Mathieu equation (2.170) the reader is directed to [9] which is without doubt the best textbook treating it, and [10] which is (paradoxically) the seminal *research* paper on the subject and one of the clearer and more detailed in the literature.

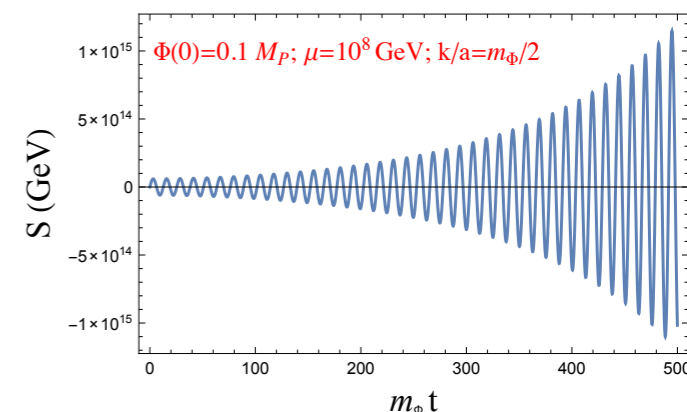


Fig. 2.8 Illustration of the parametric (also called *narrow*) resonance in the context of preheating. We can see clearly the exponential envelop of the periodic solution.

500+ pages, from inflation to dark matter detection.  
All what is needed to compute cross-sections, relic abundance,  
and retrace the history of a Dark Universe.

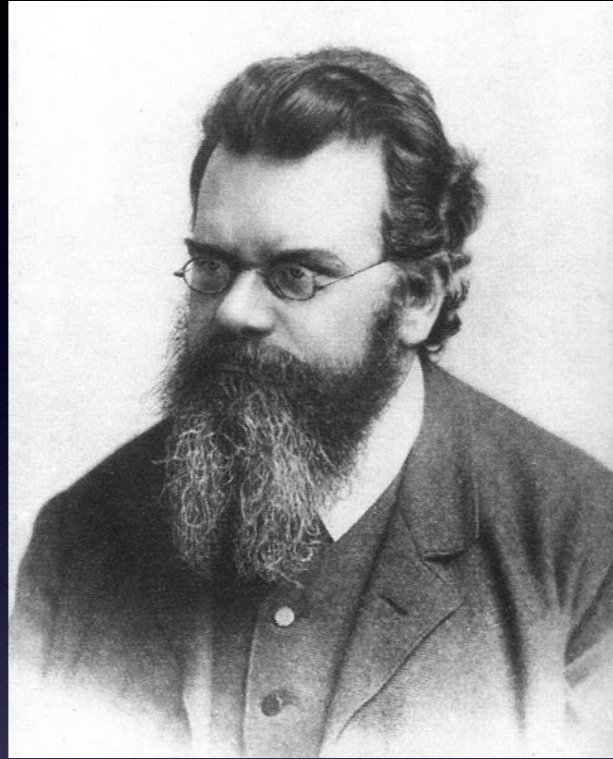
Preface and forewords by K. Olive, J. Peebles and J. Silk

# Plan

- 1) Thermal production (WIMP...)
- 2) Non Thermal production (FIMP...)
- 3) Non-perturbative production (resonances...)
- 4) Gravitational production

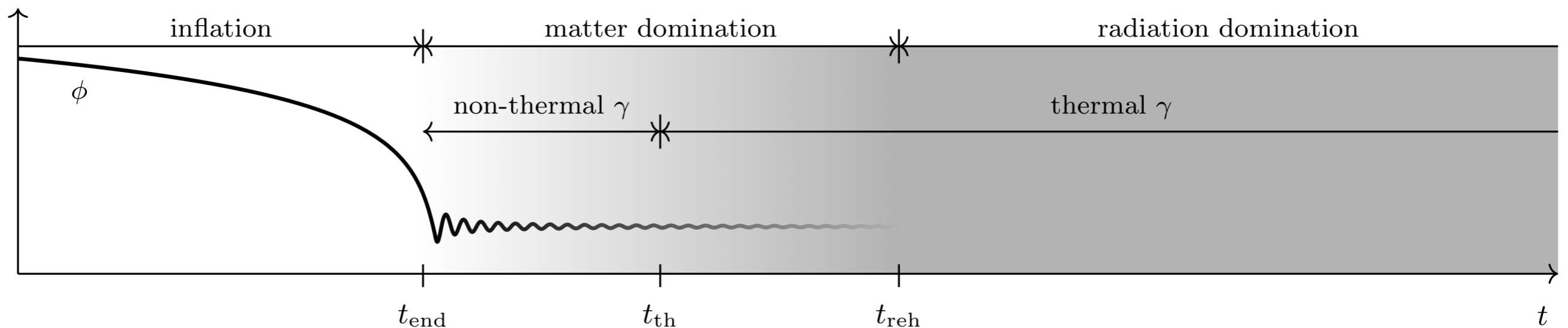
How and when to produce (dark) matter?

# A brief history of the energy in the Early Universe

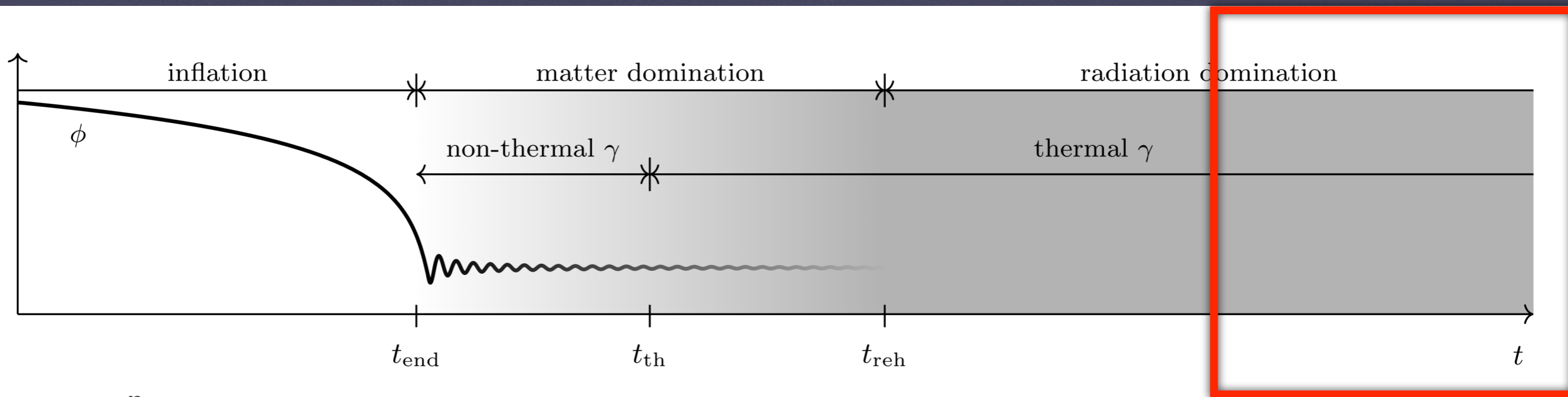


*« Available energy is the main object at stake in the struggle for existence and the evolution of the world. »*

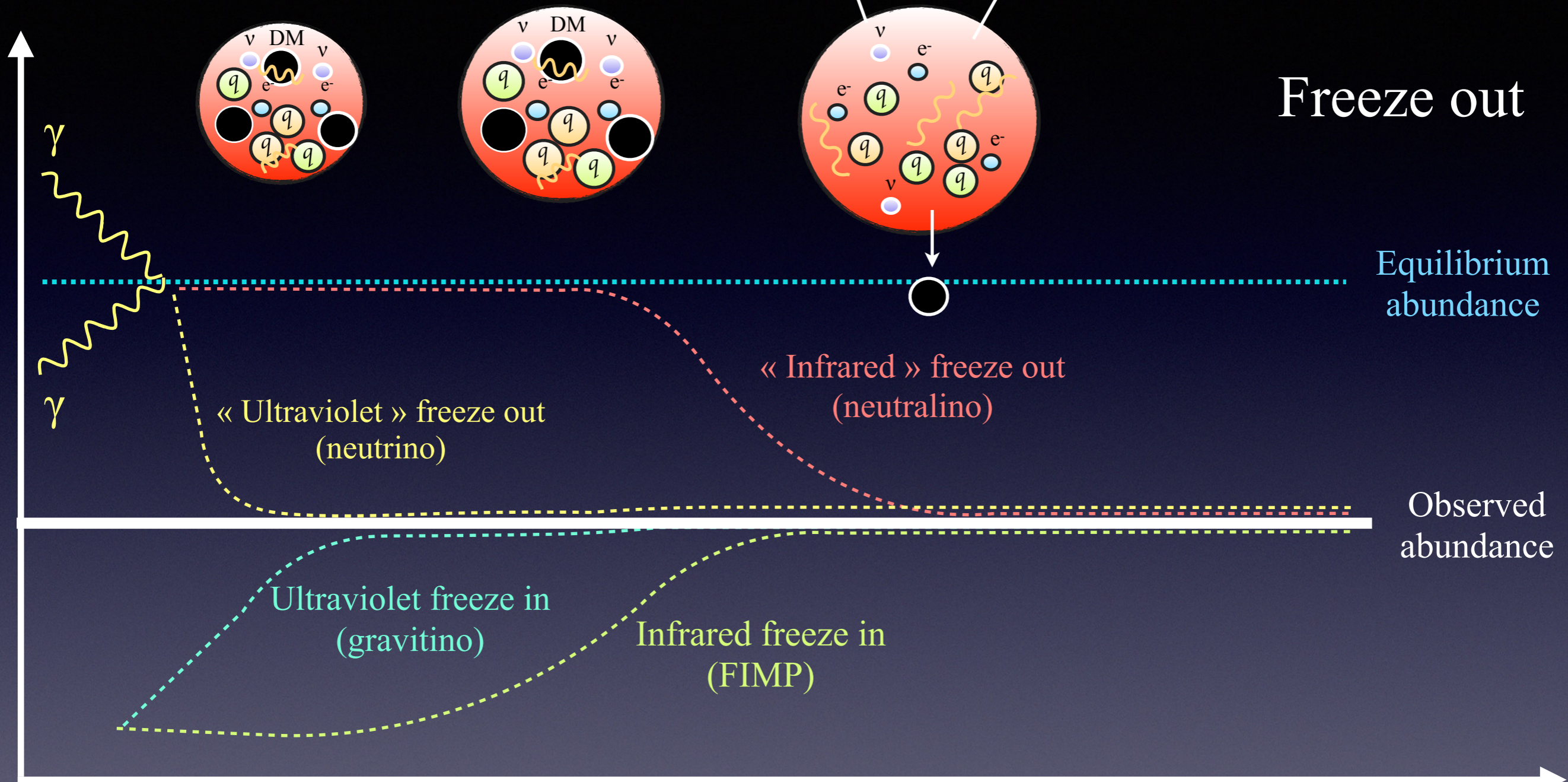
***L. Boltzmann***



# Producing (dark) matter from thermal equilibrium (WIMP)



abundance



Freeze out

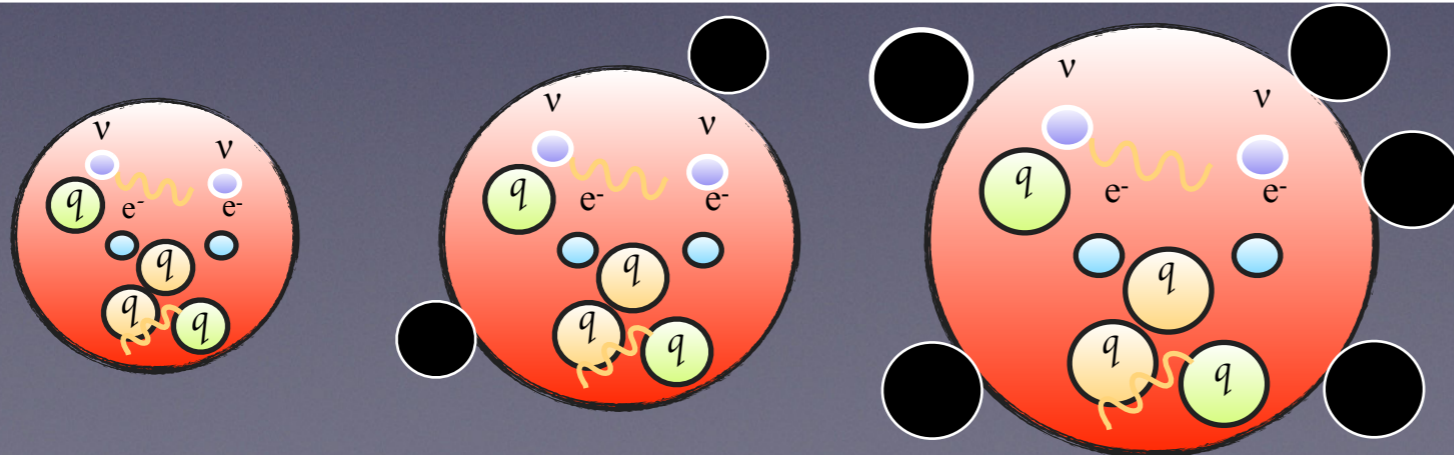
Equilibrium  
abundance

Observed  
abundance

Ultraviolet freeze in  
(gravitino)

Infrared freeze in  
(FIMP)

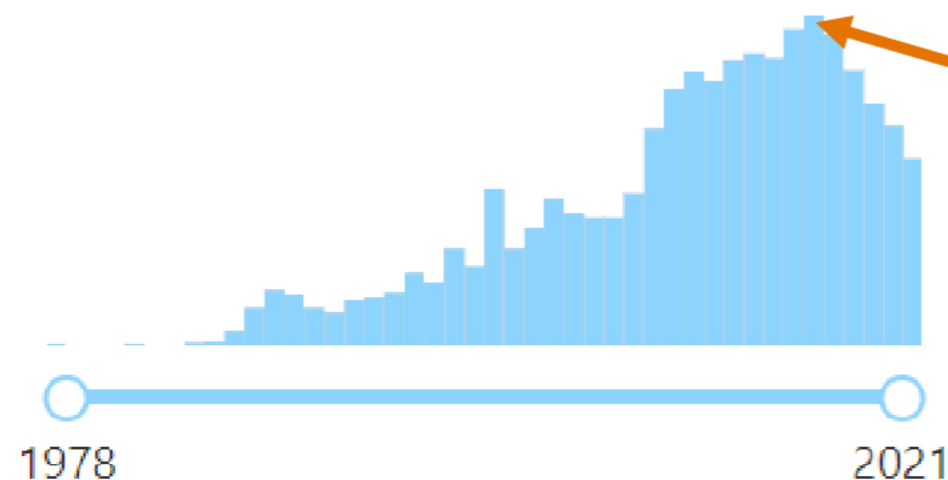
time=  $1/T$



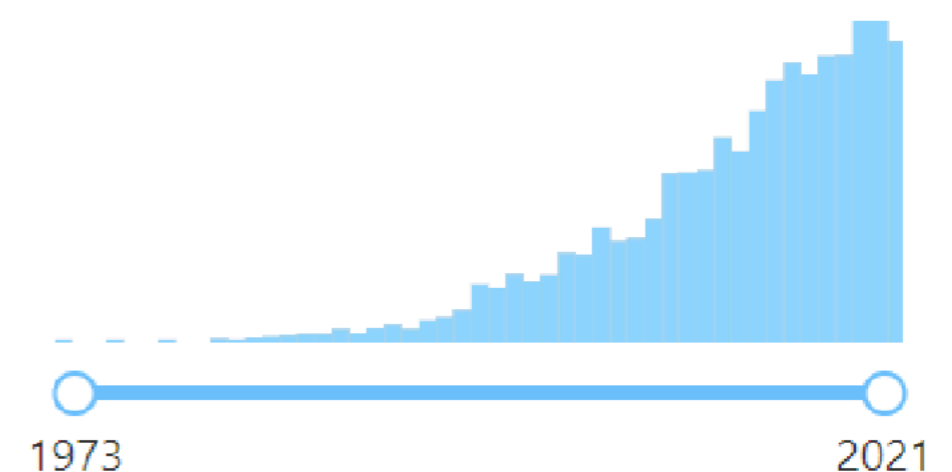
# « *The Waning of the WIMP?* *Review of Models, Searches and Constraints* »

**G. Arcadi, M. Dutra**, P. Ghosh, M. Lidner, Y.M.,  
**M. Pierre**, S. Profumo and F. Queiroz;

Eur. Phys. J. C **78** (2018) no.3, 203  
arXiv:1703.07364



Inspire-HEP papers “WIMP”



Inspire-HEP papers “freeze-in”

# « *The Dawn of FIMP Dark Matter: A Review of Models and Constraints* »

**N. Bernal**, M. Heikinheimo, T. Tenkanen, K. Tuominen and V. Vaskonen

Int. J. Mod. Phys. A **32** (2017) no.27, 1730023

# WIMP : the « issue » of the Yield

Even if the *energy density* of dark matter dominates the energy density of photons, the *number density* of dark matter is suppressed:

$$\frac{n_{DM}^0}{n_\gamma^0} = Y_{DM}^0 \simeq 10^{-9} \left( \frac{m_{DM}}{1 \text{ GeV}} \right)$$

Remark: the number density of dark matter is similar to the *number density of baryons* (roughly 5 times more) :

$$\eta_b \simeq 6 \times 10^{-10}$$

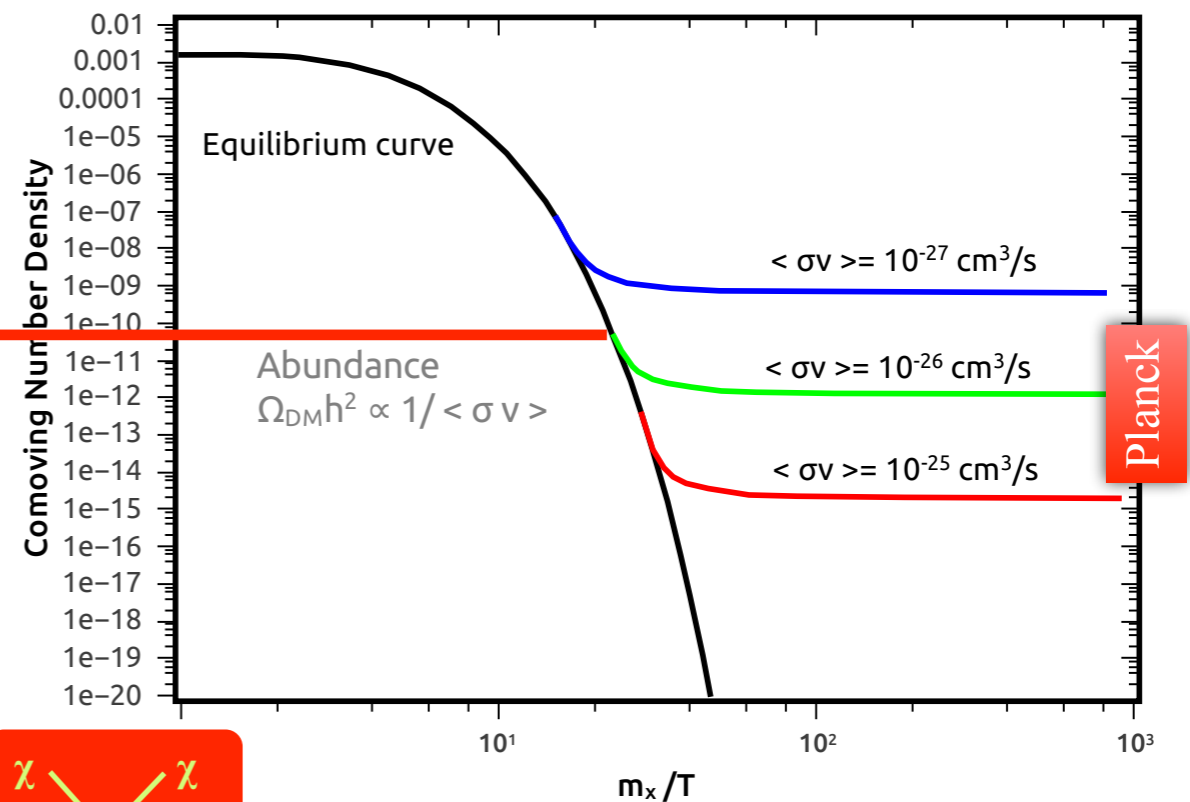
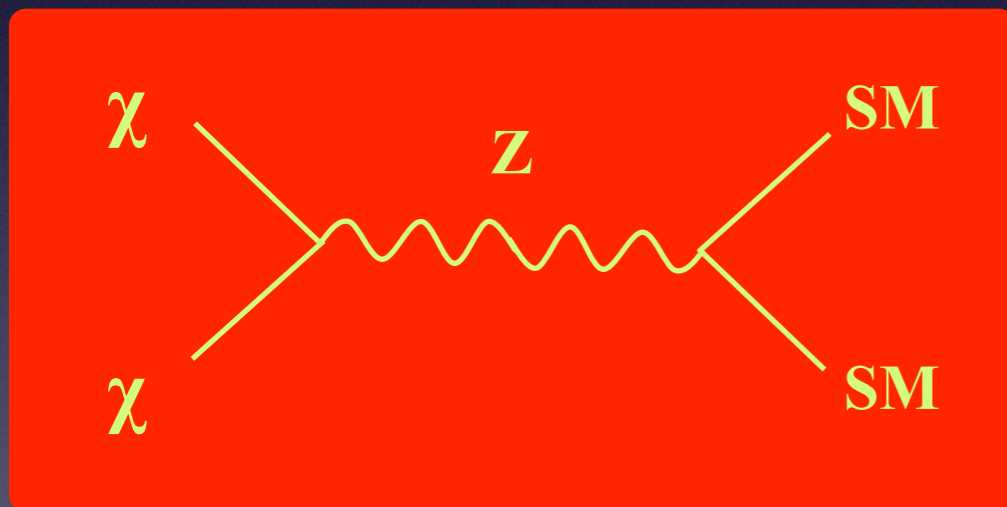
Thus, any process explaining the relic abundance, also explain the (lepto)baryogenesis, thermal or non-thermal.

# The Boltzmann equation

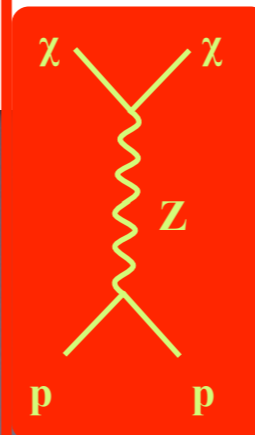
$$\frac{dn}{dt} = -3Hn - \langle \sigma v \rangle (n^2 - n_{eq}^2)$$

$$\Omega_A h^2 \simeq \frac{0.17}{\frac{\langle \sigma v \rangle}{(1.2 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1})}}$$

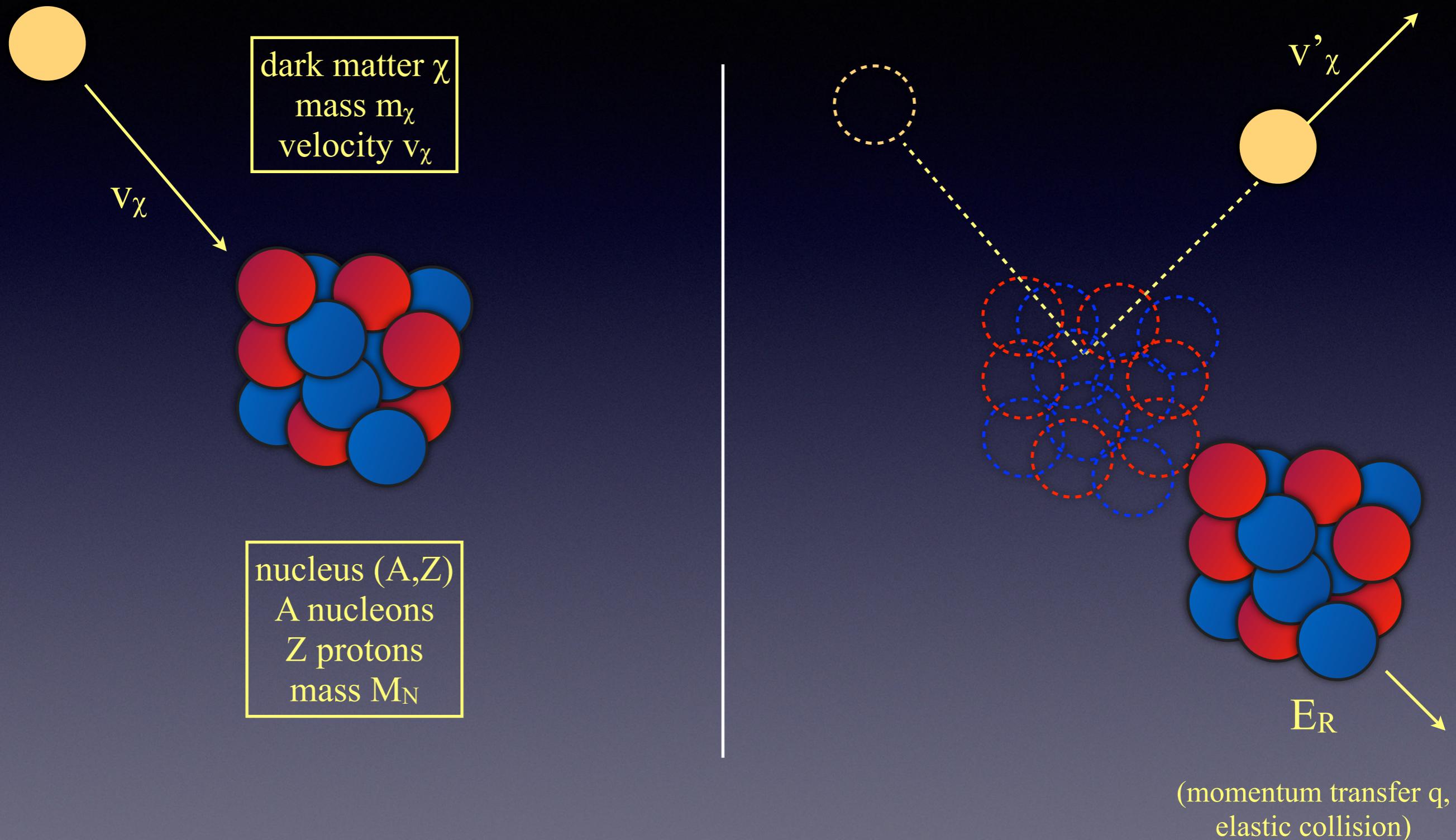
$$\begin{aligned} \langle \sigma v \rangle &= 1.2 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \\ &= 10^{-9} \text{ GeV}^{-2} \left( \frac{m_{DM}}{1 \text{ GeV}} \right)^2 \simeq G_F^{-2} \left( \frac{m_{DM}}{1 \text{ GeV}} \right)^2 \end{aligned}$$

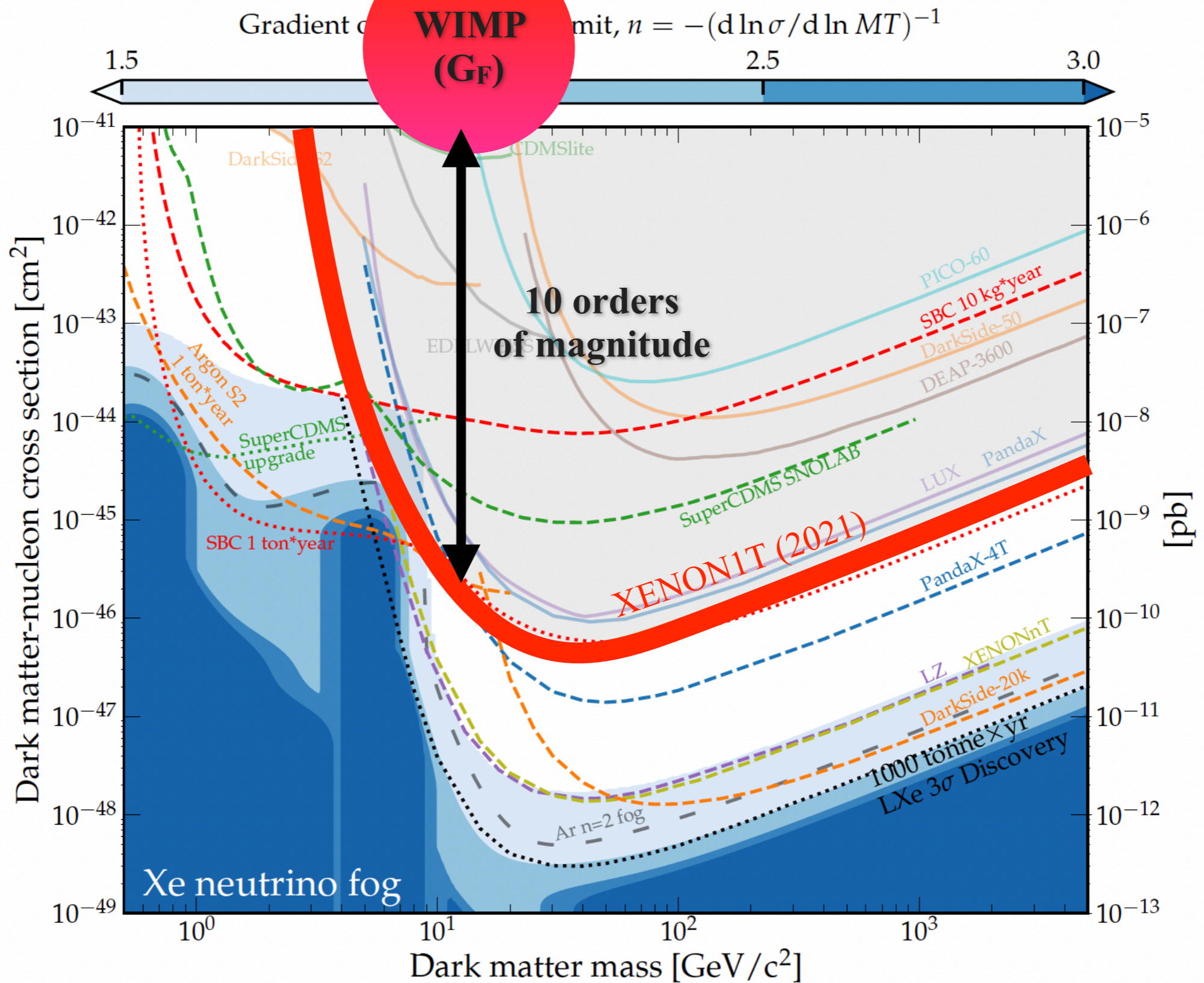


One needs a phase of depletion of dark matter, annihilating to SM to avoid the overabundance. Can we deplete it without even coupling to the SM, and thus avoiding the direct detection conflict?



# Direct detection of dark matter (basic principle)





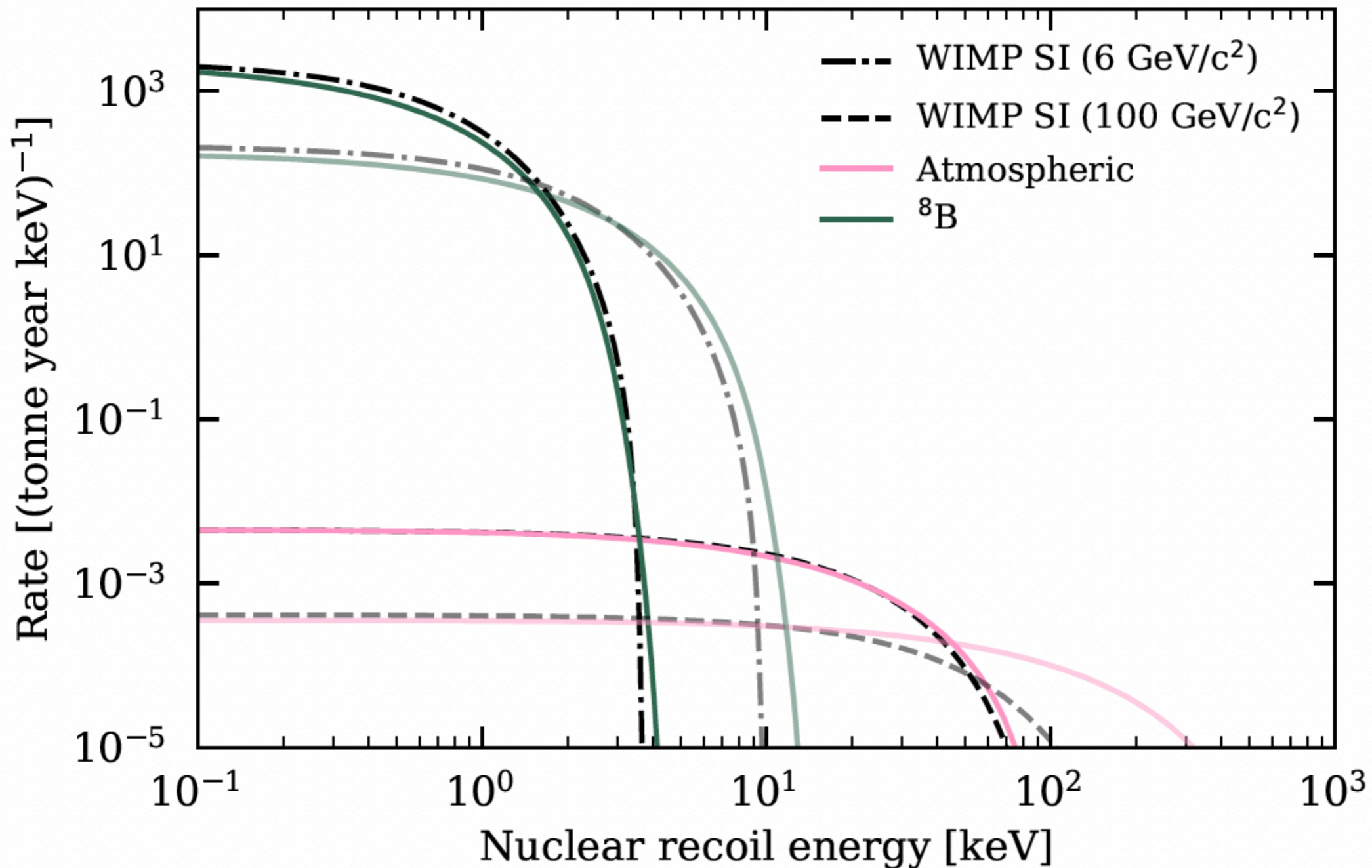
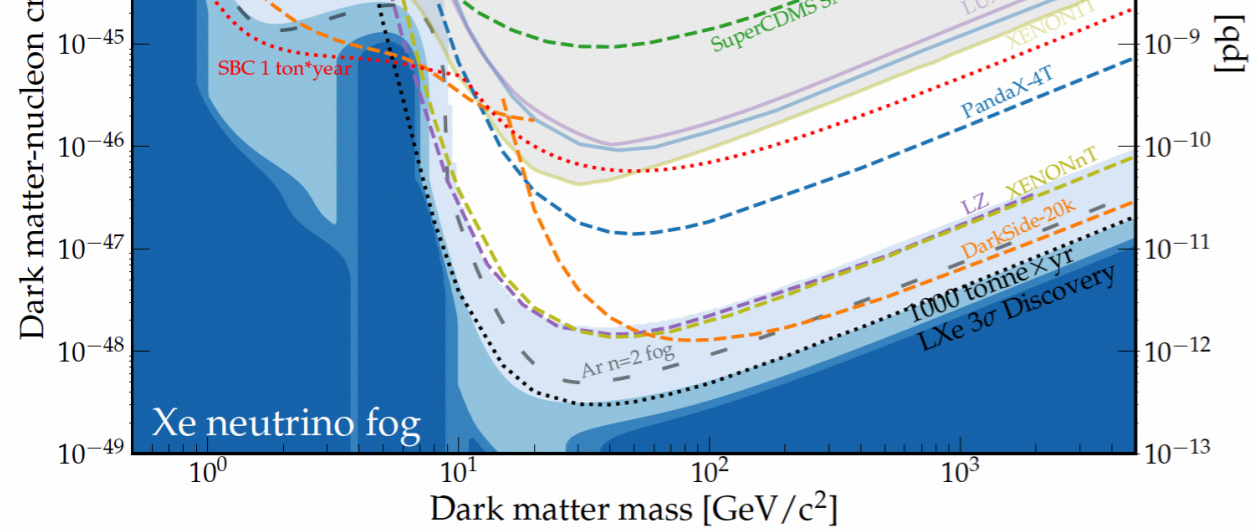
$$g_1 = 0.46; \quad g_2 = 0.65; \quad g_3 = 1.22; \quad \sigma_{dm} \lesssim 10^{-36} \times 10^{-10} \text{ cm}^2 \Rightarrow g_{dm} \lesssim 10^{-5}$$

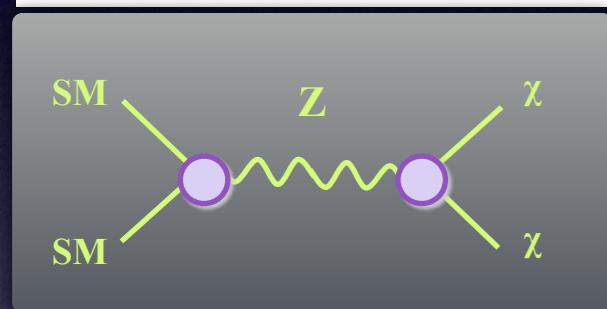
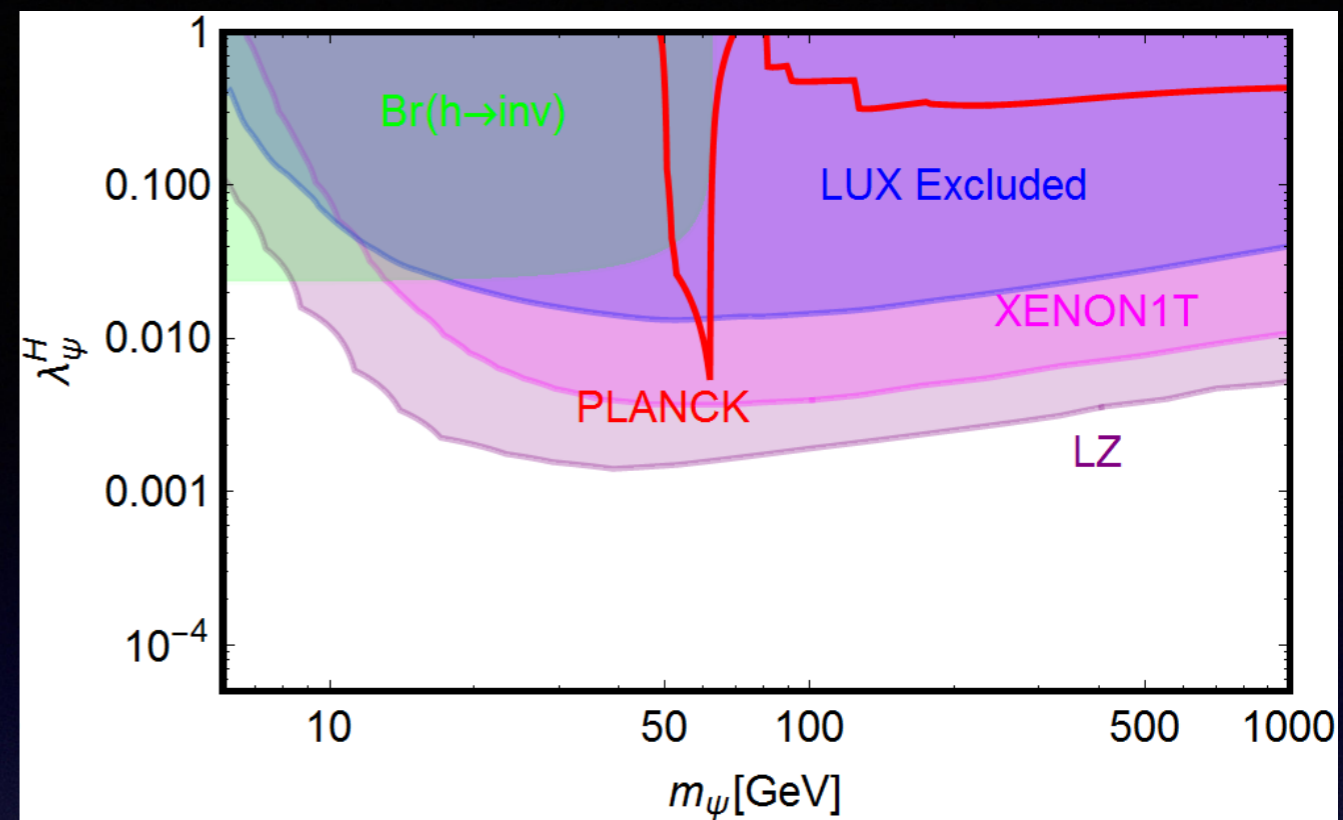
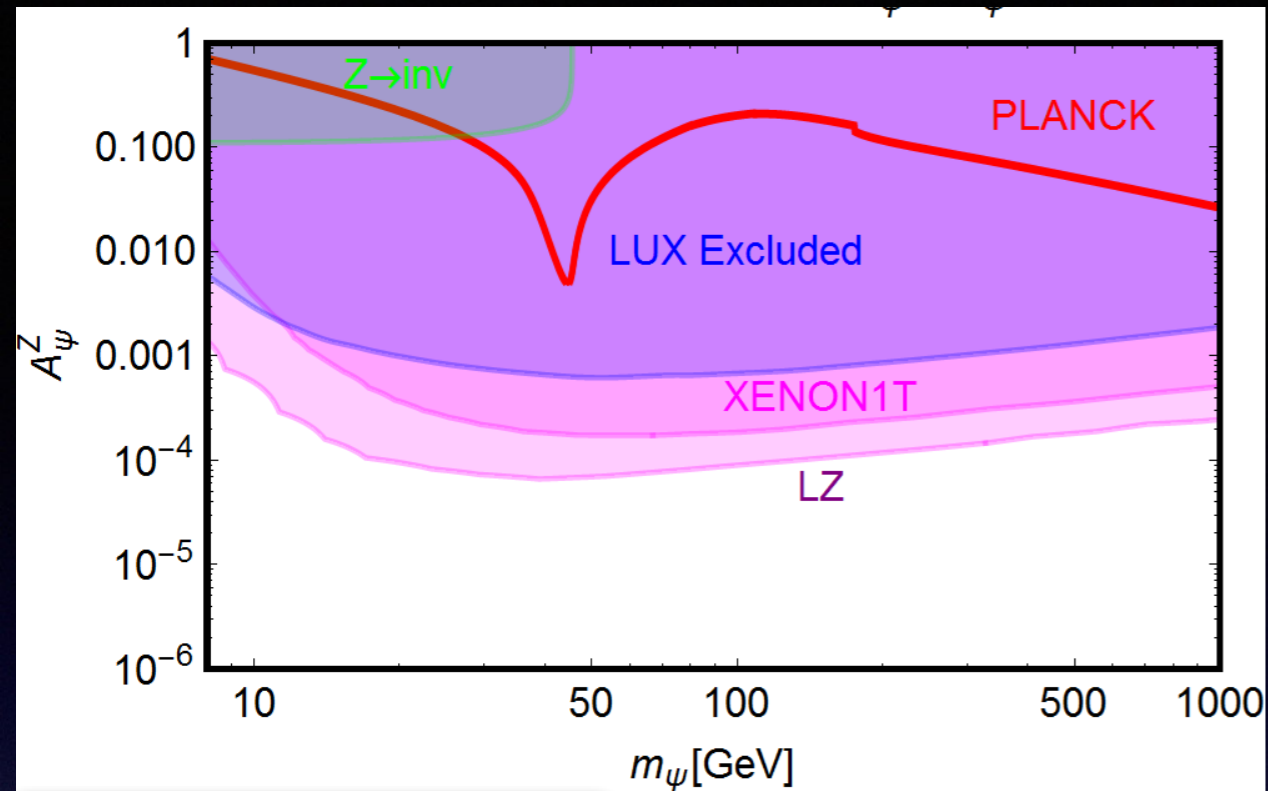
$$\text{or } g_{dm} \simeq 1 \text{ and } M_{med} \gtrsim 2 \text{ TeV}$$

# The neutrino fog

$$E_R^\nu = \frac{|p_\nu^{^8B}|^2}{2m_{Xe}} \simeq \frac{|10 \text{ MeV}|^2}{100 \text{ GeV}} \simeq 1 \text{ keV}$$

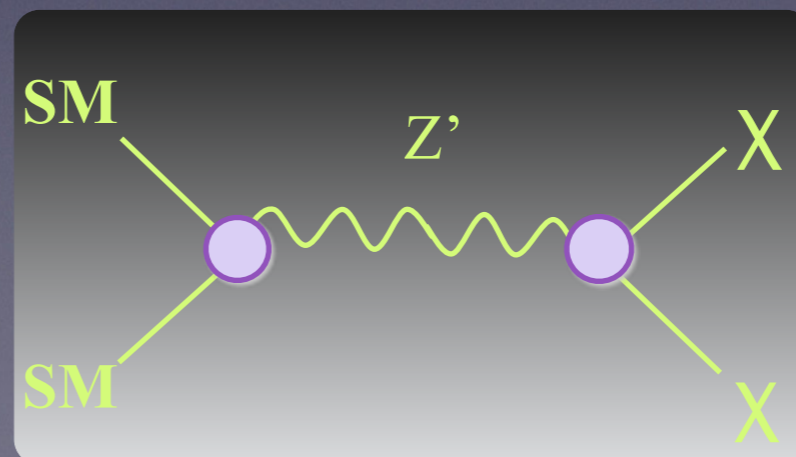
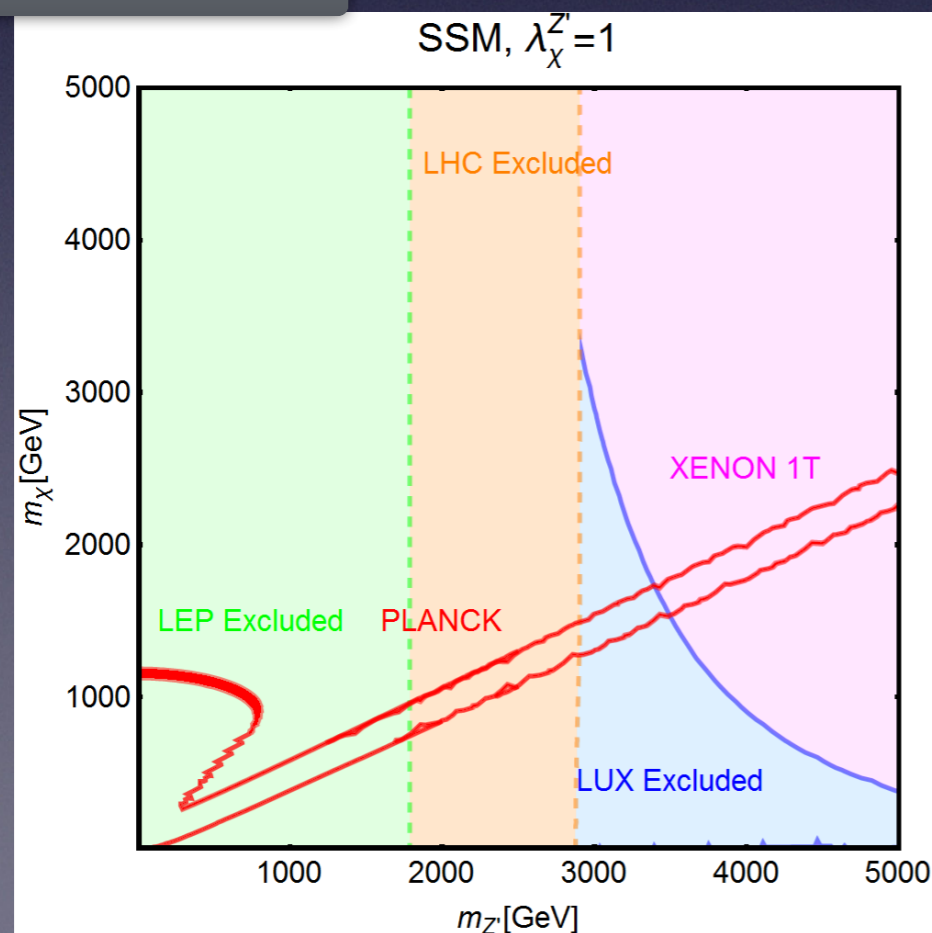
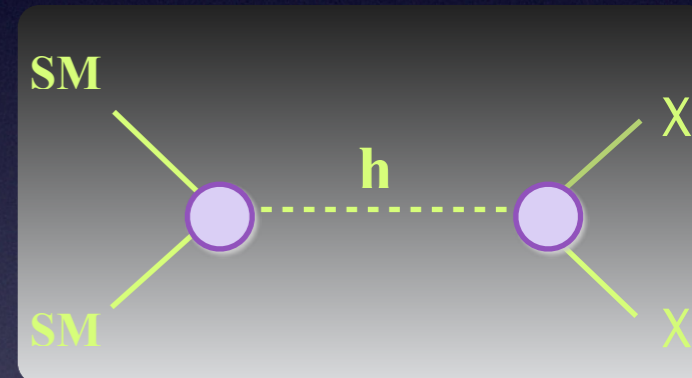
$$E_R^{DM} = \frac{1}{2} m_{DM} \left( \frac{v_{DM}}{c} \right)^2 \simeq \frac{m_{DM}}{2 \text{ GeV}} \left( \frac{300}{300000} \right)^2 \simeq 1 \text{ keV}$$





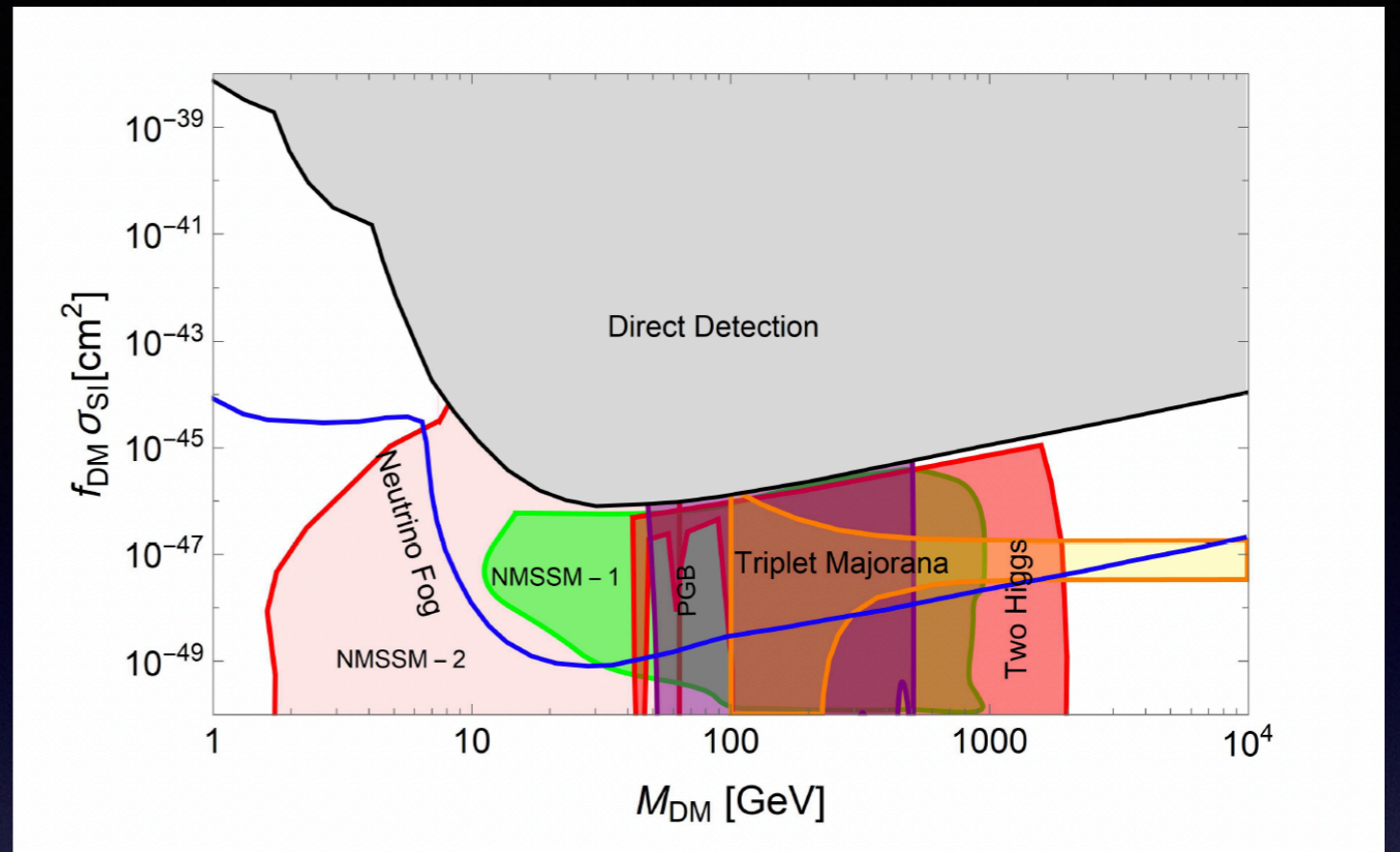
Z-portal

Higgs-portal



Z'-portal

# How to escape?



« Shy » dark matter :  $\mathcal{L} = \chi \gamma^5 \chi H \Rightarrow |\mathcal{M}|^2 \propto (p - m_\chi)^2 \Rightarrow \sigma_p \propto \frac{v^2}{c^2}$

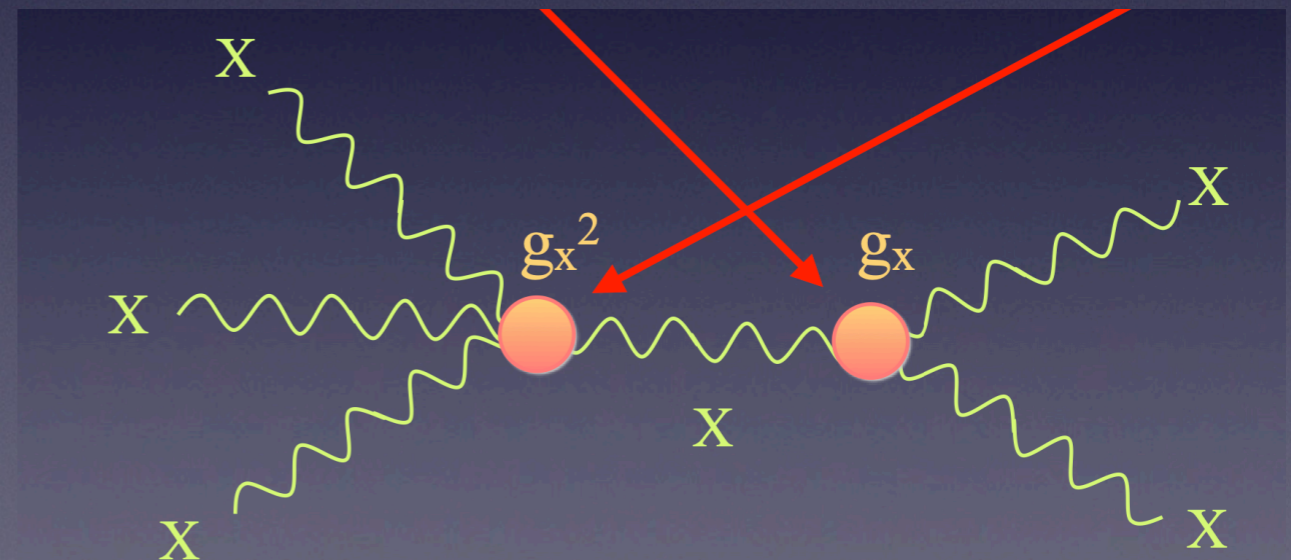
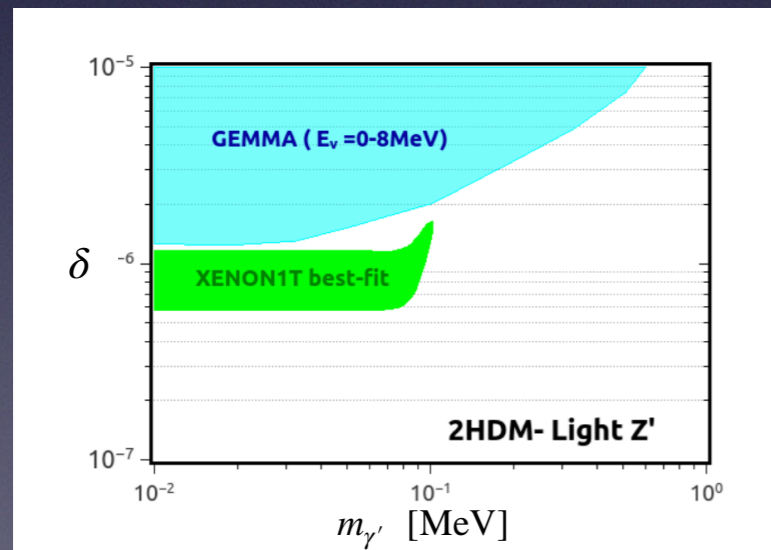
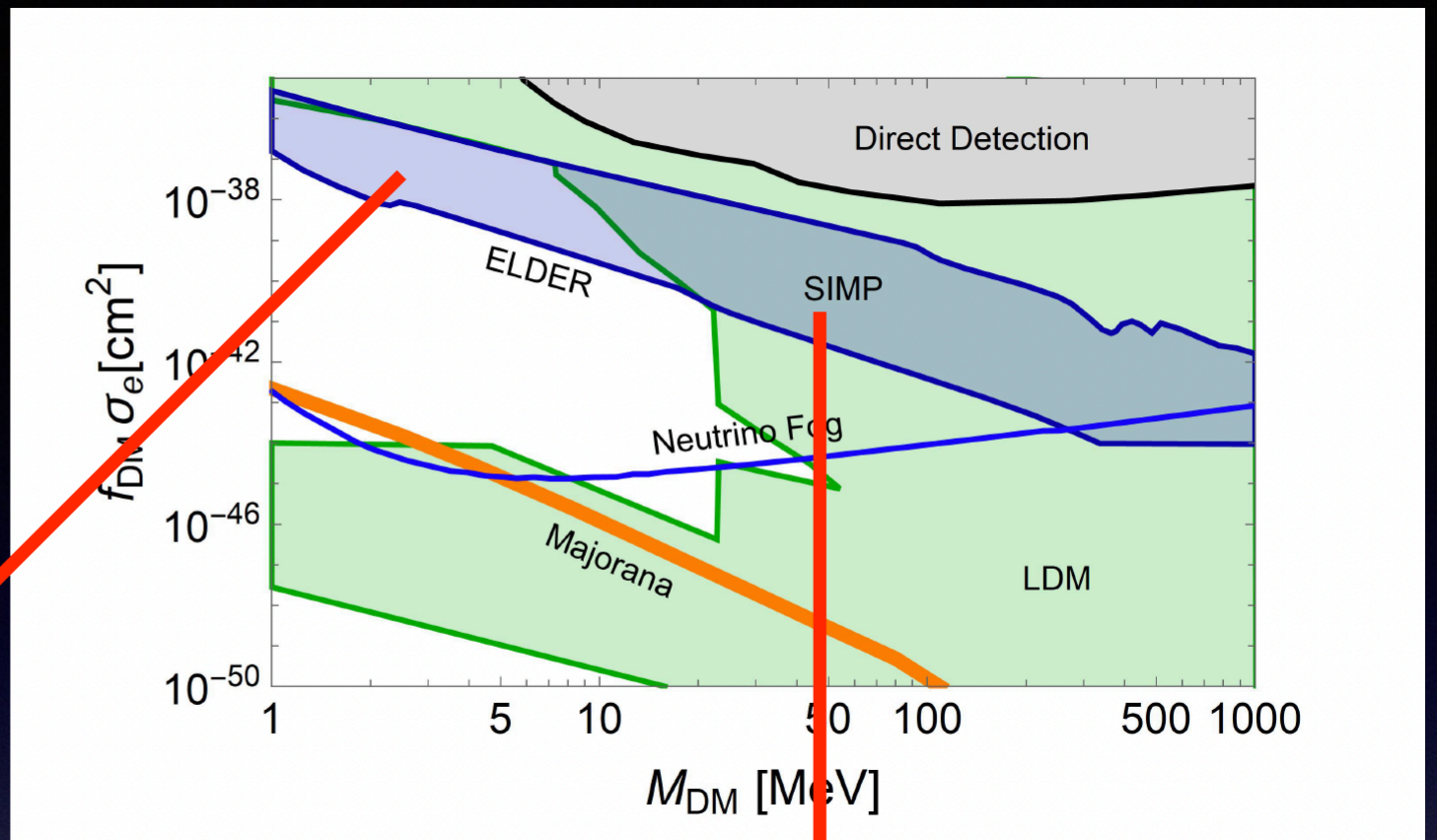
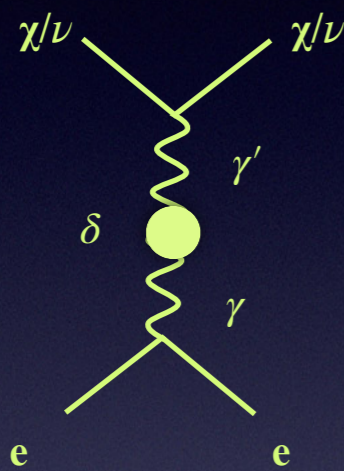
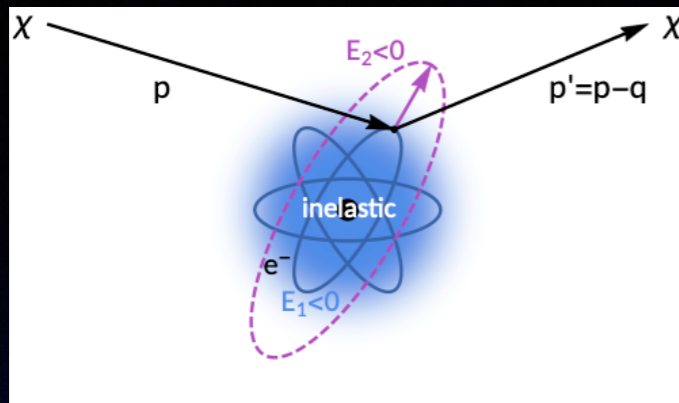
Pseudoscalar dark matter ... same

Pseudo-Goldstone dark matter :  $\mathcal{L} = (\partial_\mu a) \bar{p} p \Rightarrow |\mathcal{M}|^2 \propto m_p^2 \Rightarrow \sigma_p \propto \frac{m_p^2}{m_{DM}^2}$

Double Higgs portal ... cancelation in the amplitude

...

