

Spinning Primordial Black Holes from First Order Phase Transitions

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Based on *Spinning Primordial Black Holes from First Order Phase Transition*, **IKB**, U.
K. Dey, JHEP 07 (2024) 006, arXiv: 2311.03406

- ① Introduction
- ② Creation Mechanism
- ③ Generation of PBH Spin: Broad PS
- ④ Dependence on FOPT Parameters
- ⑤ Future Scope

① Introduction

② Creation Mechanism

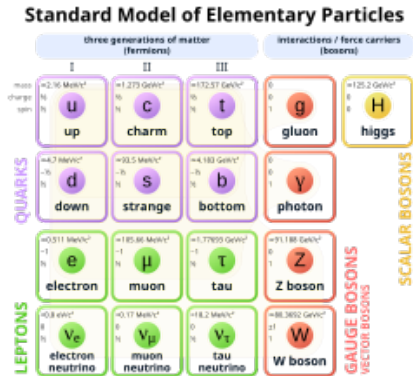
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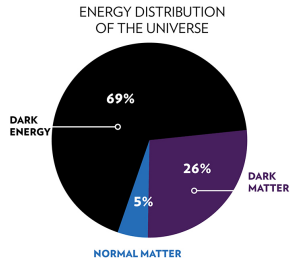
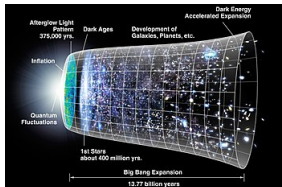
The current status: we know

- The standard model



The current status: we think we know

- The standard cosmology



FLRW metric

$$ds^2 = a^2(-d\eta^2 + dx^2 + dy^2 + dz^2)$$

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- many more ...

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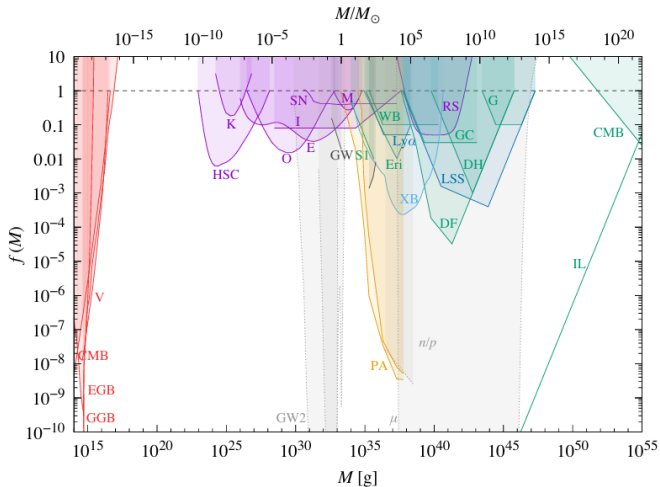
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- Can be expressed by its mass, spin and charge.

Constraints



(taken from arXiv: 2002.12778)

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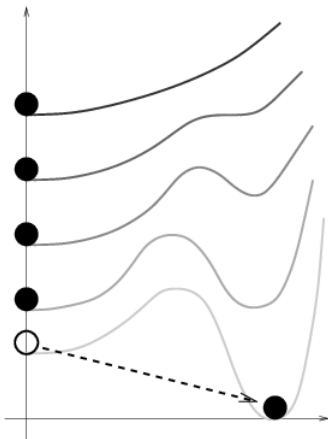
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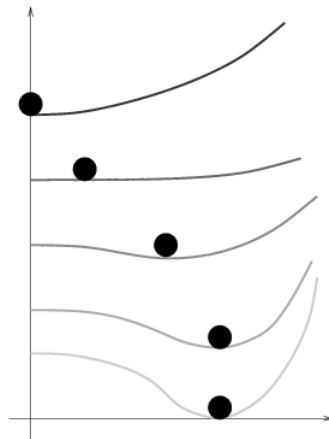
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- Physically, this corresponds to nucleation of true vacuum bubbles.

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FOPT



SOPT

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- ③ **Nucleation Temperature:**

$$T_n: (\Gamma/V)|_{T=T_n} \sim \mathcal{O}(1)$$

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Creation of PBHs

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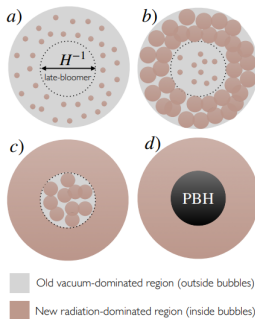
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- Curvature perturbations from FOPTs can arise through the difference in nucleation time of the true vacuum bubble in different Hubble patches.

