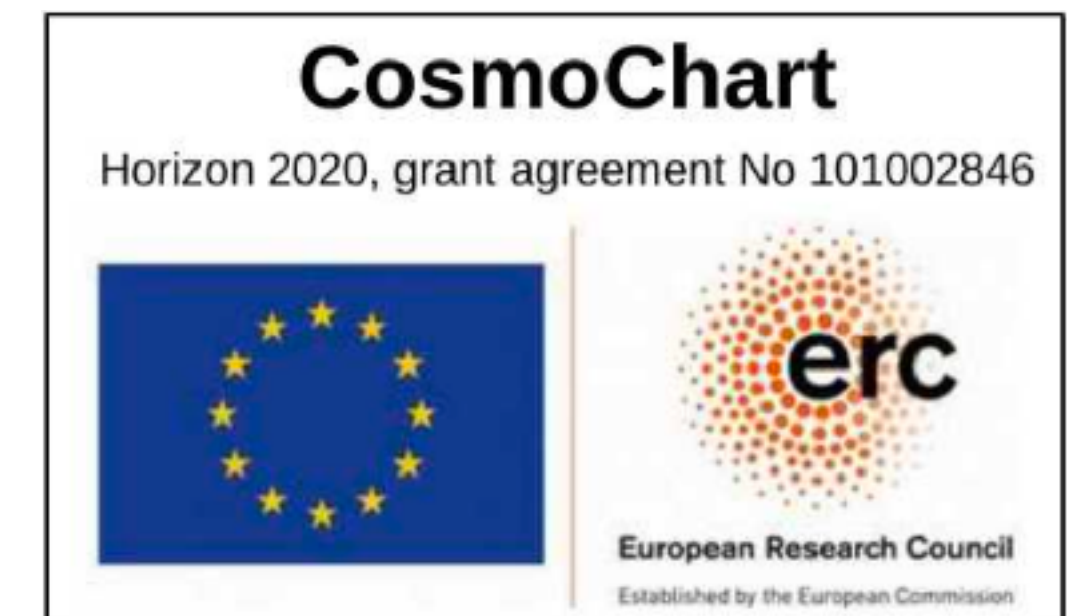


# Long-range interactions and the early Universe

**Marcos M. Flores**

**Theory Seminar, University of Oslo**

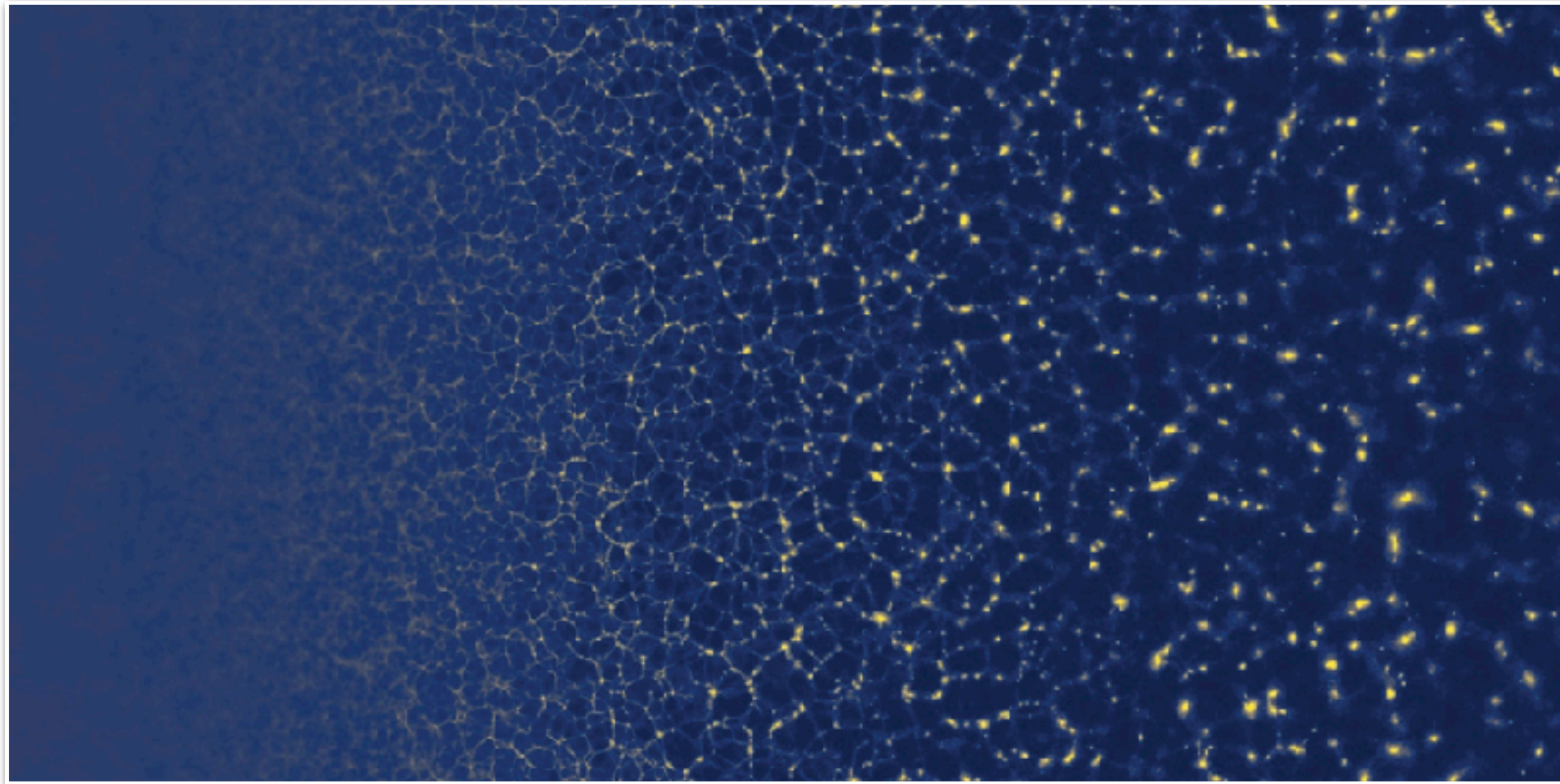
**5th, Feb., 2025**



# Long-range interactions & cosmology

$$V(r) \sim 1/r$$

primordial structure formation



[Domenech, et. al., arXiv:2303.13053]

macroscopic



partial wave unitarity