# Electron recoil



M. Lindner, Y. Mambrini, T. B. de Melo and F. S. Queiroz,  
Phys. Lett. B 811 (2020), 135972

Y. Farzan and M. Rajaei,  
Phys. Rev. D 102 (2020) no.10, 103532

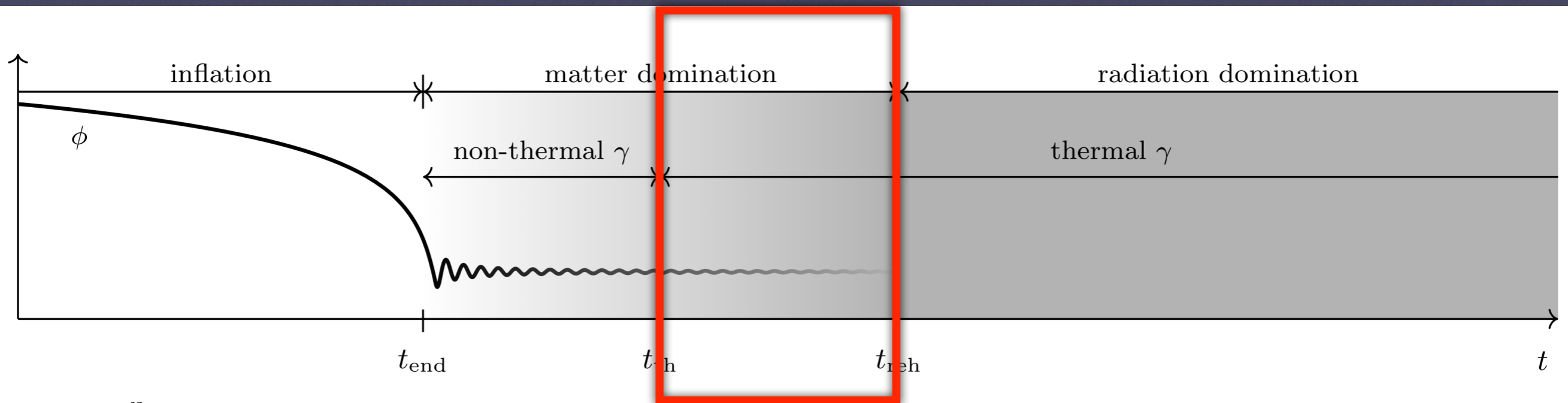
G. Alonso-Alvarez, F. Ertas, J. Jaeckel, F. Kahlhoefer and L. J. Thormaehlen,  
JCAP 11 (2020), 029

S. M. Choi, H. M. Lee, Y. Mambrini and M. Pierre,  
"Vector SIMP dark matter with approximate custodial symmetry,"  
JHEP 07 (2019), 049

# Indirect detection limit

→ see spin  $3/2$

# Mechanisms to produce (dark) matter at the **thermal reheating stage**



# Infrared FIMP

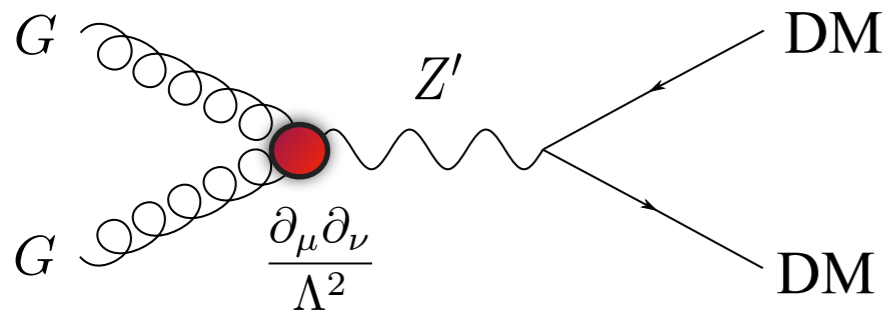
$$\frac{dn}{dt} + 3Hn = R(t) \quad \Rightarrow \quad \frac{dY}{dT} = \frac{R(T)}{T^4 H}, \quad Y = \frac{n}{T^3}, \quad H = \frac{T^2}{\sqrt{3} M_P}$$

$$R(T) = \alpha T^4 \quad \Rightarrow \quad Y^0 = \int_{T_{RH}}^{m_{DM}} \frac{\alpha}{T^2} \simeq \frac{\alpha}{m_{DM}} \quad \Rightarrow \quad \Omega h^2 \simeq 0.1 \left( \frac{\alpha}{10^{-11}} \right)$$

**FIMP miracle** : no dependance on the dark matter mass.

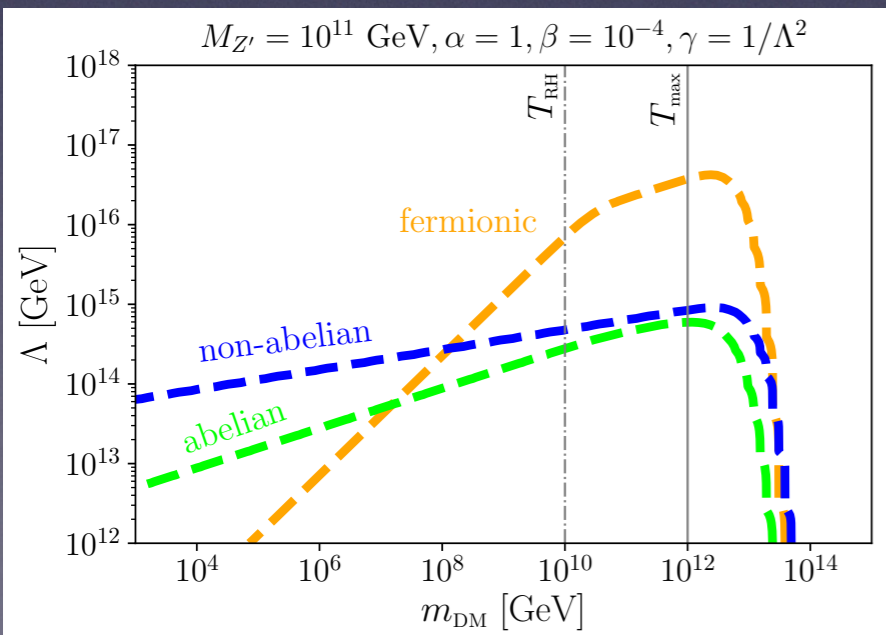
UV freeze in

# Spin-1 mediator

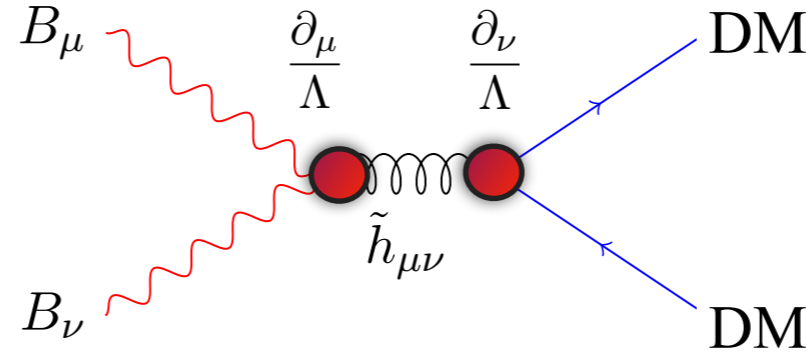


$$\mathcal{L} = \frac{\tilde{g}}{M^2} \partial^\alpha Z'_\alpha \epsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu^a \partial_\rho A_\sigma^a$$

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \partial^\alpha Z'_\alpha \epsilon^{\mu\nu\rho\sigma} \text{Tr}[G_{\mu\nu}^a G_{\rho\sigma}^a]$$



# Spin-2 mediator



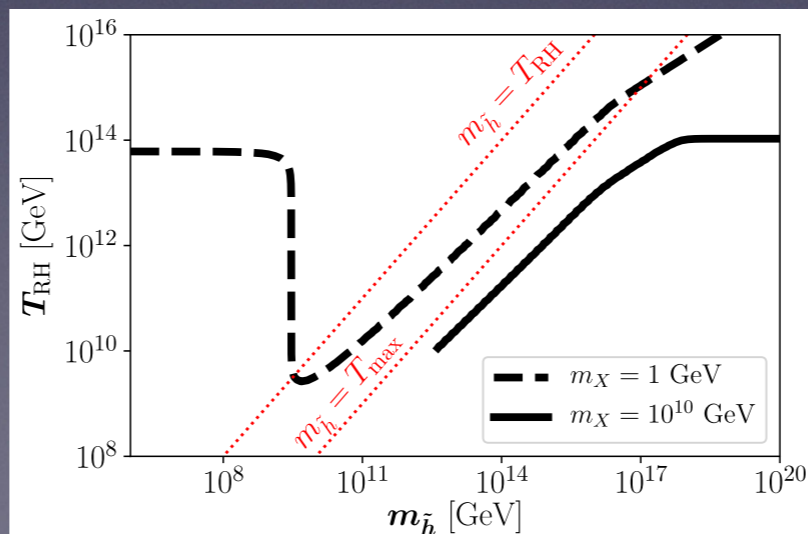
$$\mathcal{L}_{\text{int}}^1 = \frac{1}{2M_P} h_{\mu\nu} (T_{\text{SM}}^{\mu\nu} + T_{\text{X}}^{\mu\nu})$$

$$\mathcal{L}_{\text{int}}^2 = \frac{1}{\Lambda} \tilde{h}_{\mu\nu} (g_{\text{SM}} T_{\text{SM}}^{\mu\nu} + g_{\text{DM}} T_{\text{X}}^{\mu\nu})$$

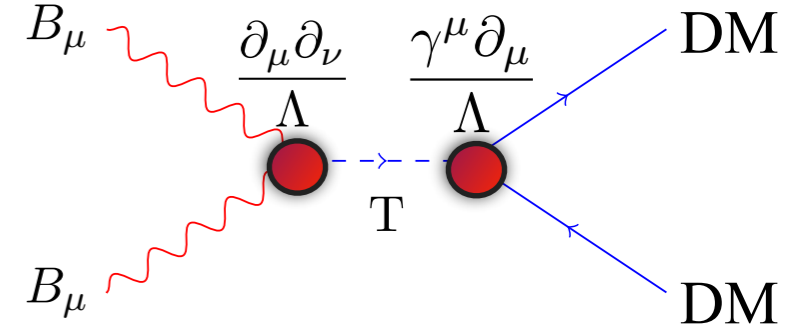
$$T_{\mu\nu}^0 = \frac{1}{2} (\partial_\mu \phi \partial_\nu \phi + \partial_\nu \phi \partial_\mu \phi - g_{\mu\nu} \partial^\alpha \phi \partial_\alpha \phi),$$

$$T_{\mu\nu}^{1/2} = \frac{i}{4} \bar{\psi} (\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu) \psi - \frac{i}{4} (\partial_\mu \bar{\psi} \gamma_\nu + \partial_\nu \bar{\psi} \gamma_\mu) \psi$$

$$T_{\mu\nu}^1 = \frac{1}{2} \left[ F_\mu^\alpha F_{\nu\alpha} + F_\nu^\alpha F_{\mu\alpha} - \frac{1}{2} g_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right].$$



# Moduli mediator

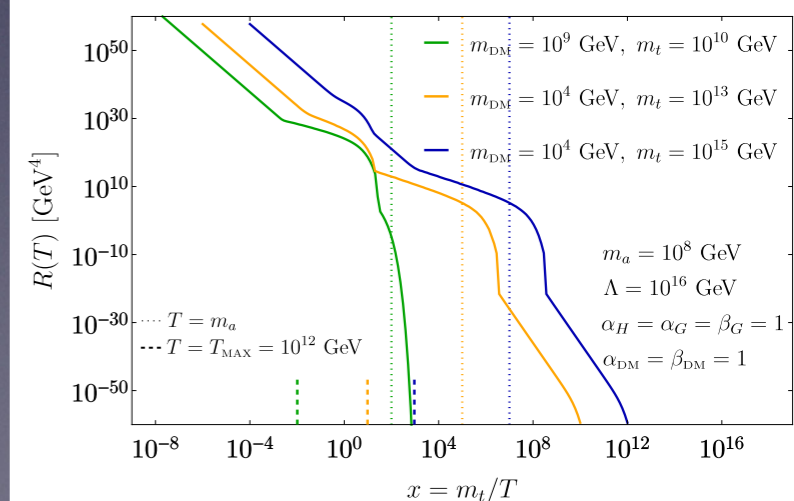


$$\mathcal{L}_{\mathcal{T}}^{SM} \supset \frac{\alpha_H}{\Lambda} t |D_\mu H|^2 - \frac{\alpha_H}{\Lambda} \mu_0^2 t |H|^2$$

$$+ \left( \frac{1}{2\Lambda} t \bar{f} i \gamma^\mu (\alpha_V^f - \alpha_A^f \gamma_5) D_\mu f + \text{h.c.} \right)$$

$$+ \frac{1}{2\Lambda} \partial_\mu a \bar{f} \gamma^\mu (\beta_V^f - \beta_A^f \gamma_5) f$$

$$- \frac{1}{4} \frac{\alpha_G}{\Lambda} t G_{\mu\nu} G^{\mu\nu} + 2 \frac{\beta_G}{\Lambda} \partial_\mu a \epsilon^{\mu\nu\rho\sigma} G_\nu \partial_\rho G_\sigma$$



# Ultraviolet FIMP

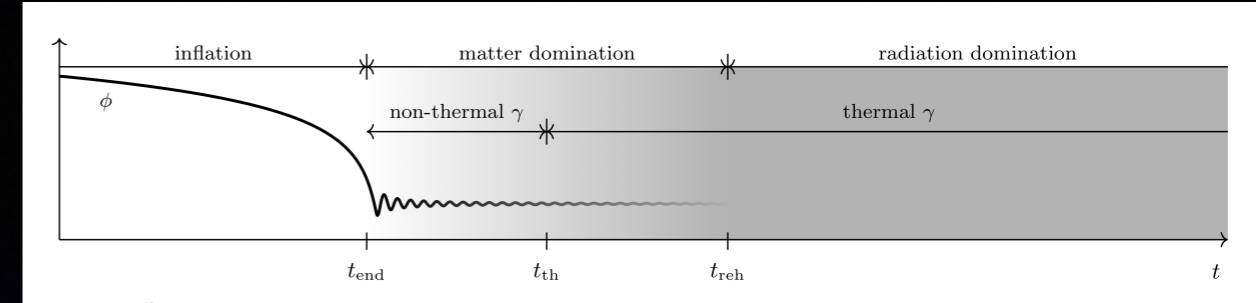
$$\frac{dn}{dt} + 3Hn = R(t) \quad \Rightarrow \quad \frac{dY}{dT} = \frac{R(T)}{T^4 H}, \quad Y = \frac{n}{T^3}, \quad H = \frac{T^2}{\sqrt{3} M_P}$$

$$R(T) = \alpha \frac{T^{n+4}}{\Lambda^n} \quad \Rightarrow \quad Y^0 = \int_{T_{RH}}^{m_{DM}} \alpha \frac{M_P}{\Lambda^n} T^{n-2} \simeq \alpha T_{RH}^{n-1} \quad \Rightarrow \quad \Omega h^2 \simeq T_{RH}^{n-1} m_{DM}$$

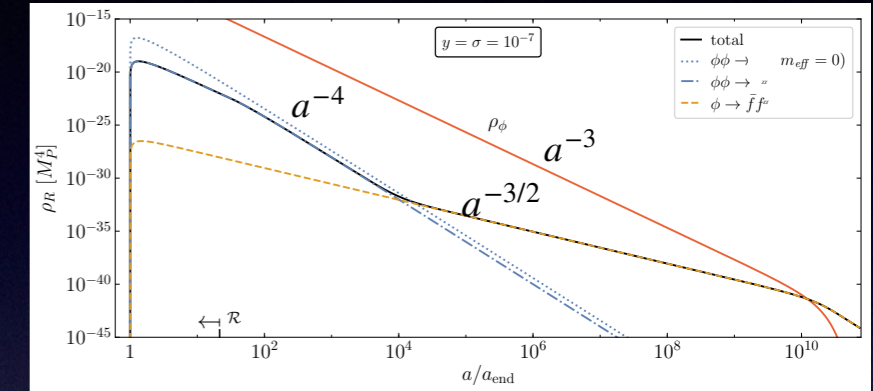
To know « n », we need to know the evolution of

$T = f(t)$ , or equivalently  $T=f(a)$

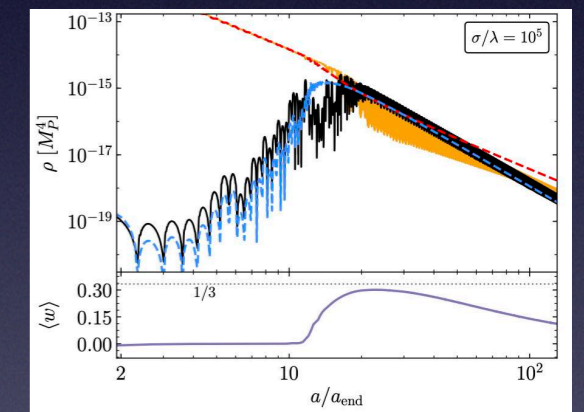
# 1). Reheating : generalities



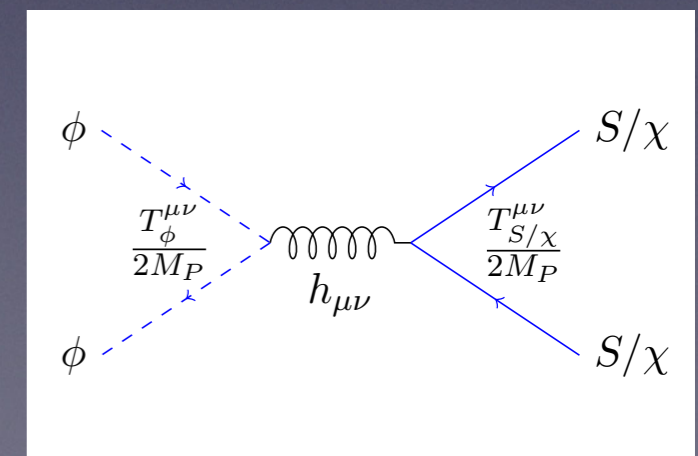
# 2). Non-instantaneous reheating



# 3). Preheating phase



# 4). Application to gravitational production



# 5). Conclusion

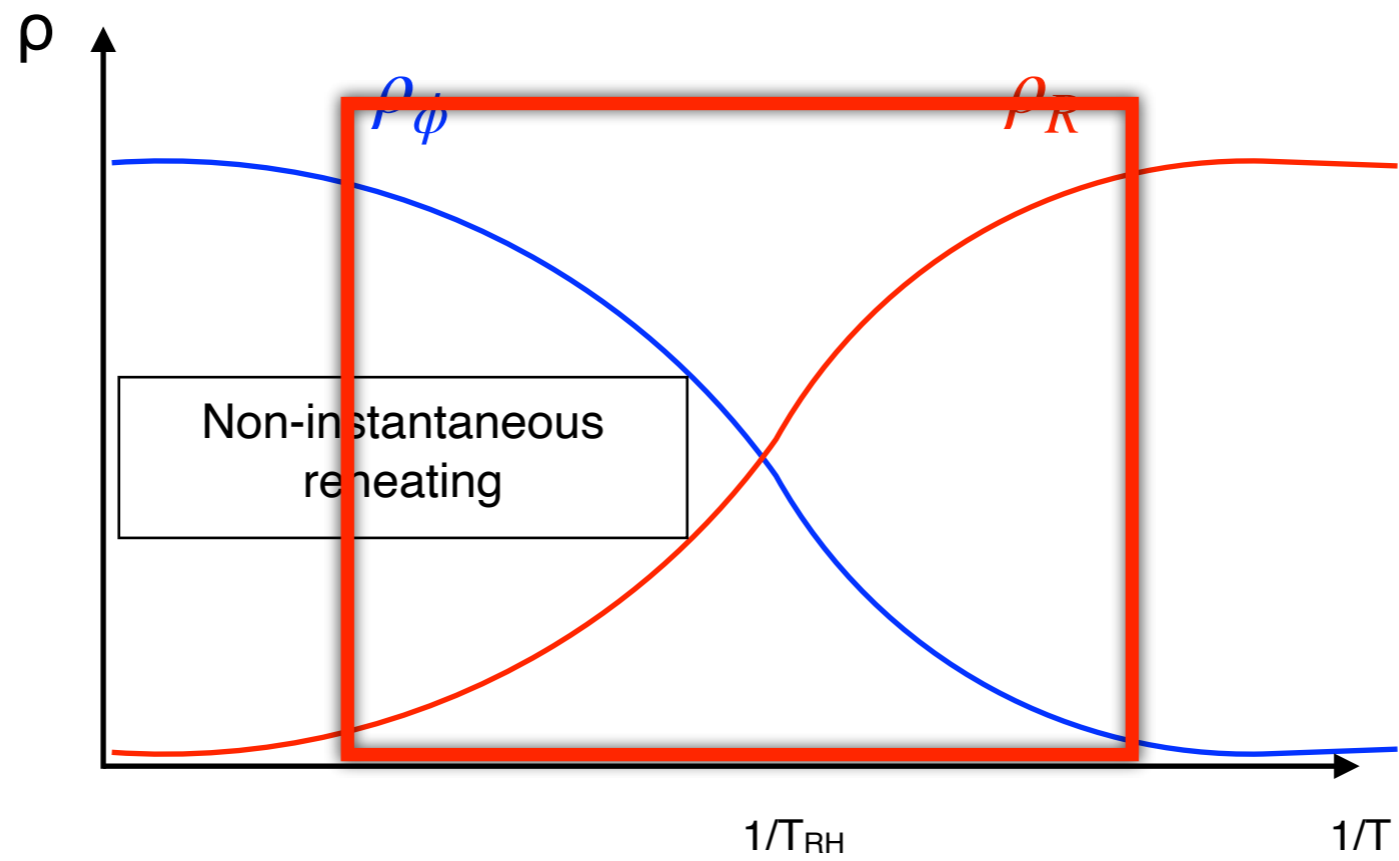
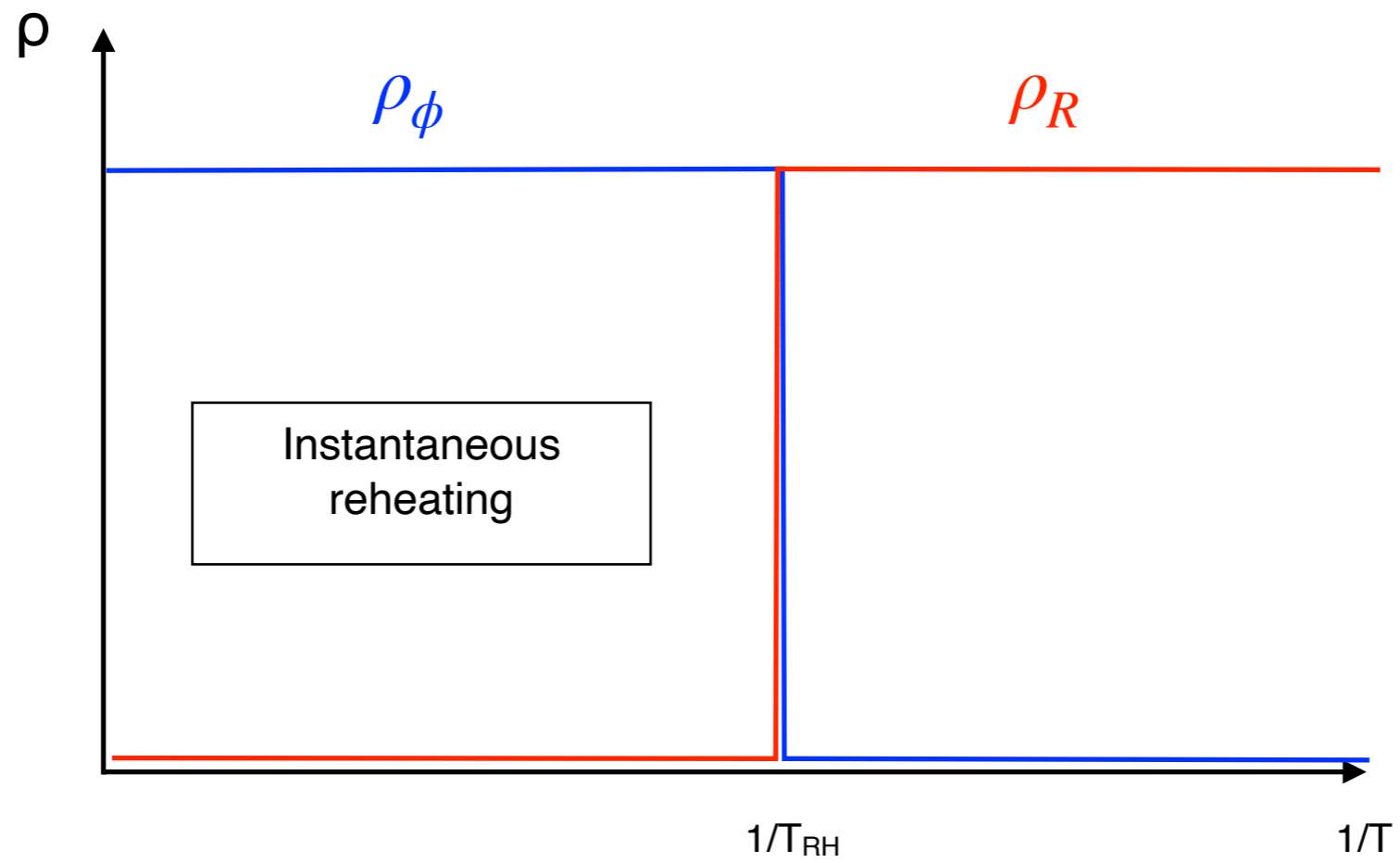
Introducing the inflaton  $\Phi$

# A progressive approach



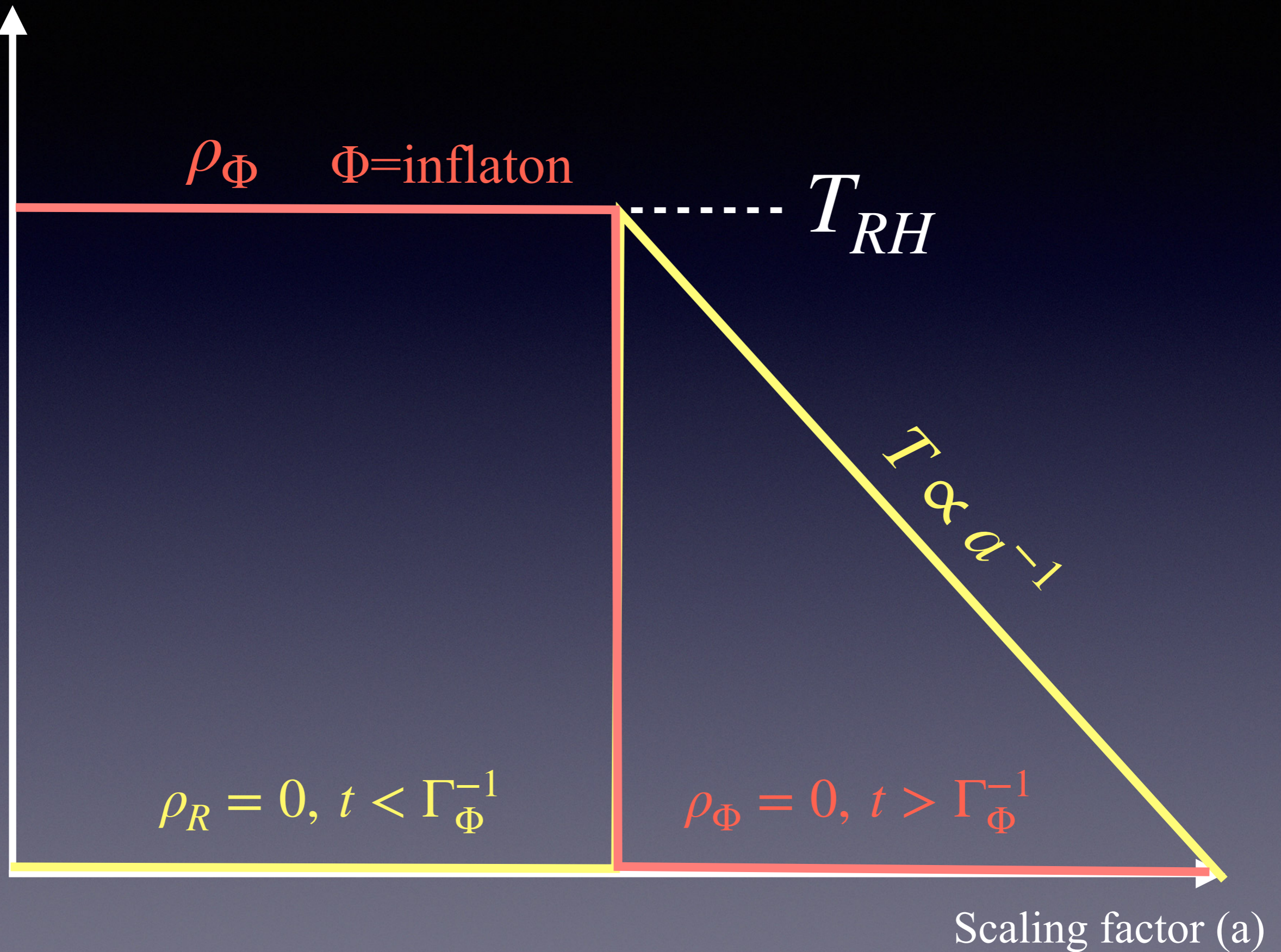
**“All things being equal, the simplest solution tends to be the best one.”**

**William of Ockham**



Temperature  
(T)

Basic : instantaneous decay

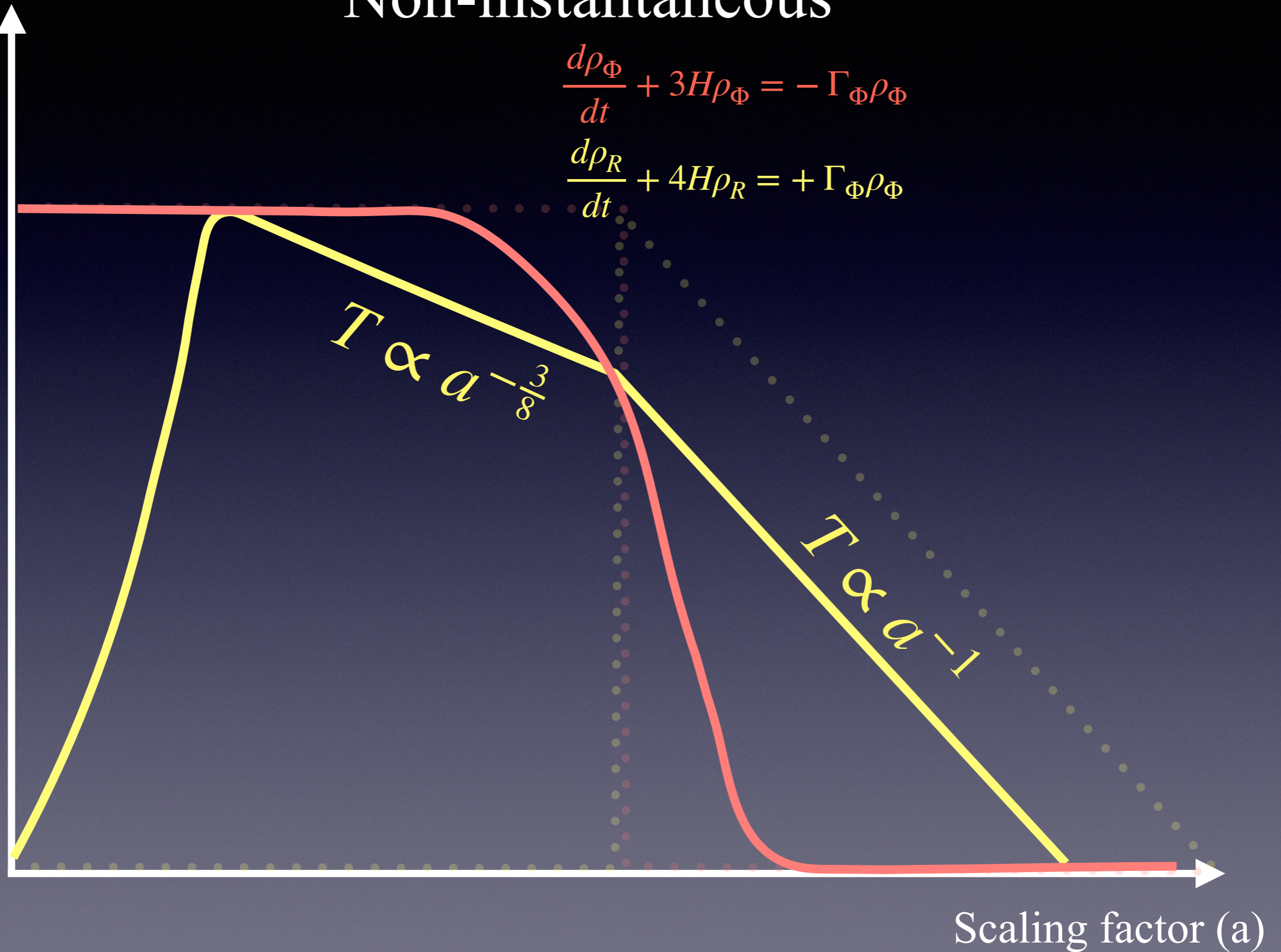


Temperature  
(T)

# Refinement I : Non-instantaneous

$$\frac{d\rho_{\Phi}}{dt} + 3H\rho_{\Phi} = -\Gamma_{\Phi}\rho_{\Phi}$$

$$\frac{d\rho_R}{dt} + 4H\rho_R = +\Gamma_{\Phi}\rho_{\Phi}$$



Temperature  
(T)

## Refinement II :

Taking into account the potential

$$\frac{d\rho_\Phi}{dt} + \frac{6k}{k+2}H\rho_\Phi = -\Gamma_\Phi\rho_\Phi$$

$$\frac{d\rho_R}{dt} + 4H\rho_R = +\Gamma_\Phi\rho_\Phi$$

$$\frac{d\rho_\Phi}{dt} + 3H(\rho_\Phi + P_\Phi) = -\Gamma_\Phi\rho_\Phi$$

$$\rho_\Phi = \frac{1}{2}\dot{\Phi} + V(\Phi)$$

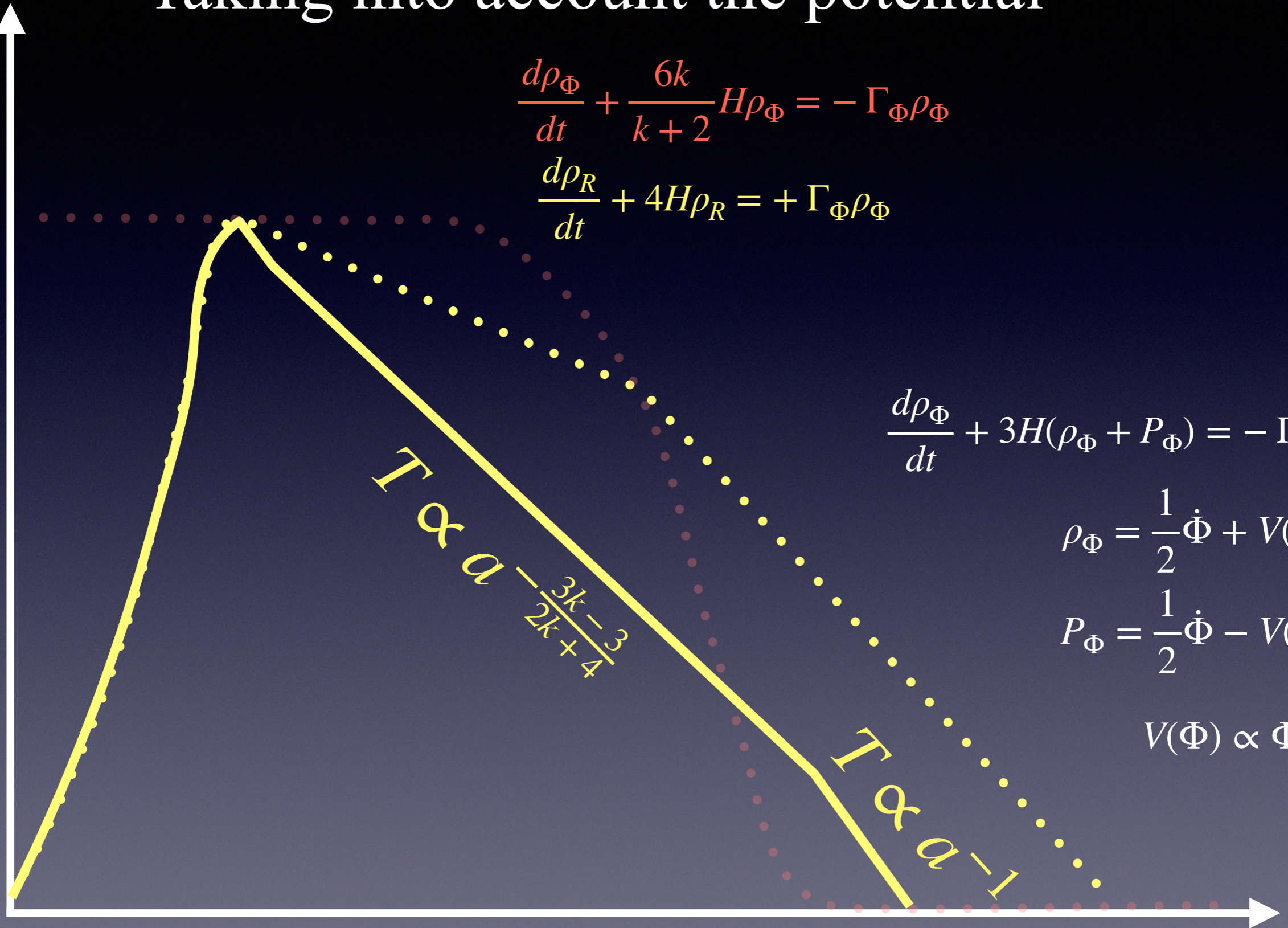
$$P_\Phi = \frac{1}{2}\dot{\Phi} - V(\Phi)$$

$$V(\Phi) \propto \Phi^k$$

$$T \propto a^{-\frac{3k-3}{2k+4}}$$

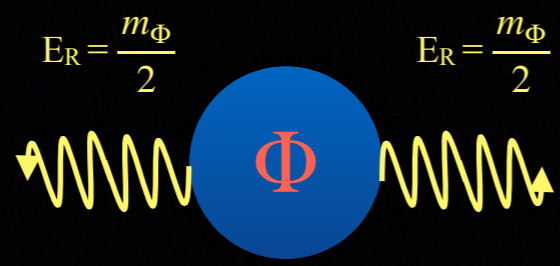
$$T \propto a^{-1}$$

Scaling factor (a)

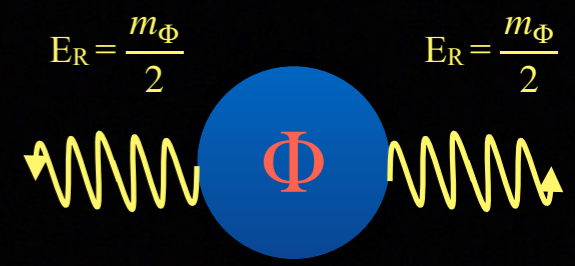


<Energy>

$$\frac{m_\Phi}{2}$$



**Refinement III :**  
Non thermal bath

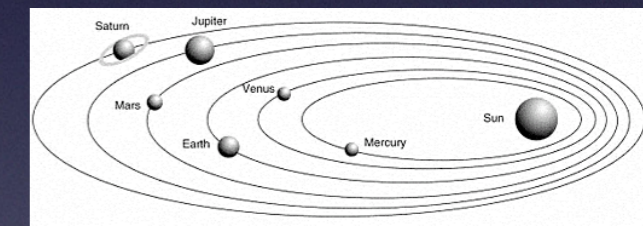


$$n\langle\sigma v\rangle \propto a^{-\frac{3}{2}} \times \frac{a^2}{m_\Phi^2} \sim \sqrt{a}$$



$n\langle\sigma v\rangle \gtrsim H \Rightarrow \text{thermal equilibrium}$

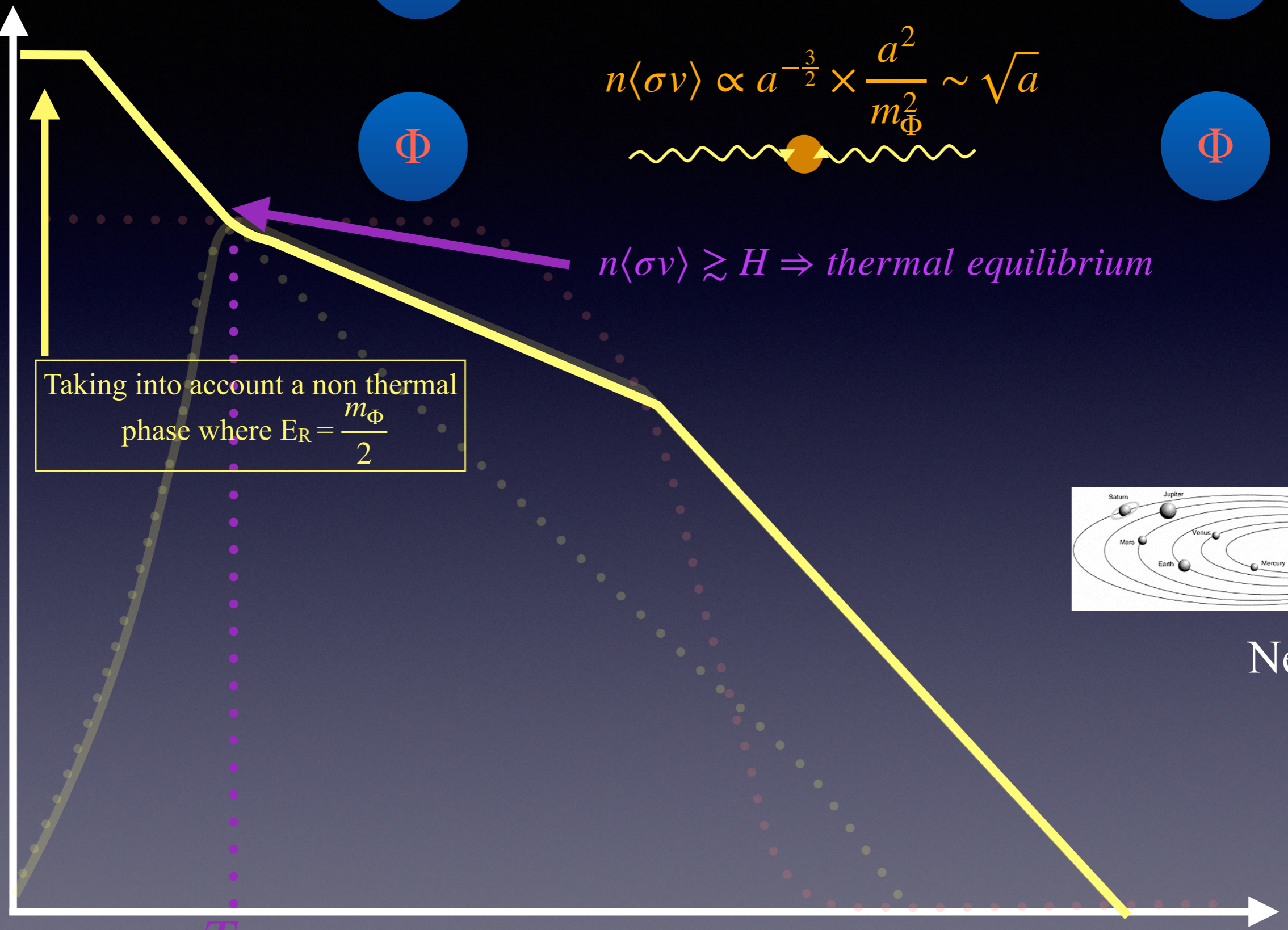
Taking into account a non thermal phase where  $E_R = \frac{m_\Phi}{2}$

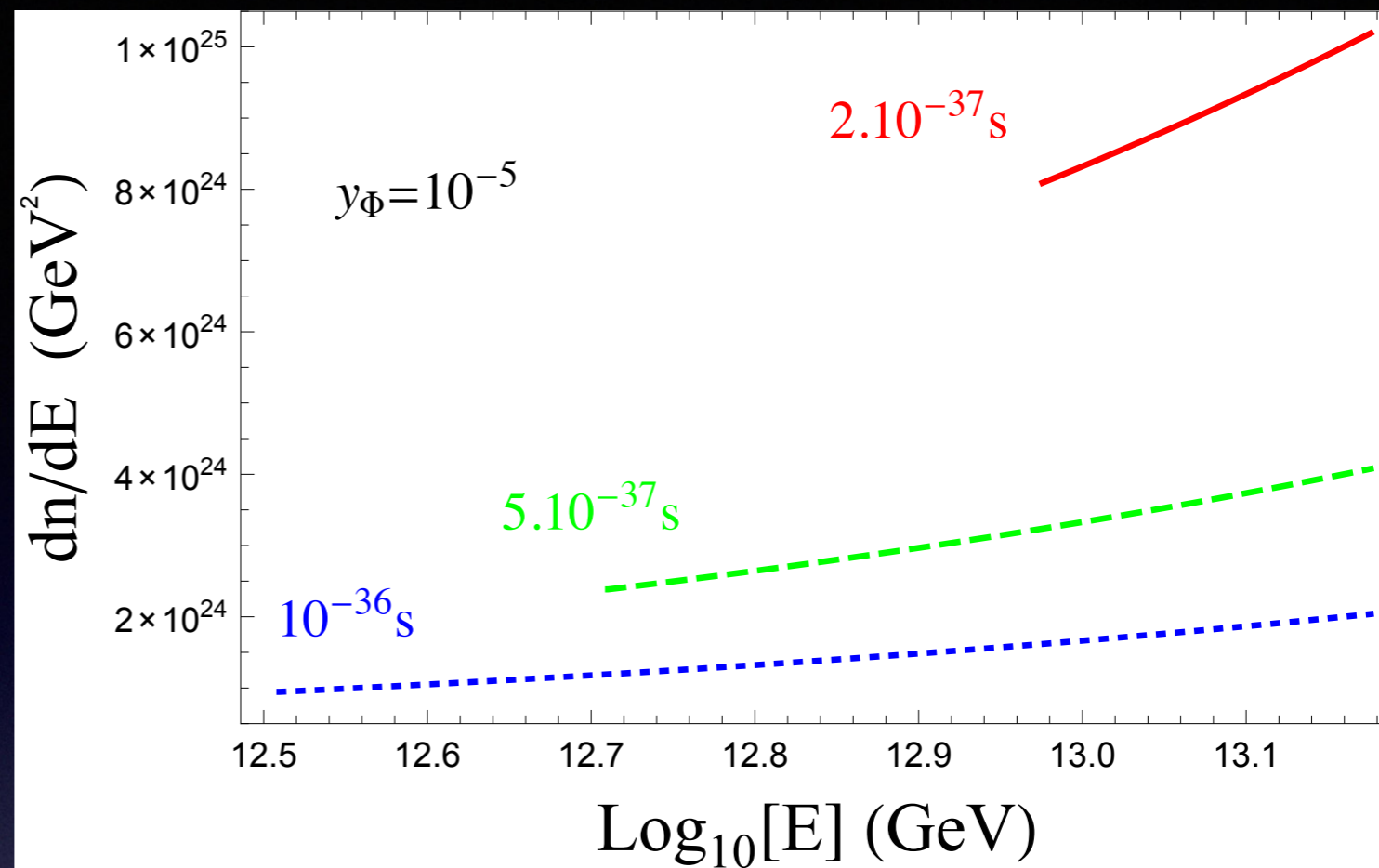
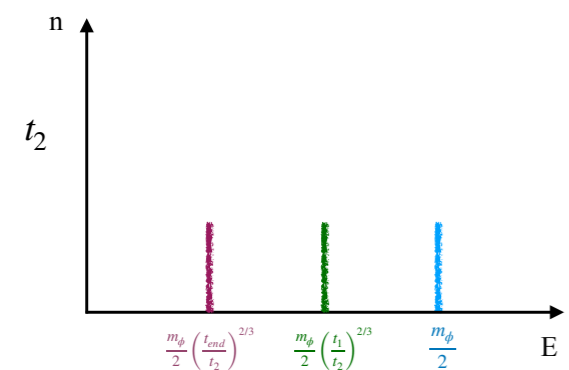
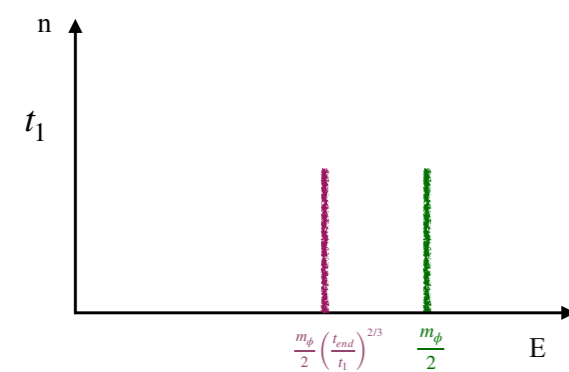
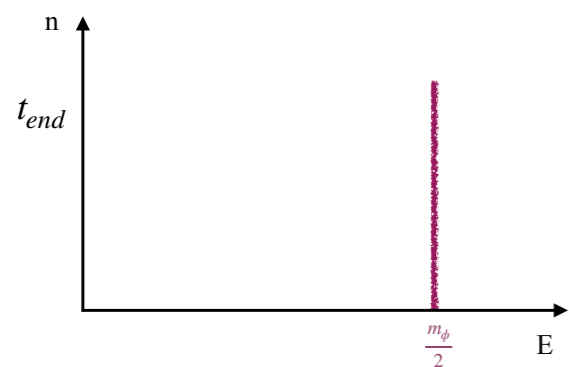


Newton

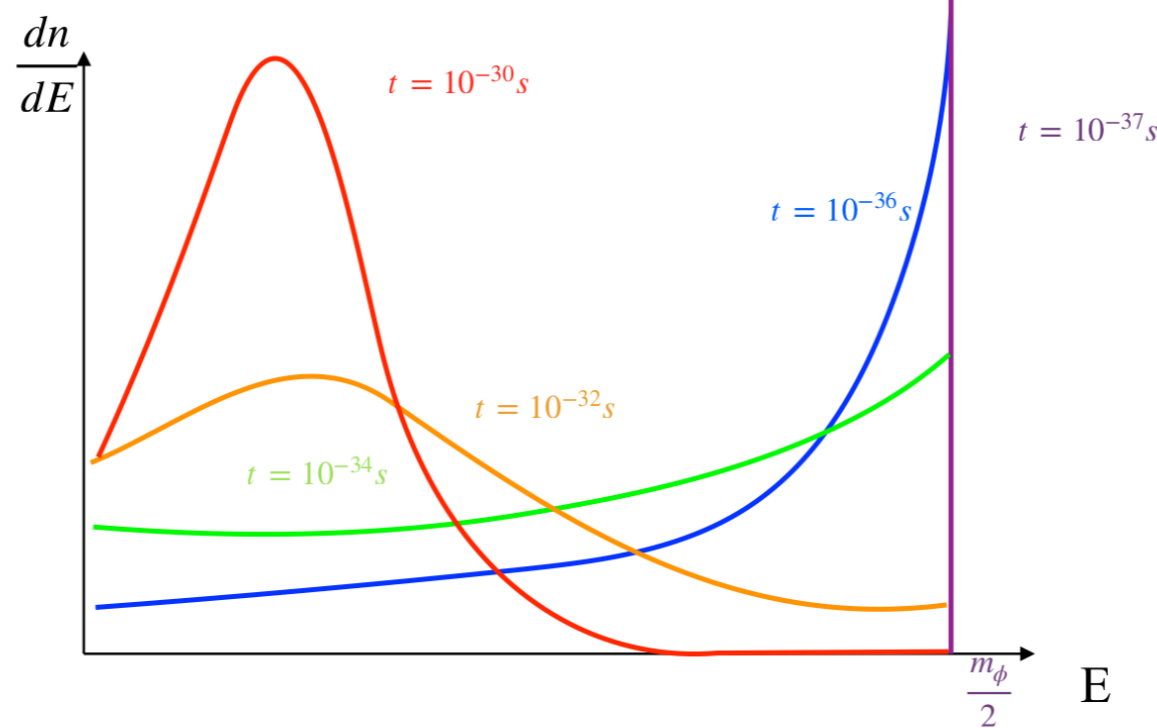
$T_{Eq}$

Scaling factor (a)



