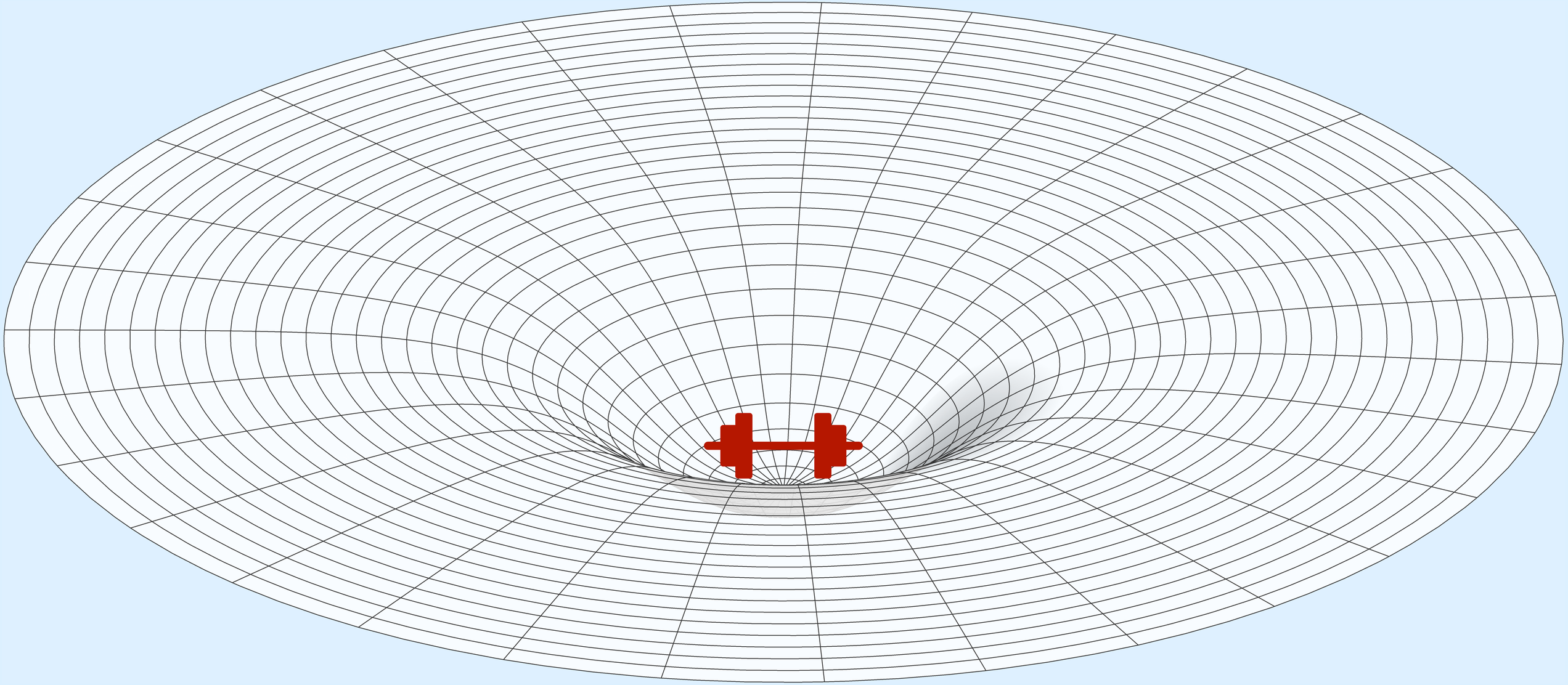
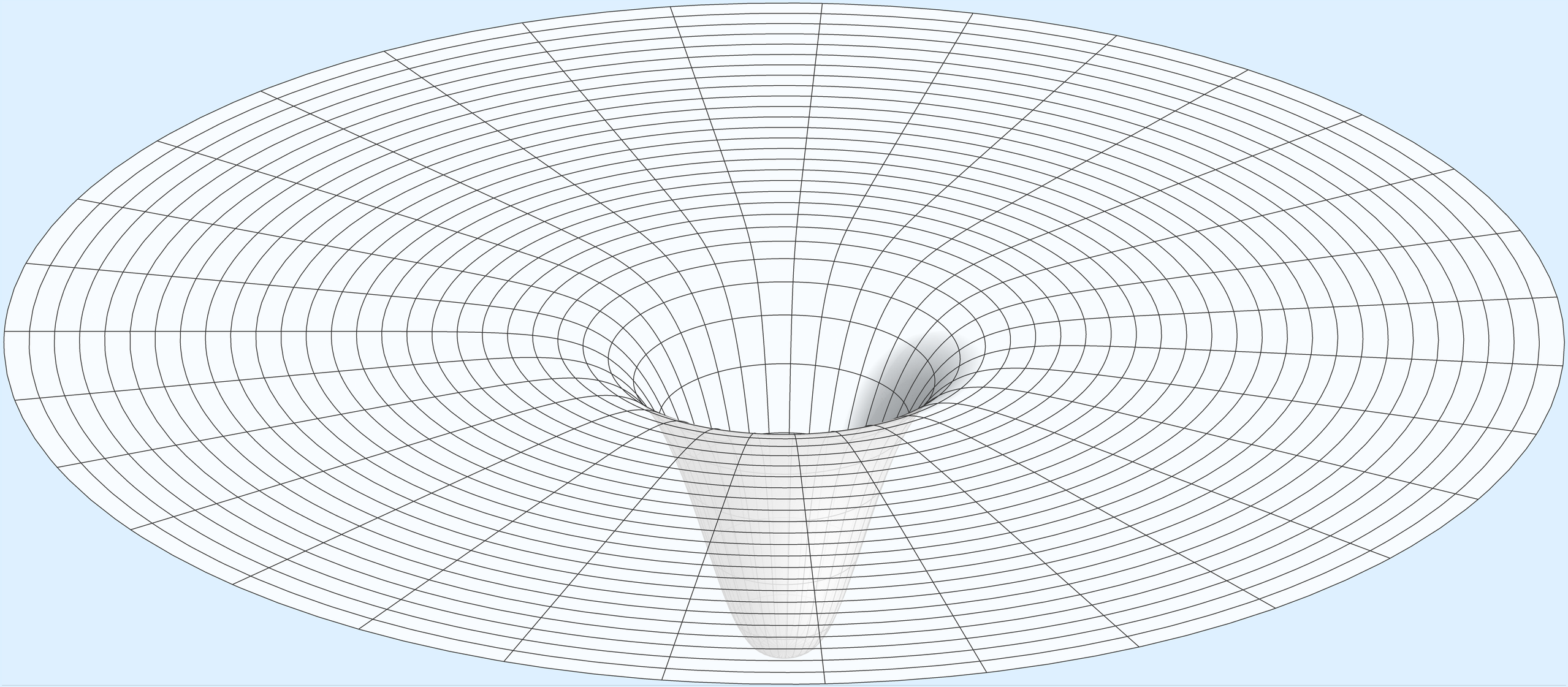


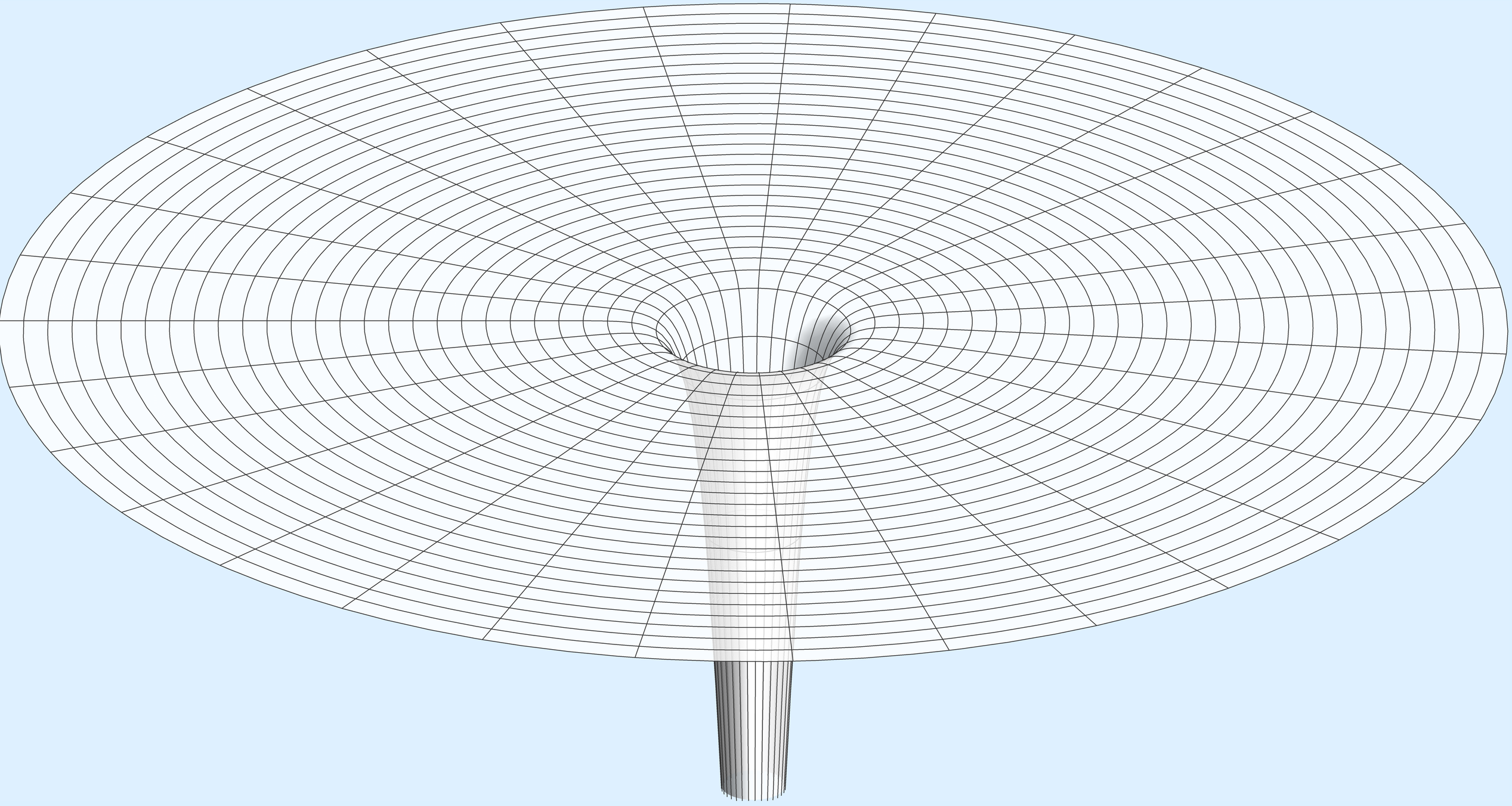
Is the end of the Information Problem on the Horizon?

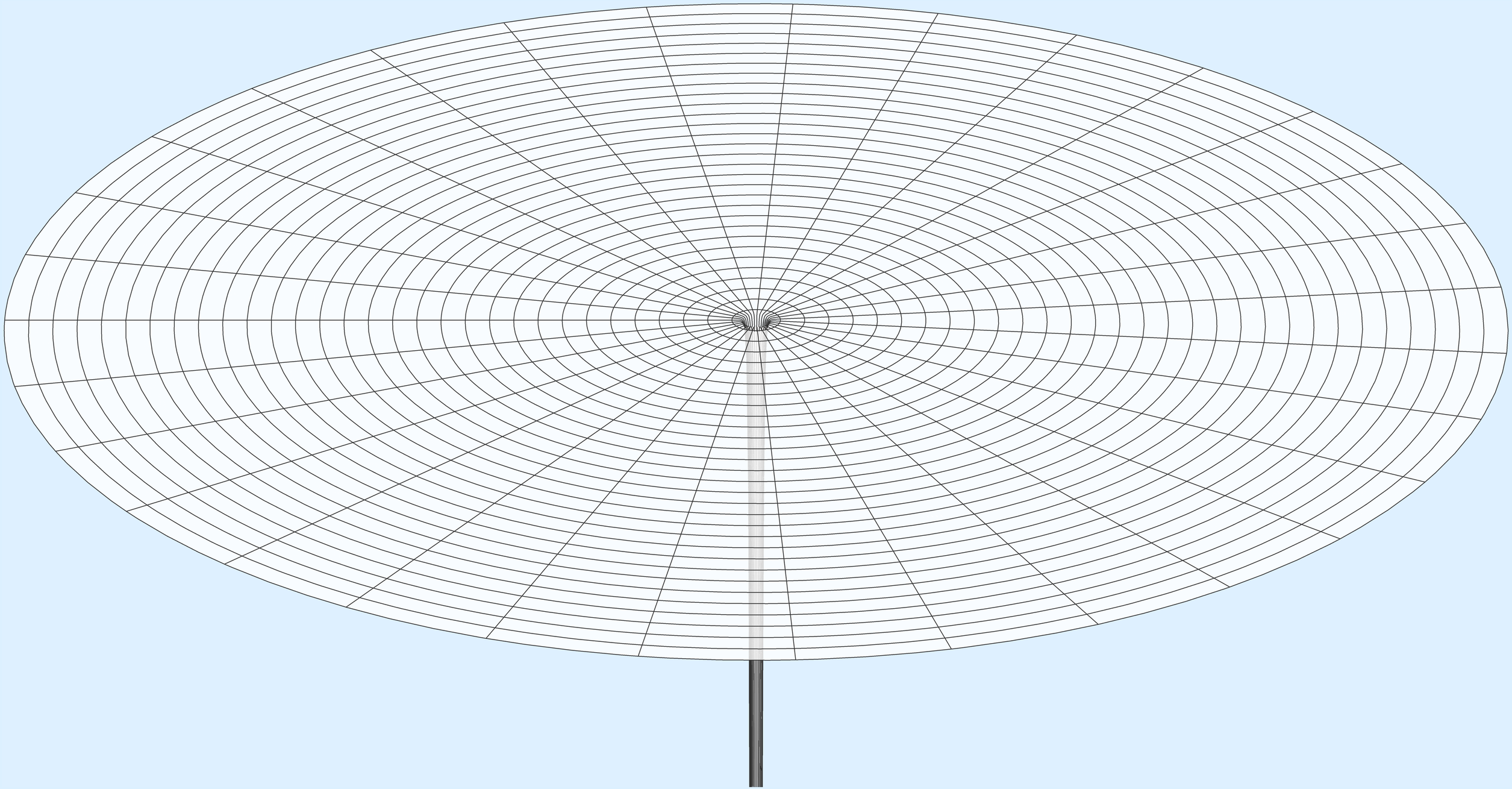
Åsmund Folkestad (UC Davis)

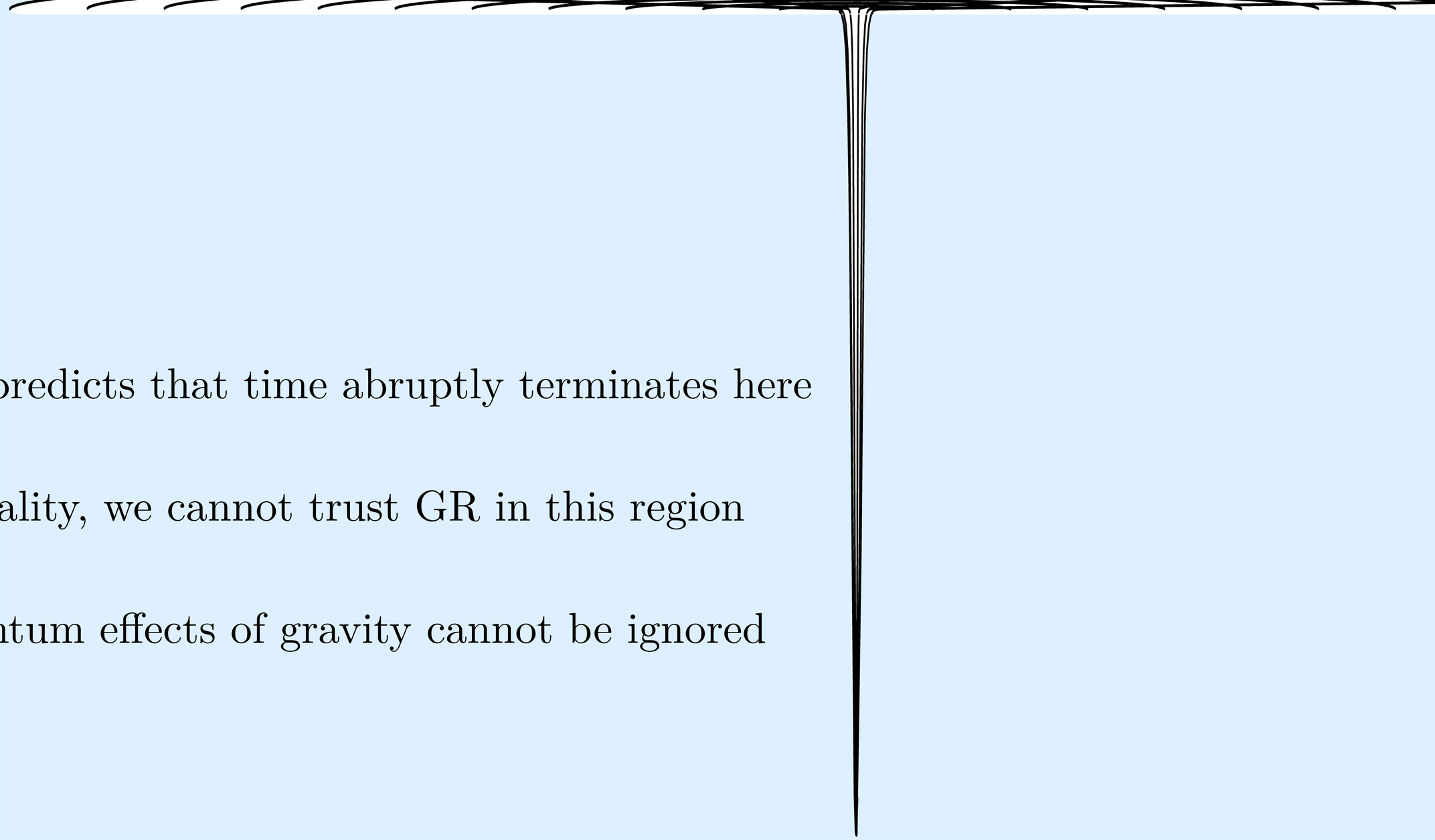
UiO seminar | October 9, 2024







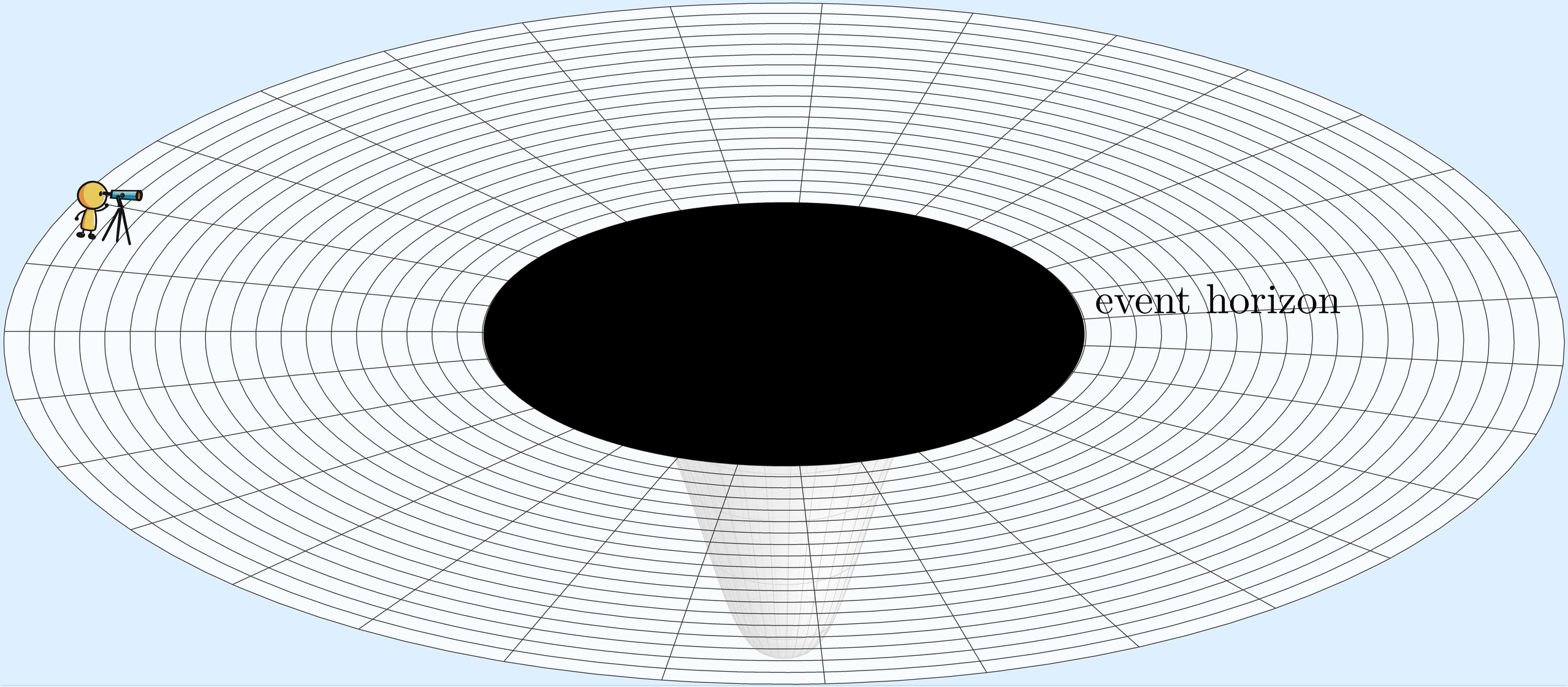




GR predicts that time abruptly terminates here

In reality, we cannot trust GR in this region

Quantum effects of gravity cannot be ignored


event horizon

If we completely understand the quantum nature of black holes, we likely will gain significant insight into the early universe as well

Right now: an exciting time for the study of quantum black holes

Plan

- Refresher: Information in quantum mechanics
- The Page curve
- Black holes and holography
- New computations of information preservation

Refresher: Information in Quantum Mechanics

Assume perfect knowledge about a closed quantum mechanical system

$$|\psi\rangle \in \mathcal{H}$$

$$|\psi(t)\rangle = U(t)\,|\psi(0)\rangle \qquad\qquad U(t)^\dagger U(t) = \mathbf{1}$$