Curvature Perturbation from FOPT

- FOPTs in the radiation dominated universe generates a curvature perturbation of the form

$$\mathcal{P}_\zeta = A^2(\alpha, \beta/H, \dots)(kR_{\mathcal{H}})^3.$$



taken from arXiv: 2305.04942

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- For super-strong (supercooled) and slow FOPTs ($\alpha \gg 100$) (2402.04158),

$$A = f(\beta/H) \propto (\beta/H)^{-5/2}$$

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PBH spin: Broad Curvature PS

Based on *Spin of Primordial Black Holes from Broad Power Spectrum: Radiation Dominated Universe*, **IKB**, T. Harada, 2409.06494

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 - 4 Reference and RMS Spin

Spectral Moments

- The spectral index of the density perturbation can be expressed as

$$\sigma_n^2 = \frac{4}{9} \eta_{\text{init}}^4 \int_{k_{\text{max}}/r_k}^{k_{\text{max}}} \frac{dk}{k} k^{2n+4} \mathcal{P}(k),$$

where $r_k = k_{\text{max}}/k_{\text{min}}$.

- For our power spectrum, the spectral moments take the form,

$$\sigma_n^2 = \frac{4}{9} \eta_{\text{init}}^4 A^2 k_{\text{max}}^{4+2n} \frac{1 - r_k^{-7-2n}}{7 + 2n}.$$

- Furthermore, if we consider $k_{\text{min}} \ll k_{\text{max}}$,

$$\sigma_n^2 \approx \frac{4}{9} \eta_{\text{init}}^4 A^2 k_{\text{max}}^{4+2n} \frac{1}{7 + 2n}.$$

Profile Shape

- The density perturbation,

$$\delta_{\text{CMC}}(\eta, \mathbf{r}) = \delta_{pk} g_{\delta}(r; k_{\delta}).$$

- The density profile for the case of the broad power spectrum,

$$g_{\delta}(r; k_{\delta}) = a_{\delta}^* - b_{\delta}^*(k_{\text{max}} r)^2,$$

where

$$a_{\delta}^* \approx 13.76 - 16.5\alpha_{\delta},$$

$$b_{\delta}^* \approx 3.21\alpha_{\delta} - 2.33,$$

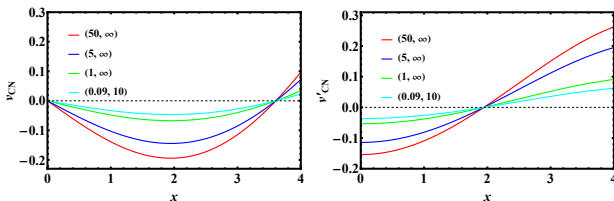
and $\alpha_{\delta} = k_{\delta}^2/k_{\text{max}}^2$.

- Density perturbation at $k_{\delta} = k_{c\delta} = \sigma_1/\sigma_0$,

$$g_{\delta}(r; k_{c\delta}) = \psi_{\delta}(r) = 1 - \frac{7}{54}(k_{\text{max}} r)^2 = 1 - \frac{(rk_{c\delta})^2}{6}.$$

Turn Around Point

- The point in time when the overdense region decouples from the background and the process of the collapse start.
- In case of radiation domination, irrespective of the dependence on k , $x_{\text{ta}} = 1.95$.



Dependence of the velocity and the change in velocity on $x = k\eta$. The different curves represent different values of (j, r_k) where $\mathcal{P}_\zeta \propto k^j$.

Reference and RMS Spin

- The reference spin of the overdense region at turn around point,

$$A_{\text{ref}}(\eta_{\text{ta}}) = \frac{\frac{4}{3} [a^4 \rho_b g_{\text{CN}}]_{\eta=\eta_{\text{ta}}} (1-f)^{5/2} R_*^5}{GM_{\text{ta}}^2}.$$

where,

$$R_* = \sqrt{3} \frac{\sigma_1}{\sigma_2},$$

$$g_{\text{CN}} = \frac{2}{3} A k_{\text{max}} G,$$

$$G^2 = \int_0^1 dx x T_{v_{\text{CN}}}^2(x) x^3,$$

$$T_{v_{\text{CN}}}^2(x) = \frac{\sqrt{3}}{8} \frac{((x/\sqrt{3})^2 - 2) \sin(x/\sqrt{3}) + 2(x/\sqrt{3}) \cos(x/\sqrt{3})}{(x/\sqrt{3})^2}.$$