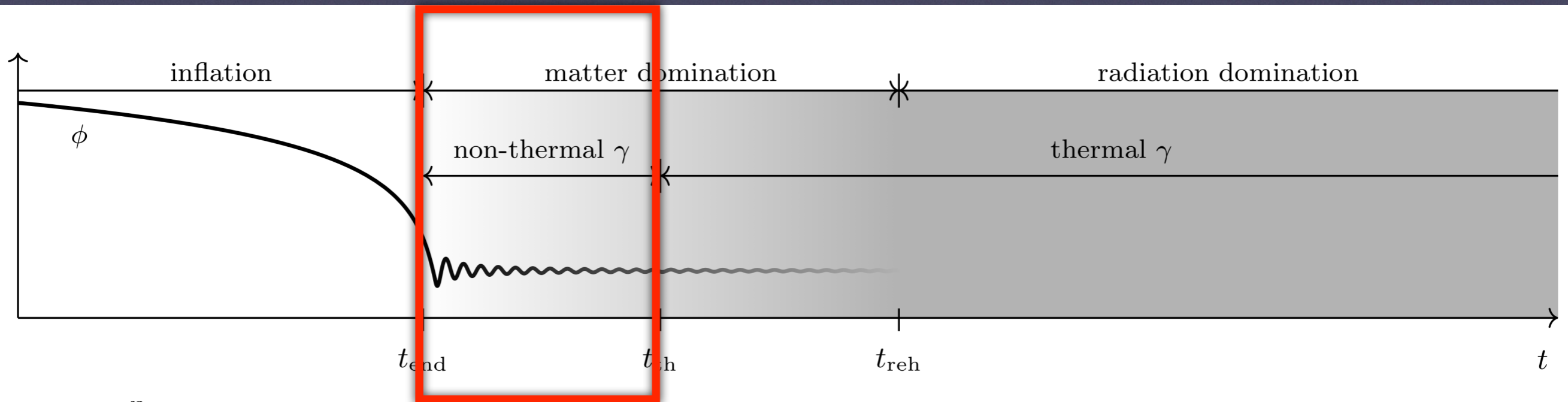


$$f(p) = \frac{8\sqrt{2}\pi^2\Gamma_\phi M_P^2}{p^{3/2}m_\phi^{5/2}t}$$



$$f(p) = \frac{1}{e^{\frac{p}{kT}} \pm 1}$$

# Mechanisms to produce (dark) matter at the **end of inflation**



After several oscillations ( $N \gtrsim 100$ ) :  
The reheating phase

$$\rho_\phi = T_{00}^\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

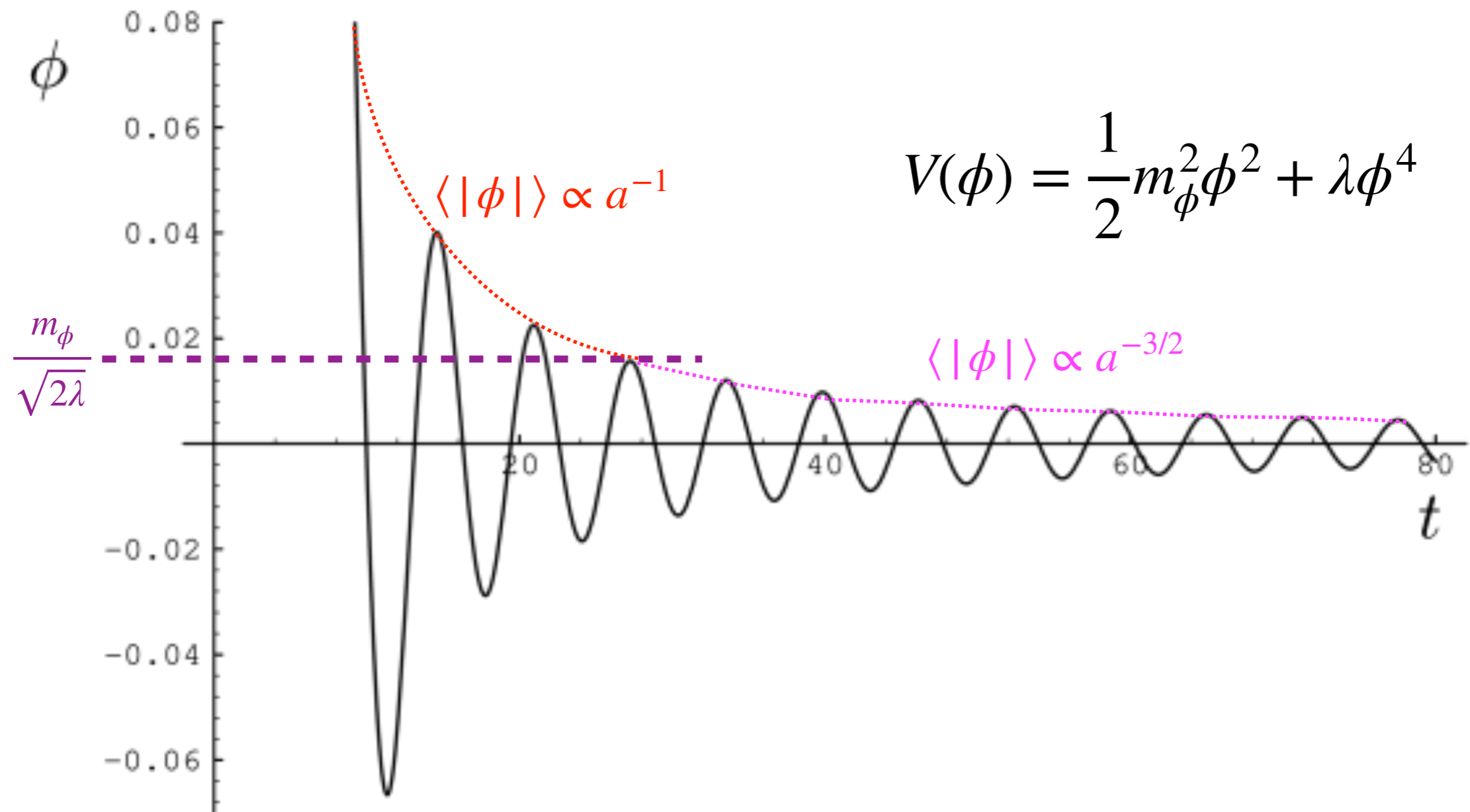
$$P_\phi = T_{ii}^\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\ddot{\phi}(t) + 3H\dot{\phi} - \frac{\nabla}{a^2}\phi(t) + V'(\phi) = 0$$

$$V(\phi) = \lambda_k \phi^k$$

$$\dot{\rho}_\phi + \frac{6k}{k+2}H\rho_\phi = 0$$

$$\Rightarrow \rho_\phi \propto a^{-\frac{6k}{k+2}} = \langle V(\phi) \rangle$$



$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 + \lambda\phi^4$$

# Adding a coupling to matter (1)

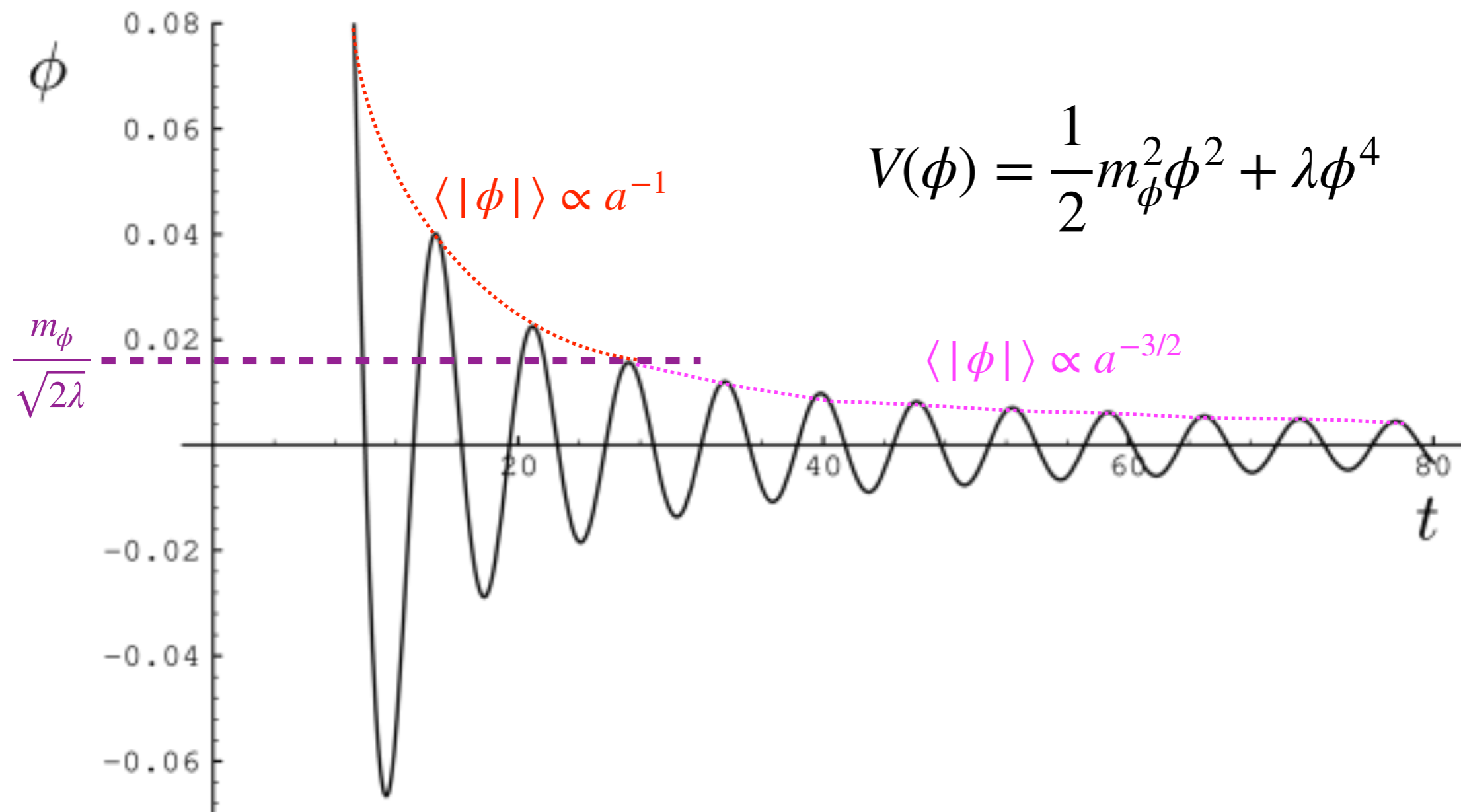
$$V = V(\phi) + y\phi\bar{f}f + \sigma\phi^2\chi^2$$

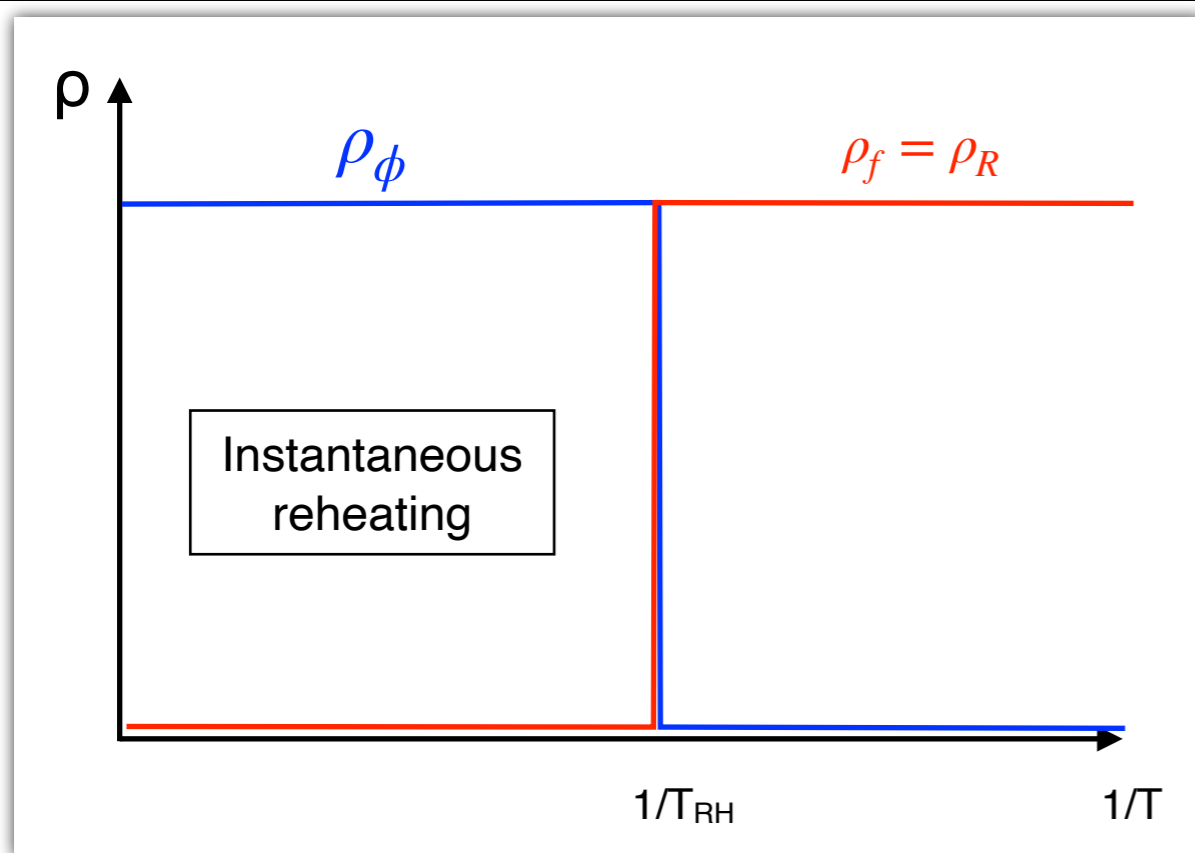
$$\sigma = 0$$

$$\ddot{\phi}(t) + 3H\dot{\phi} - \frac{\nabla}{a^2}\phi(t) + V'(\phi) = 0$$

$$\dot{\rho}_\phi + \frac{6k}{k+2}H\rho_\phi = 0$$

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 + \lambda\phi^4$$

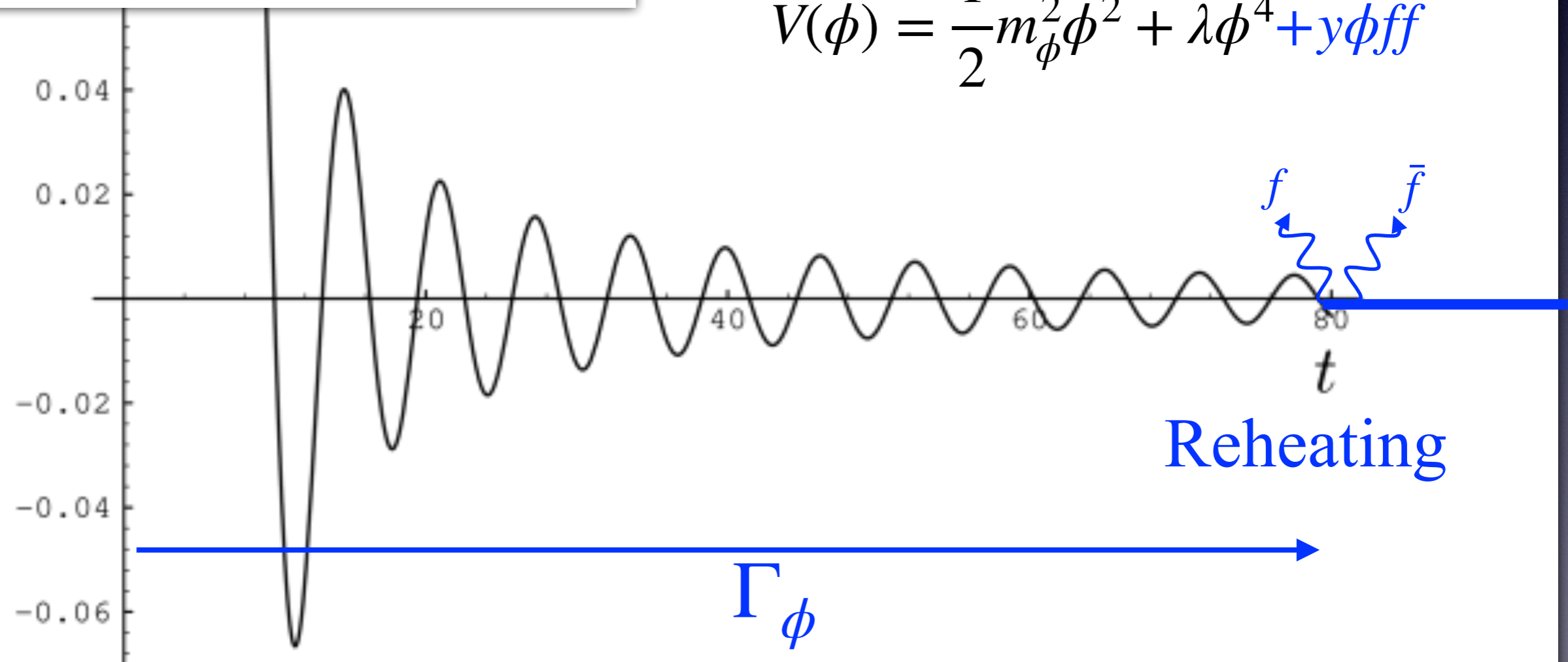


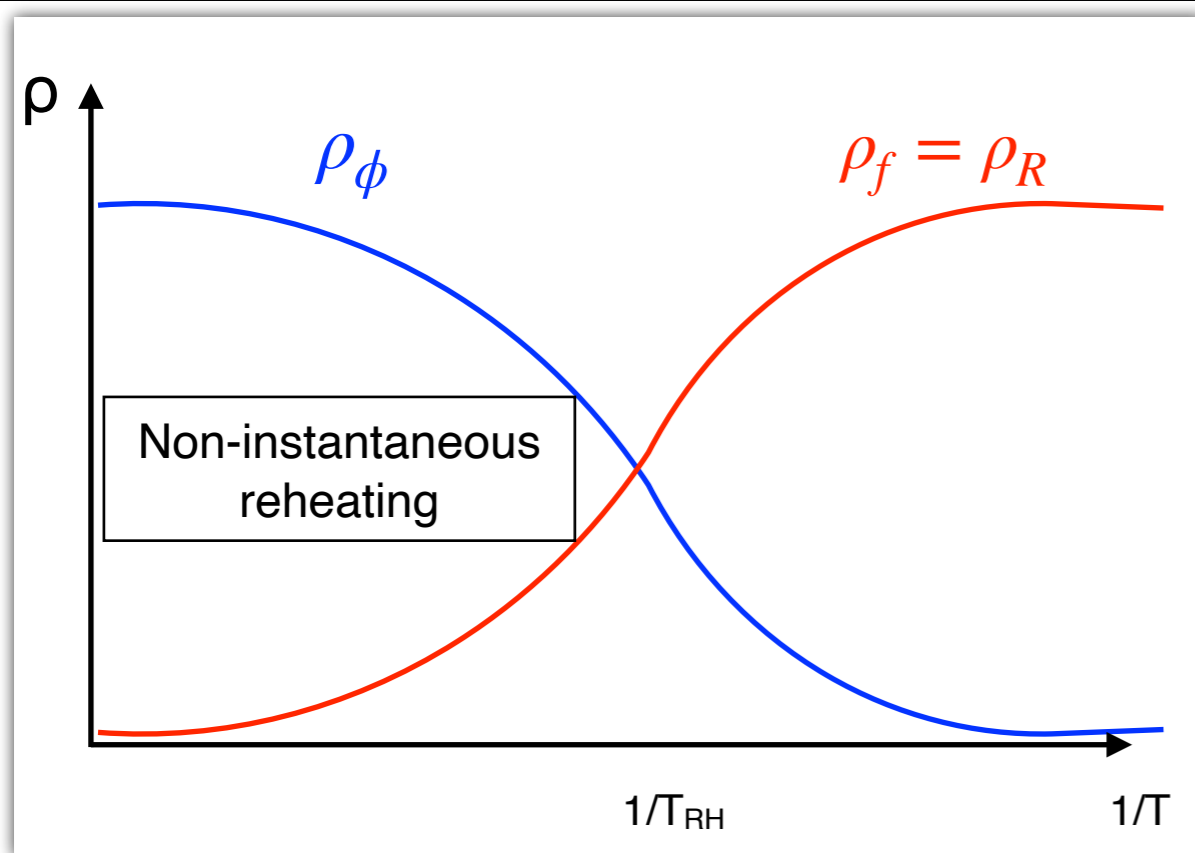


$$\ddot{\phi}(t) + 3H\dot{\phi} - \frac{\nabla}{a^2}\phi(t) + V'(\phi) = -\Gamma_\phi\dot{\phi}$$

$$\dot{\rho}_\phi + \frac{6k}{k+2}H\rho_\phi = -\Gamma_\phi\rho_\phi$$

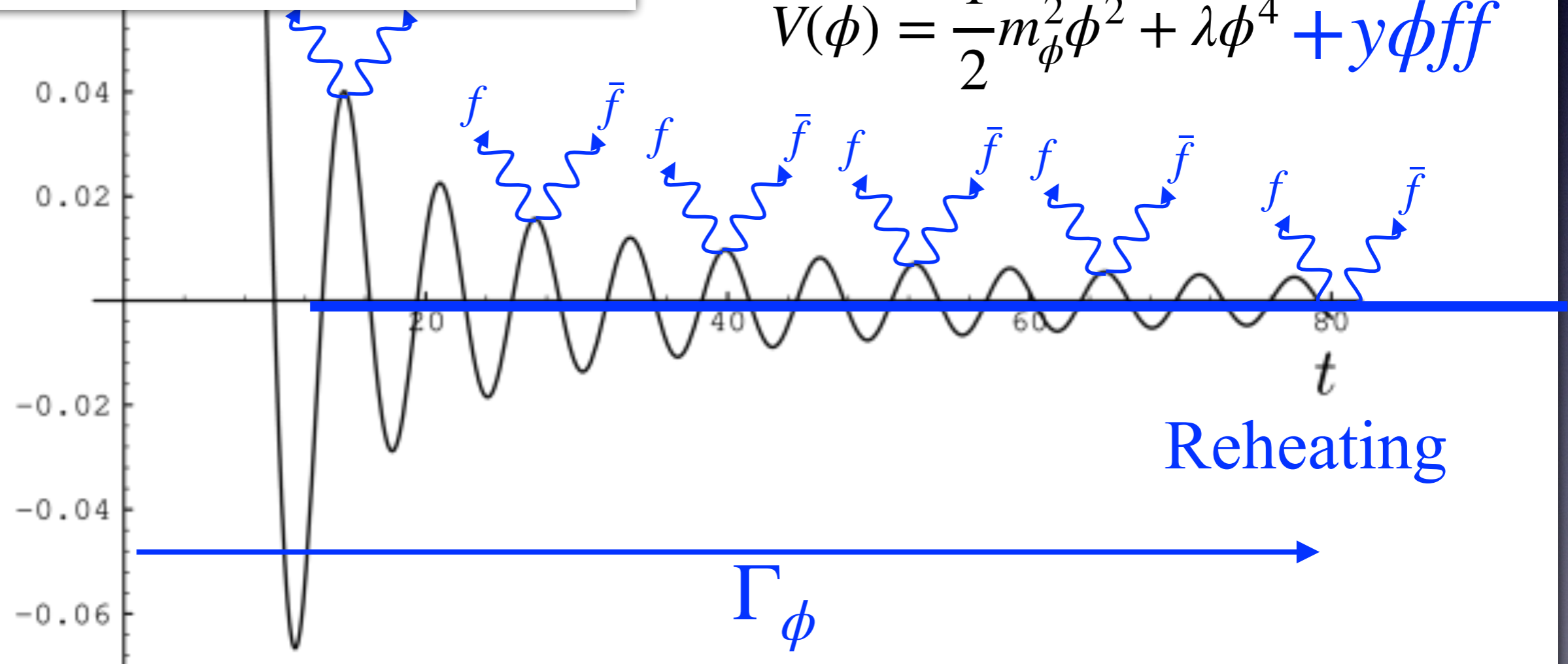
$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 + \lambda\phi^4 + y\phi\bar{f}f$$

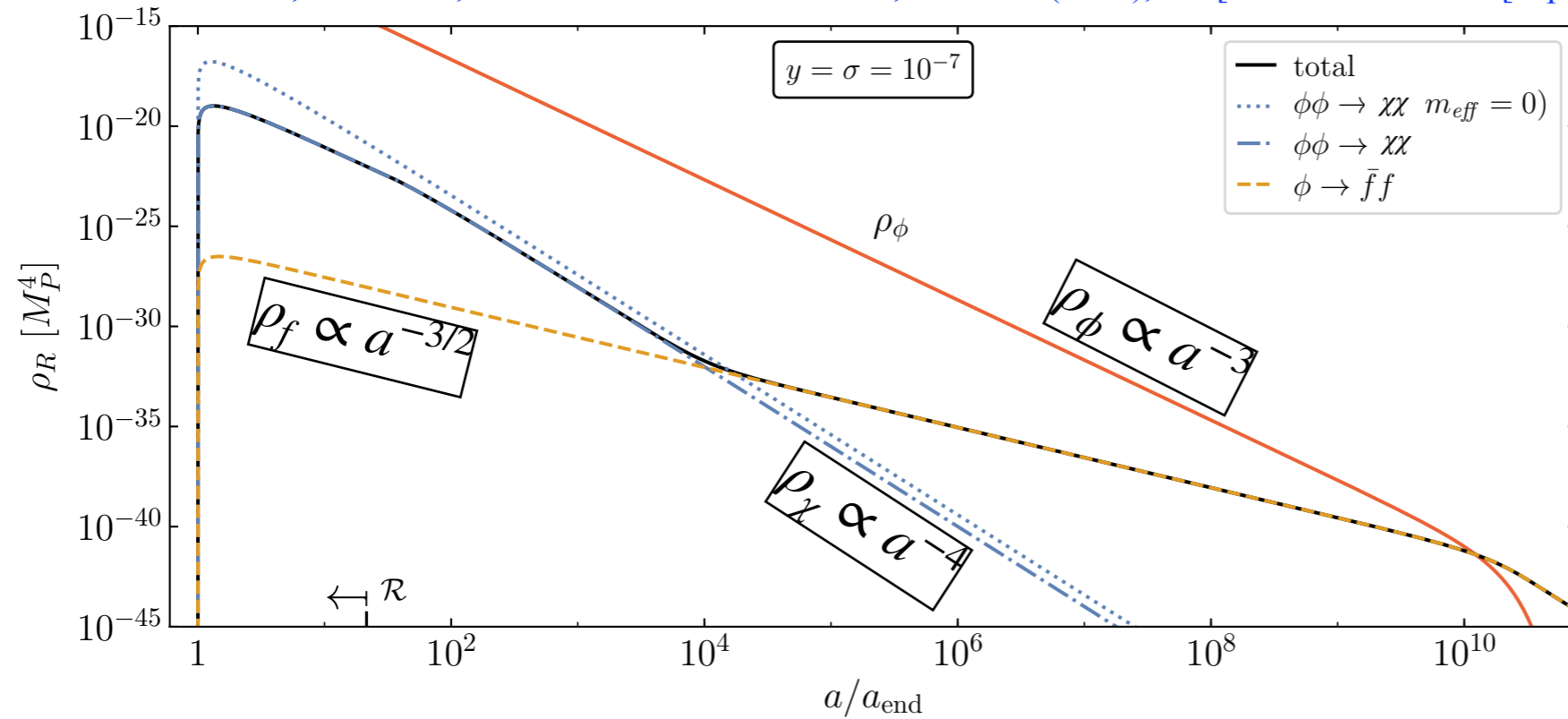




$$\ddot{\phi}(t) + 3H\dot{\phi} - \frac{\nabla}{a^2}\phi(t) + V'(\phi) = -\Gamma_\phi\dot{\phi}$$

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 + \lambda\phi^4 + y\phi\bar{f}f$$

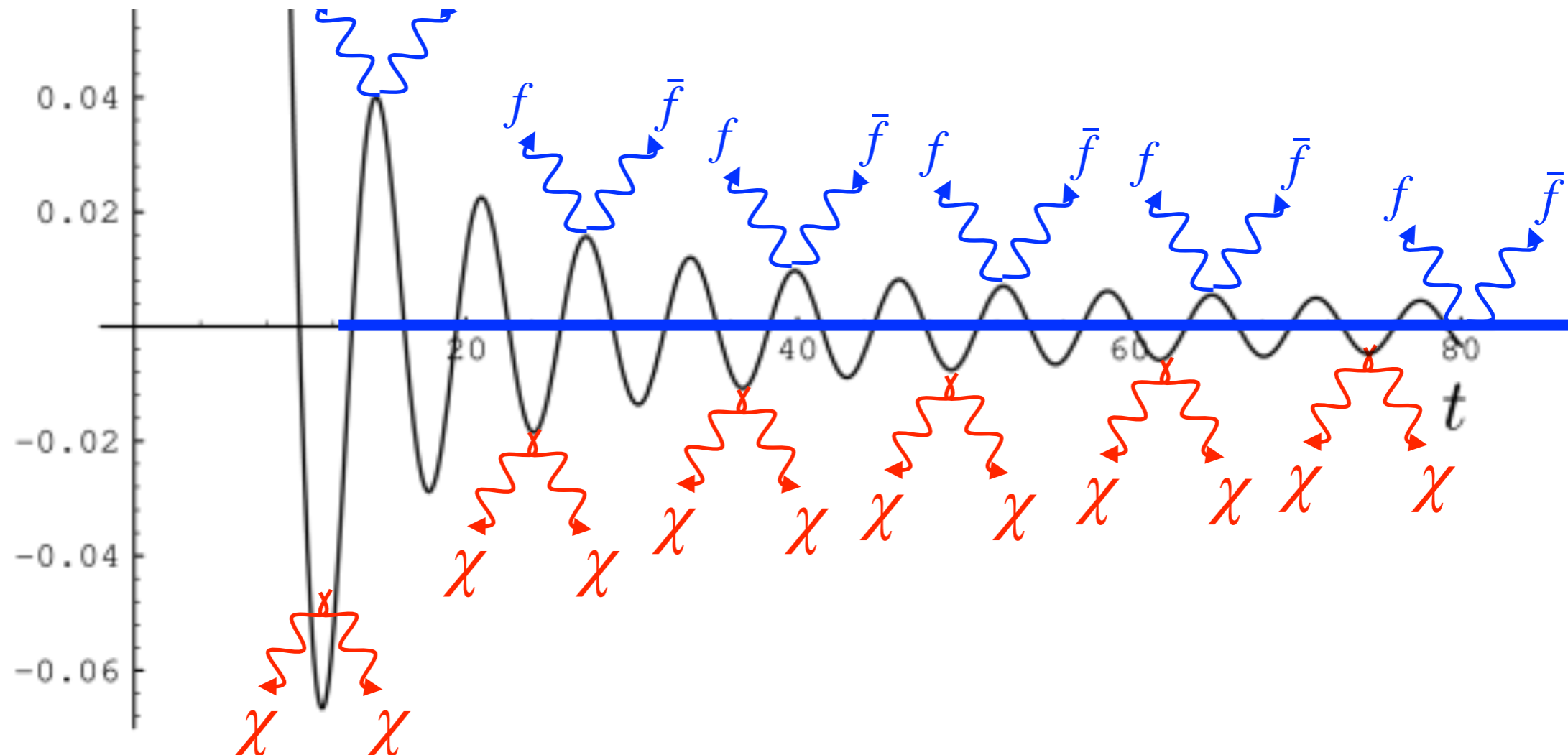




$$\frac{1}{2}m_\phi^2\phi^2 + y\phi f\bar{f} + \sigma\phi^2\chi^2$$

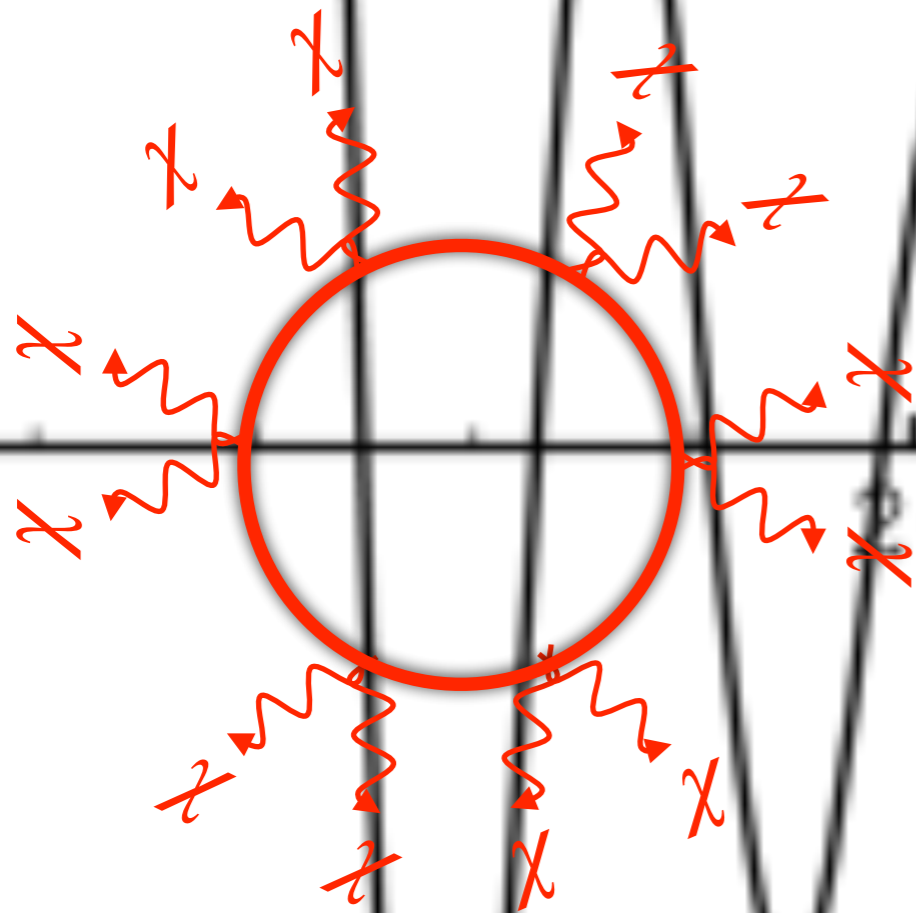
$$3H\dot{\chi} - \frac{\nabla}{a^2}\chi(t, x) + 2\sigma\phi^2\chi = 0$$

$$m_\chi^{\text{eff}} = \sqrt{2\sigma}\phi$$



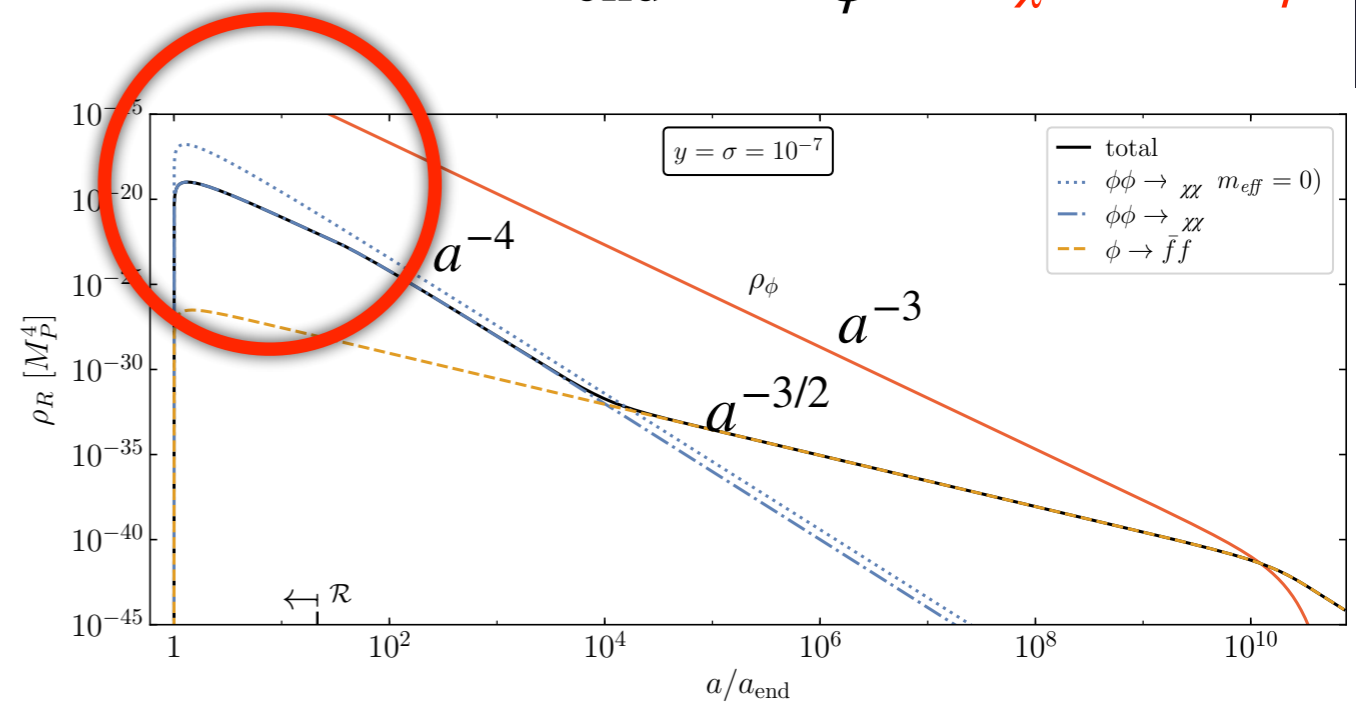
$$\ddot{\chi}(t, x) + 3H\dot{\chi}(t, x) - \frac{\nabla^2}{a^2}\chi(t, x) + 2\sigma\phi^2\chi(t, x) = 0$$

$$m_\chi^{\text{eff}} = \sqrt{2\sigma}\Phi$$



Resonant  
production

$$\sigma \neq 0, \quad \sigma \times \phi_{\text{end}}^2 \gtrsim m_\phi^2 \quad [m_\chi^{\text{eff}} \gtrsim m_\phi]$$



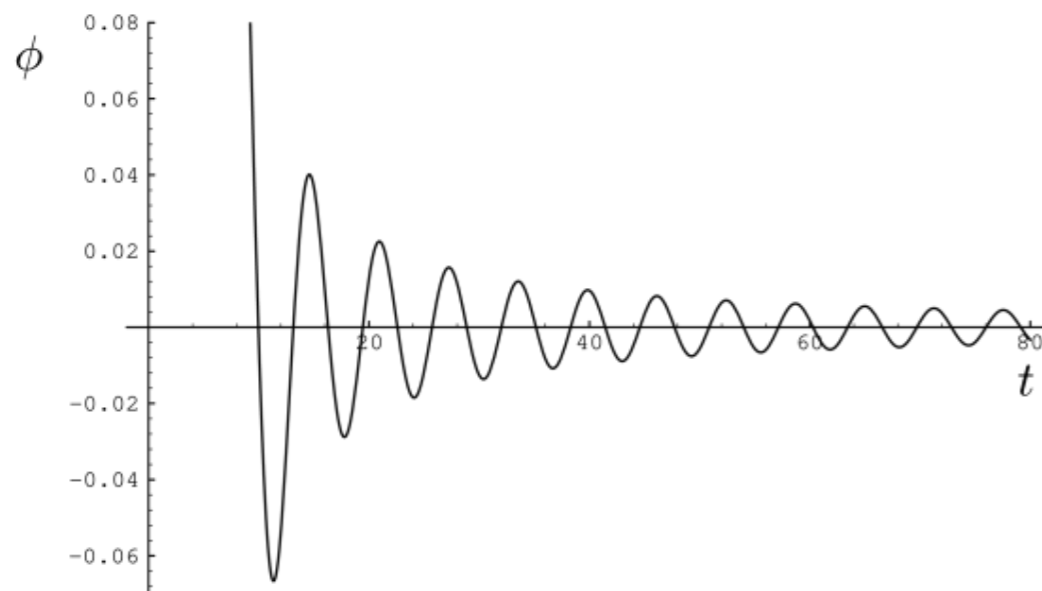
$$\ddot{\chi}(t, x) + 3H\dot{\chi}(t, x) - \frac{\nabla}{a^2}\chi(t, x) + 2\sigma\phi^2\chi(t, x) = 0 \quad m_{\chi}^{\text{eff}} = \sqrt{2\sigma}\Phi$$

$$\chi = \int \frac{d^3p}{(2\pi)^{3/2}} \left[ e^{-ipx} \chi_p(t) a_p + e^{ipx} \chi_p^*(t) a_p^\dagger \right]$$

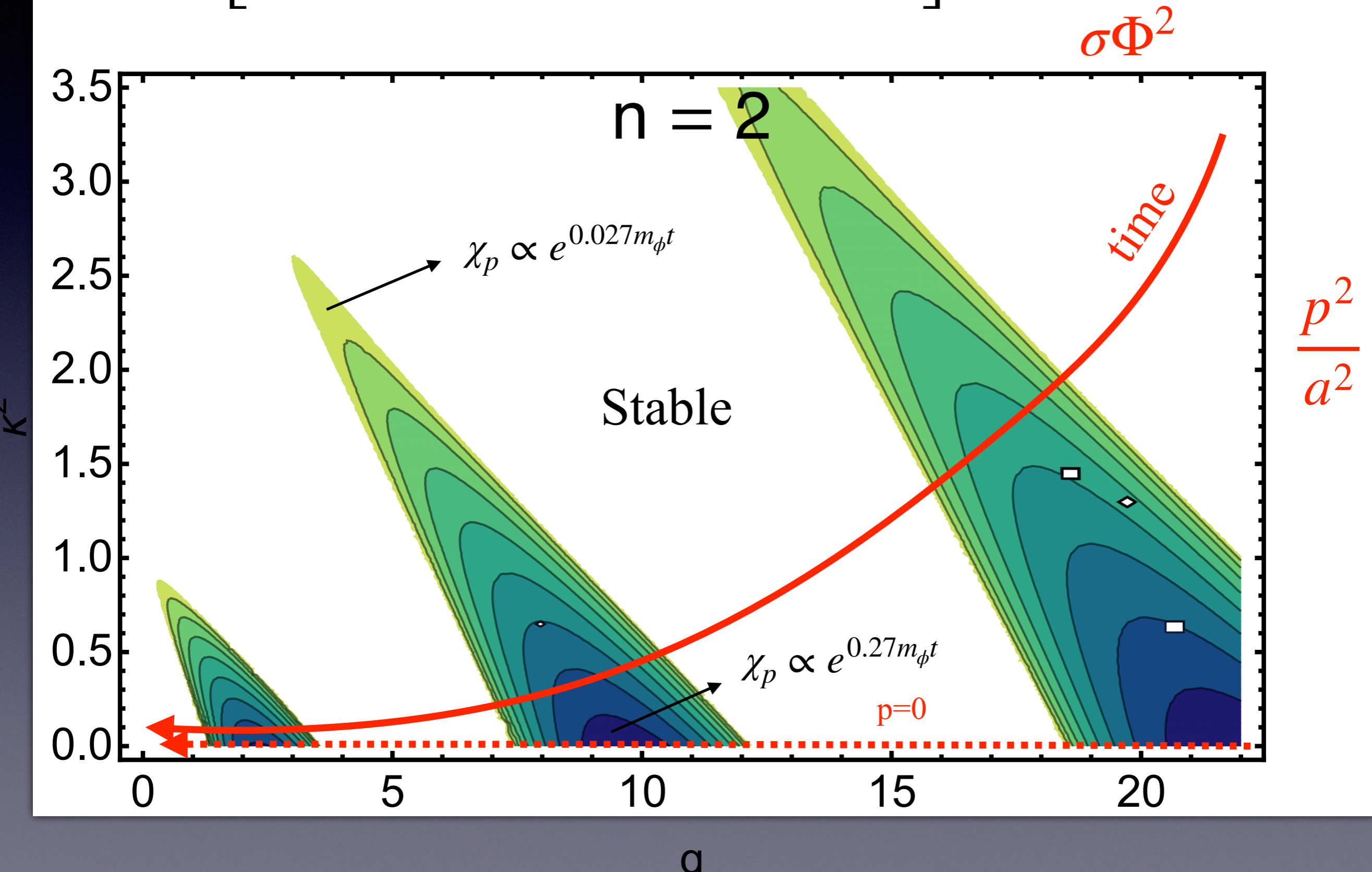
$$\ddot{\chi}_p(t) + \left[ \frac{p^2}{a^2} + (m_{\chi}^{\text{eff}})^2 + \sigma\Phi^2(t) \times \cos 2m_{\phi}t \right] \chi_p(t) = 0 \quad \text{Mathieu equation}$$

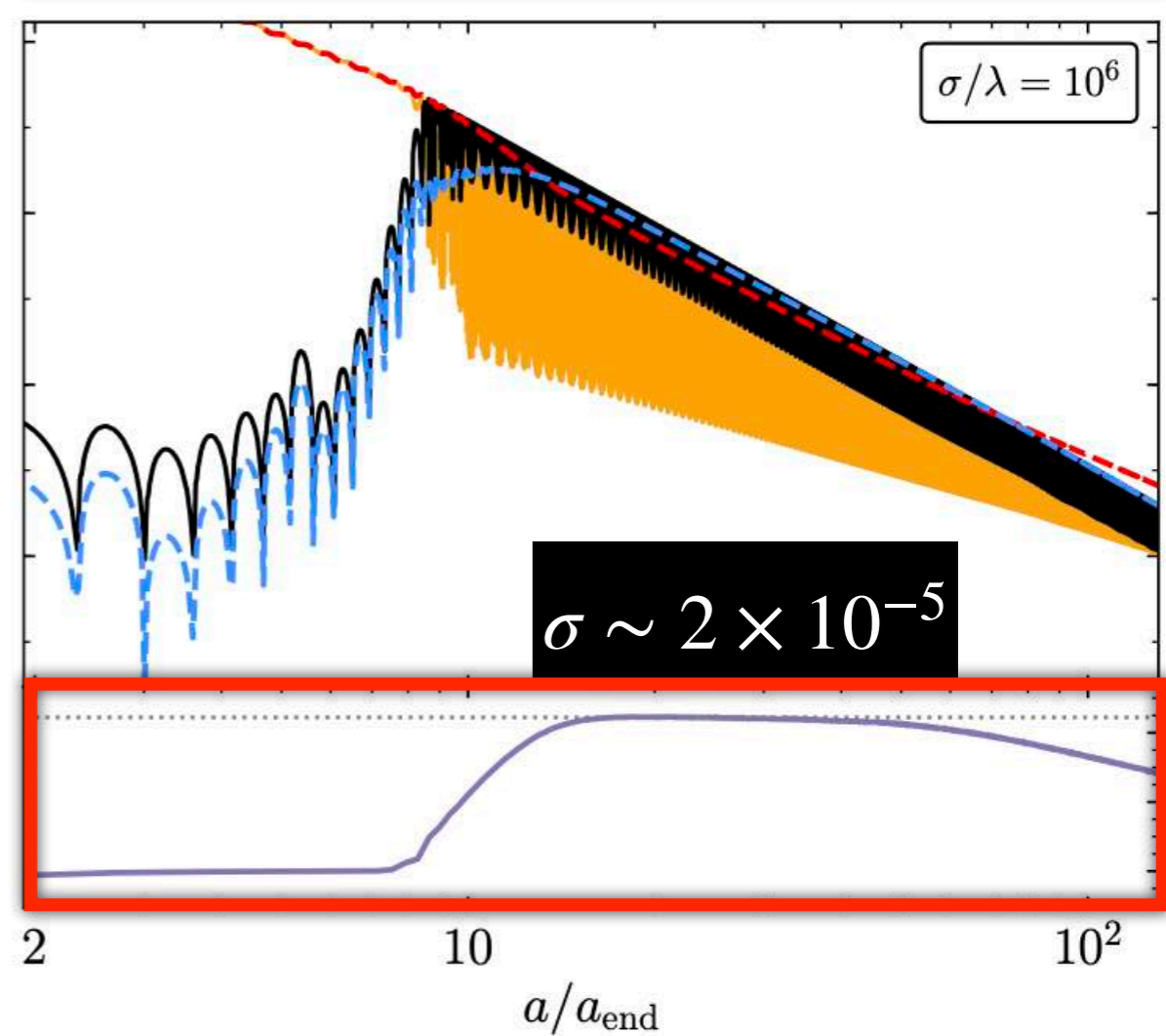
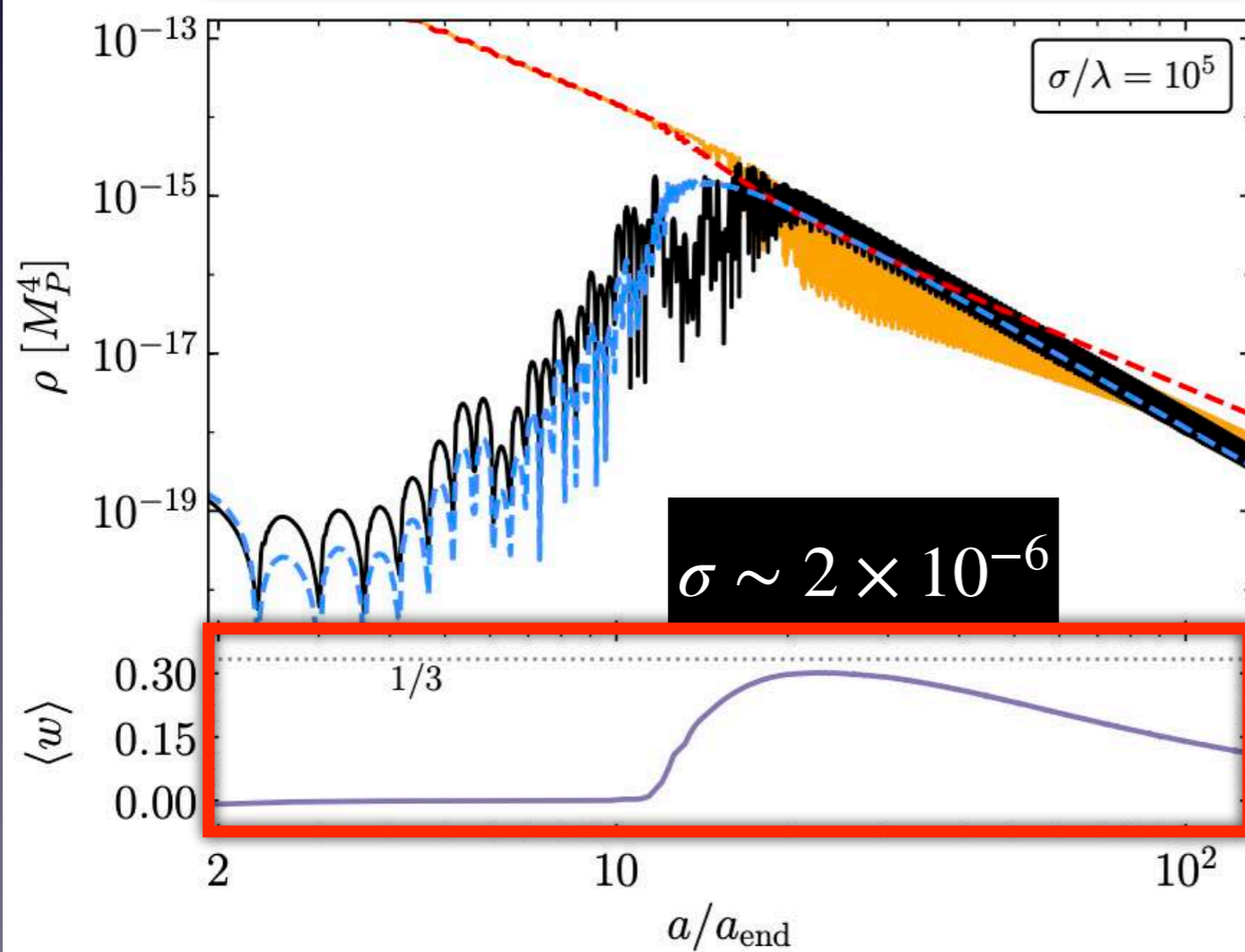
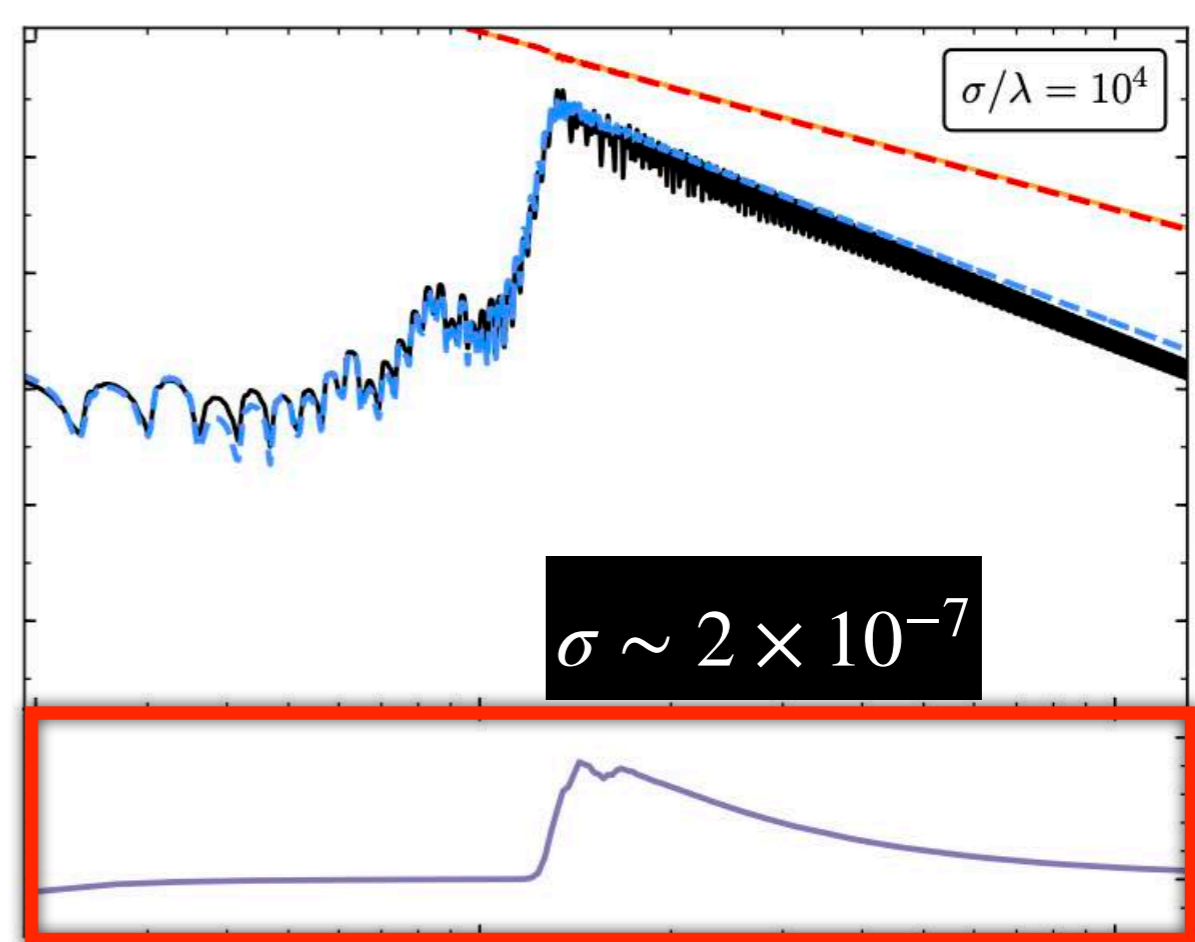
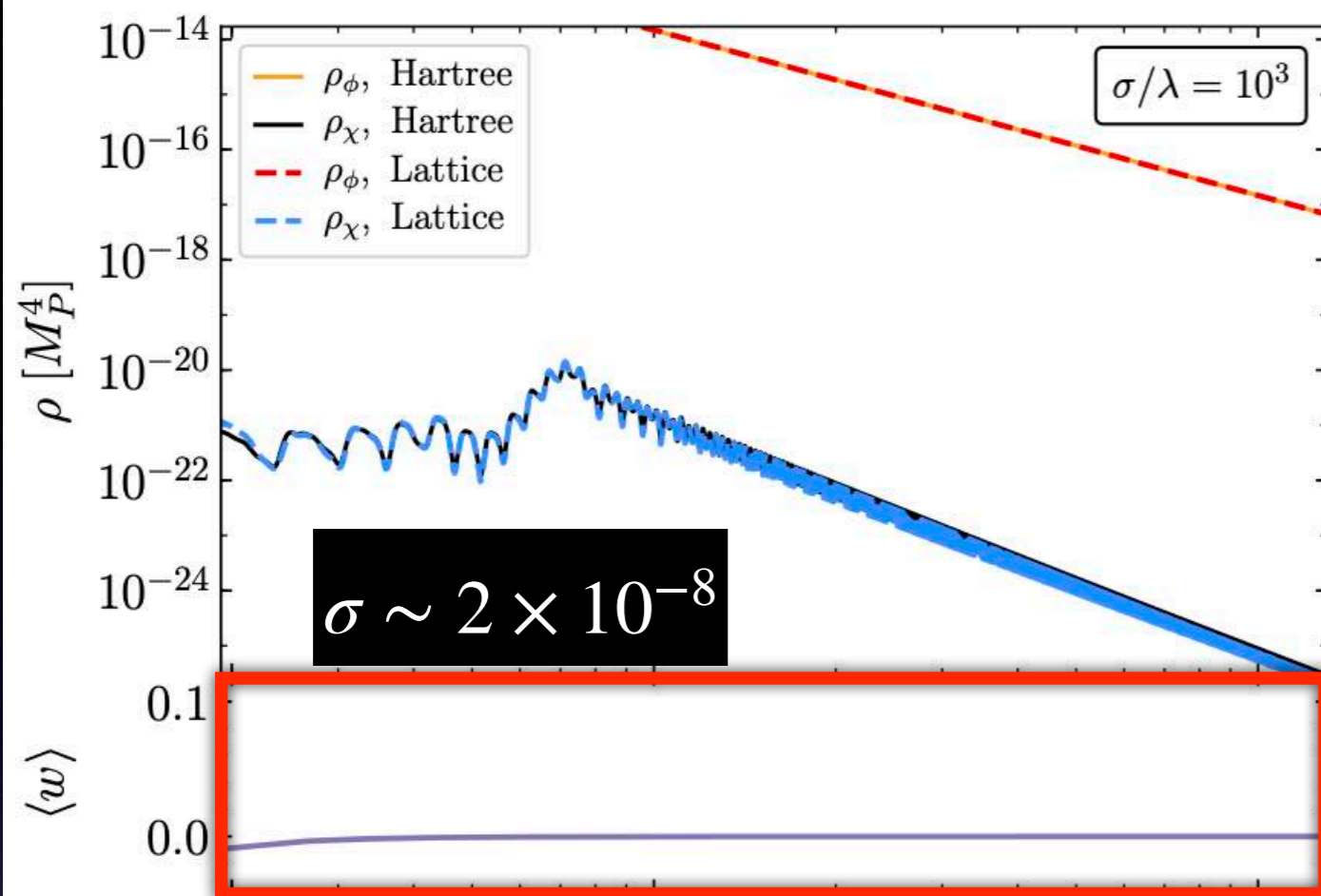
The Mathieu equation is present in any system with a periodical source of energy.