Imperfect info: we must replace pure states with density matrices

$$\rho = \sum_i p_i \left|i\right\rangle \left\langle i\right|$$

$$\rho \geq 0, \qquad \rho^\dagger = \rho, \qquad \mathrm{Tr}(\rho) = 1$$

Let Alice prepare a quantum state for Bob by flipping a coin

Heads: spin up, tails: spin down

$$\rho_{\text{Bob}} = \frac{1}{2} \left|\uparrow_z\right\rangle \left\langle\uparrow_z\right| + \frac{1}{2} \left|\downarrow_z\right\rangle \left\langle\downarrow_z\right| = \frac{1}{2}\mathbb{I}$$

$$S_{\mathrm{vN}}[\rho] = -\operatorname{tr}(\rho \log \rho)$$

- Measures the amount of information we lack about the state ρ
- $S[|\psi\rangle \langle\psi|] = 0$
- $S[\rho_{\mathrm{Bob}}] = 1$
- $S[\rho_{\mathrm{general}}] \leq \log \dim \mathcal{H}$

for a **closed** system

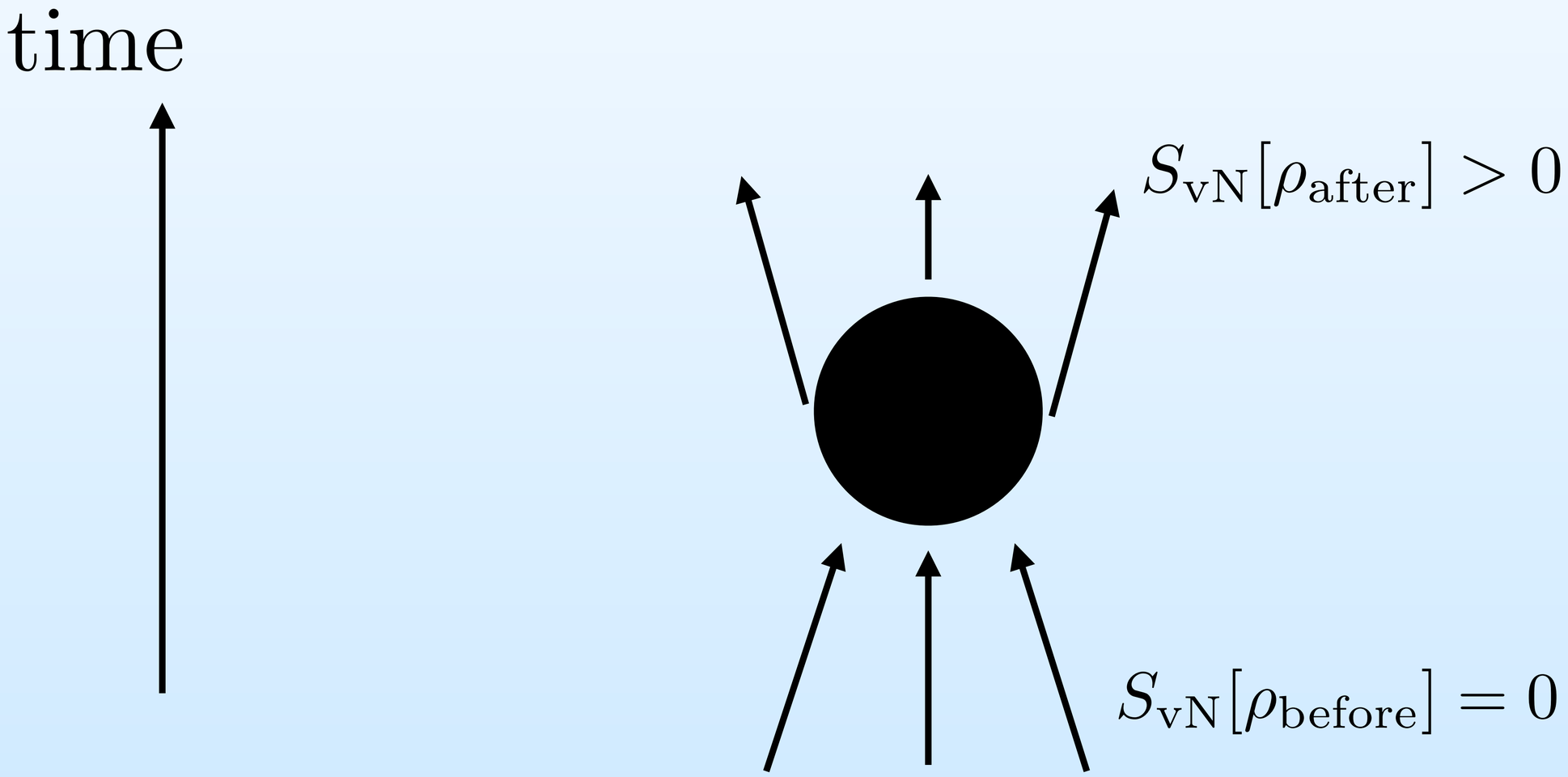
$$\rho(t) = U(t)\rho(0)U(t)^\dagger$$

A theorem

$$S_{\mathrm{vN}}(U(t)^{\dagger}\rho U(t)) = S_{\mathrm{vN}}(\rho)$$

If you keep track of all degrees of freedom, information is never lost

The info problem in a nutshell



[Hawking '74, '75, '76]

contradicts

$$\rho_{\text{after}} = U \rho_{\text{before}} U^{\dagger}$$

The Page curve

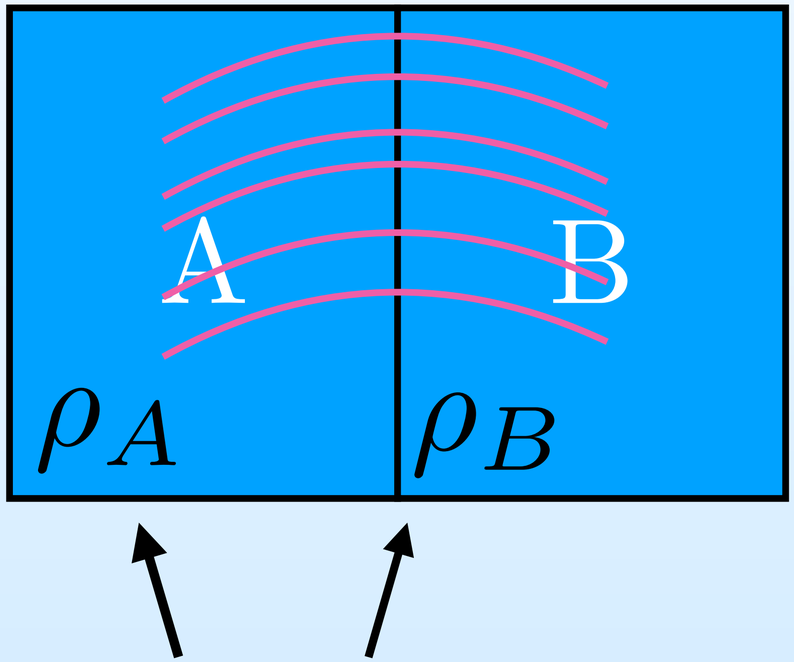
Example: let Alice and Bob each hold a spin that jointly is in an entangled Bell state

$$|\psi\rangle = |\uparrow\rangle_A |\uparrow\rangle_B + |\downarrow\rangle_A |\downarrow\rangle_B \in \mathcal{H}_A \otimes \mathcal{H}_B$$

$$\rho_B = \mathrm{tr}_A |\psi\rangle\langle\psi| = \sum_i \langle i|_A \rho |i\rangle_A = \frac{1}{2}\mathbb{I}$$

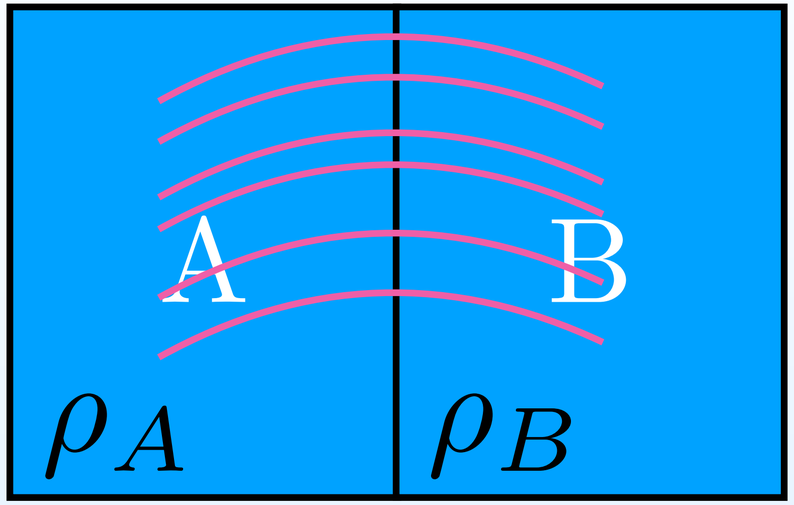
From Bob's perspective, who has access to only one of the qubits in the entangled pair, the state is completely equivalent to the probabilistically generated state from earlier

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$



Almost always mixed!

$$|\psi\rangle_{AB}$$

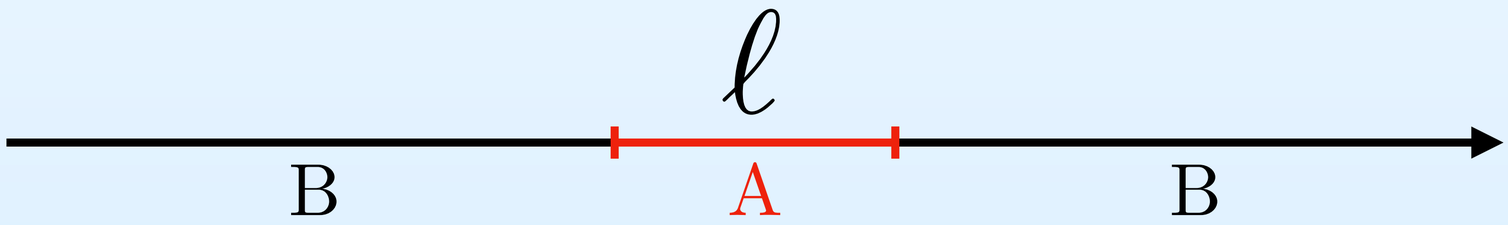


Thm: state on AB pure $\Rightarrow S_A = S_B$

$S_A = S_B$ measures the effective number of entangled qubits between A and B

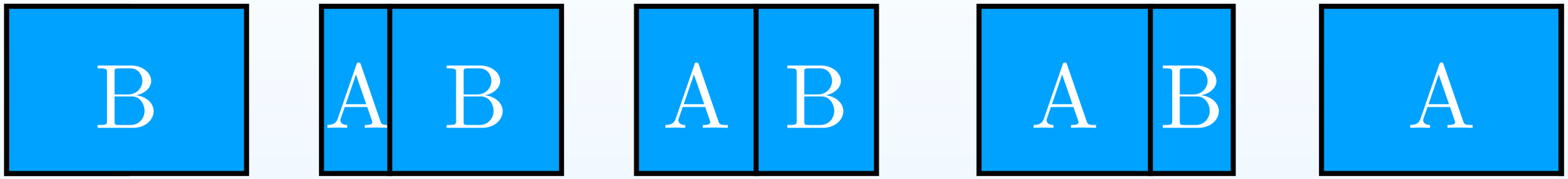
An example

Consder a 1+1 dimensional conformally invariant QFT (a CFT) in the vacuum state



$$S_{\mathrm{vN}}[\rho_A] = \frac{c}{3} \log \frac{\ell}{\epsilon_{\mathrm{UV}}}$$

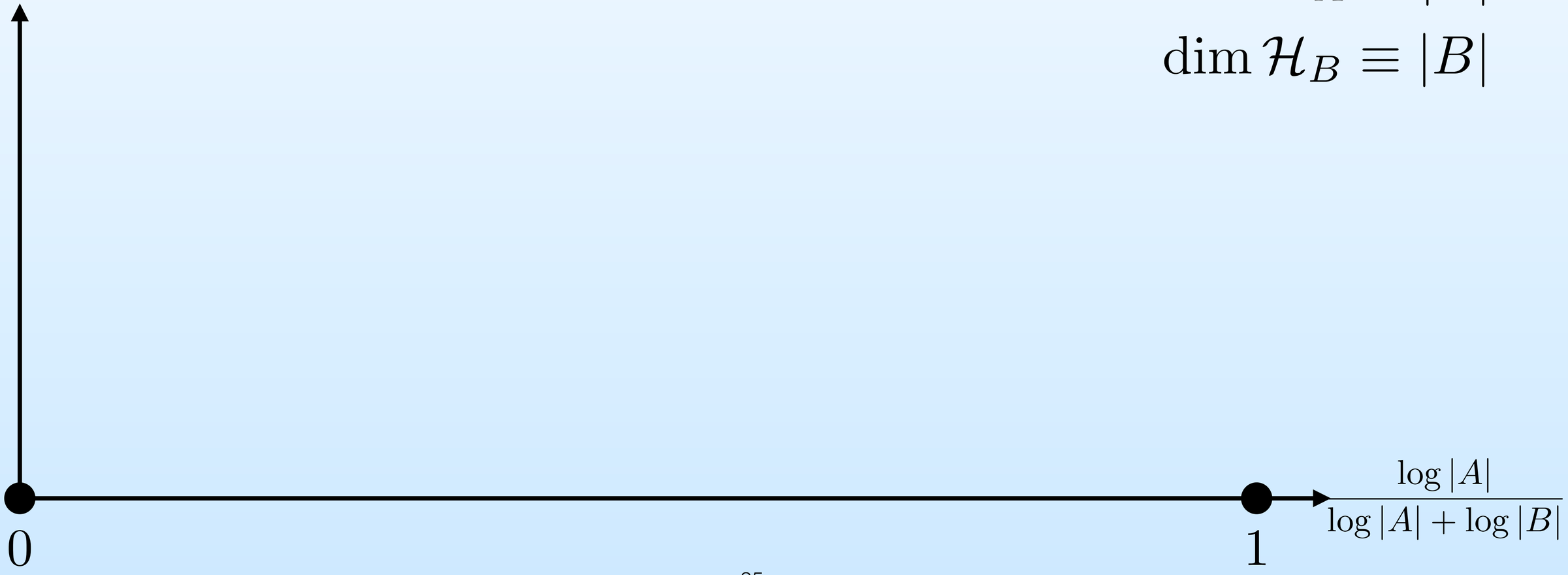
[Holzhey et al. '94]



$S_A = S_B$

$\dim \mathcal{H}_A \equiv |A|$

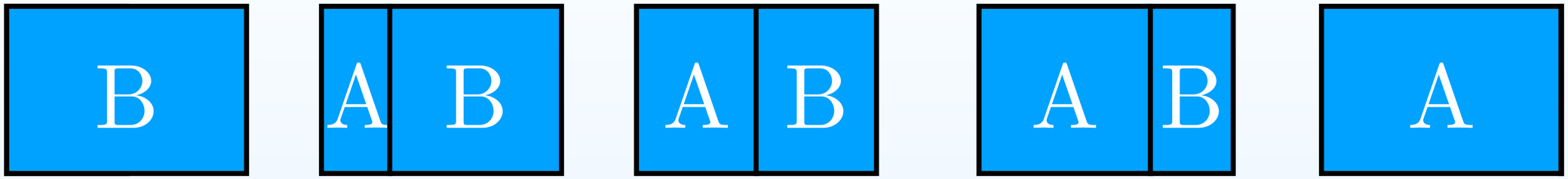
$\dim \mathcal{H}_B \equiv |B|$

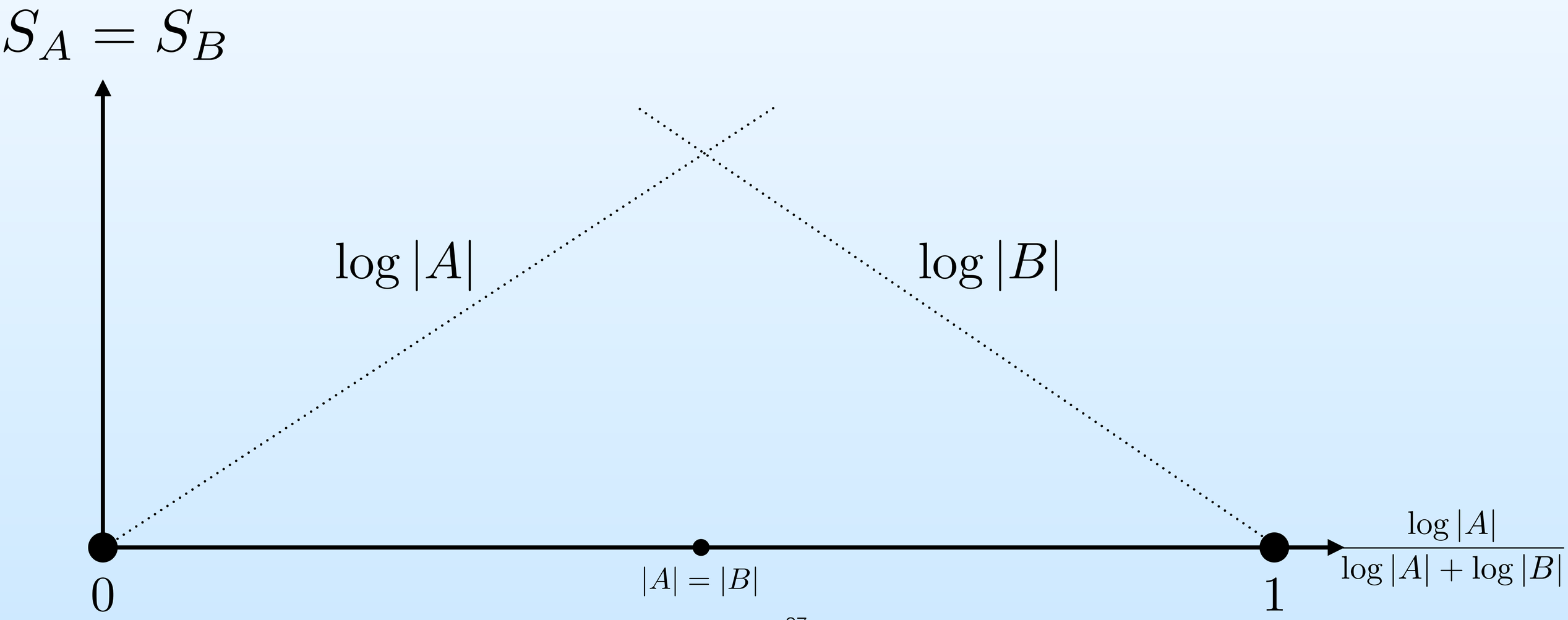


$$S_A = S_B \leq \min(\log|\mathrm{A}|, \log|\mathrm{B}|)$$



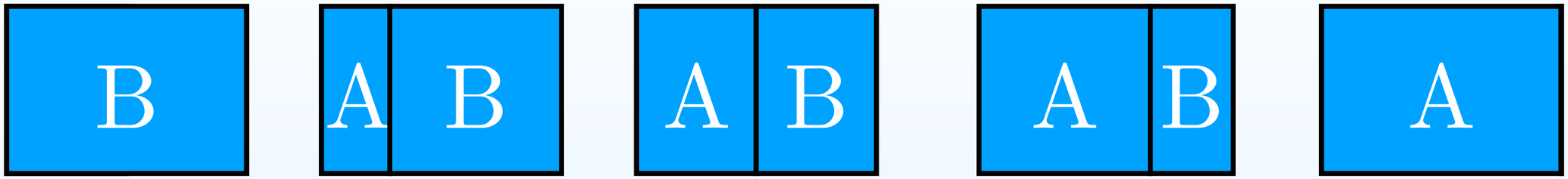
$$S[\rho] \leq \log \dim \mathcal{H}$$

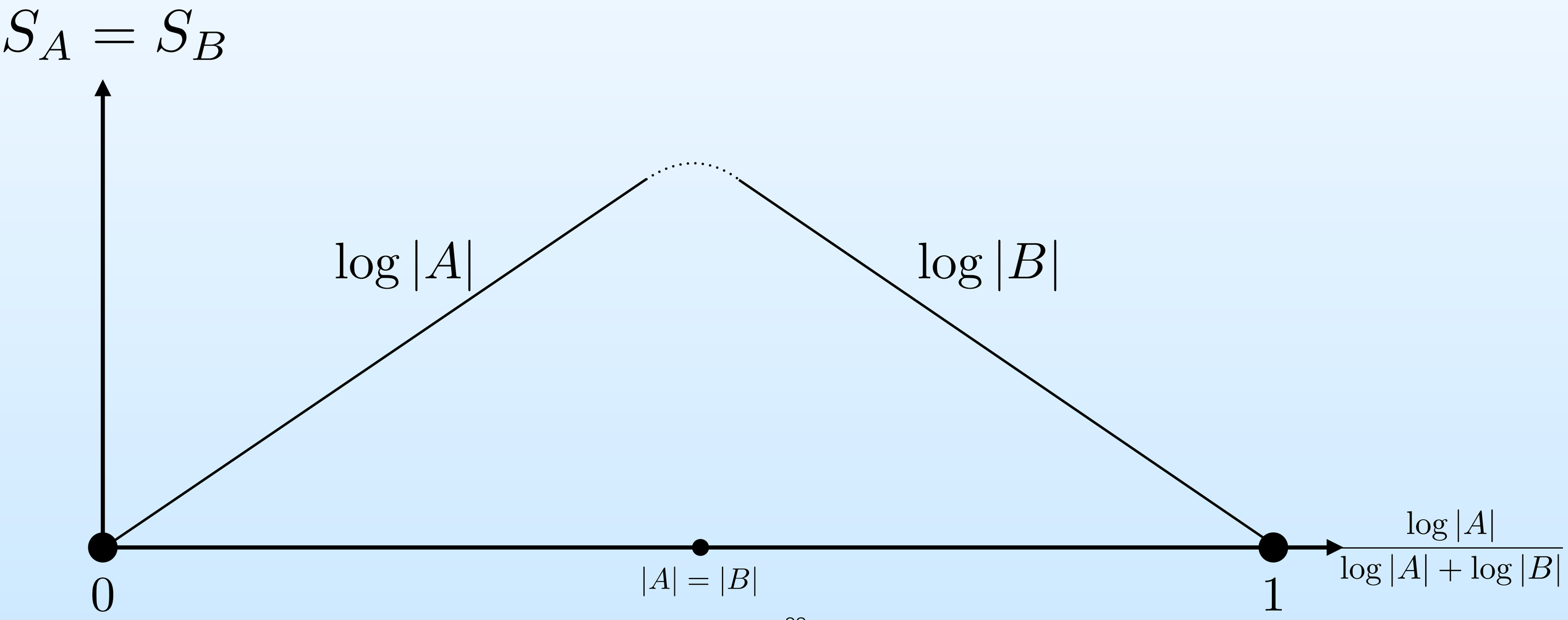

B
A
B
A
B
A
B
A

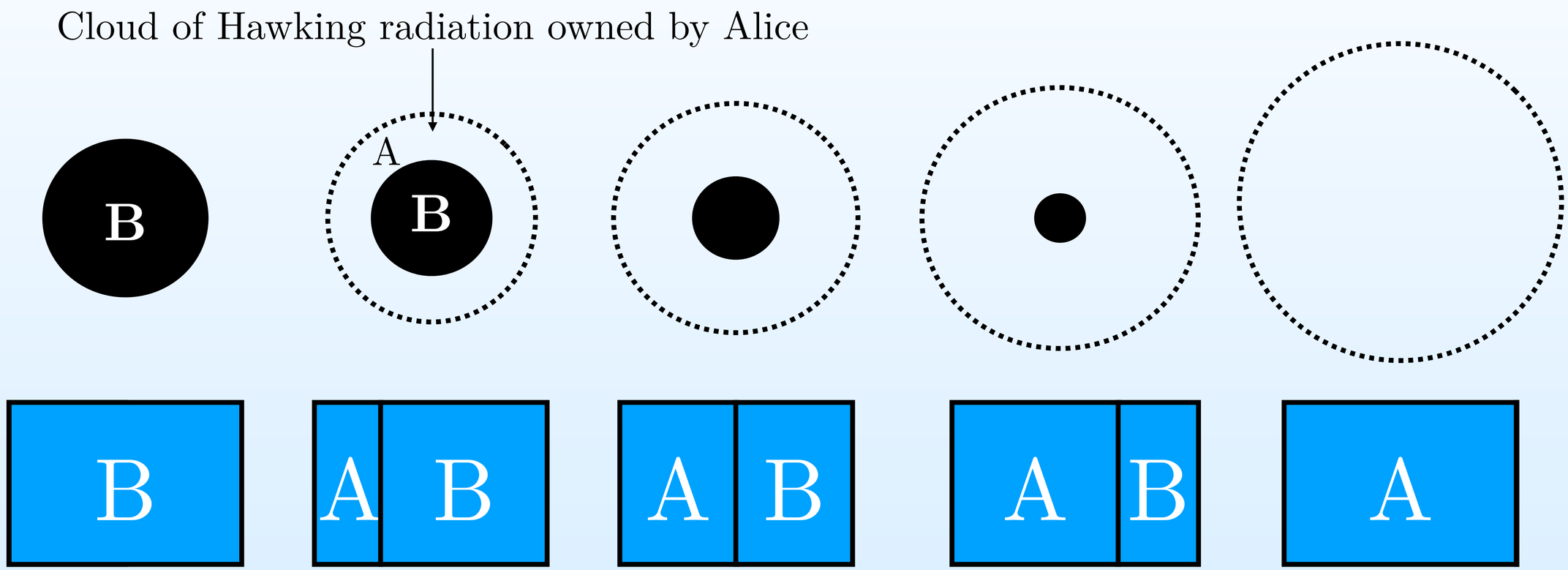

$S_A = S_B$
$\log|A|$
$\log|B|$
0
$|A| = |B|$
1
$\frac{\log|A|}{\log|A| + \log|B|}$

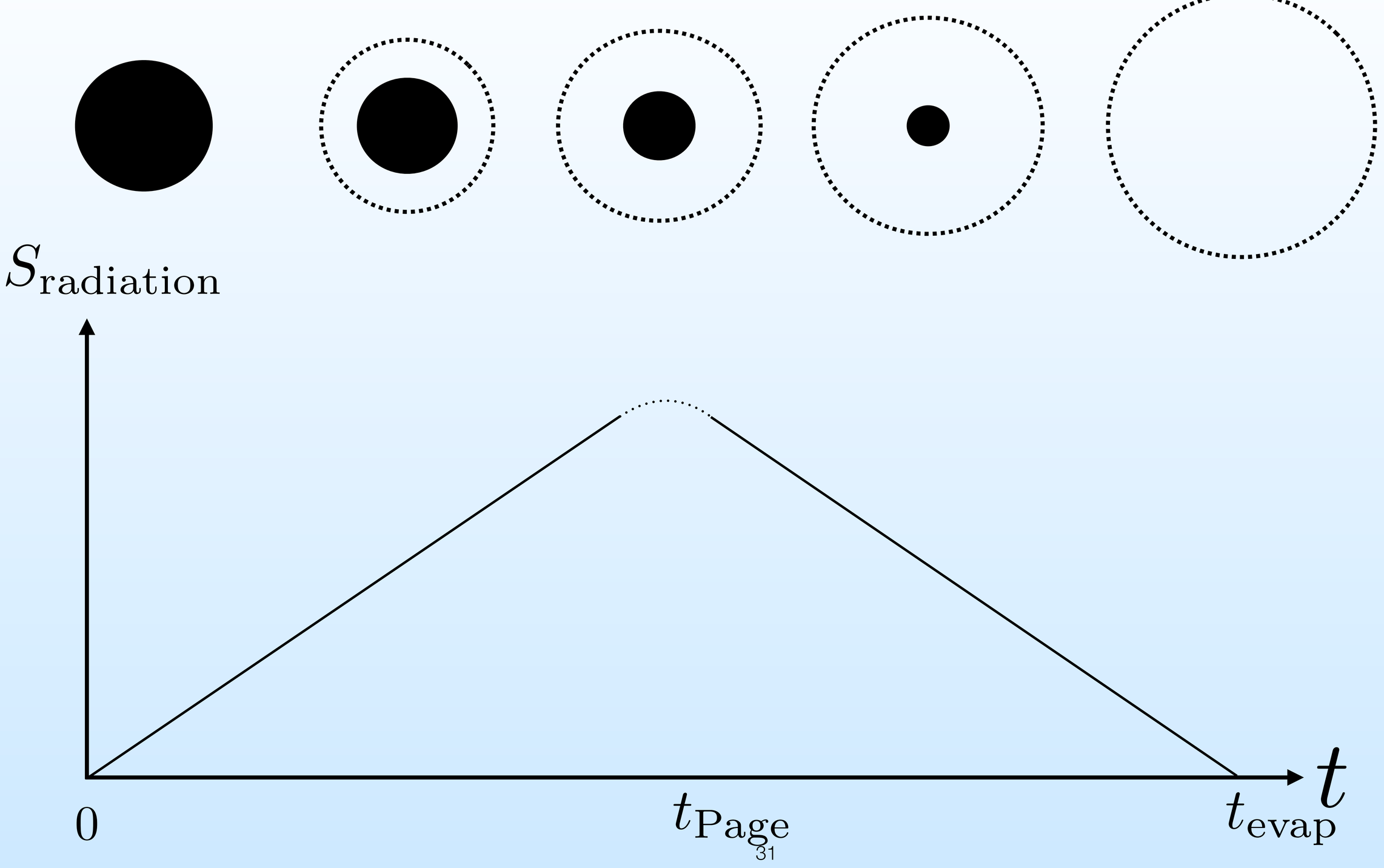
[Page ’93]

