

QCD in the cores of neutron stars

HOW PQCD CONSTRAINS THE EQUATION OF STATE AT NEUTRON STAR DENSITIES

KOMOLTSEV & AK, PRL128 (2022) 20, 2111.05350

AB-INITIO QCD CALCULATIONS IMPACT THE INFERENCE OF NEUTRON-STAR EQUATION OF STATE

GORDA, KOMOLTSEV & AK, ASTROPHYS.J. 950 (2023), 2204.118

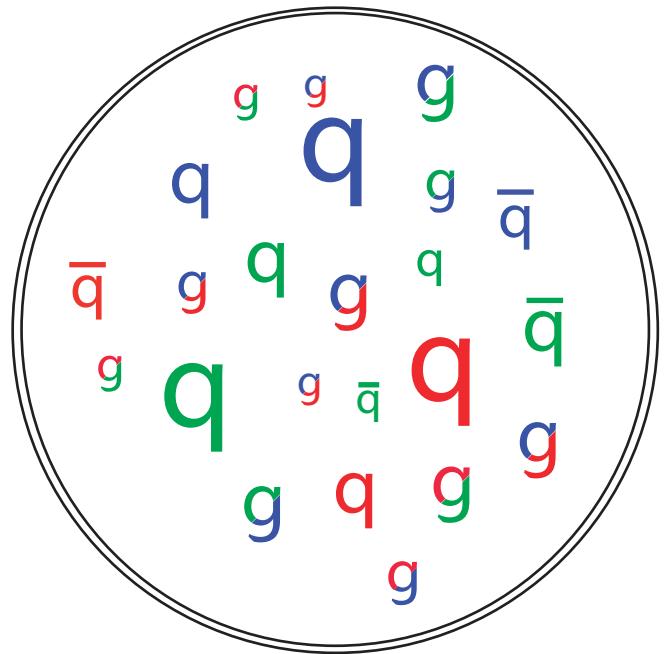
BAYESIAN UNCERTAINTY QUANTIFICATION OF PERTURBATIVE QCD INPUT TO THE NEUTRON-STAR EQUATION OF STATE

GORDA, KOMOLTSEV, AK, MAZELIAUSKAS, JHEP 06 (2023), 2303.02175

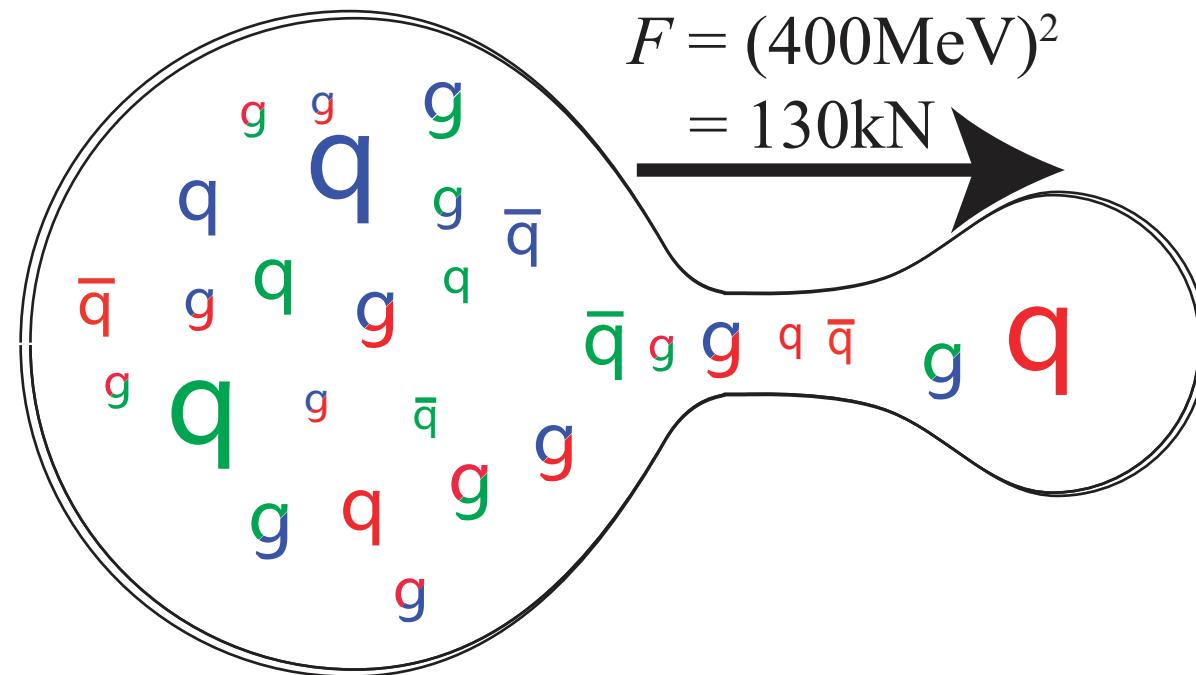
STRONGLY INTERACTING MATTER EXHIBITS DECONFINED BEHAVIOR IN MASSIVE NEUTRON STARS

ANNALA, GORDA, HIRVONEN, KOMOLTSEV, AK, ET AL 2303.11356

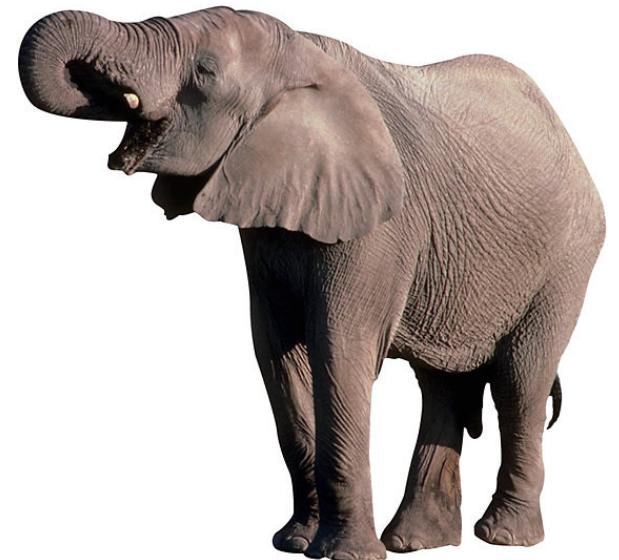
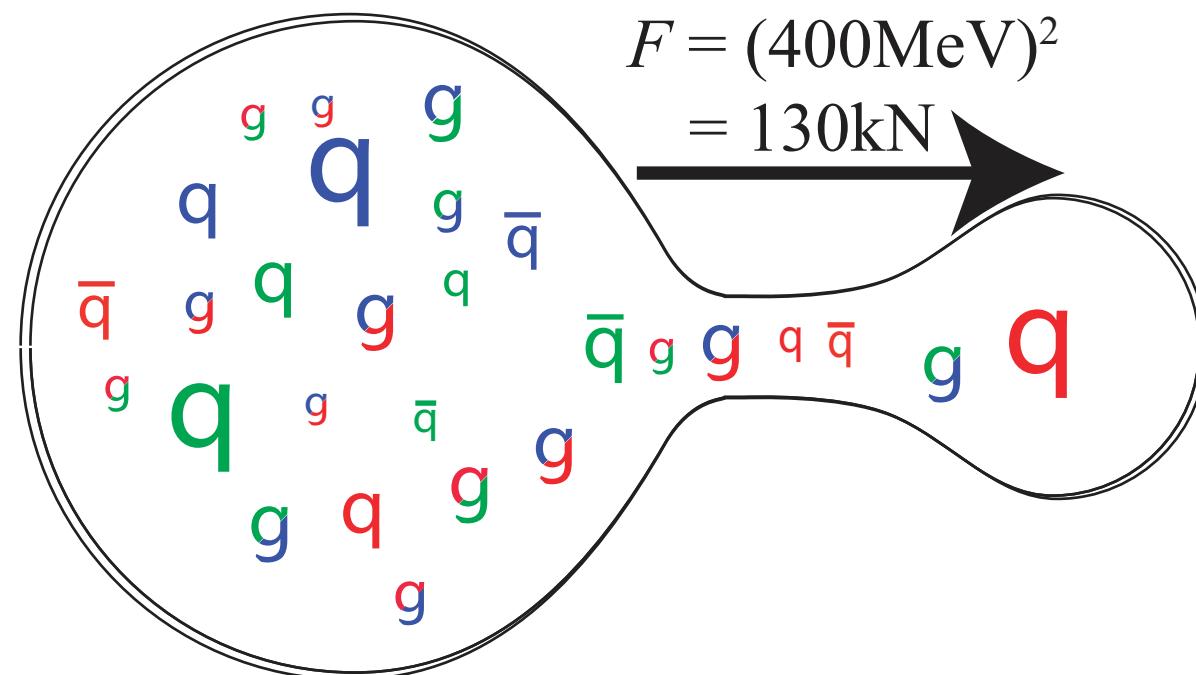
Nuclear substructure



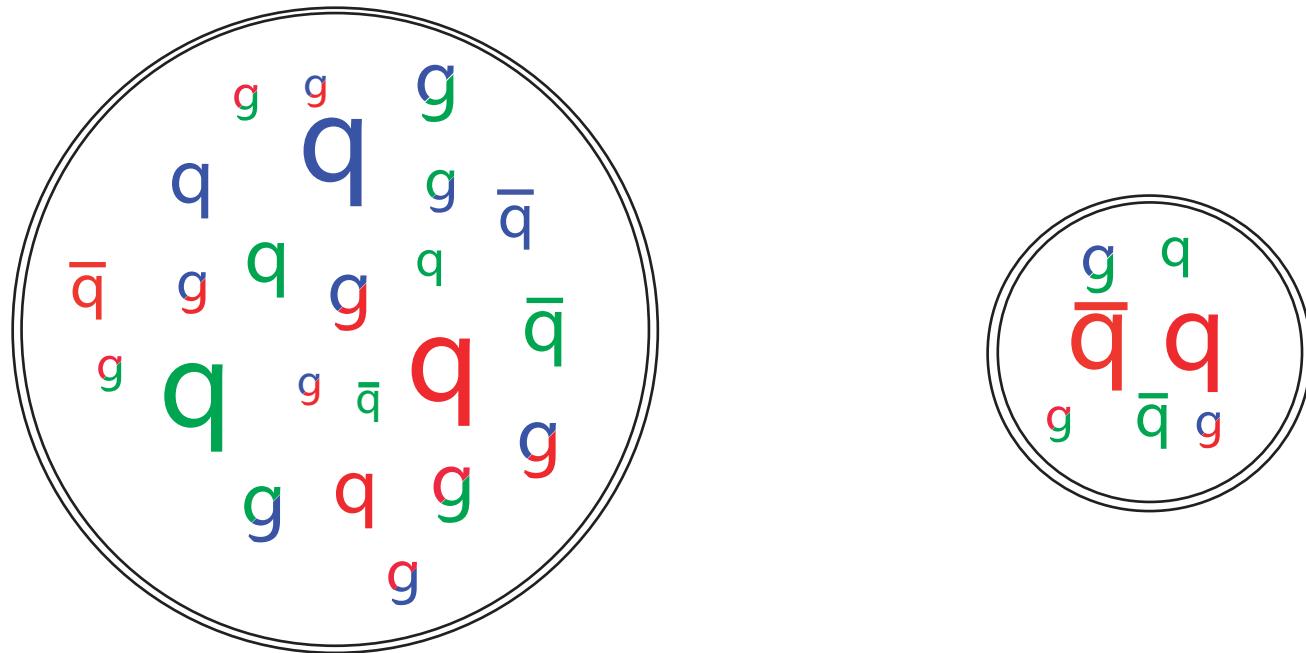
Nuclear substructure



Nuclear substructure

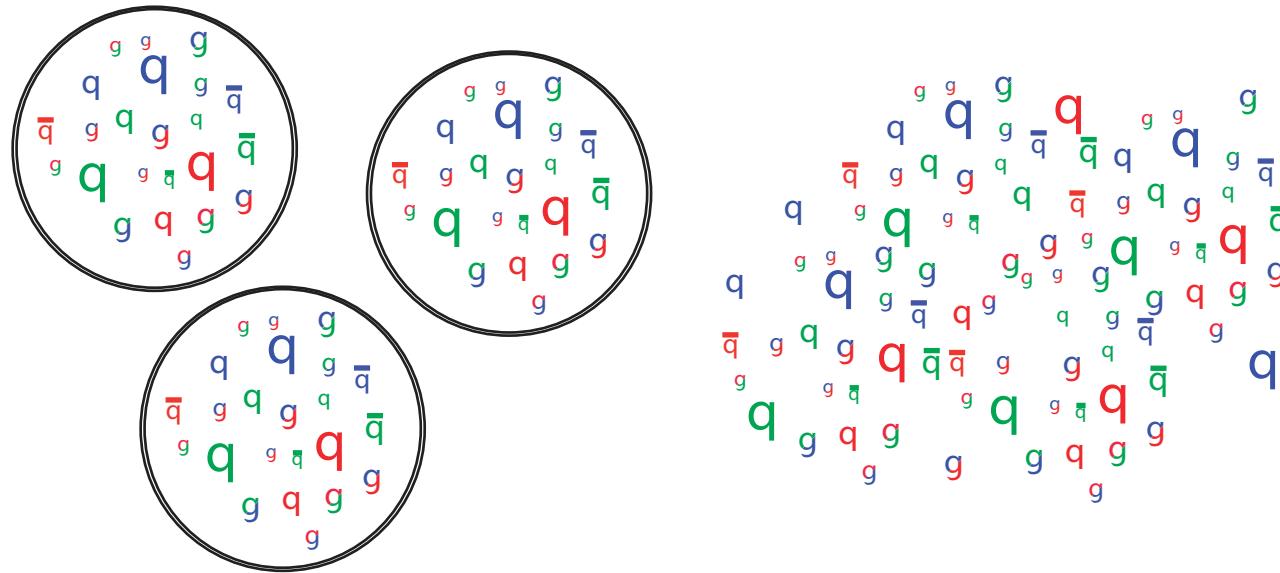


Nuclear substructure



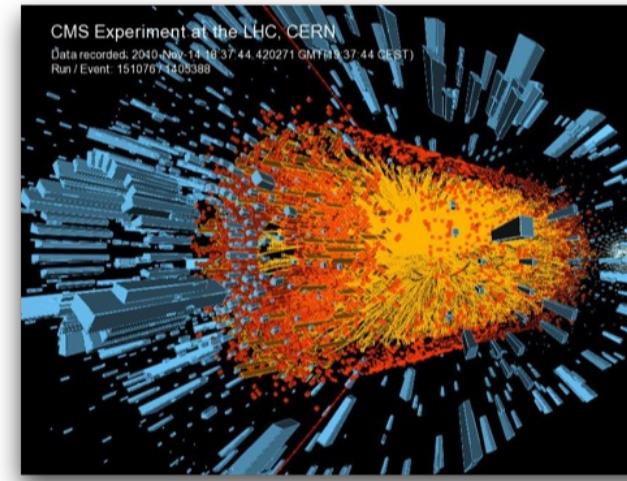
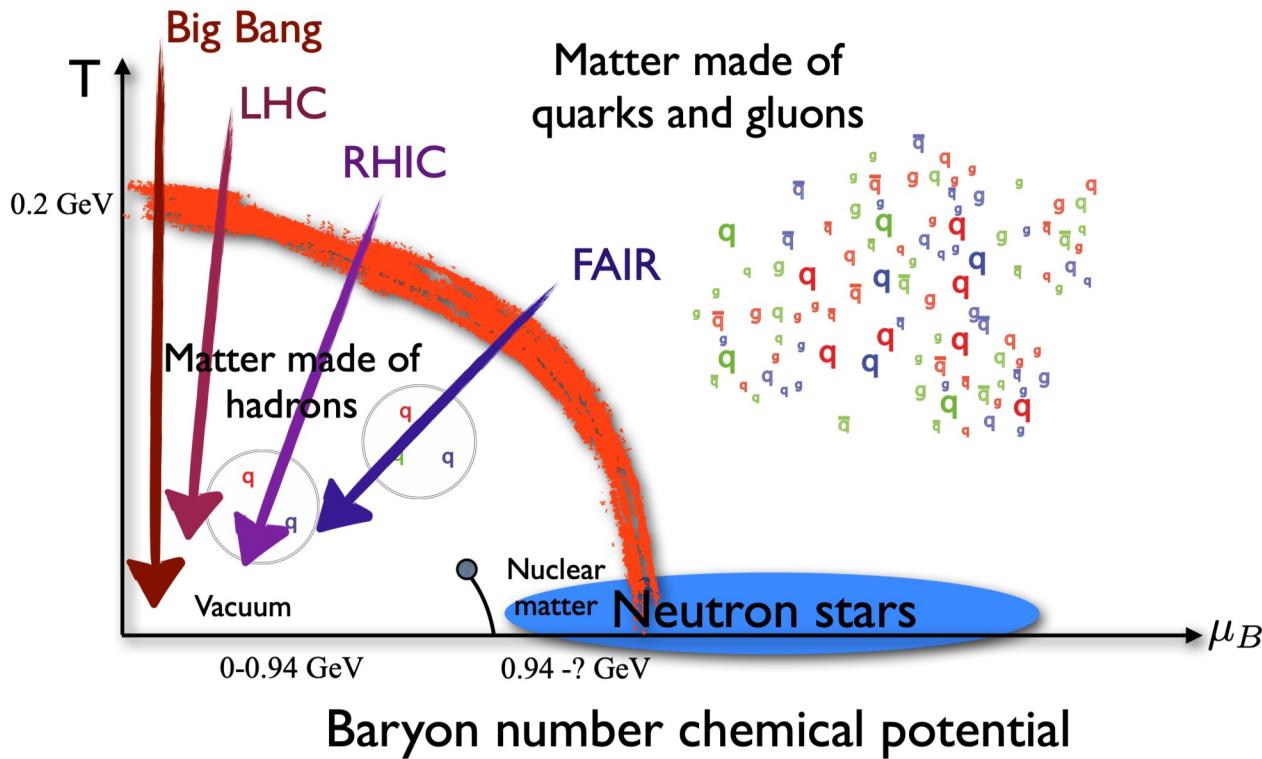
Quarks and gluons are confined into hadrons

Nuclear substructure



... unless the conditions get really extreme

Elementary particle matter

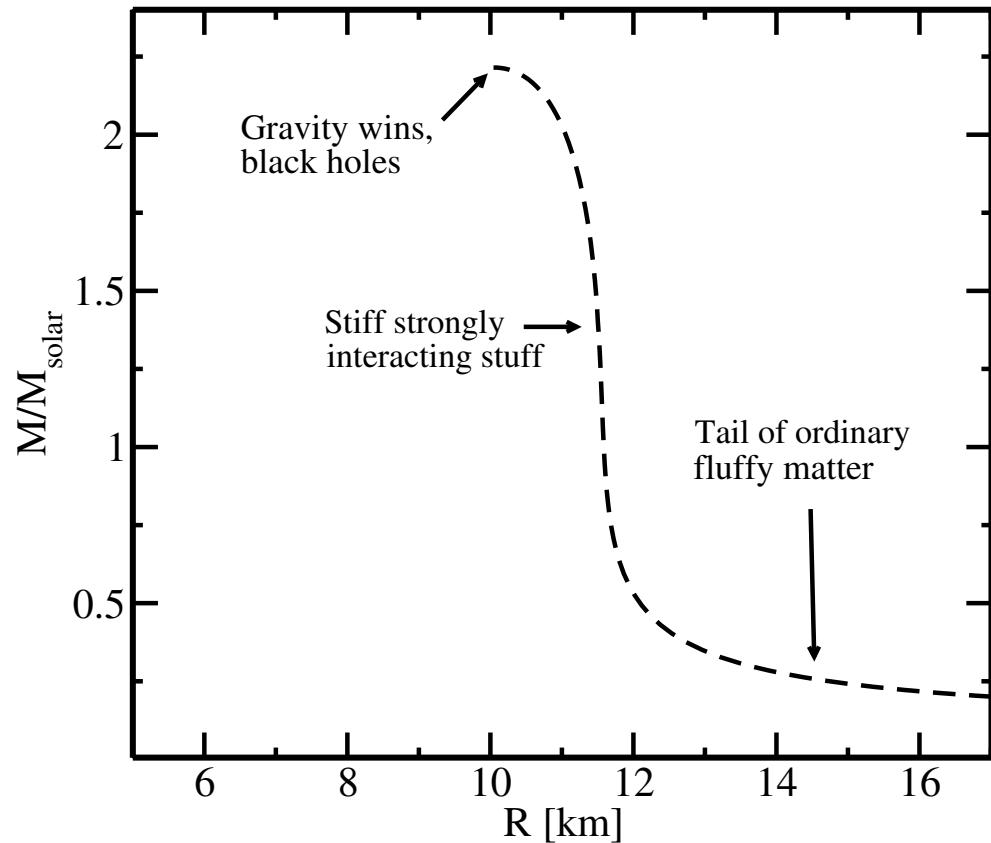


LHC, RHIC, FAIR, EIC,...



LIGO+Virgo+Kagra, NICER, eXTP,...