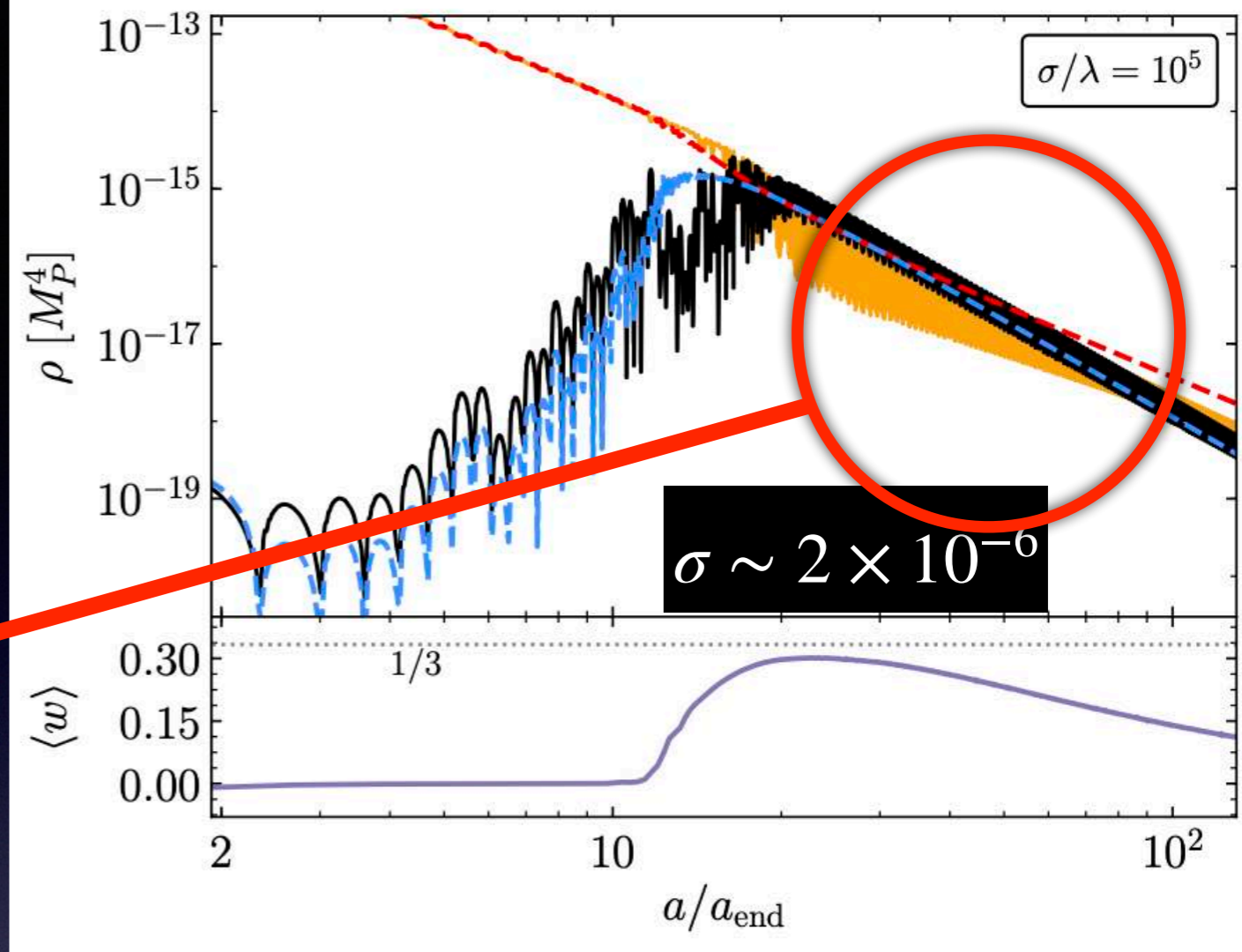
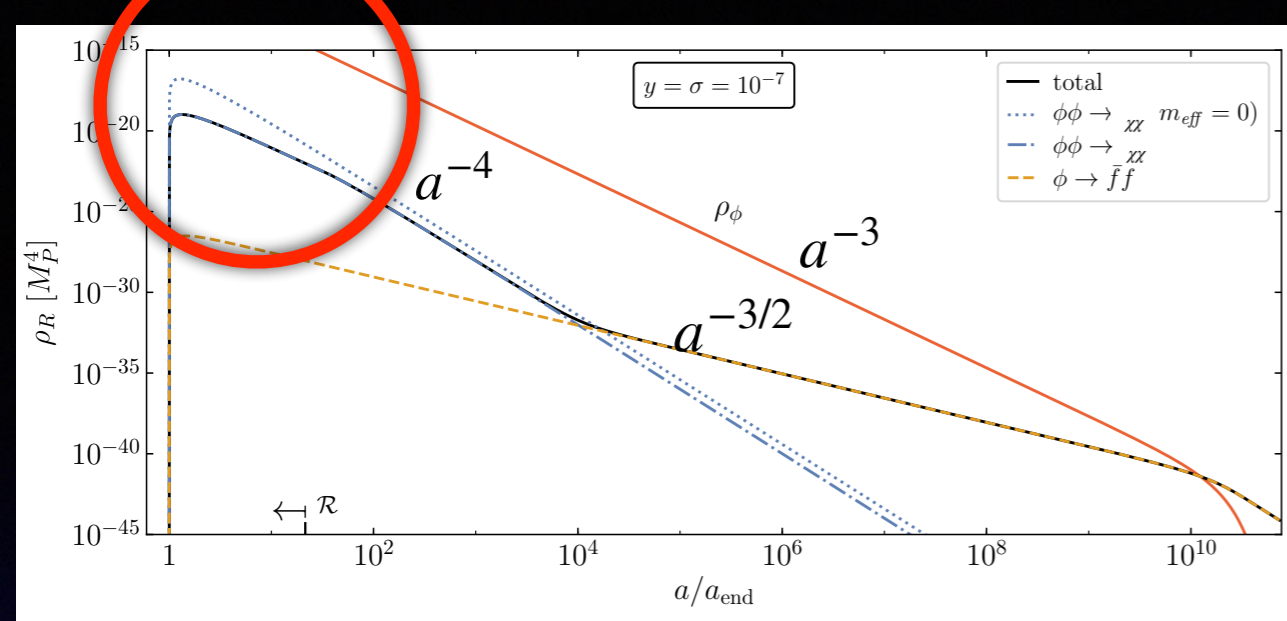
From electric circuit to mechanical balance, spring excitations...



$$\ddot{\chi}_p(t) + \left[ \frac{p^2}{a^2} + (m_\chi^{\text{eff}})^2 + \sigma\Phi^2(t) \times \cos 2m_\phi t \right] \chi_p(t) = 0 \quad \text{Mathieu equation}$$







$$\sigma \Phi^2 \lesssim H^2$$

$$\dot{\chi}(t) + 3H\dot{\chi} - \frac{\nabla}{a^2}\chi(t) + 2\sigma\phi^2\chi = 0$$

Back-reaction effects of  $\chi$  on  $\phi$

# Backreactions

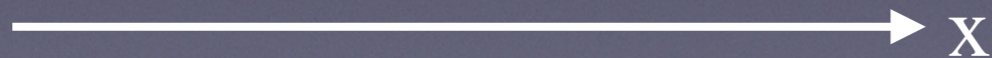
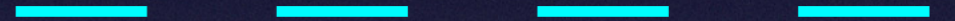
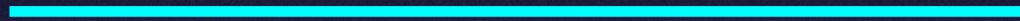
Formally speaking, this is the effect of  $\chi$  on the condensate  $\phi$  through the equation of motion

The net effect is the destruction of the  $\phi$ -condensate into  $\phi$ -particles

$$\ddot{\chi} + 3H\dot{\chi} - \frac{\nabla^2}{a^2}\chi + 2\sigma\phi^2\chi = 0$$

$$\ddot{\phi}(t, \mathbf{x}) + 3H\dot{\phi}(t, \mathbf{x}) + m_\phi^2\phi(t, \mathbf{x}) - \frac{\nabla^2}{a^2}\phi(t, \mathbf{x}) + 2\sigma\chi^2\phi = 0$$

$$\ddot{\phi}(t, \mathbf{x}) + 3H\dot{\phi}(t, \mathbf{x}) + m_{\phi}^2\phi(t, \mathbf{x}) - \frac{\nabla^2}{a^2}\phi(t, \mathbf{x}) + 2\sigma\chi^2\phi = 0$$



Frequency  $m_{\phi}$

$\Leftrightarrow$  N coherent oscillators of density

$n_{\phi} = \frac{\rho_{\phi}}{m_{\phi}}$  and frequency  $m_{\phi}$

$$\ddot{\phi}(t, \mathbf{x}) + 3H\dot{\phi}(t, \mathbf{x}) + m_{\phi}^2\phi(t, \mathbf{x}) - \frac{\nabla^2}{a^2}\phi(t, \mathbf{x}) + 2\sigma\chi^2\phi = 0$$

$\chi(p_1)$                        $\chi(p_2)$                        $\chi(p_3)$                        $\chi(p_4)$

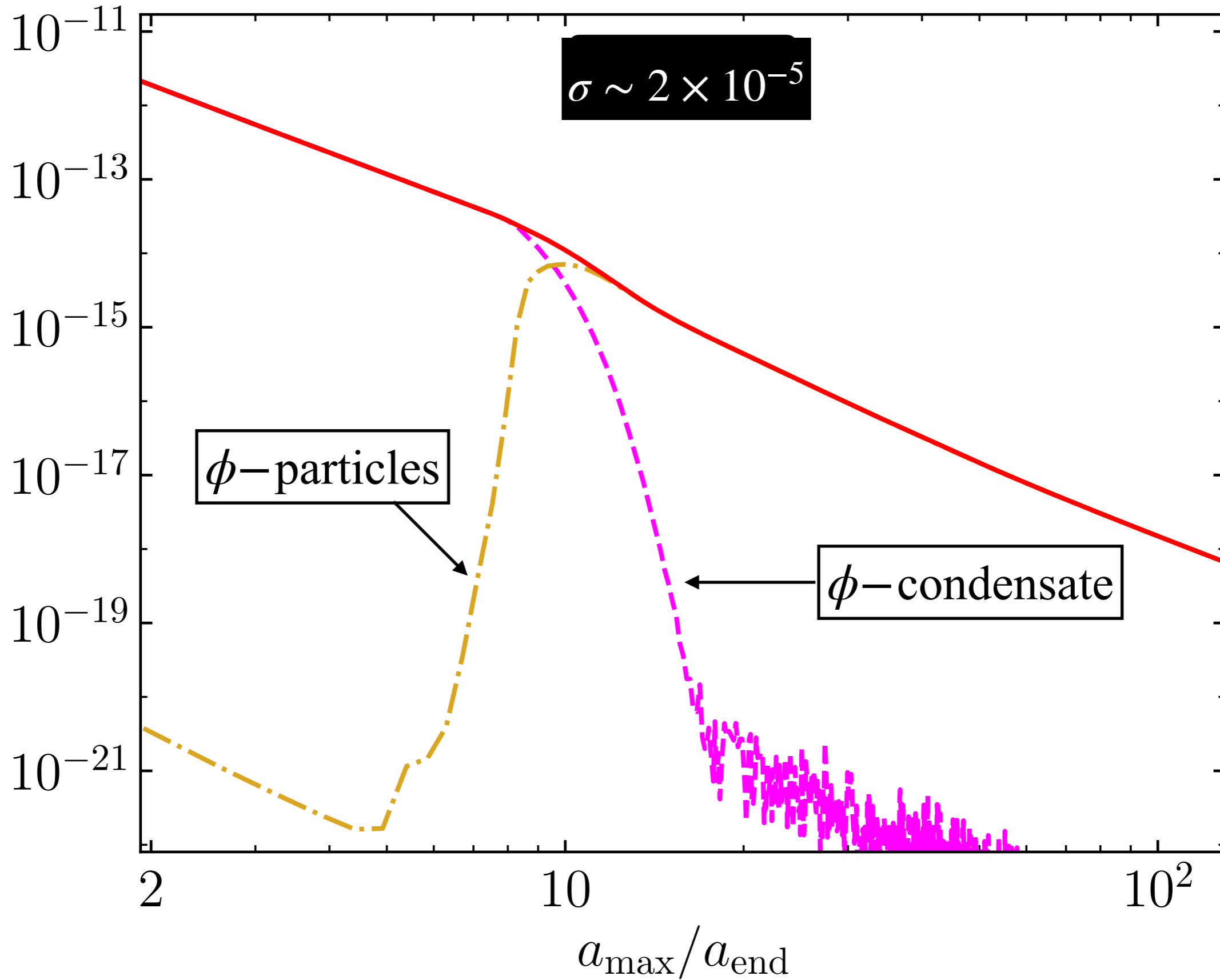


Frequency  $m_{\phi}$

$\Leftrightarrow$  N coherent oscillators of density

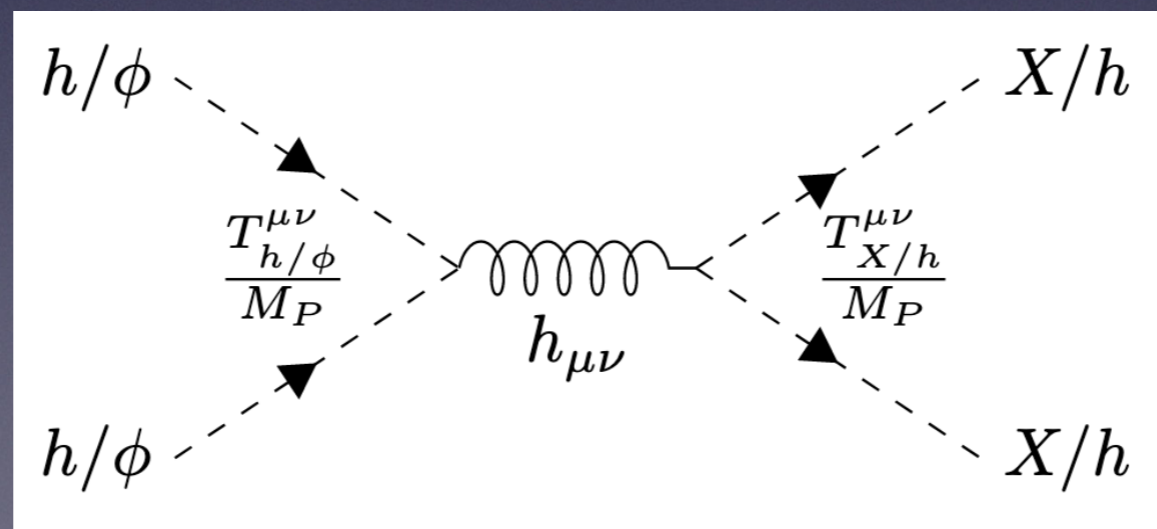
$n_{\phi} = \frac{\rho_{\phi}}{m_{\phi}}$  and frequency  $m_{\phi}$

$\ddot{\phi}(t, \mathbf{x})$

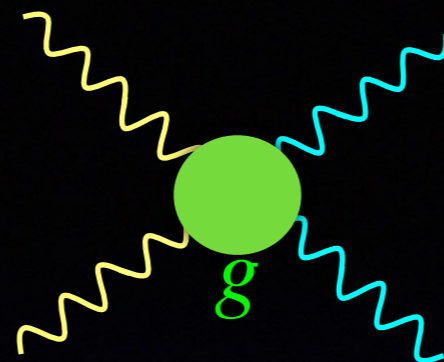
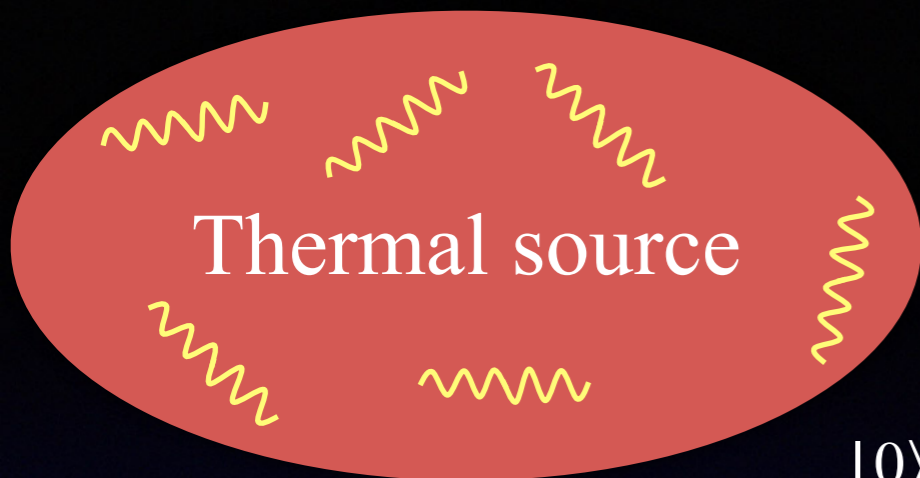


an

# (Dark) matter from gravitational scattering of the inflaton



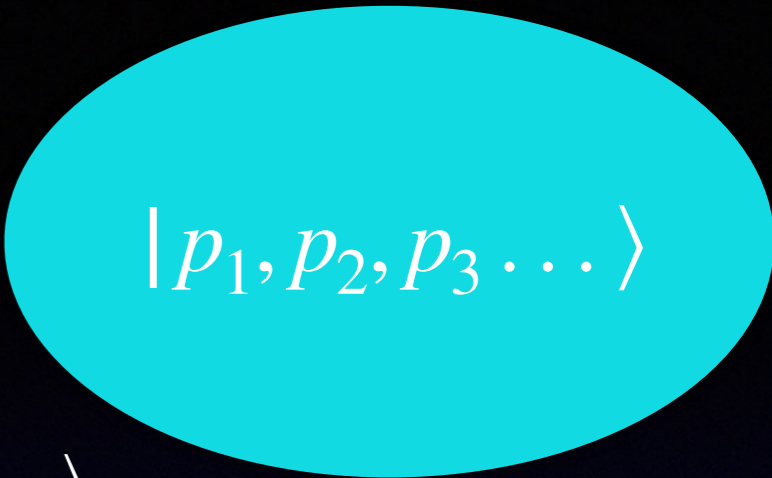
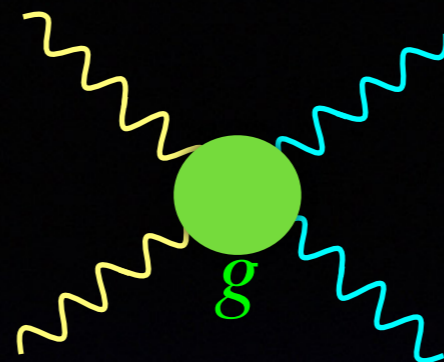
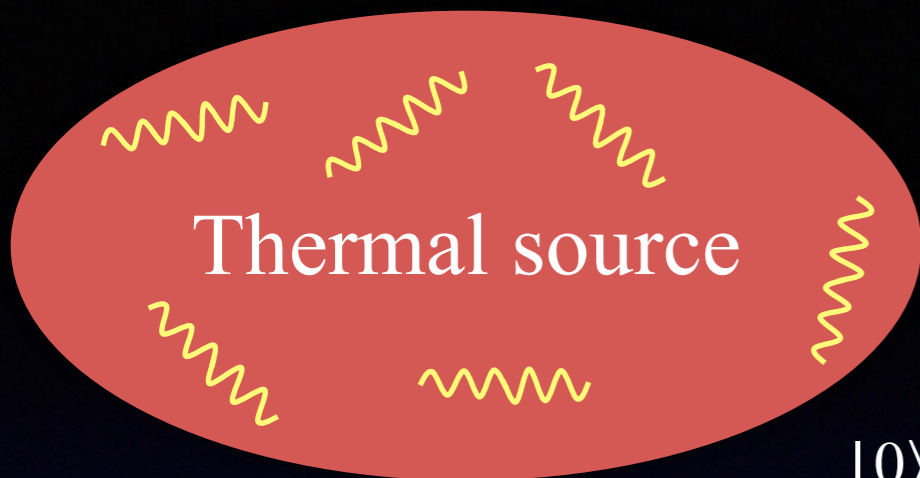
The meaning of  
« gravitational production »



$$|0\rangle \rightarrow_t |0\rangle + \epsilon |0\rangle = |0\rangle + g^4 T^n |p_1 p_2 p_2 \dots\rangle$$

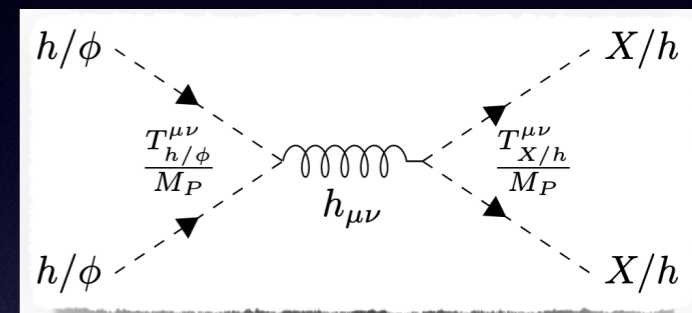
Perturbative approach

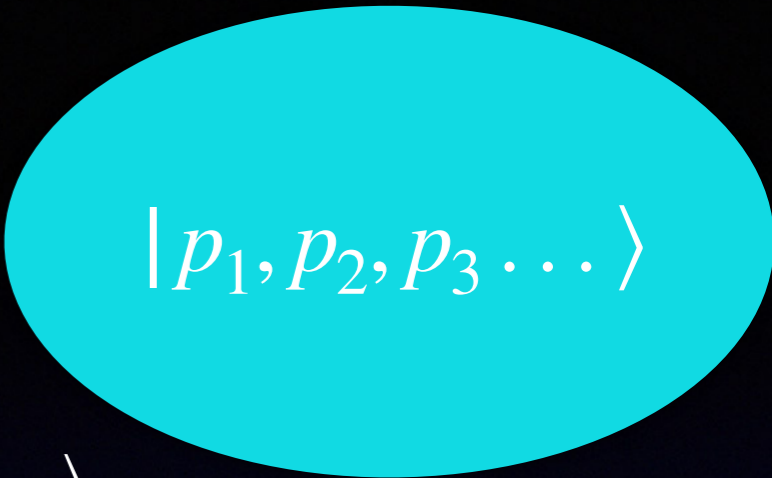
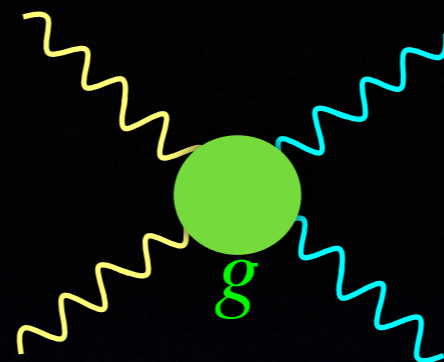
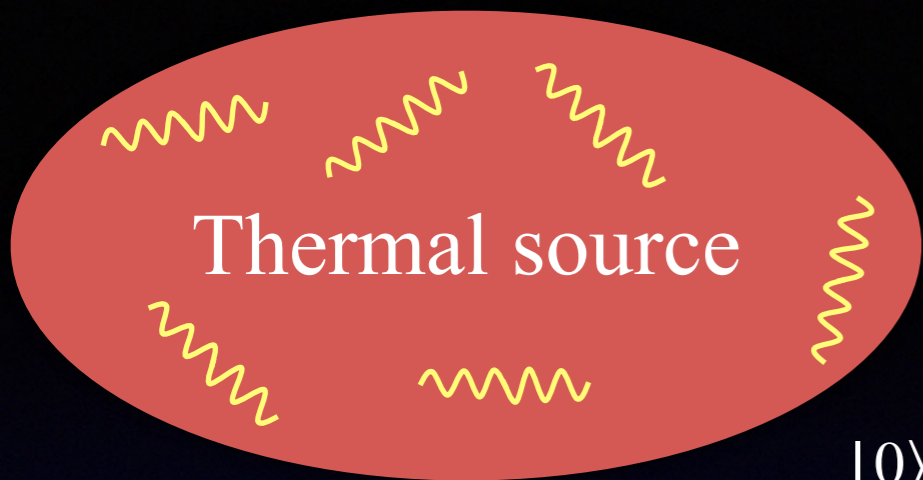
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$$|0\rangle \rightarrow_t |0\rangle + \epsilon |0\rangle = |0\rangle + g^4 T^n |p_1 p_2 p_2 \dots\rangle$$

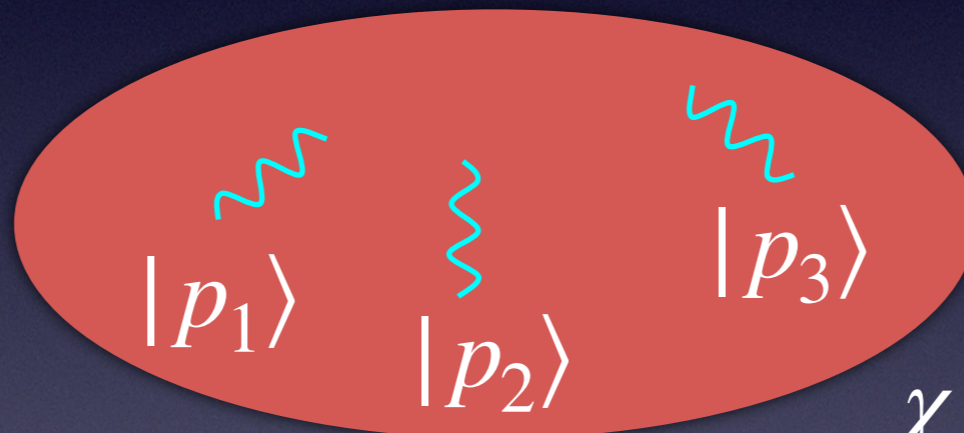
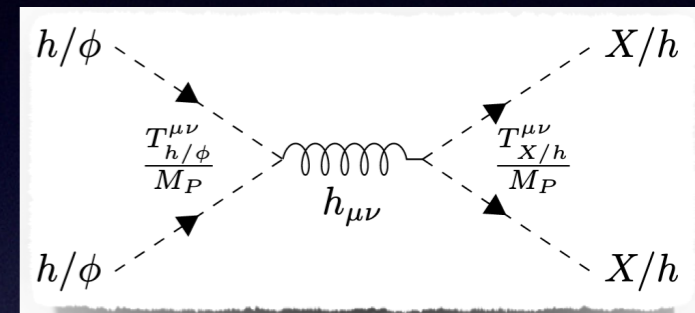
## Perturbative approach





$$|0\rangle \rightarrow_t |0\rangle + \epsilon |0\rangle = |0\rangle + g^4 T^n |p_1 p_2 p_2 \dots\rangle$$

## Perturbative approach



## Bogoliubov approach

$$\chi \sim \int e^{-i\omega_p t} a_p + e^{i\omega_p t} a_p^\dagger$$

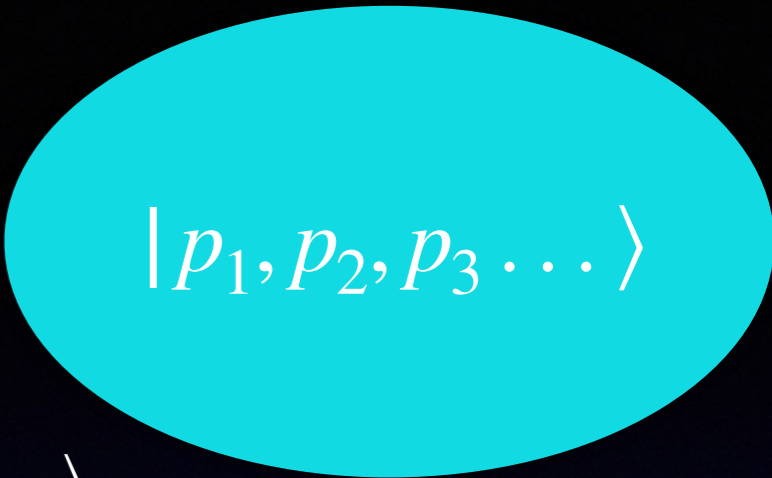
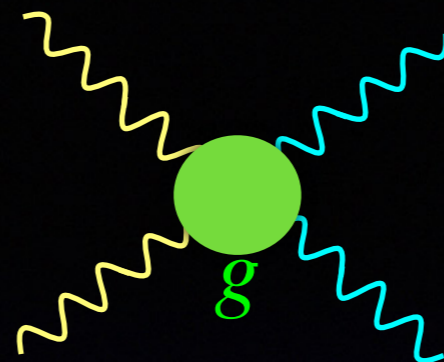
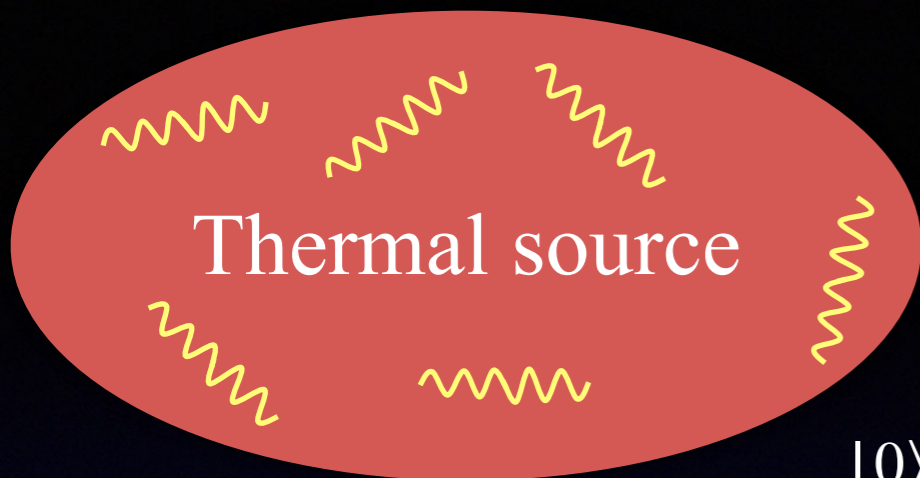
$$a_p |0\rangle = 0$$

$$a_p \rightarrow_{a_1 \rightarrow a_2} \alpha_p \times a_p + \beta_p \times a_p^\dagger$$

$$\chi'' - \nabla^2 \chi + a^2 m_\chi^2 \chi \left( -\frac{a''}{a} \chi \right) = 0$$

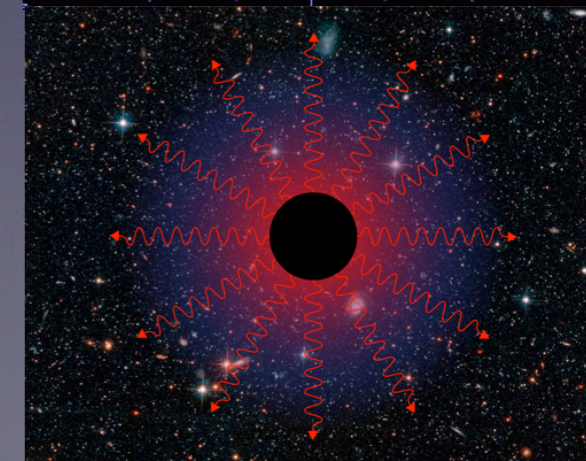
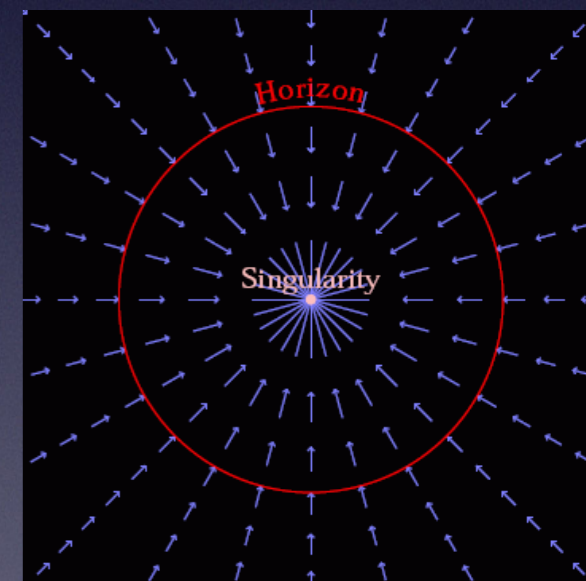
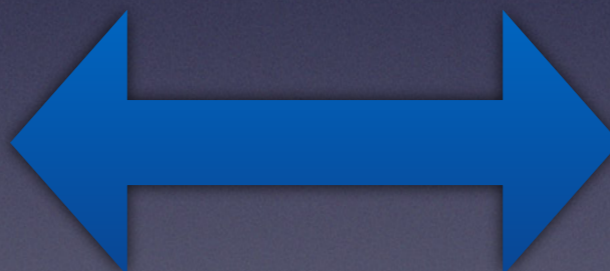
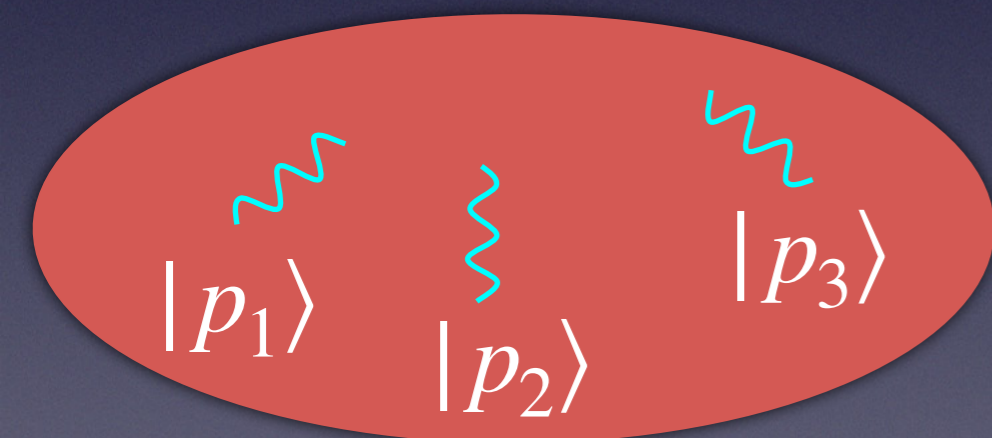
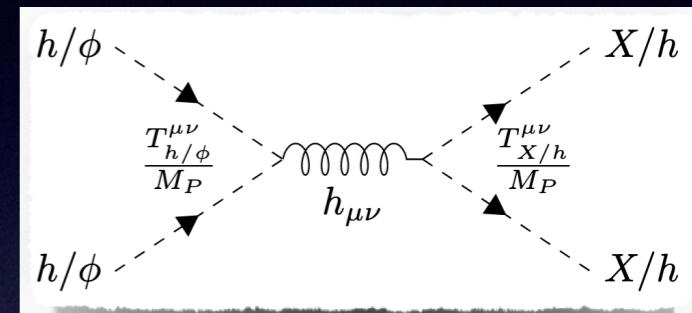
$$\frac{a''}{a} = -\frac{1}{6} \mathcal{R} a^2 = \frac{1}{2} a^2 H^2 (1 - 3w)$$

$$\beta_p = \int \frac{\omega'_p}{2\omega_p} e^{2i \int \omega_p}$$



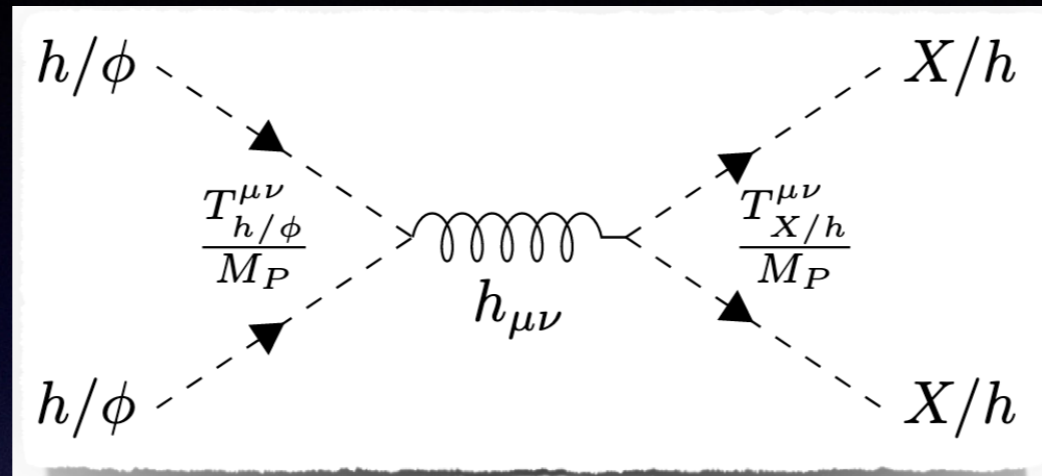
$$|0\rangle \rightarrow_t |0\rangle + \epsilon |0\rangle = |0\rangle + g^4 T^n |p_1 p_2 p_2 \dots\rangle$$

Perturbative approach



Bogoliubov approach

Hawking radiation



$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_P^2} h_{\mu\nu}$$

$$\mathcal{L} = \frac{\sqrt{-g}}{2} g_{\mu\nu} \partial^\mu \Phi \partial^\nu \Phi \quad \Rightarrow \quad \delta \mathcal{L} = \frac{1}{12} \mathcal{R} \Phi^2$$

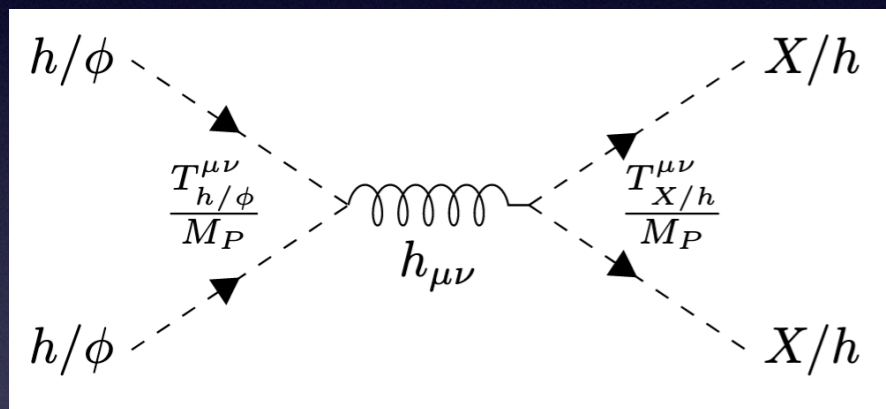
Can we **gravitationally** produce  
DM in a sufficiently large amount?

**Yes!**

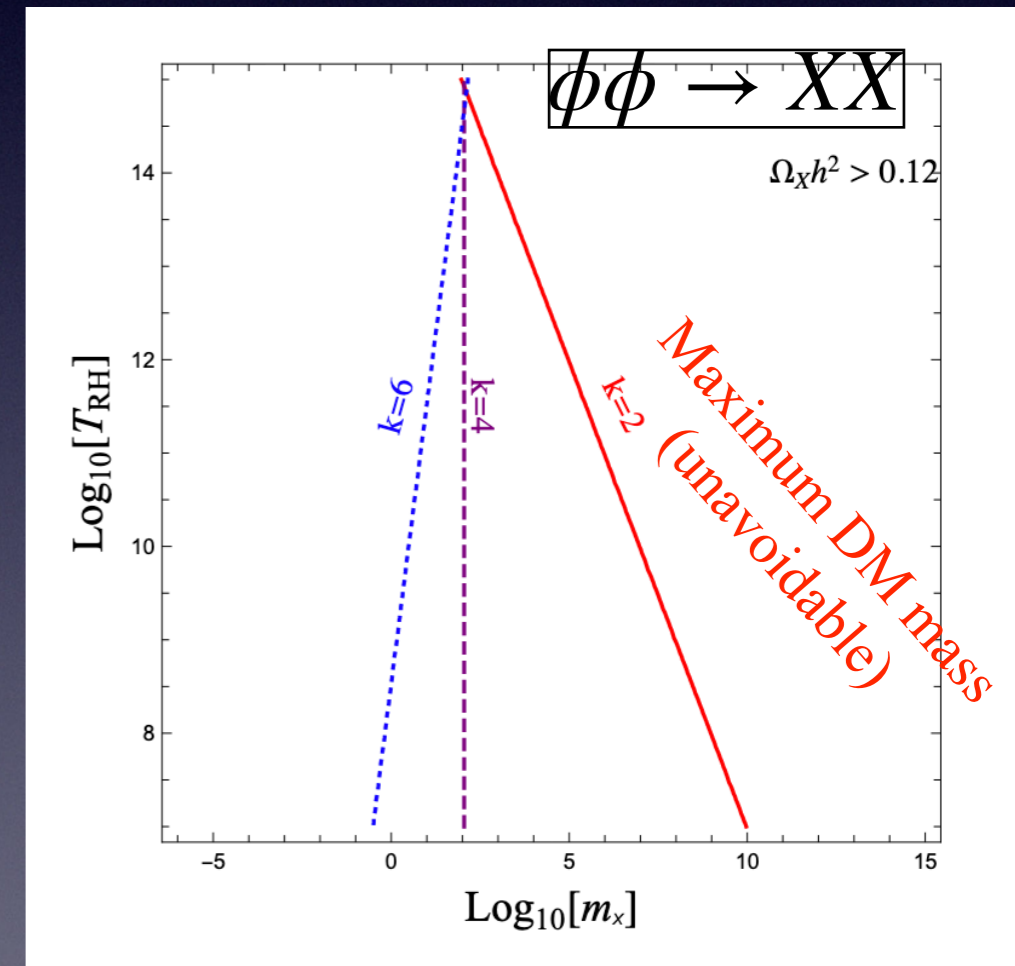
2019 → 2022 : Gravitational (and thus unavoidable) production of (dark) matter in the early Universe

$$\mathcal{L} = \sqrt{-g} \left( -\frac{M_P^2}{2} R + \mathcal{L}_\phi + \mathcal{L}_h + \mathcal{L}_X \right) ; \quad g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_P} h_{\mu\nu}$$

$$\Rightarrow \quad \mathcal{L} \supset \frac{1}{M_P} h_{\mu\nu} \left( T_\phi^{\mu\nu} + T_h^{\mu\nu} + T_X^{\mu\nu} \right)$$



$$V(\phi) = \lambda M_P^4 \left( \frac{\phi}{M_P} \right)^k$$



Y. Mambrini and K. A. Olive, Phys. Rev. D **103** (2021) no.11, 115009 [arXiv:2102.06214]

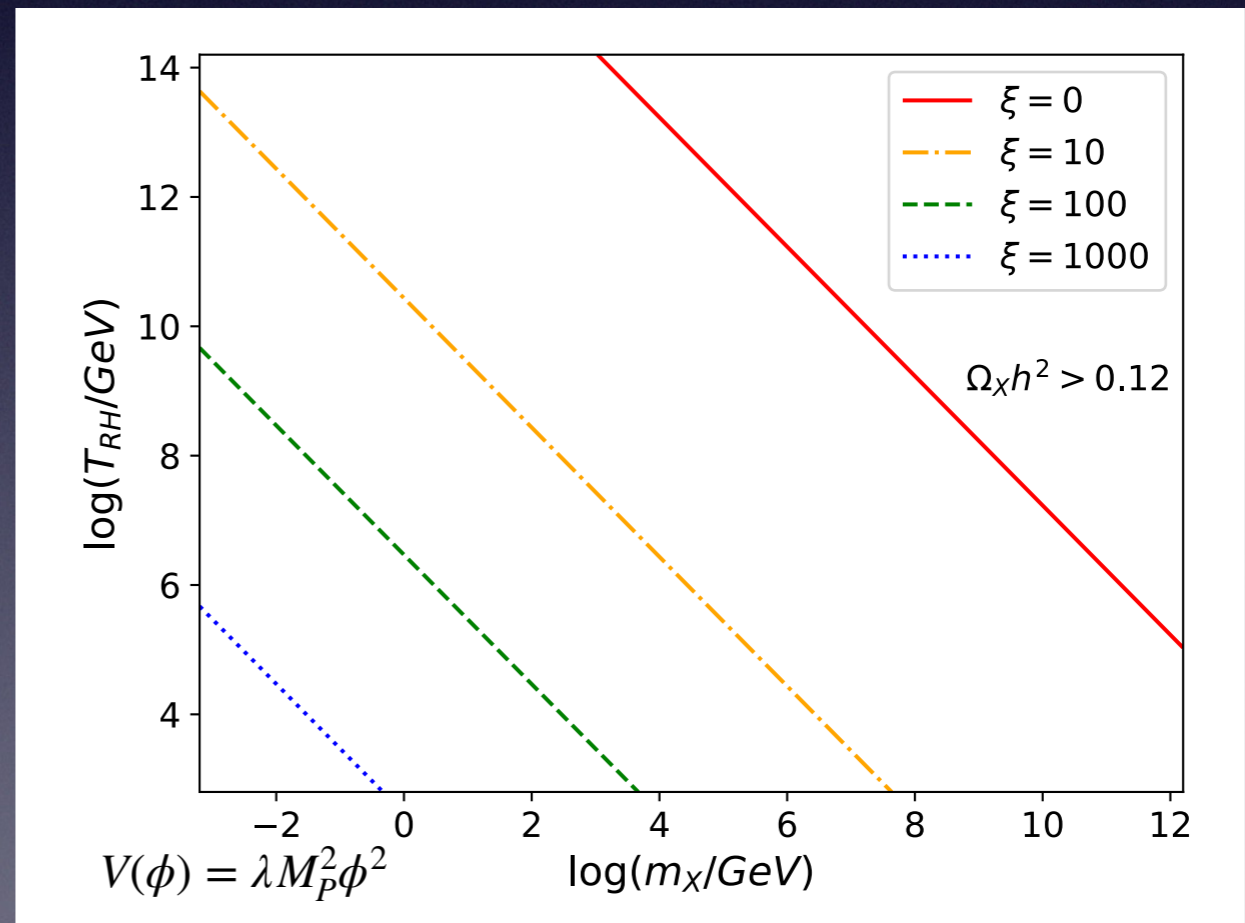
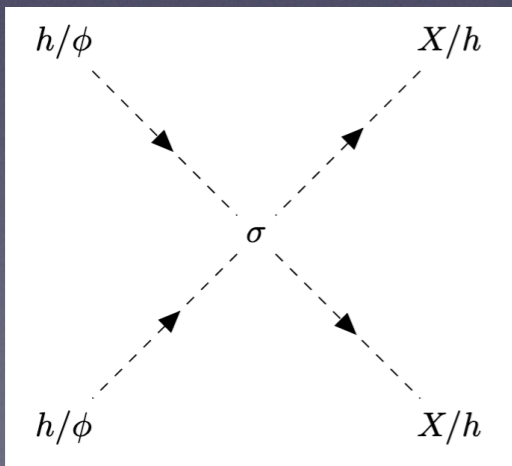
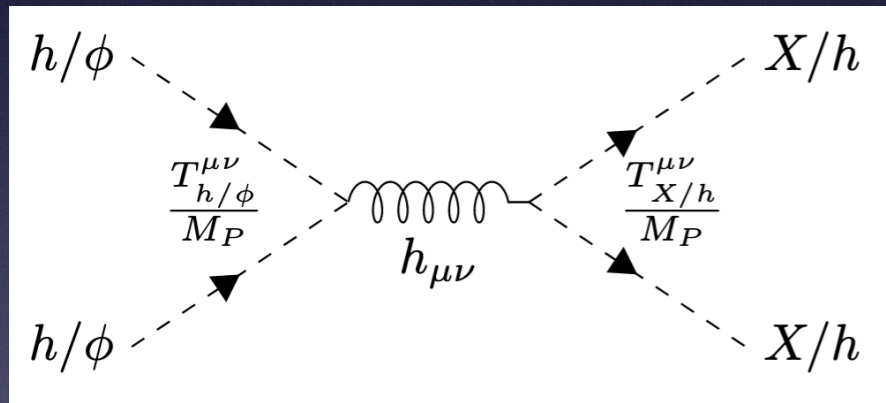
S. Clery, Y. Mambrini, K. A. Olive and S. Verner, Phys. Rev. D **105** (2022) no.7, 075005 ; [arXiv:2112.15214]

See also : B. Barman and N. Bernal, "Gravitational SIMPs," JCAP **06** (2021), 011

Adding the possibility for non-minimal gravitational coupling

$$\mathcal{L} = \sqrt{-g} \left( -\frac{M_P^2}{2} \left[ 1 + \xi_\phi \frac{\phi^2}{M_P^2} + \xi_h \frac{h^2}{M_P^2} + \xi_X \frac{X^2}{M_P^2} \right] R + \mathcal{L}_\phi + \mathcal{L}_h + \mathcal{L}_X \right)$$

$$\Rightarrow \mathcal{L} \supset \frac{1}{M_P} h_{\mu\nu} \left( T_\phi^{\mu\nu} + T_h^{\mu\nu} + T_X^{\mu\nu} \right) + \sigma_{\phi h}^\xi \phi^2 h^2 + \sigma_{\phi X}^\xi \phi^2 X^2 + \sigma_{hX}^\xi h^2 X^2$$



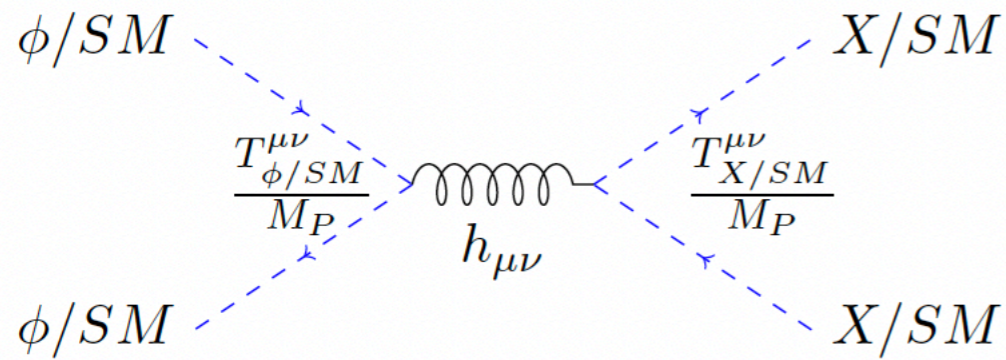
Can we reheat?

Yes!

$$\mathcal{L} = \frac{1}{M_P} h_{\mu\nu} T_{\phi}^{\mu\nu} + \frac{1}{M_P} h_{\mu\nu} T_S^{\mu\nu}$$

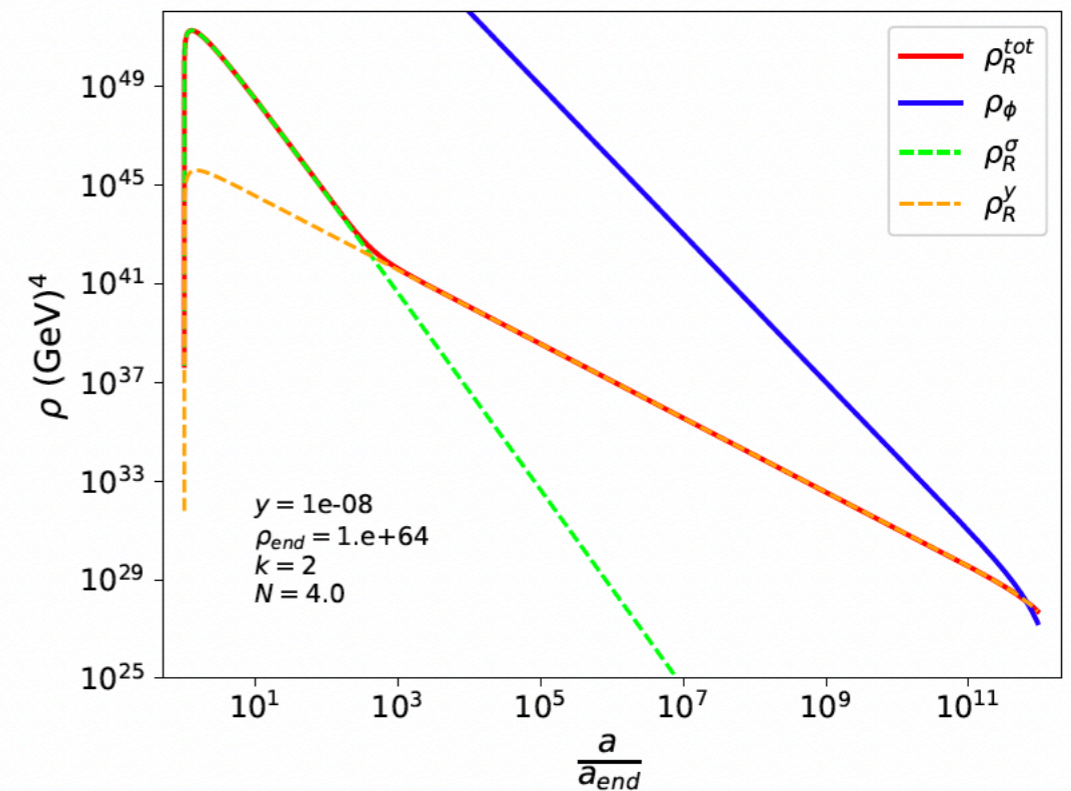
Y. Mambrini and K. A. Olive, Phys. Rev. D **103** (2021) no.11, 115009 [arXiv:2102.06214].

