

UNIVERSITY OF OSLO

Next-to-leading power
resummation in slepton pair
production

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Theory seminar

October 23



Outline

- ① Background: Large logarithms in perturbative QCD
- ② NLP resummation of large logarithms
- ③ Numerical impact
- ④ Conclusions and outlook

Background: Large logarithms in perturbative QCD

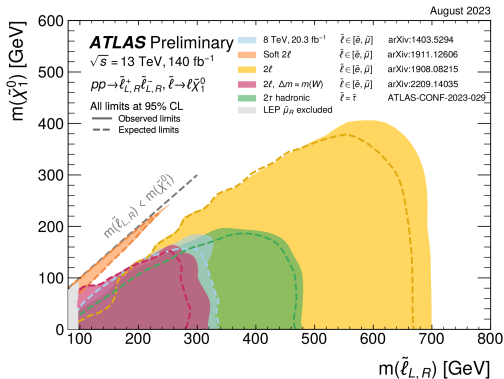
Prologue: A very short introduction to sleptons and the MSSM

- Minimal Supersymmetric Standard Model:
 - Poincaré symmetry group of Special Relativity extended by (broken) boson-fermion symmetry
 - Standard Model particles obtain supersymmetric partners with opposite statistics
- Standard Model lepton $\ell \rightarrow$ MSSM sleptons $\tilde{\ell}_L, \tilde{\ell}_R$
- In general different from mass eigenstates $\tilde{\ell}_A$, $A = 1, 2$
- Mass mixing described by 2×2 matrix L

$$\begin{pmatrix} \tilde{\ell}_1 \\ \tilde{\ell}_2 \end{pmatrix} = L \begin{pmatrix} \tilde{\ell}_L \\ \tilde{\ell}_R \end{pmatrix}$$

The need for precision theory predictions

- $X\%$ CL exclusion based on combined theoretical and experimental uncertainty
- Large theoretical uncertainties smear out the likelihood $\Rightarrow$ less stringent exclusion
- High-precision theoretical calculations needed, *and an accurate estimate of the theory uncertainty*
- Initial attempt: Perturbation theory



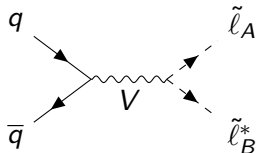
Slepton pair production in perturbative QCD

Writing the cross-section differential in the slepton-pair invariant mass Q^2 (for center-of-mass energy $\sqrt{s}$):

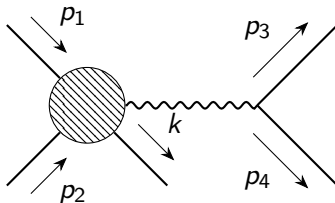
$$\frac{d\sigma}{dQ^2} = \sum_{ij} \int dx_1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \frac{d\hat{\sigma}_{ij}}{dQ^2} \left(z = \frac{Q^2}{x_1 x_2 s}, Q^2, \underbrace{\mu_R, \mu_F}_{\text{Renormalization/factorization scales}} \right)$$

$\Rightarrow$ Partonic cross-sections $\frac{d\hat{\sigma}_{ij}}{dQ^2}$ (in principle) computable by order in α_s

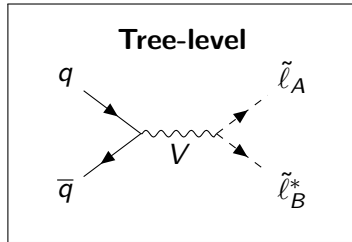
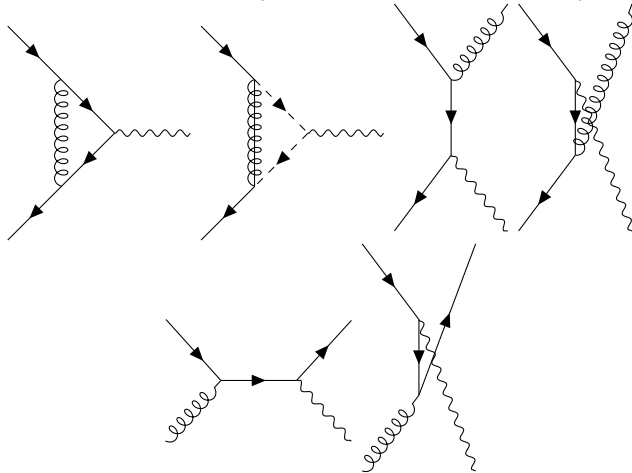
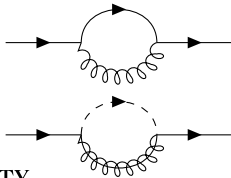
Leading-order partonic diagram:



Higher-order kinematics:



- $\hat{s} \equiv (p_1 + p_2)^2$
- $Q^2 \equiv (p_3 + p_4)^2$
- $z \equiv \frac{Q^2}{\hat{s}}$
- $k^0 = (1 - z) \frac{\sqrt{\hat{s}}}{2}$

Partonic contributions to $\mathcal{O}(\alpha_s)$ **One-loop level (slepton part unchanged)****Counterterms**

Threshold limit

$z \rightarrow 1$ ($Q^2 \rightarrow x_1 x_2 s$): Nearly all available energy goes to the sleptons, radiation becomes soft. In this limit,

$$\frac{d\hat{\sigma}_{q\bar{q}}^{\text{NLO}}}{dQ^2} \sim \frac{C_F \alpha_s}{\pi} \left\{ 4 \left(\frac{\log(1-z)}{1-z} \right)_+ - 2 \log \frac{\mu_F^2}{Q^2} \frac{1}{(1-z)_+} - 4 \log(1-z) + c_1^{(\delta)} \delta(1-z) \right\}$$

$$\frac{d\hat{\sigma}_{qg}^{\text{NLO}}}{dQ^2} \sim \frac{T_F \alpha_s}{\pi} \log(1-z) \quad \left[\int_0^1 dz g(z) f_+(z) \equiv \int_0^1 dz (g(z) - g(1)) f(z) \right]$$

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These terms may dominate in this limit and spoil the stability of the result!

Large logarithmic terms: Classification

Such terms appear at all orders; generically we have

$$\frac{d\sigma}{dQ^2} \sim \sigma_0 \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \left[\sum_{m=0}^{2n-1} \left[\underbrace{c_{nm}^{(-1)} \left(\frac{\log^m(1-z)}{1-z}\right)}_{\text{Leading power (LP) terms}} + \underbrace{c_{nm}^{(0)} \log^m(1-z)}_{\text{NLP terms}} + \dots \right] + c_n^{(\delta)} \delta(1-z) \right]$$

$$m = \begin{cases} 2n-1 : & \text{Leading logarithmic (LL) terms} \\ 2n-2 : & \text{Next-to-leading logarithmic (NLL) terms} \\ 2n-k : & \text{N}^{k-1}\text{LL terms} \end{cases}$$

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- Perturbation theory is insufficient for precise predictions
- Resummation methods needed to properly capture threshold effects

NLP resummation of large logarithms

Mellin transforms

$$\mathcal{M}(f) \equiv \int_0^1 dx \, x^{N-1} f(x)$$

$$\mathcal{M}^{-1}(F) \equiv \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN \, x^{-N} F(N)$$

Under Mellin transforms, convolution integrals factorize:

$$f(x) \equiv \int_0^1 dx_1 dx_2 f_1(x_1) f_2(x_2) \delta(x - x_1 x_2) \longrightarrow \mathcal{M}(f) = \mathcal{M}(f_1) \mathcal{M}(f_2)$$

Factorization and exponentiation in QCD

In Mellin space near threshold, the cross-section factorizes:

$$\mathcal{M} \left(\frac{d\hat{\sigma}_{ij}}{dQ^2} \right) = (\text{Ultraviolet (hard) effects}) \times (\text{Infrared (soft) effects})$$