Properties of neutron stars reflect properties of dense matter



Competition between pressure and gravity

$$\frac{dP}{dr} = -\frac{G\epsilon(r)M(r)}{r^2} \left[1 + \frac{P(r)}{\epsilon(r)} \right] \left[1 + \frac{4\pi r^3 P(r)}{M(r)} \right] \left[1 - \frac{2GM(r)}{r} \right]^{-1}$$
$$\frac{dM}{dr} = 4\pi r^2 \epsilon(r)$$

Macroscopic properties determined by the EoS

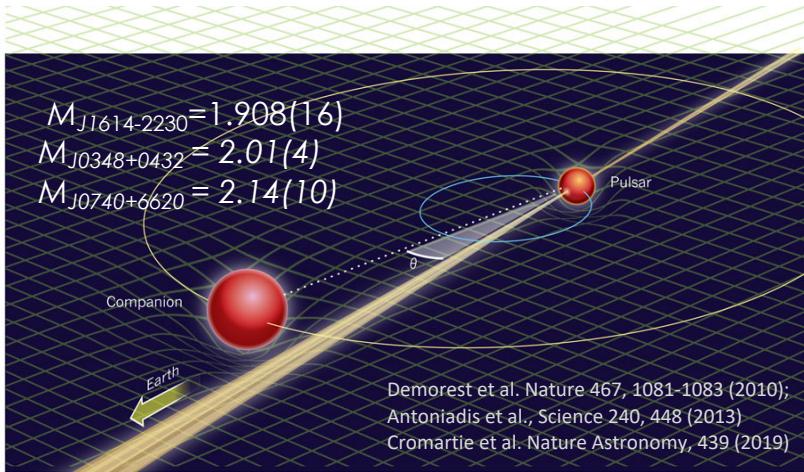
$$\epsilon(P) \Leftrightarrow R(M)$$

EOS reflects the microscopic structure,
phase diagram

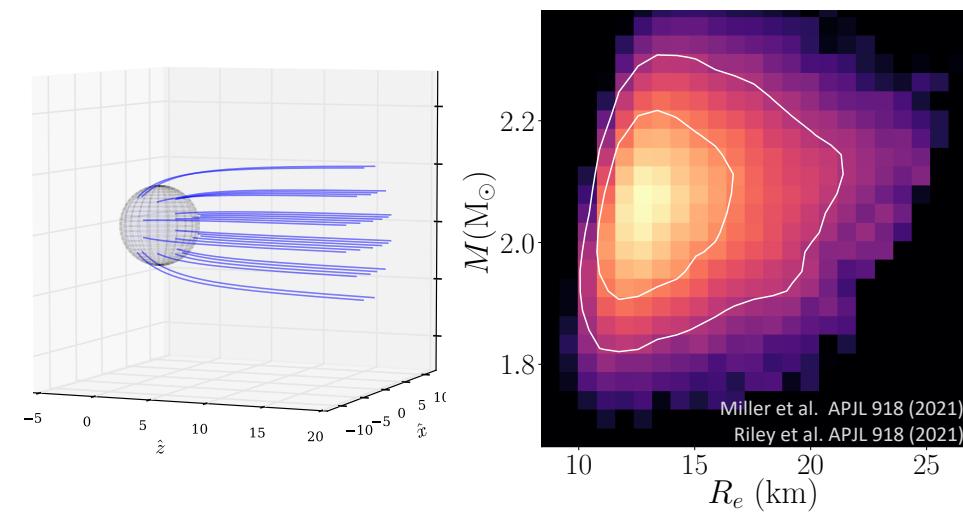
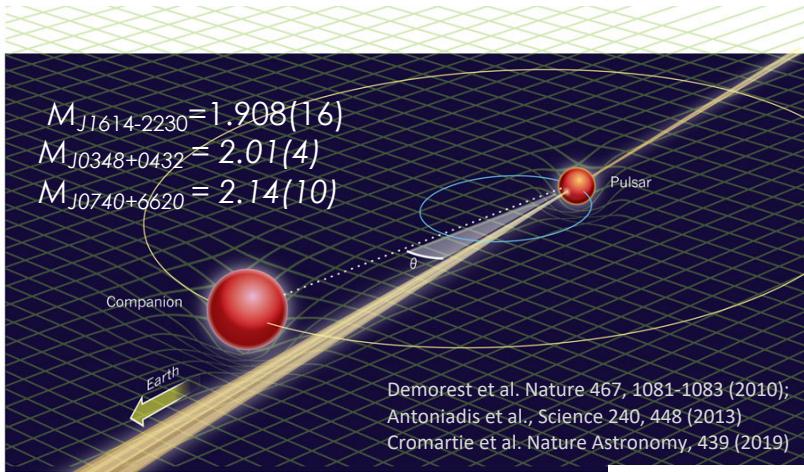
Properties of neutron stars reflect properties of dense matter



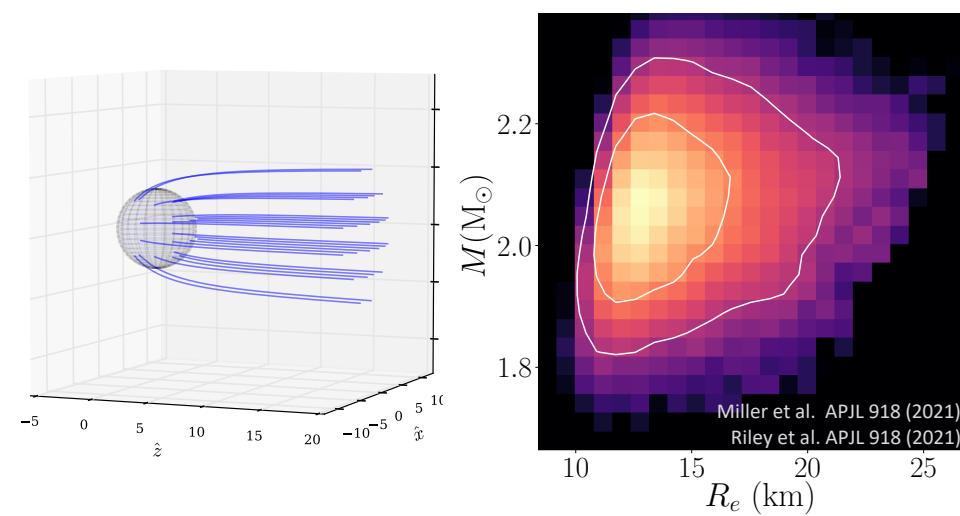
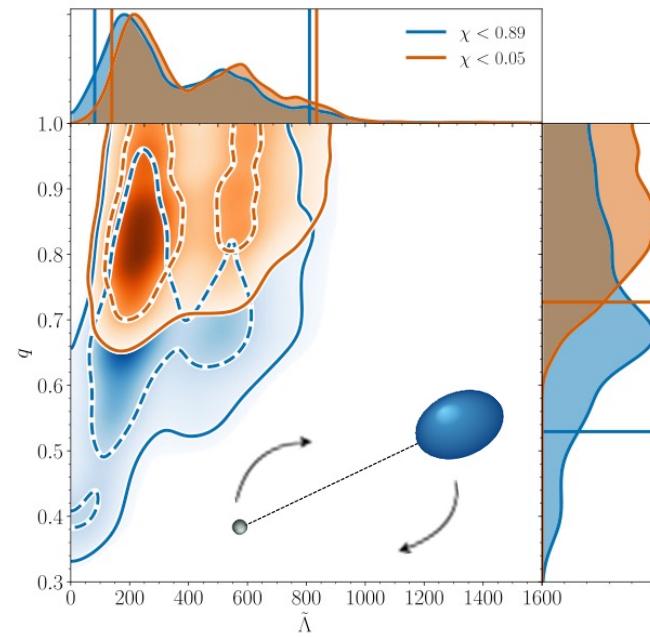
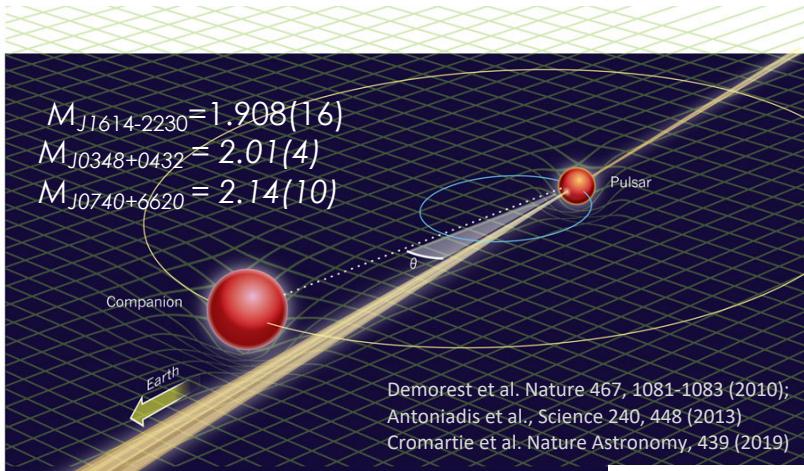
Properties of neutron stars reflect properties of dense matter

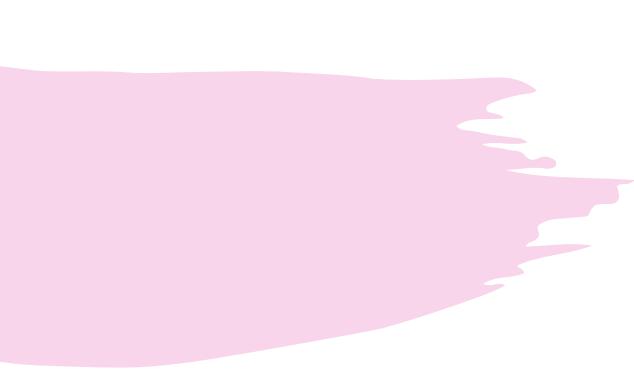


Properties of neutron stars reflect properties of dense matter



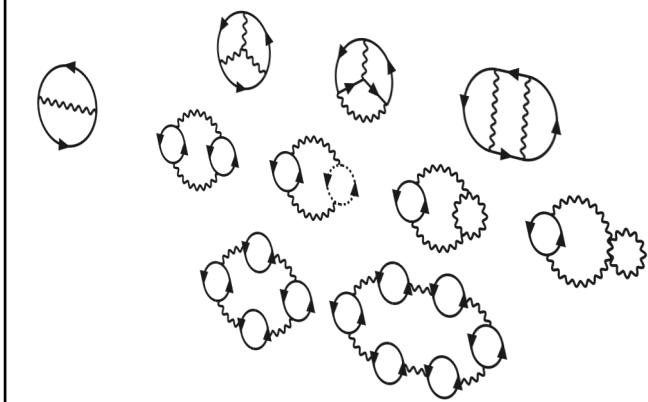
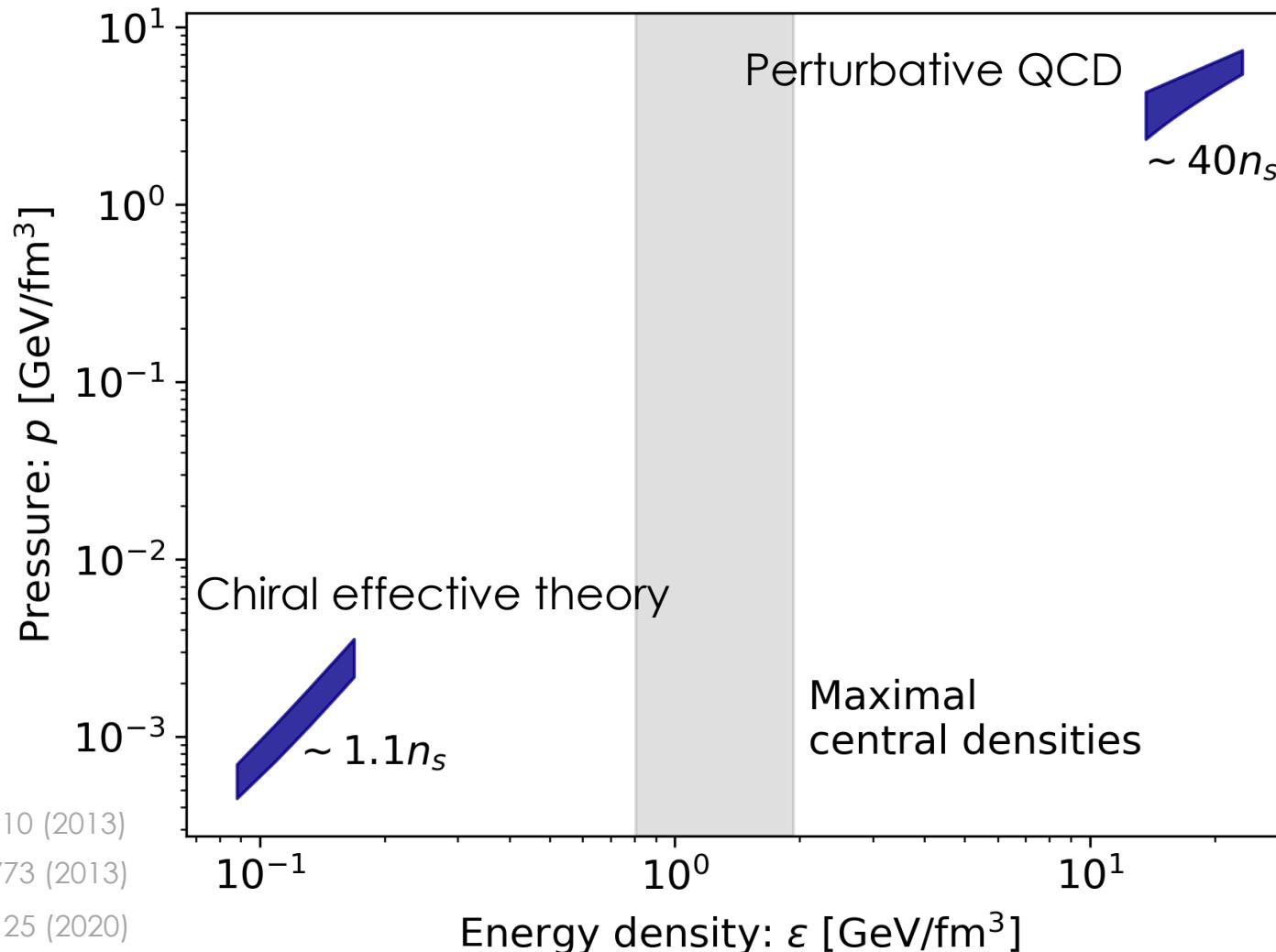
Properties of neutron stars reflect properties of dense matter





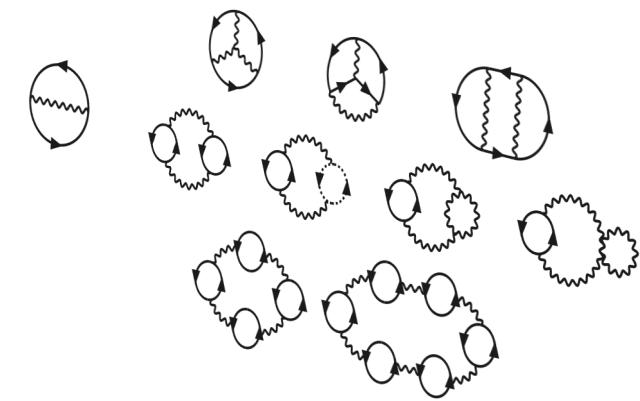
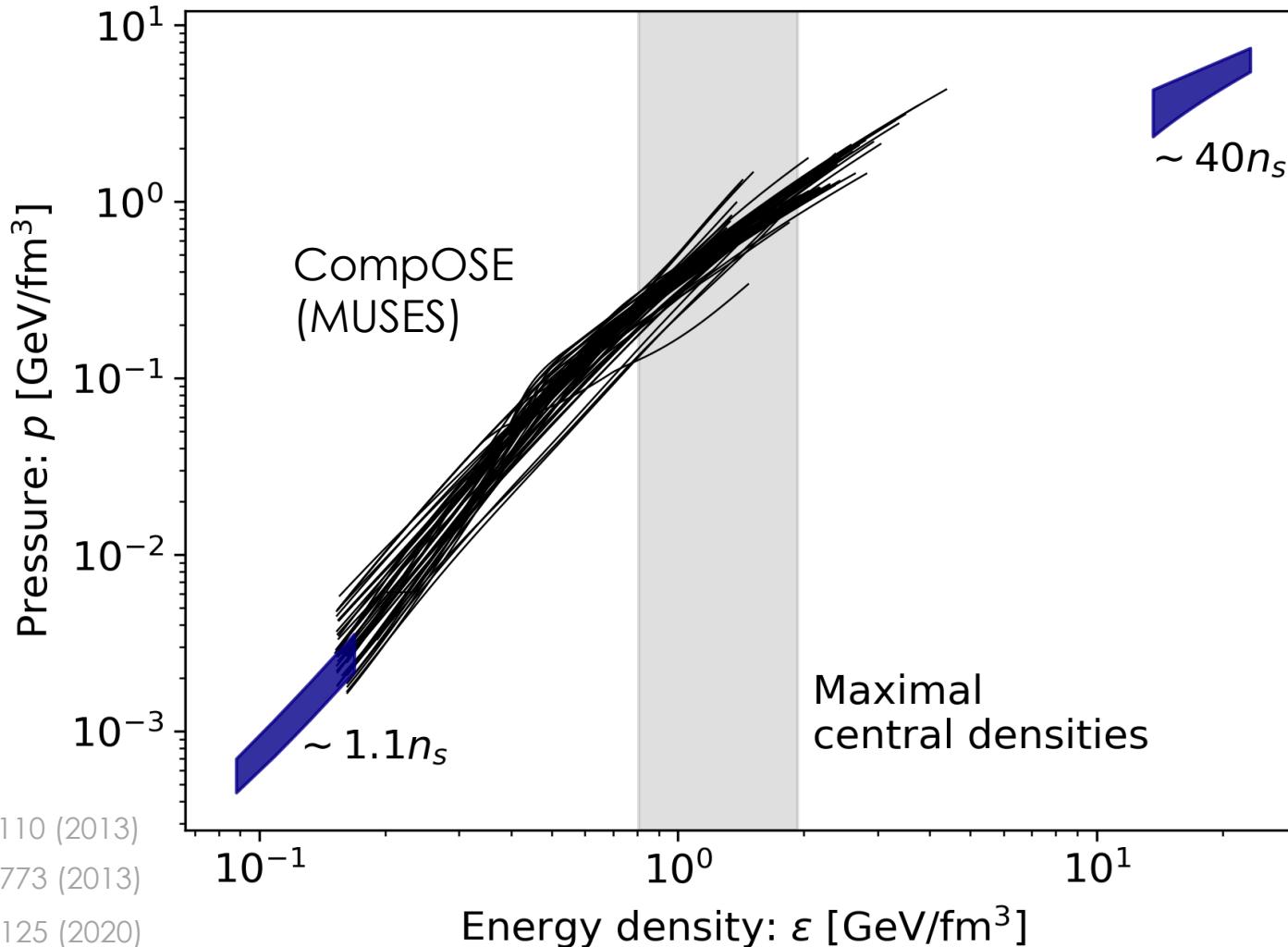
What do we know about the neutron star equation of state?

Equation of state, theoretically:



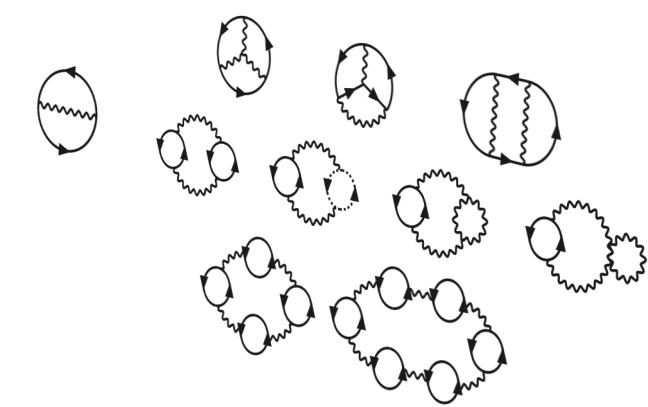
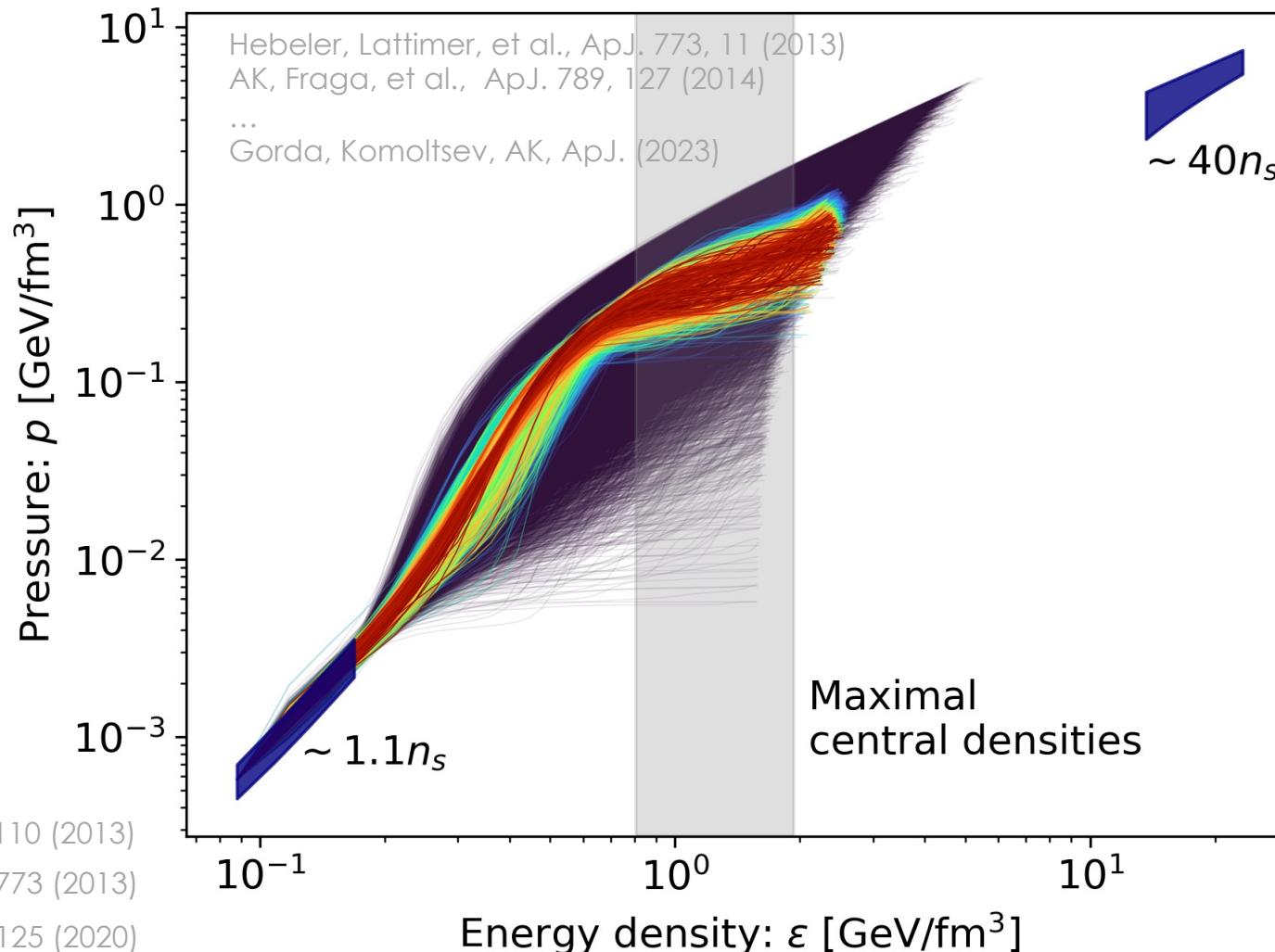
Freedman, McLerran, PRD 16 (1977)
Kurkela et al. PRD 81 (2010)
Kurkela, Vuorinen PRL 117 (2016)
Gorda et al. PRL 121 (2018)
Gorda et al. PRD 104 (2021)
Gorda et al. PRL 127 (2021)
Gorda et al. PRD 107 (2023)