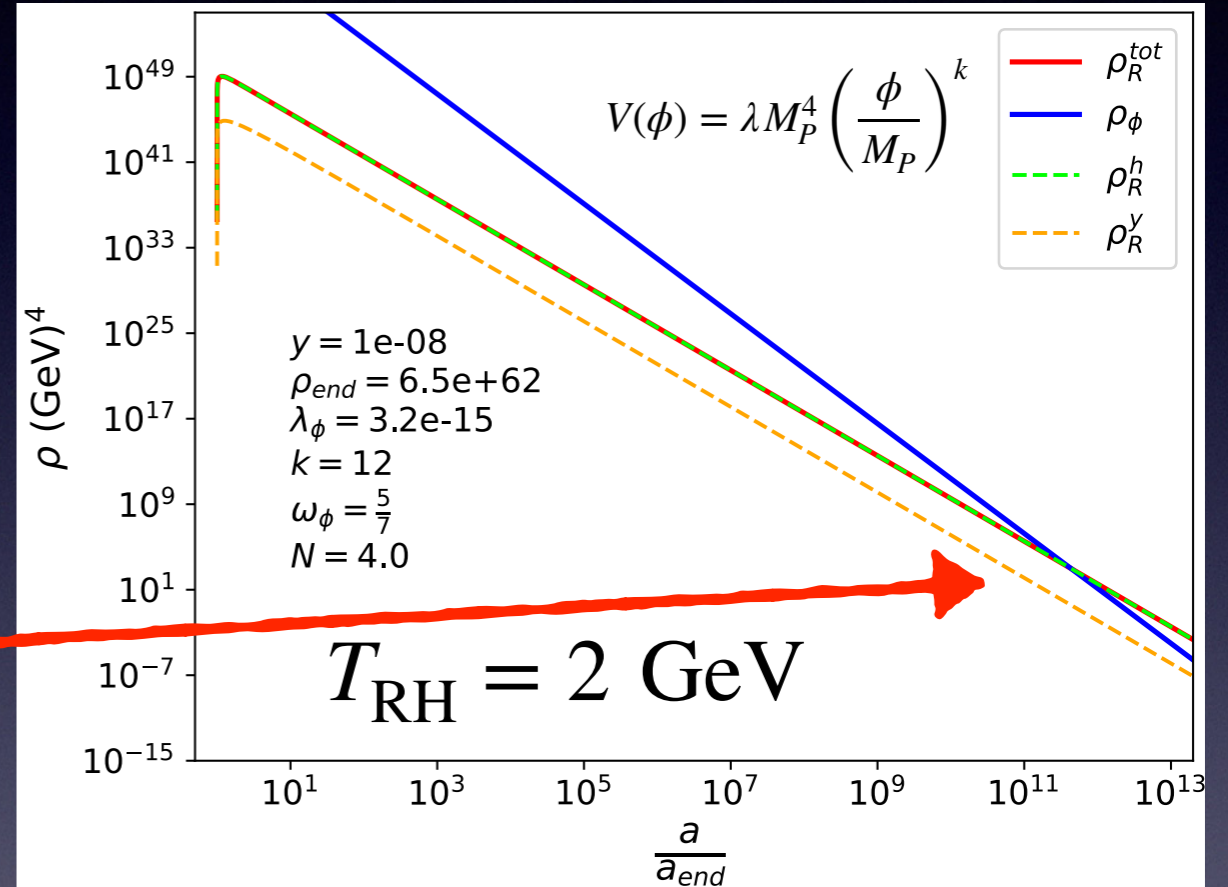
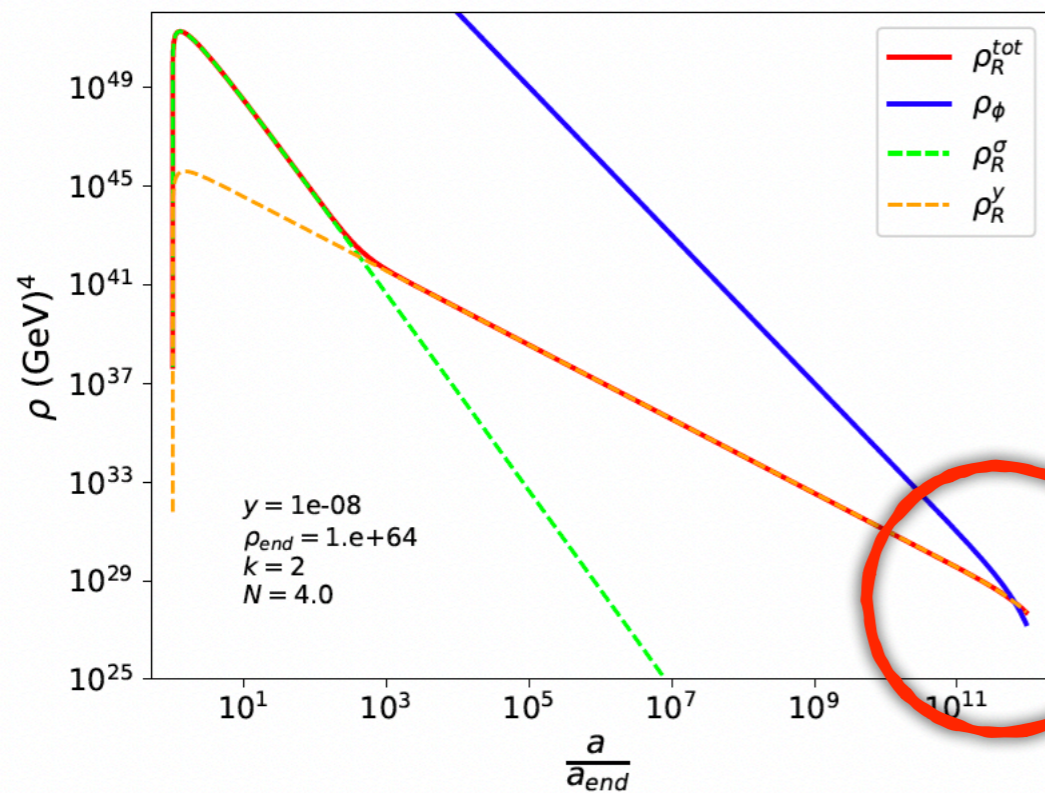
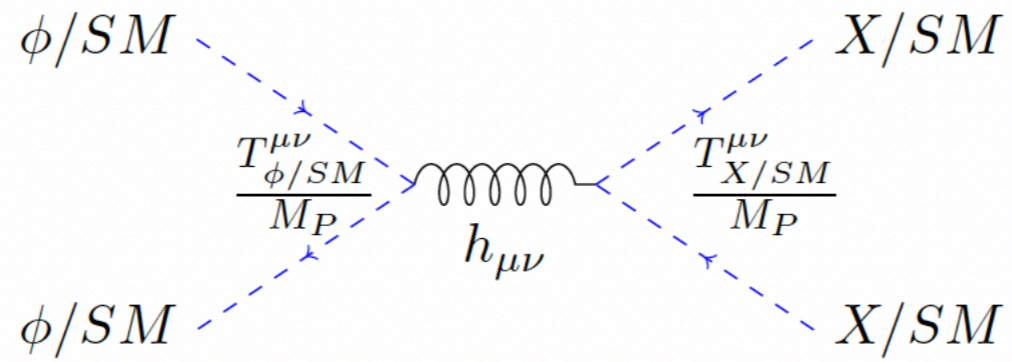
S. Clery, Y. Mambrini, K. A. Olive, and S. Verner, Phys. Rev. D **105** (2022) no.9, 095042 [arXiv:2112.15214].



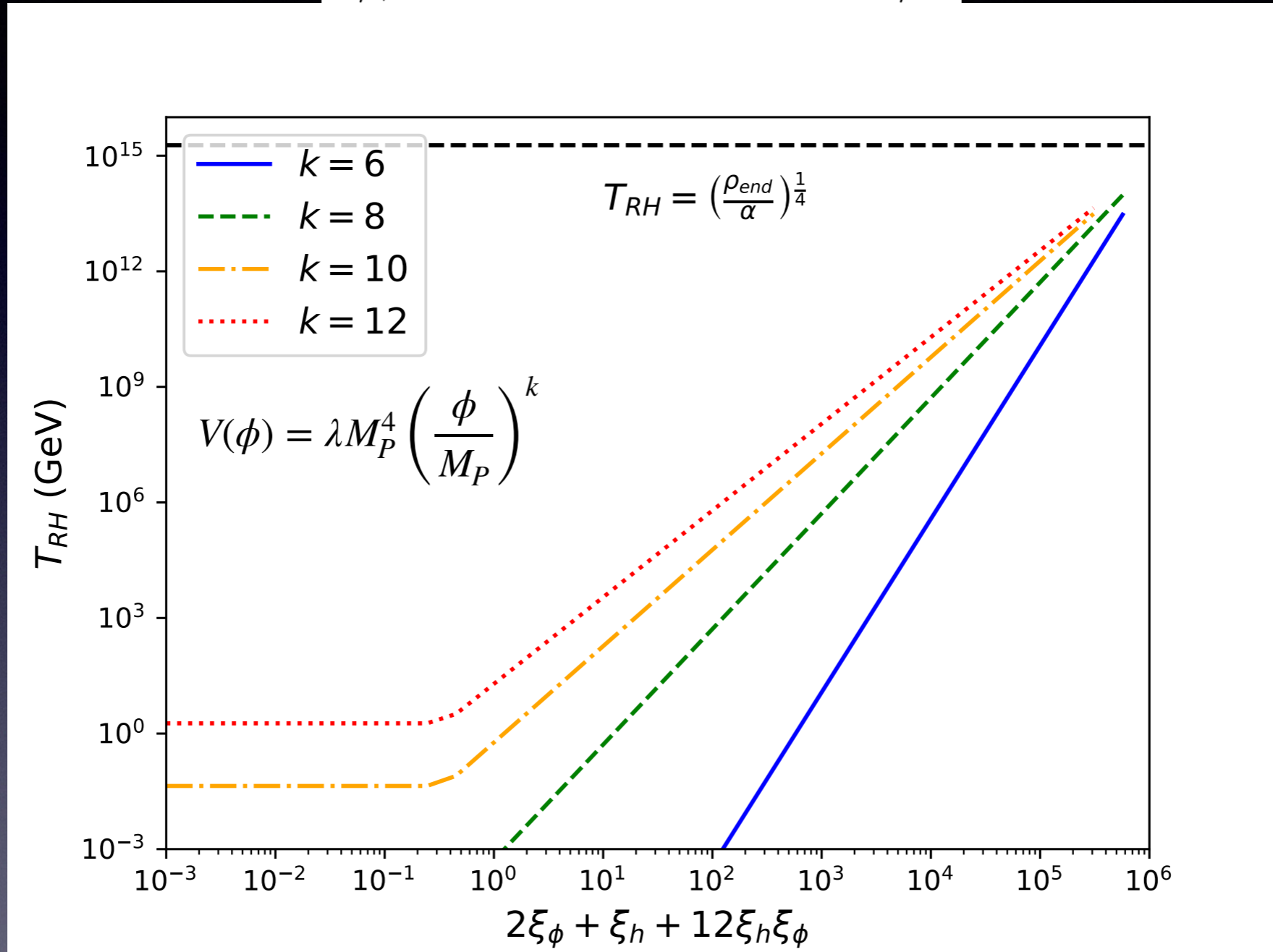
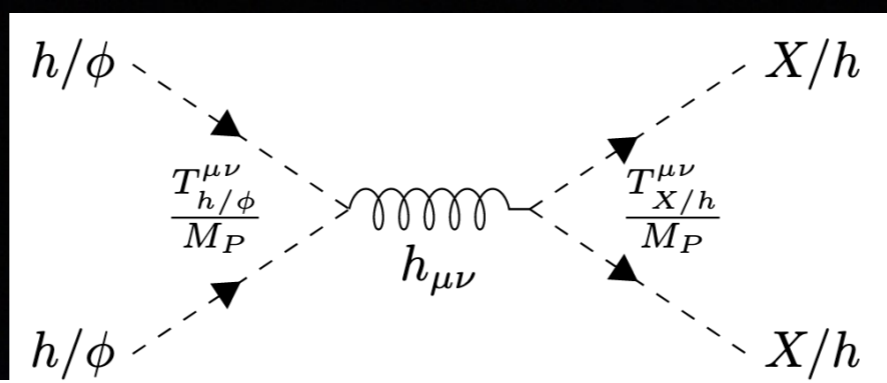
There exists a minimal maximal temperature in the Universe  $\sim 10^{12}$  GeV

	$k = 2$	$k = 4$	$k = 6$
$T_{\max}$	$1.0 \times 10^{12}$ GeV	$7.5 \times 10^{11}$ GeV	$6.5 \times 10^{11}$ GeV
$y_{\max}$	$1.8 \times 10^{-6}$	$1.4 \times 10^{-6}$	$1.1 \times 10^{-6}$
$T_{\text{RHmax}}$	$7.9 \times 10^8$ GeV	470 GeV	$9.7 \times 10^{-4}$ GeV





**We can even gravitationally reheat!!**



$$\mathcal{L} = \sqrt{-g} \left( -\frac{M_P^2}{2} \left[ 1 + \xi_\phi \frac{\phi^2}{M_P^2} + \xi_h \frac{h^2}{M_P^2} + \xi_X \frac{X^2}{M_P^2} \right] R + \mathcal{L}_\phi + \mathcal{L}_h + \mathcal{L}_X \right)$$

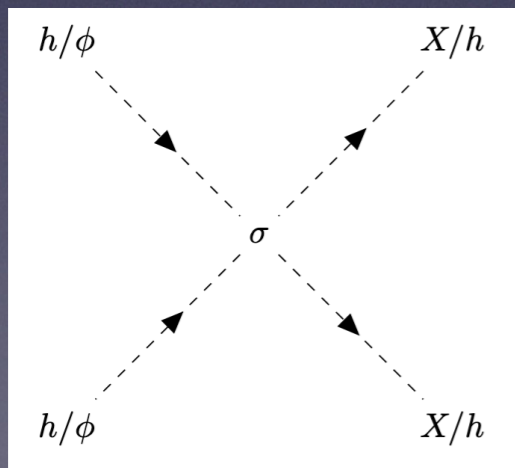
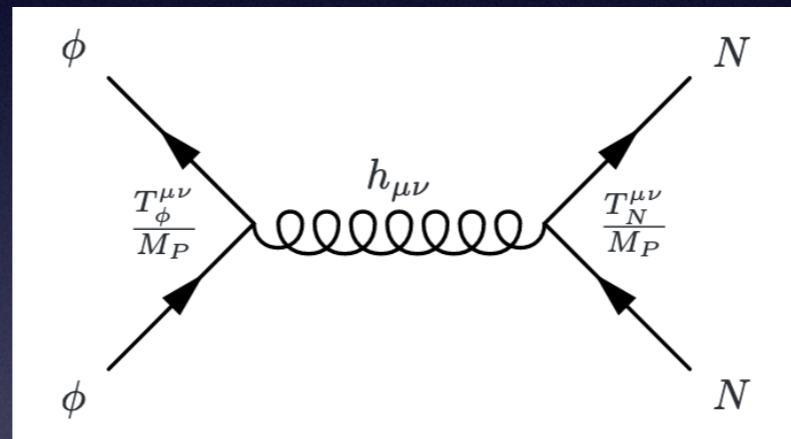
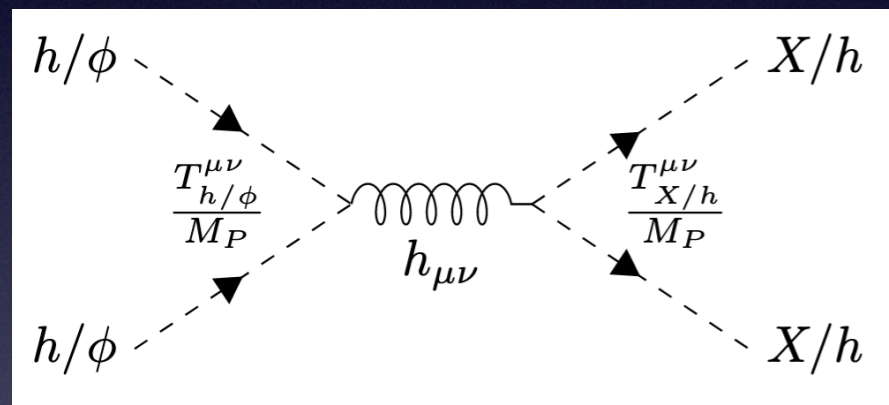
Can we generate leptogenesis?

Yes!!

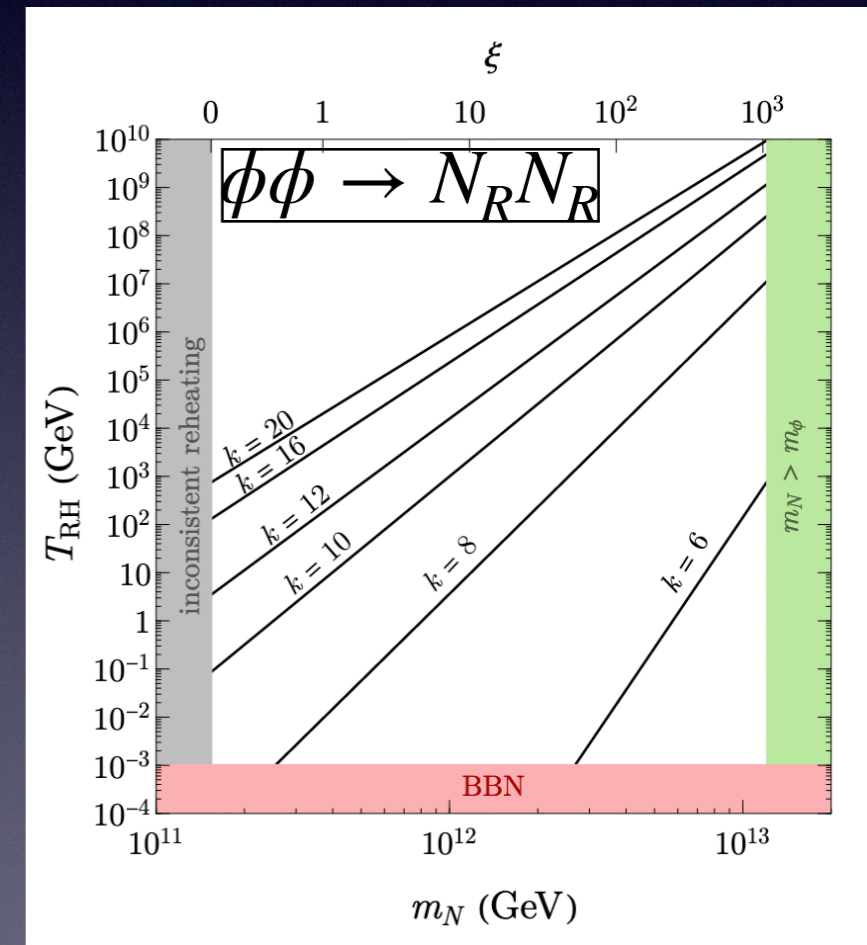
# Adding the possibility for **Leptogenesis**

$$\mathcal{L} = \sqrt{-g} \left( -\frac{M_P^2}{2} \left[ 1 + \xi_\phi \frac{\phi^2}{M_P^2} + \xi_h \frac{h^2}{M_P^2} + \xi_X \frac{X^2}{M_P^2} \right] R + \mathcal{L}_\phi + \mathcal{L}_h + \mathcal{L}_X - M_R \bar{N}_R^c N_R \right)$$

$$\Rightarrow \mathcal{L} \supset \frac{1}{M_P} h_{\mu\nu} \left( T_\phi^{\mu\nu} + T_h^{\mu\nu} + T_X^{\mu\nu} + T_{N_R}^{\mu\nu} \right) + \sigma_{\phi h}^\xi \phi^2 h^2 + \sigma_{\phi X}^\xi \phi^2 X^2 + \sigma_{hX}^\xi h^2 X^2$$



$$V(\phi) = \lambda M_P^4 \left( \frac{\phi}{M_P} \right)^k$$

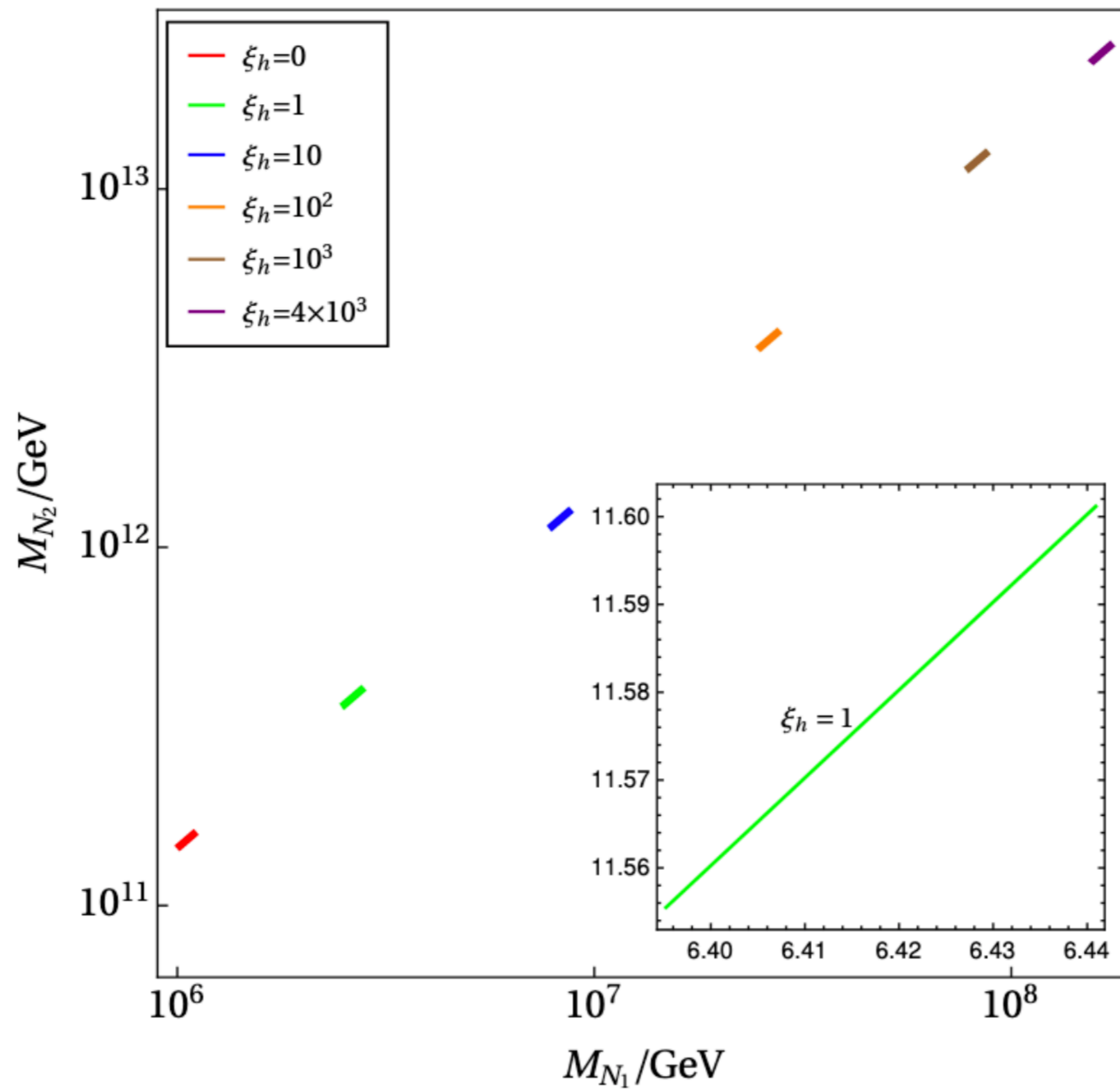


R. T. Co, Y. Mambrini and K. A. Olive, "Inflationary Gravitational Leptogenesis," ; [arXiv:2205.01689 [hep-ph]].

See also : N. Bernal and C. S. Fong, "Dark matter and leptogenesis from gravitational production," JCAP **06** (2021), 028

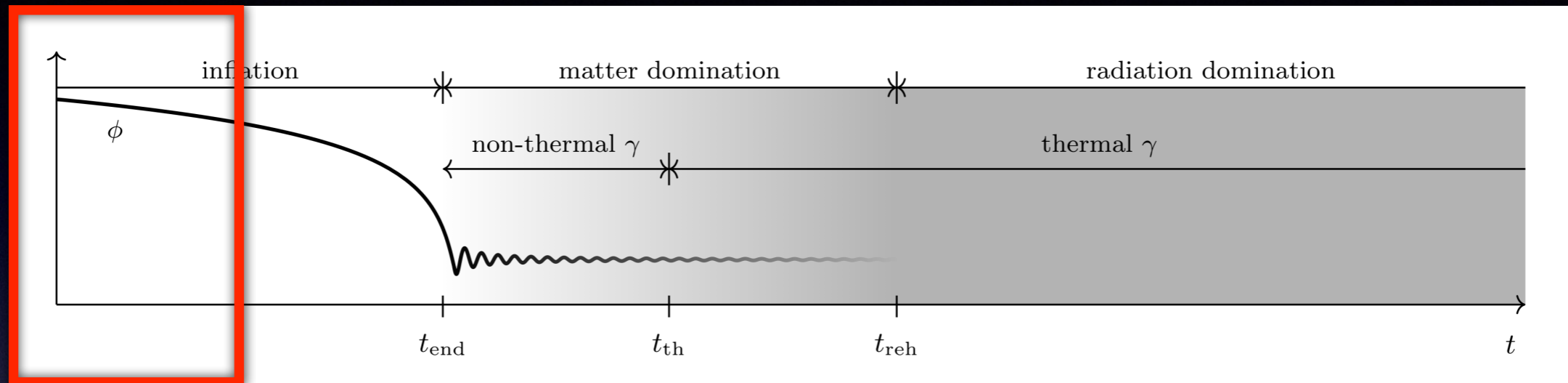
Can we combine everything?  
(Gravitational reheating + relic  
abundance + leptogenesis)

Yes!!!

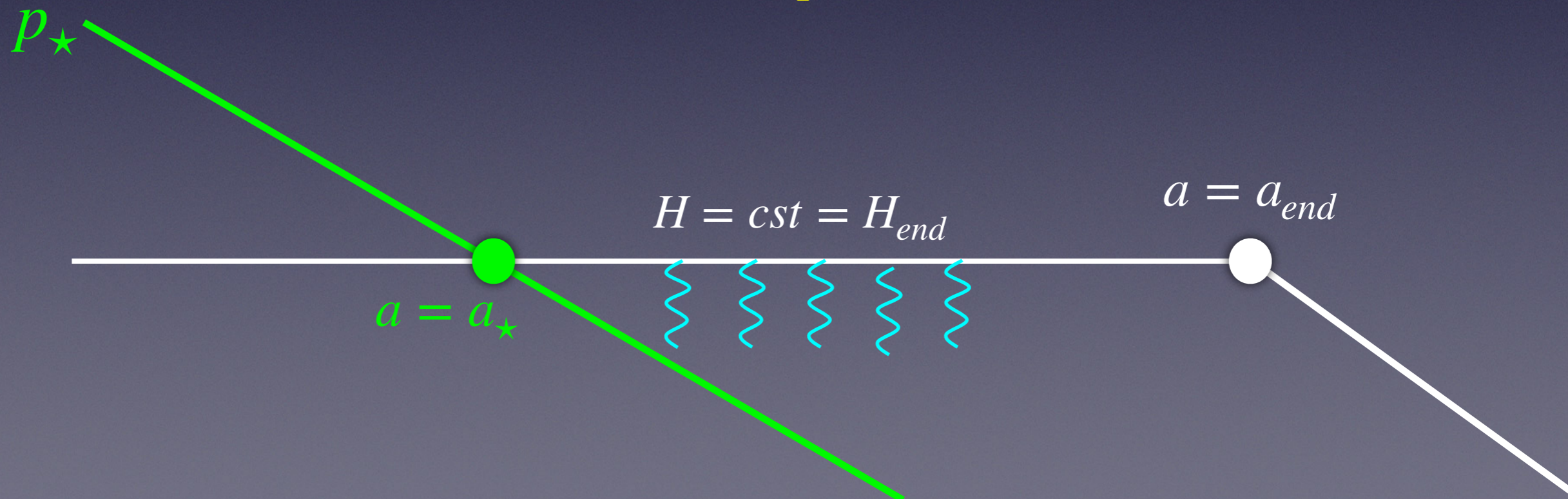


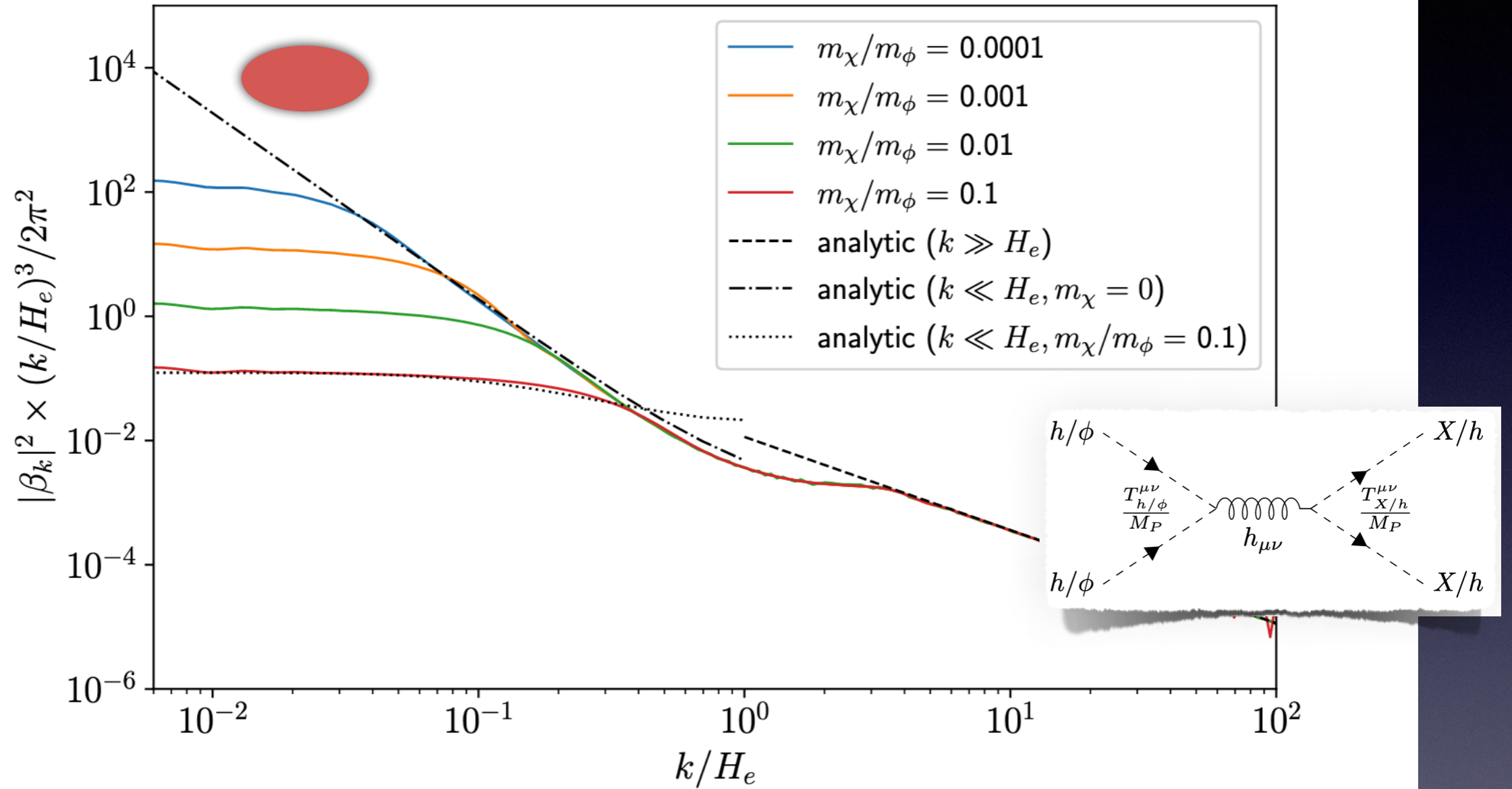
Low mode regime

$$\chi'' - \nabla^2 \chi + a^2 m_\chi^2 \chi - \frac{a''}{a} \chi = 0 \Leftrightarrow \chi_p'' + (\cancel{p^2 + a^2 m_\chi^2} - \frac{a''}{a}) \chi = 0$$



large production diverging  $\propto \frac{1}{p}$  for massless modes



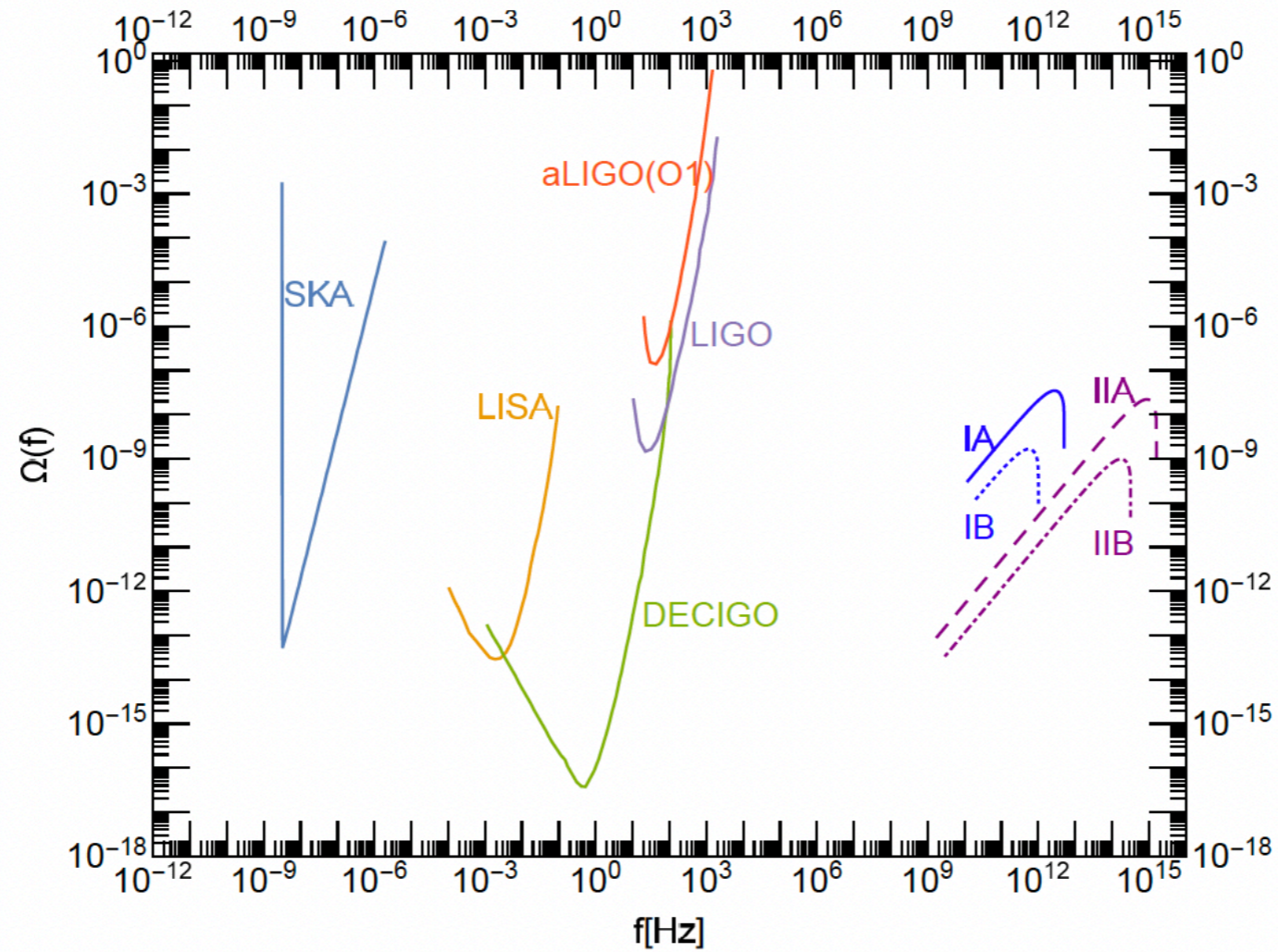
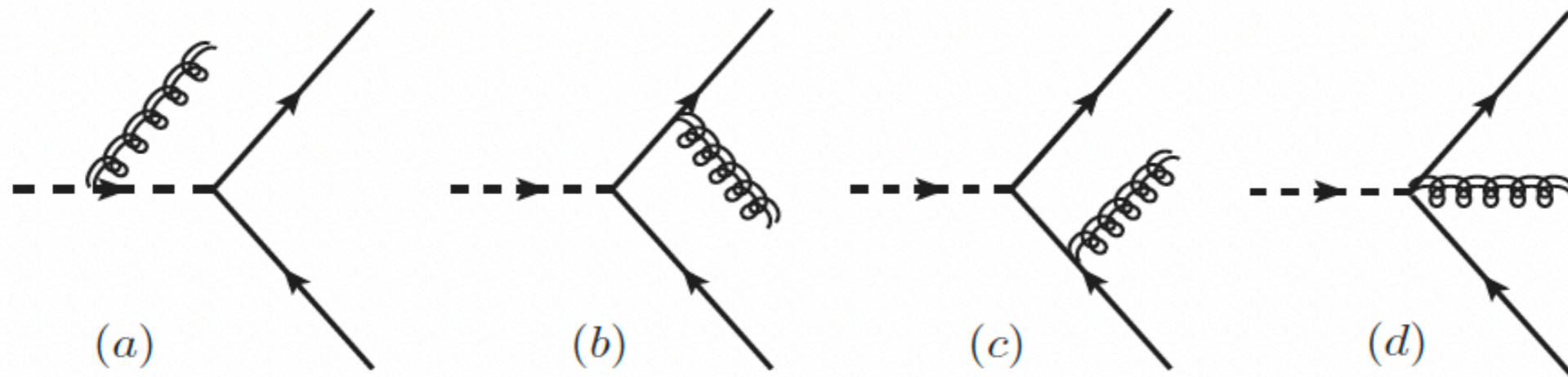


K. Kaneta, S. M. Lee and K. Y. Oda, JCAP 09 (2022), 018

M. A. G. Garcia, M. Pierre and S. Verner, [arXiv:2206.08940 [hep-ph]].

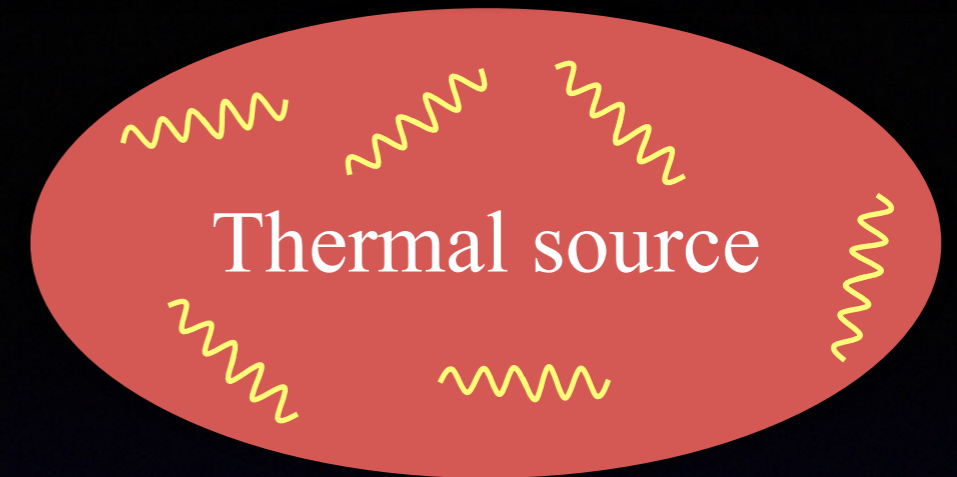
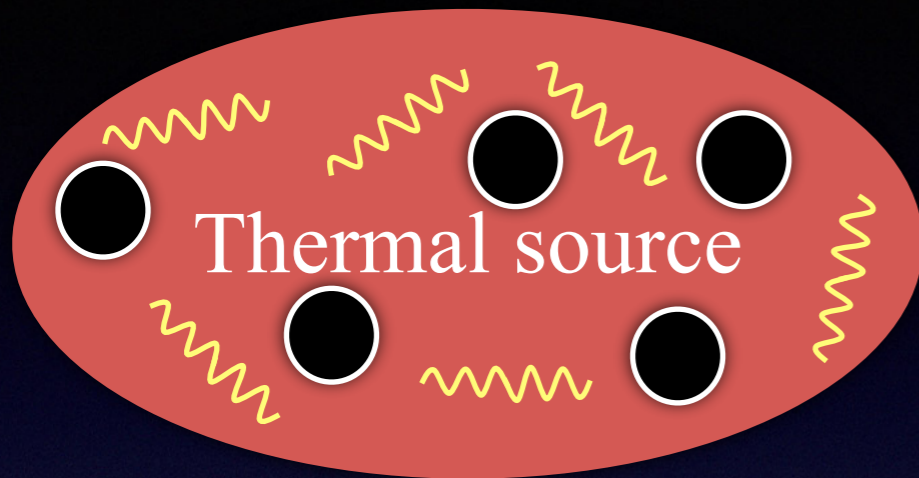
Can we observe it?

...

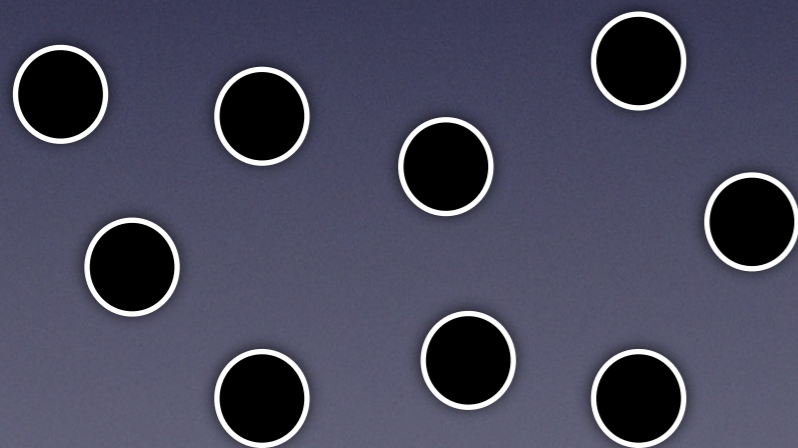


# Conclusions

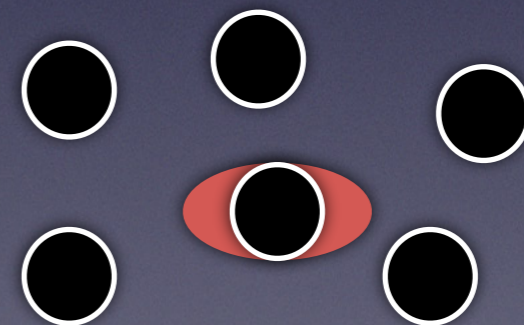
WIMP : thermal source



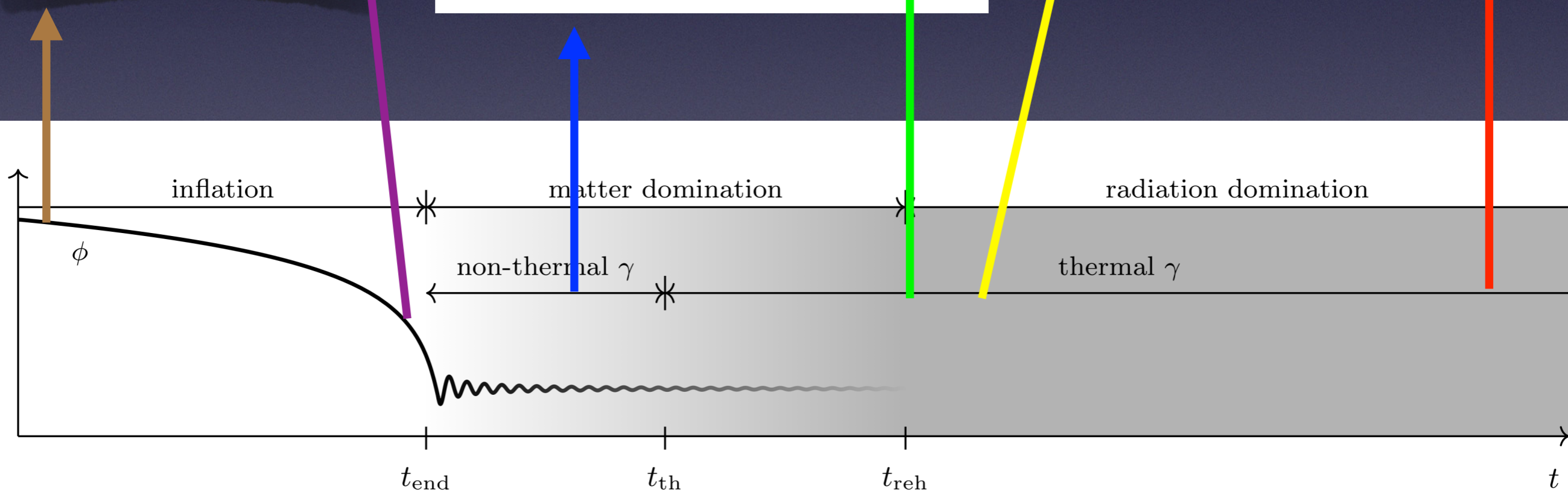
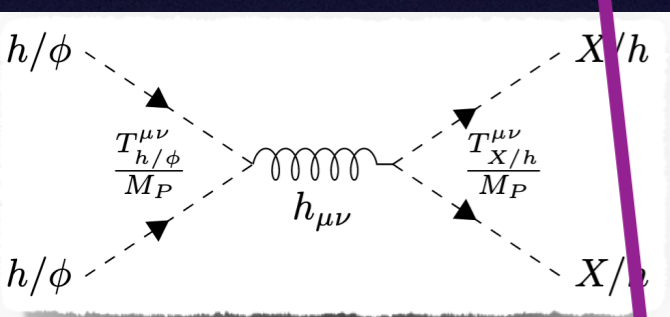
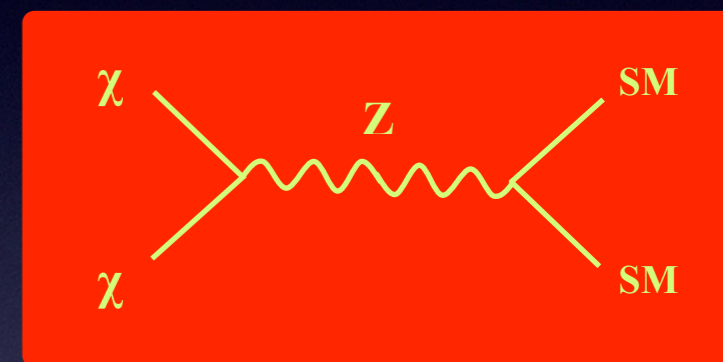
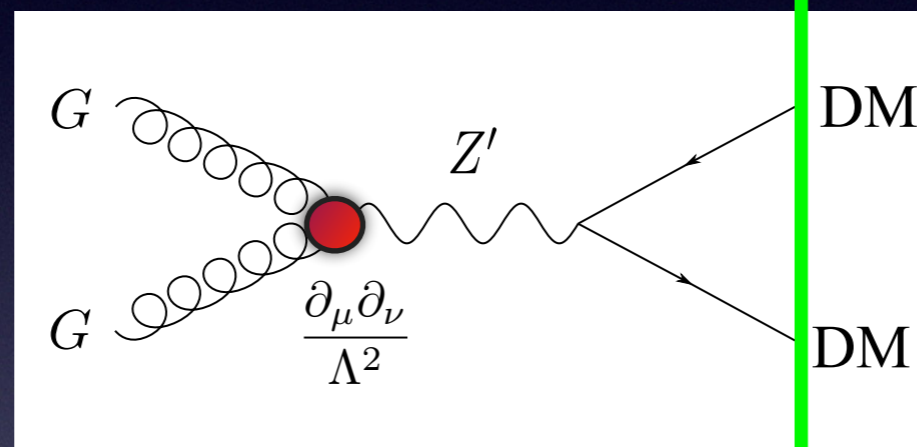
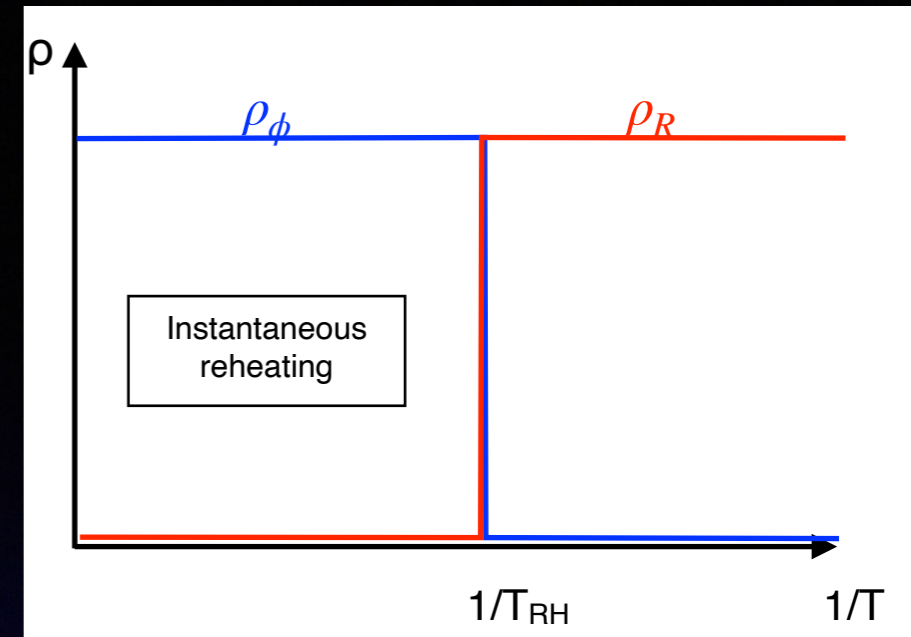
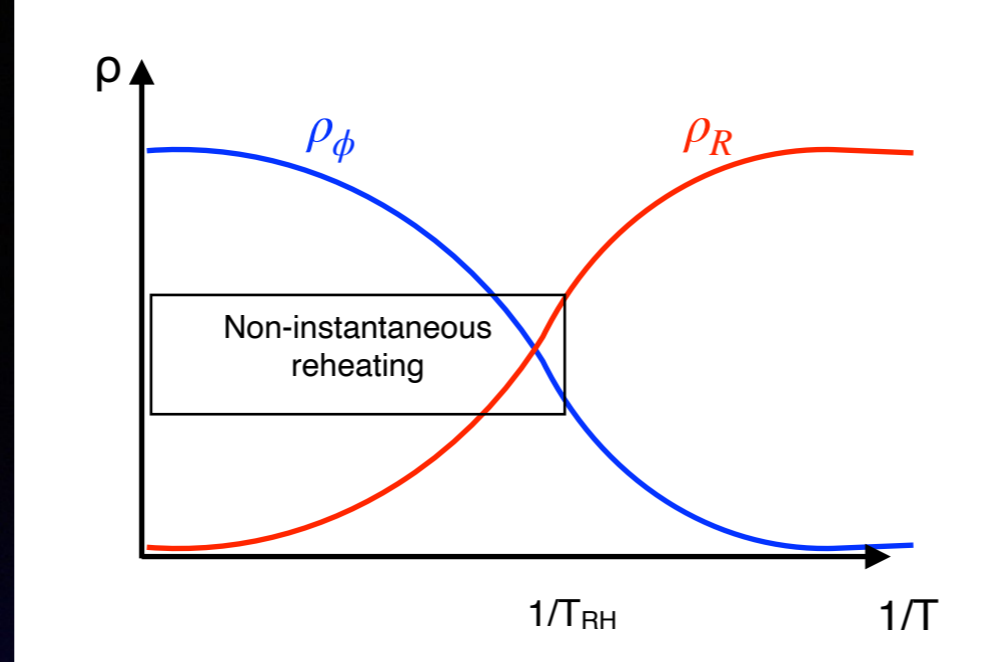
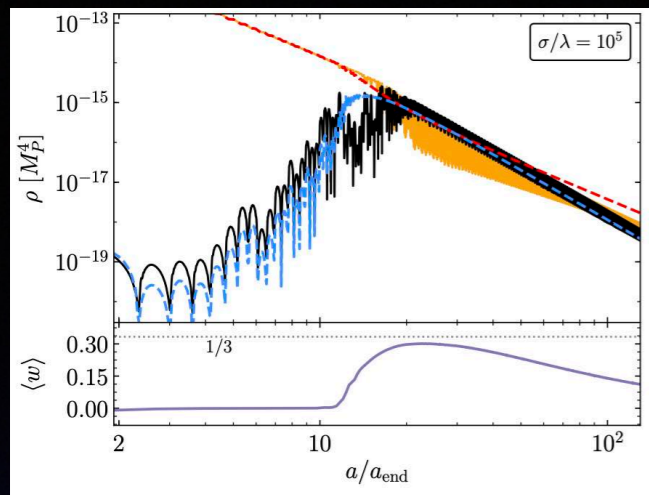
FIMP : external source



Oscillating background



Gravitational production



Studying the details of dark matter production in the earliest phase of the Universe is important for DM production in **freeze-in** scenario

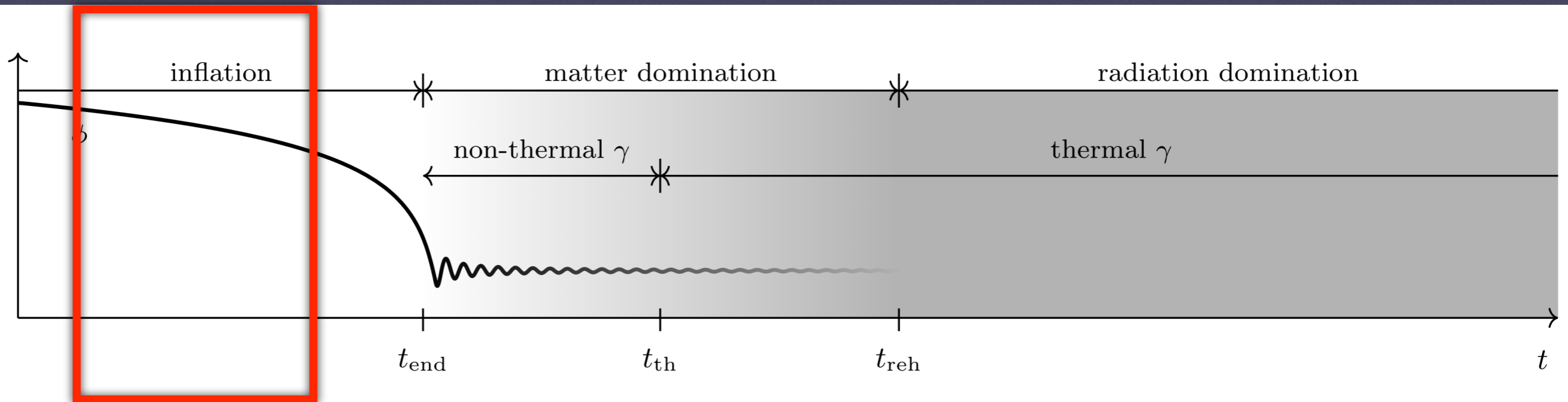
Preheating effect important for couplings  $\gtrsim 10^{-7}$

Gravitational production can make it

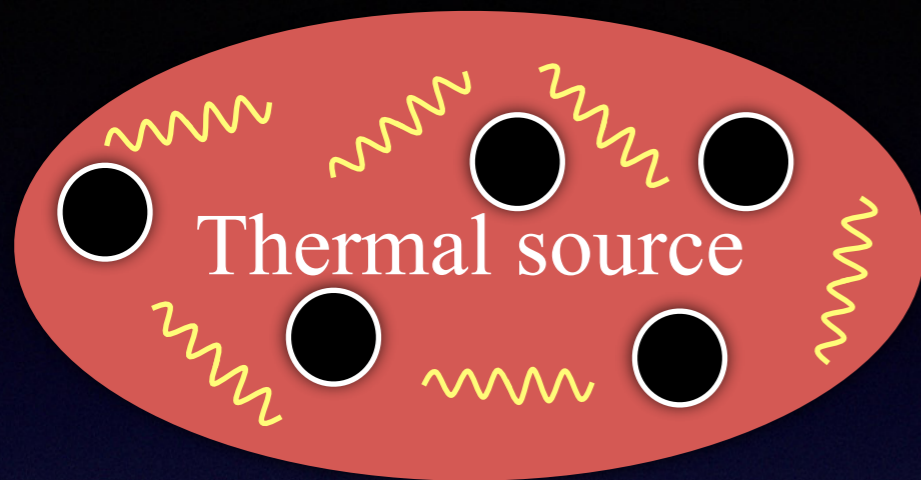
# To do

- 1) Barman work
- 2) Grav. Bremsstrahlung
- 3) Expansion, Minkowski, Bogoliubov
- 4)  $R \Leftrightarrow \text{graviton} \Leftrightarrow \text{expansion}$
- 5) Hawking  $H=T$

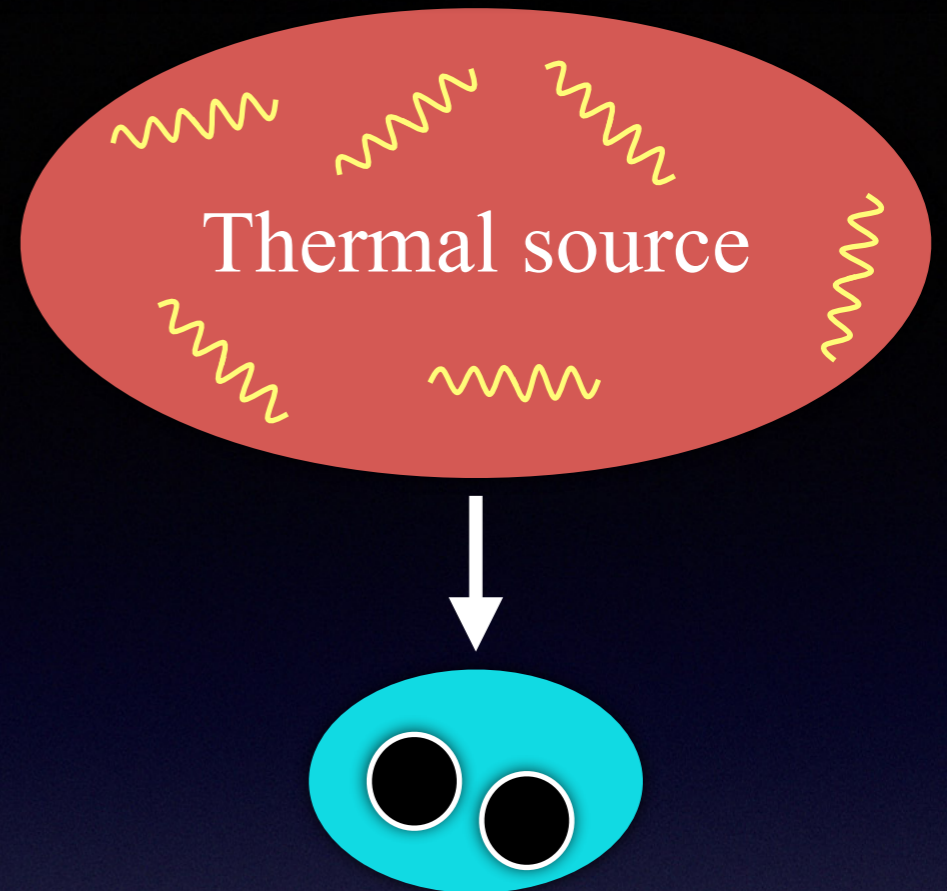
# Mechanisms to produce (dark) matter during a phase of inflation



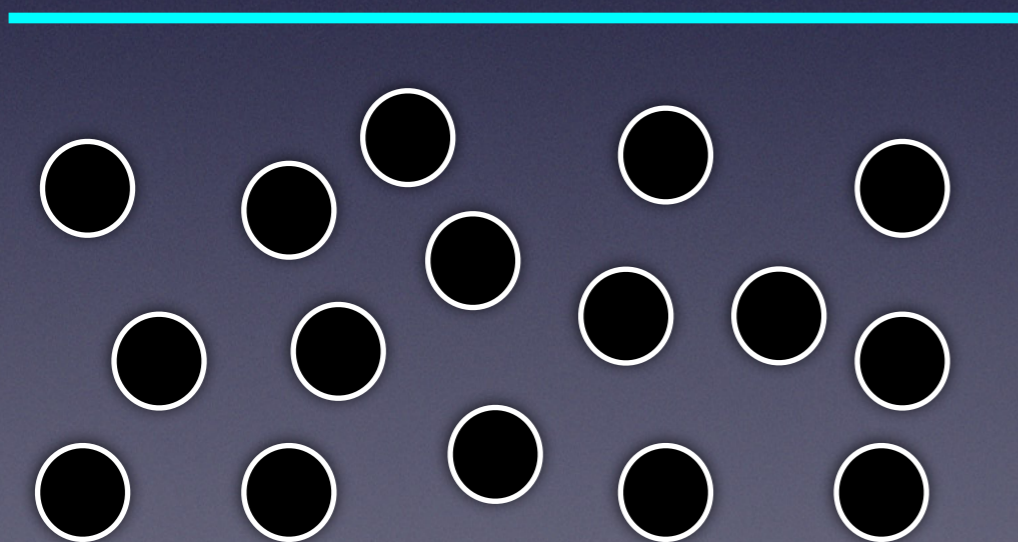
WIMP : thermal source



Thermal source



FIMP : external source



Oscillating background



# Résumé production gravitationnelle

# Gravitational production

Who believe graviton exists?