For typical states the answer looks like the following:

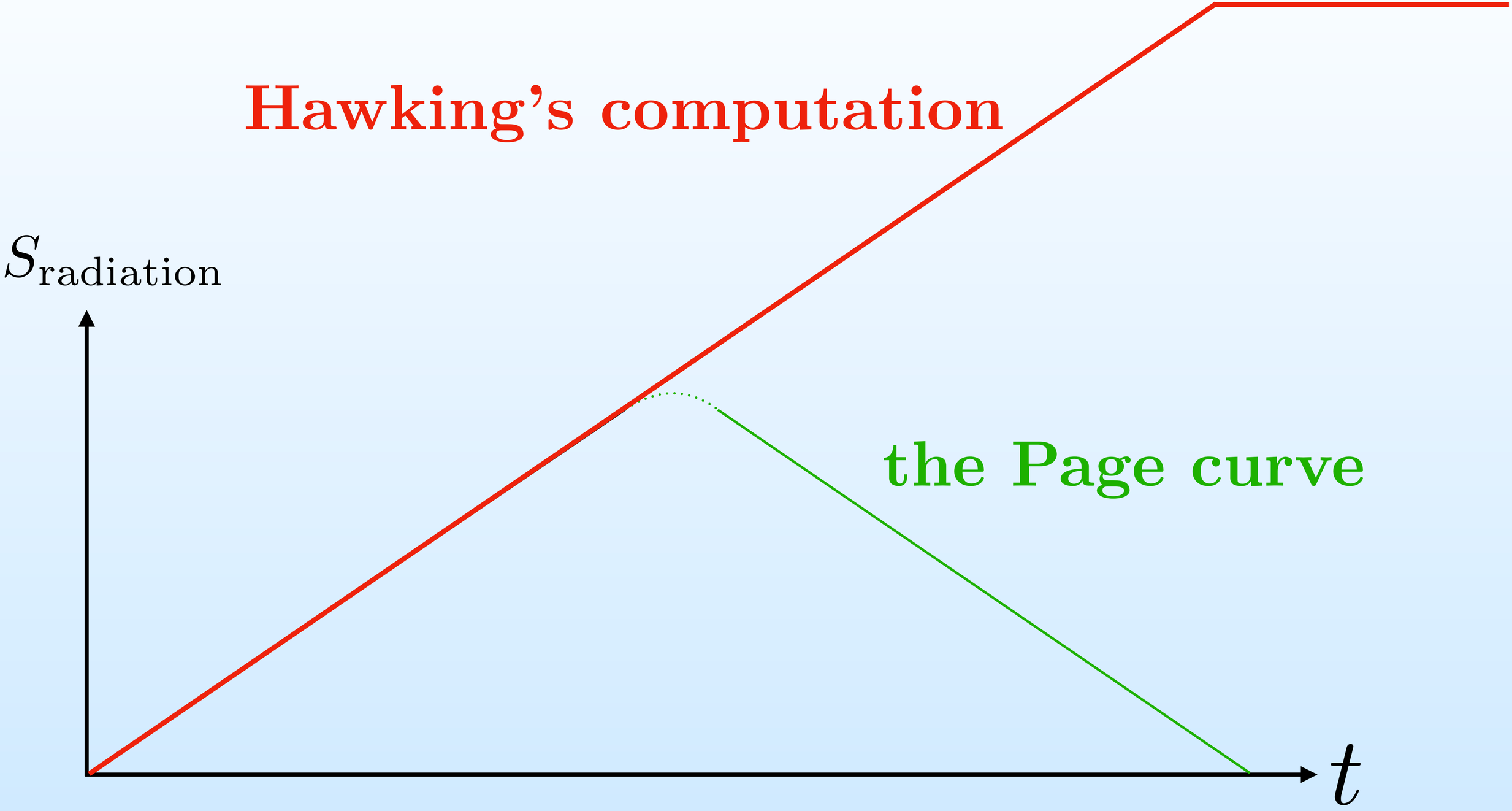

B
A
B
A
B
A
B
A

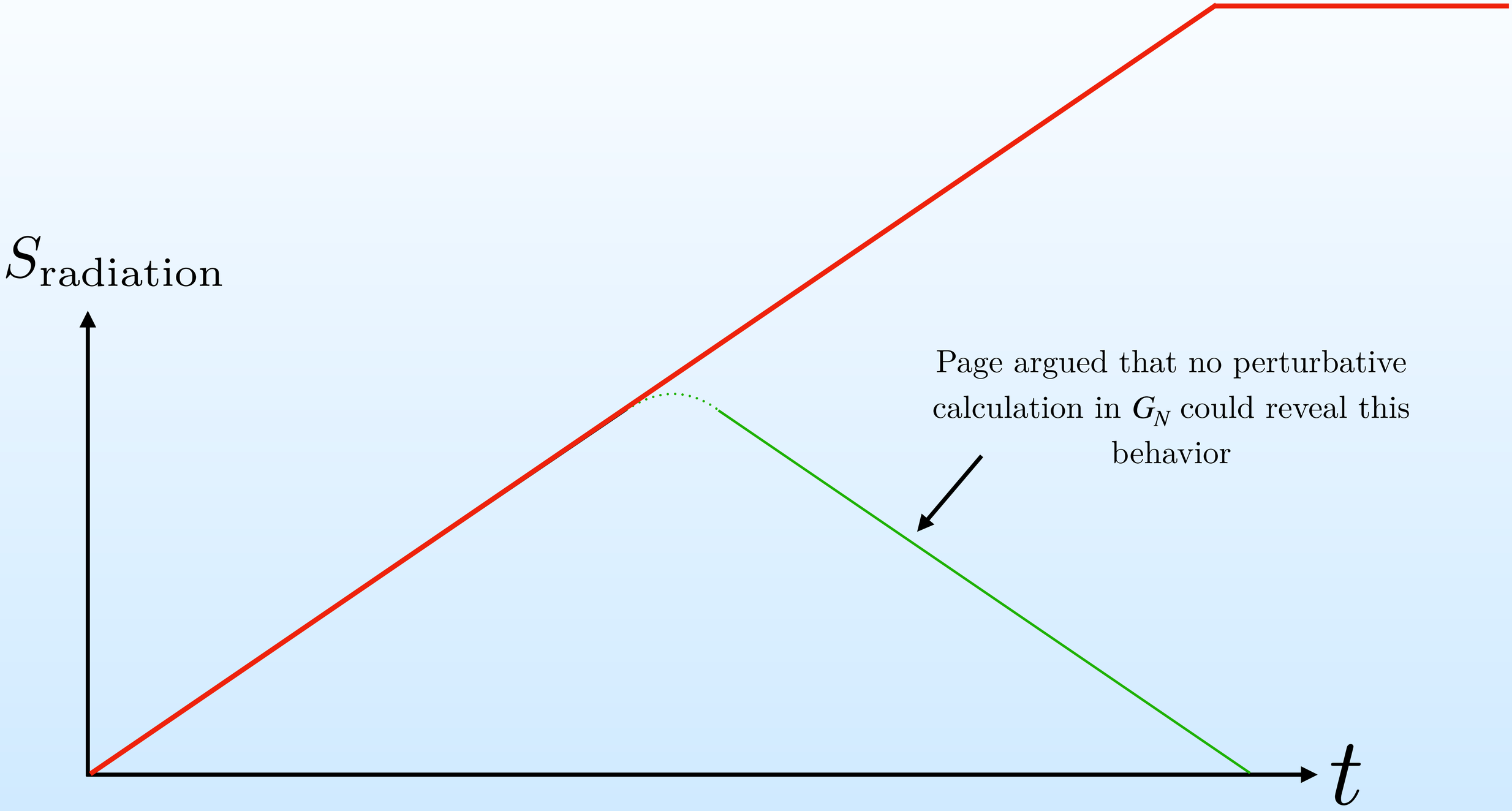

$S_A = S_B$
$\log|A|$
$\log|B|$
0
$|A| = |B|$
1
$\frac{\log|A|}{\log|A| + \log|B|}$


Cloud of Hawking radiation owned by Alice
A
B
B
A B
A B
A B
A


$S_{\text{radiation}}$
t
0
t_{Page}
t_{evap}

Hawking's computation




$S_{\text{radiation}}$
Page argued that no perturbative calculation in G_N could reveal this behavior
t

Black Holes and Holography

Black hole: a region of spacetime from which no signal can escape

Event horizon: the boundary of this region

The study of black hole thermodynamics show that black holes behave as thermodynamical systems with finite entropy and temperature:

$$S_{\mathrm{BH}} = \frac{\mathrm{Area[horizon]}}{4G_N\hbar}$$

$$T_{\mathrm{BH}} = \frac{\hbar}{8\pi G_N M}$$

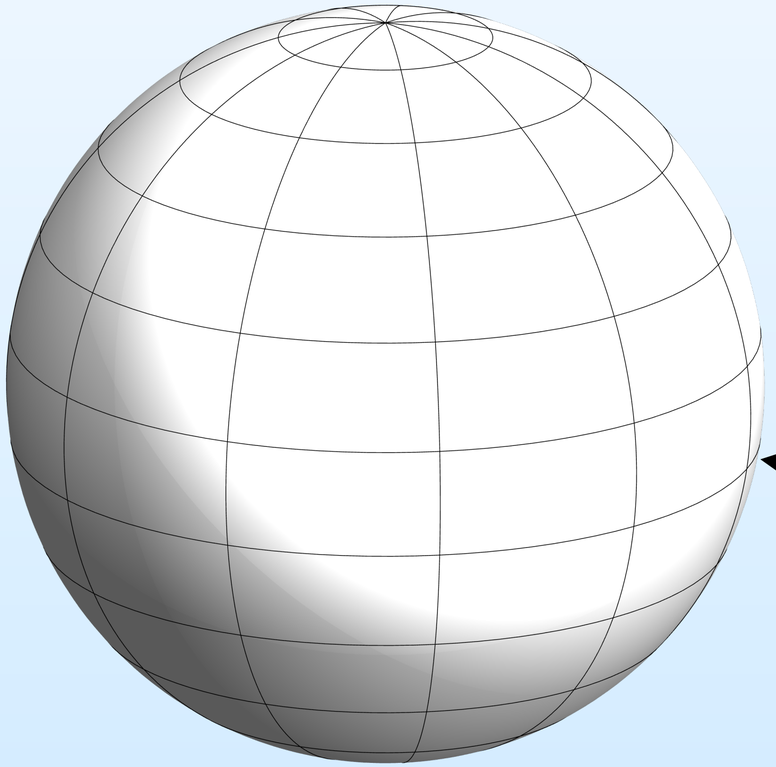
[Bekenstein '72, Bardeen et. al '73, Hawking '74, ...]

$$S_{\text{microcanonical}} = \log(\#\text{ energy eigenstates in relevant energy window})$$

Suggests that a Hilbert space of dimension $e^{S_{\text{BH}}} = e^{\frac{A}{4G_N\hbar}}$ suffices to describe all possible microstates of a black hole

Very different from non-gravitational QFT, where $\log \dim \mathscr{H} \propto$ Volume

The holographic principle



Conjecture that in quantum gravity, all the info about the interior of a region is available on its surface

['t Hooft '93, Susskind '94]

- In certain universes known as Anti-de Sitter (AdS) spacetimes, we have a quite precise understanding of holography

- AdS: universes having a negative cosmological constant

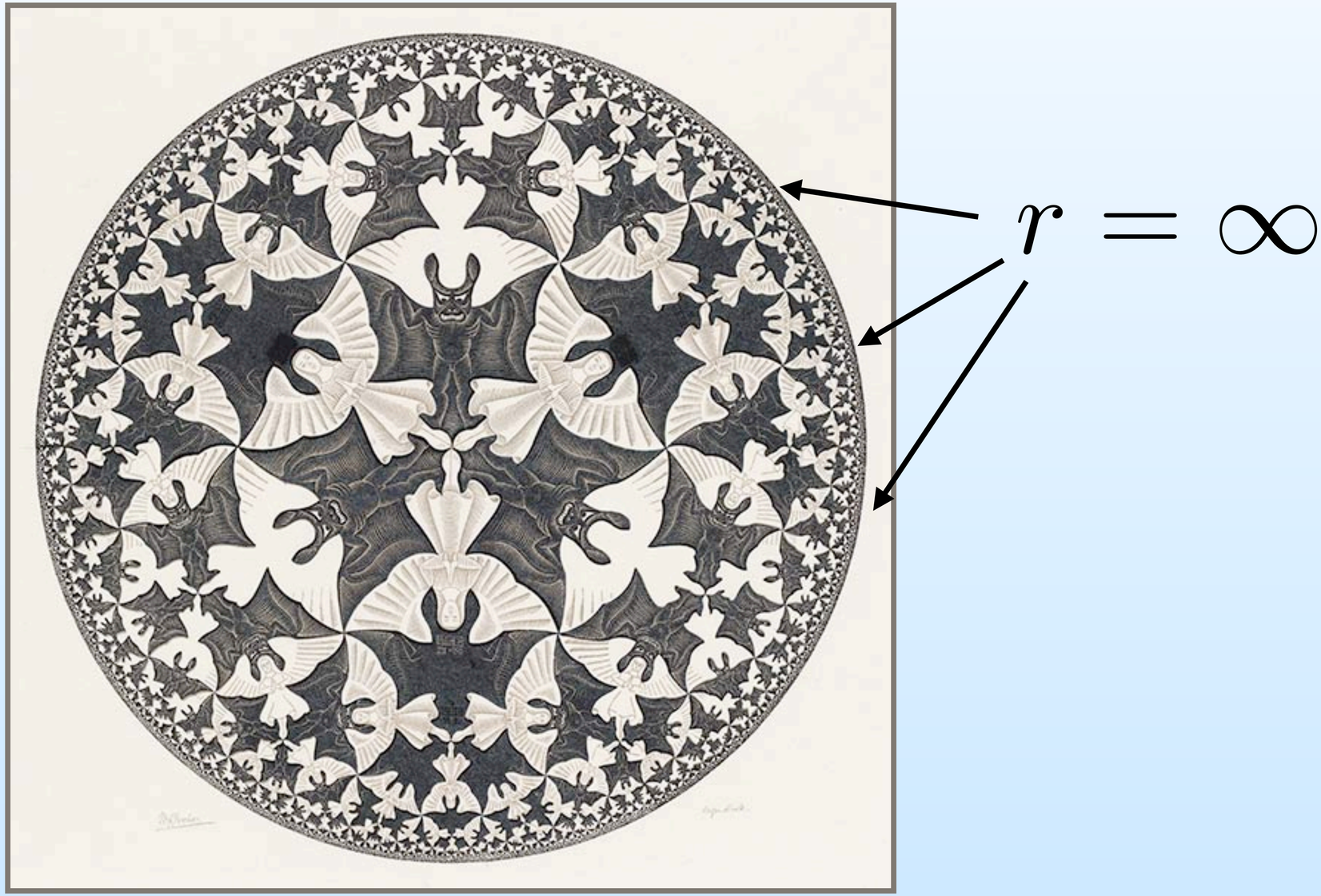
A spatial slice of AdS_{2+1}





By Daina Taimina. Photo Steve Rowell.

A spatial slice of AdS_{2+1}



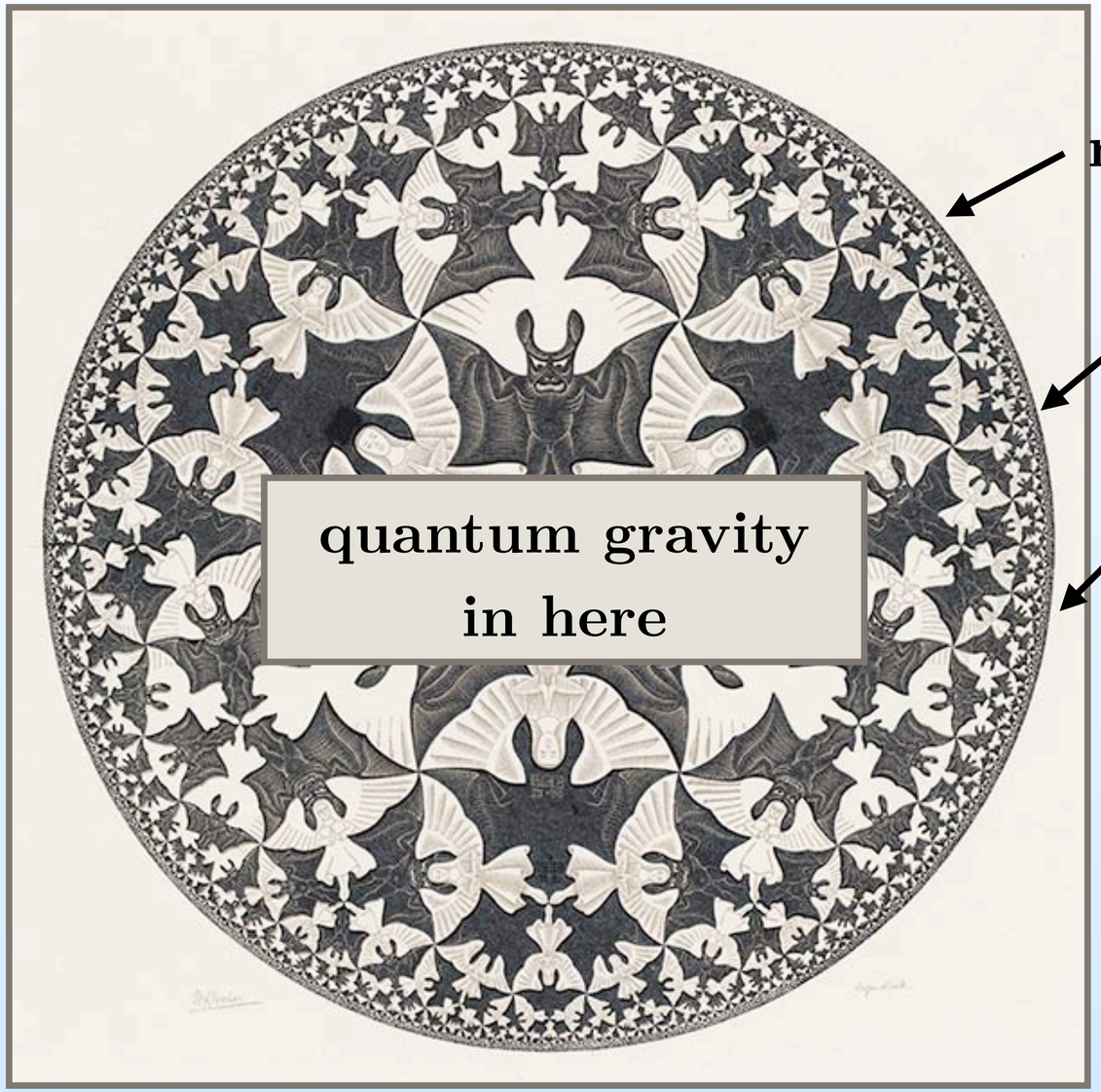
The AdS/CFT correspondence

String theory in AdS_{d+1} spacetimes (with some compact dimensions)

=

Non-gravitational d-dimensional CFT living ‘at infinity’

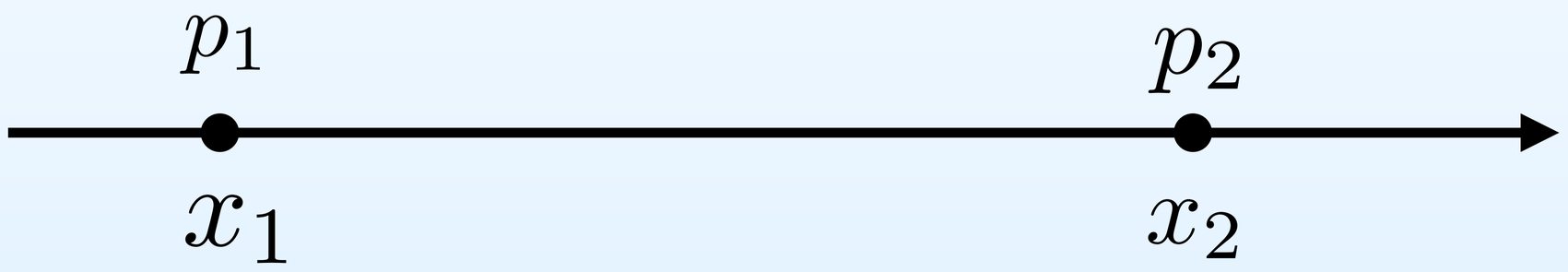
[Maldacena ’97, ...]



non-gravitational
CFT here

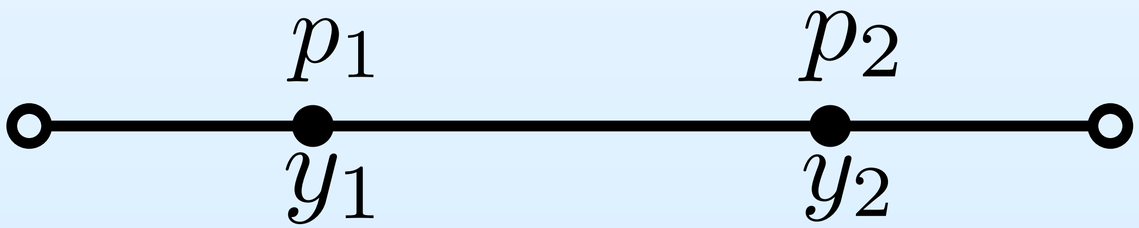
A diagrammatic interlude

$\mathbb{R}$

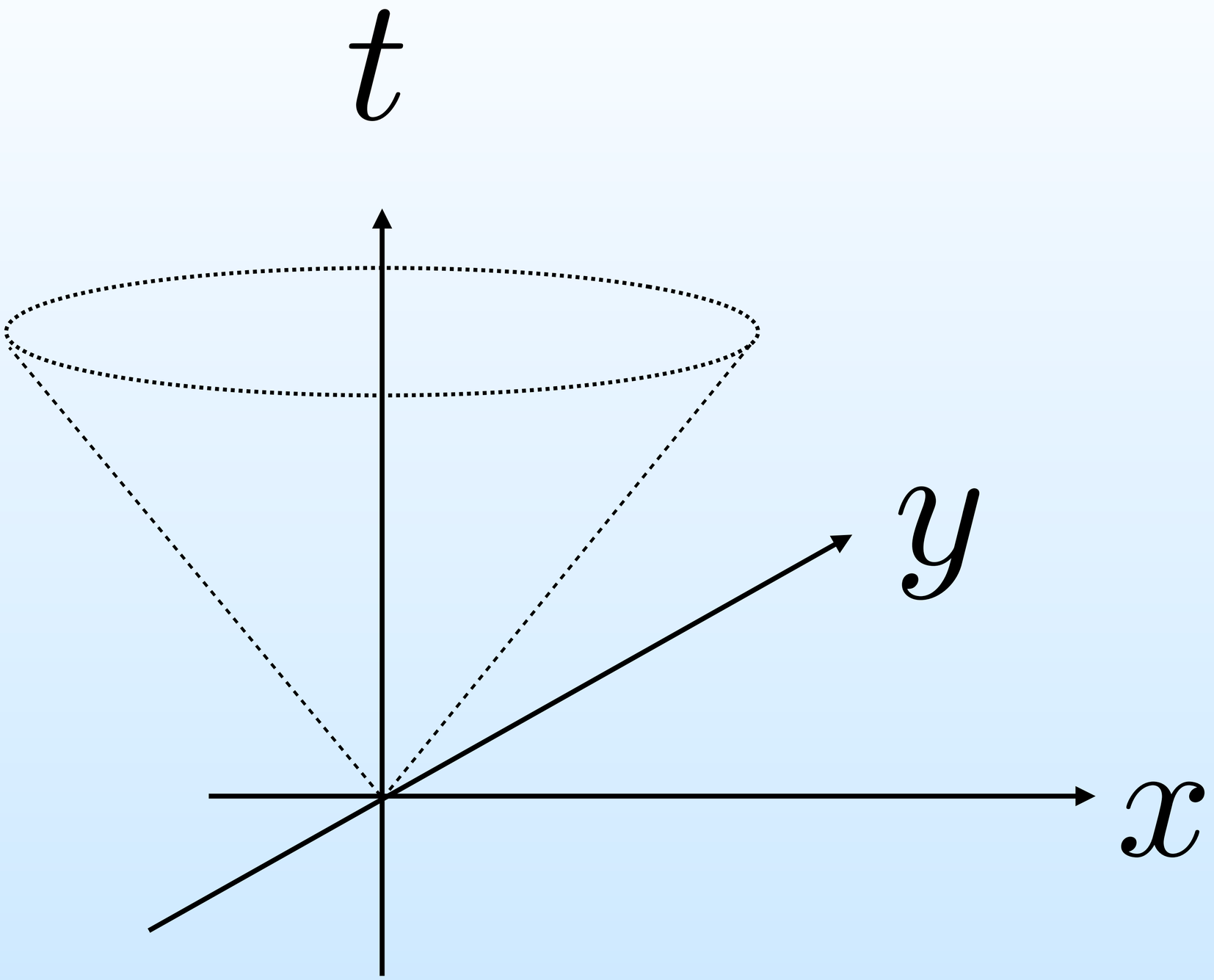


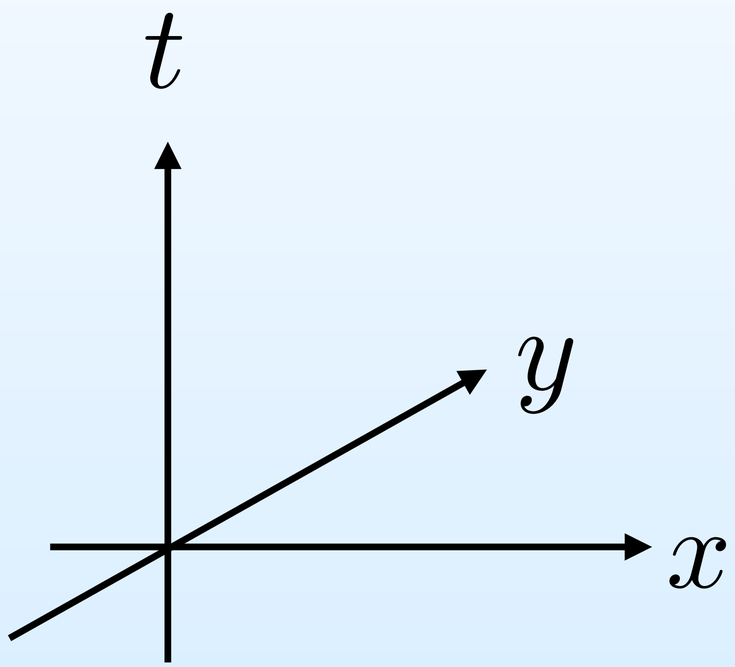
$$\mathrm{dist}(p_1, p_2) = |x_1 - x_2|$$