Equation of state, models:

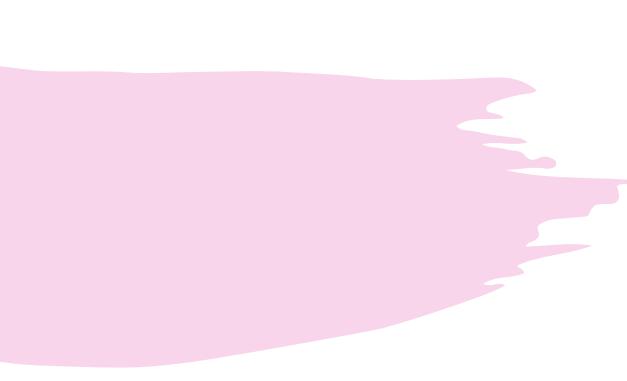


Freedman, McLellan, PRD 16 (1977)
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Gorda et al. PRL 121 (2018)
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Gorda et al. PRL 127 (2021)
Gorda et al. PRD 107 (2023)

Equation of state, inference:

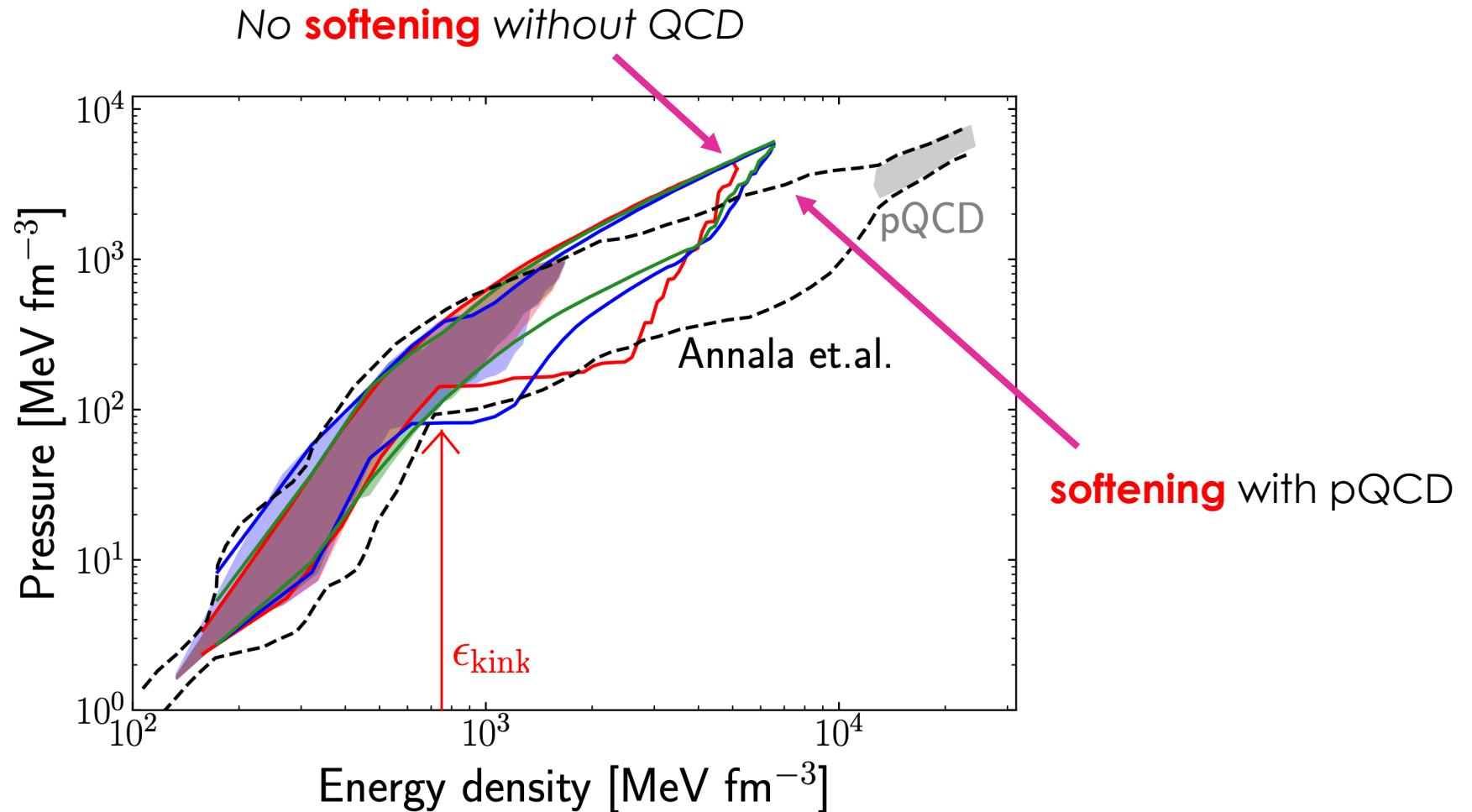


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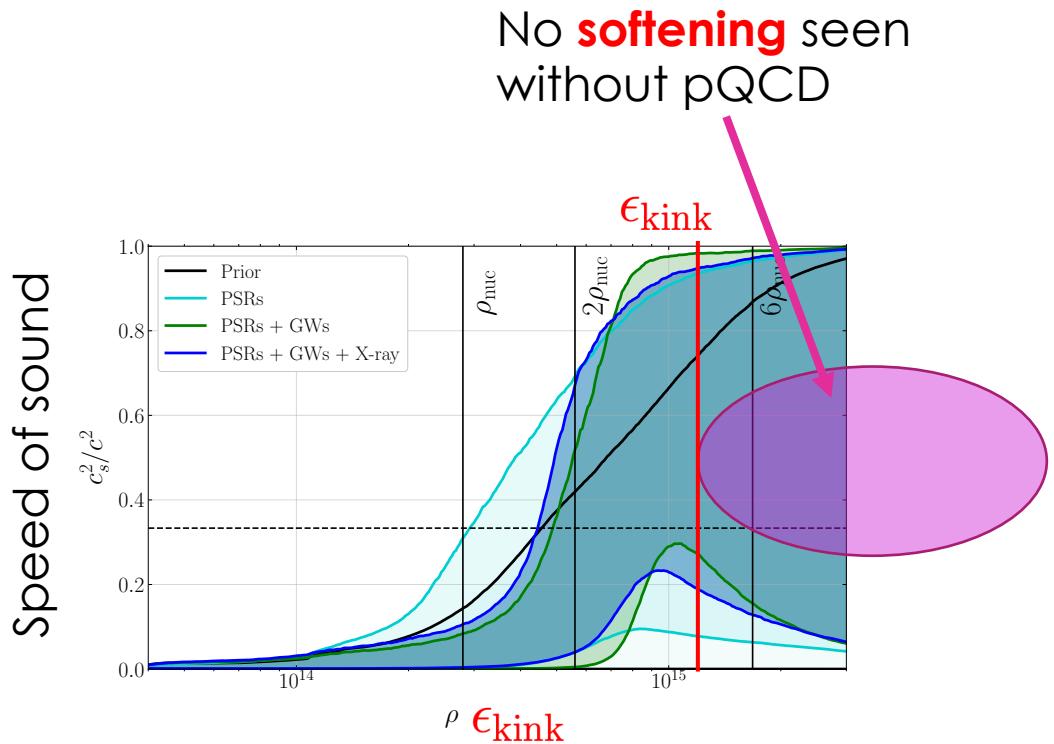
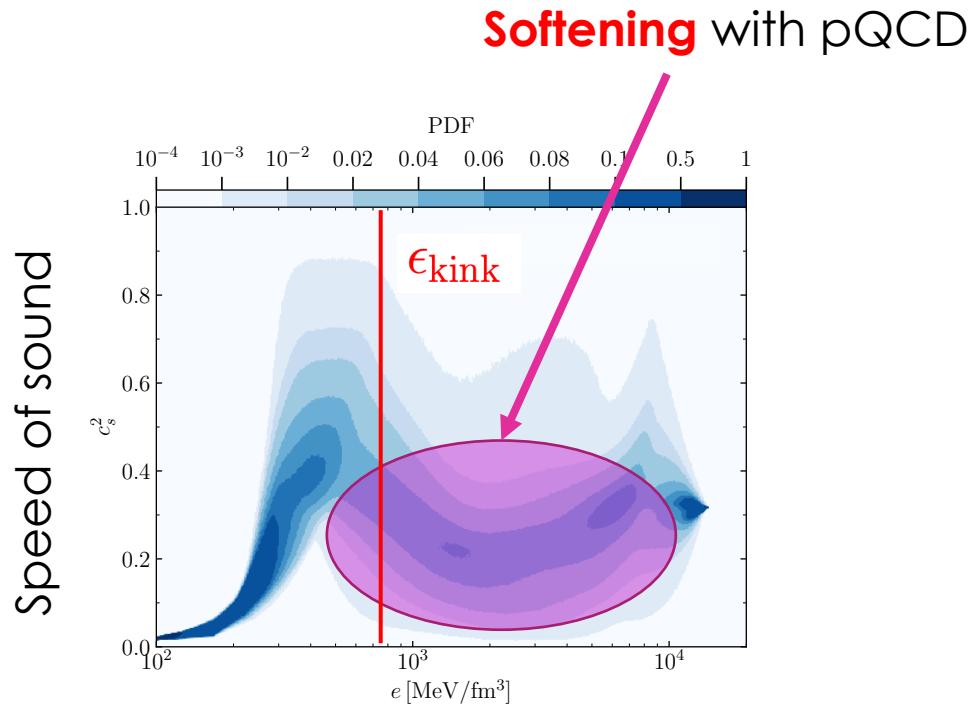


pQCD seems to **soften** the EoS at high densities

Studies with pQCD see **softening** of EoS



Studies with pQCD see **softening** of EoS



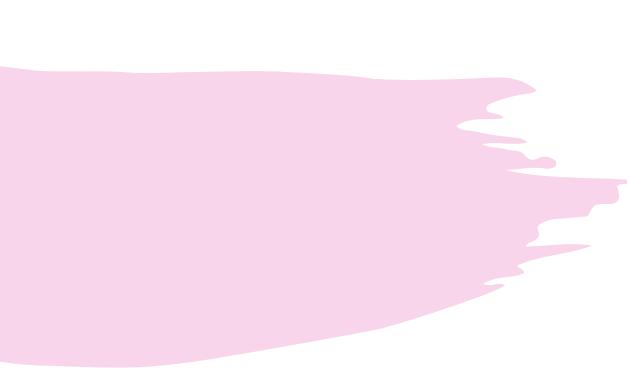
Annala, Gorda, Kurkela, Näyttälä, Vuorinen Nature Physics 16 (2020) 9

Altiparmak, Eckler, Rezzolla 2112.08157

Also: Han, Huang, Tang & Yi-Zhong Fan 2207.13613,
Marczenko, McLellan, Redlich & Sasaki 2207.13059

Landry, Essick, Chatzioannou PRD 101 (2020)
Somasundaram, Tews, Margueron 2112.08157

...



Softening may be a sign of a transition to
Quark Matter

Softening interpreted as phase change

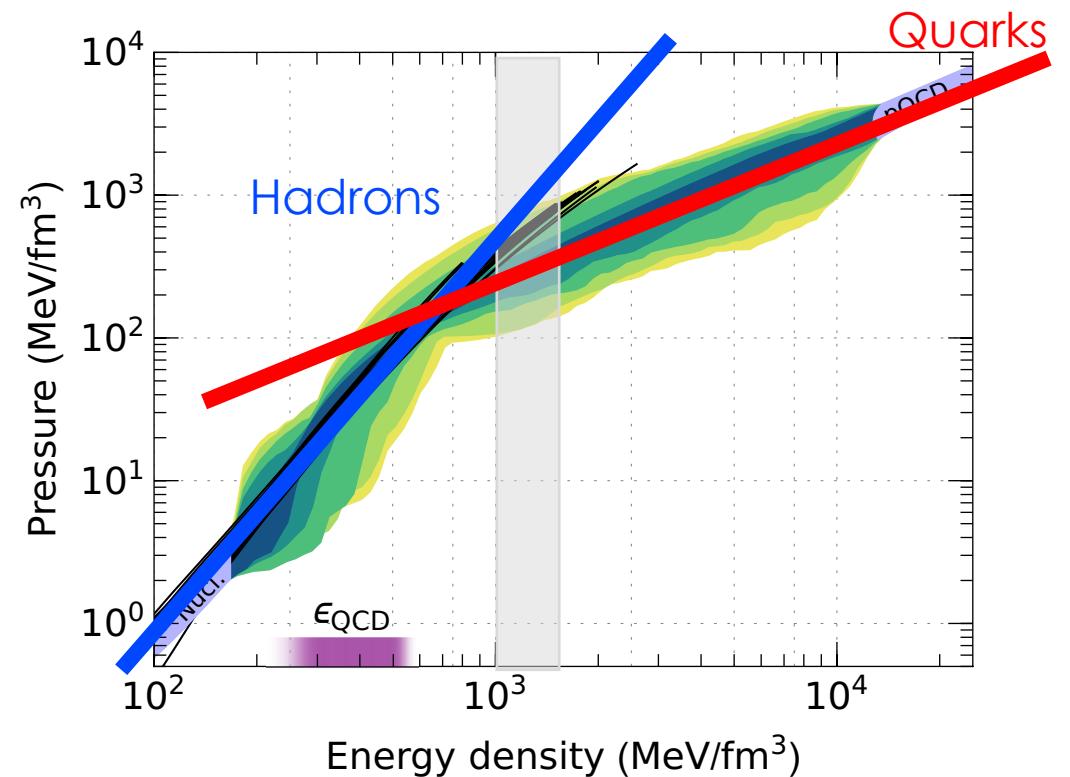
Properties of the EoS reflect the **phase structure** of the matter.

The cores of most massive NSs consistent with **deconfined, nearly conformal Quark Matter**.

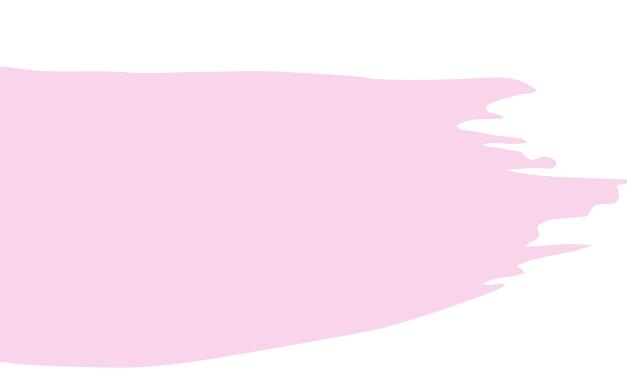
Softening

=

Conformalization



Annala, Gorda, Kurkela, Nätttilä, Vuorinen Nature Physics 16 (2020) 9
Also: Fujimoto, Fukushima, McLellan, Praszalowicz 2207.06753,
Kojo PRD 104, Annala, Gorda, Hirvonen, Komoltsev, AK, et al
2303.11356 ,
...



Is the **softening** an artefact of the
interpolation?

...or a robust prediction of QCD?

- **Why** does QCD at $40n_s$ constrain the EoS at NS densities:

How pQCD constrains the equation of state at neutron star densities

Komoltsev & AK, PRL128 (2022) 20, 2111.05350



- **How** QCD affects EoS inference

Ab-initio QCD calculations impact the inference of neutron-star equation of state

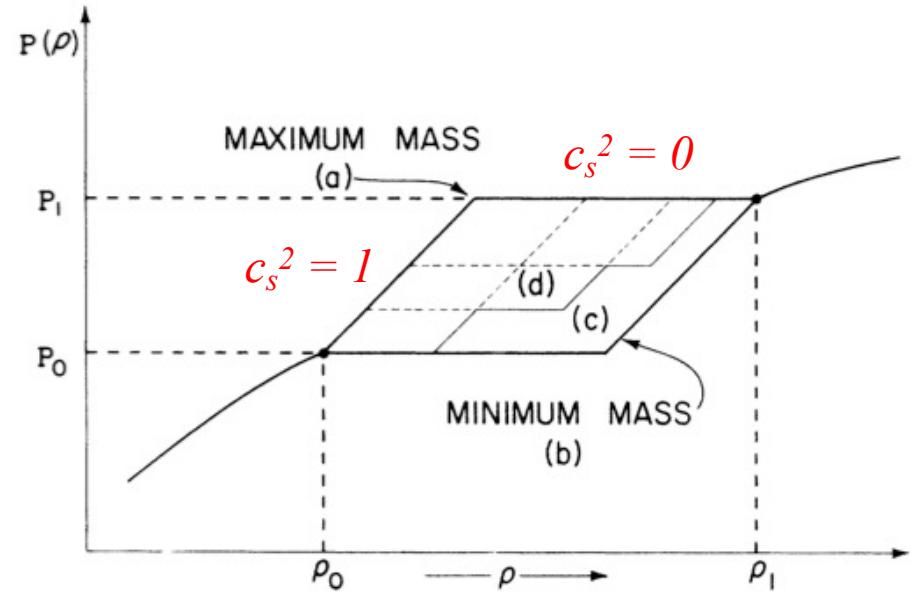
Gorda, Komoltsev & AK, Astrophys.J. 950 (2023)

Oleg

Robust EoS constraints:

General considerations:

- Mechanical stability: $c_s^2 > 0$
- Causality: $c_s^2 < 1$



Rhoades & Ruffini, Phys.Rev.Lett. 32 (1974)
Lope-Oter, Windisch, Llanes-Estrada, Alford, J. Phys. G (2019)
Lope-Oter, Llanes-Estrada, EPJA 58 (2022)

Robust EoS constraints:

General considerations:

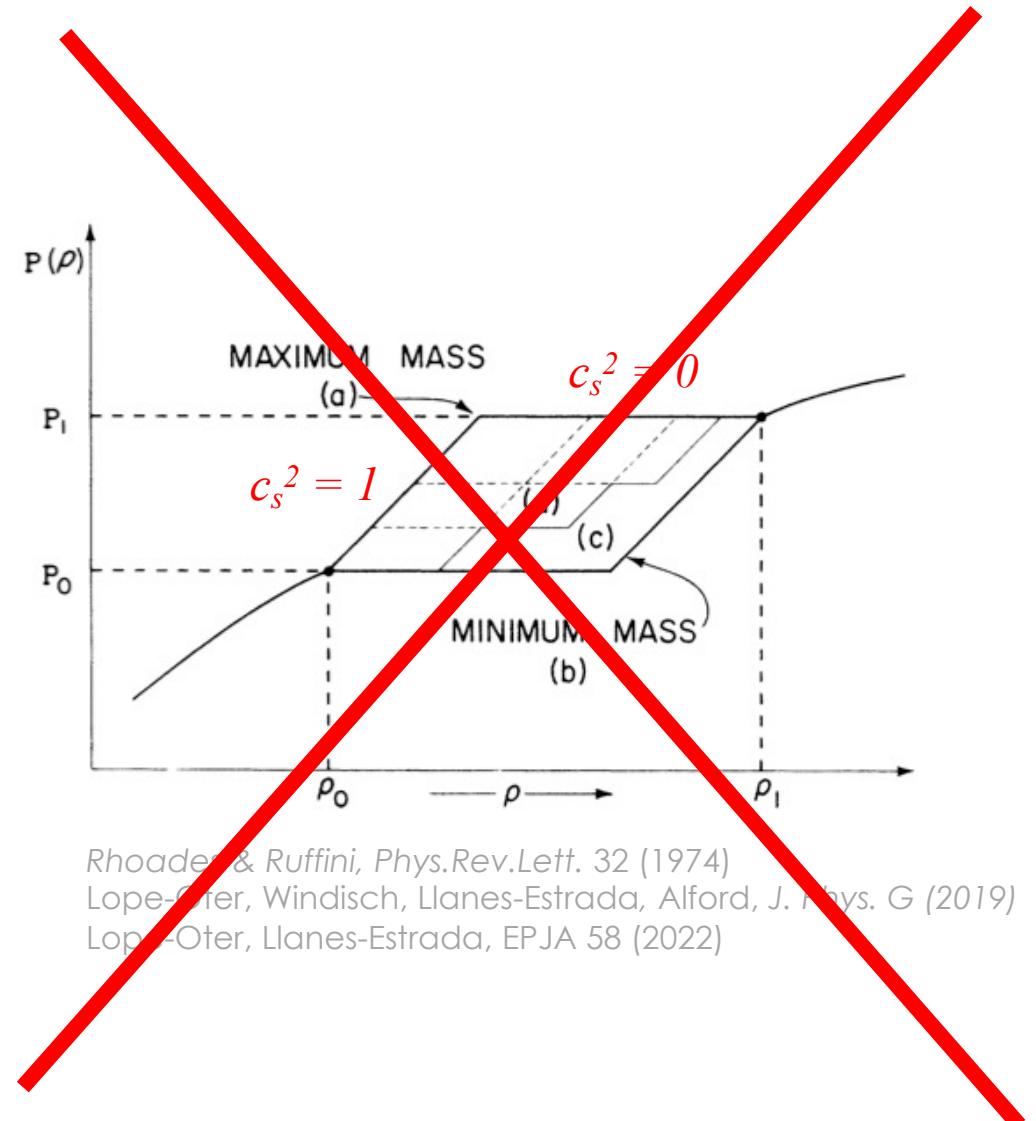
- Mechanical stability: $c_s^2 > 0$
- Causality: $c_s^2 < 1$
- Consistency:

$$P(\epsilon) \text{ vs. } \Omega(\mu)$$

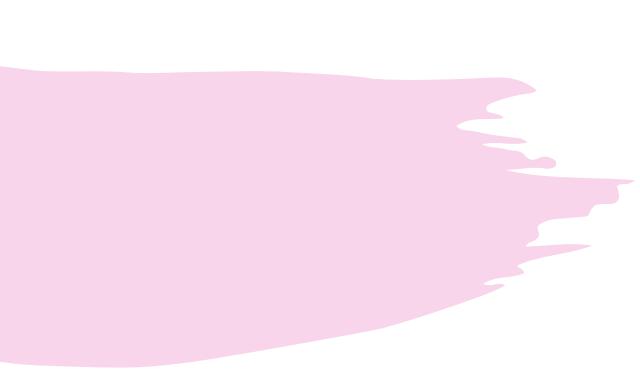
Reduced EoS

Full EoS

Information of $\{P, \epsilon, n\}$

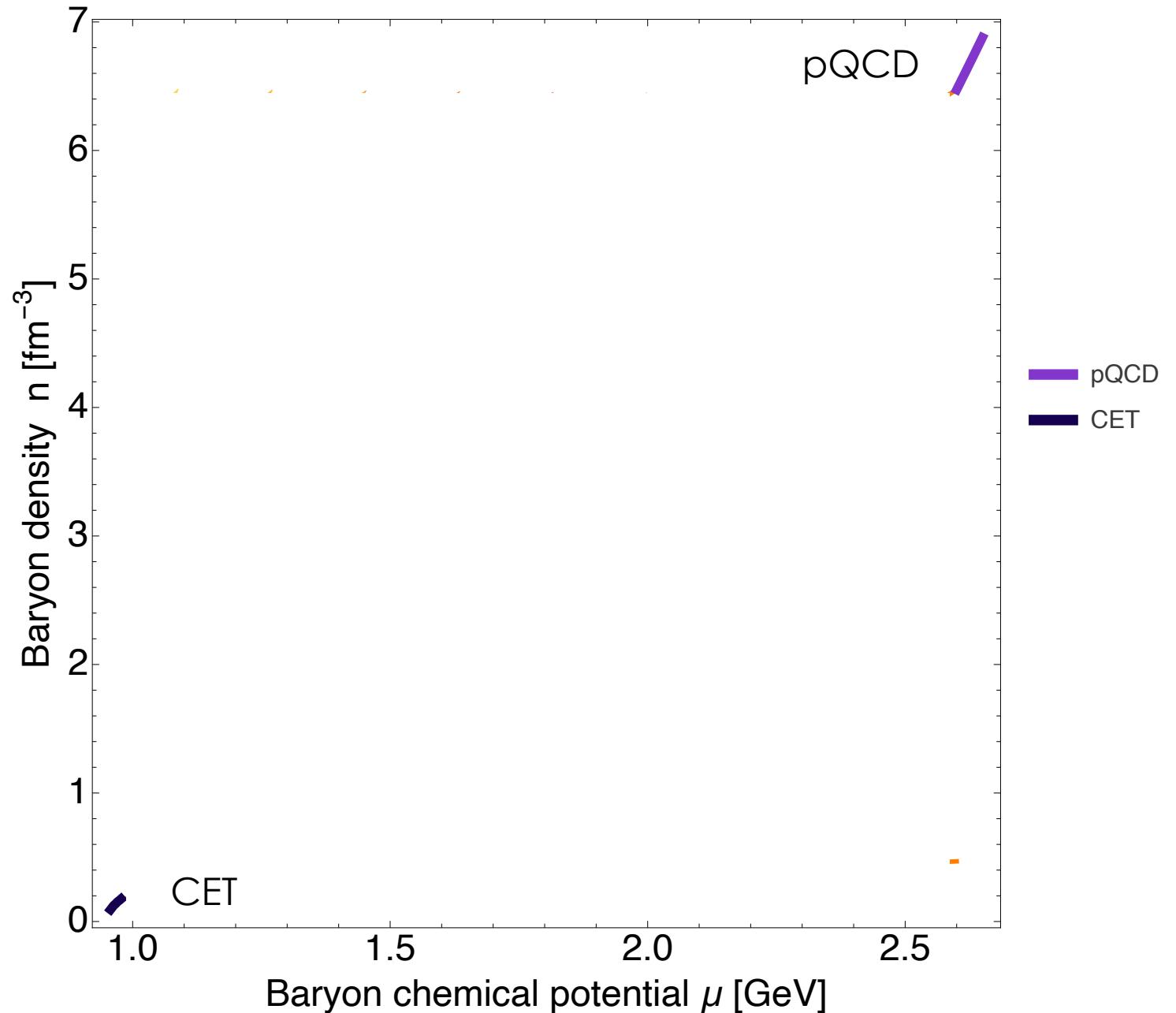


Rhoades & Ruffini, Phys.Rev.Lett. 32 (1974)
Lope-Oter, Windisch, Llanes-Estrada, Alford, J. Phys. G (2019)
Lope-Oter, Llanes-Estrada, EPJA 58 (2022)



What are **the most extreme** allowed EoSs:

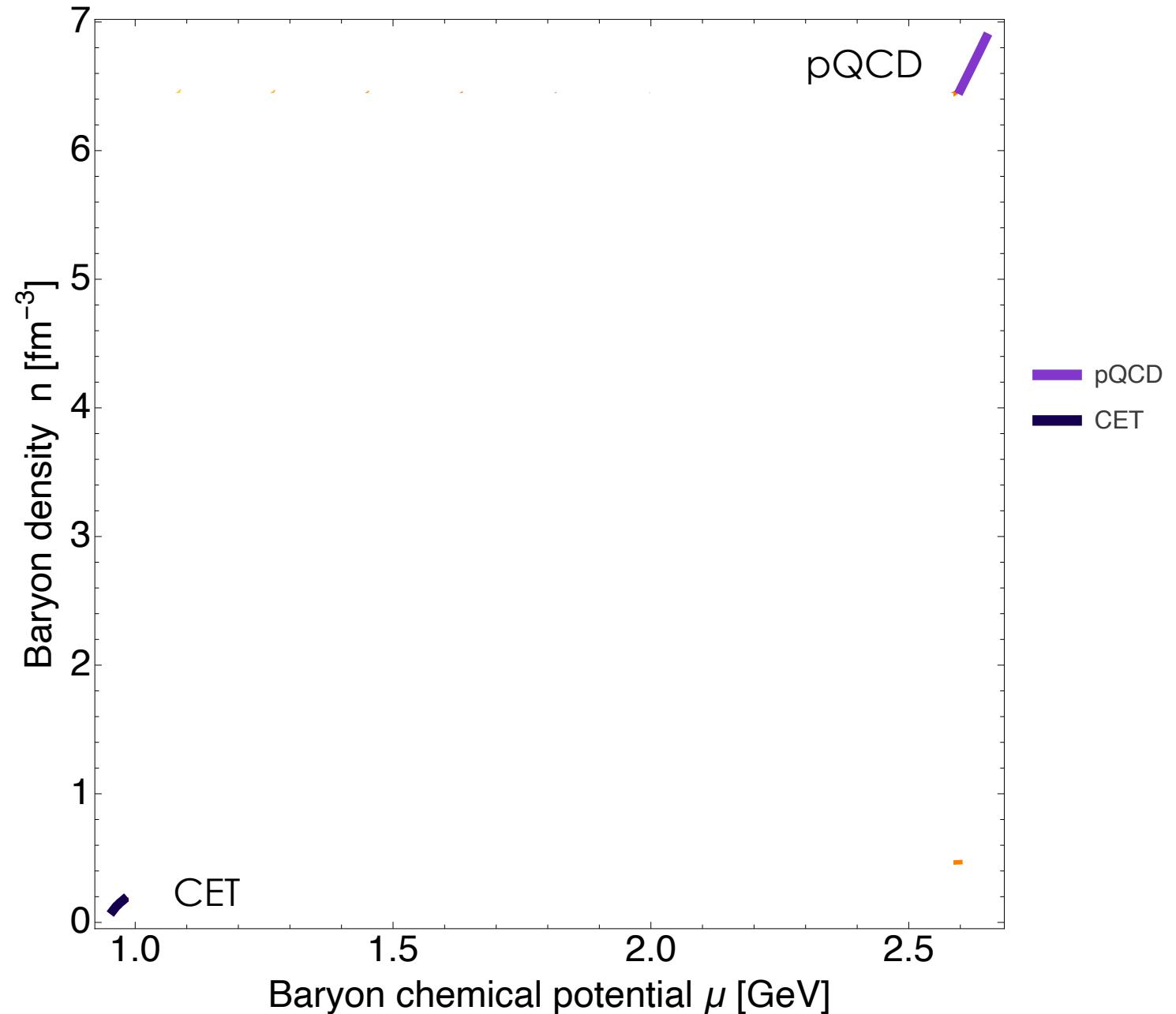
Setup:



Setup:

- Stability

$$\partial_\mu^2 \Omega(\mu) \leq 0 \quad \Rightarrow \quad \partial_\mu n(\mu) \geq 0$$



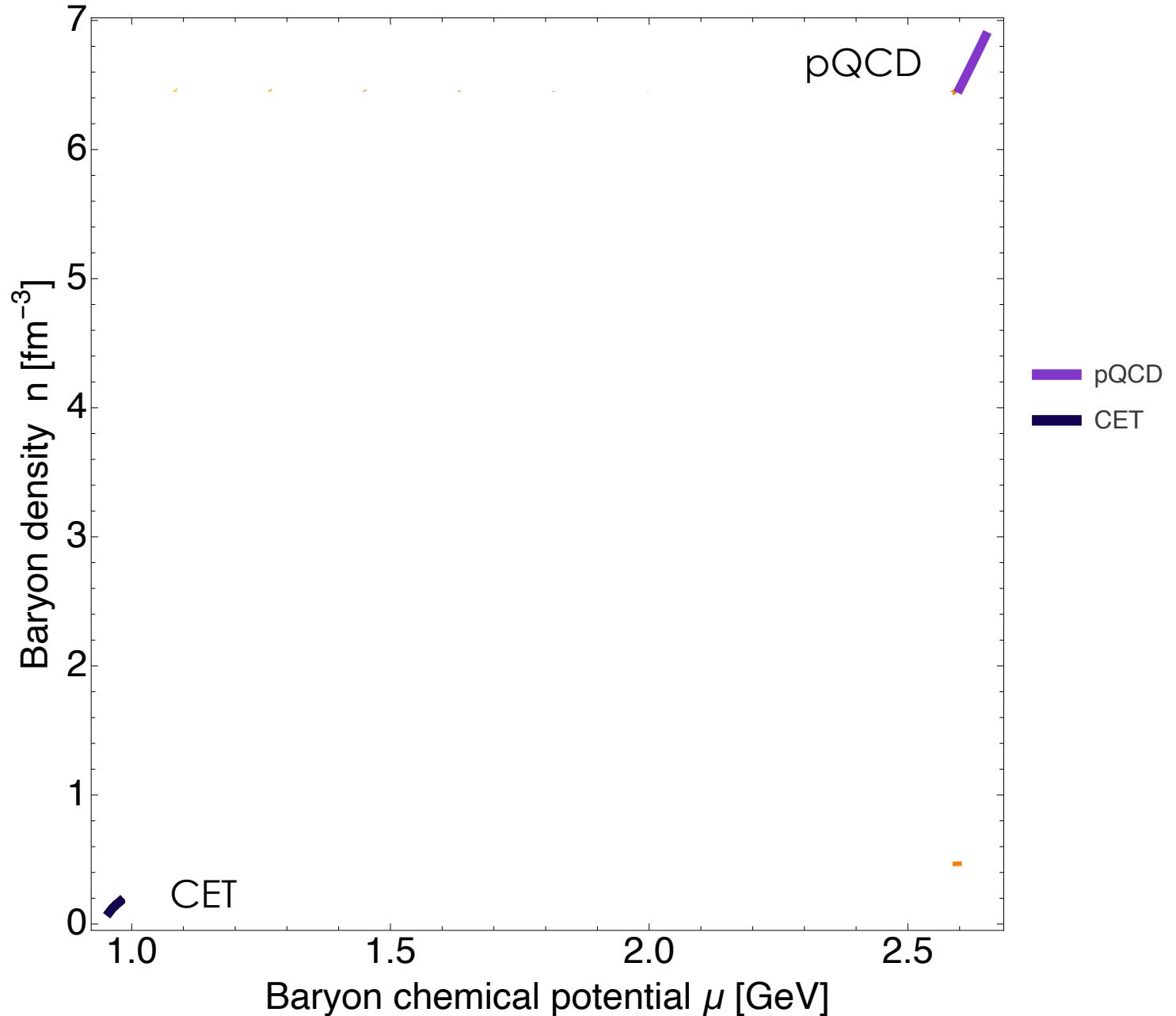
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$$\partial_\mu^2 \Omega(\mu) \leq 0 \quad \Rightarrow \quad \partial_\mu n(\mu) \geq 0$$

- Causality

$$c_s^{-2} = \frac{\mu}{n} \frac{\partial n}{\partial \mu} \geq 1 \quad \Rightarrow \quad \partial_\mu n(\mu) \geq \frac{n}{\mu}$$



Setup:

- Stability

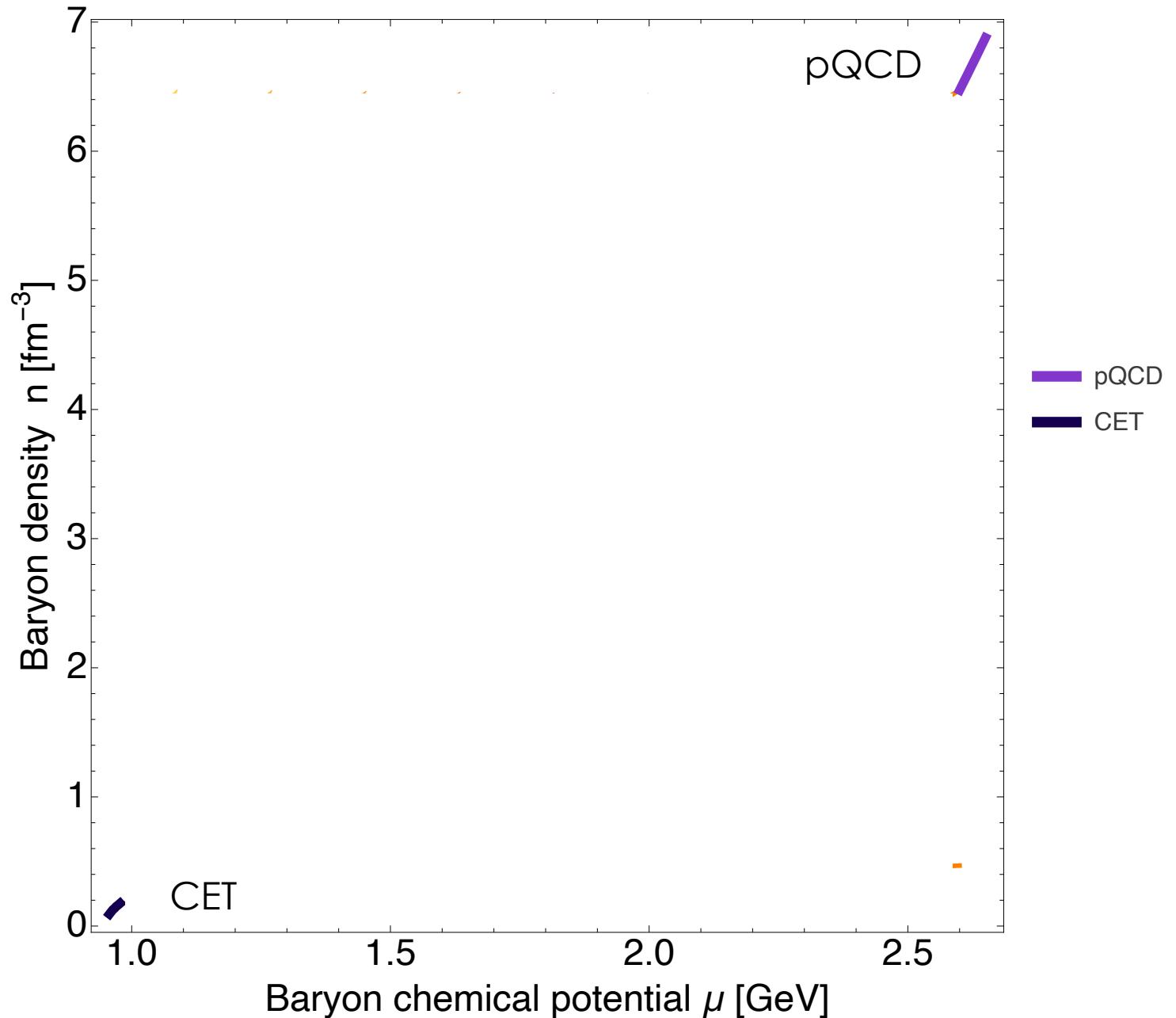
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- Consistency

$$\int_{\mu_{CET}}^{\mu_{QCD}} n(\mu) d\mu = p_{QCD} - p_{CET} = \Delta p$$



Setup:

- Stability

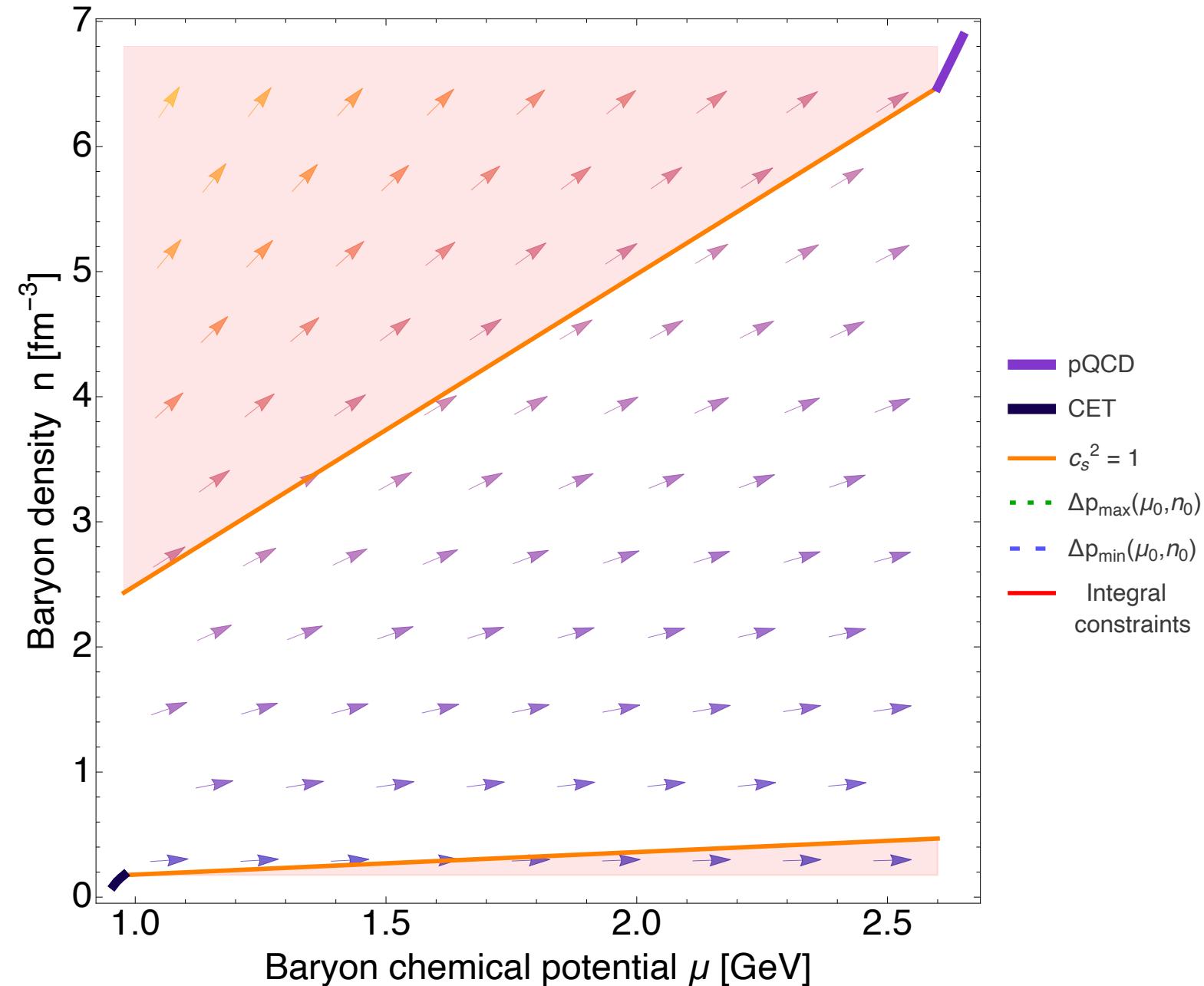
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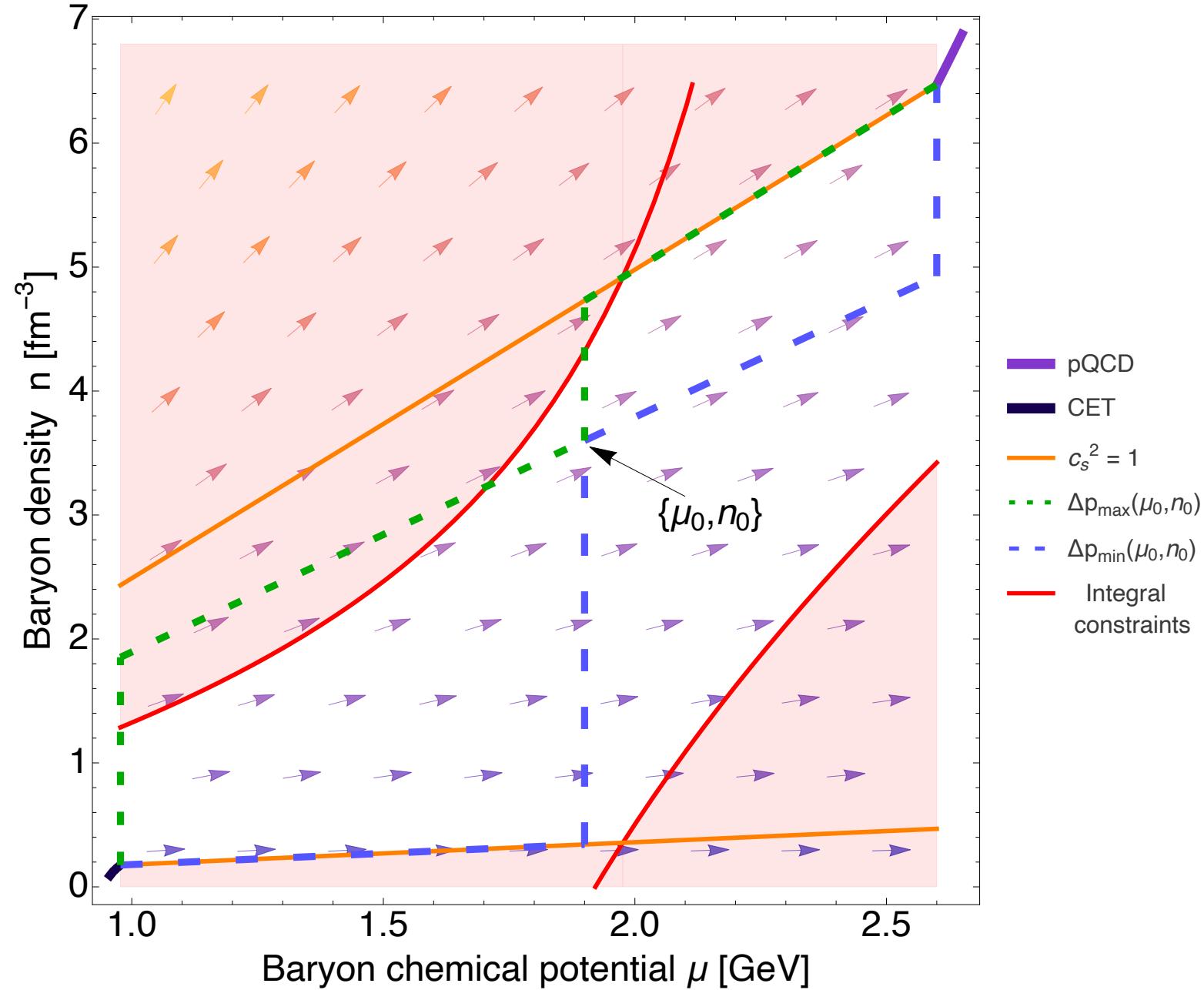
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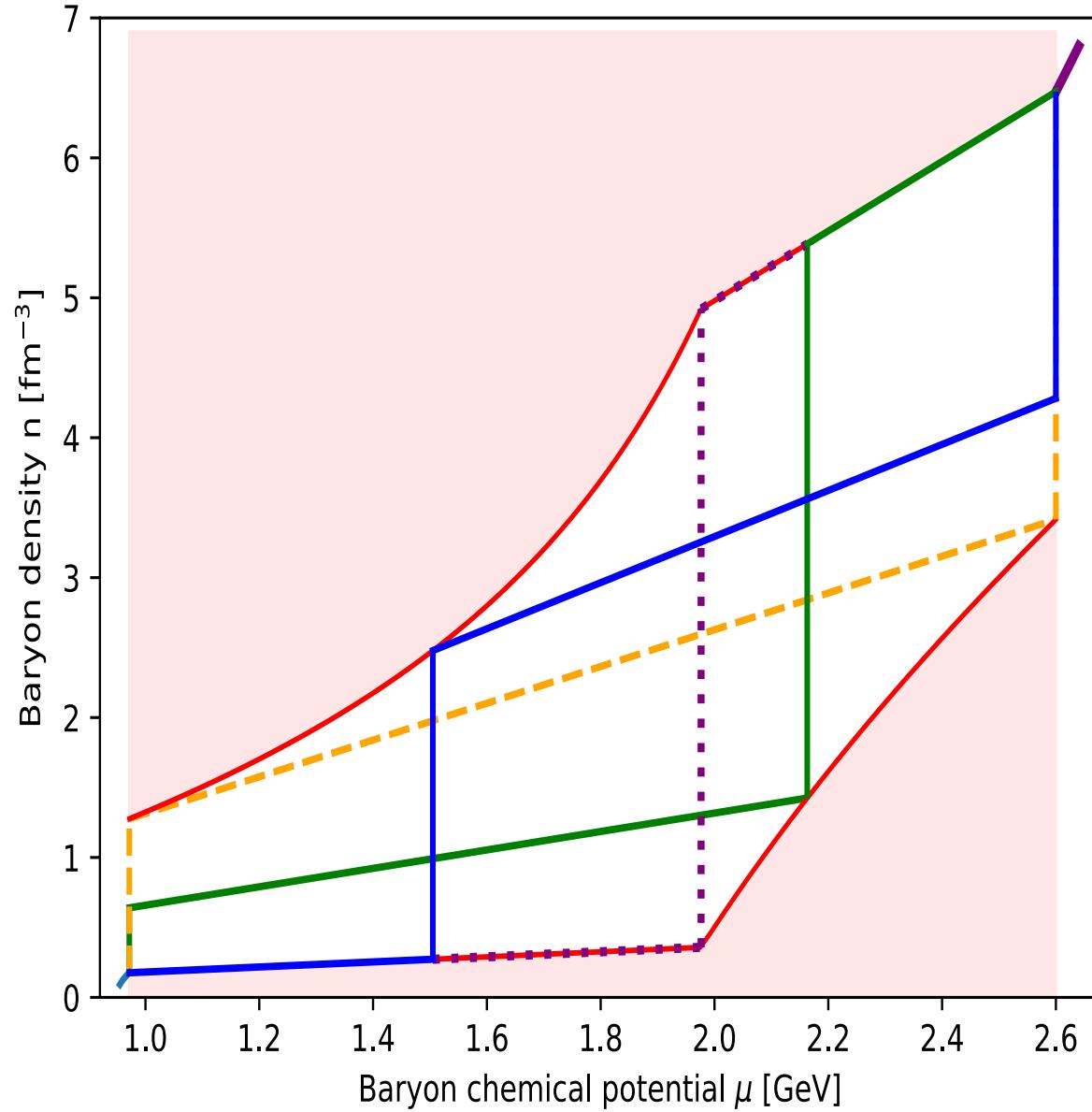
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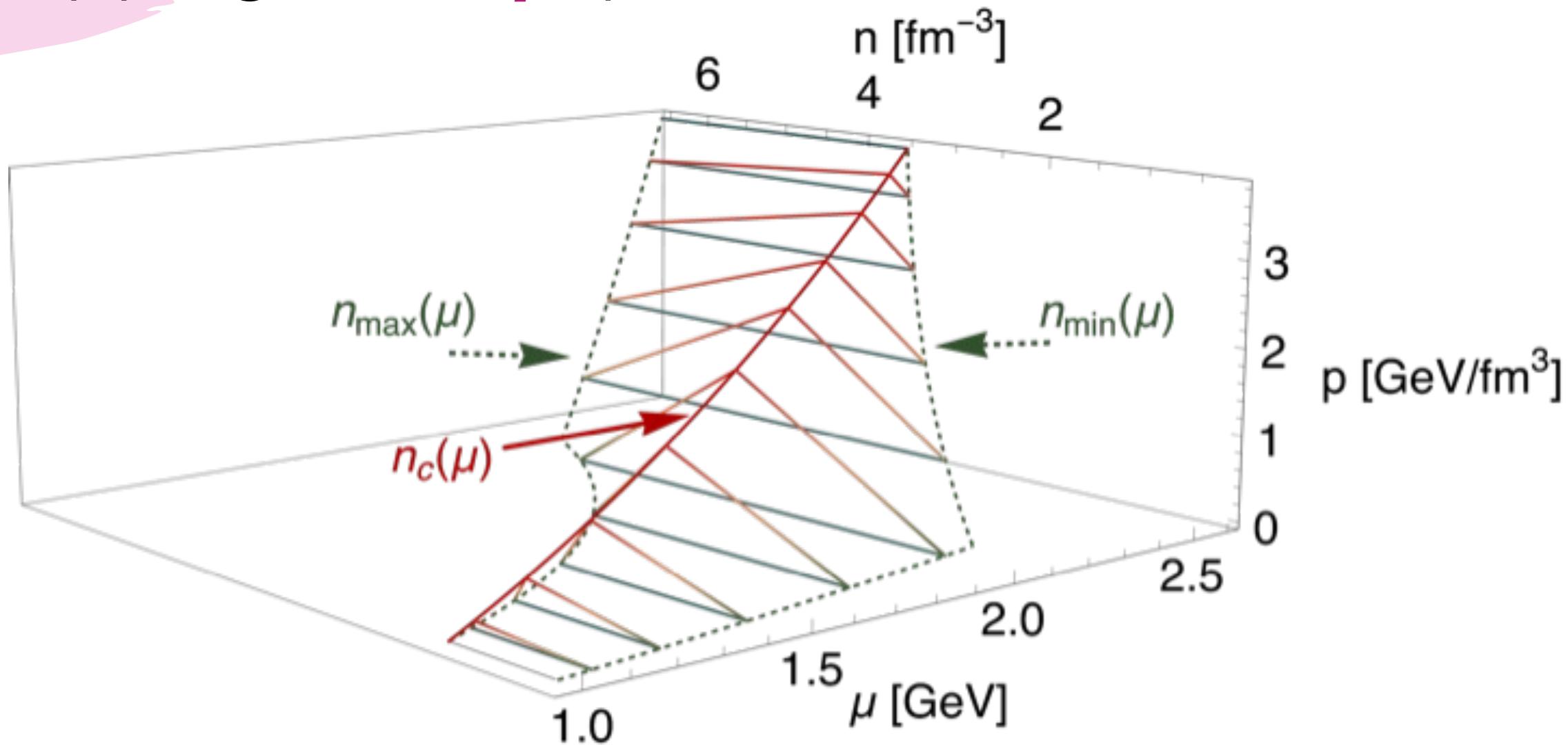
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- Consistency

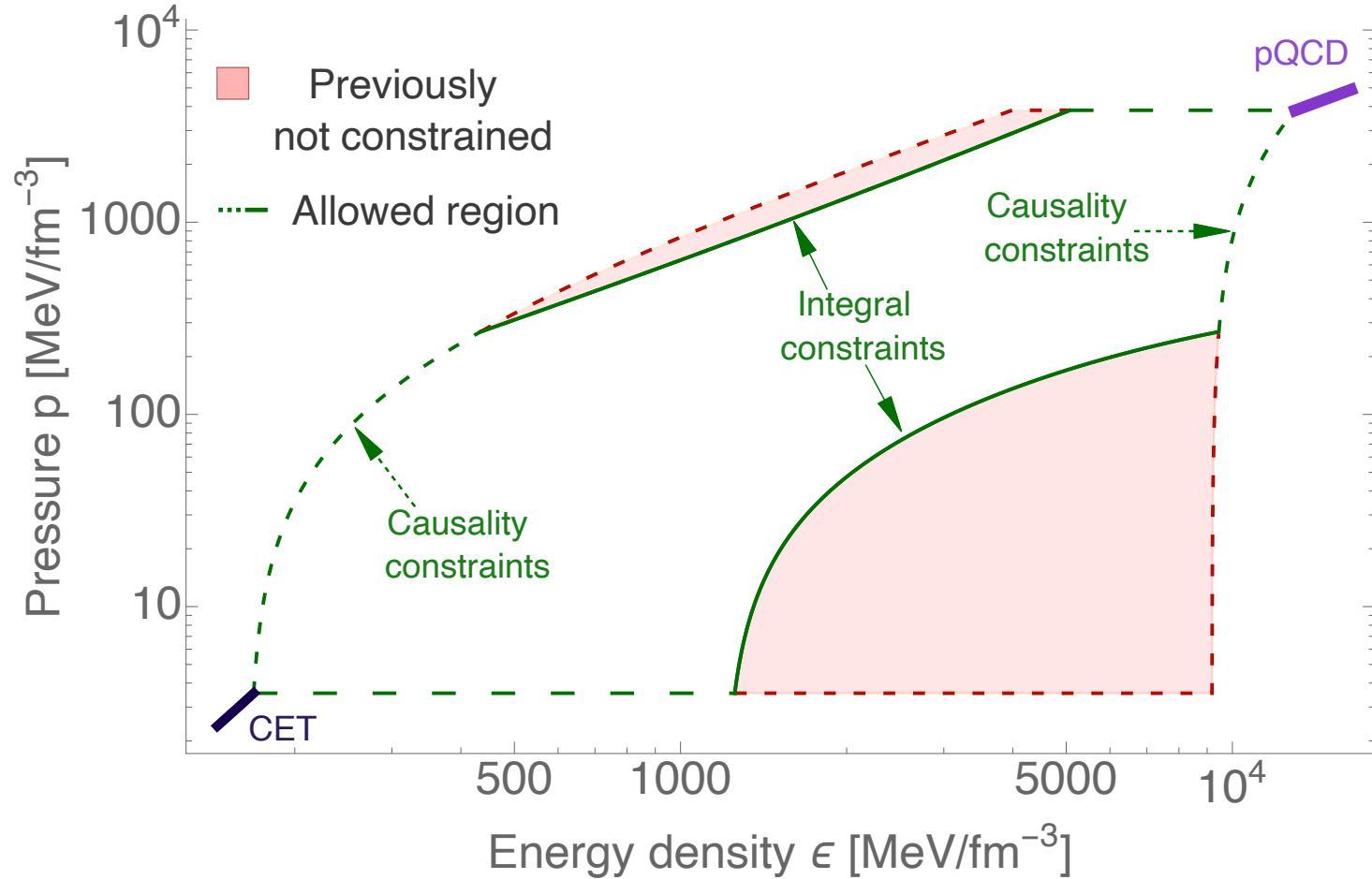
$$\int_{\mu_{CET}}^{\mu_{QCD}} n(\mu) d\mu = p_{QCD} - p_{CET} = \Delta p$$



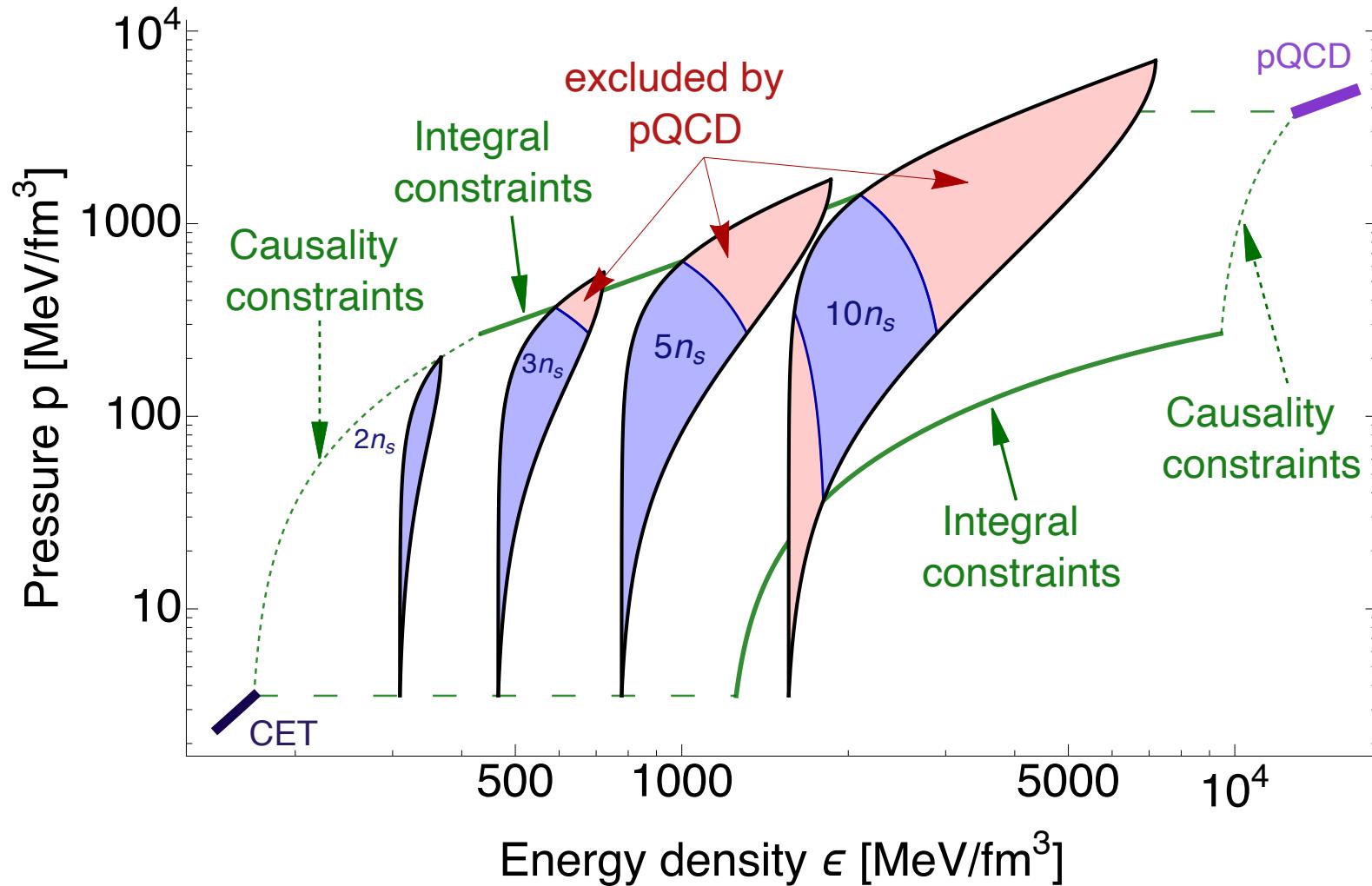
Mapping to $\epsilon - p$ -plane

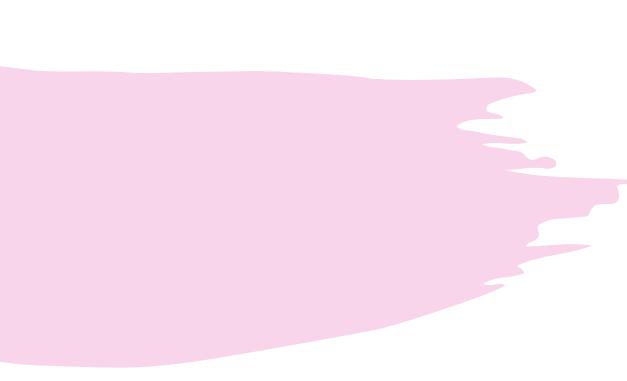


Mapping to $\epsilon - p$ -plane



Constraints for fixed n on $\epsilon - p$ -plane

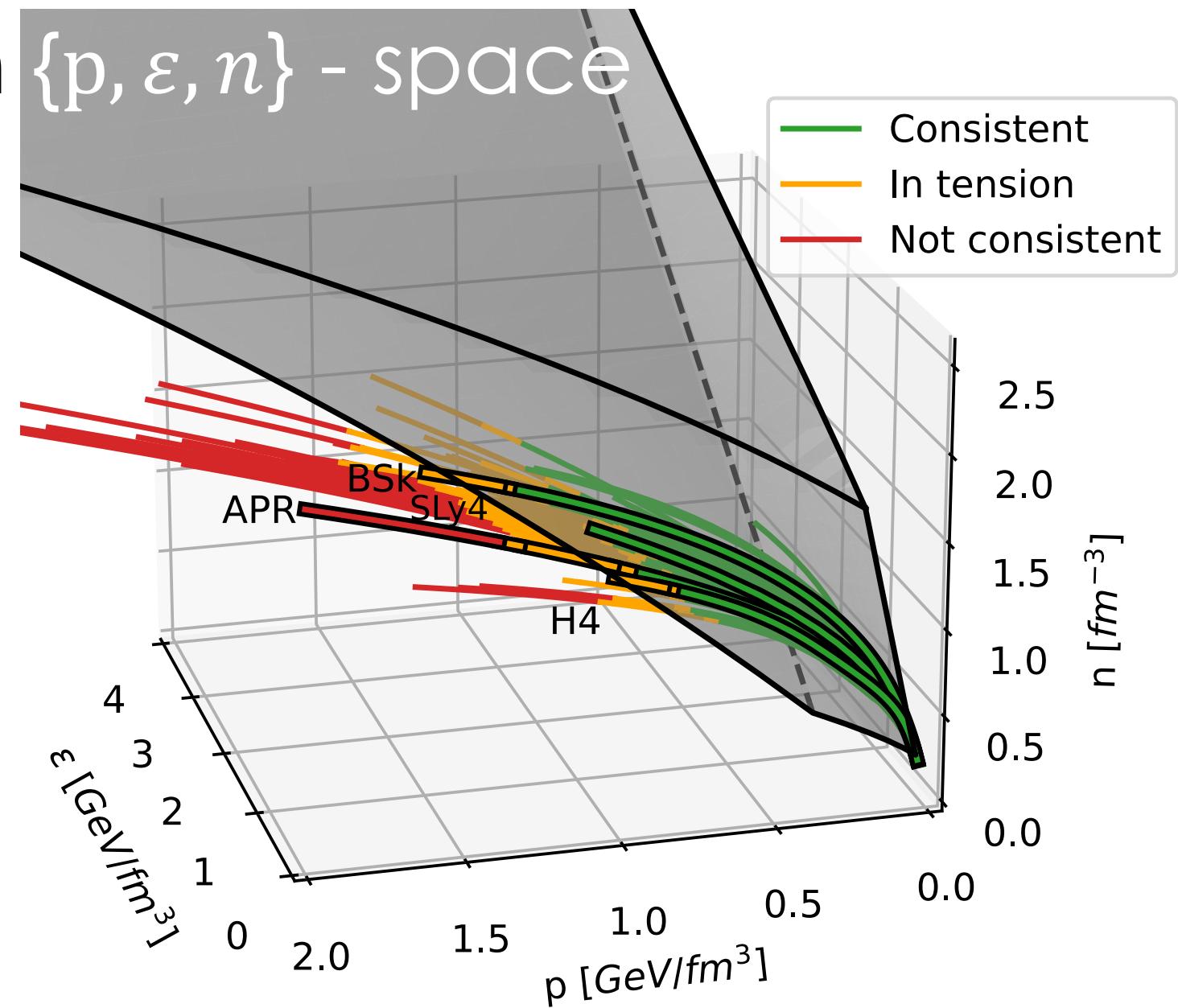




Does any of this matter?