Who Believe Einstein = effective theory?

Who believe energy is conserved?

Tell the 3 (2) possibilities

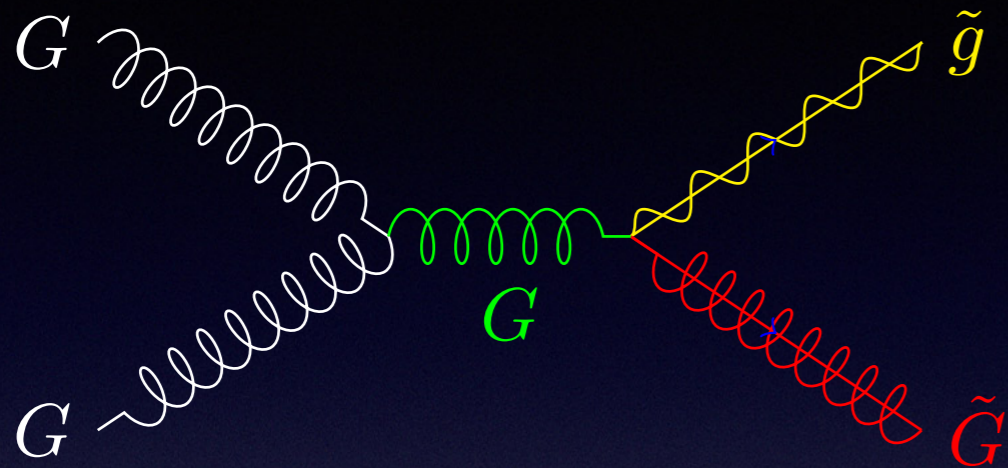
Graviton = perturbation of metric

Inflaton oscillations  $\rightarrow$  gravitational current  $\rightarrow$  production of matter

MP=effective scale, coupling to Ricci equivalent Higgs

Example: decaying spin  $3/2$

Ellis, Kim and Nanopoulos (84) then considered for the first time the dominant process (in fact, they listed 10 processes)



# COSMOLOGICAL GRAVITINO REGENERATION AND DECAY

John Ellis, Jihn E. Kim<sup>\*</sup>) and D.V. Nanopoulos

CERN - Geneva

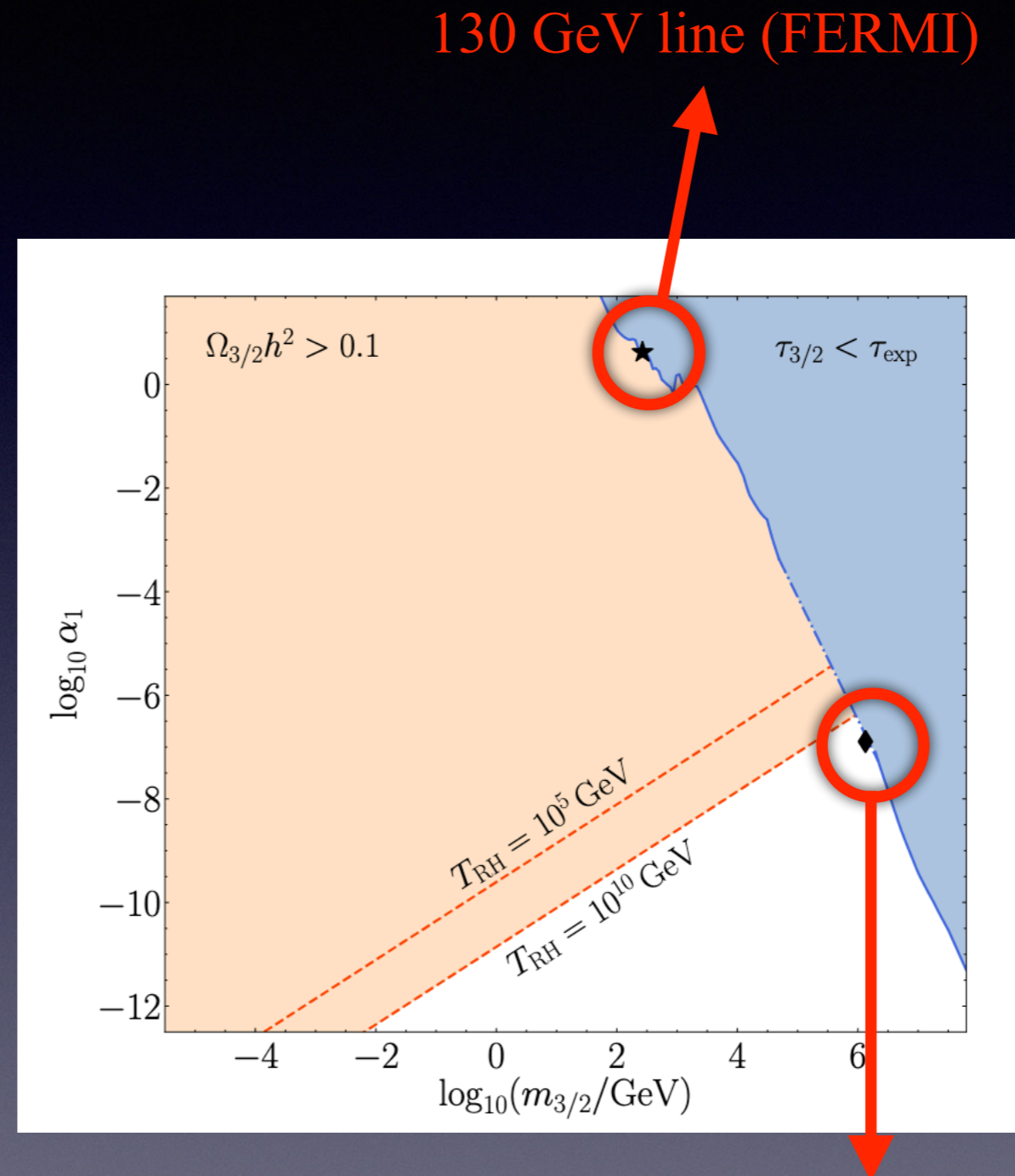
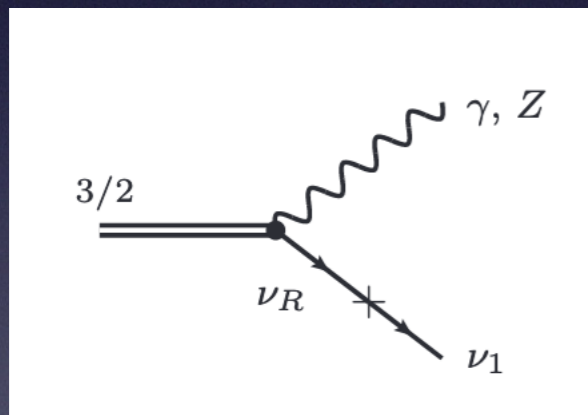
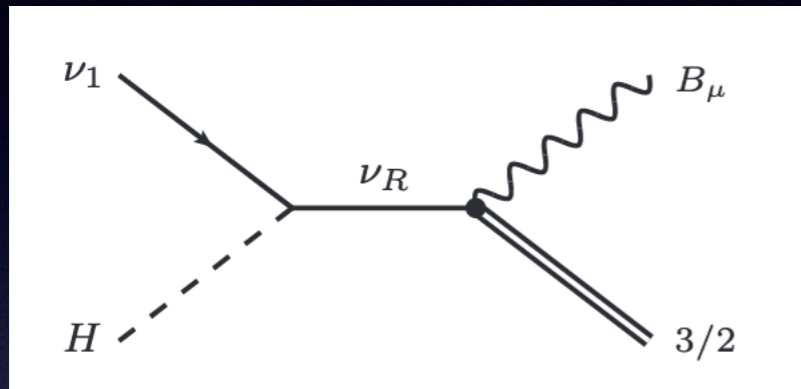
careful analyses of their decay products' disruptive effects on light nuclei and on the microwave background radiation suggest  $T_{\text{max}} < 10^9 \sim 10^{10} \text{ GeV}$ .

$$\mathcal{L} = \frac{1}{4M_{\text{Pl}}} \underbrace{\bar{\psi}^{\alpha} \gamma_{\alpha} [\gamma^{\mu}, \gamma^{\nu}]}_{\text{gravitino}} \underbrace{\tilde{G}}_{\text{gluino}} \underbrace{G_{\mu\nu}}_{\text{gluon}}$$

$$\Omega_{3/2} h^2 \sim 0.3 \left( \frac{1 \text{ GeV}}{m_{3/2}} \right) \left( \frac{T_{\text{RH}}}{10^{10} \text{ GeV}} \right) \sum \left( \frac{m_{\tilde{G}}}{100 \text{ GeV}} \right)^2$$

# Generic spin 3/2

$$\mathcal{L}_{3/2} = i \frac{\alpha_1}{2M_P} \bar{\nu}_R \gamma^\mu [\gamma^\rho, \gamma^\sigma] \Psi_\mu F_{\rho\sigma} + \text{h.c.}$$



# Early Universe physics

$T_{RH}$  dependance

$\Rightarrow$

one needs to understand the physics of particle creation in the early Universe

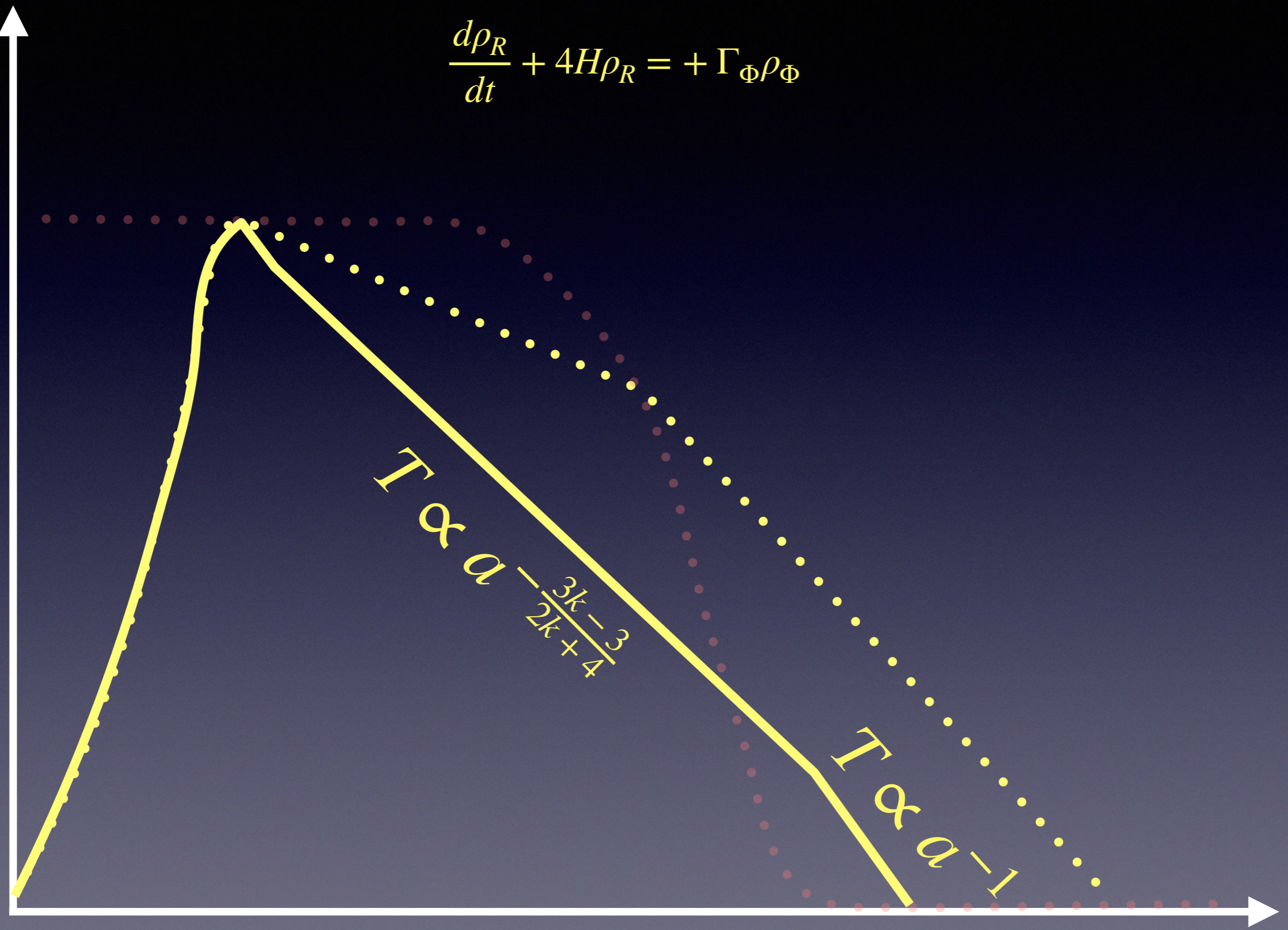


Temperature  
(T)

$$\frac{d\rho_\Phi}{dt} + \frac{6k}{k+2}H\rho_\Phi = -\Gamma_\Phi\rho_\Phi$$

$$\frac{d\rho_R}{dt} + 4H\rho_R = +\Gamma_\Phi\rho_\Phi$$

$$V(\phi) = \lambda_k \phi^k$$

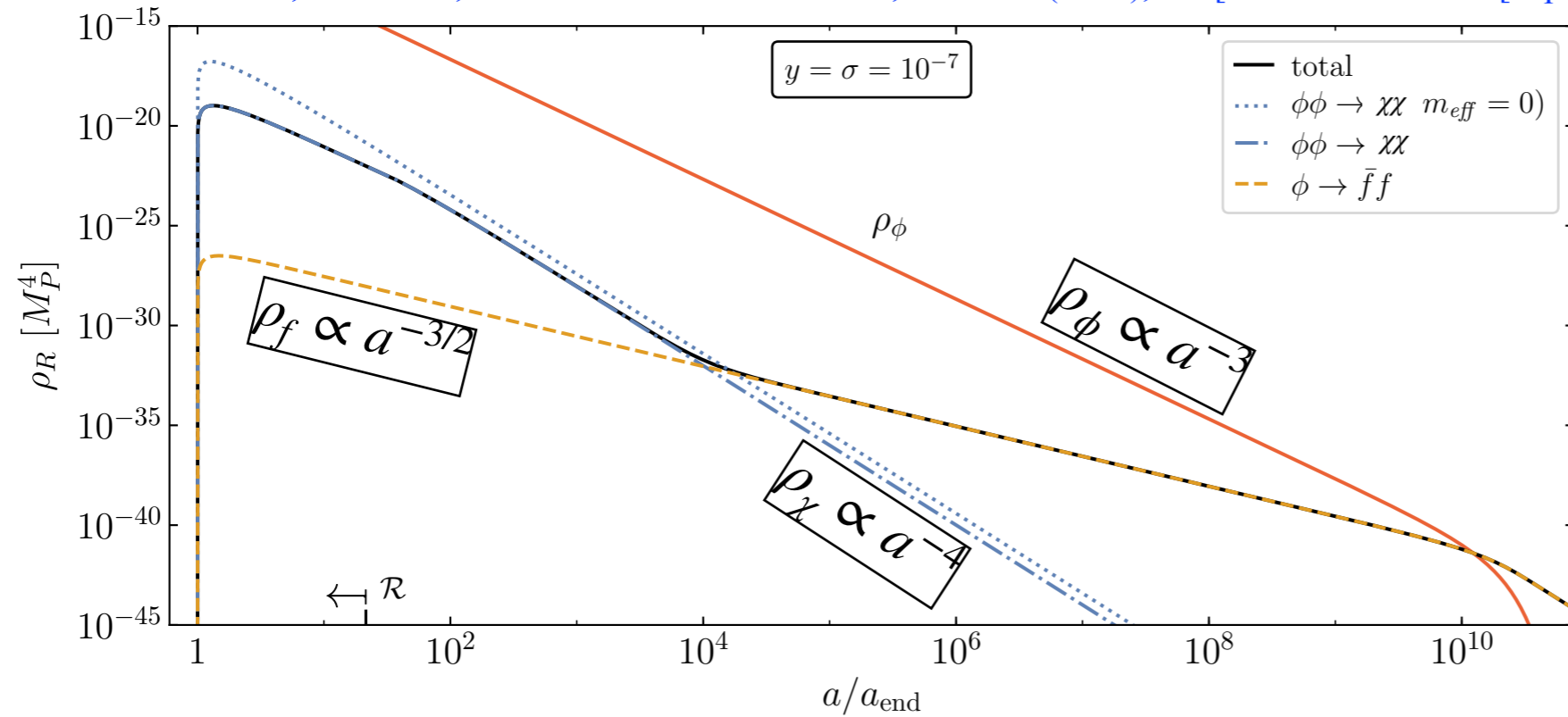


Scaling factor (a)

# Adding a coupling to matter (2)

$$V = V(\phi) + y\phi\bar{f}f + \sigma\phi^2\chi^2$$

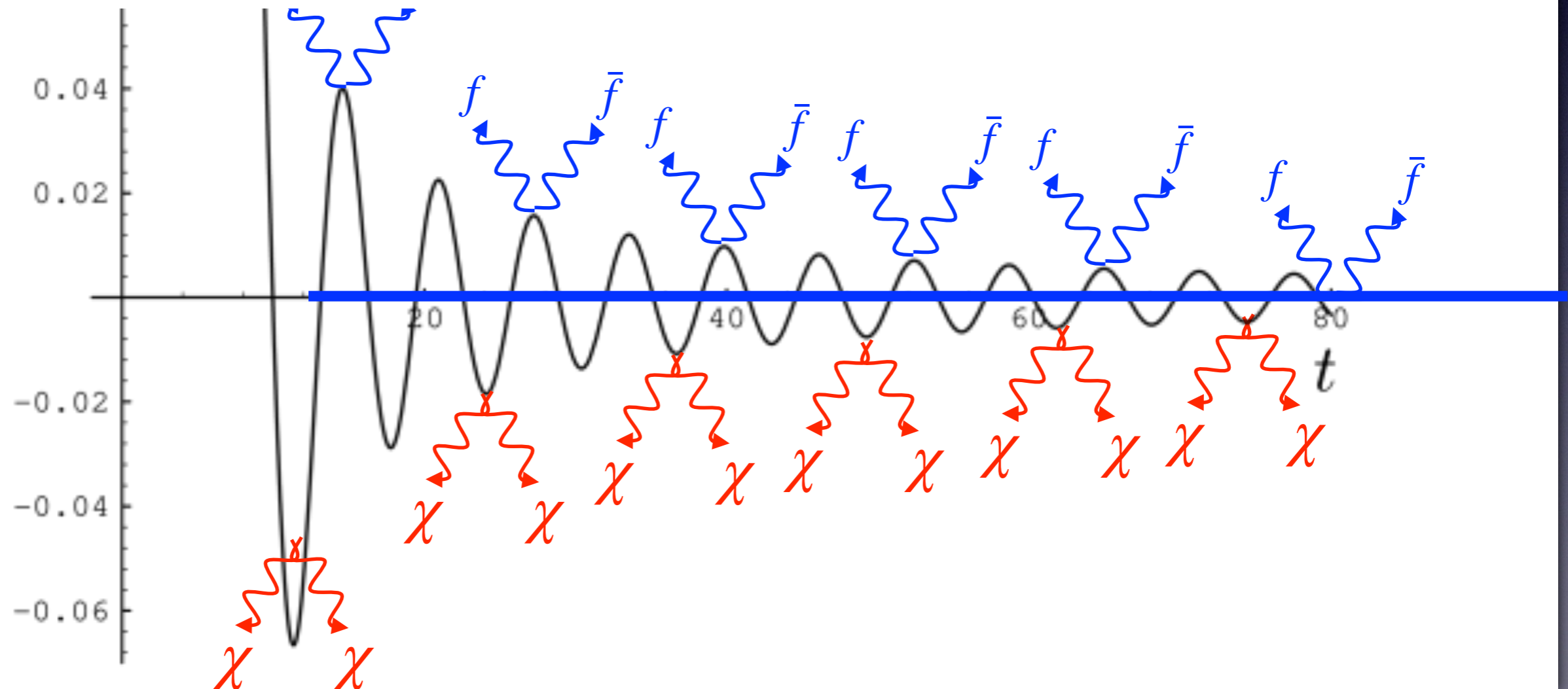
$$\sigma \neq 0, \quad \sigma \times \phi_{\text{end}}^2 \ll m_\phi^2$$



$$\frac{1}{2}m_\phi^2\phi^2 + y\phi f\bar{f} + \sigma\phi^2\chi^2$$

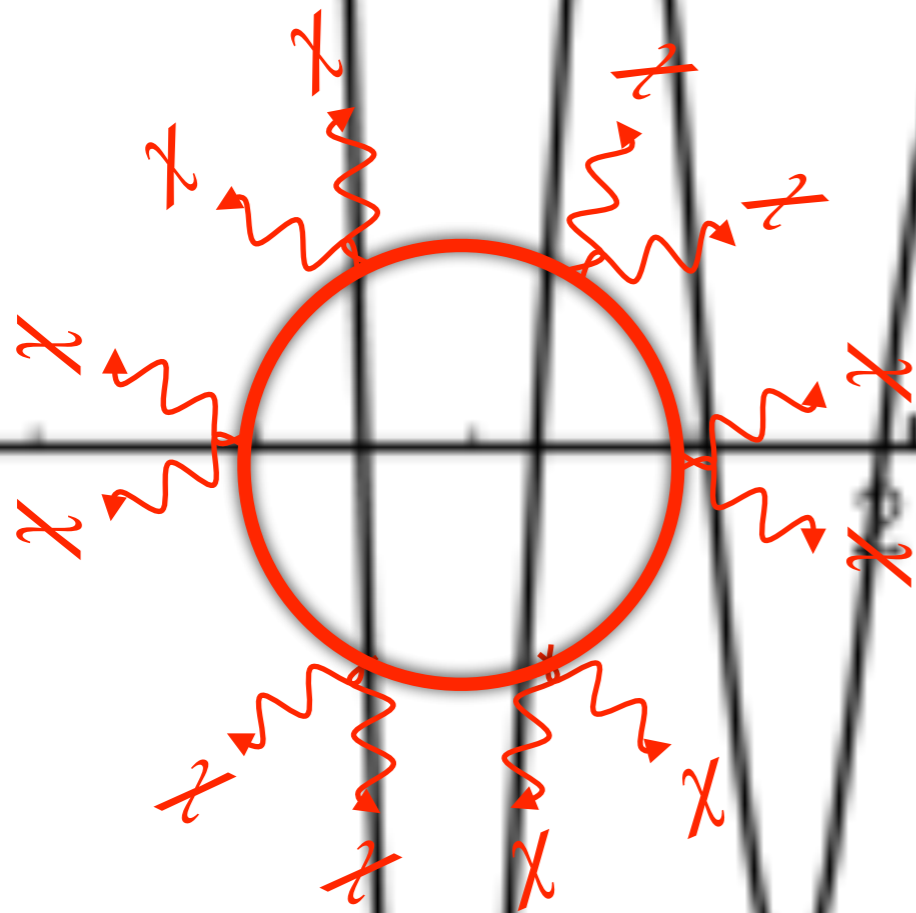
$$3H\dot{\chi} - \frac{\nabla}{a^2}\chi(t, x) + 2\sigma\phi^2\chi = 0$$

$$m_\chi^{\text{eff}} = \sqrt{2\sigma}\phi$$



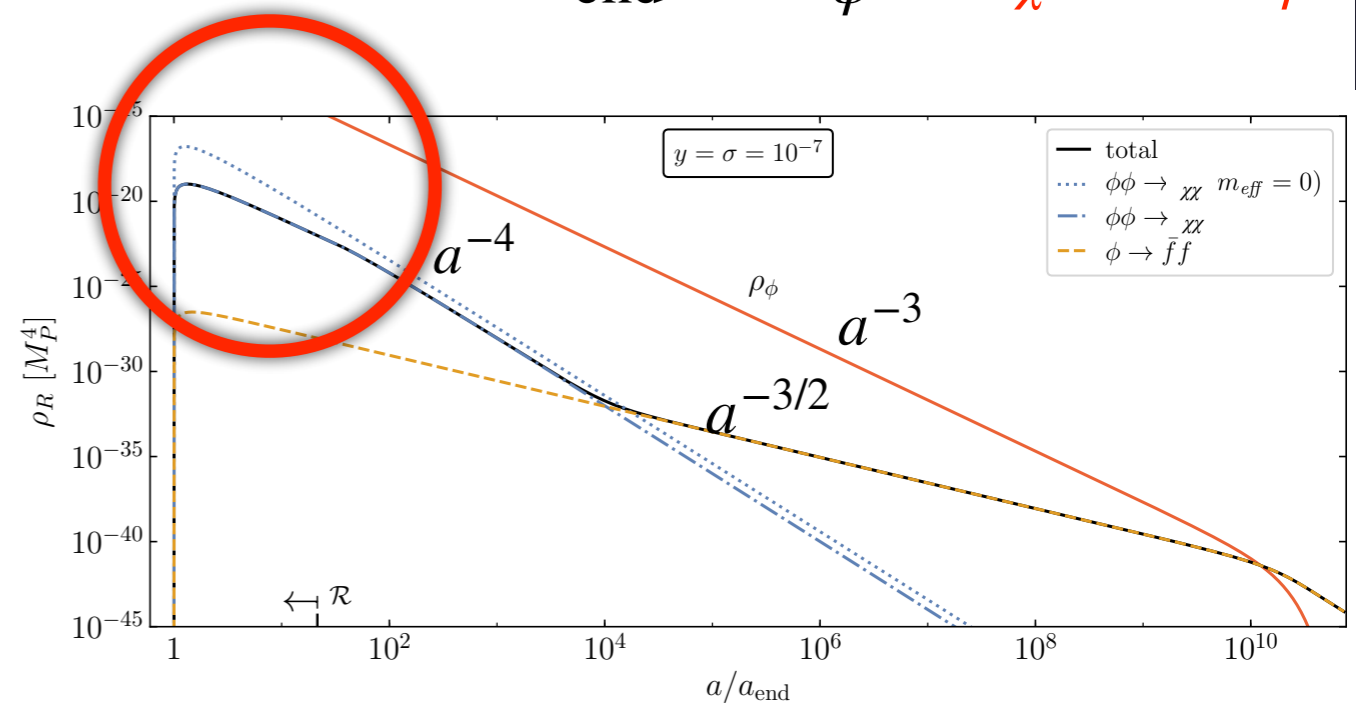
$$\ddot{\chi}(t, x) + 3H\dot{\chi}(t, x) - \frac{\nabla^2}{a^2}\chi(t, x) + 2\sigma\phi^2\chi(t, x) = 0$$

$$m_\chi^{\text{eff}} = \sqrt{2\sigma}\Phi$$

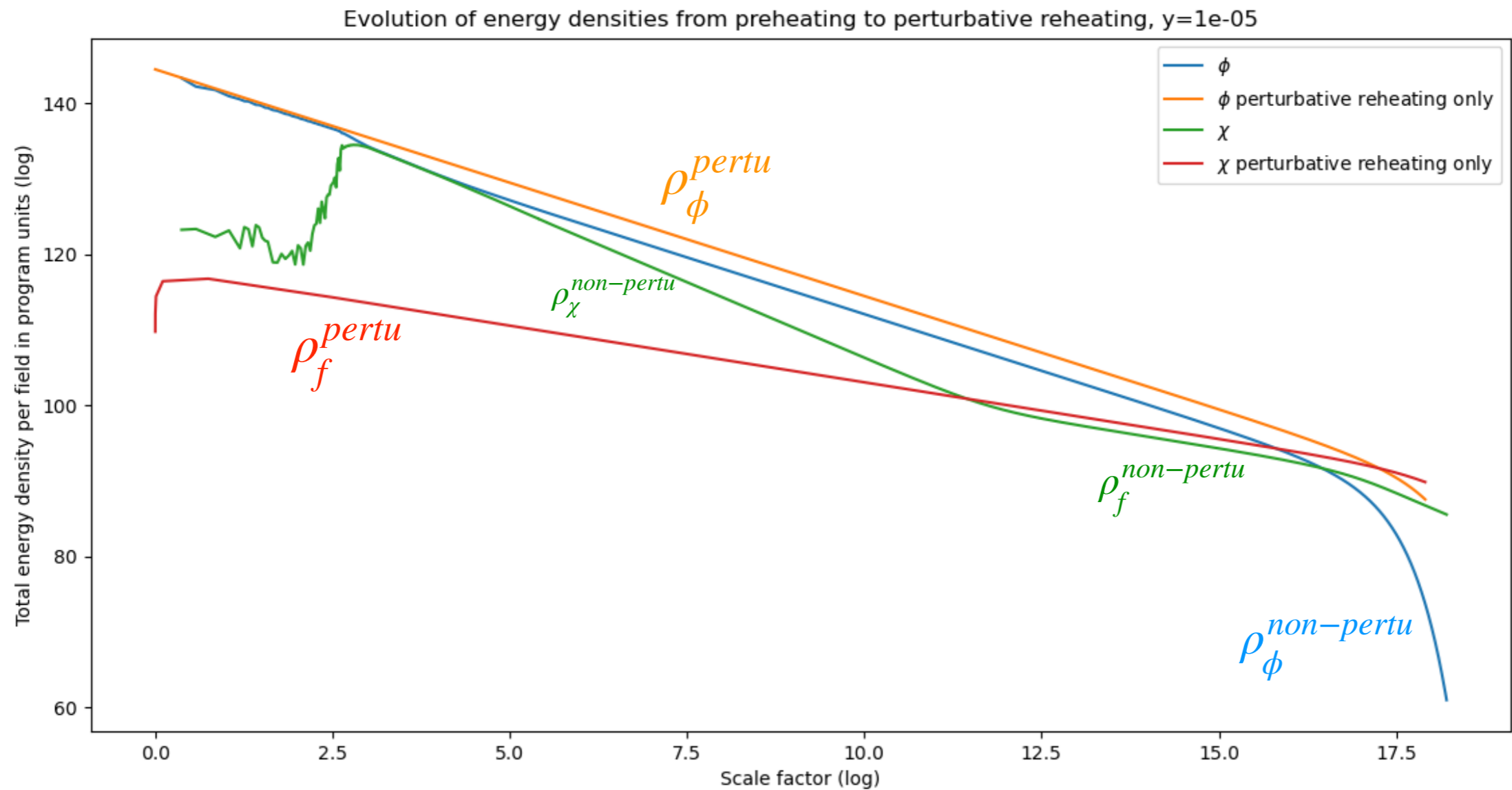
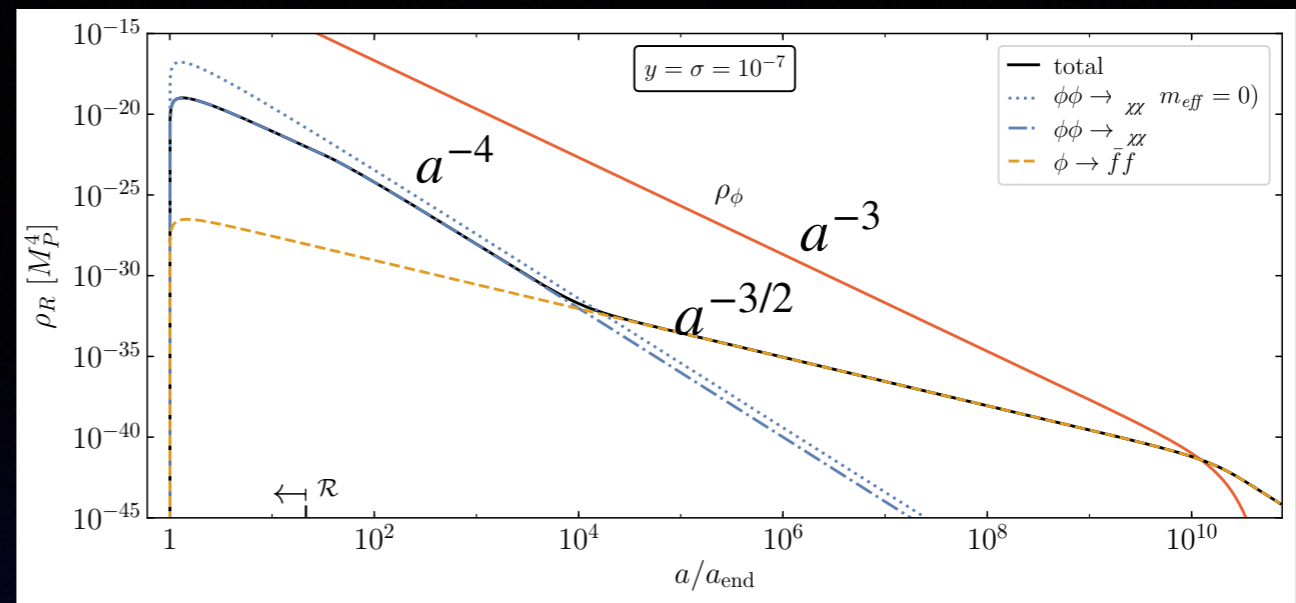
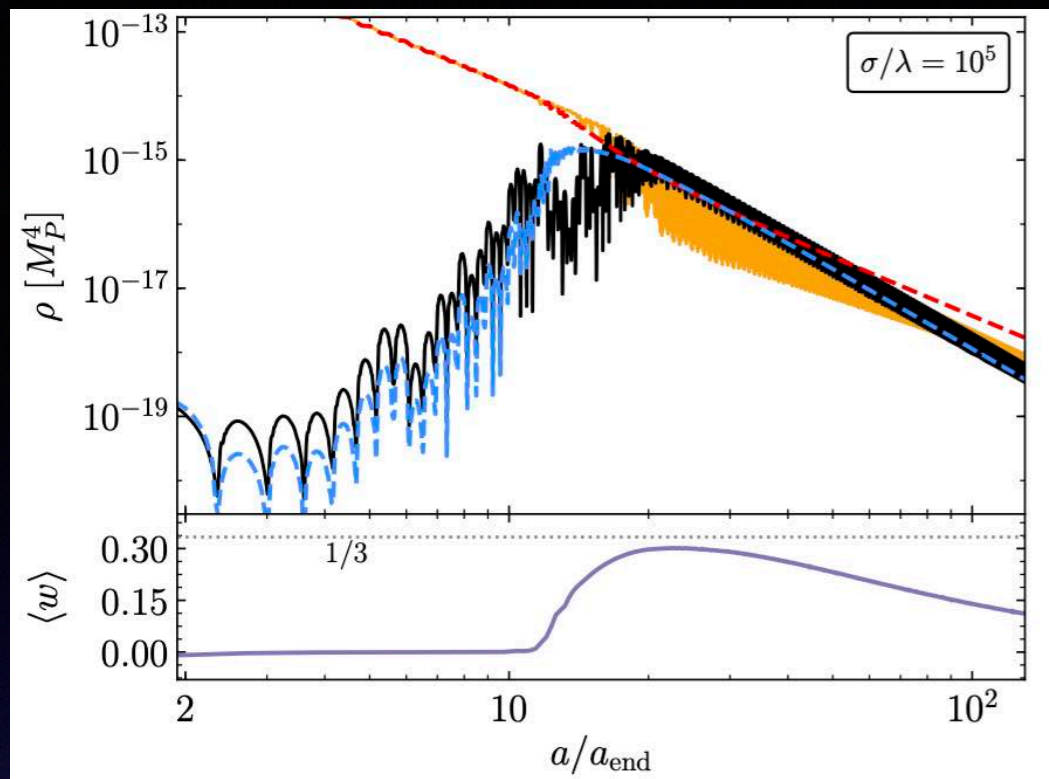


Resonant  
production

$$\sigma \neq 0, \quad \sigma \times \phi_{\text{end}}^2 \gtrsim m_\phi^2 \quad [m_\chi^{\text{eff}} \gtrsim m_\phi]$$



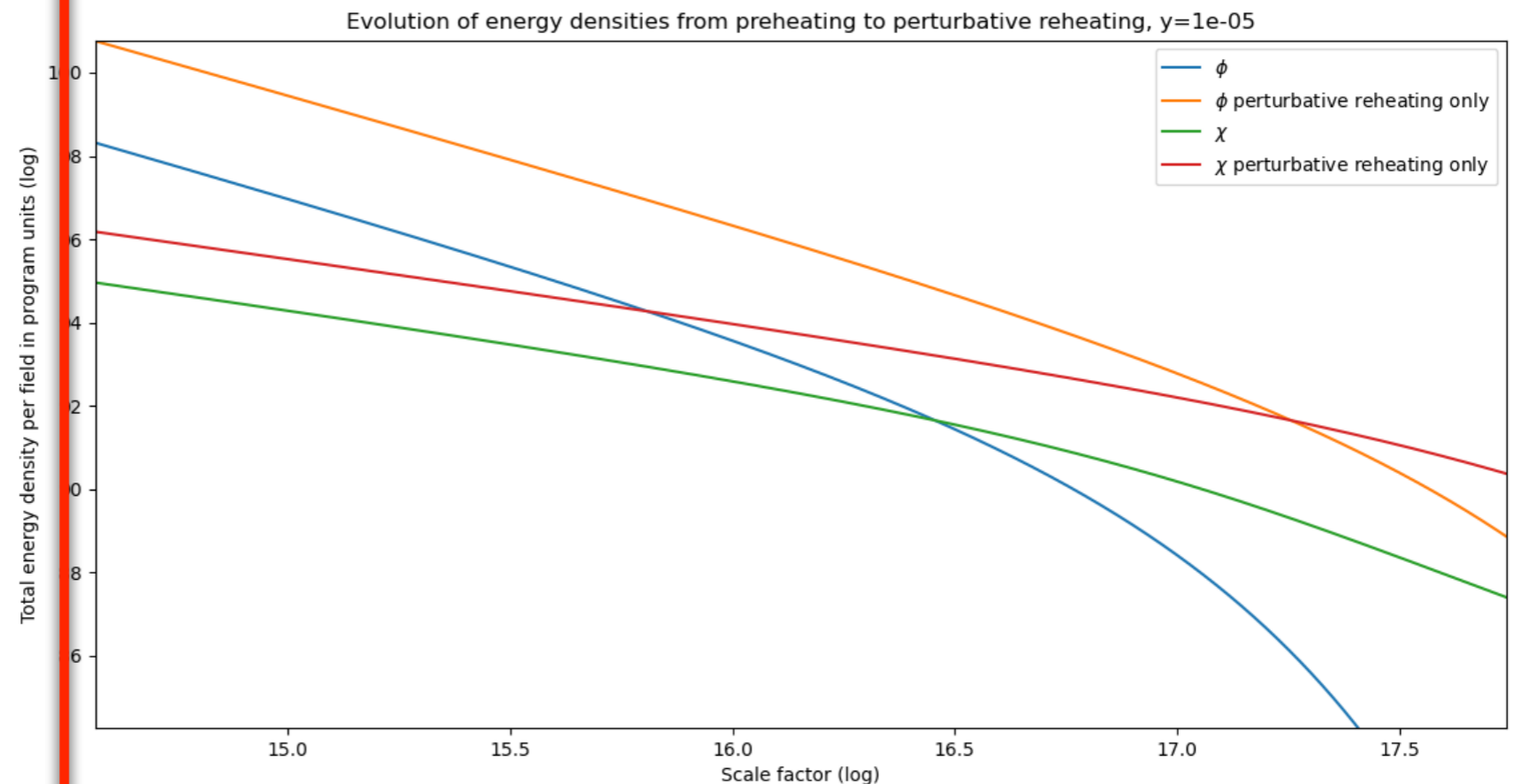
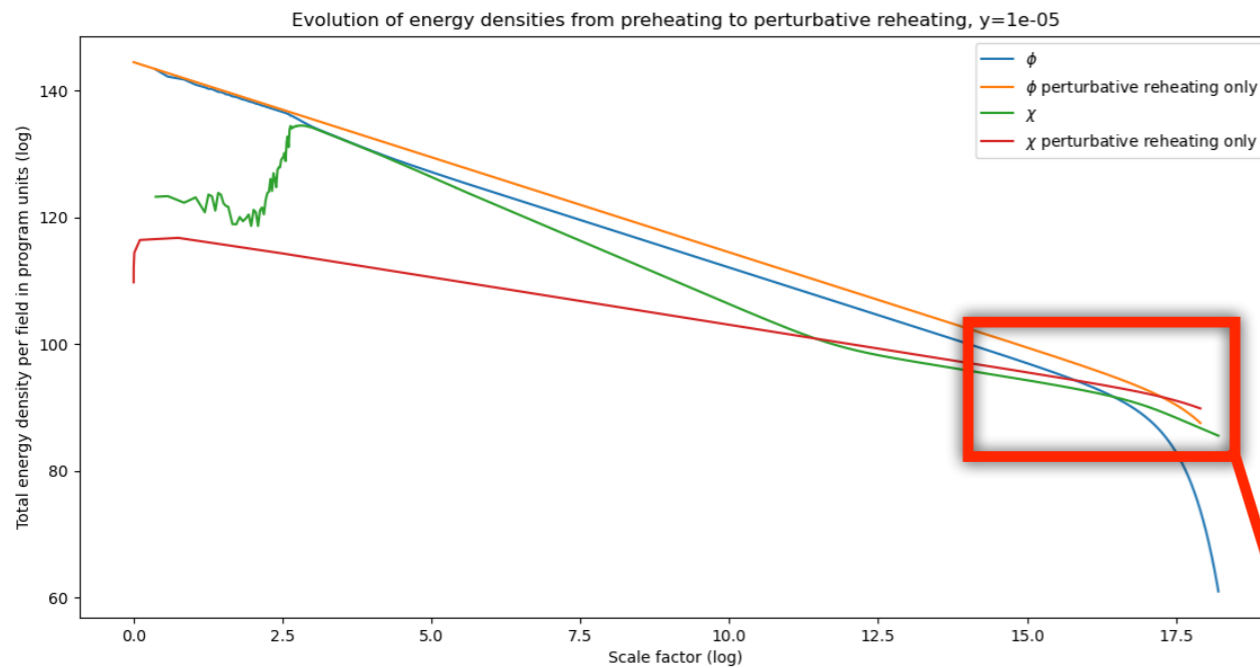
# Summary



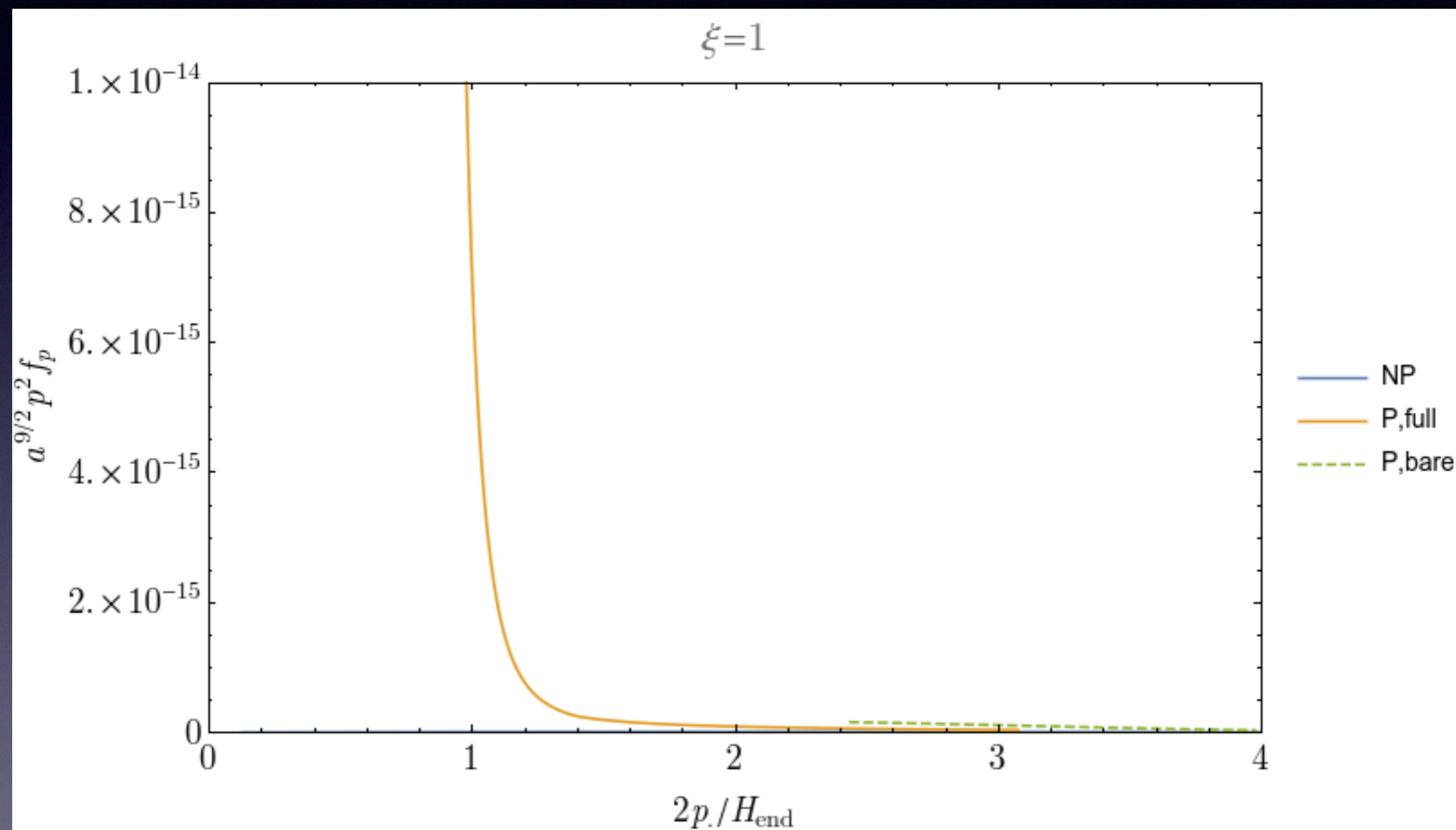
Important remark : the preheating *says nothing about reheating*, and especially gives the same reheating temperature than a perturbative treatment.

This comes from the fact that the reheating happens for  

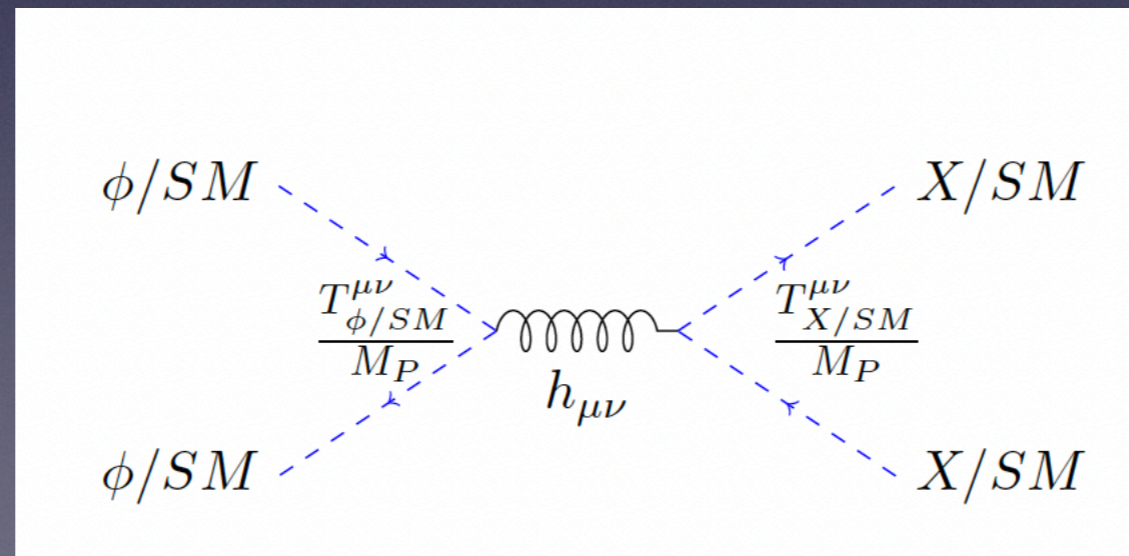
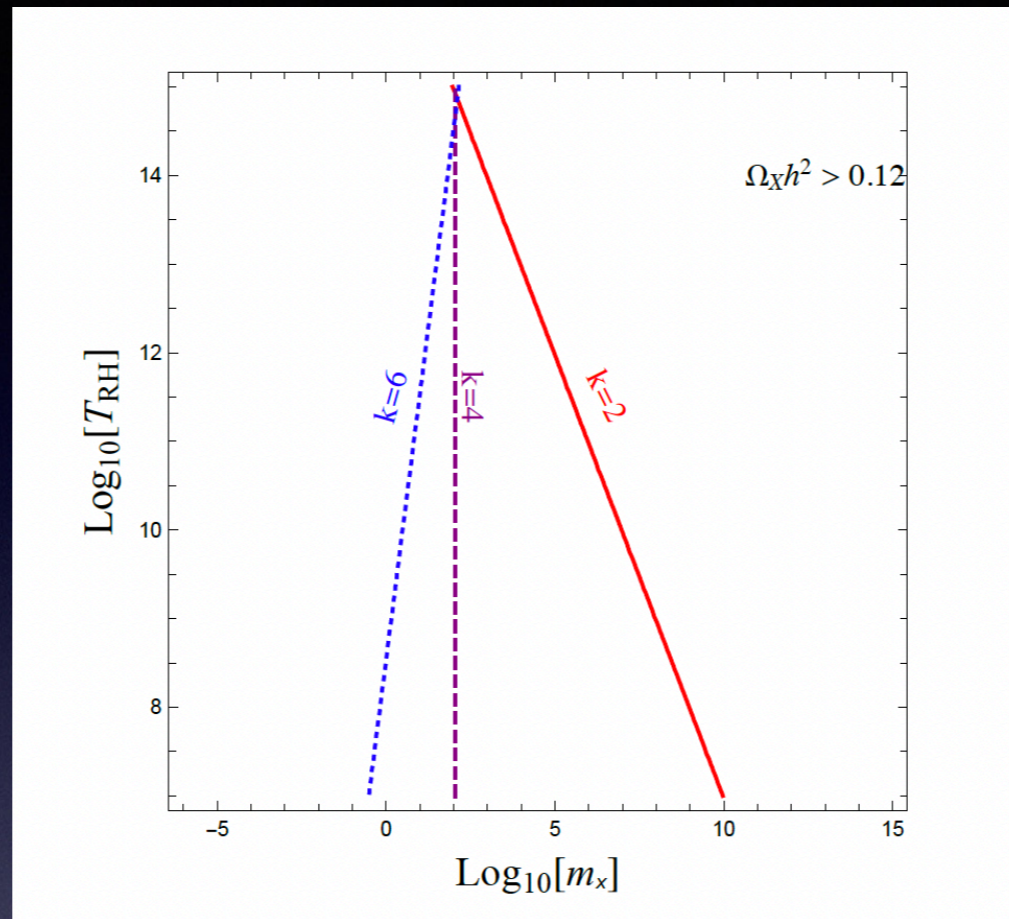
$$\rho_R = \rho_\phi \simeq \sqrt{\Gamma_\phi M_P} \sim T_{RH}^4$$
 It just happens *before*.