$$\sigma_{\text{inel}}^{\ell} v_{\text{rel}} \leq \sigma_{\text{uni}}^{\ell} v_{\text{rel}} = \frac{4\pi(2\ell + 1)}{M_{\text{DM}}^2 v_{\text{rel}}}$$

microscopic



# Long-range interactions and *primordial structure formation*

[MMF, A. Kusenko; 2404.05430]

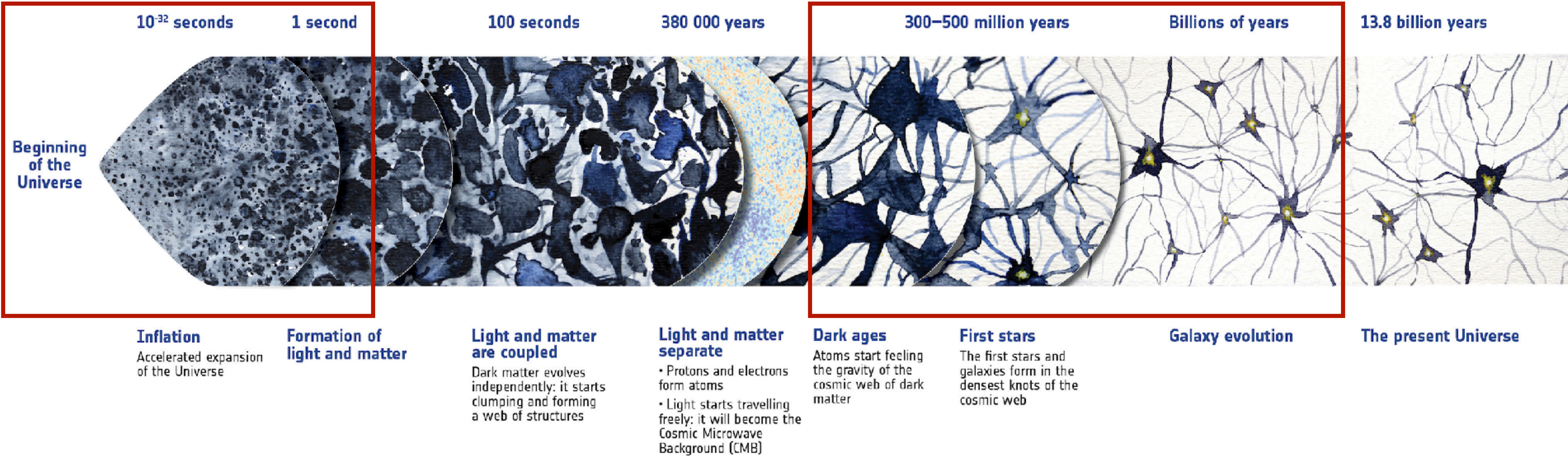
*To appear in the book “Primordial Black Holes”, ed. Chris Byrnes, Gabriele Franciolini,  
Tomohiro Harada, Paolo Pani, Misao Sasaki;*