Constraints in $\{p, \varepsilon, n\}$ - space

Models from CompOSE
database



- Why does QCD at $40n_s$ constrain the EoS at NS densities:

How pQCD constrains the equation of state at neutron star densities

Komoltsev & AK, PRL128 (2022) 20, 2111.05350

- **How** QCD affects EoS inference

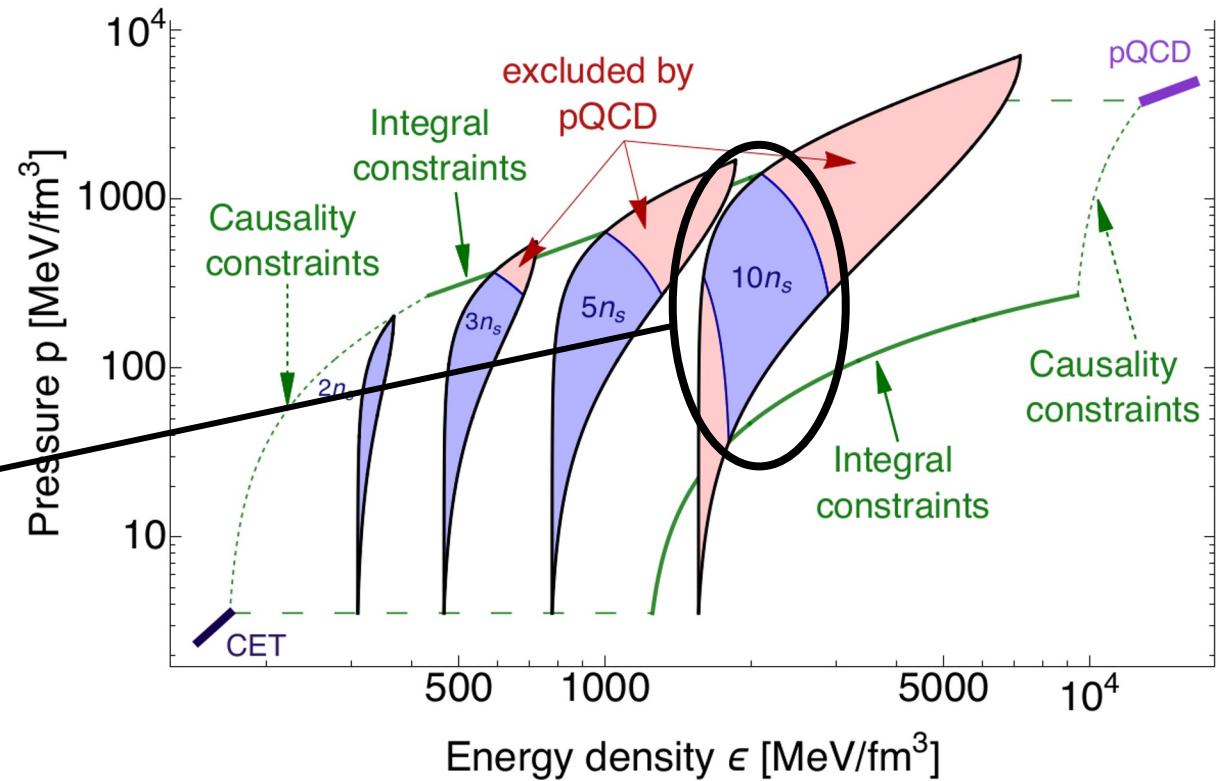
Ab-initio QCD calculations impact the inference of neutron-star equation of state

Gorda, Komoltsev & AK APJ (2023)

Implementing pQCD to EoS inference:

- Inference setup where QCD can be turned on/off
- Easily implemented to any other extrapolation setup

Use this area to condition an extrapolation

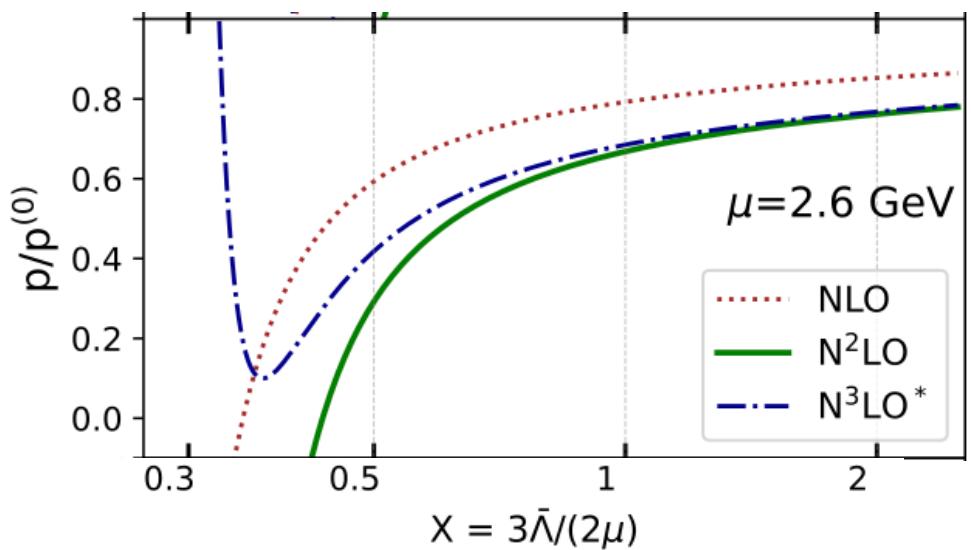


pQCD likelihood function:

Scale variation uncertainty

$$\frac{p}{p_0} = 1 + d_1 \alpha_s(\bar{\Lambda}) + d_2(\bar{\Lambda}) \alpha_s^2(\bar{\Lambda}) + d_3(\bar{\Lambda}) \alpha_s^3(\bar{\Lambda}) + \dots$$

Missing Higher Order errors

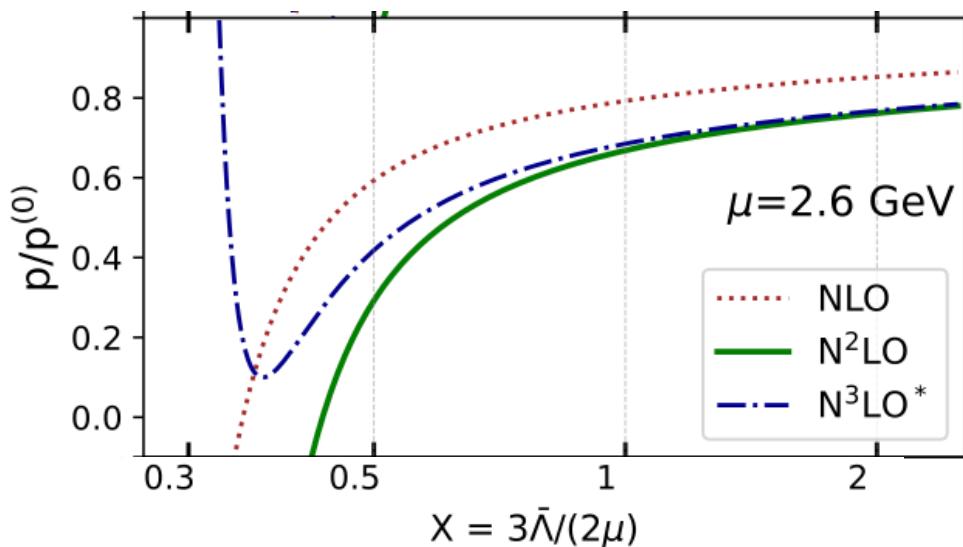


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- **Machine learning** based Bayesian interpretation of **Scale variation** and **Missing Higher Order** errors

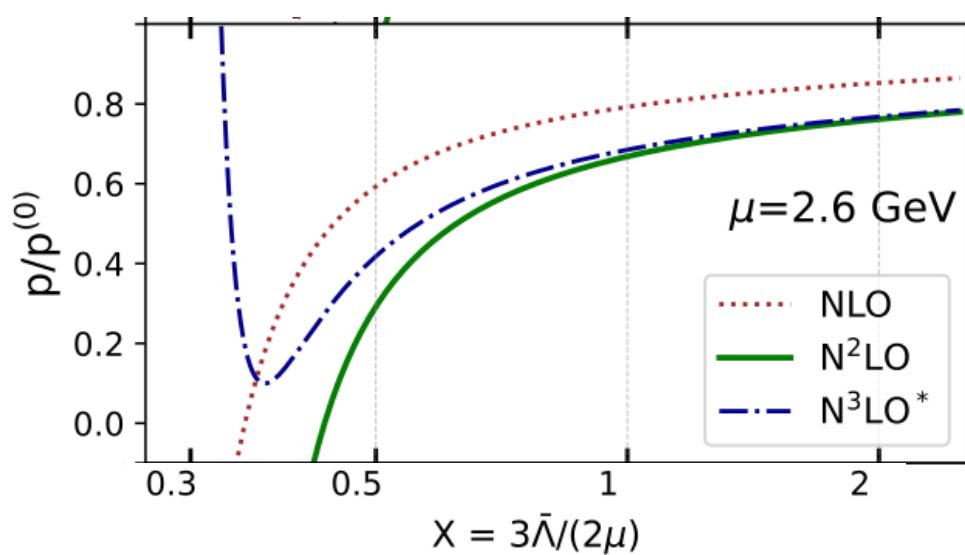
Cacciari & Houdeau, JHEP 09, (2011), Duhr et al. JHEP 122, (2021), Gorda, Komoltsev, AK, Mazeliauskas, 2303.02175

- Perturbative series modelled as draw from statistical model that is trained with the available terms

pQCD likelihood function:

Scale variation uncertainty

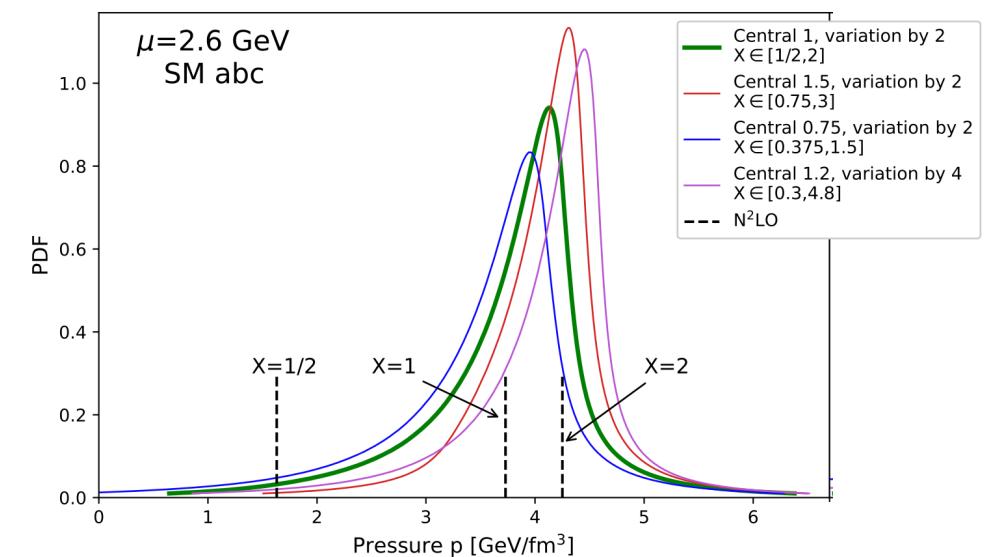
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Missing Higher Order errors

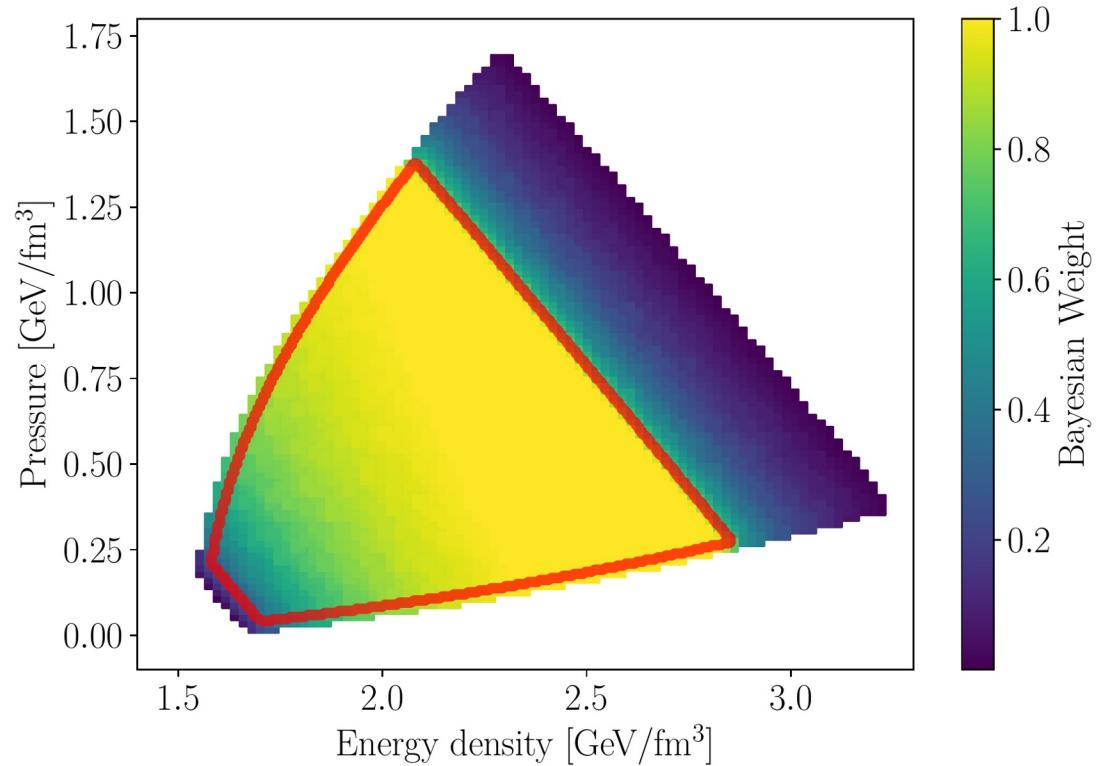
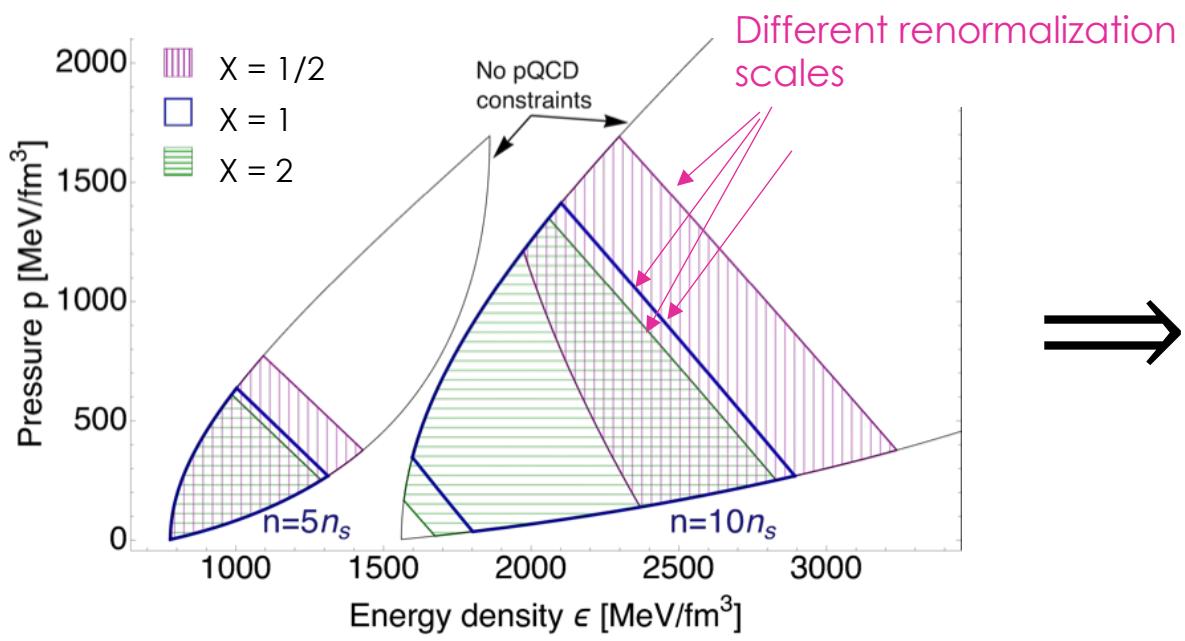


Probability distribution: $P(p | \{d_n(\bar{\Lambda})\})$



- **Machine learning** based Bayesian interpretation of **Scale variation** and **Missing Higher Order** errors
- Cacciari & Houdeau, JHEP 09, (2011), Duhr et al. JHEP 122, (2021), Gorda, Komoltsev, AK, Mazeliauskas, 2303.02175
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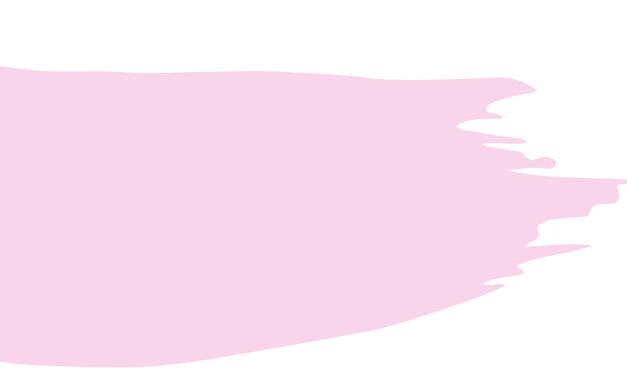
QCD likelihood function:



- **Machine learning** based Bayesian interpretation of **Scale variation** and **Missing Higher Order** errors

Cacciari & Houdeau, JHEP 09, (2011), Duhr et al. JHEP 122, (2021), Gorda, Komoltsev, AK, Mazeliauskas, 2303.02175

- Perturbative series modelled as draw from statistical model, that is trained with the available terms



Using QCD likelihood in EoS inference

<https://github.com/OKomoltsev/QCD-likelihood-function>

Implementing pQCD to EoS inference:

- Bayesian inference setup:

$$P(\text{EoS} \mid \text{data}) = \frac{P(\text{EoS}) P(\text{data} \mid \text{EoS})}{P(\text{data})}$$

Prior

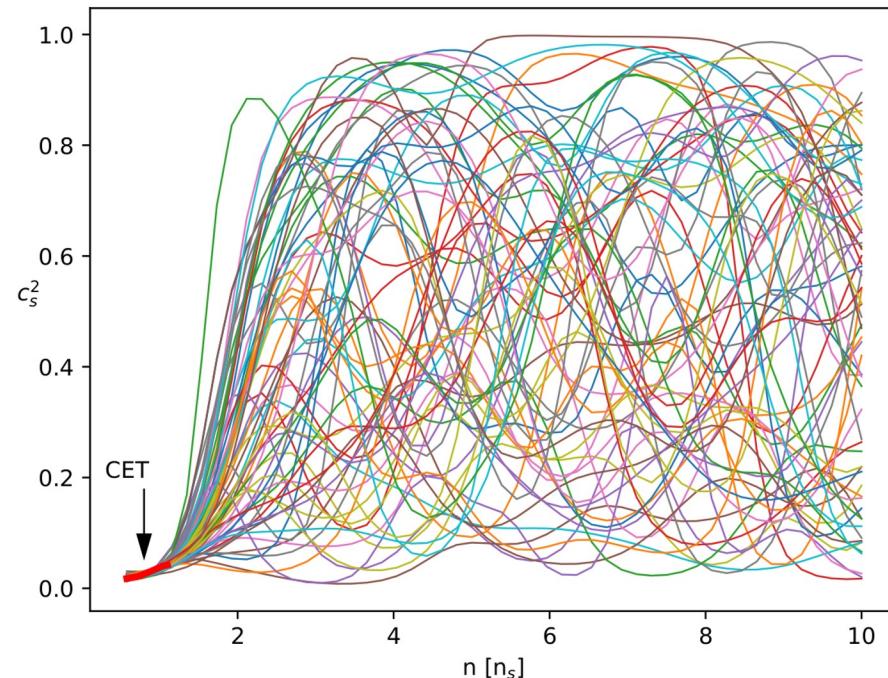
Likelihood of EoS
given data

```
graph TD; Prior --> P_EoS[P(EoS)]; Likelihood --> P_Data_EoS[P(data | EoS)];
```

Implementing pQCD to EoS inference:

- **Gaussian-process** based inference:

Similar to Landry & Essick PRD 99 (2019), but for function of n instead of ε



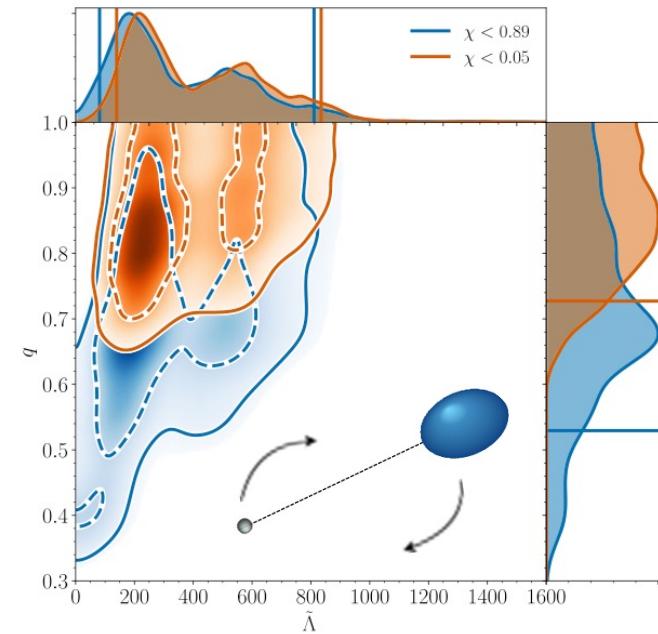
$$P(\text{EoS} \mid \text{data}) = \frac{P(\text{EoS}) P(\text{data} \mid \text{EoS})}{P(\text{data})}$$

Ask for back-up slide for more details ...