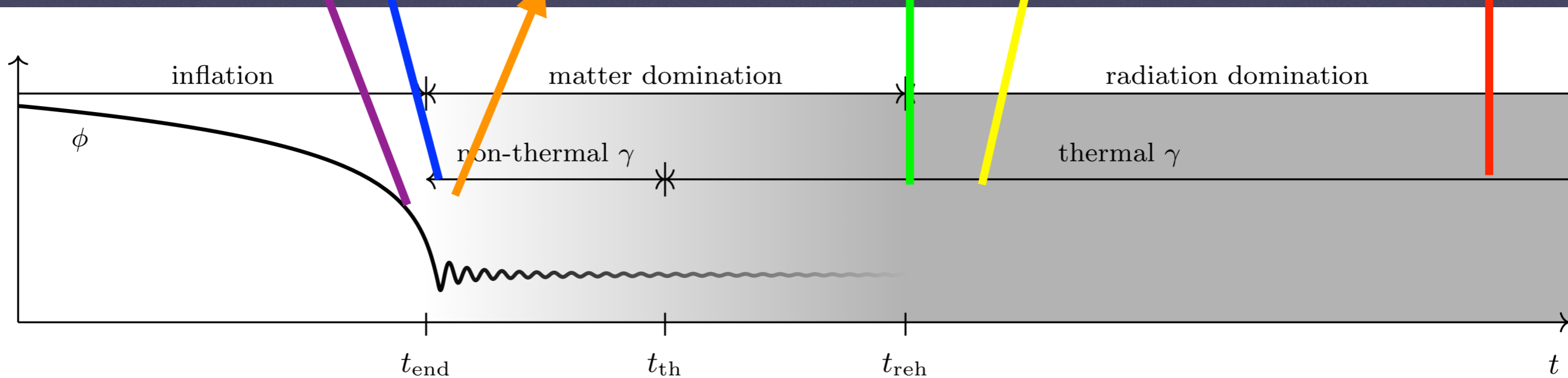
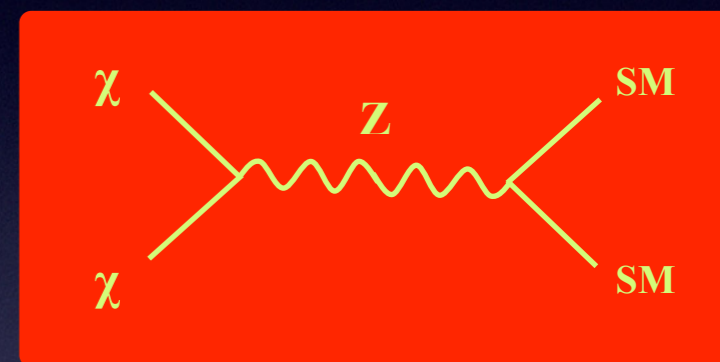
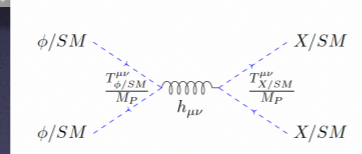
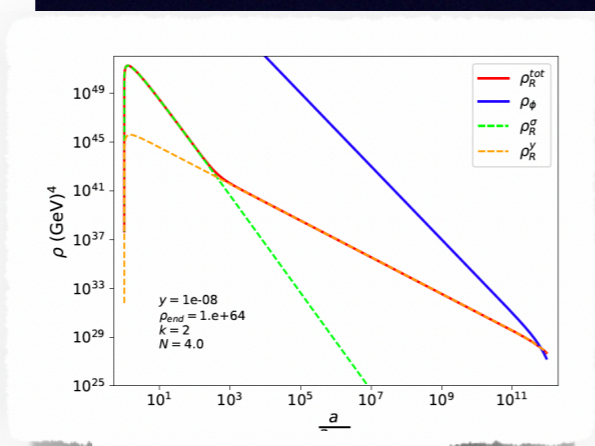
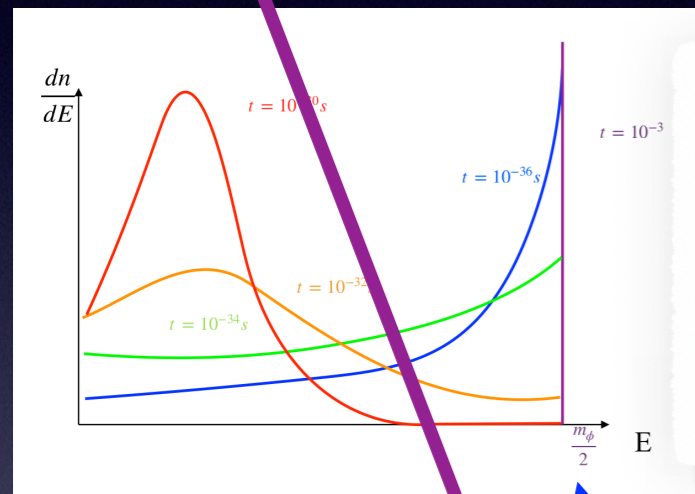
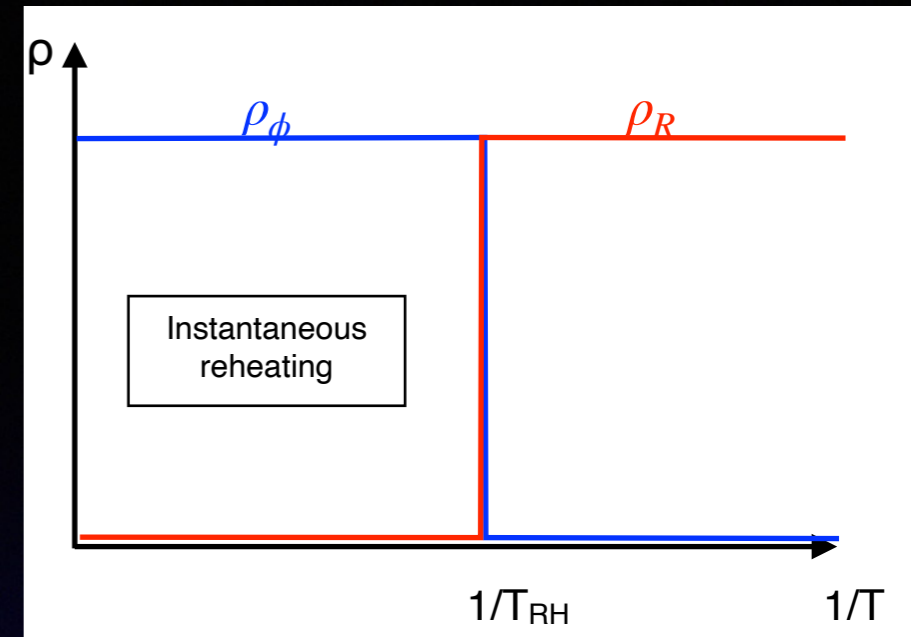
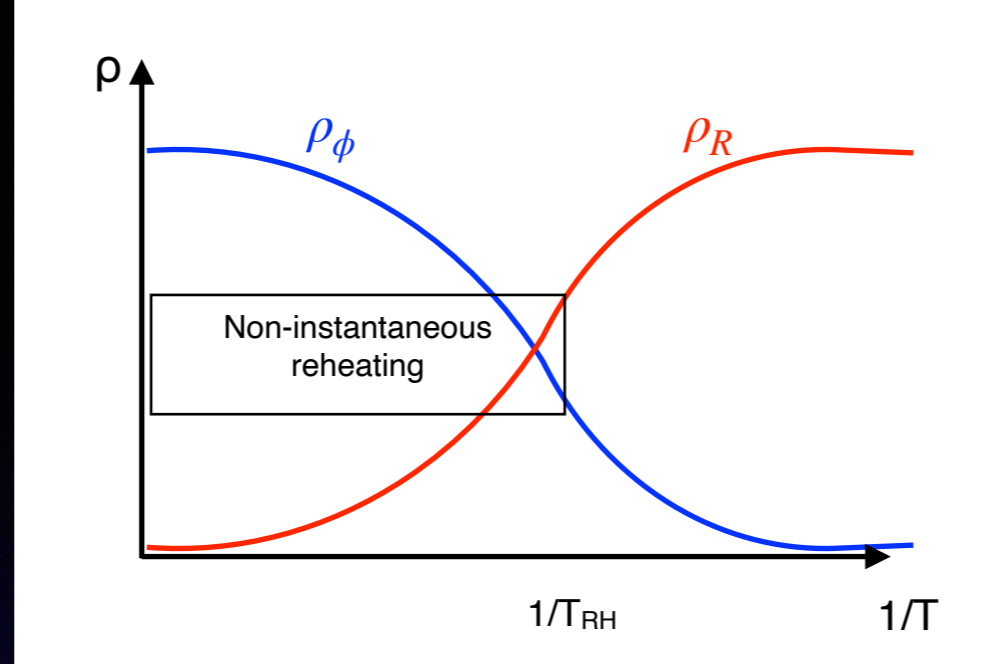
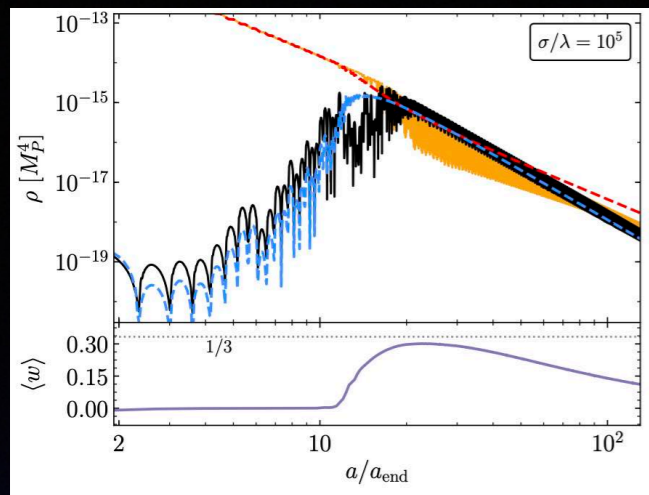


# Distribution functions

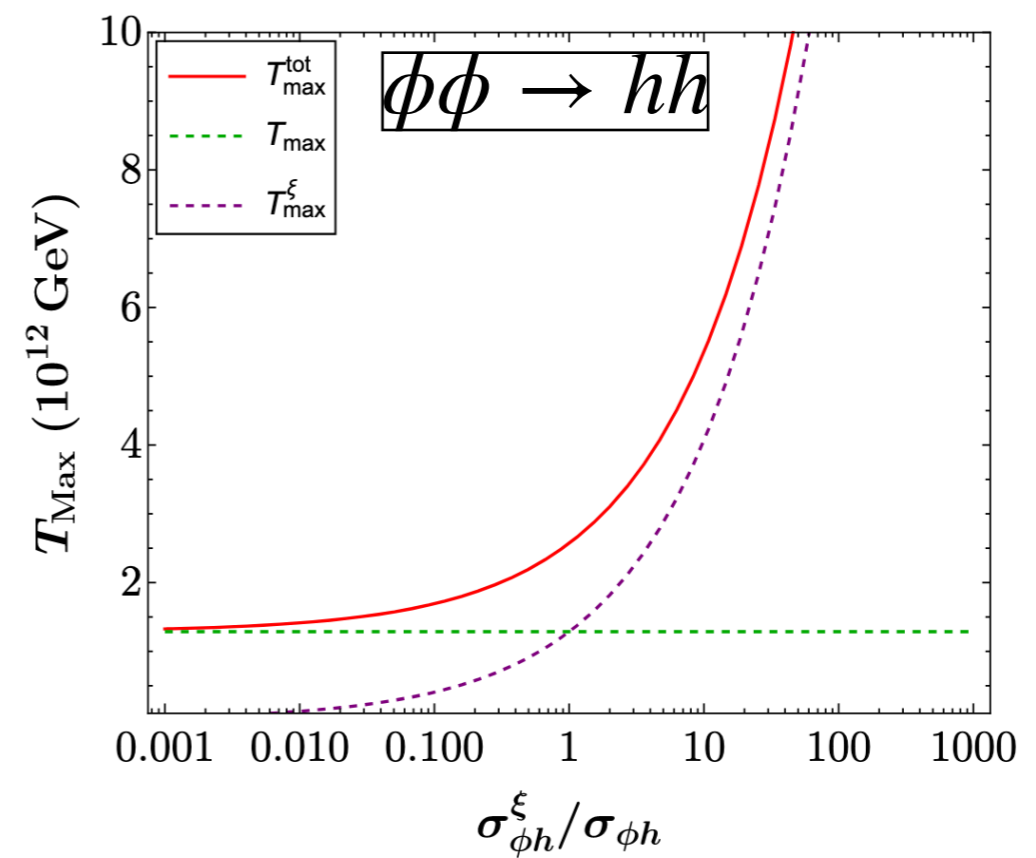
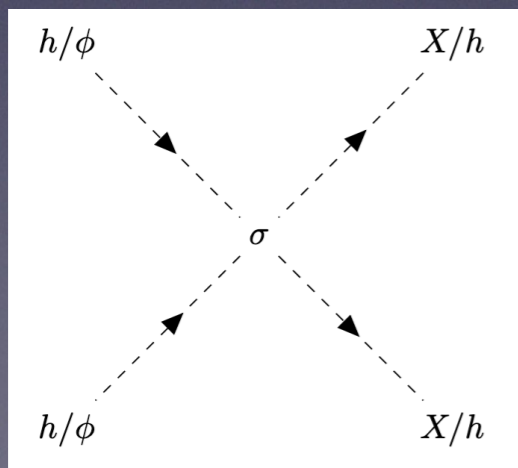


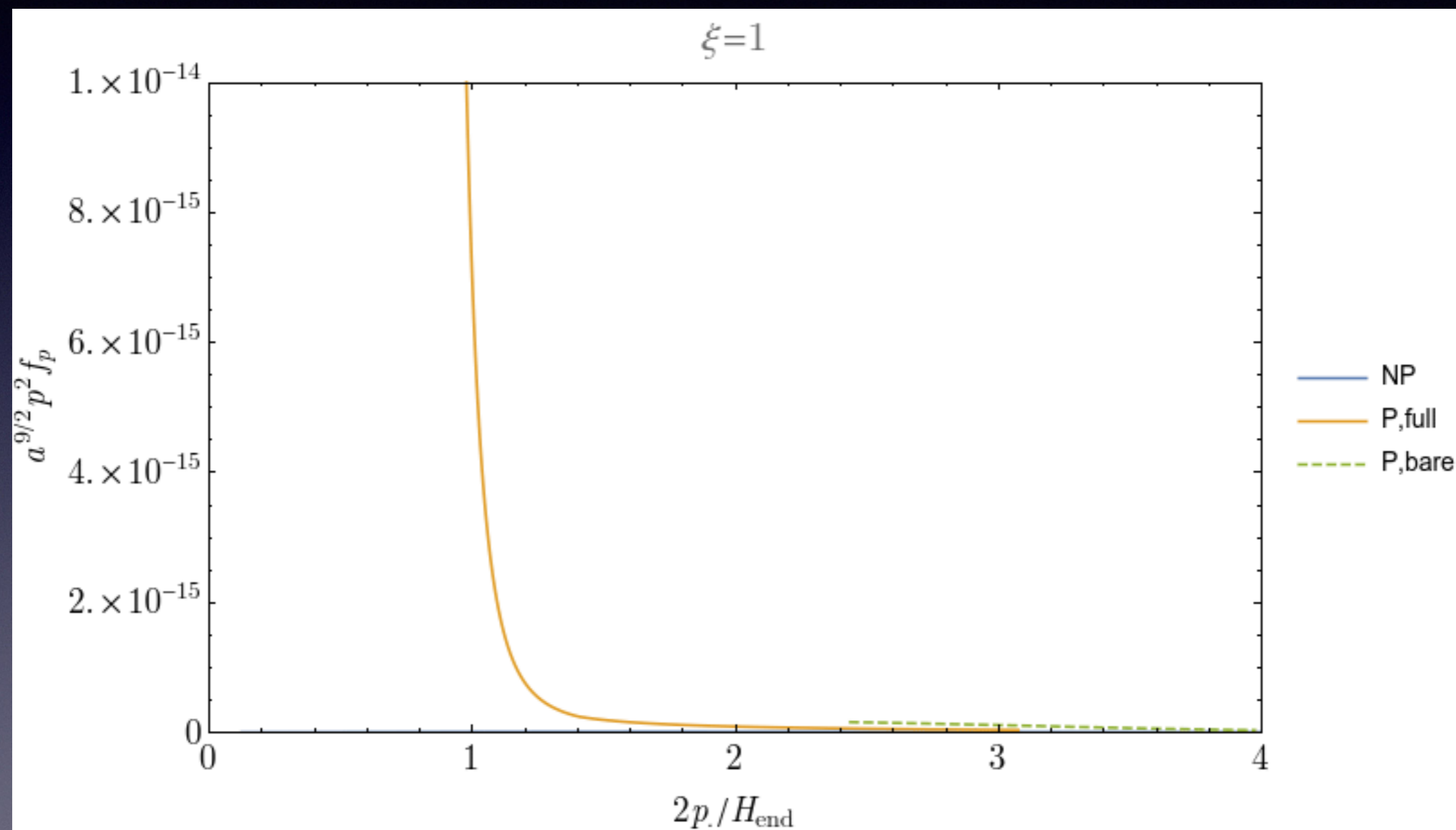
# Also working for Dark Matter



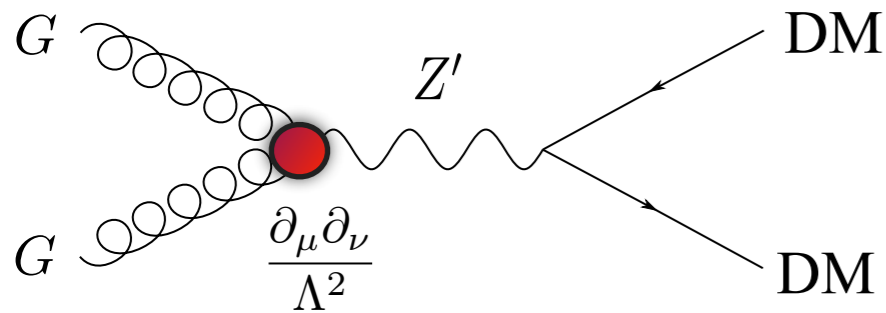


# Distribution Functions



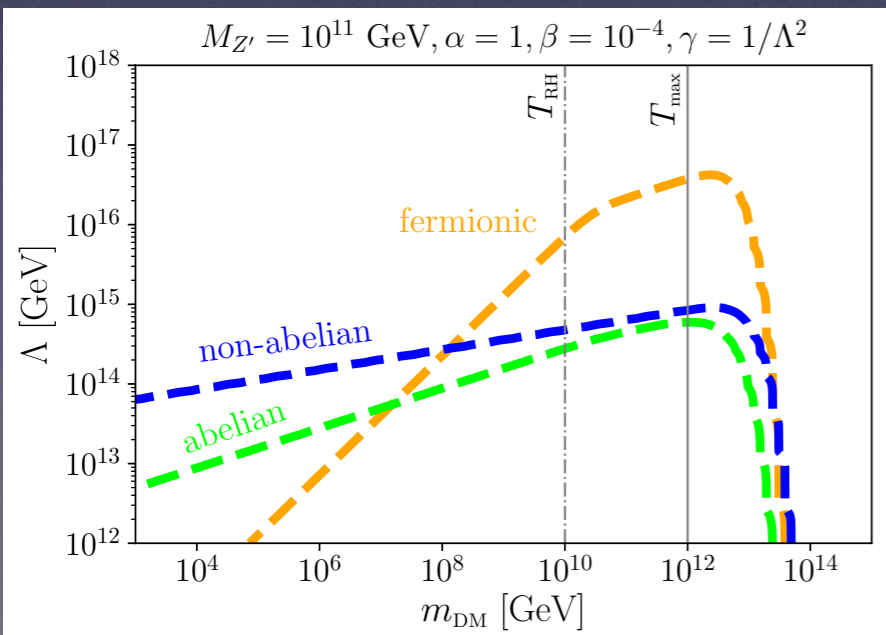


# Spin-1 mediator

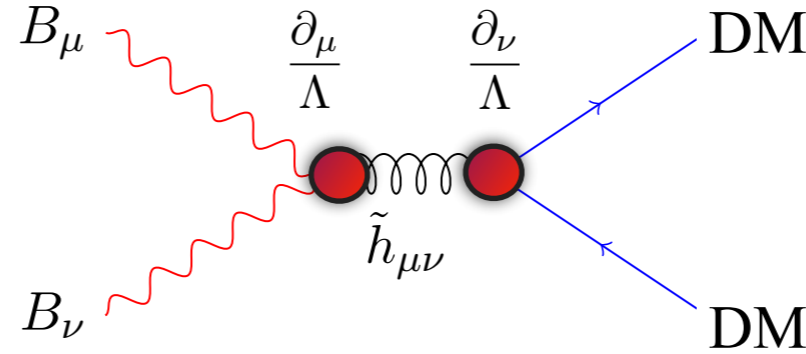


$$\mathcal{L} = \frac{\tilde{g}}{M^2} \partial^\alpha Z'_\alpha \epsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu^a \partial_\rho A_\sigma^a$$

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \partial^\alpha Z'_\alpha \epsilon^{\mu\nu\rho\sigma} \text{Tr}[G_{\mu\nu}^a G_{\rho\sigma}^a]$$



# Spin-2 mediator



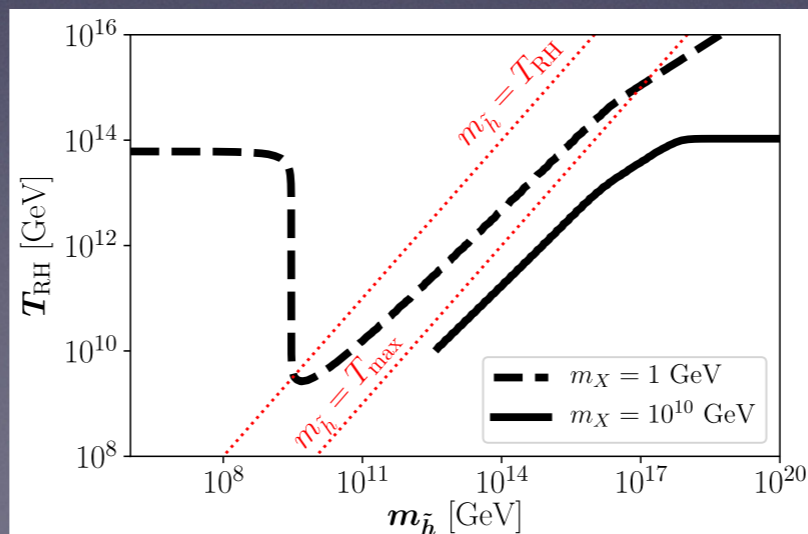
$$\mathcal{L}_{\text{int}}^1 = \frac{1}{2M_P} h_{\mu\nu} (T_{\text{SM}}^{\mu\nu} + T_{\text{X}}^{\mu\nu})$$

$$\mathcal{L}_{\text{int}}^2 = \frac{1}{\Lambda} \tilde{h}_{\mu\nu} (g_{\text{SM}} T_{\text{SM}}^{\mu\nu} + g_{\text{DM}} T_{\text{X}}^{\mu\nu})$$

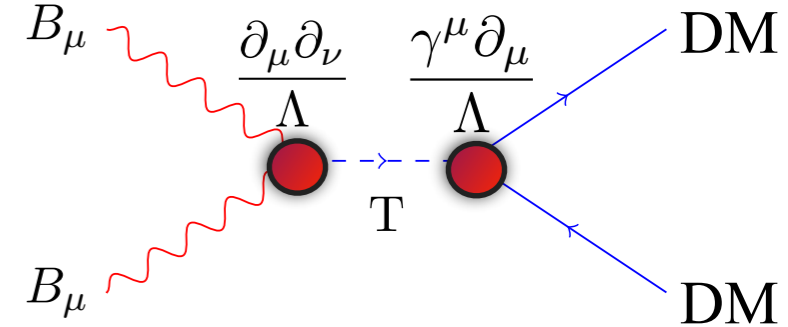
$$T_{\mu\nu}^0 = \frac{1}{2} (\partial_\mu \phi \partial_\nu \phi + \partial_\nu \phi \partial_\mu \phi - g_{\mu\nu} \partial^\alpha \phi \partial_\alpha \phi),$$

$$T_{\mu\nu}^{1/2} = \frac{i}{4} \bar{\psi} (\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu) \psi - \frac{i}{4} (\partial_\mu \bar{\psi} \gamma_\nu + \partial_\nu \bar{\psi} \gamma_\mu) \psi$$

$$T_{\mu\nu}^1 = \frac{1}{2} \left[ F_\mu^\alpha F_{\nu\alpha} + F_\nu^\alpha F_{\mu\alpha} - \frac{1}{2} g_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right].$$



# Moduli mediator

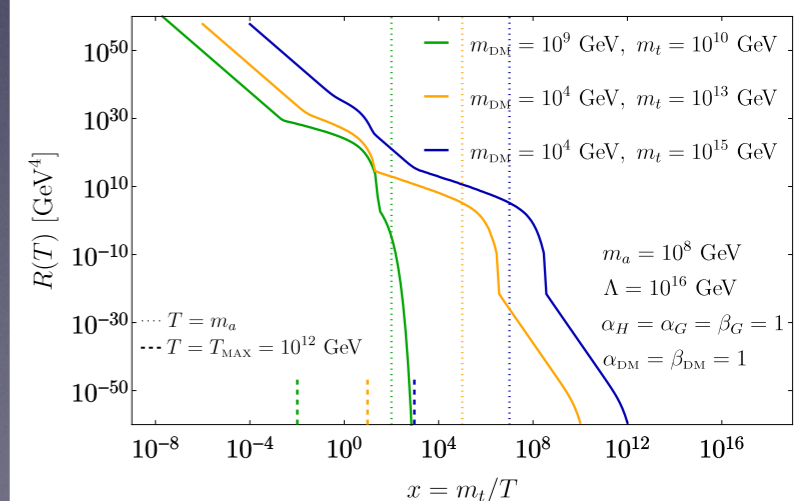


$$\mathcal{L}_{\mathcal{T}}^{SM} \supset \frac{\alpha_H}{\Lambda} t |D_\mu H|^2 - \frac{\alpha_H}{\Lambda} \mu_0^2 t |H|^2$$

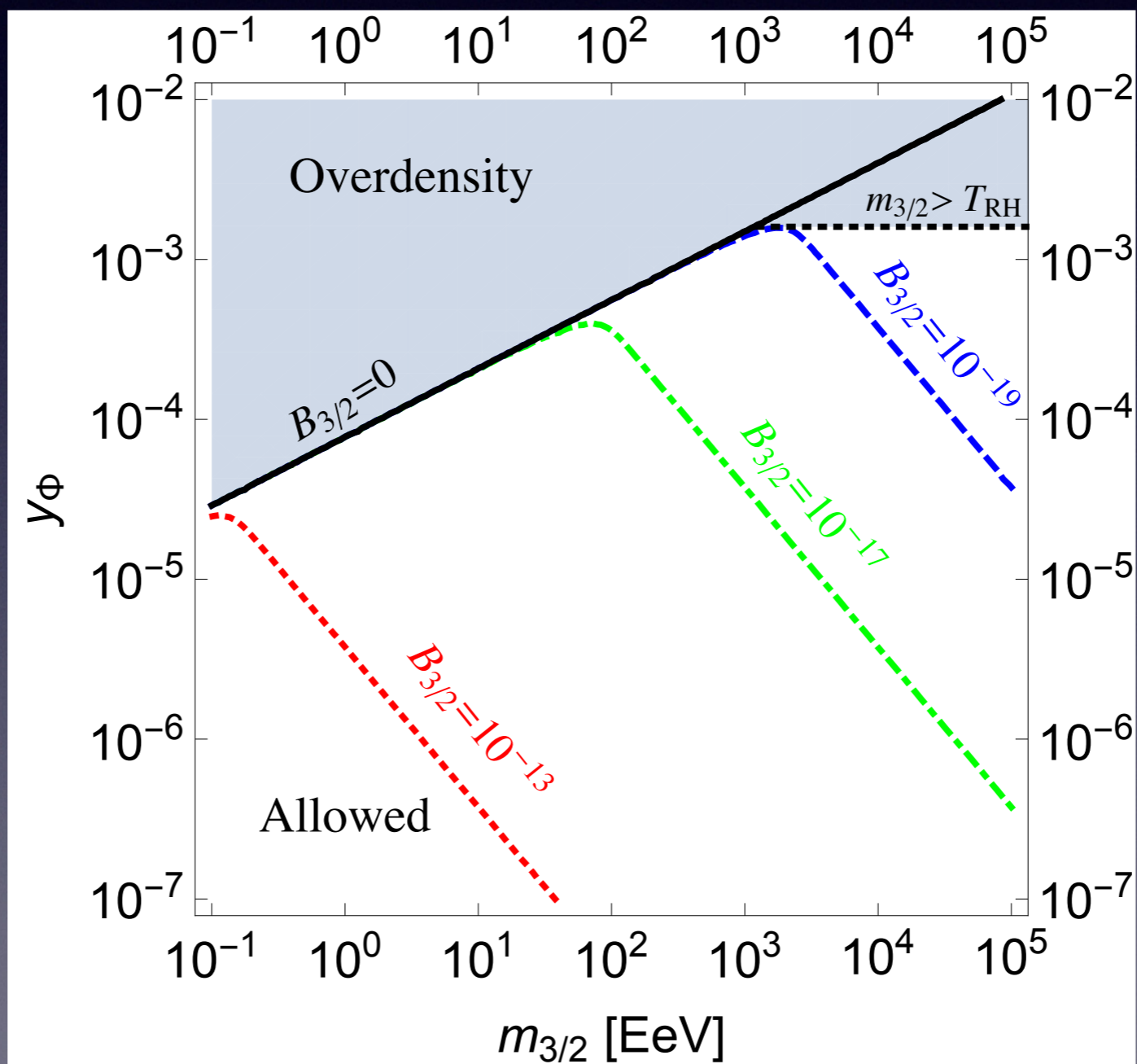
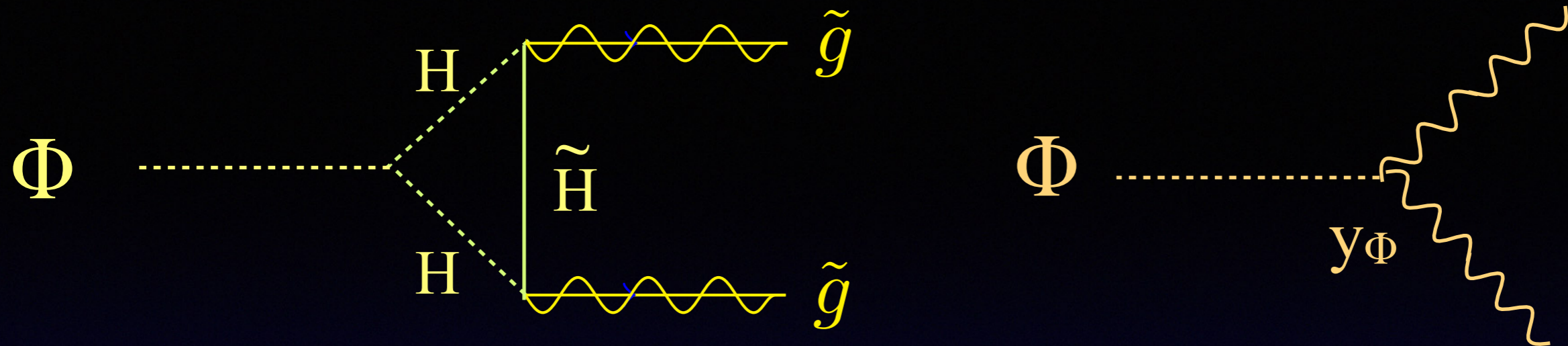
$$+ \left( \frac{1}{2\Lambda} t \bar{f} i \gamma^\mu (\alpha_V^f - \alpha_A^f \gamma_5) D_\mu f + \text{h.c.} \right)$$

$$+ \frac{1}{2\Lambda} \partial_\mu a \bar{f} \gamma^\mu (\beta_V^f - \beta_A^f \gamma_5) f$$

$$- \frac{1}{4} \frac{\alpha_G}{\Lambda} t G_{\mu\nu} G^{\mu\nu} + 2 \frac{\beta_G}{\Lambda} \partial_\mu a \epsilon^{\mu\nu\rho\sigma} G_\nu \partial_\rho G_\sigma$$



Adding the contribution from  
radiative decay of the inflaton



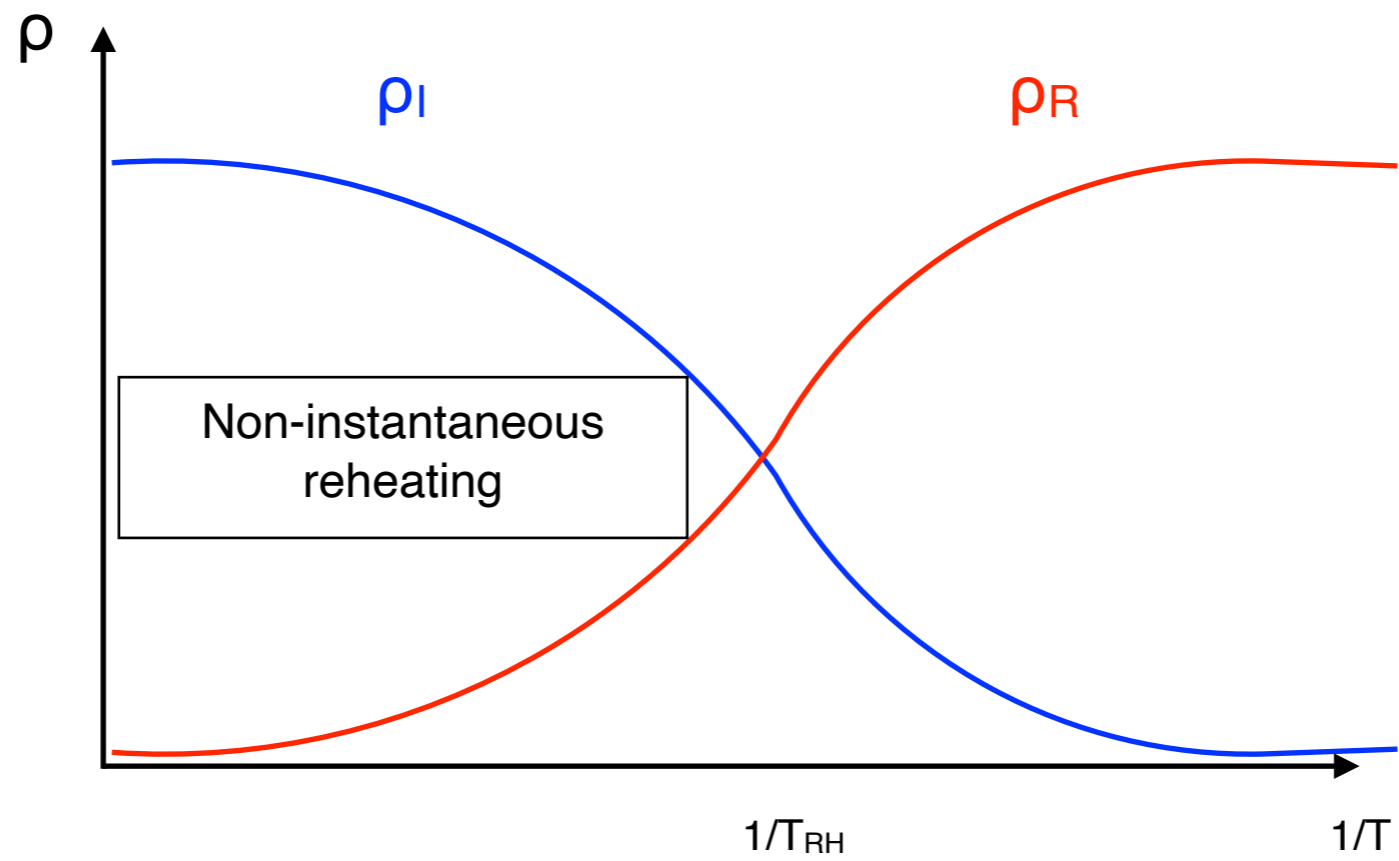
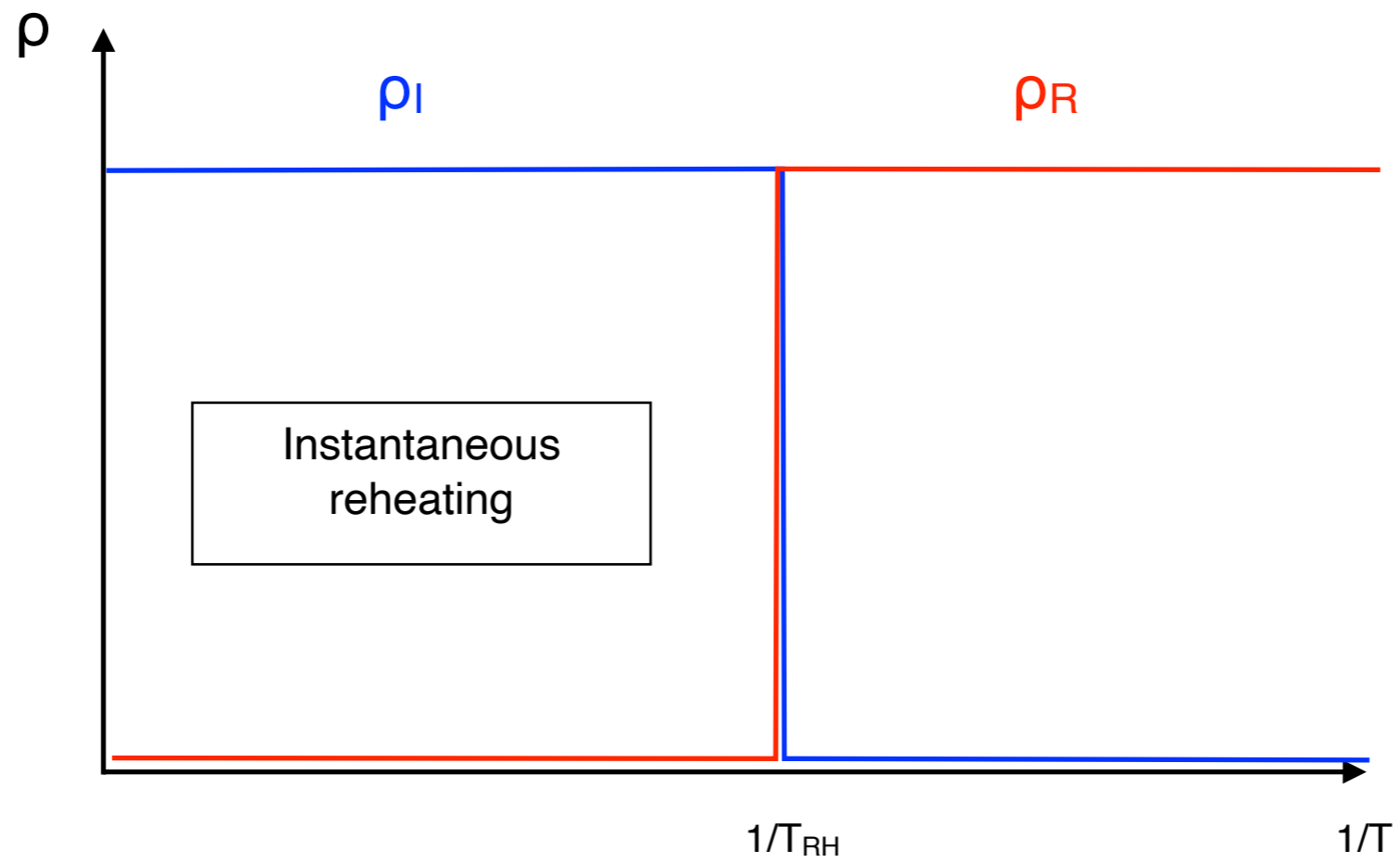
*“I wish I could show you,  
When you are lonely or in darkness,*

*The Astonishing Light  
Of your own Being!”*

— Hafez, The Divan

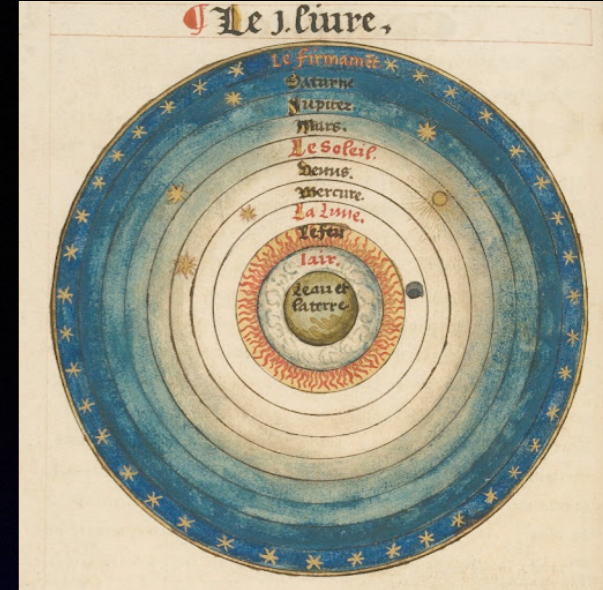




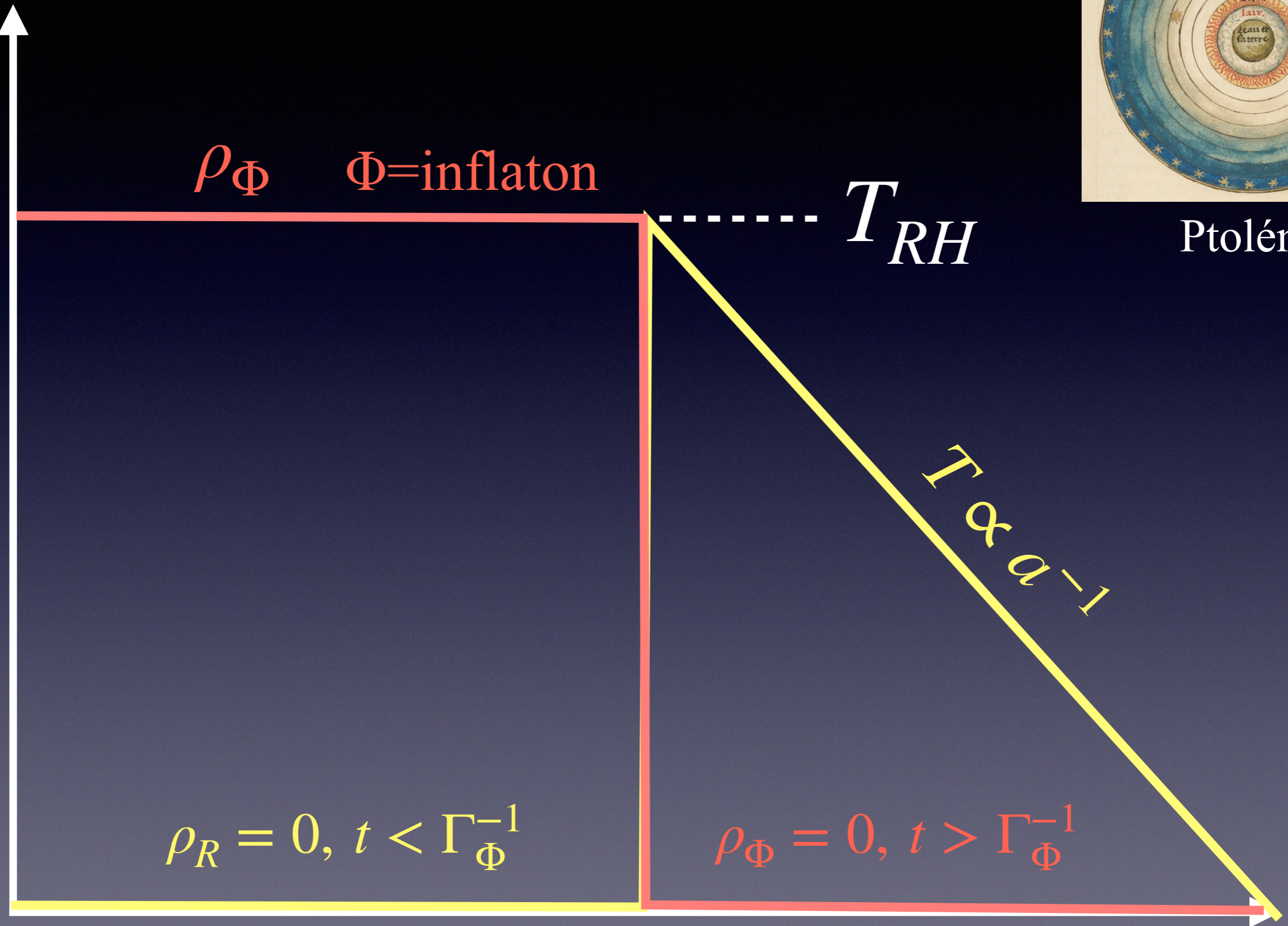


Temperature  
(T)

Basic



Ptolémée



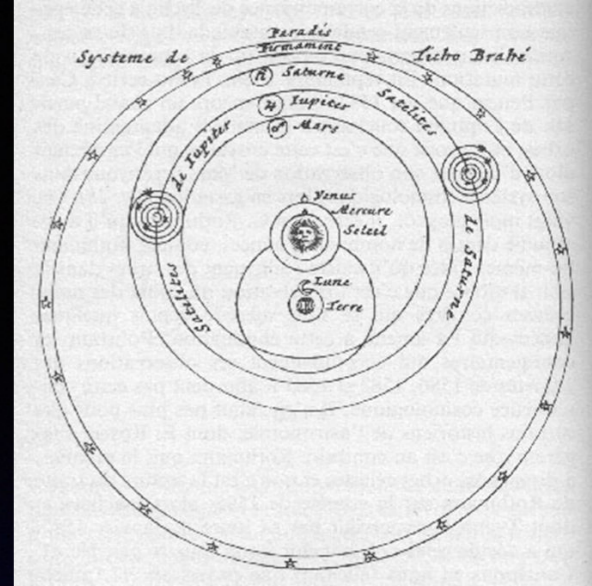
Scaling factor (a)

Temperature  
(T)

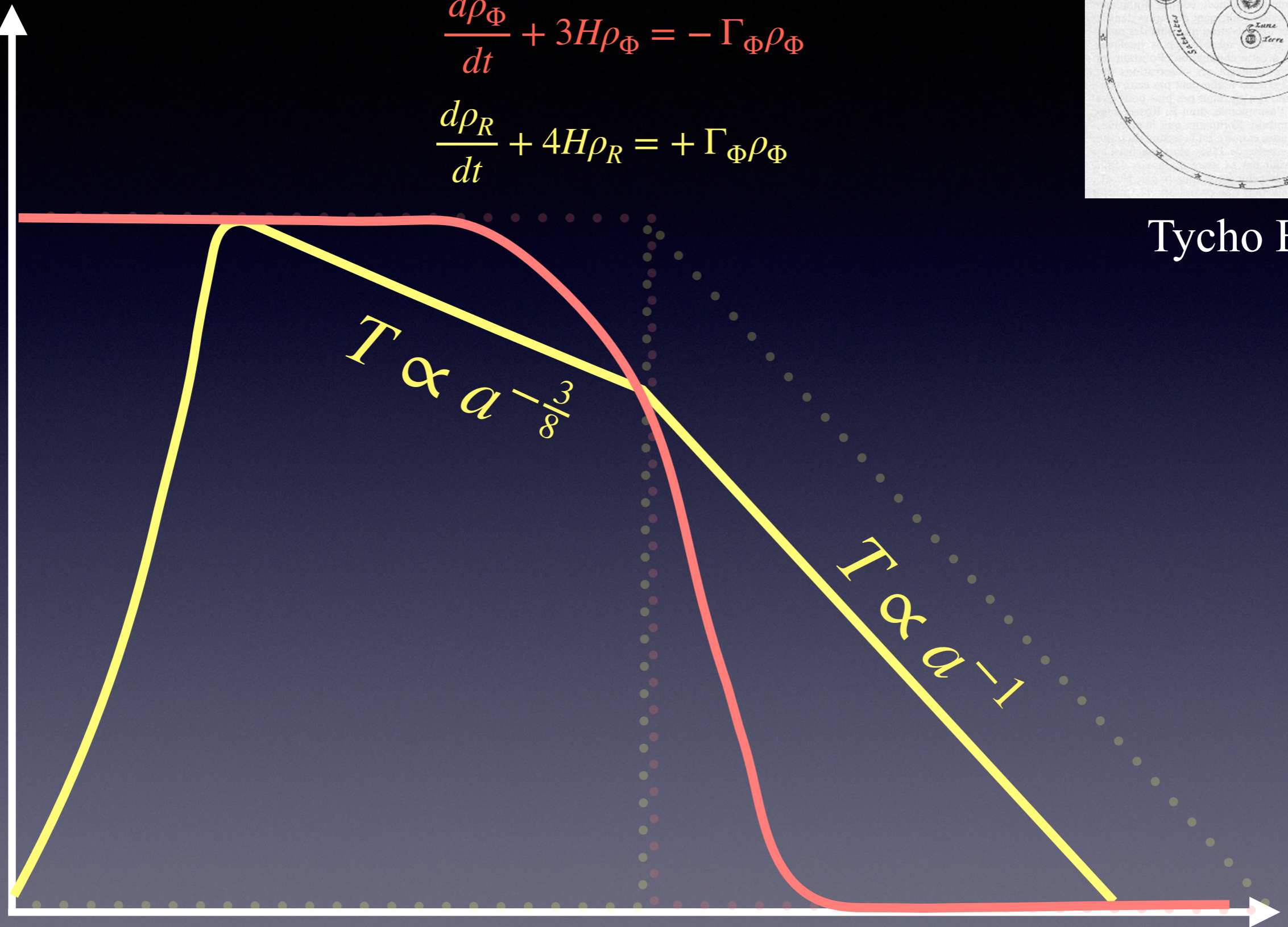
# Refinement I

$$\frac{d\rho_{\Phi}}{dt} + 3H\rho_{\Phi} = -\Gamma_{\Phi}\rho_{\Phi}$$

$$\frac{d\rho_R}{dt} + 4H\rho_R = +\Gamma_{\Phi}\rho_{\Phi}$$



Tycho Brahe

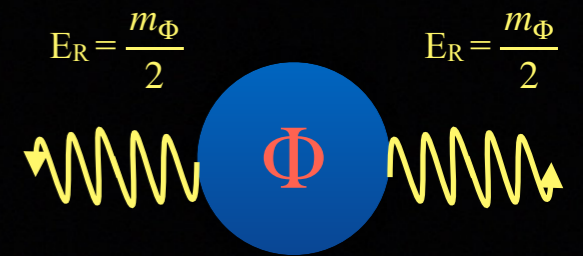
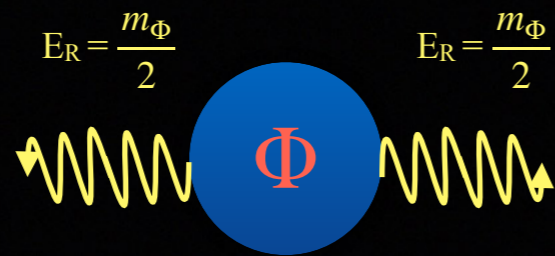


Scaling factor (a)

$\langle \text{Energy} \rangle$

$$\frac{m_\Phi}{2}$$

# Refinement III



$$n\langle\sigma v\rangle \propto a^{-\frac{3}{2}} \times \frac{a^2}{m_\Phi^2} \sim \sqrt{a}$$

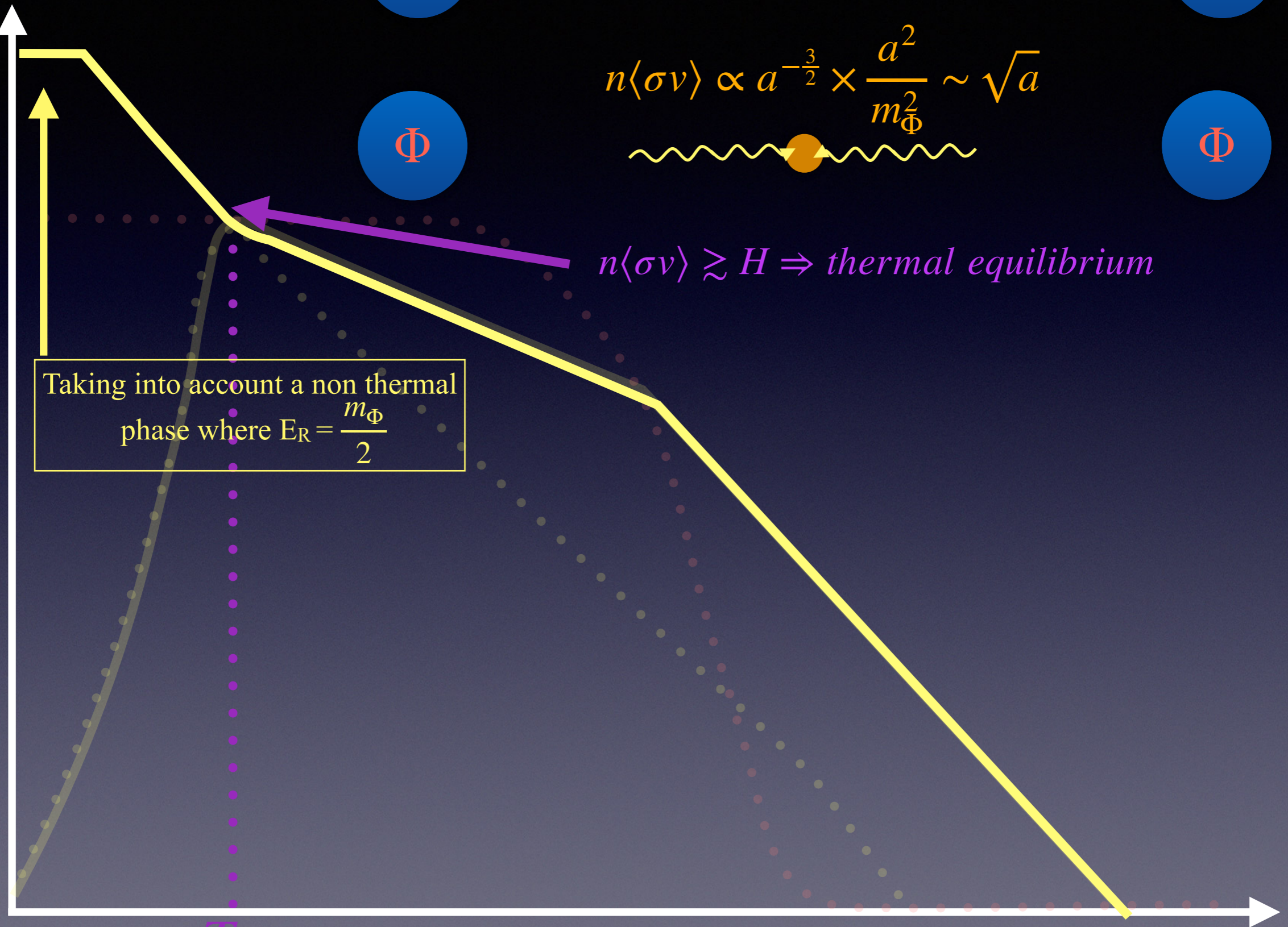


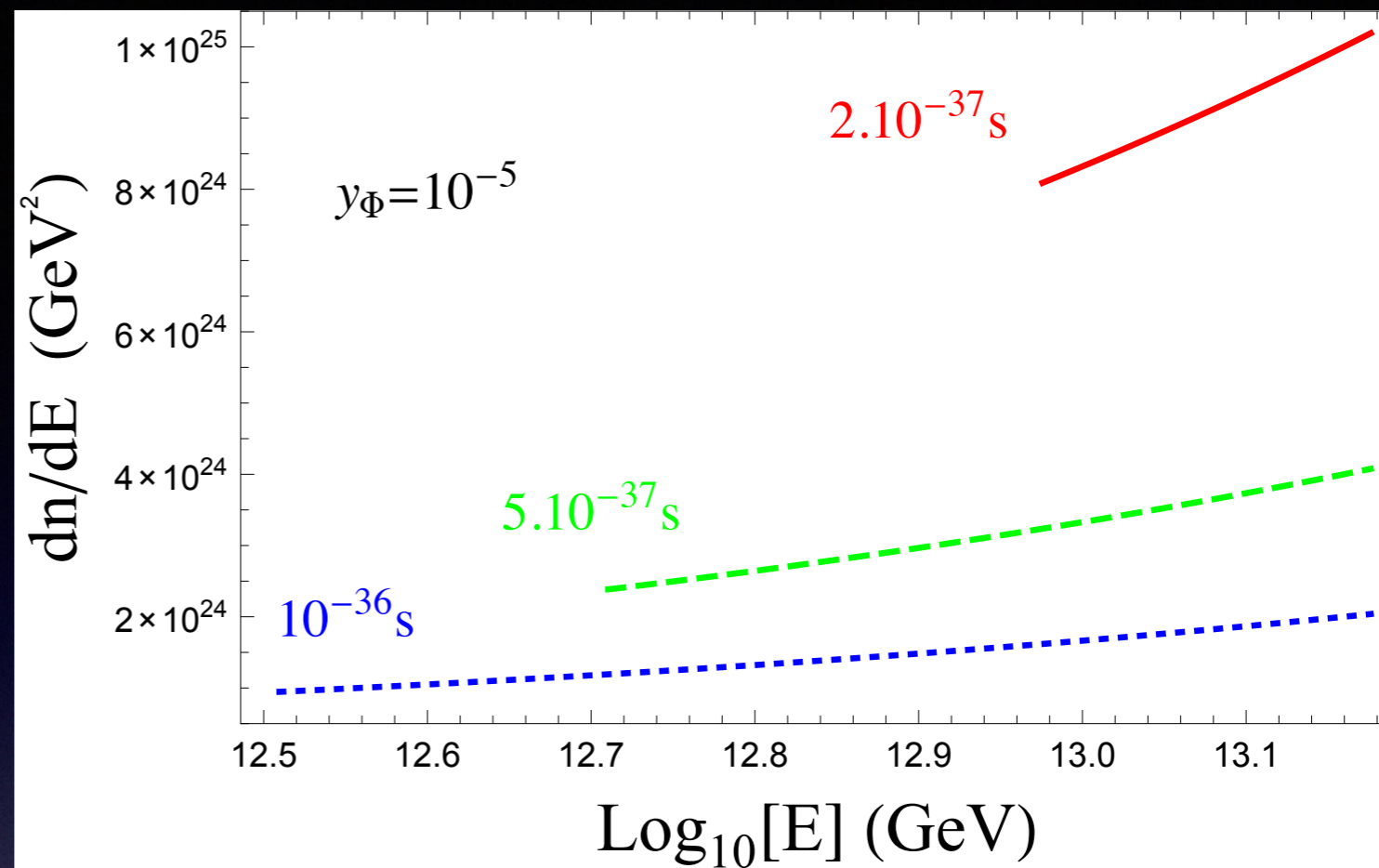
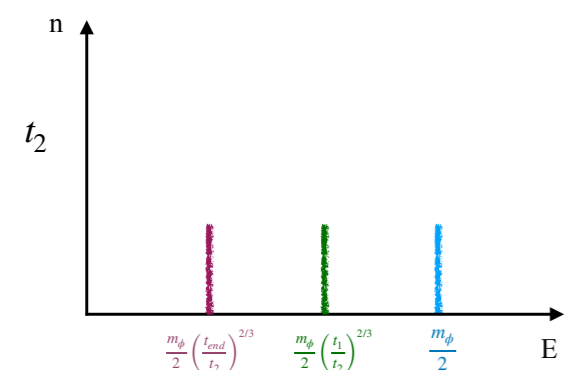
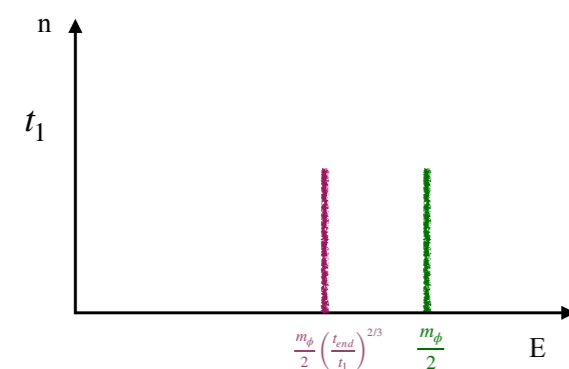
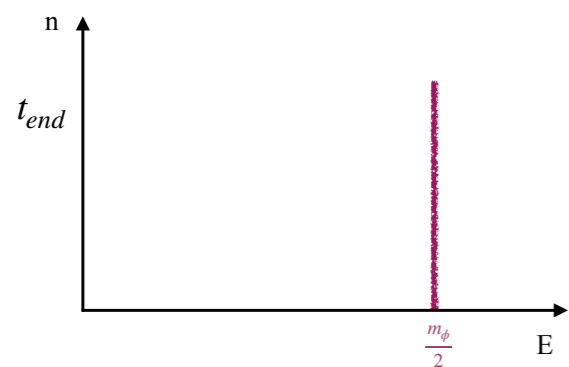
$n\langle\sigma v\rangle \gtrsim H \Rightarrow \text{thermal equilibrium}$