Further, the infrared effects exponentiate, leaving

$$\mathcal{M} \left(\frac{d\hat{\sigma}_{ij}}{dQ^2} \right) = H_{ij} (Q^2, \mu_F, \mu_R) \exp (W_{ij} (N, Q^2, \mu_F, \mu_R))$$

Resummation strategies

Logarithms from the two partonic channels are resummed in different ways:

- $q\bar{q}$: Generalized Wilson lines
- qg ($\bar{q}g$): Diagrammatic methods

Wilson lines

Arise as *connections* in non-Abelian gauge theories;

$$W_P(x, y) \equiv P \left\{ \exp \left(ig_s \int_y^x A_\mu^a(z) T^a dz^\mu \right) \right\}$$

- $W_P(x, y) \psi(y) - \psi(x) \rightarrow e^{i\alpha^a(x)T^a} (W_P(x, y) \psi(y) - \psi(x))$
- $D_\mu \psi(x) \equiv \lim_{\delta x^\mu \rightarrow 0} \frac{W_P(x, x + \delta x) \psi(x + \delta x) - \psi(x)}{\delta x^\mu}$
 $= \partial_\mu \psi(x) - ig_s A_\mu^a T^a \psi(x)$

Wilson lines and the eikonal approximation

Semi-infinite Wilson lines can be used to describe soft radiation from high-energy fermions:

Letting $z^\mu(\lambda) = p^\mu \lambda$, with $\lambda \in (-\infty, 0)$, $p^2 = 0$, we find

$$W(p) = P \left\{ \exp \left(\int \frac{d^4 k}{(2\pi)^4} \tilde{A}_\mu^a(k) g_s T^a \frac{p^\mu}{p \cdot k} \right) \right\}$$

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$\Rightarrow$ Exponentiated expression containing all-order soft radiation!

Cross-sections from Wilson lines

The eikonal Drell-Yan cross-section can be constructed from the *Drell-Yan Wilson line*
 $W_{\text{DY}} \equiv W^\dagger(p_2) W(p_1)$

$$\begin{aligned} \frac{d\hat{\sigma}_{q\bar{q}}^{\text{eik}}}{dQ^2} &= \sigma_0 \left\langle |W_{\text{DY}}|^2 \right\rangle \delta\left(z - \frac{Q^2}{\hat{s}}\right) \\ &= \sigma_0 \sum_{\left\{ \begin{array}{l} n\text{-gluon final states} \\ \text{integrated over} \\ \text{phase space} \end{array} \right\}} \left\langle 0 \left| W^\dagger(p_1) W(p_2) \right| n \right\rangle \left\langle n \left| W^\dagger(p_2) W(p_1) \right| 0 \right\rangle \delta\left(z - \frac{Q^2}{\hat{s}}\right) \end{aligned}$$

$\Rightarrow$ Exponentiates, leading to NLL leading-power resummed result

Generalized Wilson lines

“Next-to-eikonal” effects captured by allowing for slight deviations from a straight-line trajectory:

$$F(p) = P \exp \left[\int \frac{d^d k}{(2\pi)^d} \tilde{A}_\mu^a(k) \underbrace{g_s T^a \left(\frac{p^\mu}{p \cdot k} - \frac{k^\mu}{2p \cdot k} + k^2 \frac{p^\mu}{2(p \cdot k)^2} + ik_\nu \frac{S^{\nu\mu}}{p \cdot k} \right)}_{\text{NE vertex}} \right]$$

$$[S^{\nu\mu} \equiv \frac{i}{4} [\gamma^\nu, \gamma^\mu]]$$

Cross-sections from generalized Wilson lines

From this we can construct the next-to-eikonal cross-section in the same way as before:

$$\frac{d\hat{\sigma}_{q\bar{q}}^{\text{NE}}}{dQ^2} = \sigma_0 \sum \left\langle 0 \left| F^\dagger(p_1) F(p_2) \right| n \right\rangle \left\langle n \left| F^\dagger(p_2) F(p_1) \right| 0 \right\rangle \delta \left(z - \frac{Q^2}{\hat{s}} \right)$$