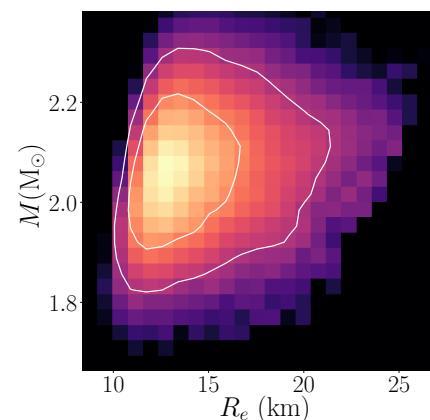
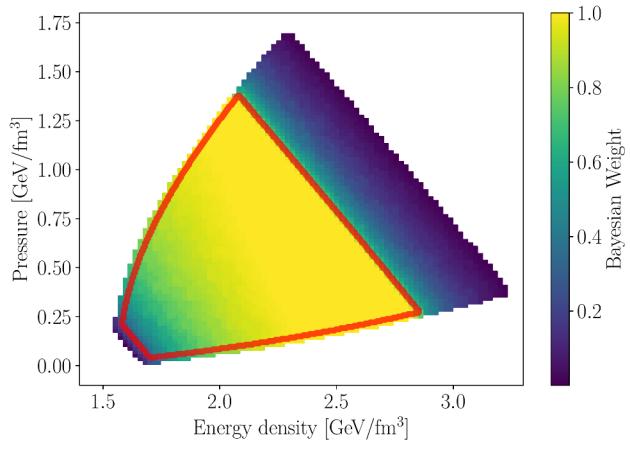
Setup:

$$P(\text{EoS} \mid \text{data}) = \frac{P(\text{EoS}) P(\text{data} \mid \text{EoS})}{P(\text{data})}$$

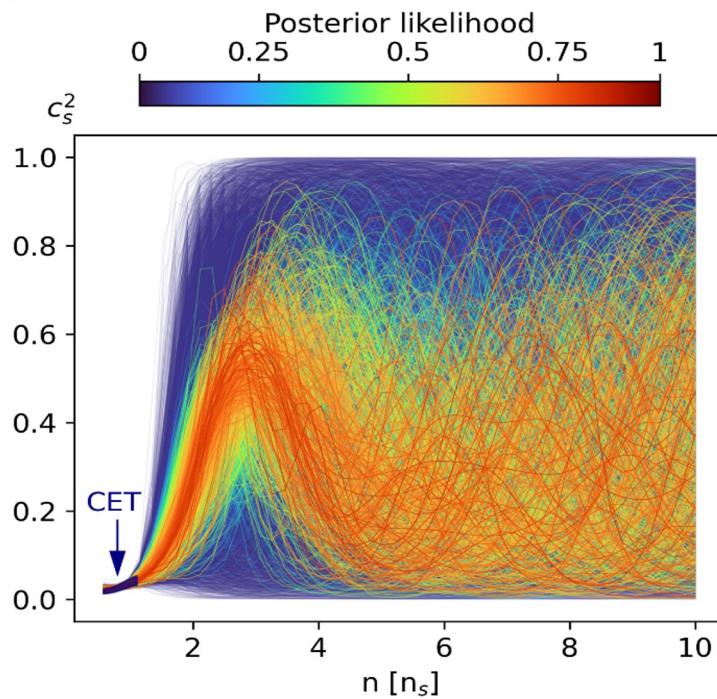
$M_{J1614-2230} = 1.908(16)$
 $M_{J048+0432} = 2.01(4)$
 $M_{J0740+6620} = 2.14(10)$



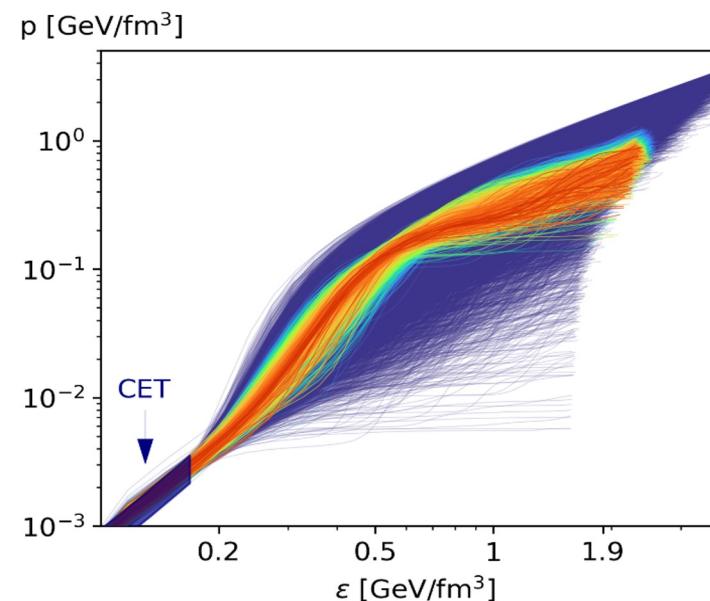
$$P(\text{data} \mid \text{EoS}) = P(\text{QCD} \mid \text{EoS}) P(\text{Mass} \mid \text{EoS}) P(\text{NICER} \mid \text{EoS}) P(\tilde{\Lambda}, \text{BH} \mid \text{EoS}).$$



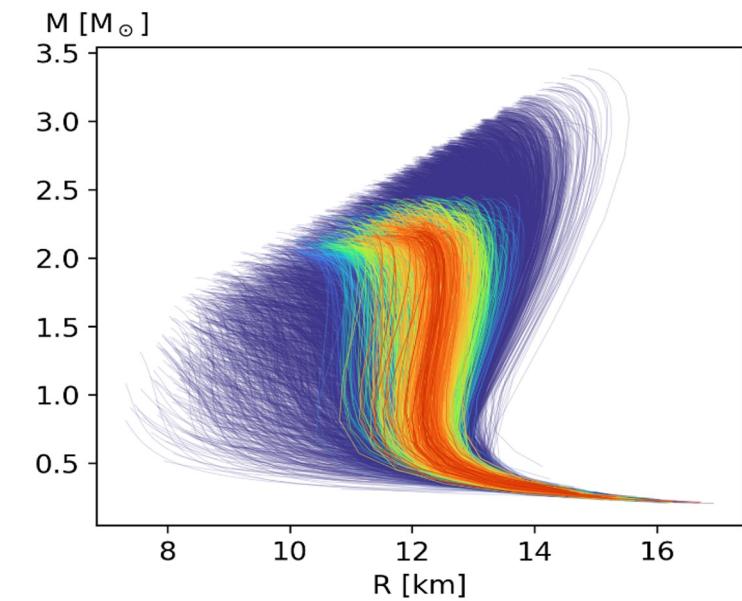
Inferred EoS:



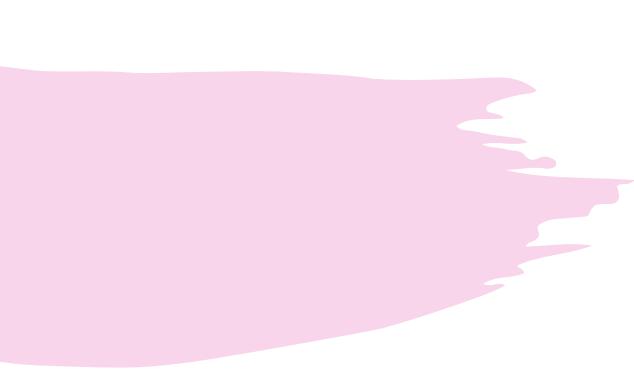
Speed of sound



Equation of state



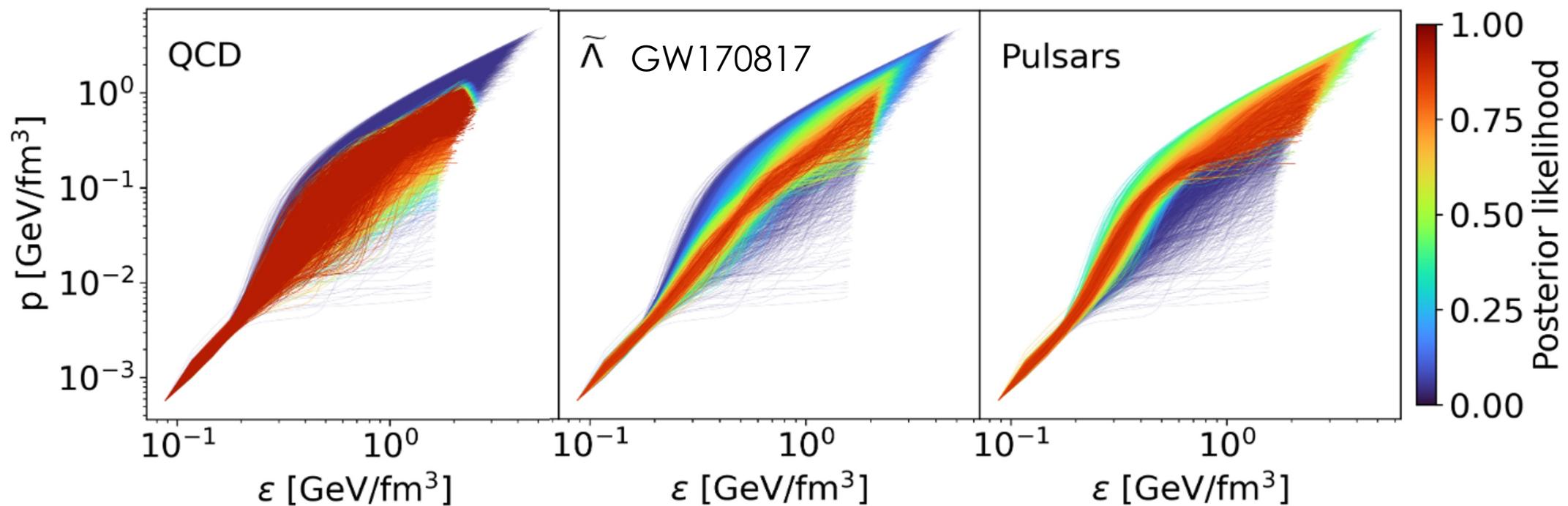
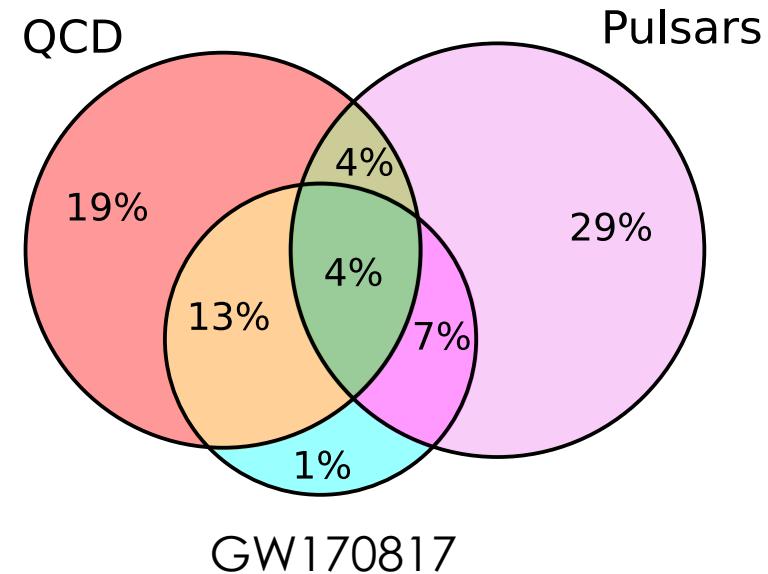
Mass-radius relation



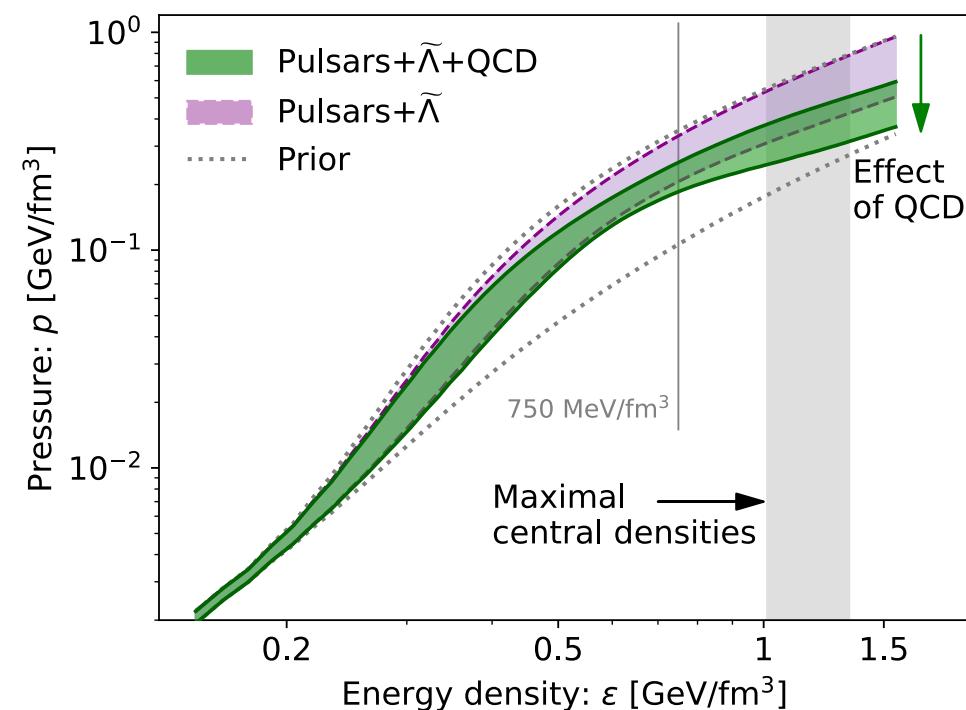
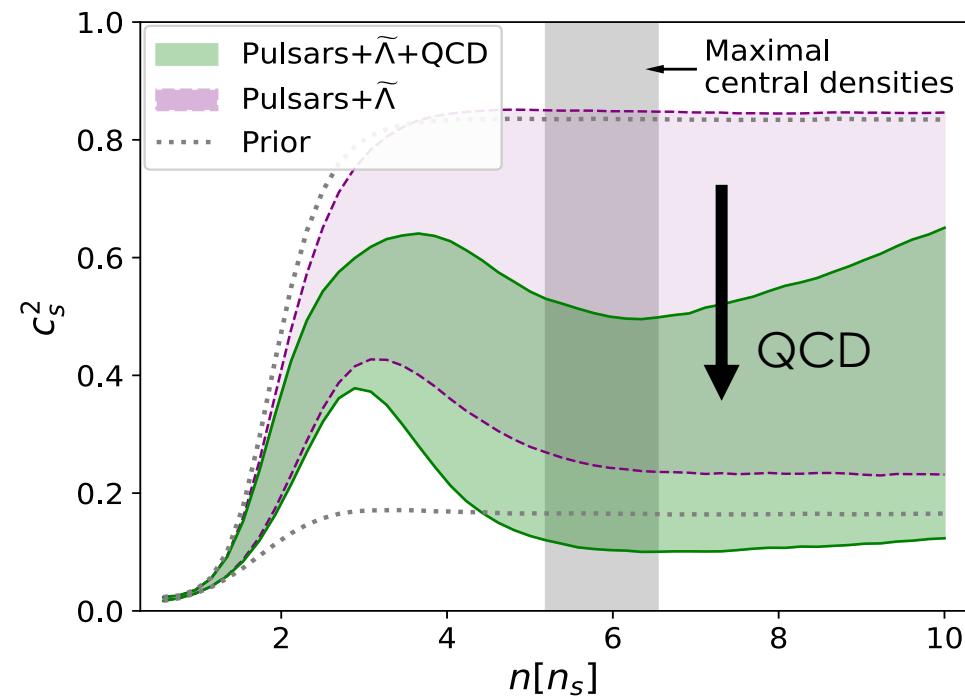
What did my QCD input do?

Effect of QCD:

QCD input complements NS observations



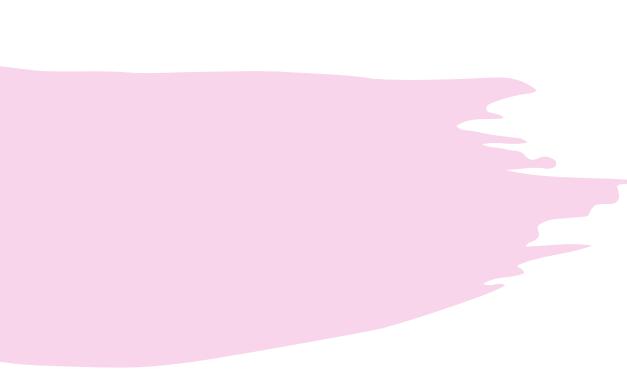
QCD responsible for the **softening**:



QCD pushes EoS towards conformality, **softening** at high densities

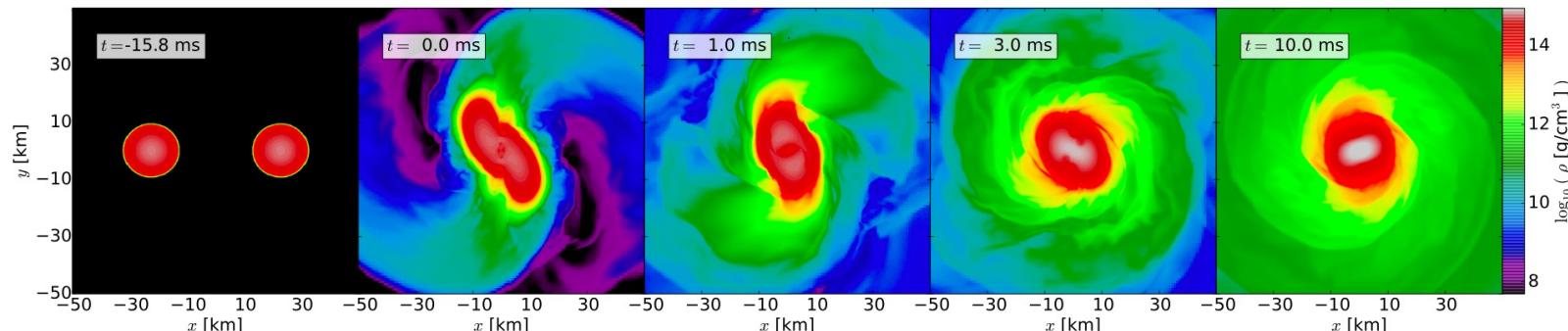
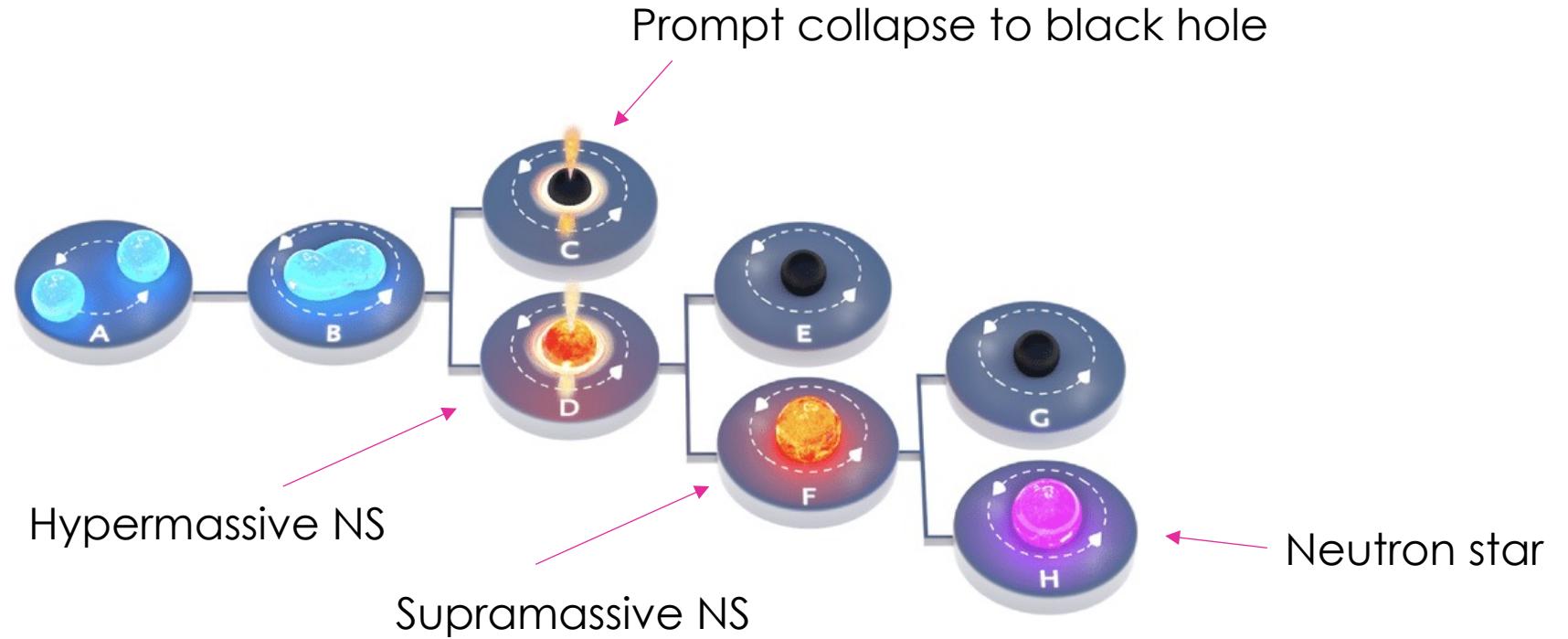
Note: QCD input knows **nothing** about c_s^2 of pQCD

c_s^2 pushed down because of the interplay of QCD and mass measurements

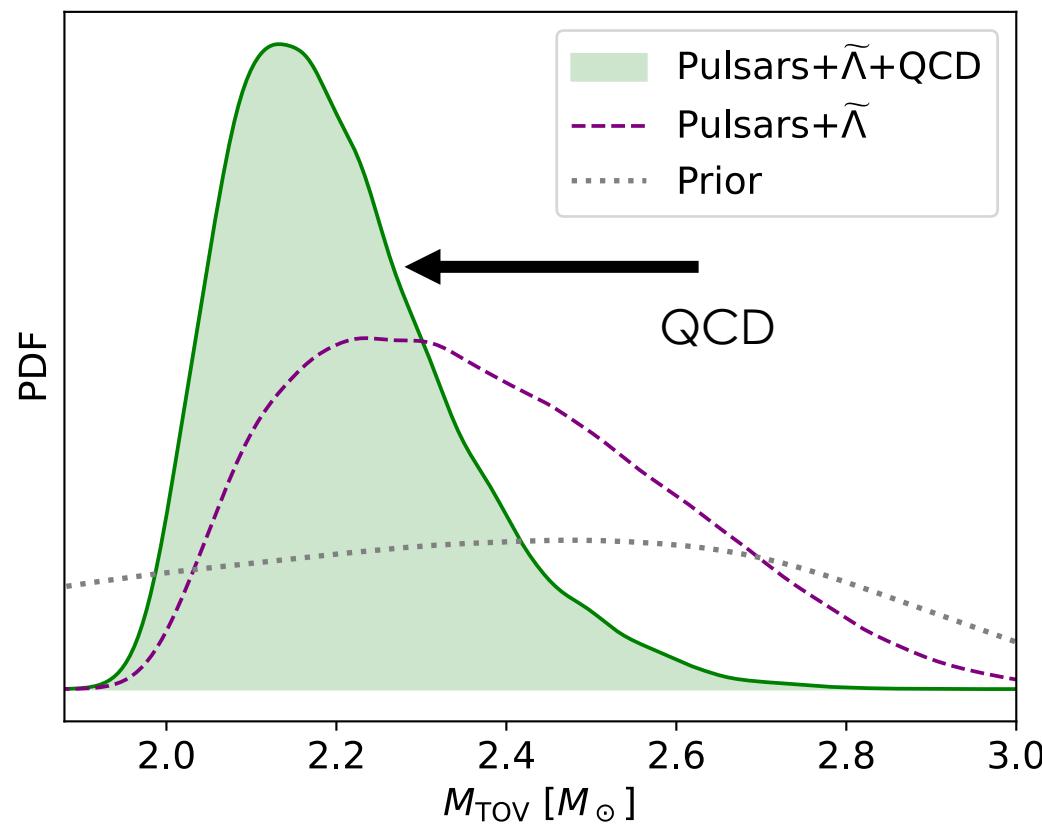


Can the **softening** be observationally verified?