$$\mathrm{y} = \arctan(\mathrm{x}) \qquad\qquad y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$



$$\operatorname{dist}(p_1, p_2) = |\arctan y_2 - \arctan y_1|$$

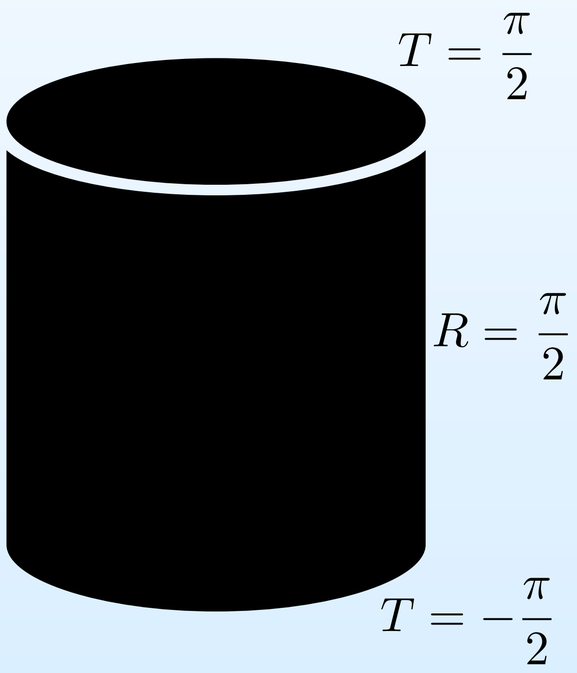

t
y
x



$$T = \arctan t$$

$$R = \arctan \sqrt{x^2 + y^2}$$

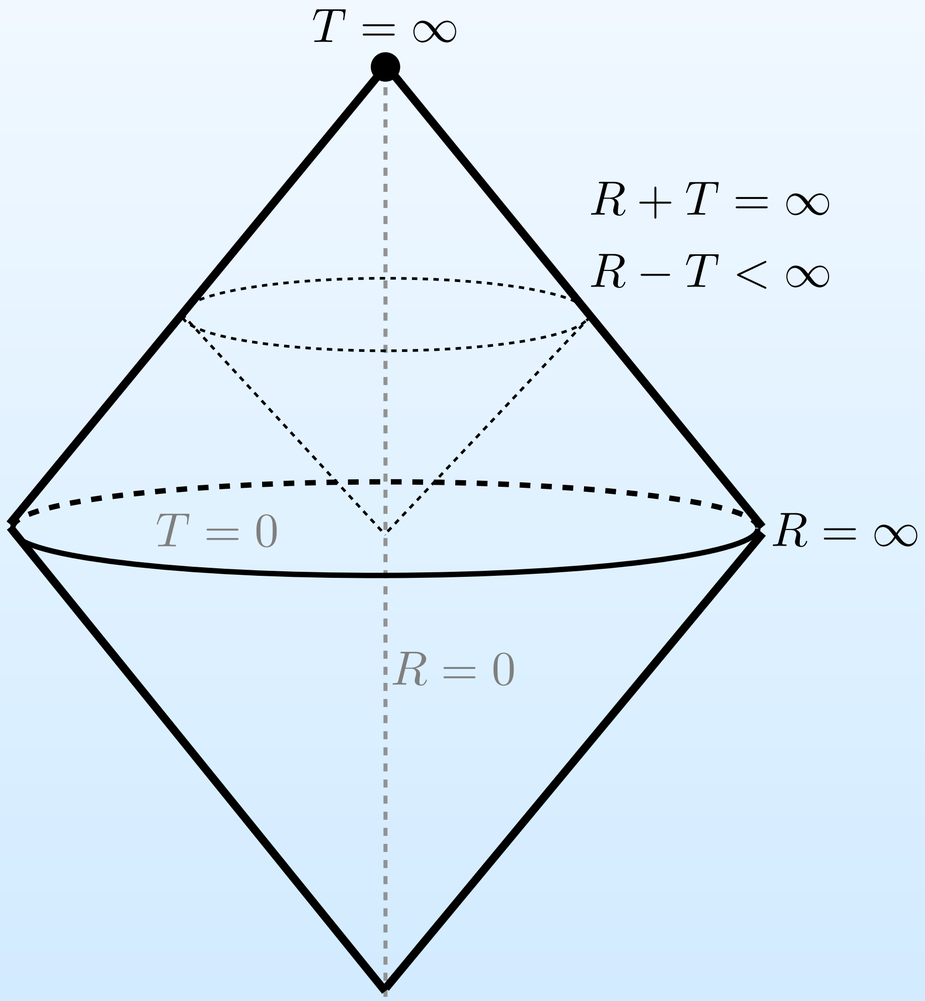
$$\Theta = \theta$$

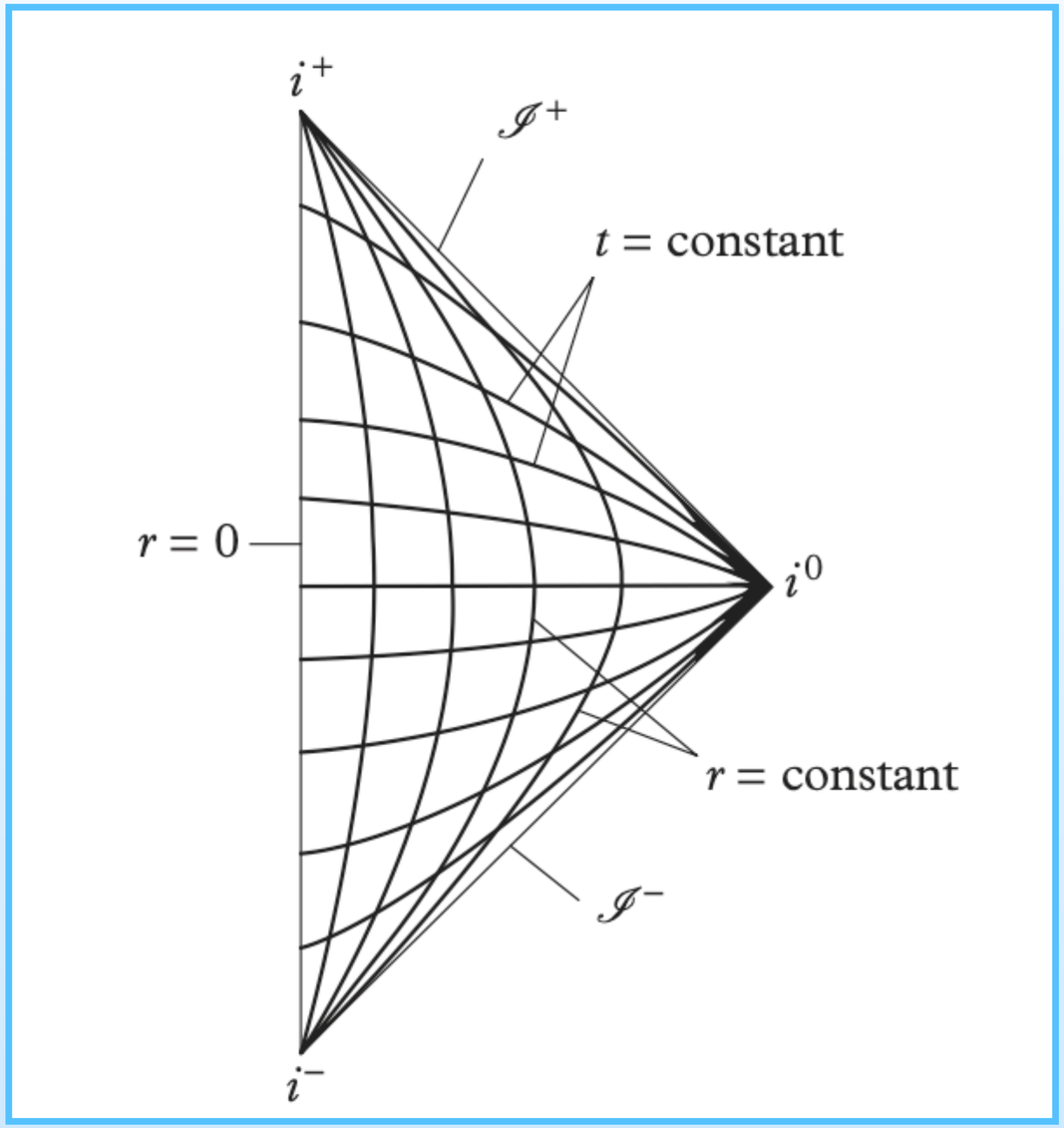




inconventient choice,
lightcones all messed up

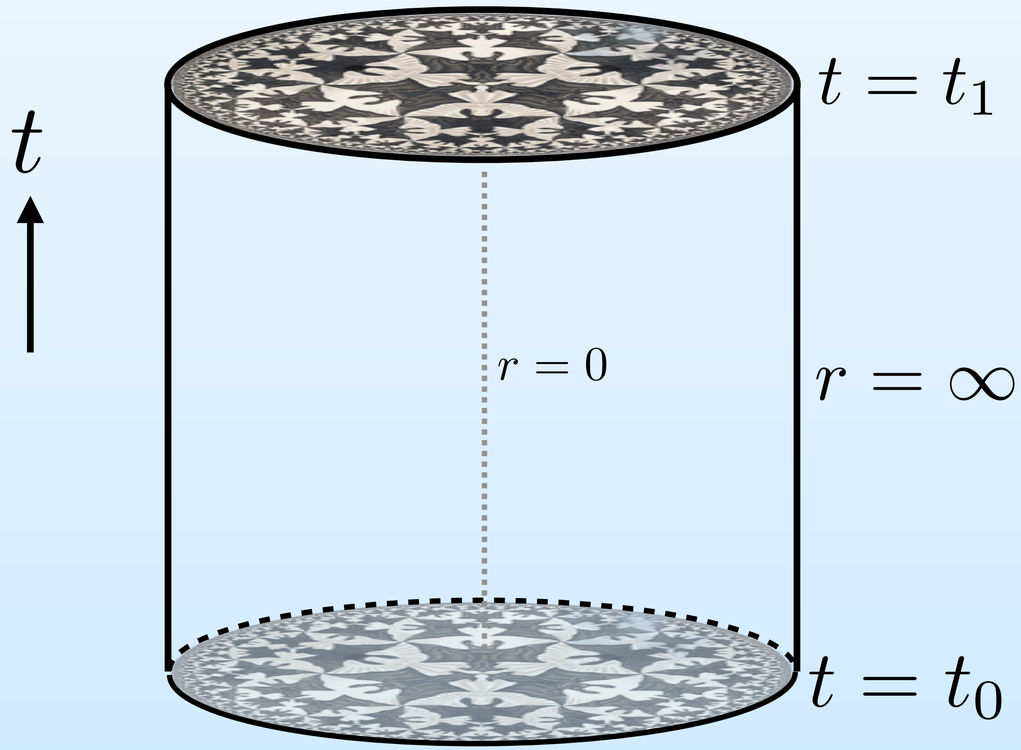
$$T \pm R = \arctan(t \pm r)$$

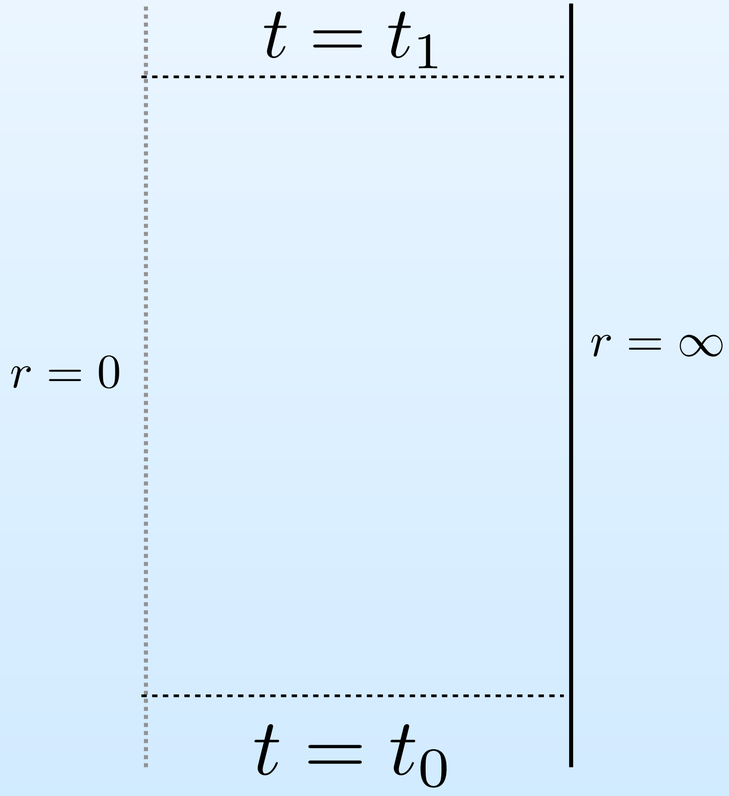


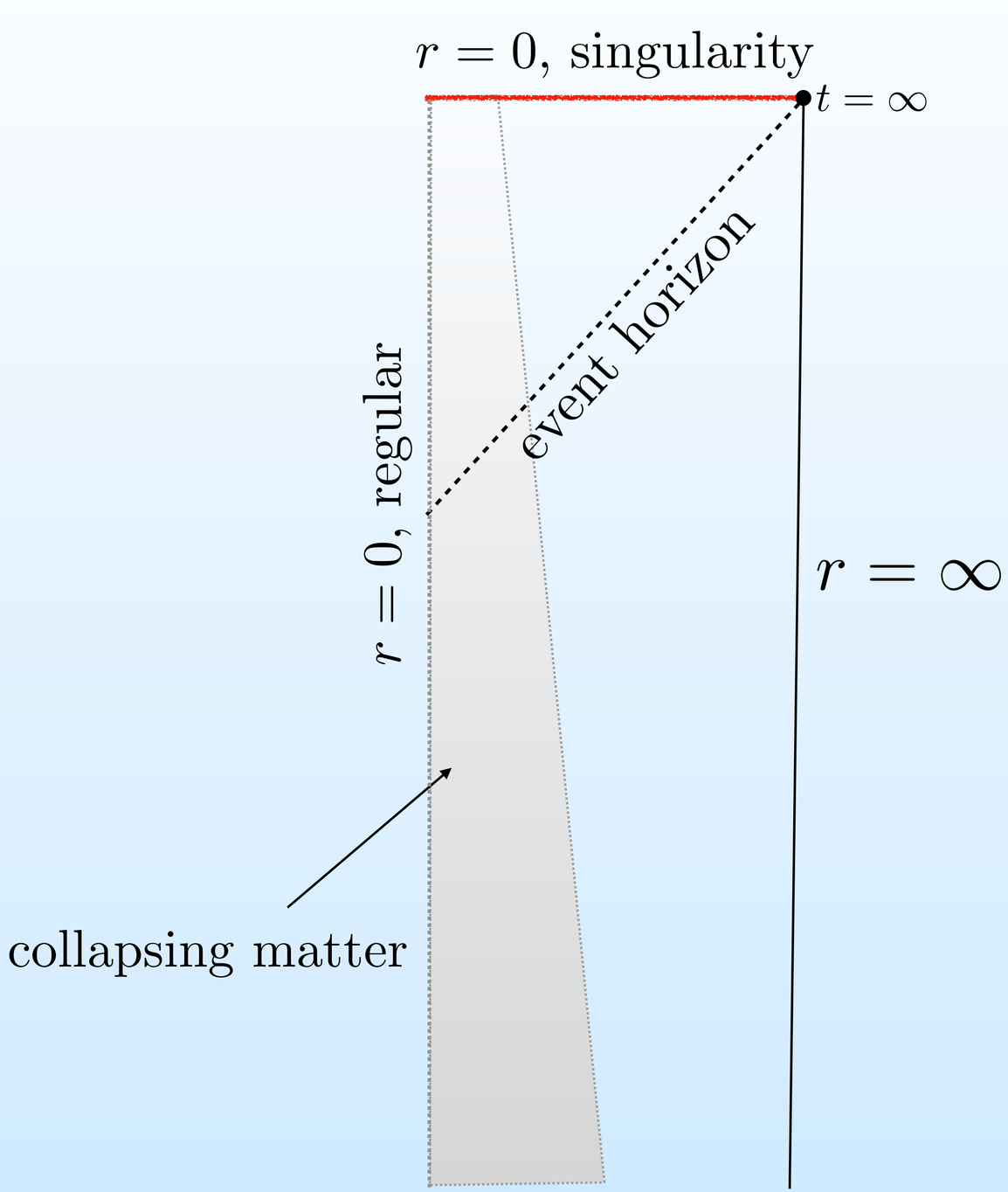


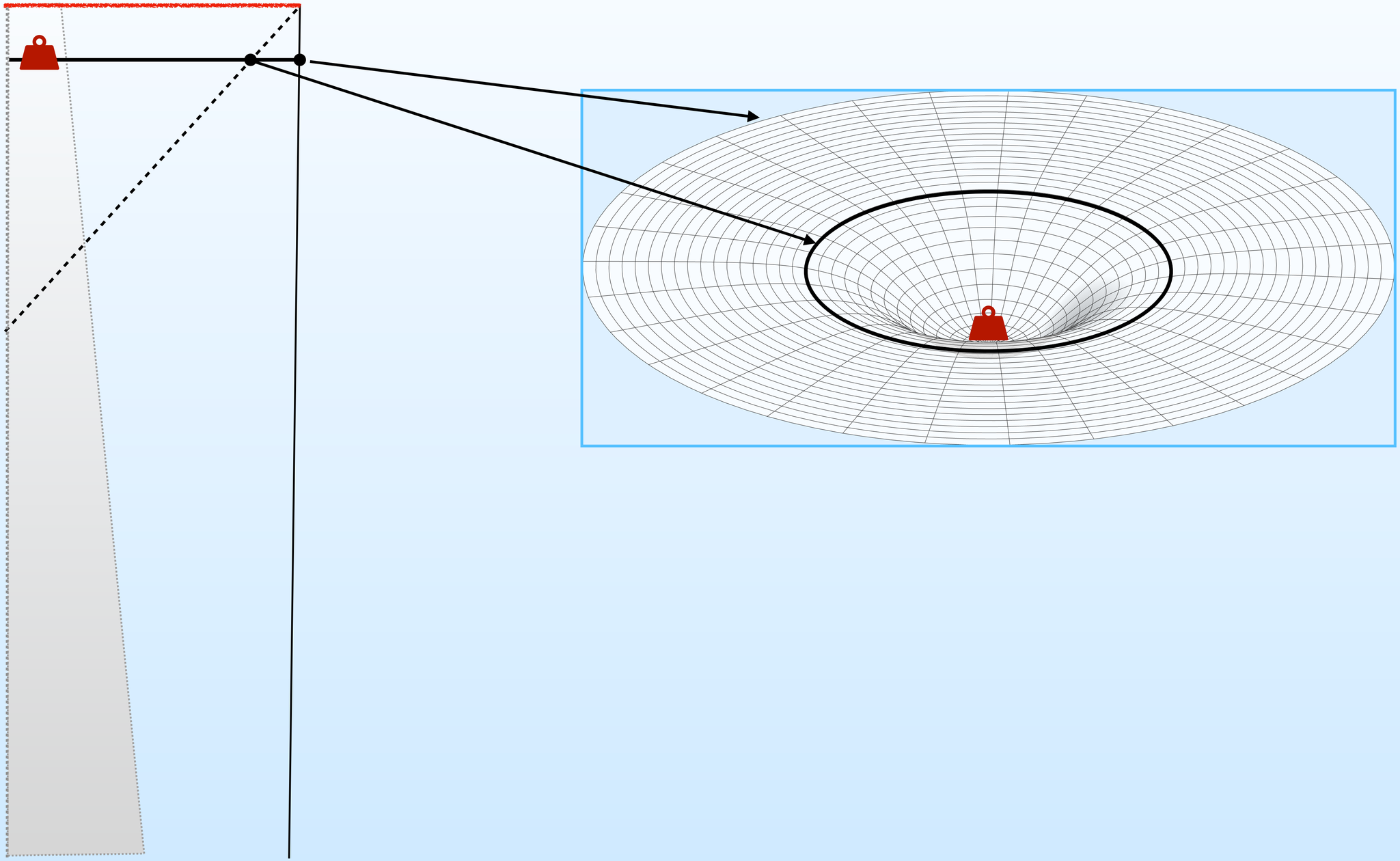
Return to AdS

Partial compactification of AdS_3



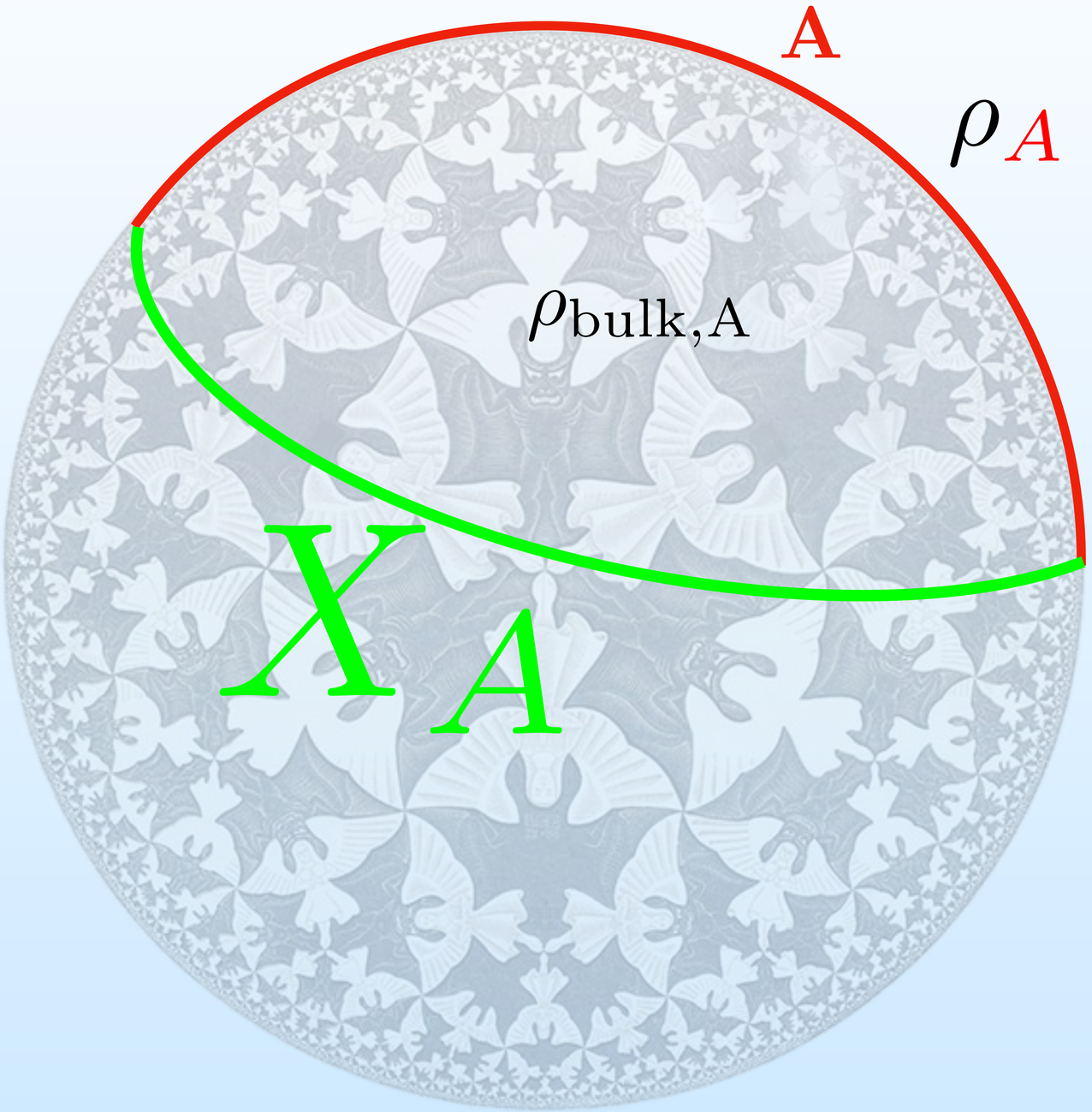



$r = 0$, singularity
$t = \infty$
event horizon
$r = 0$, regular
$r = \infty$
collapsing matter