# Cosmological timeline

*“Primordial”*

**Structure Formation**



# Conventional structure formation: Basics

$$\rho(x, t) = \bar{\rho}(t) (1 + \delta(x, t))$$

$\Downarrow$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad \nabla_{\mu} T^{\mu\nu} = 0$$

$\Downarrow$

System of Coupled Differential Equations

$$(\delta(x, t) \ll 1)$$

# Conventional structure formation by example

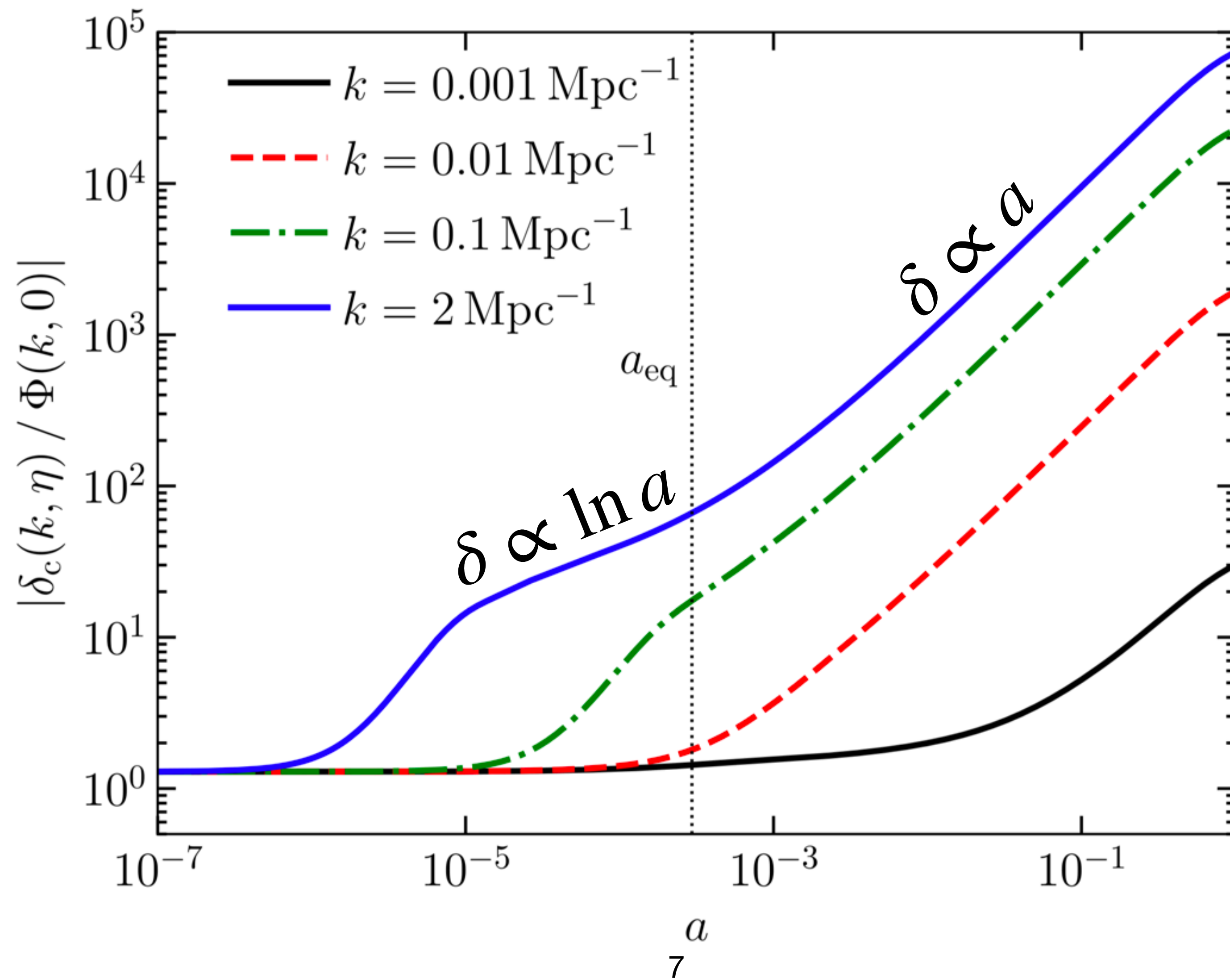
$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\bar{\rho}_m\delta, \quad H = \frac{1}{2t}, \quad \bar{\rho}_m \sim \Omega_m \sim 0$$