Different binary merger products:

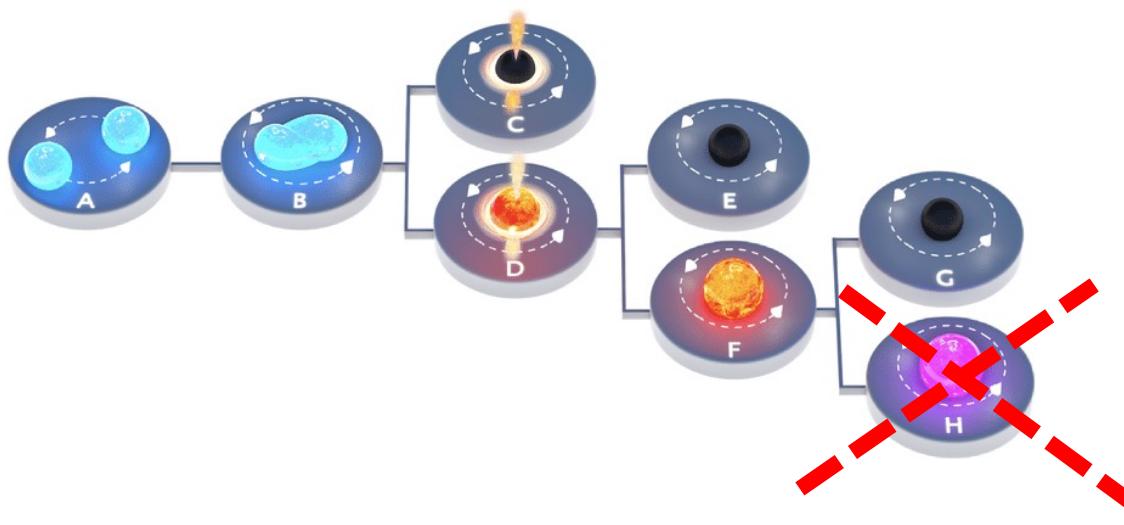
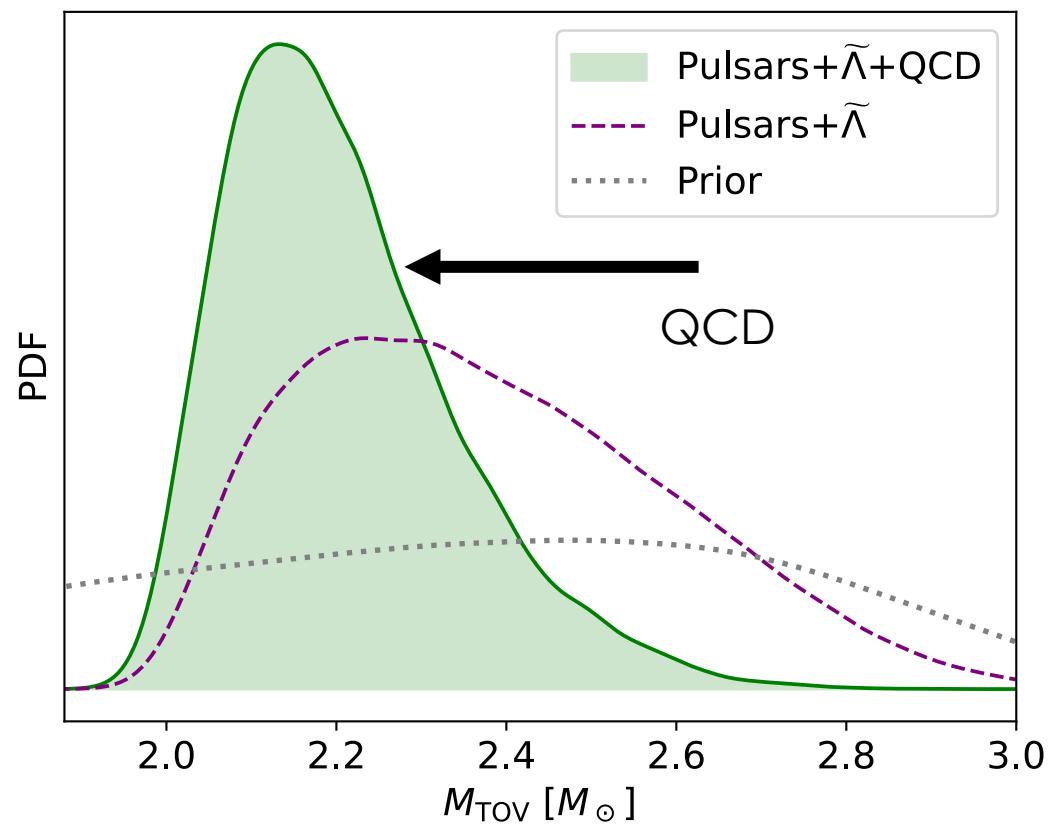


Effect of QCD:



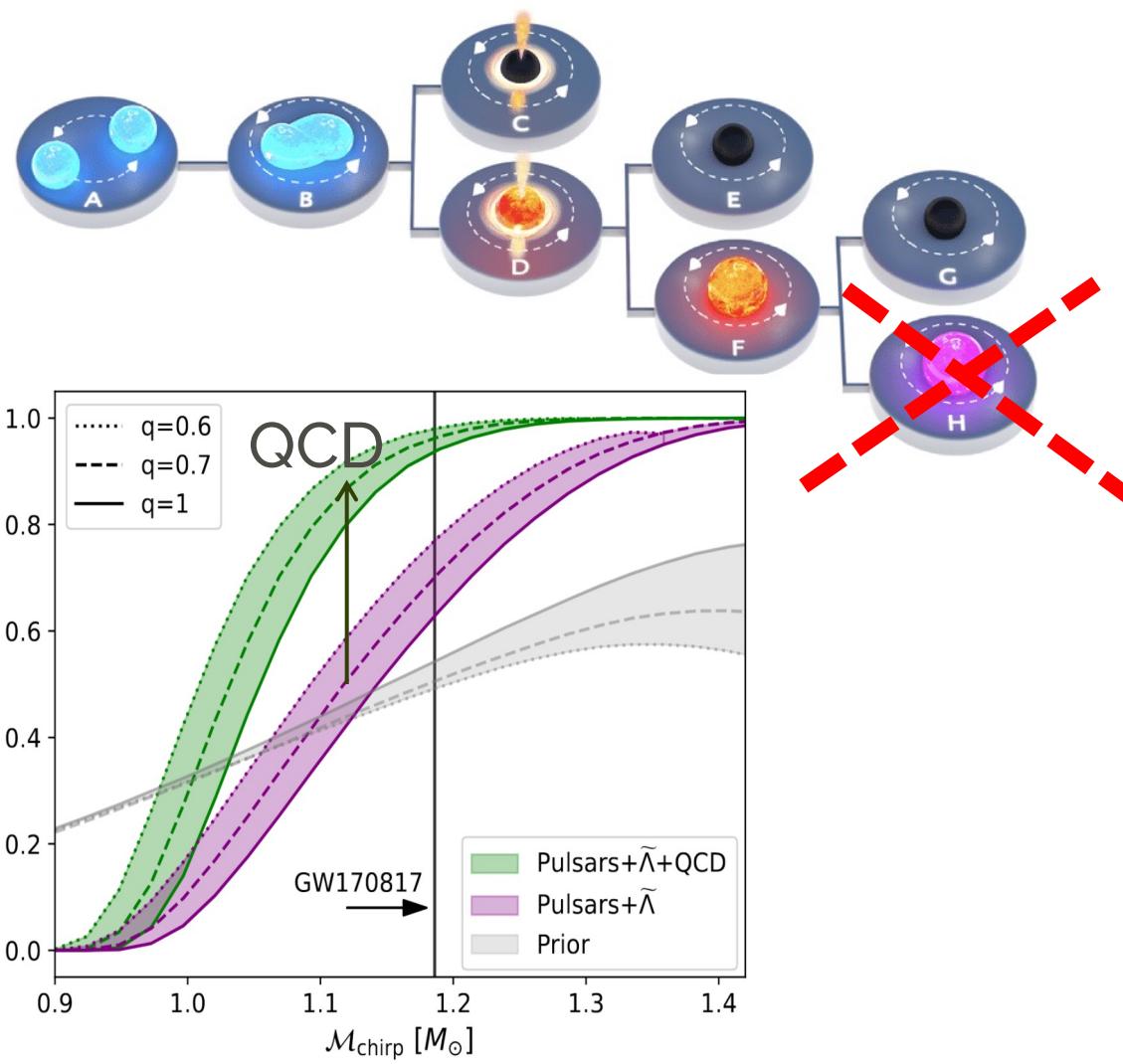
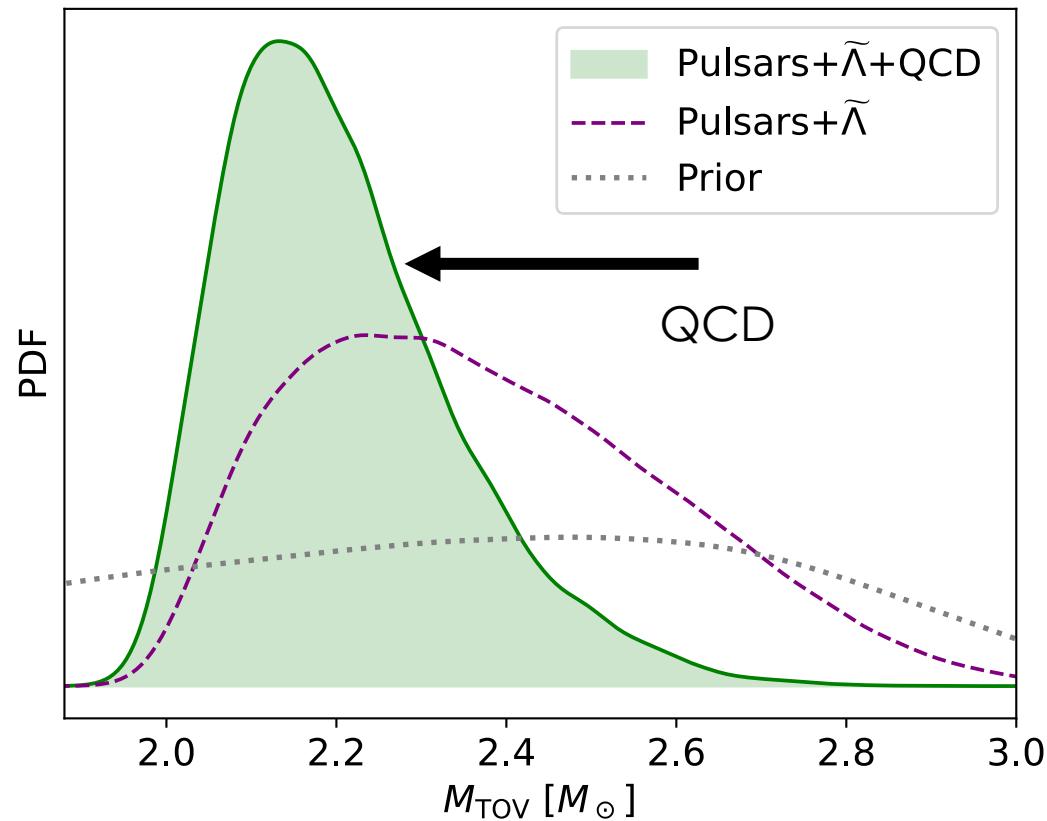
- **Softening** implies an upper limit for the maximal mass

Effect of QCD:



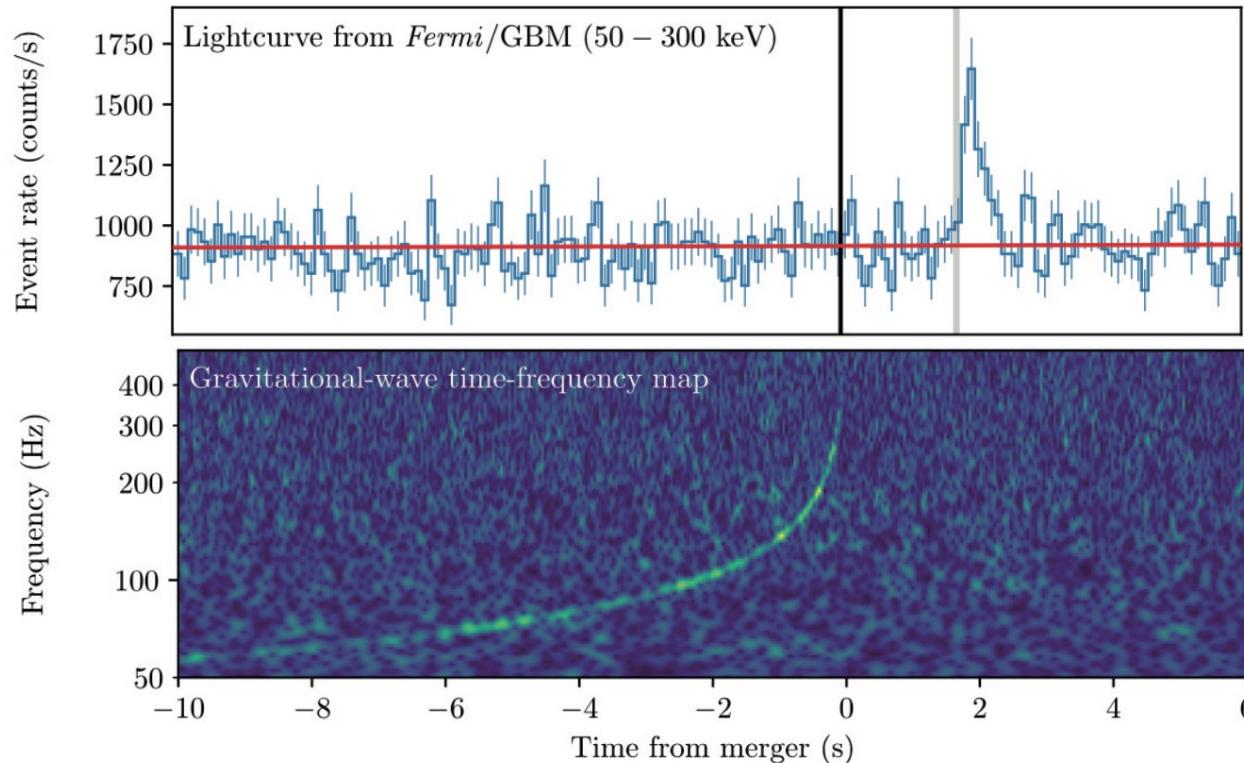
- **Softening** implies an upper limit for the maximal mass
- **Softening** implies BH formation in most mergers

Effect of QCD:



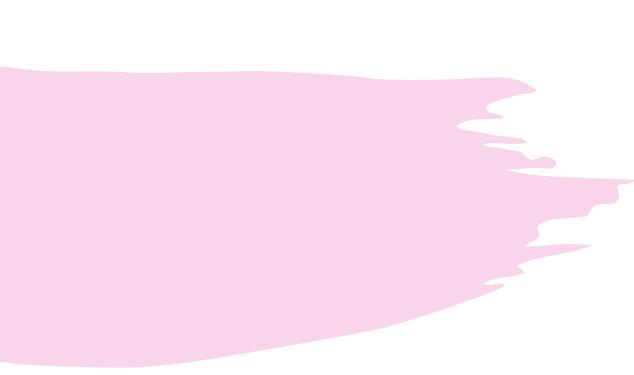
- **Softening** implies an upper limit for the maximal mass
- **Softening** implies BH formation in most mergers

Binary merger product of GW170817



- Astrophysical modelling suggests that GW170817 is consistent with collapse to BH
GRB \Rightarrow Jet \Rightarrow Ergoregion \Rightarrow Black Hole

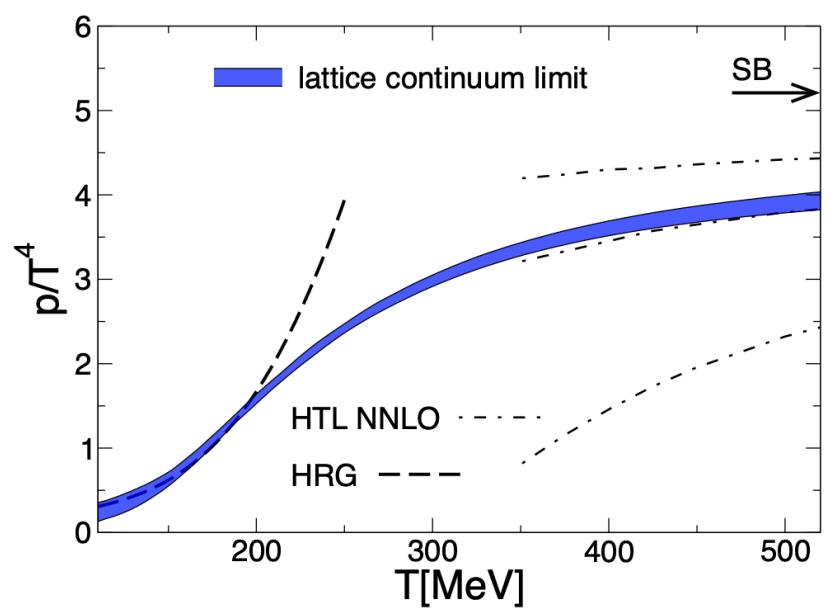
Margalit & Metzger ApJL 850 (2017)
Rezzolla, Most & Weih ApJL 852 (2018)



Is there **quark matter** in neutron stars?

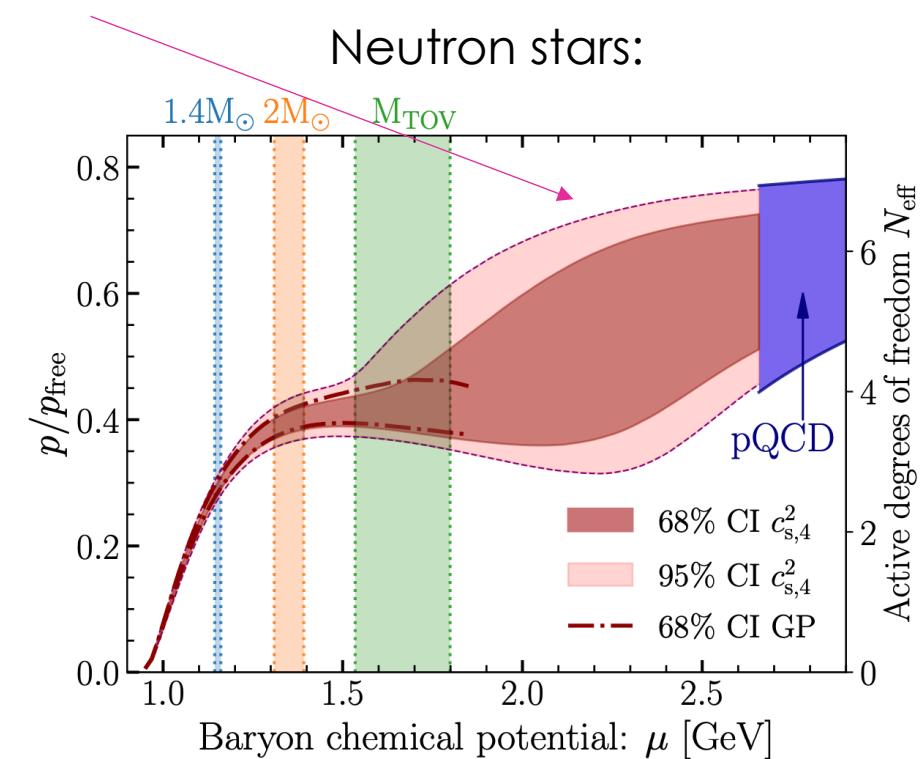
Softening interpreted as Quark Matter:

Heavy-ion collisions:



Nearly conformal matter with
d.o.f.s consistent with
Quark Matter

Neutron stars:



- Either QM or strong 1st order Phase Transition at M_{TOV}
 $P(QM) = 88\%, \quad P(PT) = 11\%$

Summary:

- $\{n, p, \varepsilon\}$ carries more information than $p(\varepsilon)$
- Stability, causality, and consistency
- QCD at $n = 40 n_s$ offers a **robust** constraint down to $n = 2.3 n_s$
- Results support findings of (some) earlier works with QCD
Annala et al. Phys.Rev.X 12 (2022), Altiparmak, Ecker, Rezzolla 2112.08157, ...
- QCD **softens** the EoS at high densities. **Quark Matter?**
Annala, Gorda, Kurkela, Nättilä, Vuorinen, Nature Phys. 16 (2020)
- QCD predicts that binary mergers lead to BHs

Discussion:

- QCD Complementary to astro. **No model uncertainites**
no transport models, no stellar models, no extrapolation in isospin, ...
- Can be used to test hypothesis (BSM, beyond GR)

Lope-Oter, Windisch, Llanes-Estrada, Alford, *J. Phys. G* (2019)

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Conclusion:

QCD and neutron stars offer interesting intersection of theoretically solid calculations combined with rapidly evolving observational field.