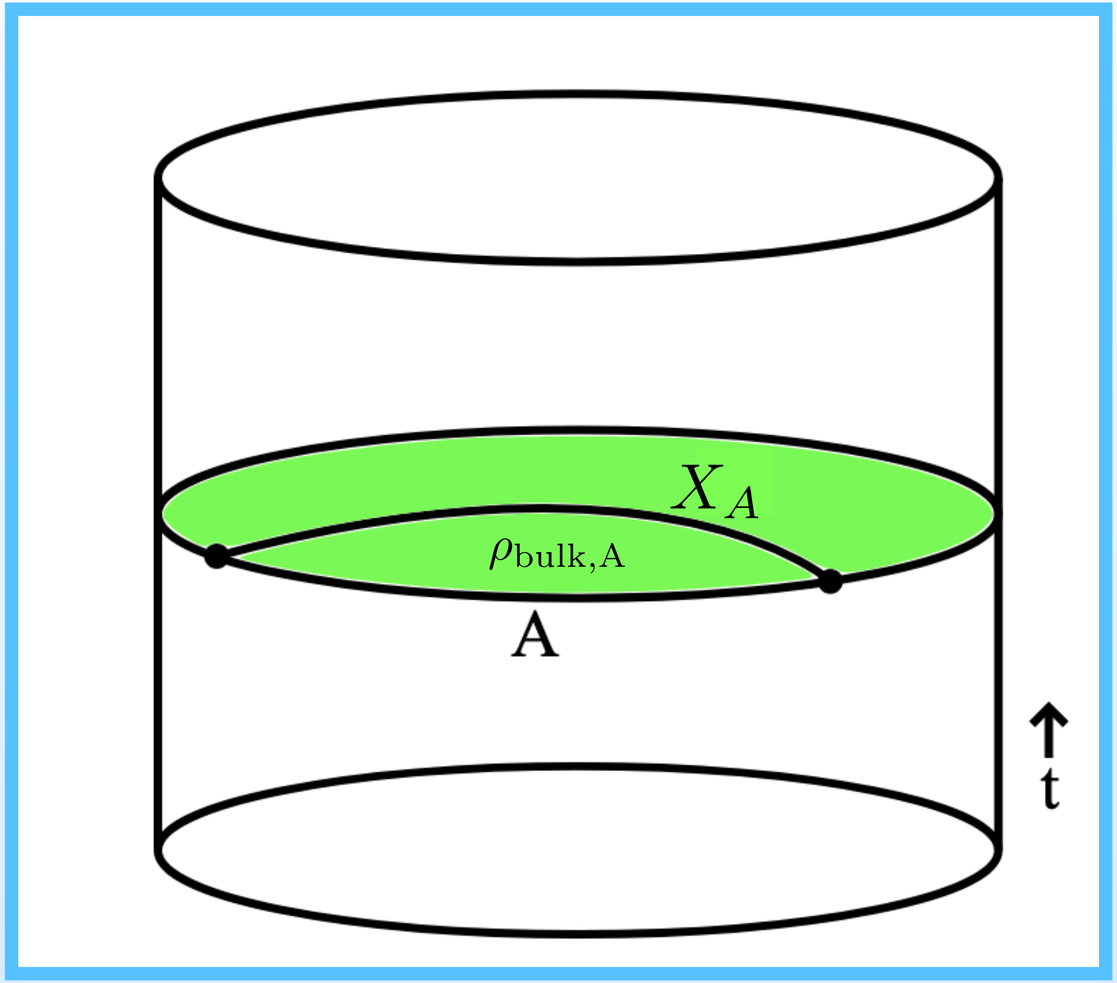
Arguably the most important discovery about AdS/CFT:

The gravitational theory gives a simplified way to compute von Neumann entropies of spatial subsystems of the CFT



$$\rho_A = \text{tr}_{\bar{A}} |0\rangle \langle 0|$$

$$S_{\text{vN}}[\rho_A] = \frac{\text{Area}[X_A]}{4G_N\hbar} + S_{\text{vN}}[\rho_{\text{bulk,A}}]$$



$$S_{\text{vN}}[\rho_A] = \underbrace{\frac{\text{Area}[X_A]}{4G_N\hbar} + S_{\text{vN}}[\rho_{\text{bulk,A}}]}_{S_{\text{gen}}[X_A]}$$

Quantum extremal surface formula / Ryu-Takayanagi formula

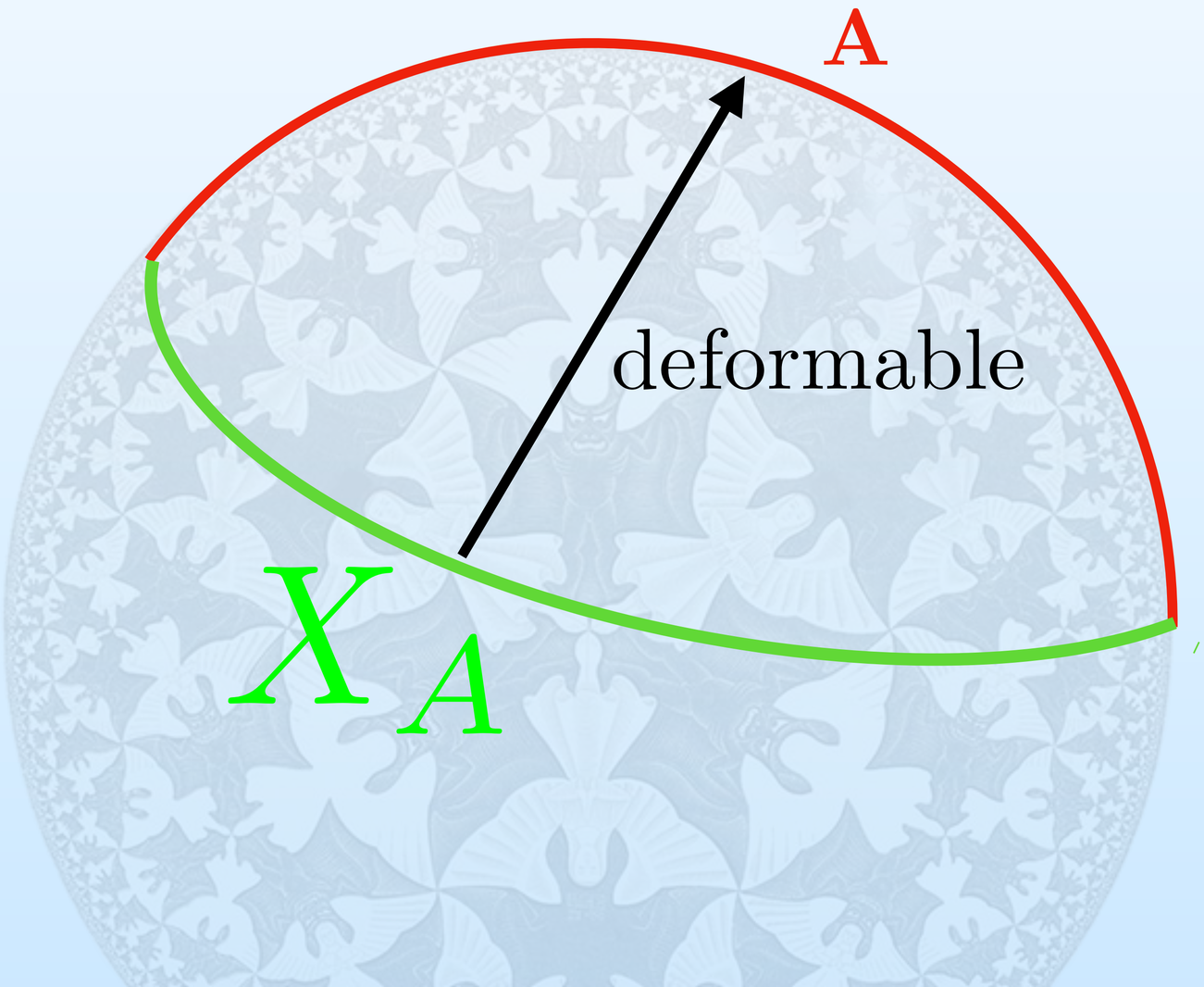
$$S_{\mathrm{vN}}[\rho_A] = \frac{\mathrm{Area}[X_A]}{4G_N\hbar} + S_{\mathrm{vN}}[\rho_{\mathrm{bulk,A}}]$$

X_A is the surface that extremizes the RHS

if there are multiple, X_A is the one with smallest RHS

[Ryu, Takayanagi '06, Hubeny, Rangamani, Takayanagi '07, Lewkowycz, Maldacena '13, Faulkner, Lewkowycz, Maldacena '13, Engelhardt, Wall '14, … many more]

additional condition



$$S_{\mathrm{vN}}[\rho_A] = \frac{\mathrm{Area}[X_A]}{4G_N\hbar} + S_{\mathrm{vN}}[\rho_{\mathrm{bulk,A}}]$$

- Has been compared with direct CFT computations and found to match

- For some states we can give a direct derivation from the Euclidean gravitational path integral (under certain assumptions) [Lewkowycz, Maldacena '13]

- The Bekenstein-Hawking entropy is a special case of this formula applied to a eternal two-sided black hole (a wormhole)

Computing the Page curve

The entropy of bulk quantum fields and the entanglement wedge of an evaporating black hole

Ahmed Almheiri,[a] **Netta Engelhardt,**[b,c] **Donald Marolf,**[d] **Henry Maxfield**[d]

[a] *Institute for Advanced Study, Princeton, NJ 08540, USA*

[b] *Department of Physics, Princeton University, Princeton, NJ 08544, USA*

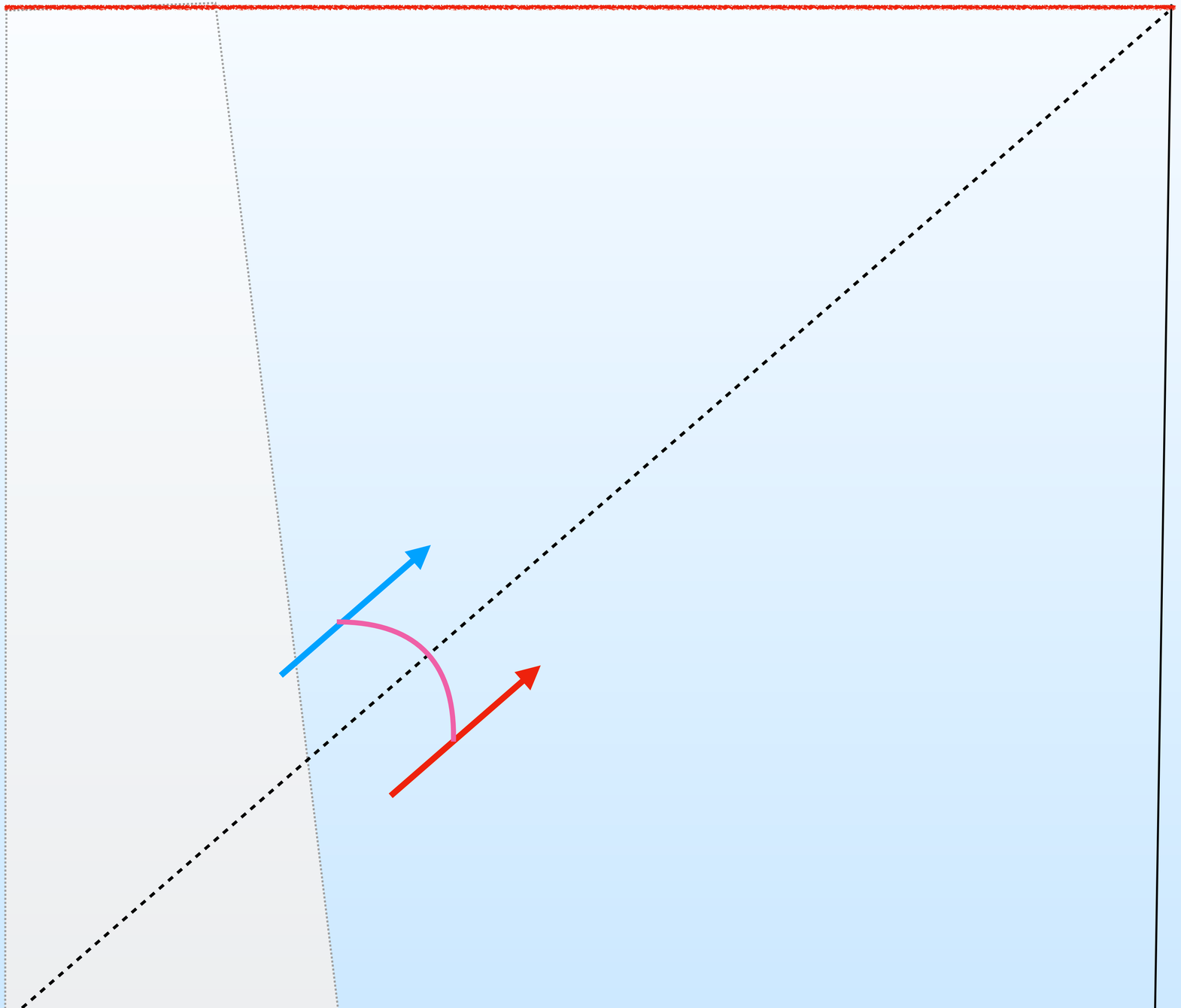
[c] *Gravity Initiative, Princeton University, Princeton NJ 08544, USA*

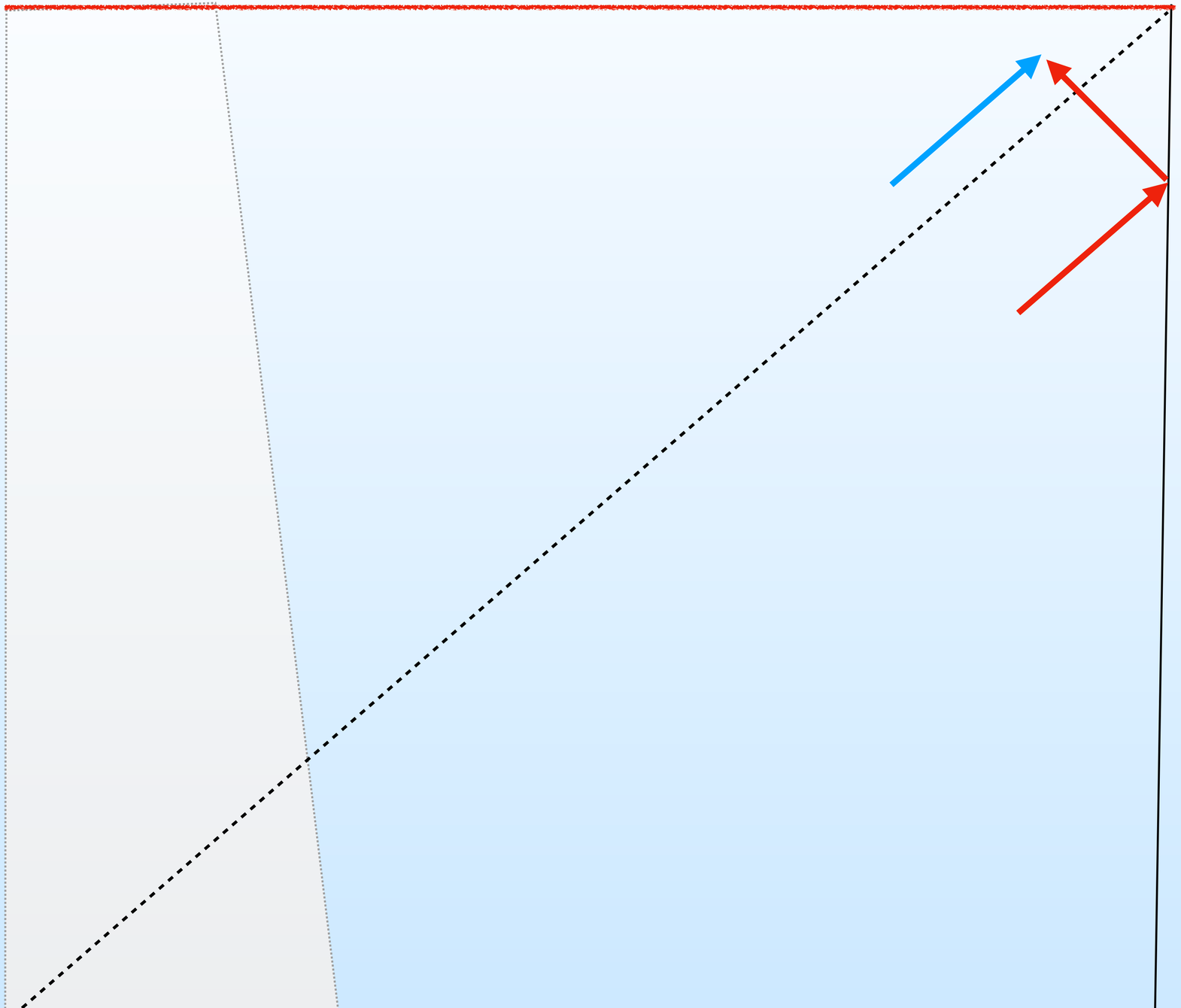
[d] *Physics Department, University of California, Santa Barbara, CA 93016*

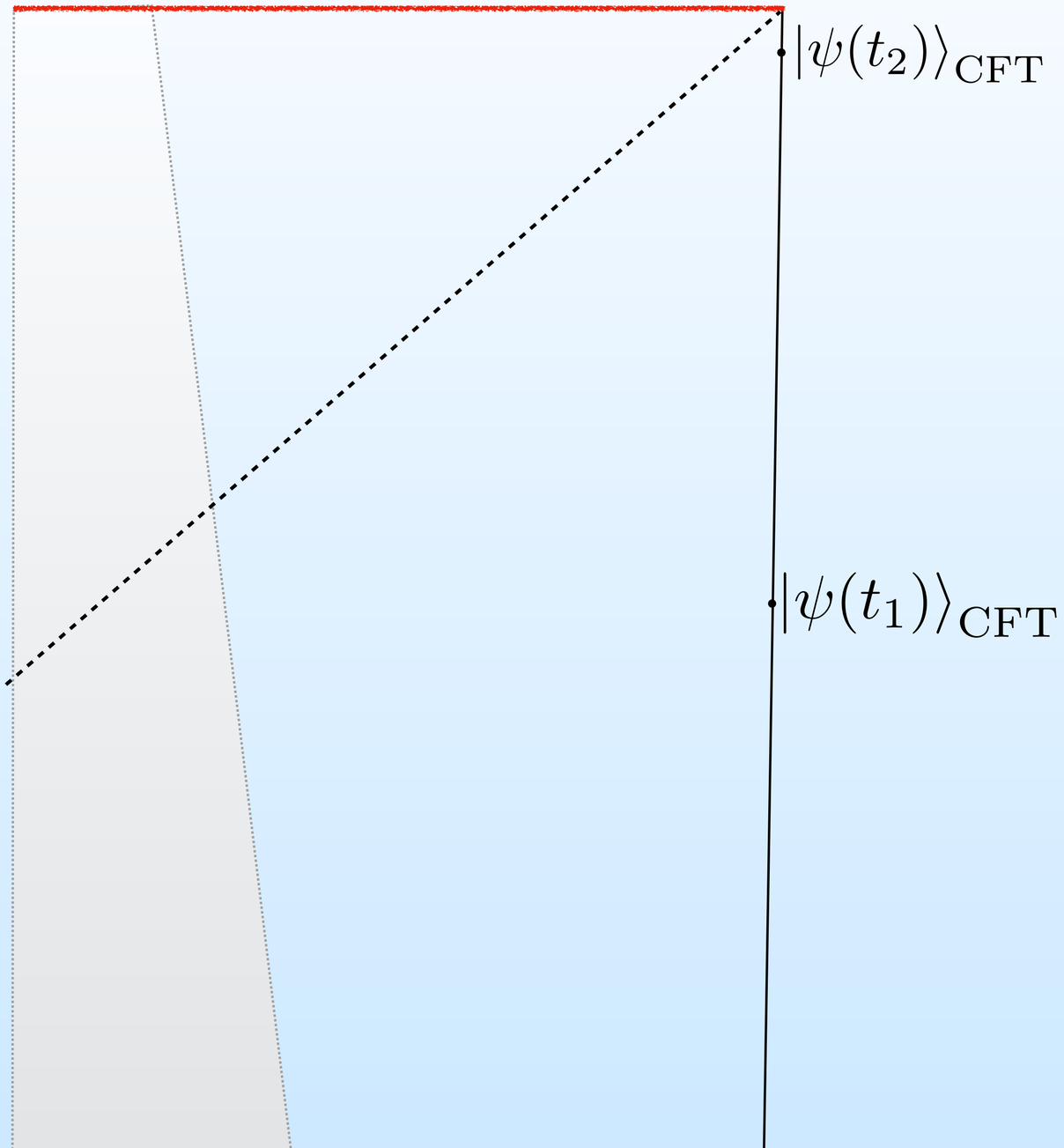
Entanglement wedge reconstruction and the information paradox

Geoffrey Penington[1]

[1] *Stanford Institute for Theoretical Physics, Stanford University, Stanford CA 94305 USA*

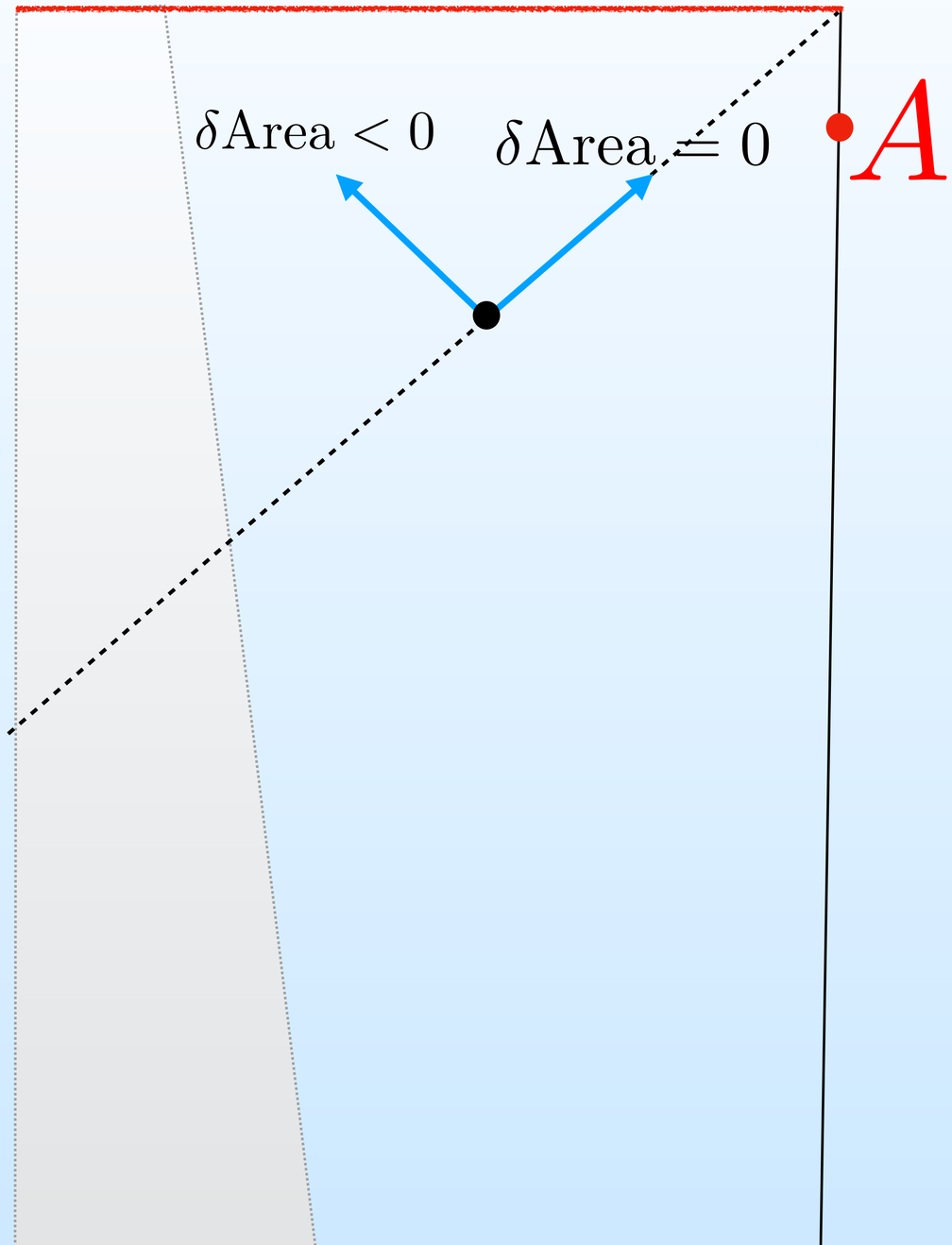






- The black hole never evaporates - it remains in thermal equilibrium with a bath of radiation