$$\delta(t) = c_1 + c_2 \ln t$$

**Conclusion:** *Matter perturbations only grow logarithmically during a radiation dominated era*

# Conventional structure formation by example



[Credit: Dodelson, Modern Comology]

How can you have *primordial* structure formation?

How can you have primordial structure formation?

$$\mathcal{L} \supset y\chi\bar{\psi}\psi \implies V(r) = -\frac{y^2}{4\pi r}\exp\left(-m_\chi r\right)$$

$$r \ll m_\chi^{-1} \longleftrightarrow \text{long-range force}$$

$$\implies H^{-1} \ll m_\chi^{-1}$$

# How can you have primordial structure formation?

$$\mathcal{L} \supset y\chi\bar{\psi}\psi \implies V(r) = -\frac{y^2}{4\pi r}\exp\left(-m_\chi r\right)$$

Yukawa interactions are *always* attractive\*

$$\beta \equiv y \left( M_{\text{Pl}}/m_\psi \right)$$

# Growth of perturbations

- For large scalar forces, the perturbations grow quickly as demonstrated by the approximate solution:

$$\Delta_k(t) \approx \frac{\Delta_k(t_0)}{\sqrt{8\pi}} \frac{\exp\left(4\sqrt{p}(t/t_{\text{eq}})^{1/4}\right)}{p^{1/4}(t/t_{\text{eq}})^{1/8}}, \quad p = \frac{3}{8} \underline{(1 + \beta^2)}$$

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For  $p \gg 1 \implies \Delta_k / \dot{\Delta}_k \ll H^{-1} \implies$  rapid structure formation

# Growth of perturbations

$$\Delta_k \ll 1 \implies \Delta_k \gtrsim 1 \iff \text{nonlinear regime} \implies \text{virialize}$$

***Without dissipation***, halos will remain viralized until the constituent particles decay

# Energy dissipation through scalar radiation

The same long-range force that cause the growth of structure will also cause accelerating particles to emit scalar waves

There are *five* possible dissipation channels:

1. Coherent motion
2. Incoherent motion
3. Bremsstrahlung (free-free) emission
4. Bound state formation
5. Surface radiation

Bremsstrahlung and surface radiation will be the most important channels for our discussion