Taking into account a non thermal phase where  $E_R = \frac{m_\Phi}{2}$

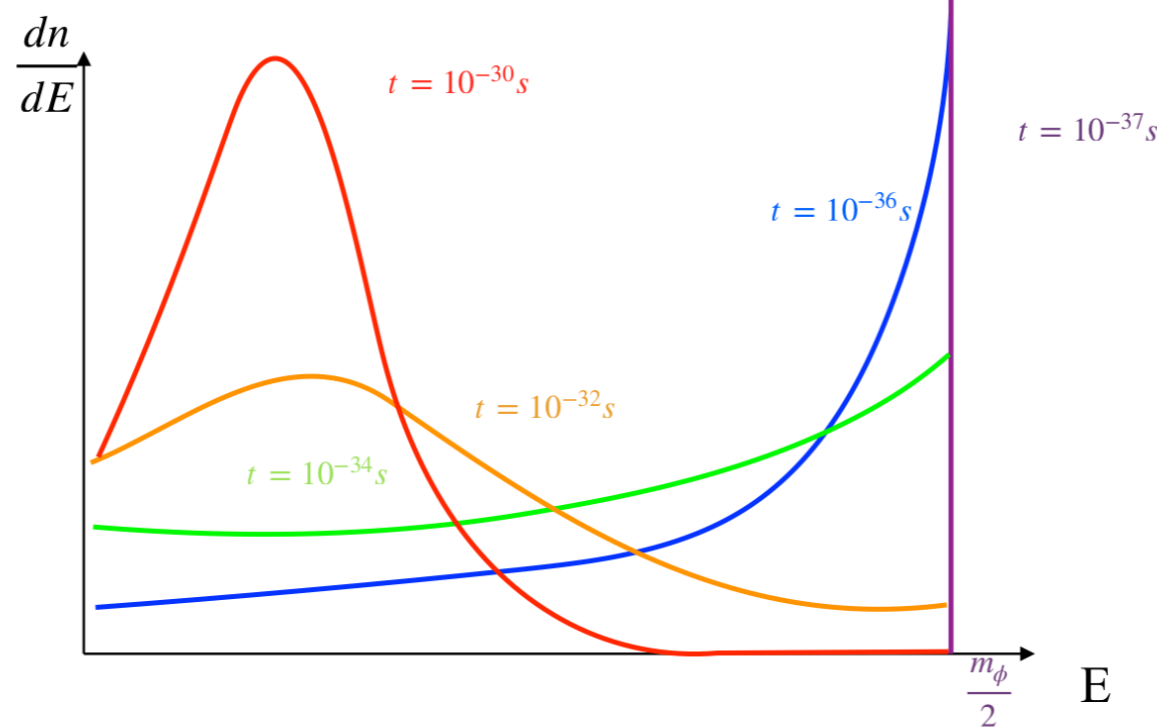
$T_{Eq}$

Scaling factor (a)





$$f(p) = \frac{8\sqrt{2}\pi^2\Gamma_\phi M_P^2}{p^{3/2}m_\phi^{5/2}t}$$



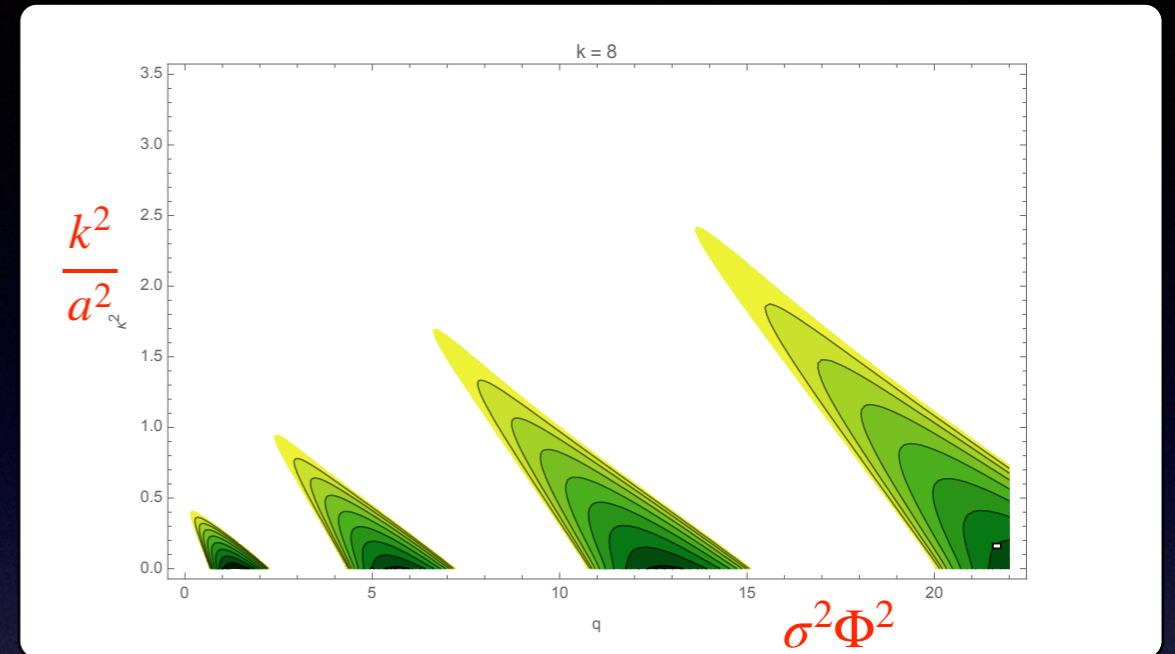
$$f(p) = \frac{1}{e^{\frac{p}{kT}} \pm 1}$$

Before even thermalization, there exists a non-perturbative phase, where we observe an explosive resonant production of particles through the parametric (or tachyonic) resonance, highly dependent on the inflationary potential.

$$\mathcal{L} = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} m_\phi^2 \Phi^2 + \sigma |\phi|^2 |S|^2$$

$$\Rightarrow \ddot{S}_k + 3H\dot{S}_k + \left( \frac{k^2}{a^2} + \sigma^2 \Phi^2 \sin^2(m_\Phi t) \right) S_k = 0$$

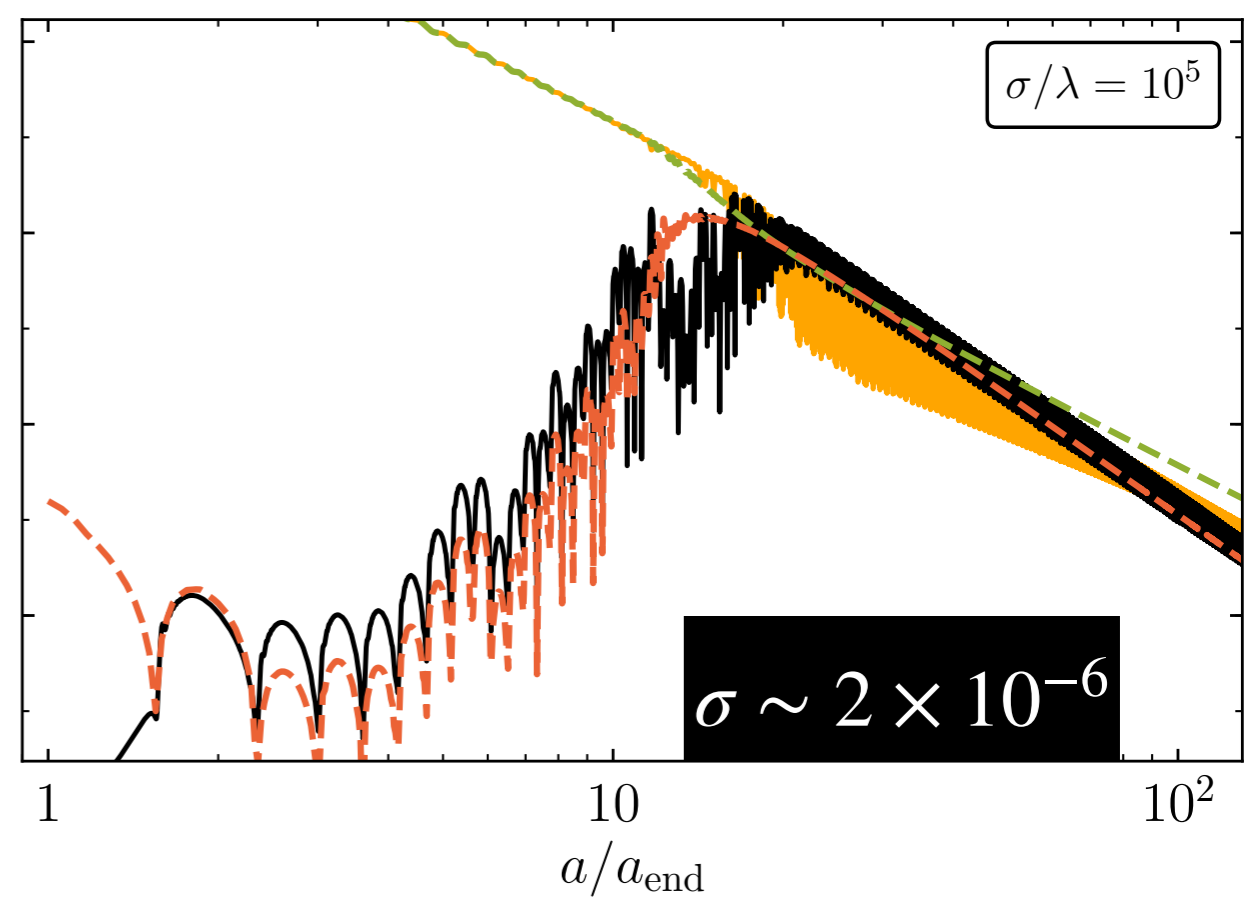
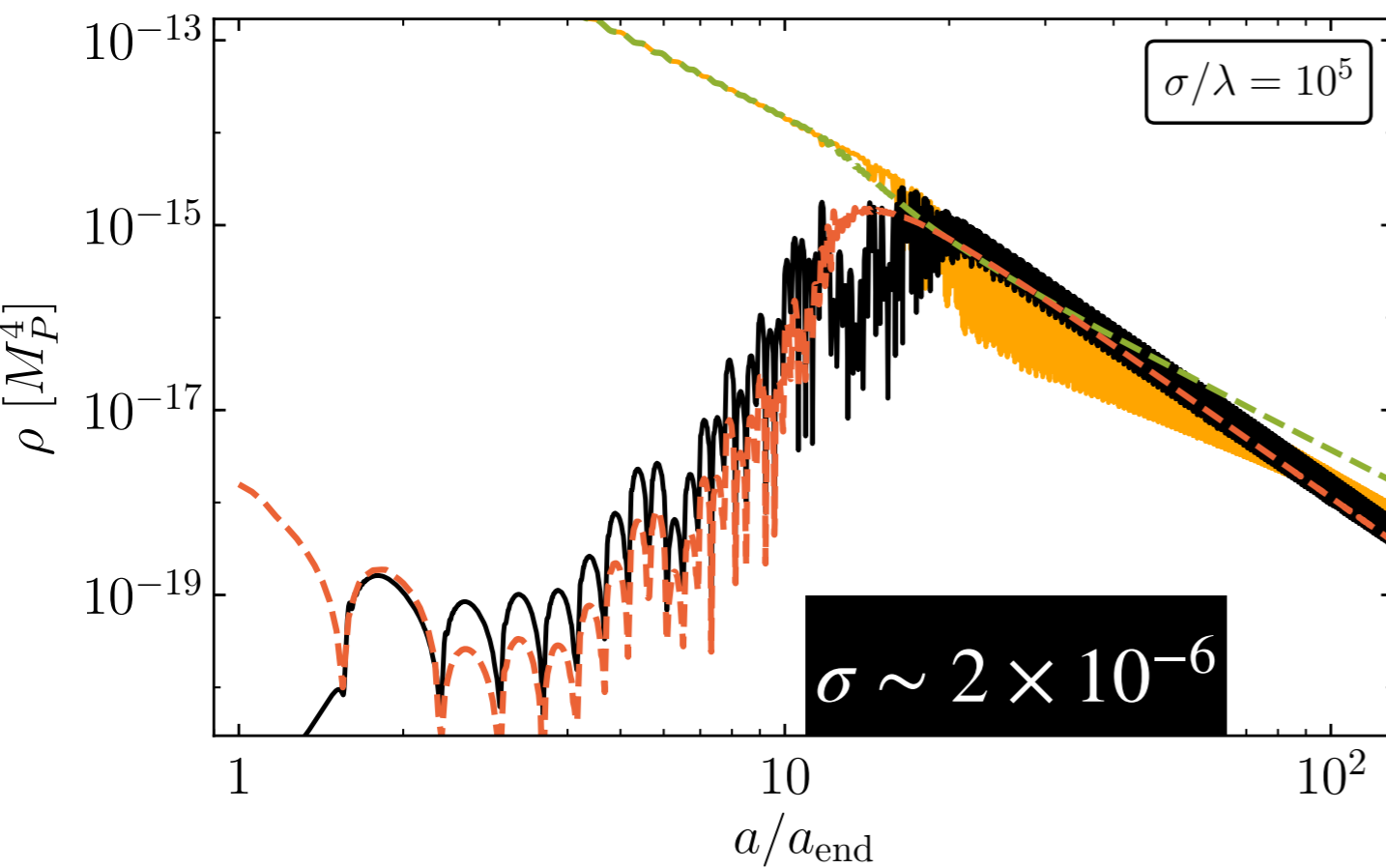
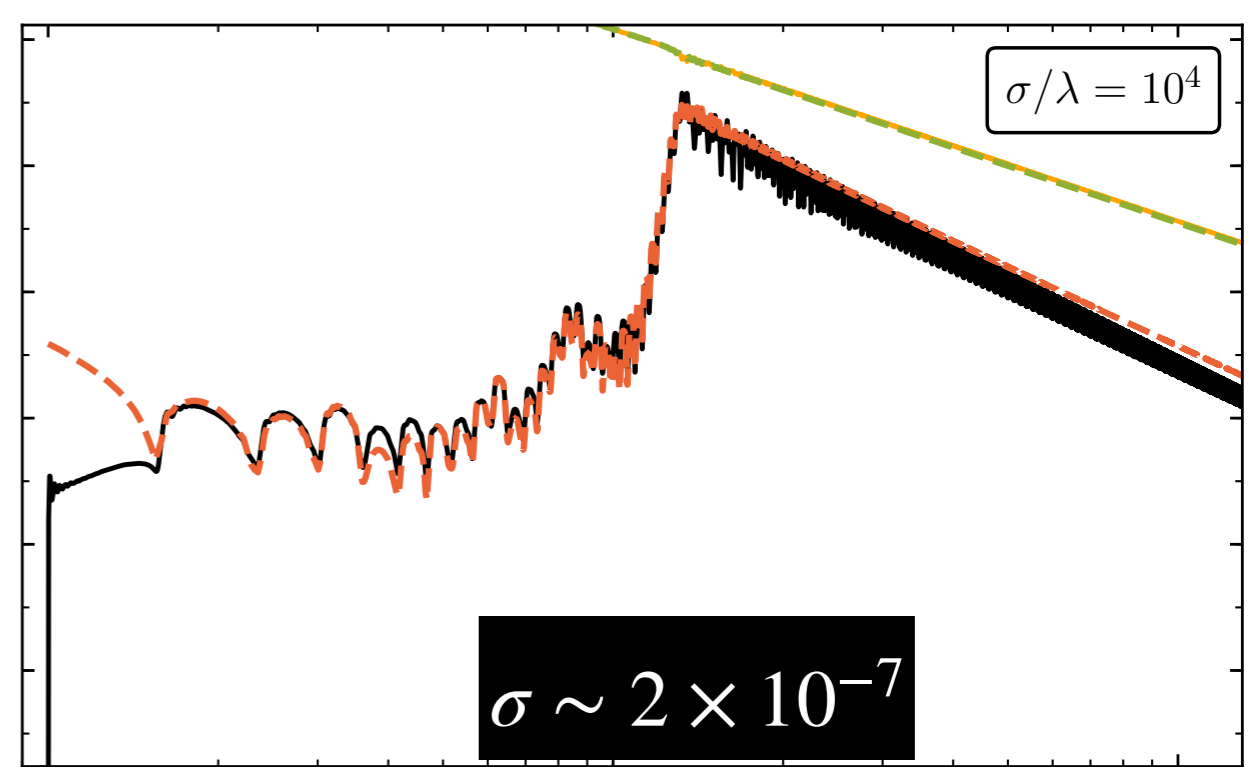
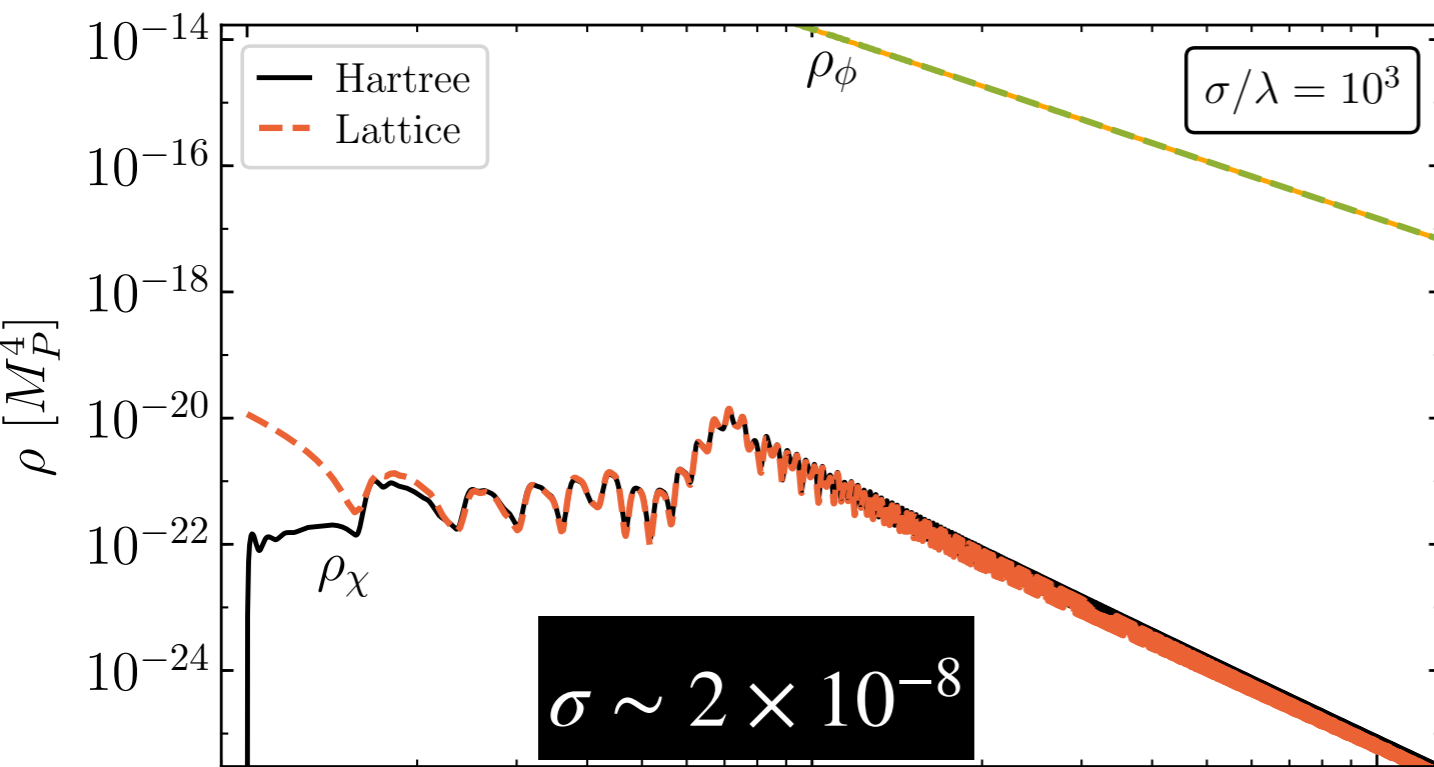
Mathieu equation.  $\Rightarrow$  résonance bands

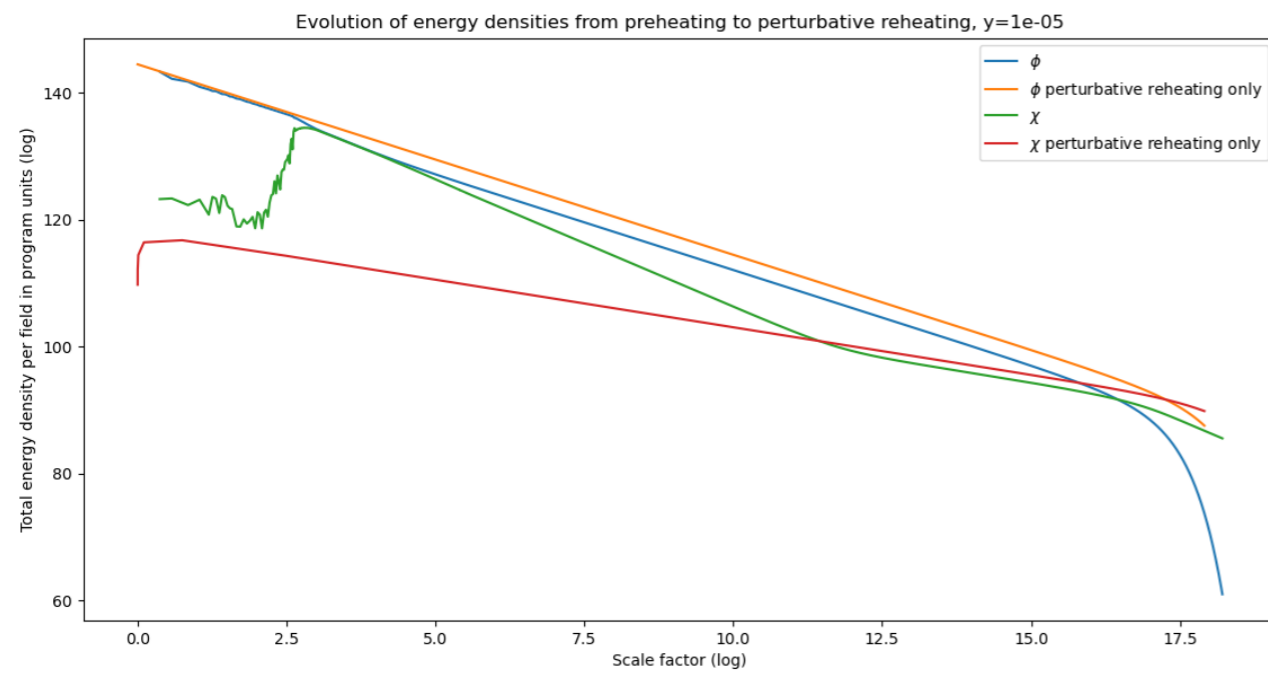


UNIVERSITY OF  
CAMBRIDGE

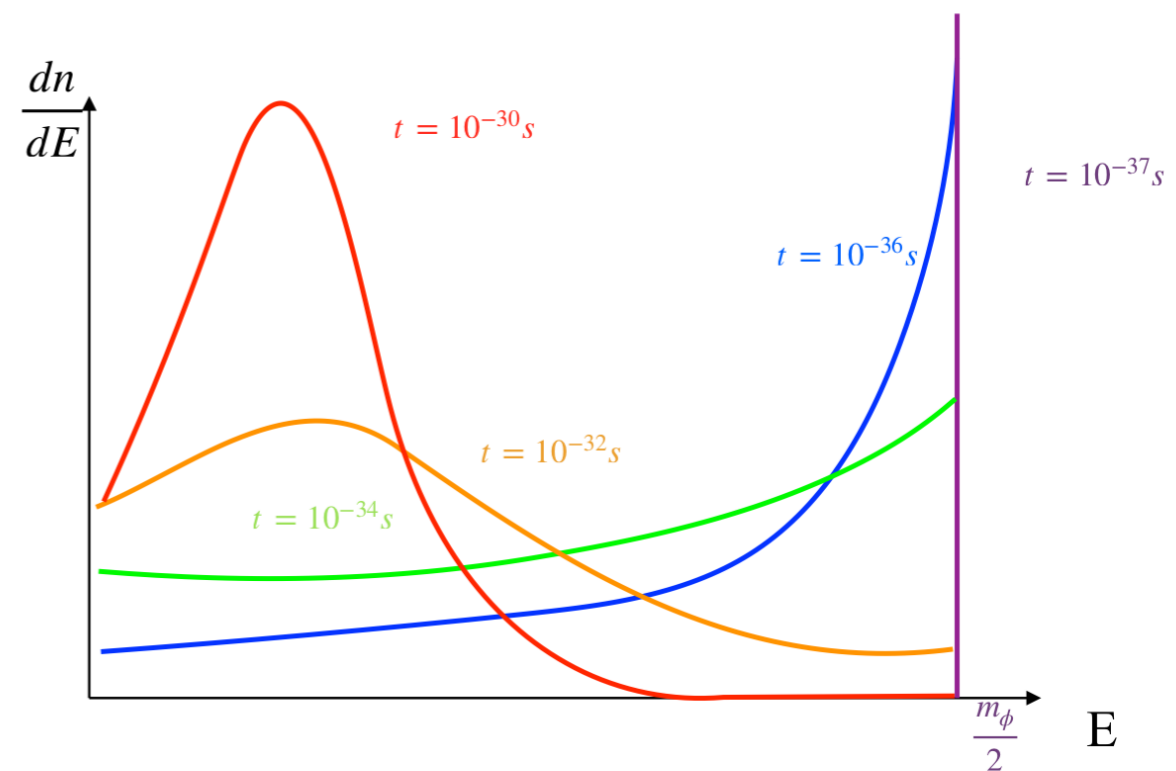


Cambridge Centre for  
Smart Infrastructure  
and Construction





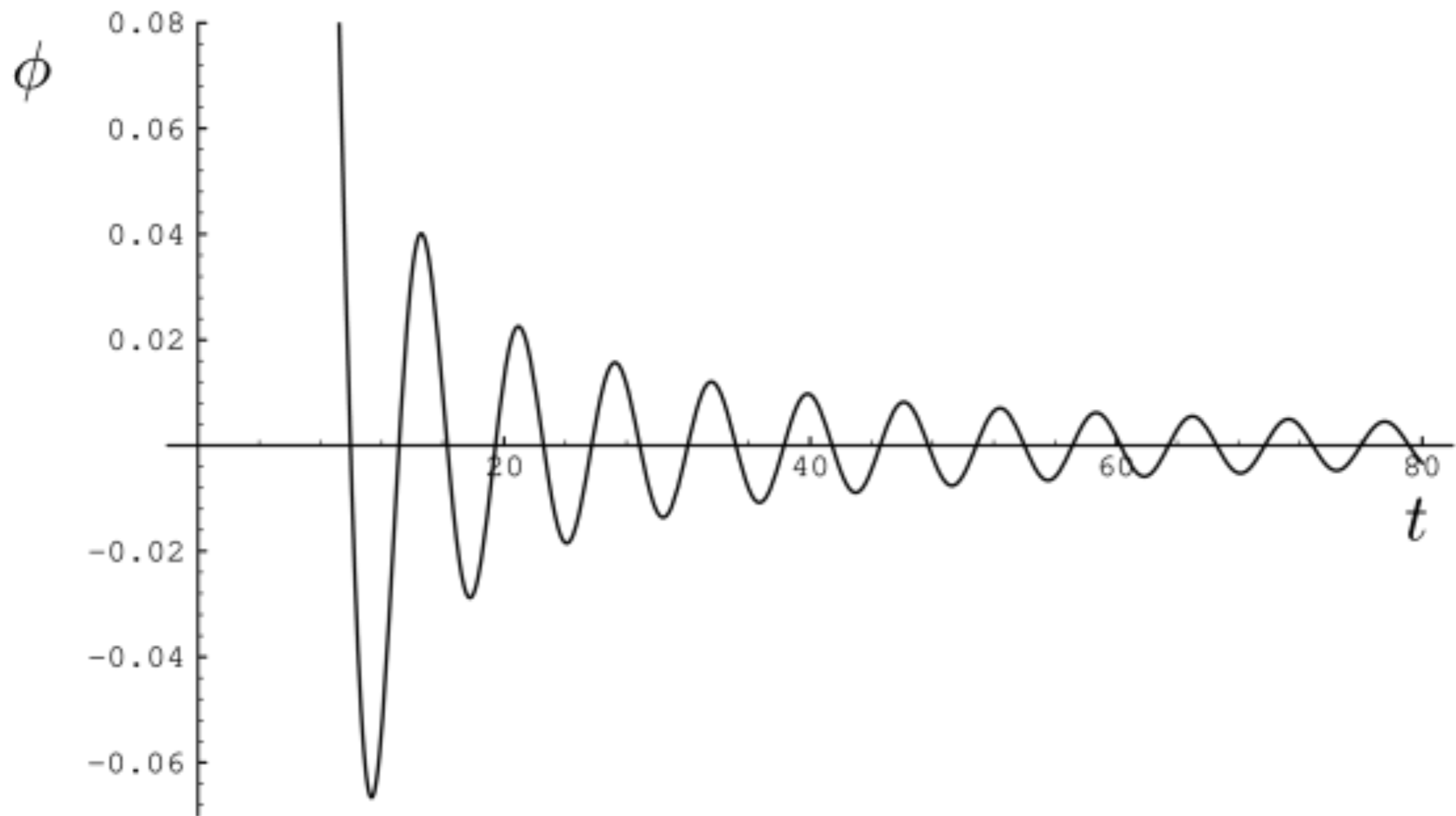
Preheating  
+  
thermalization...



Backreaction effect

# Application to DM 1 : Wimpflation

$$V(\phi) = \frac{1}{2}m_\phi^2 + \lambda\phi^4 + \sigma\phi^2 H^2$$



Application to DM 2 :  
Gravitational scattering

$$\frac{d\rho_\Phi}{dt} + 3H\rho_\Phi = -\Gamma_\Phi\rho_\Phi \quad [\text{inflaton } \Phi]$$

$$\frac{d\rho_R}{dt} + 4H\rho_R = +\Gamma_\Phi\rho_\Phi \quad [\text{radiation } R]$$

$$\longrightarrow \quad \frac{dn_\chi}{dt} + 3Hn_\chi = \boxed{R(T)} \quad [\text{dark matter } \chi]$$

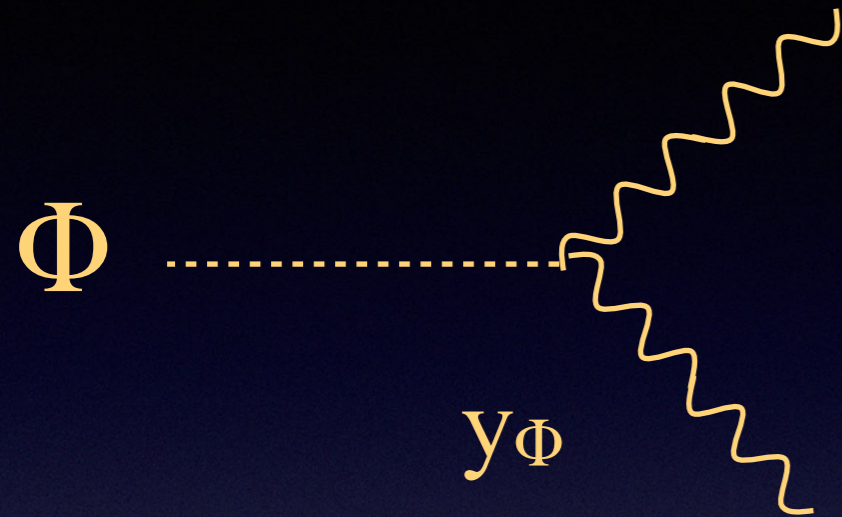
$$H^2 = \frac{\rho_\Phi}{3M_P^2} + \frac{\rho_R}{3M_P^2} \quad [\text{scale } a]$$

$$\Rightarrow \quad H(T) = \frac{5}{6} \frac{\alpha}{\Gamma_I M_P^2} T^4$$

$$[H(T) = \frac{\alpha}{3M_P} T^2 \text{ in radiation dominated universe}]$$

$$\boxed{\frac{dY_\chi}{dT} = -\frac{8}{3} \frac{\boxed{R(T)}}{H T^9} \quad \text{with } Y_\chi = \frac{n_\chi}{T^8}}$$

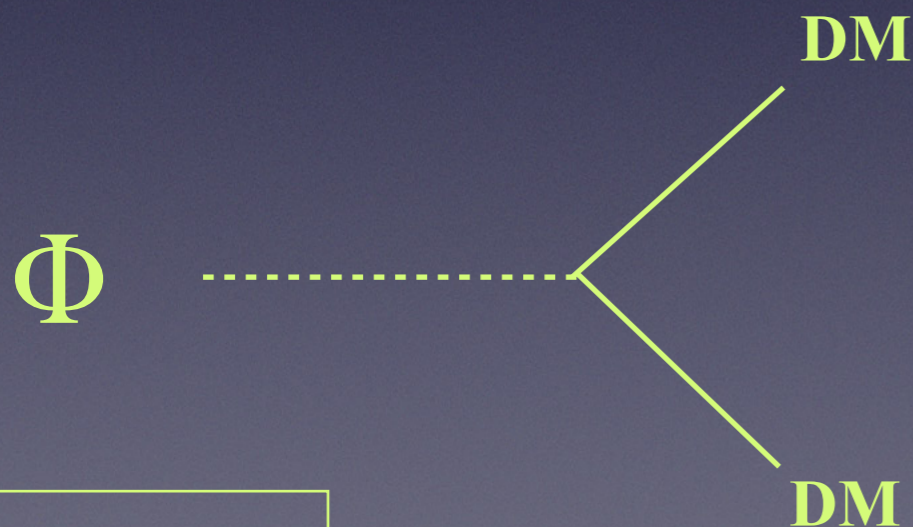
# Another DM source: The inflaton decay



$T_{RH}$

$$T_{RH} = \left( \frac{10}{g_s} \right)^{1/4} \left( \frac{2\Gamma_\phi M_P}{\pi c} \right)^{1/2} = 0.55 \frac{y_\phi}{2\pi} \left( \frac{m_\phi M_P}{c} \right)^{1/2}$$

$$\Omega_{3/2} h^2 \simeq 0.11 \left( \frac{0.1 \text{ EeV}}{m_{3/2}} \right)^3 \left( \frac{m_\phi}{3 \times 10^{13} \text{ GeV}} \right)^{7/2} \left( \frac{y_\phi}{2.9 \times 10^{-5}} \right)^7$$



$\Omega h^2$

$$\Omega h^2 = \left( \frac{B_R}{2.0 \times 10^{-9}} \right) \left( \frac{T_{RH}}{M_\Phi} \right) \left( \frac{M_{DM}}{10^{10} \text{ GeV}} \right)$$

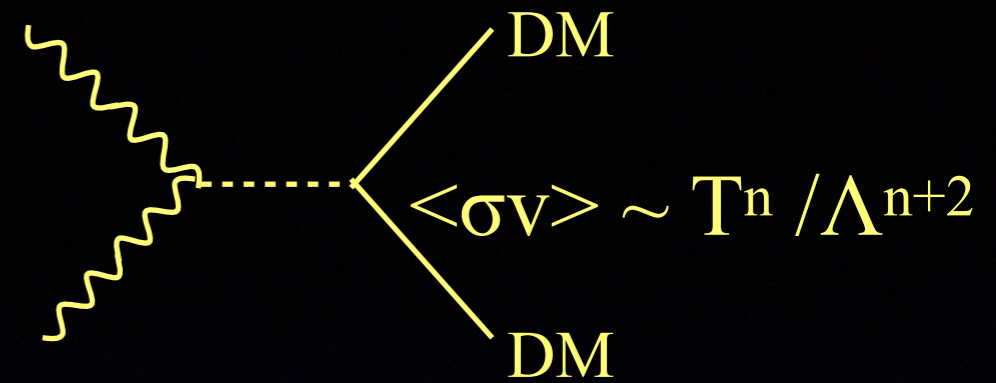
$$B_R = \frac{\Gamma_{\Phi \rightarrow DM \text{ DM}}}{\Gamma_\Phi}$$

Temperature

mixed universe

$$H \propto T^4$$

$$T = \beta a^{-3/8}$$



$T_{RH}$

DM production

10% if  $\langle \sigma v \rangle \sim T^2 / \Lambda^4$

50% if  $\langle \sigma v \rangle \sim T^4 / \Lambda^6$

99.996% if  $\langle \sigma v \rangle \sim T^6 / \Lambda^8$

Radiative universe

$$H \propto T^2$$

$$T = \beta a^{-1}$$

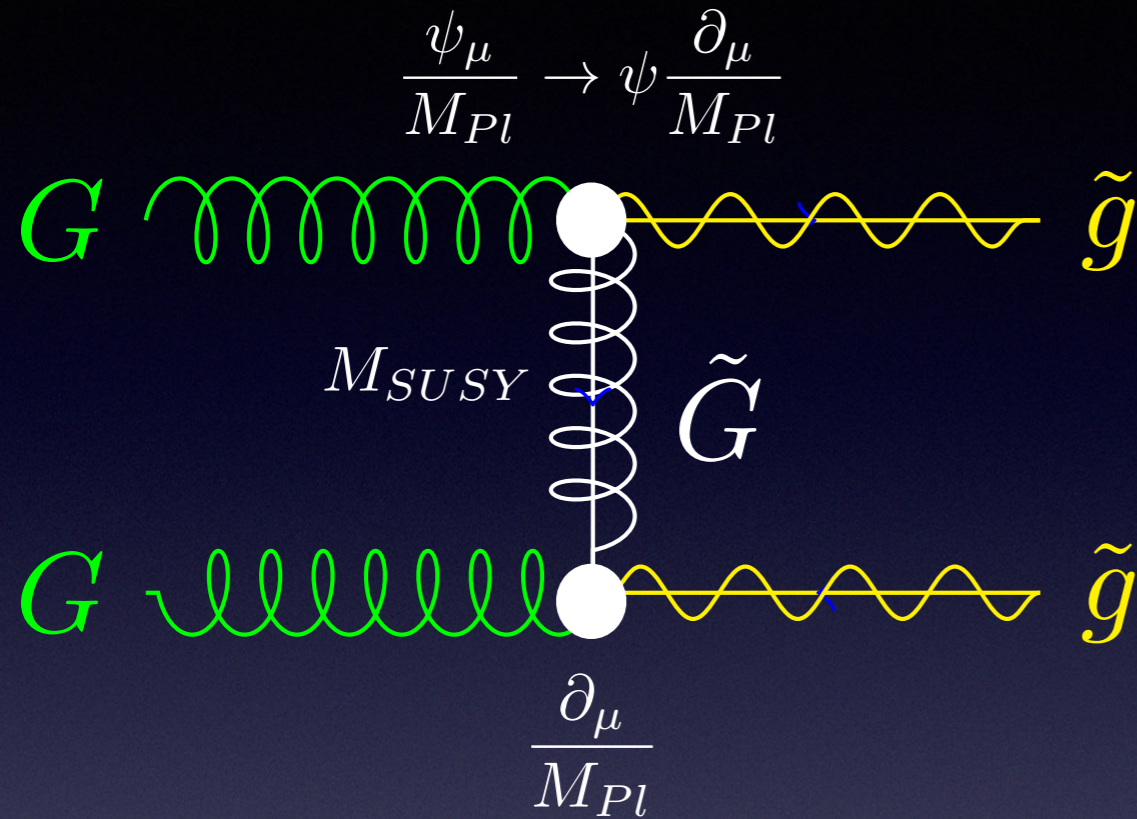
inflaton

inflaton

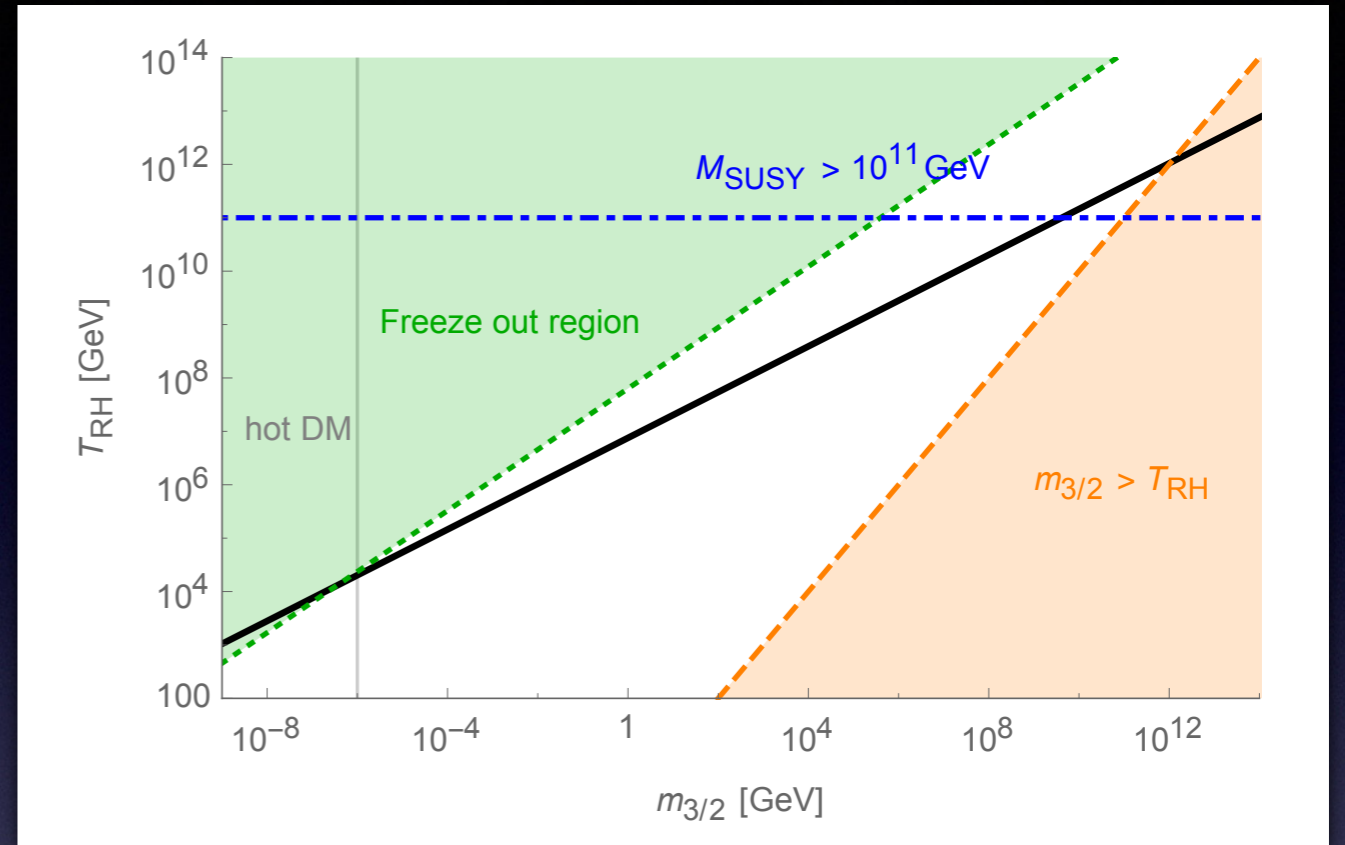
time

# Models

$$R(T) = \frac{T^{12}}{M_{SUSY}^4 M_{Pl}^4}$$



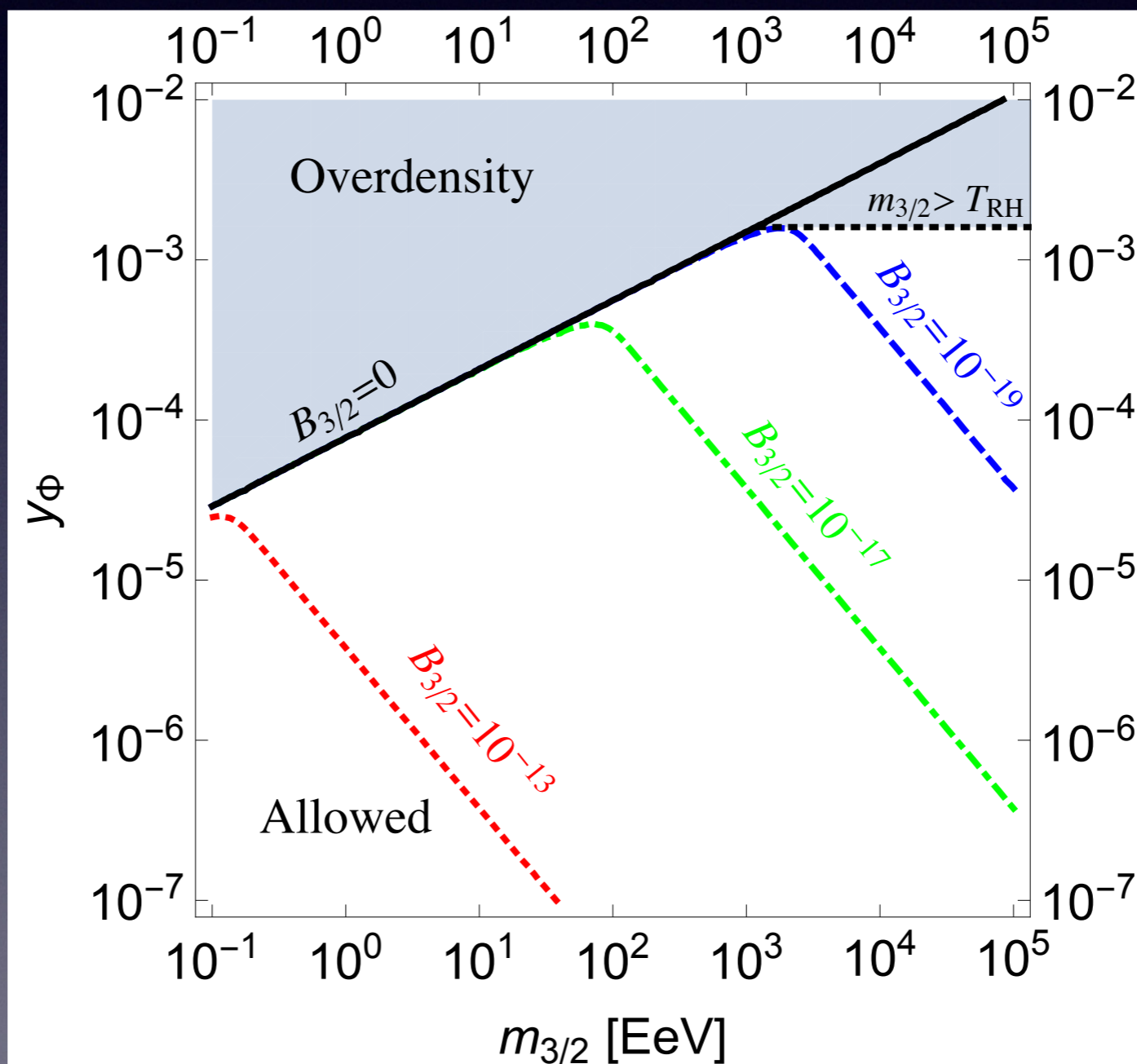
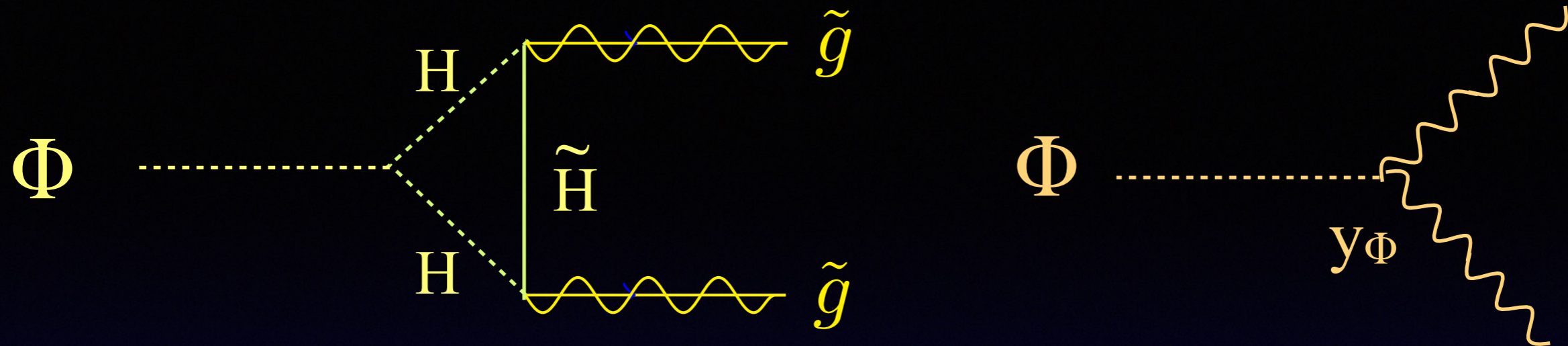
# High scale supergravity



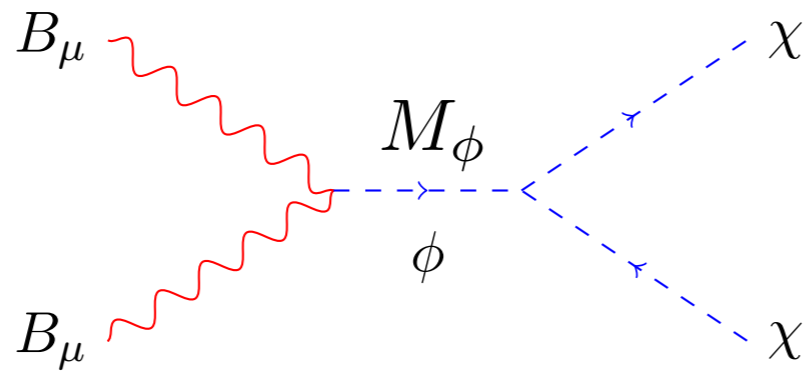
$$\Omega_{3/2} h^2 \simeq 0.11 \left( \frac{0.1 \text{ EeV}}{m_{3/2}} \right)^3 \left( \frac{T_{RH}}{2 \times 10^{10}} \right)^7 \times \frac{56}{5} \ln \left( \frac{T_{max}}{T_{RH}} \right)$$

Not taking into account reheating :  $\Omega_{3/2} h^2 \sim 0.3 \left( \frac{1 \text{ GeV}}{m_{3/2}} \right) \left( \frac{T_{RH}}{10^{10} \text{ GeV}} \right) \sum \left( \frac{m_{\tilde{G}}}{100 \text{ GeV}} \right)^2$

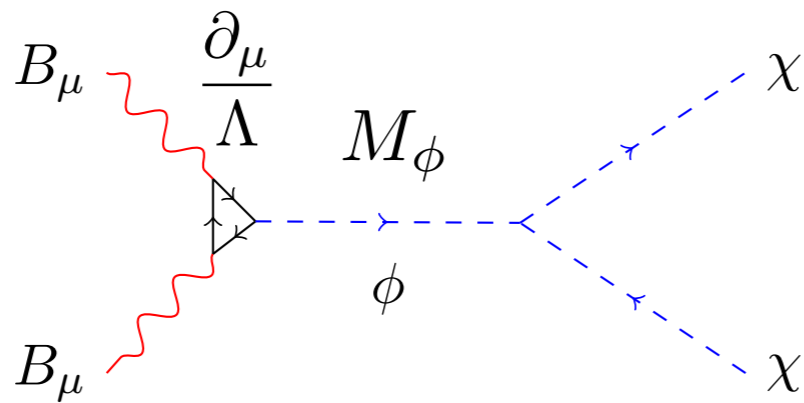
Adding the contribution from  
radiative decay of the inflaton



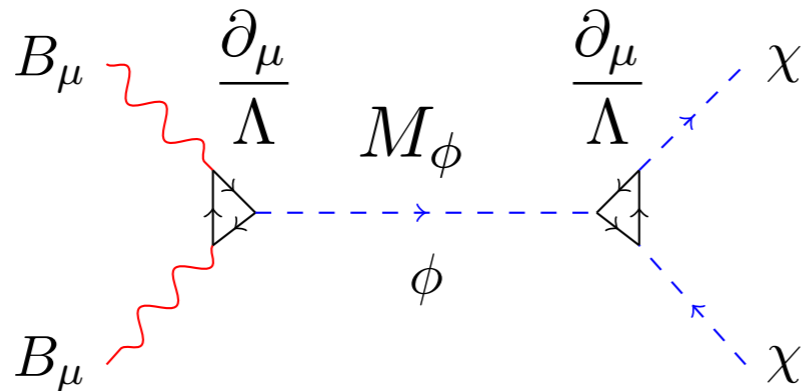
# Example of rates



$$R(T) = \frac{T^8}{M_\phi^4}$$



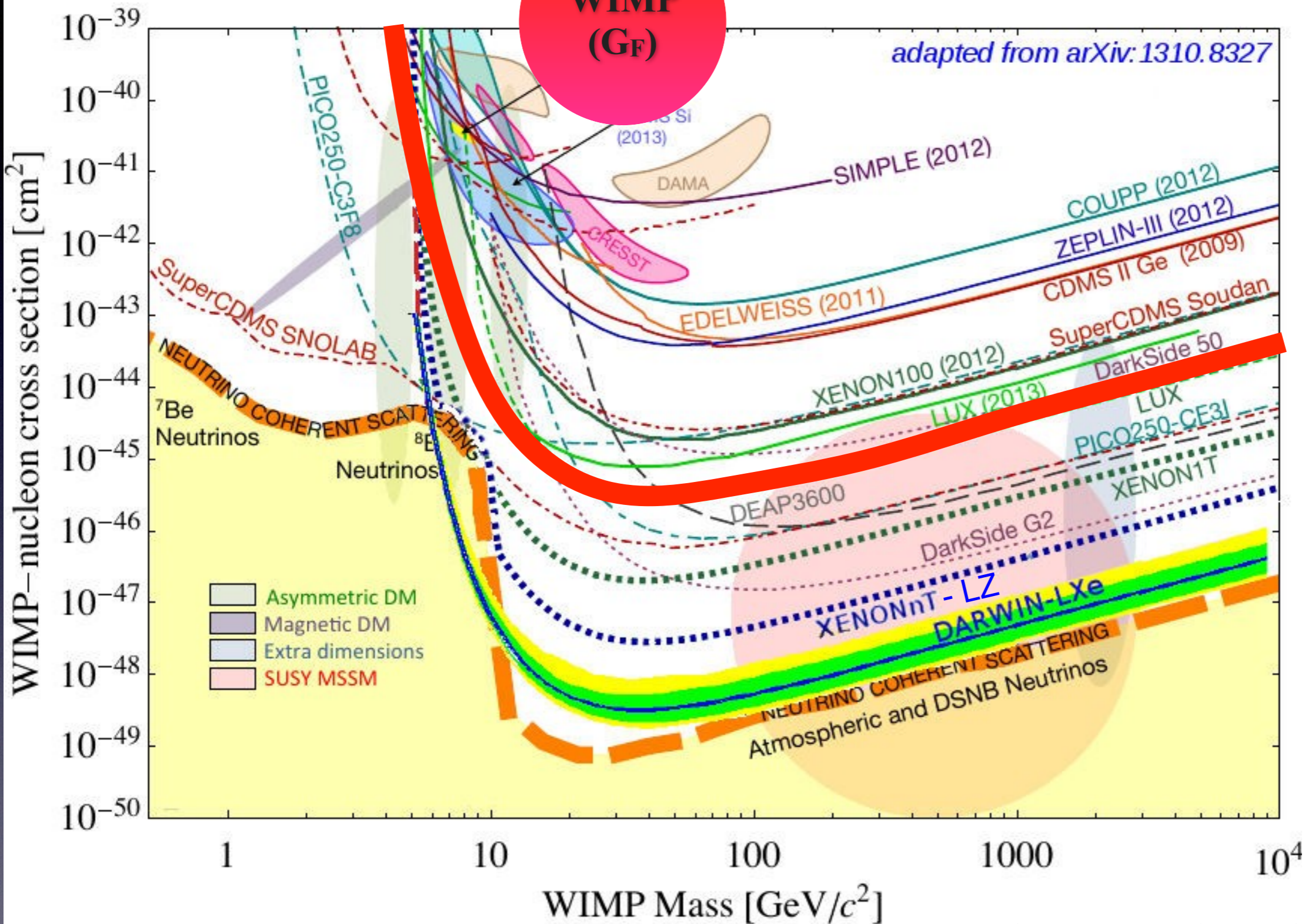
$$R(T) = \frac{T^{10}}{M_\phi^4 \Lambda^2}$$



$$R(T) = \frac{T^{12}}{M_\phi^4 \Lambda^4}$$

# Perspectives

WIMP  
( $G_F$ )



$$g_1 = 0.46; \quad g_2 = 0.65; \quad g_3 = 1.22; \quad \sigma_{dm} \lesssim 10^{-37} \times 10^{-10} \text{ cm}^2 \Rightarrow g_{dm} \lesssim 10^{-5}$$

$$\text{or } g_{dm} \simeq 1 \text{ and } M_{med} \gtrsim 2 \text{ TeV}$$