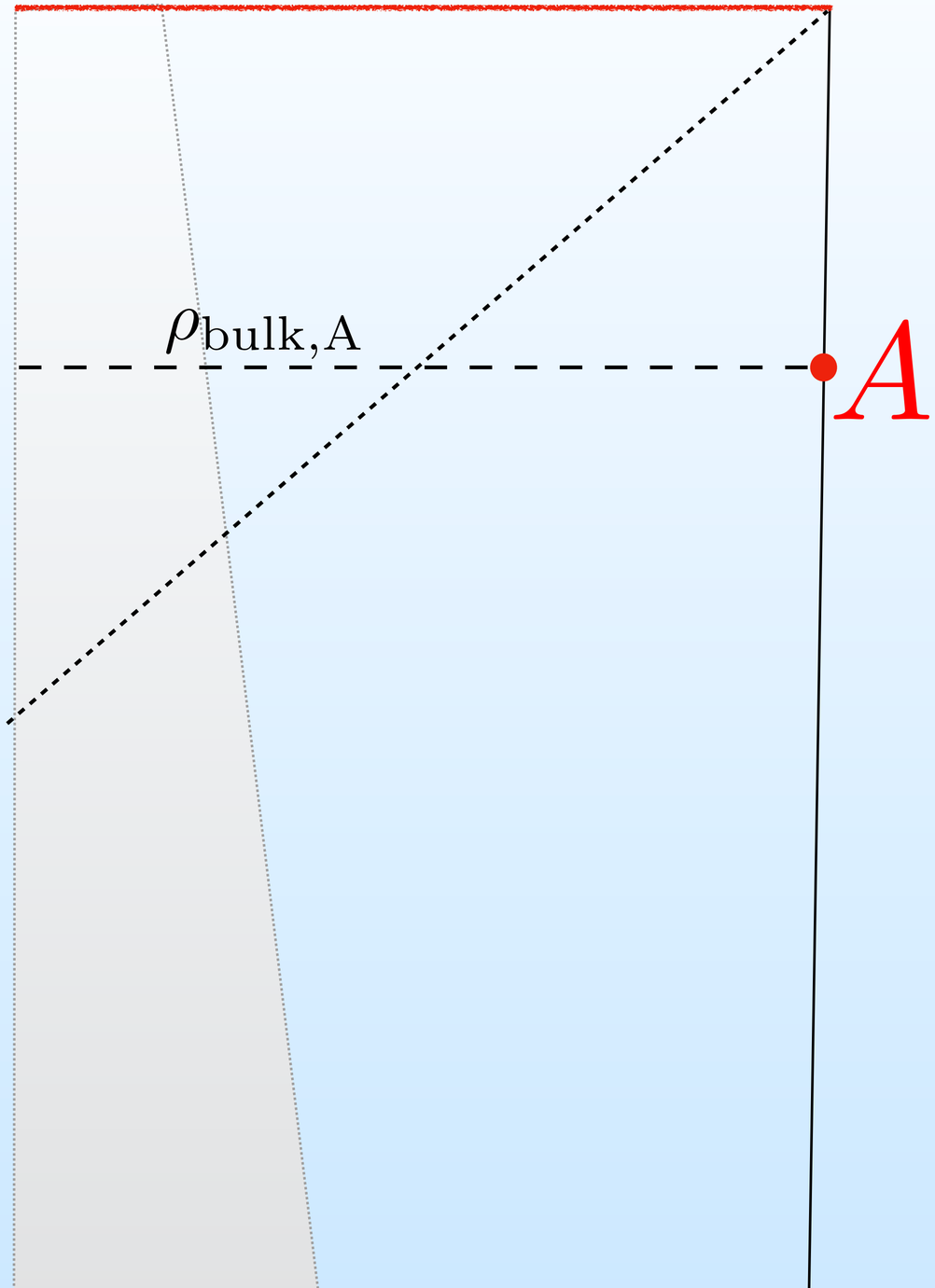
- The CFT state describes both the radiation and the black hole, remains pure for all time



perturbatively suppressed

$$S_{\text{vN}}[\rho_A] = \frac{\text{Area}[X_A]}{4G_N\hbar} + S_{\text{vN}}[\text{bulk,A}]$$

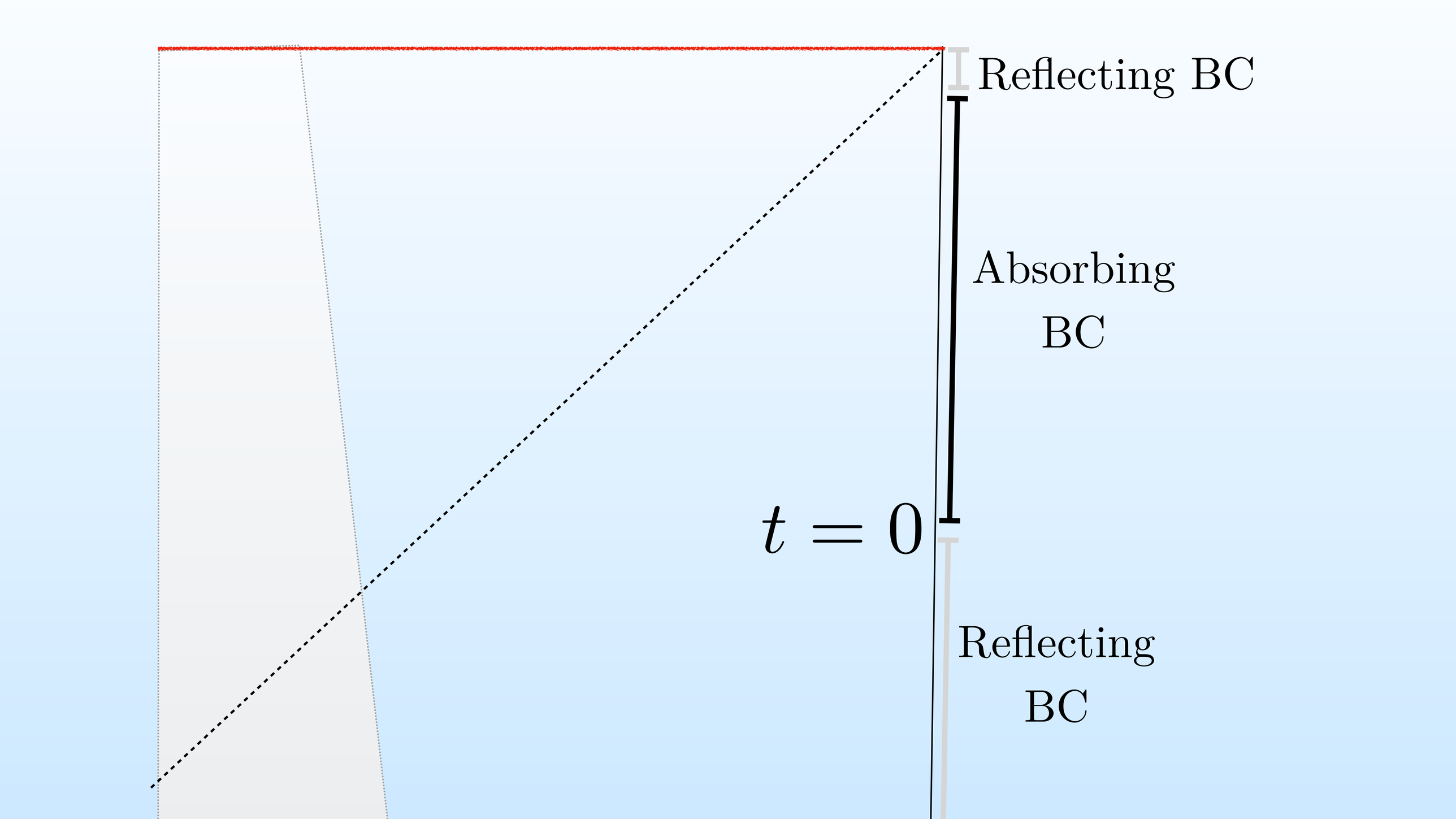
Need to find a surface whose area does not change under small perturbations. No non-trivial such surface exists in this spacetime

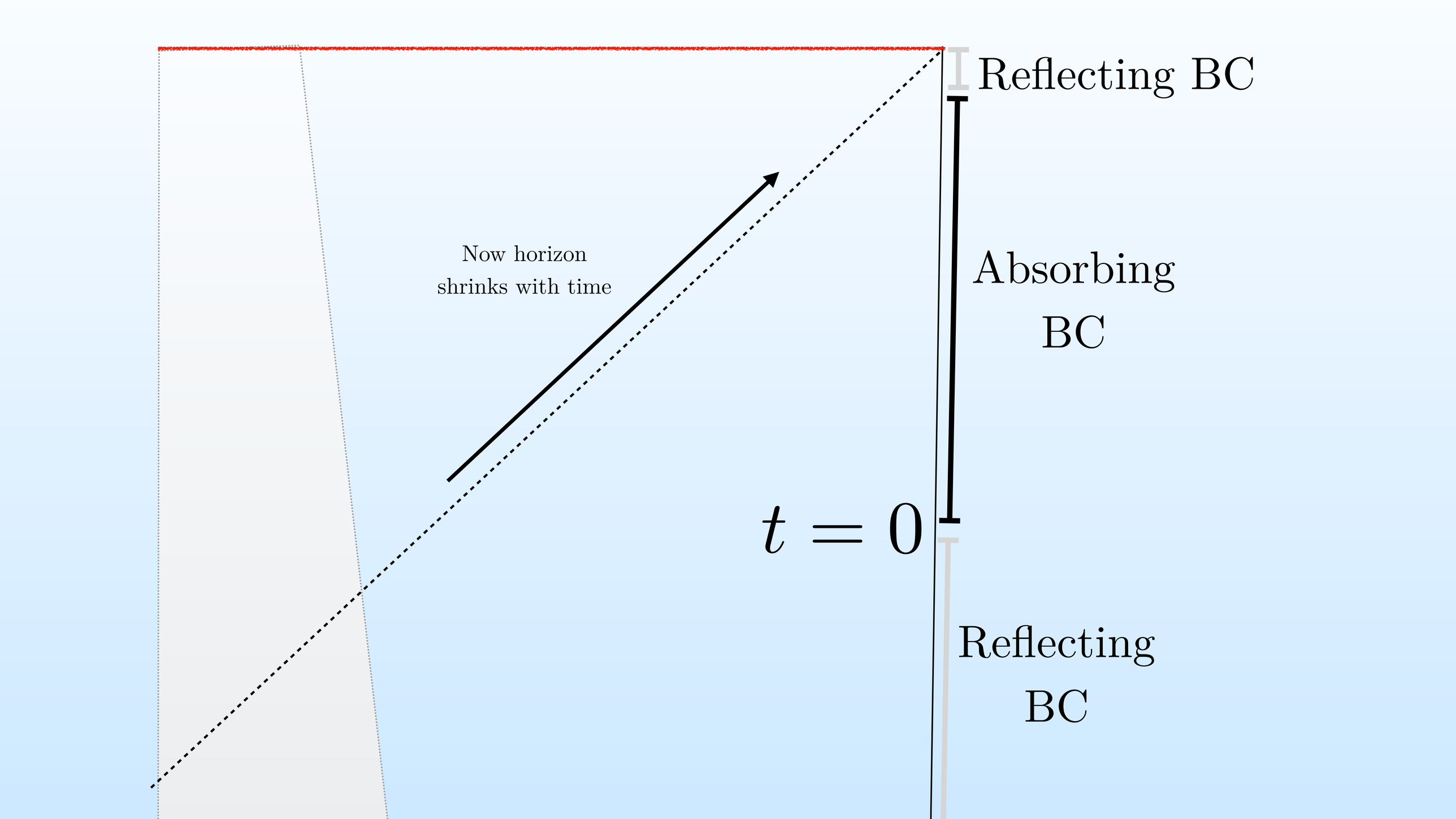


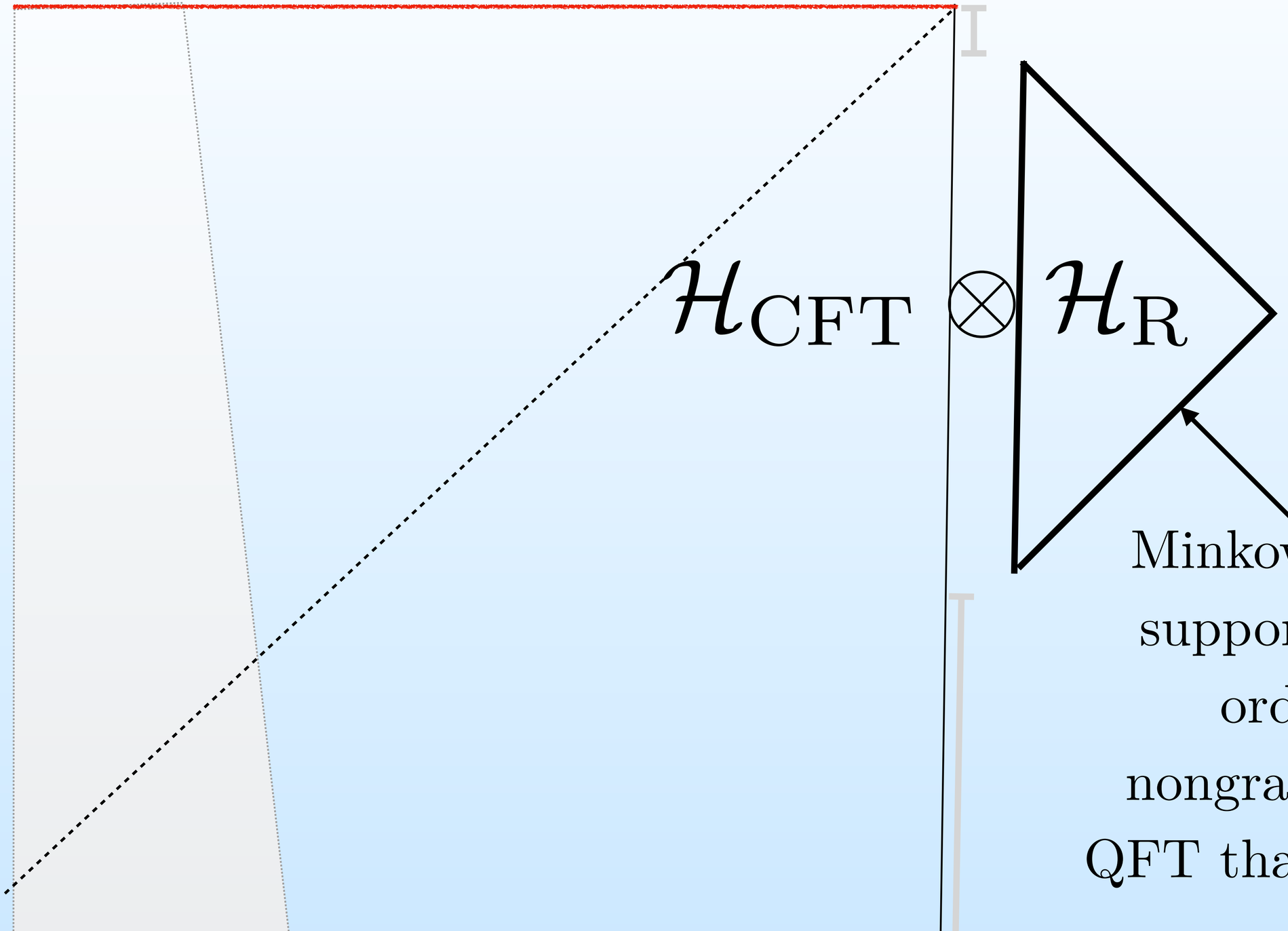
$$S_{\text{vN}}[\rho_A] = \frac{\text{Area[empty set]}}{4G_N\hbar} + S_{\text{vN}}[\rho_{\text{bulk,A}}] = 0$$

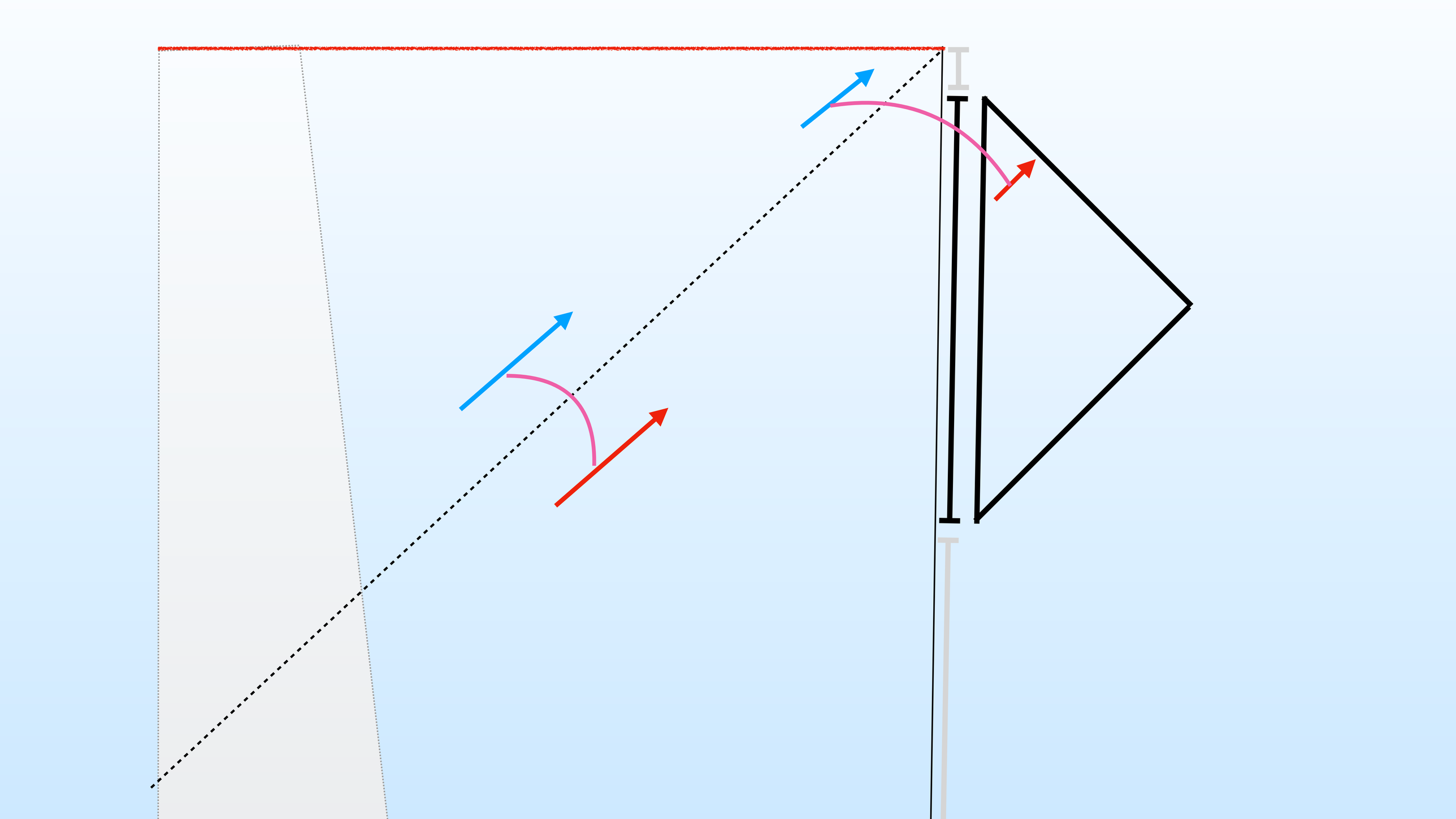
$$S_{\text{vN}}[\rho_A] = S_{\text{vN}}[|BH(t)\rangle\langle BH(t)|] = 0$$

Evaporating an AdS black hole


Reflecting BC
Absorbing
BC
$t = 0$
Reflecting
BC

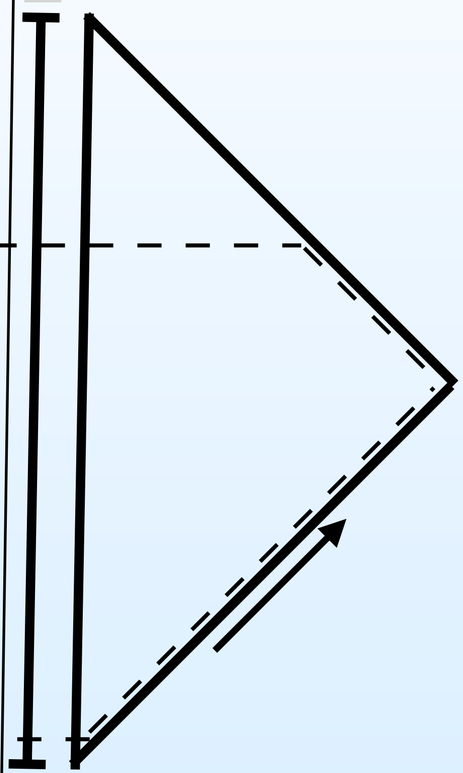

Reflecting BC
Absorbing BC
Reflecting BC
Now horizon shrinks with time
$t = 0$

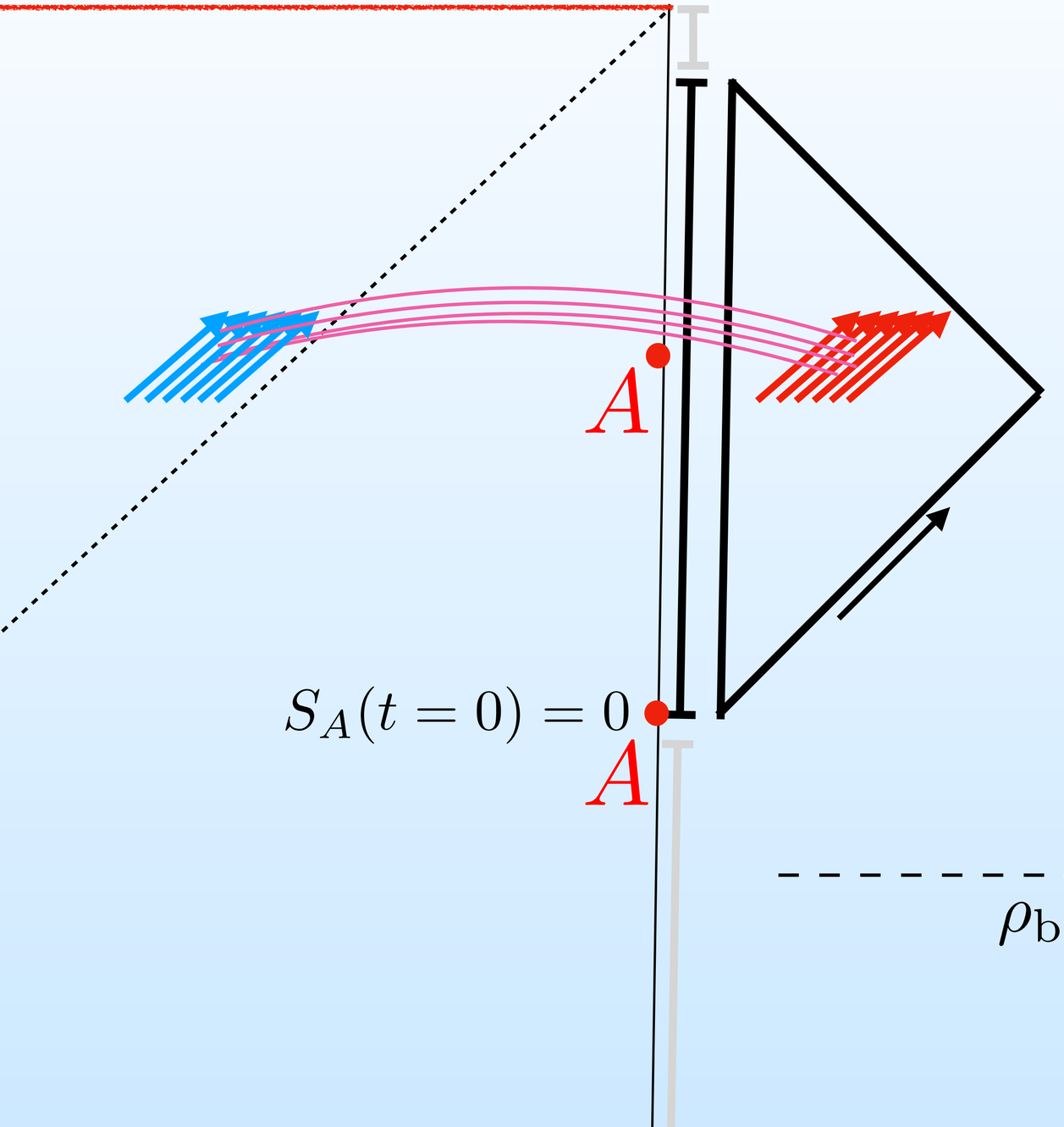

$\mathcal{H}_{\mathrm{CFT}} \otimes \mathcal{H}_{\mathrm{R}}$
Minkowski space supporting and ordinary nongravitational QFT that we call R



$$|\psi(t>0)\rangle_{\mathrm{CFT}\otimes R} = \sum_{ij} c_{ij}\,|i\rangle_{\mathrm{CFT}} \otimes |j\rangle_R$$

$$|\psi(0)\rangle_{\mathrm{CFT}\otimes R} = |\mathrm{BH}(0)\rangle \otimes |0\rangle$$