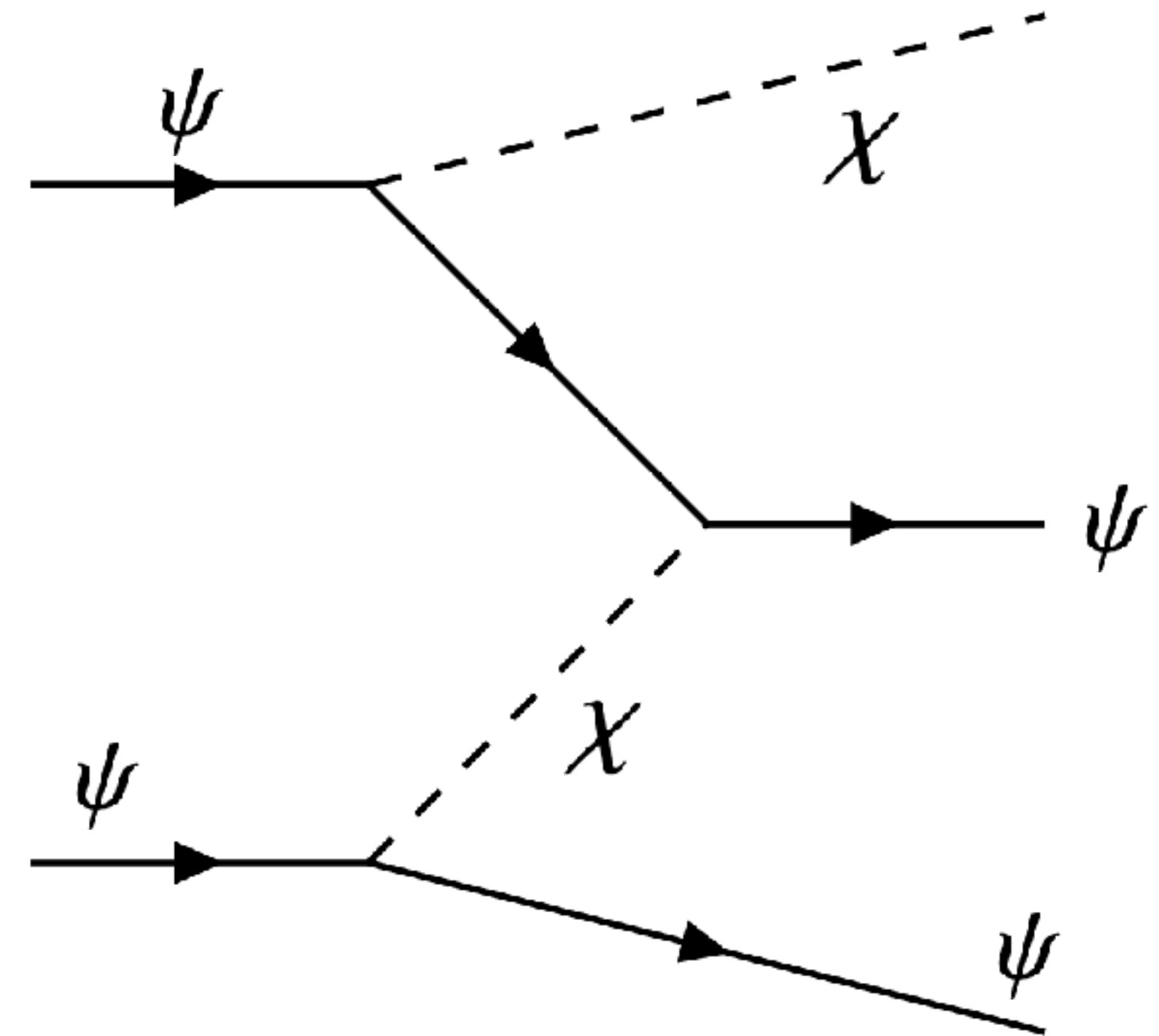
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5. Surface radiation

$$\tau_{\text{cool}} = \frac{E}{P_{\text{brem}} + P_{\text{surf}} + \dots}$$

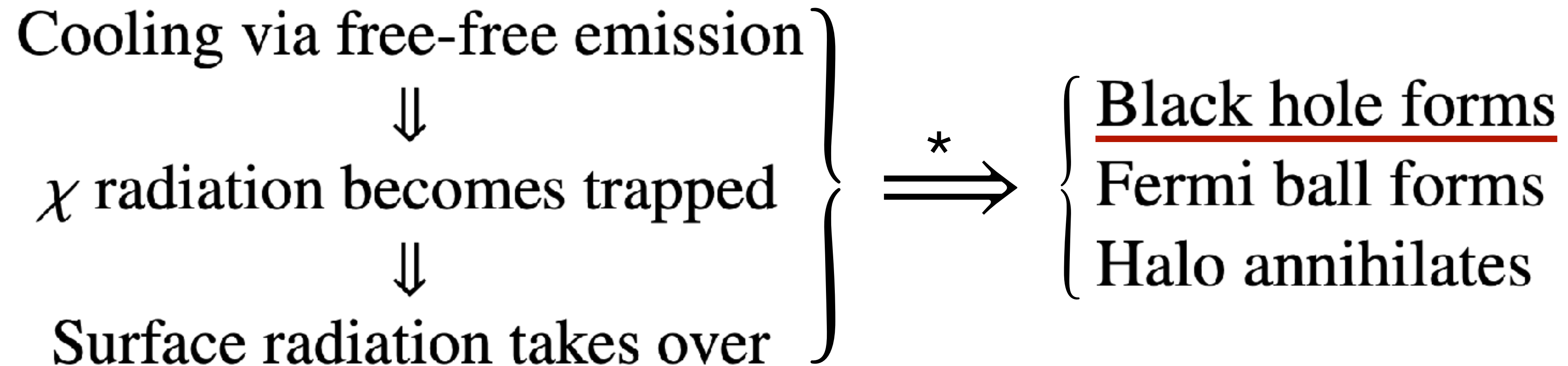
Bremsstrahlung and surface radiation will be the most important channels for our discussion