$\left\{ \begin{array}{l} n\text{-gluon final states} \\ \text{integrated over} \\ \text{LP phase space} \end{array} \right\}$

N. Bahjat-Abbas et al., *Diagrammatic resummation of leading-logarithmic threshold effects at next-to-leading power*, *JHEP* **11** (2019) 002

[1905.13710]

Soft quark emission

- Previous calculation: Only gluon emission, $\sum_{\text{spins}} \epsilon_{\mu}(k) \epsilon_{\nu}^{*}(k) \rightarrow -g_{\mu\nu}$

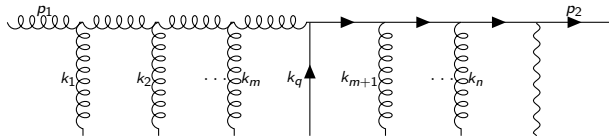
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- Previous calculation: Only gluon emission, $\sum_{\text{spins}} \epsilon_\mu(k) \epsilon_\nu^*(k) \rightarrow -g_{\mu\nu}$
- Gluon-initiated process: Quark emission, $\sum_{\text{spins}} u(k) \bar{u}(k) = \not{k}$
 - A soft quark emission leaves us at NLP in soft momenta
 - Only need to consider further eikonal radiation with the vertex $g_s T_R \frac{p^\mu}{p \cdot k}$, and with LP phase space

Resummation of qg contribution

LL NLP cross-section obtained by summing n -gluon diagrams

- Real emission diagrams evaluated directly:



- Virtual corrections found indirectly through soft gluon unitarity, factorization arguments

M. van Beekveld et al., *Threshold resummation of new partonic channels at next-to-leading power*, *JHEP* **12** (2021) 087 [2109.09752]

Numerical impact

Evaluating total cross-section

Recall factorization for the resummed part:

$$\mathcal{M} \left(\frac{d\hat{\sigma}_{ij}^{\text{res}}}{dQ^2} \right) = H_{ij} (Q^2, \mu_F, \mu_R^2) \exp (W_{ij} (N, Q^2, \mu_F, \mu_R))$$

- H_{ij} only needed to $\mathcal{O}(\alpha_s^{n-1})$ for $N^n\text{LL}$ resummation
- Results with higher-order terms in H_{ij} referred to as $N^n\text{LL}'$

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Two main sources:

- Input parameters (including PDFs)
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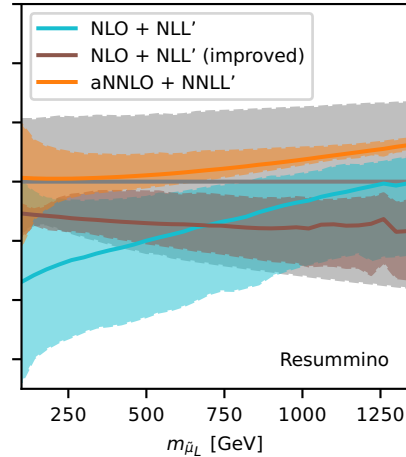
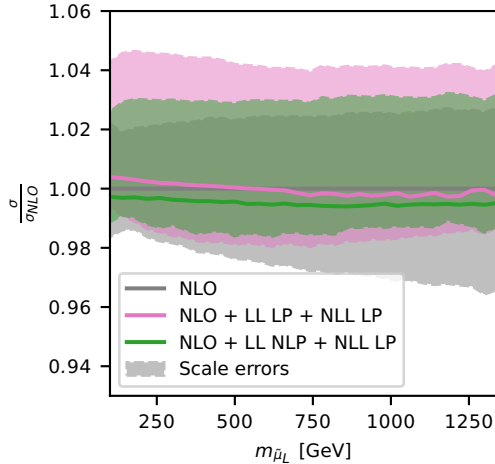
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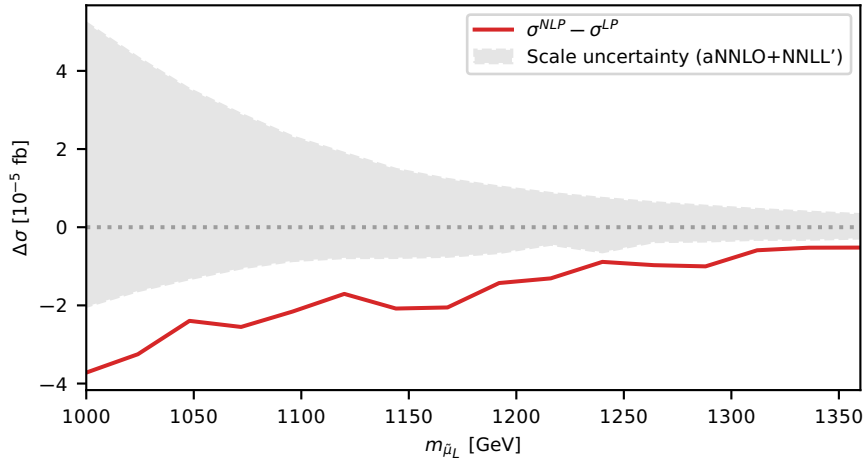
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 - We will use 7-point scale variation:
$$\left(\frac{\mu_F}{\mu_0}, \frac{\mu_R}{\mu_0}\right) \in \left\{(1, 1), \left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 1\right), \left(1, \frac{1}{2}\right), (1, 2), (2, 1)\right\}$$