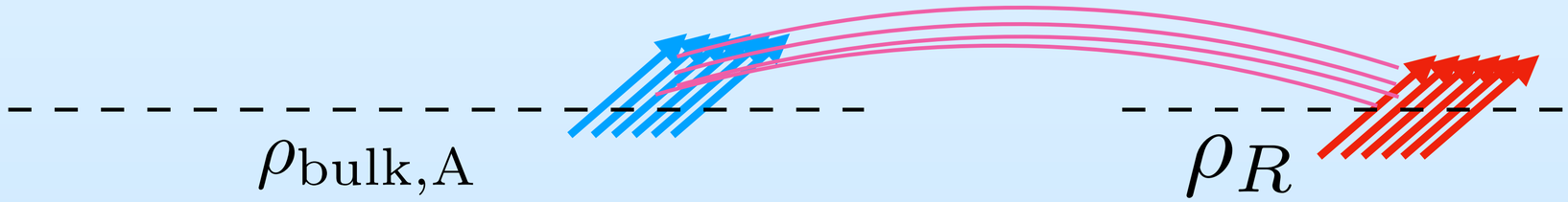




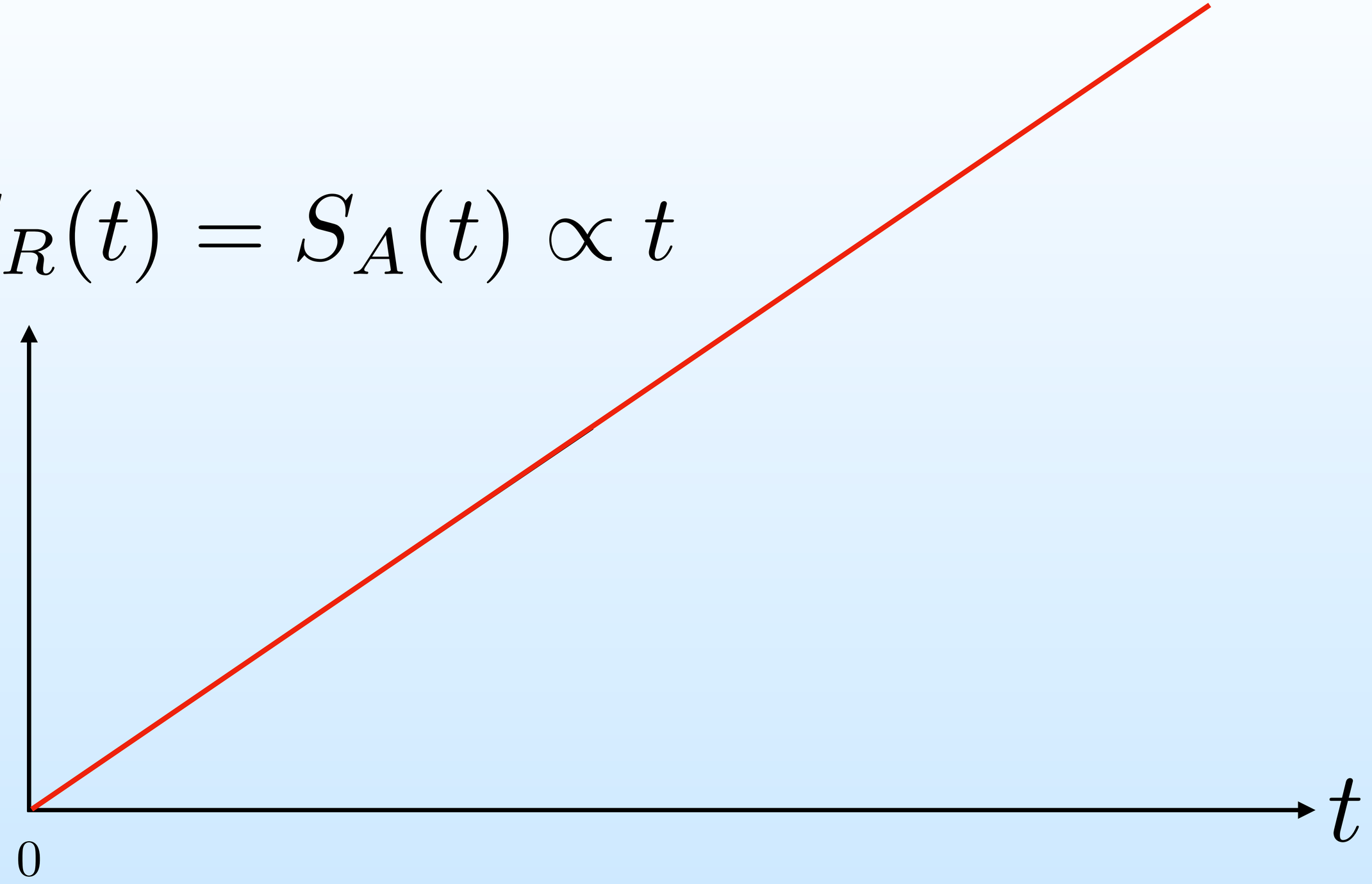
$$S_A(t) = \frac{\text{Area[empty set]}}{4G_N\hbar} + S_{\text{vN}}[\rho_{\text{bulk,A}}]$$

$$= S_{\text{vN}}[\rho_{\text{bulk,A}}]$$

$$\propto t$$



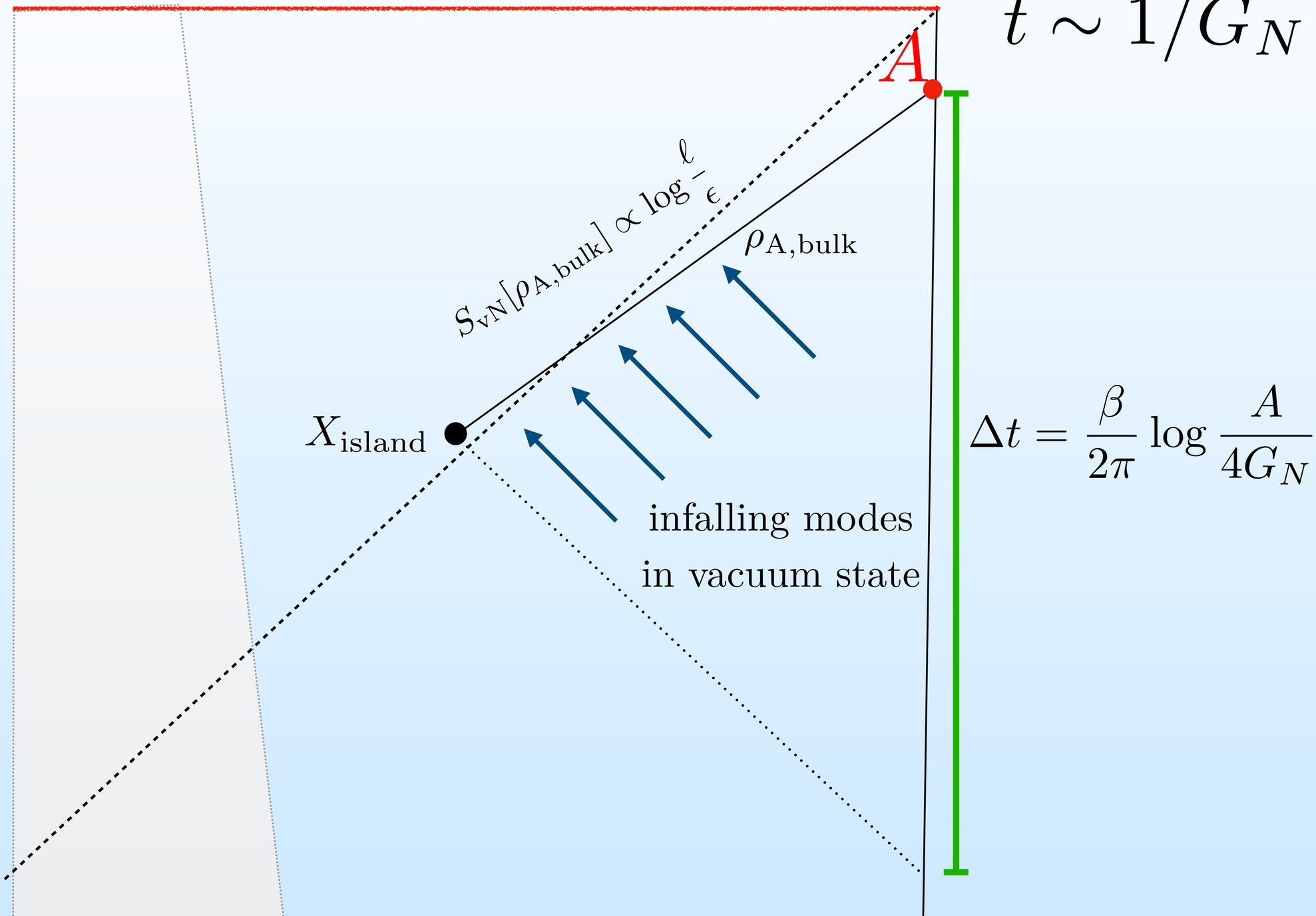
$$S_R(t) = S_A(t) \propto t$$



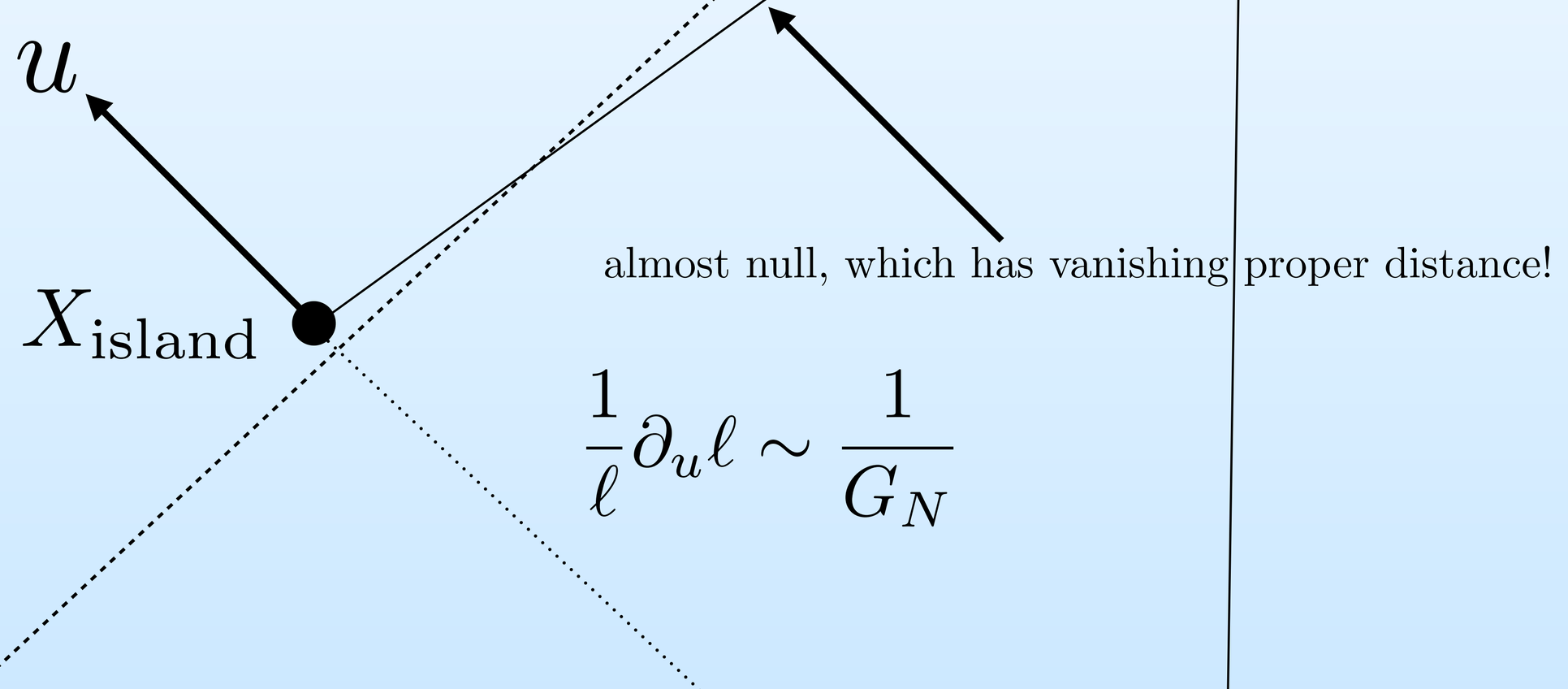
but... we made an interesting mistake for large times

$$S_{\mathrm{vN}}[\rho_A] = \frac{\mathrm{Area}[X_A]}{4G_N\hbar} + S_{\mathrm{vN}}[\rho_{\mathrm{bulk,A}}]$$

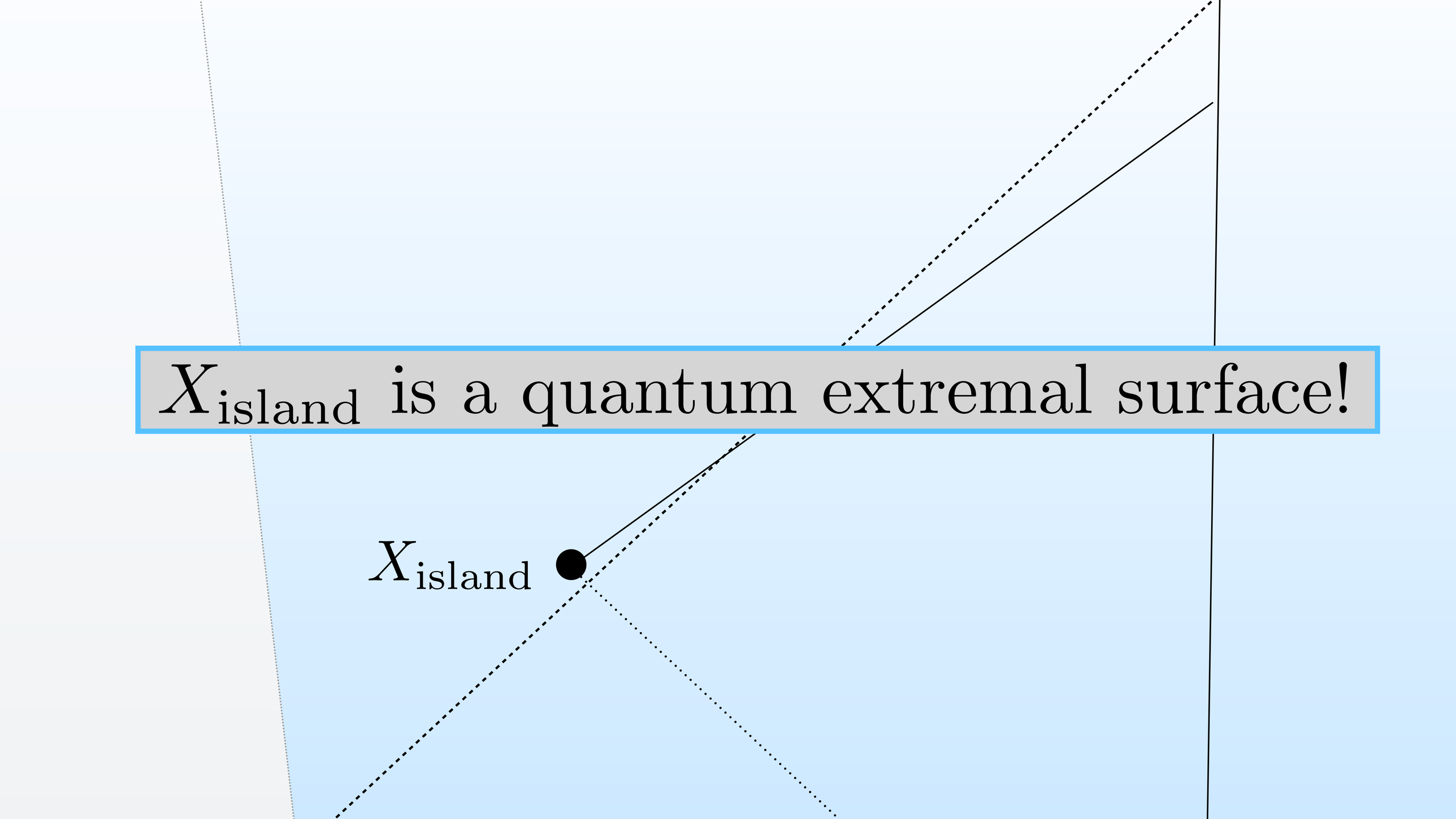
no longer perturbatively suppressed!

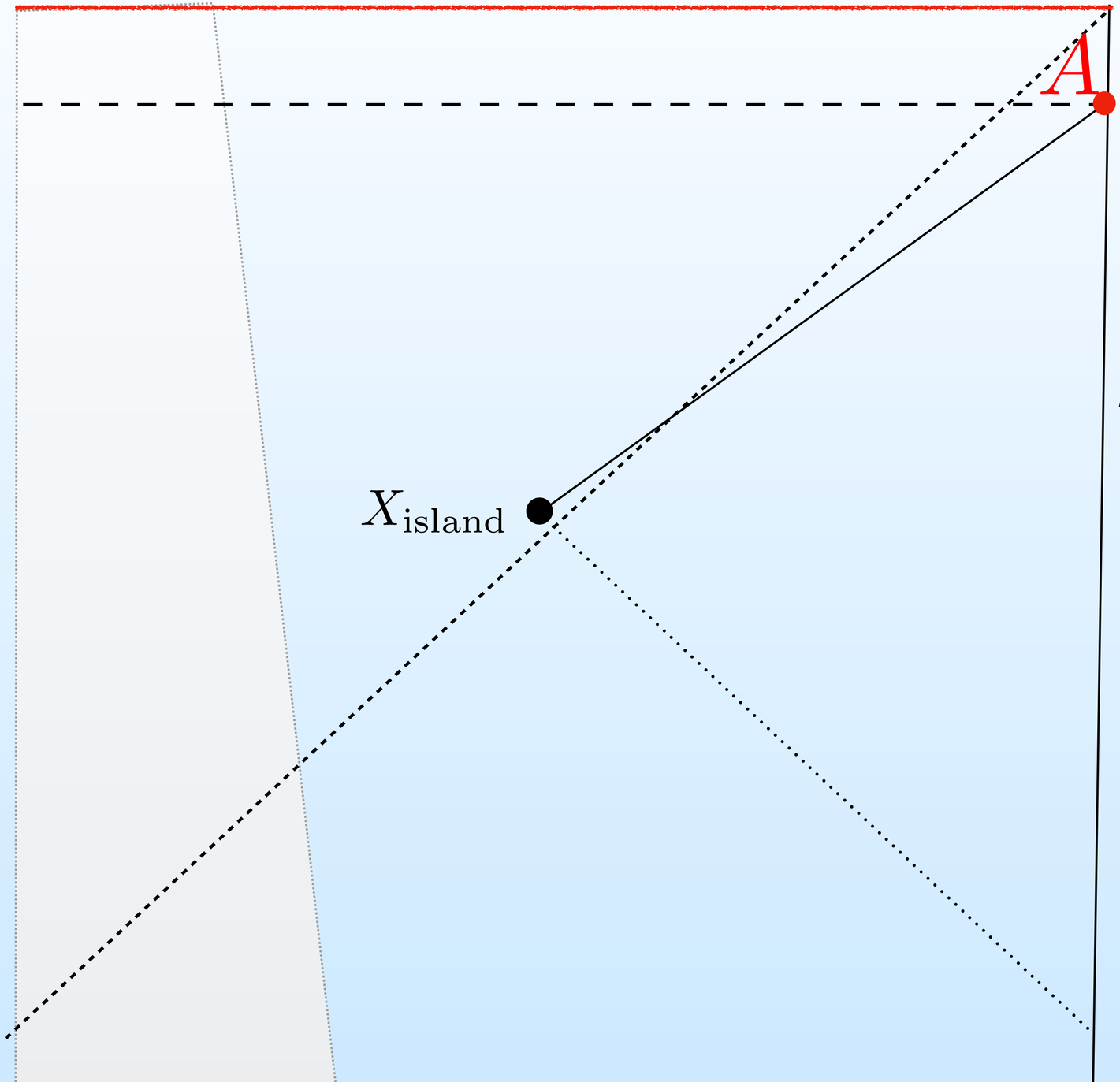

$t \sim 1/G_N$
A
$S_{\mathrm{vN}}[\rho_{\mathrm{A,bulk}}] \propto \log \frac{\ell}{\epsilon}$
$\rho_{\mathrm{A,bulk}}$
X_{island}
infalling modes
in vacuum state
$\Delta t = \frac{\beta}{2\pi} \log \frac{A}{4G_N}$

$$\partial_u S = \frac{1}{4G_N}\partial_u \text{Area}[X] + \frac{c}{3}\frac{1}{\ell}\partial_u \ell$$



X_{island} is a quantum extremal surface!