Reference and RMS Spin

- Simplifying this for our case we find,

$$A_{\text{ref}}(\eta_{\text{ta}}) = 0.00286A(1-f)^{-1/2}.$$

- The RMS spin of the region can be expressed as,

$$\sqrt{\langle a_*^2 \rangle} = A_{\text{ref}} \times 5.96 \times \frac{\sqrt{1-\gamma^2}}{\gamma^6 \nu},$$

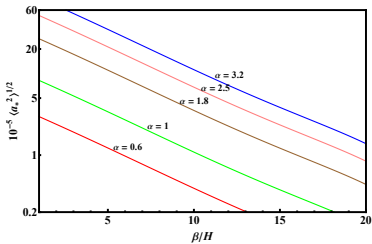
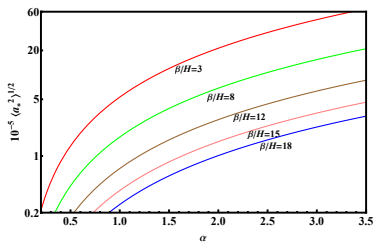
where $\nu = 1.92/\sigma_0$, and $\gamma = \sigma_1^2/(\sigma_0\sigma_2)$.

- Finally, the RMS spin can be expressed as,

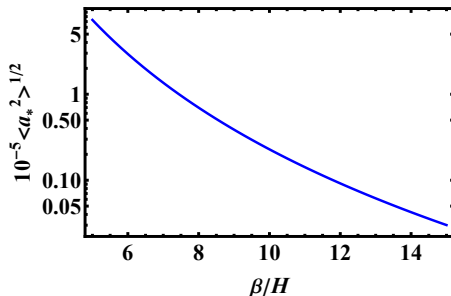
$$\sqrt{\langle a_*^2 \rangle} = 3.4 \times 10^{-4} \left(\frac{M_{\text{PBH}}}{M_H} \right)^{-1/3} A^2$$

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Case-I: $\alpha \sim \mathcal{O}(1)$



Dependence of the RMS spin of a PBH population on α and β/H .

Case-II: $\alpha \gg 1$ 

Dependence of the RMS spin of a PBH population on β/H for $\alpha \gg 1$.

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Future Scope

- Some studies have shown that the value of j may vary depending on the FOPT parameters and value of k . This scheme of calculation can be modified to account for those cases as well.
- FOPTs in Non-standard cosmology, such as some early matter-dominated era, may give rise to PBH with high initial spin, which has implications in Hawking evaporation superradiant instability, gravitational waves, etc.

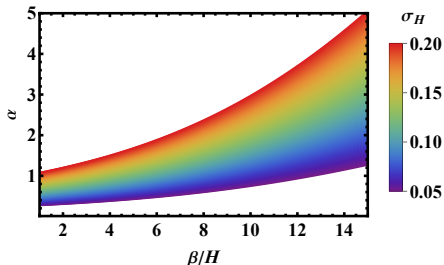
Thanks!

Behaviour of $\mathcal{P}_\zeta$ for $\alpha \sim \mathcal{O}(1)$

The form of the curvature perturbation in this case can be expressed as,

$$\mathcal{P}_\zeta(k) = 34.5[\sigma_H(\alpha, \beta/H)]^2 (kR_{\mathcal{H}})^3,$$

where the behaviour of the function $\sigma_H(\alpha, \beta/H)$ can be expressed as,



Solution from Cosmological Linear Perturbation Theory I

The gauge invariant quantities corresponding to the density and the velocity perturbations can be expressed as,

$$\Delta(x) = D\sqrt{3} \left(\frac{\sin z}{z} - \cos z \right),$$
$$V(x) = D \left[\frac{3}{4} \left(\frac{2}{z^2} - 1 \right) \sin z - \frac{3}{2} \frac{\cos z}{z} \right],$$

where D is an arbitrary constant, whose value depends on the shape of perturbation, $z = x/\sqrt{3}$, and $x = k\eta$. For the CMC

Solution from Cosmological Linear Perturbation Theory II

gauge, the density perturbation and the velocity of the region can be expressed as

$$\delta_{\text{CMC}} = D \frac{\sqrt{3} z^2}{z^2 + 2} \left(2 \frac{\sin z}{z} - \cos z \right),$$

$$v_{\text{CMC}} = -\frac{3}{4} D \frac{(z^2 - 2) \sin z + 2z \cos z}{z^2 + 2}.$$

For the conformal Newtonian gauge, the quantities take the form

$$\delta_{\text{CN}} = \sqrt{3} D \frac{2(z^2 - 1) \sin z + (2 - z^2) z \cos z}{z^4},$$

$$v_{\text{CN}} = \frac{3}{4} D \frac{(2 - z^2) \sin z - 2z \cos z}{z^2}.$$