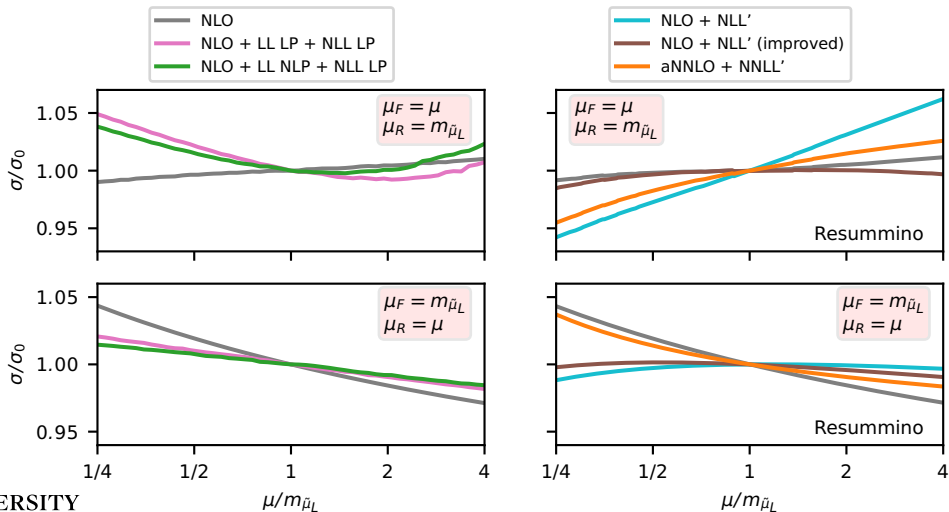
Impact and scale uncertainty



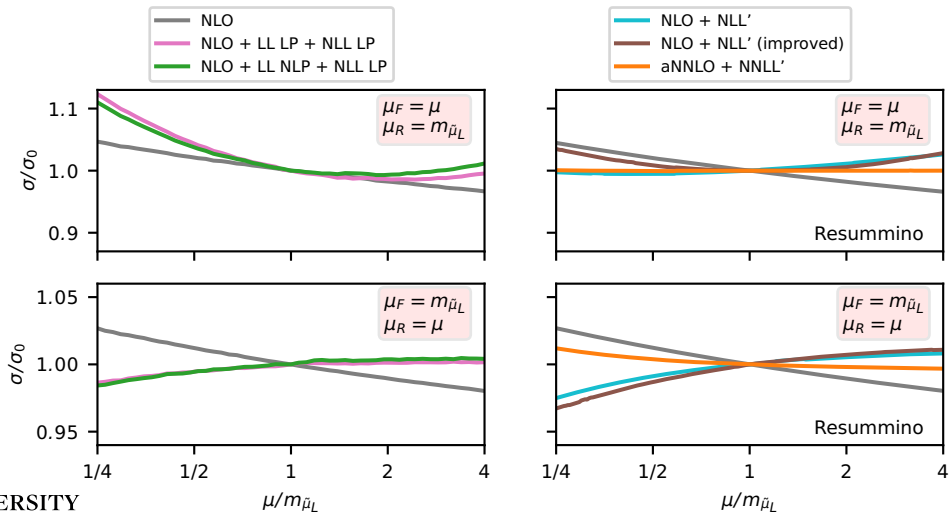
NLP impact vs NNLL' uncertainty



Scale dependence ($m_{\tilde{\mu}_L} = 120 \text{ GeV}$)



Scale dependence ($m_{\tilde{\mu}_L} = 1000 \text{ GeV}$)



Conclusions and outlook

Conclusions and outlook

- Higher-power resummation an important complement to higher logarithmic accuracy
 - Reduced scale uncertainty compared to leading-power resummation
 - Larger shift in cross-section than the scale uncertainty from existing calculations
⇒ may be underestimating impact of new corrections
- Implemented in the *Smoking* code (under development), initially for slepton pair production

Extras

Resummation results

$$W_{q\bar{q}}^{\text{res}} = \exp \left\{ \log \bar{N} g_1(\lambda) + g_2(\lambda) + h_1(\lambda, N) \right\},$$

$$\text{with } (\bar{N} \equiv N e^{\gamma_E}, \lambda \equiv b_0 \alpha_s \log \bar{N})$$

$$g_1(\lambda) = \frac{A_1}{\pi b_0 \lambda} (2\lambda + (1 - 2\lambda) \log(1 - 2\lambda)),$$

$$g_2(\lambda) = -\frac{A_2}{\pi^2 b_0^2} (2\lambda + \log(1 - 2\lambda)) \\ + \frac{A_1 b_1}{\pi b_0^3} \left(2\lambda + \log(1 - 2\lambda) + \frac{1}{2} \log^2(1 - 2\lambda) \right) \\ - \frac{A_1}{\pi b_0} (2\lambda + \log(1 - 2\lambda)) \log \frac{\mu_R^2}{Q^2} + \frac{2A_1 \lambda}{\pi b_0} \log \frac{\mu_F^2}{Q^2},$$

$$h_1(\lambda, N) = -\frac{A_1}{\pi b_0} \frac{\log(1 - 2\lambda)}{N}$$

Constants:

$$4\pi b_0 = \frac{11}{3} C_A - \frac{4}{3} T_F N_F,$$

$$(4\pi)^2 b_1 = \frac{34}{3} C_A^2 - \frac{20}{3} C_A T_F N_F - 4 C_F T_F N_F,$$

$$A_1 = C_F,$$

$$A_2 = C_F \left(C_A \left(\frac{67}{36} - \frac{\pi^2}{12} \right) - N_F \frac{5}{18} \right)$$

Resummation results

$$W_{qg}^{\text{res}} = \frac{T_F}{C_A - C_F} \frac{1}{2N \log N} \left[\exp \left\{ 2 \frac{\alpha_s}{\pi} C_F \log^2 N \right\} \mathcal{B}_0 \left(\frac{\alpha_s}{\pi} (C_A - C_F) \log^2 N \right) - \exp \left\{ \frac{\alpha_s}{2\pi} (C_F + 3C_A) \log^2 N \right\} \right],$$

where

$$\mathcal{B}_0(x) \equiv 1 - \frac{x}{2} - \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n!)^2} |B_{2n}| x^{2n}$$