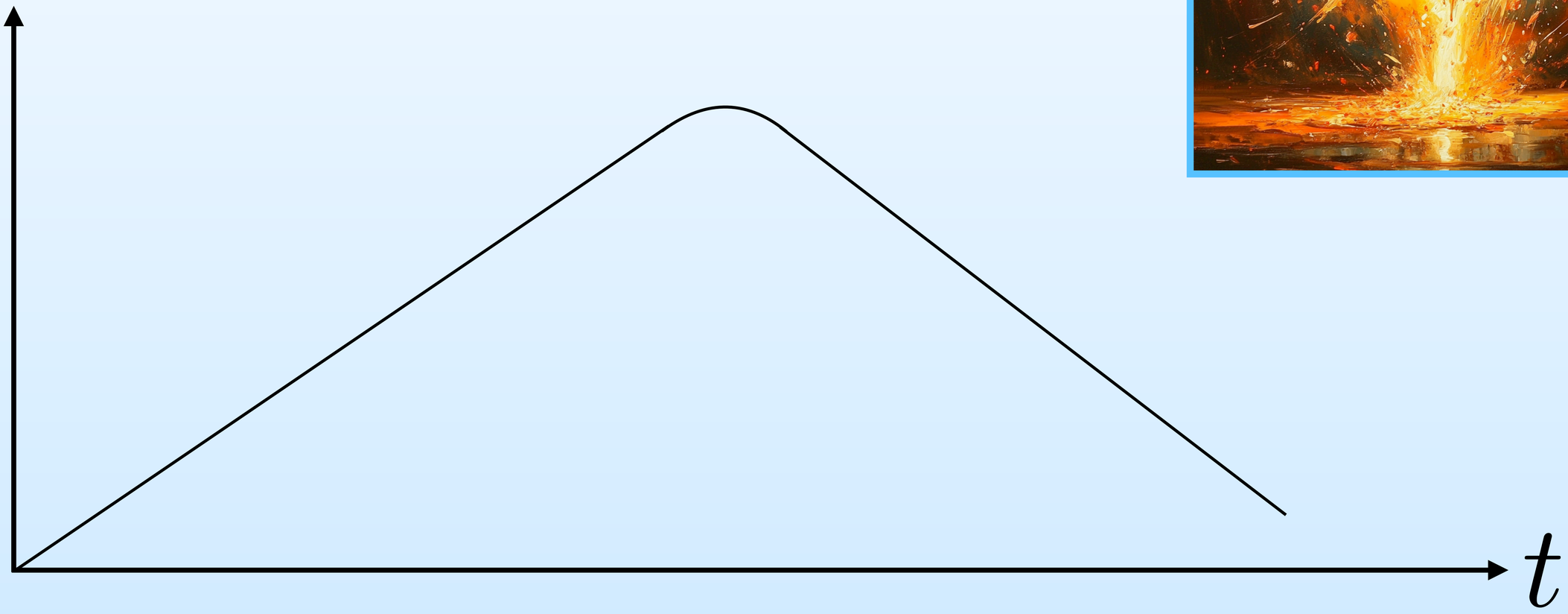


$$S_A(t) = \frac{\text{Area}[X_{\text{island}}]}{4G_N\hbar} + O(1)$$

$$\approx \frac{\text{Area}[\text{horizon}]}{4G_N\hbar}$$

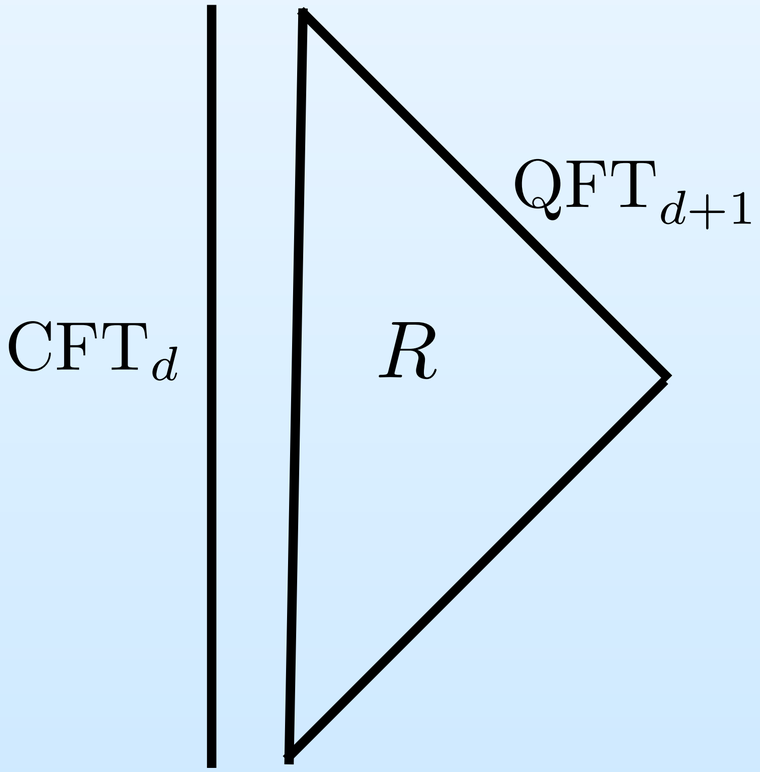
$$S_R(t) = S_A(t)$$

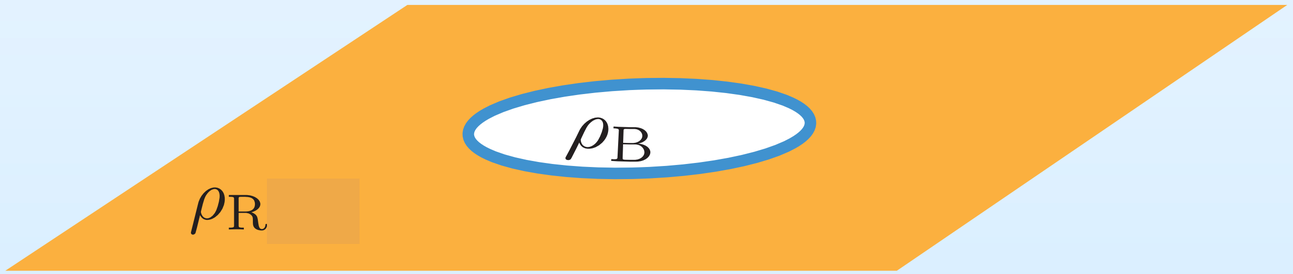




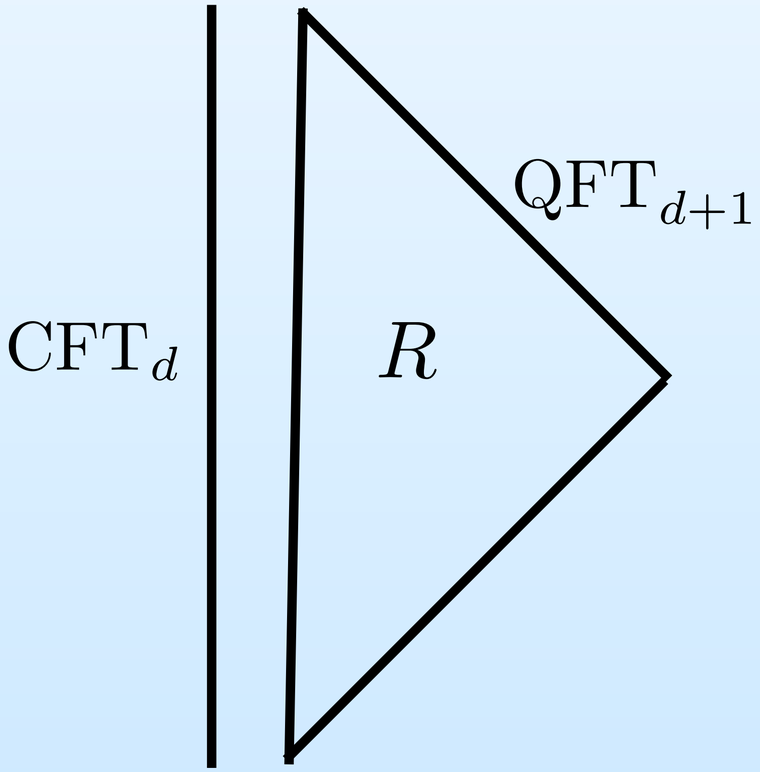
- The QES formula secretely knows about non-perturbative effects
- If we can assume the QES formula, only perturbative calculations are needed to reproduce the Page curve. Just enough non-perturbative information.

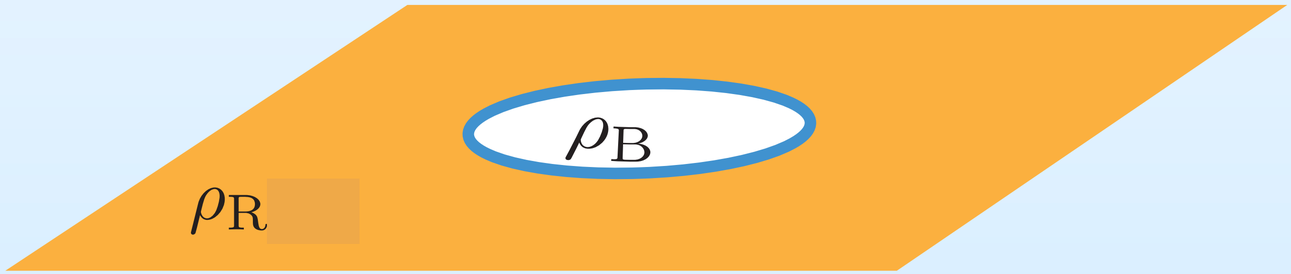
Forgetting about the gravitational perspective for a second



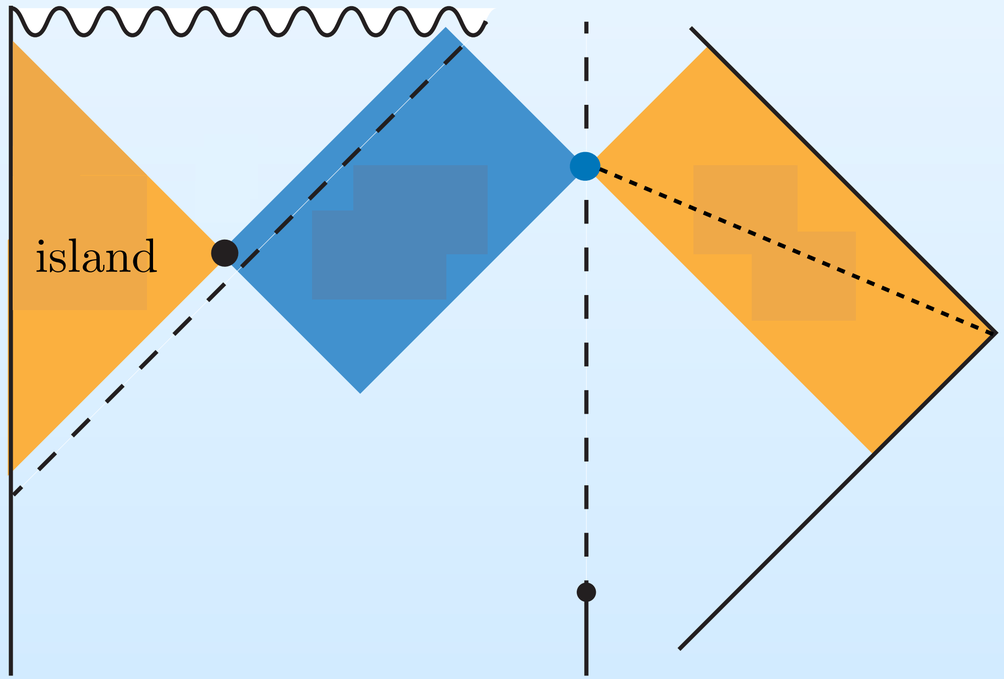


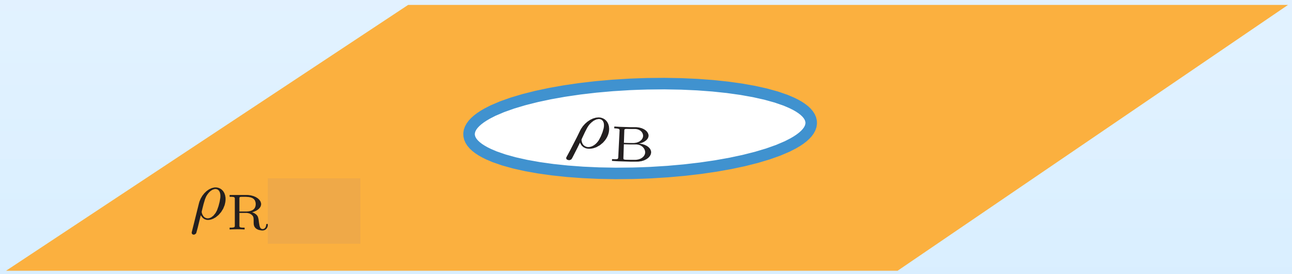
What system encodes what part of the gravitational bulk? If I want to act with an operator in the black hole interior, must I act on CFT_d or R?





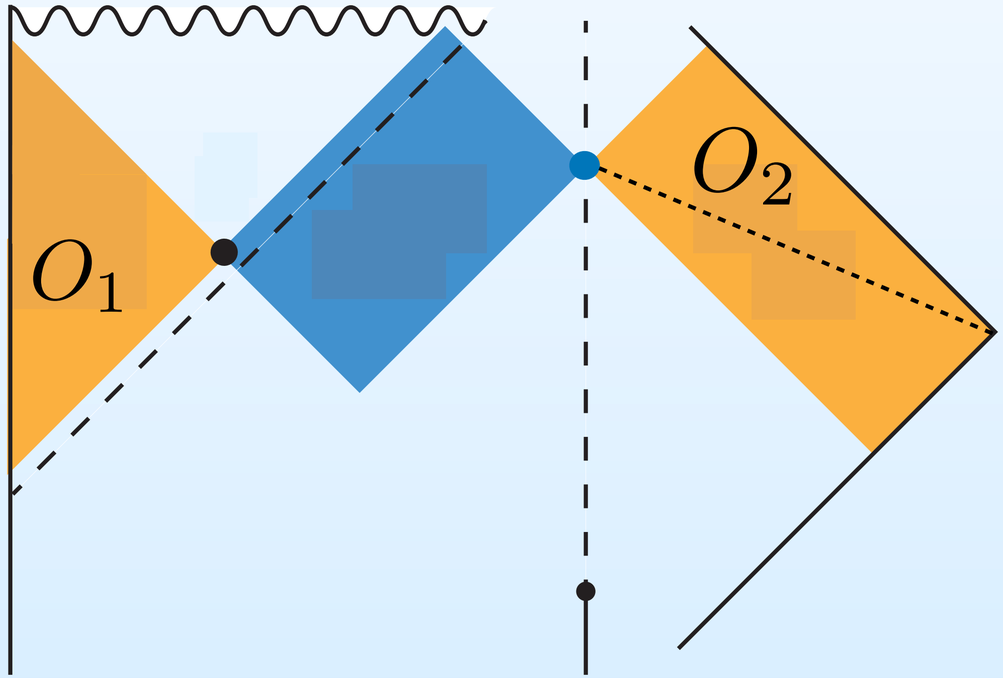
Everything points the island being encoded in the radiation





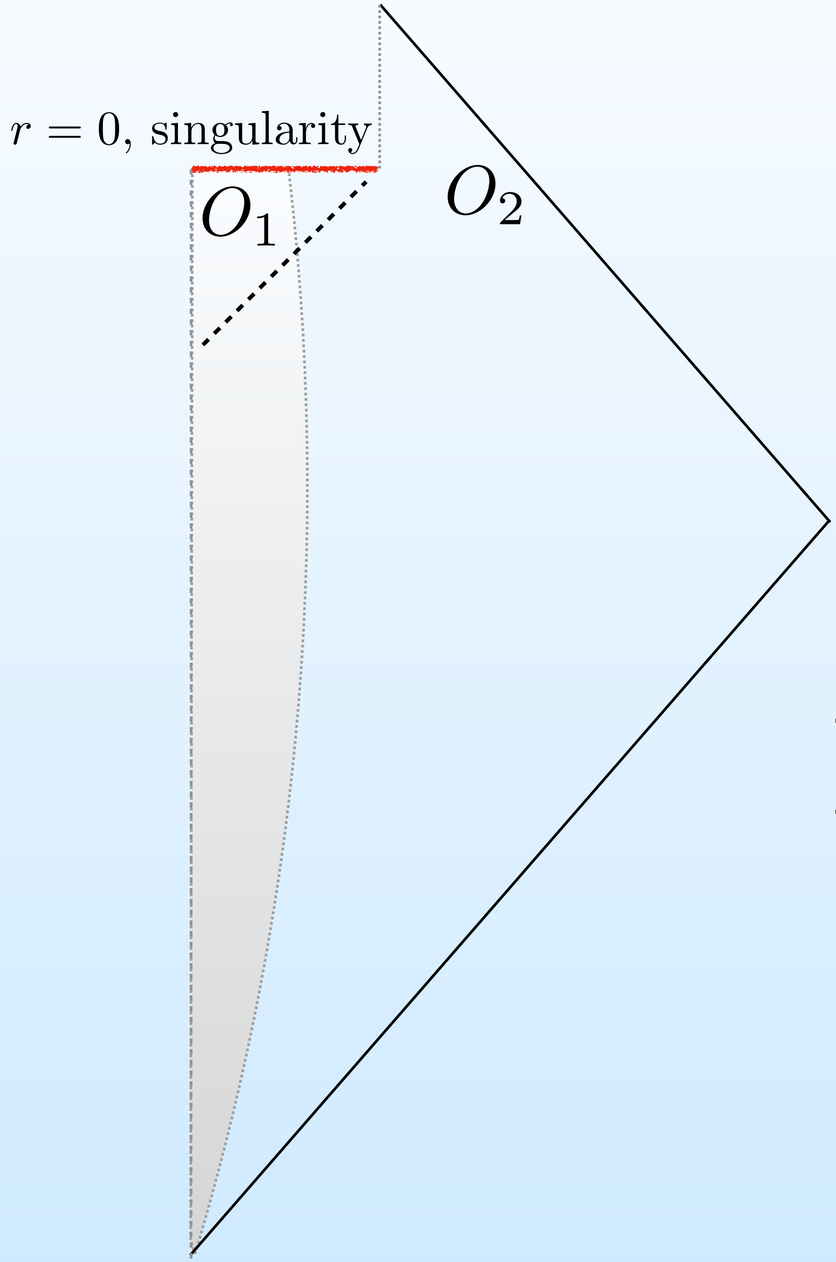
Partially solves the firewall problem

$\mathcal{H}_{\text{island}}$ should be identified as a subspace of $\mathcal{H}_{\text{R}}$



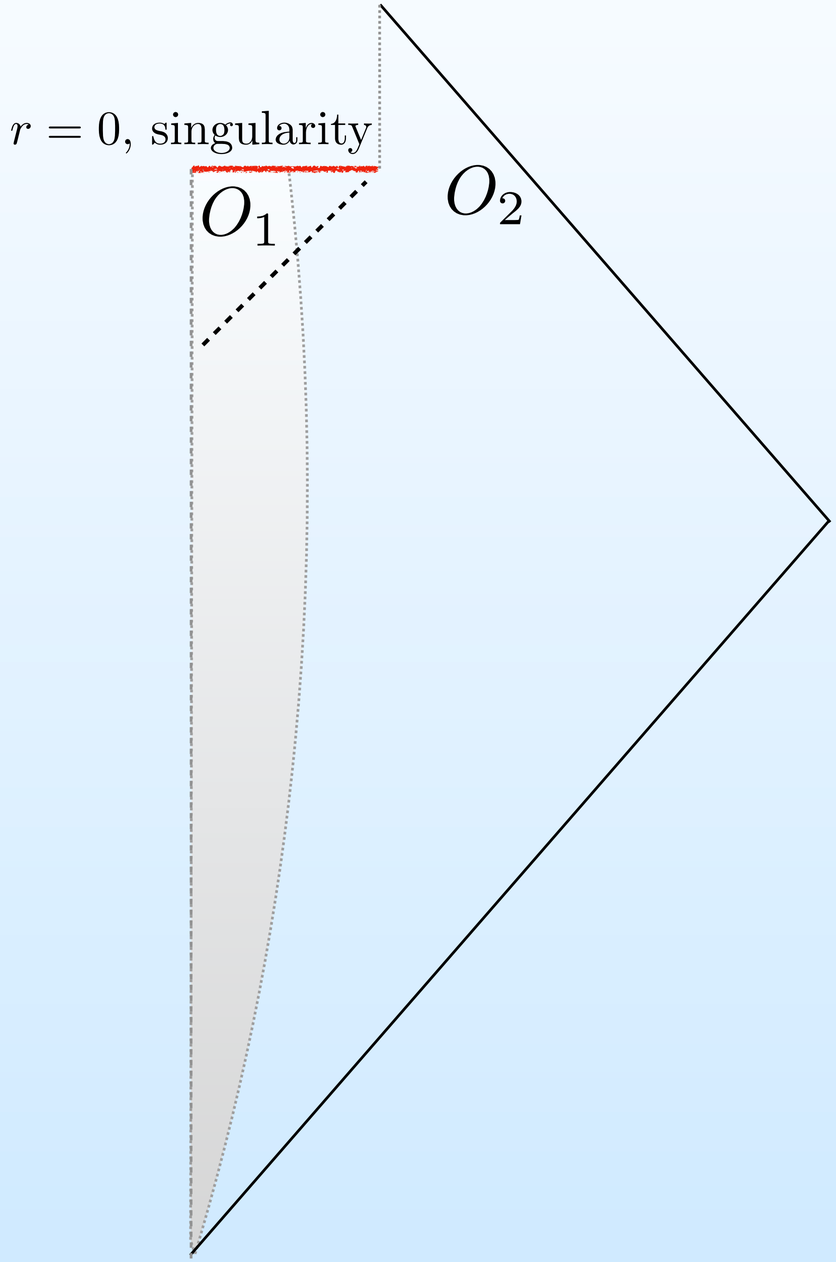
$$[O_1, O_2] \neq 0$$

"At late times, the black hole interior is secretly the same system as the radiation"



$$[O_1, O_2] \neq 0$$

Breakdown of locality/ microcausality?!



Evidence that these effects are exponentially surpressed for observers measuring operators that are simple — operators that do not require an exponentially large quantum circuit to implement [Akers et al. '22]

Spacetime locality does not seem to hold for exponentially complex operators

Black holes as mirrors: quantum information in random subsystems

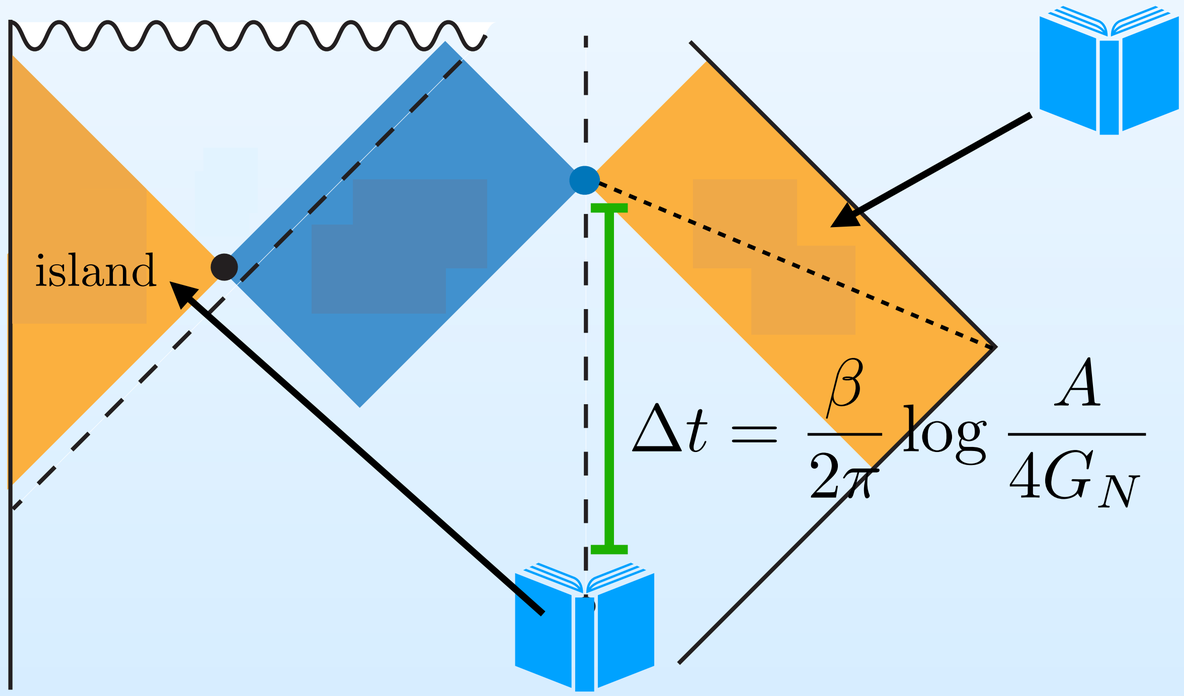
Patrick Hayden
School of Computer Science, McGill University, Montreal, Quebec, H3A 2A7, Canada

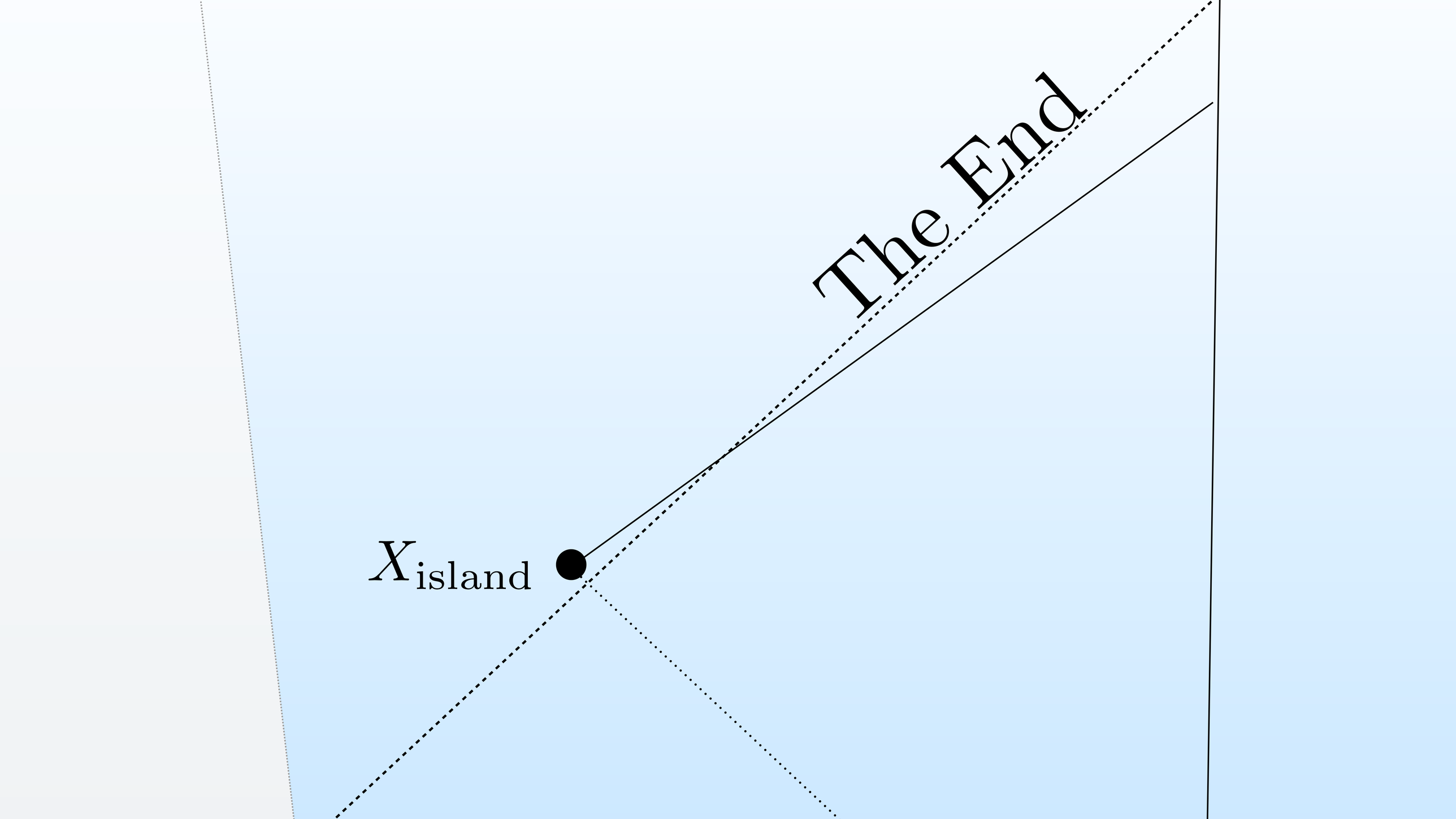
John Preskill
Institute for Quantum Information, California Institute of Technology, Pasadena CA 91125, USA

- Influential 2007 paper predicted that black holes really behave like mirrors after the Page time

- Assuming unitarity and mild assumption about chaotic nature about black holes, they predicted that the info of a diary thrown into a black hole comes back out in the emitted radiation after a time

$$t_{\text{scrambling}} = \frac{\beta}{2\pi} \log S_{\text{BH}}$$


island
$\Delta t = \frac{\beta}{2\pi} \log \frac{A}{4G_N}$


The End
X_{island}