# Energy dissipation through scalar radiation

Given a halo of size  $R$  can lose energy and contract as long as,

$$\tau_{\text{cool}}(R) \ll H^{-1}$$

General algorithm for collapse:

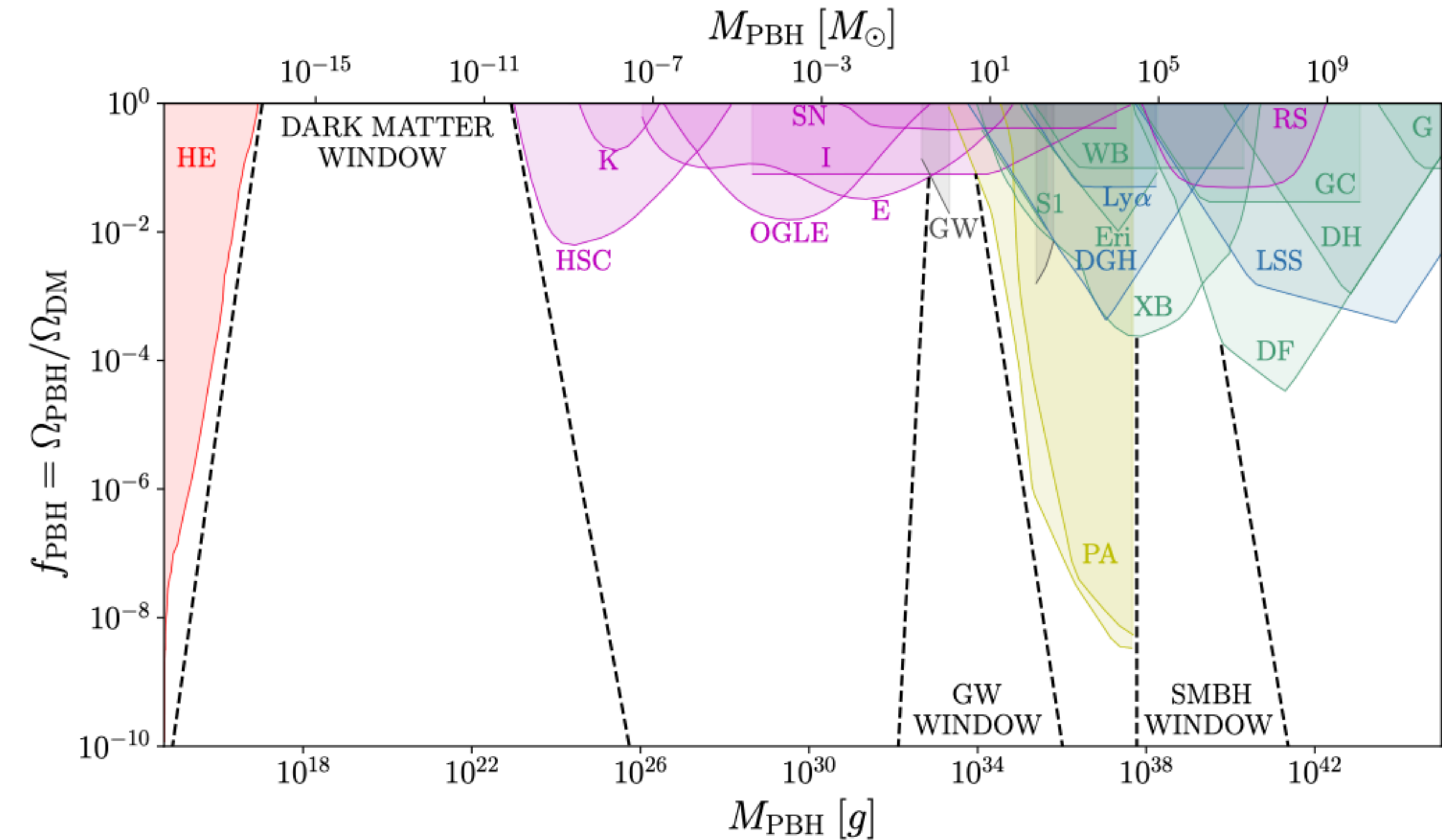


# Primordial black holes: An overview

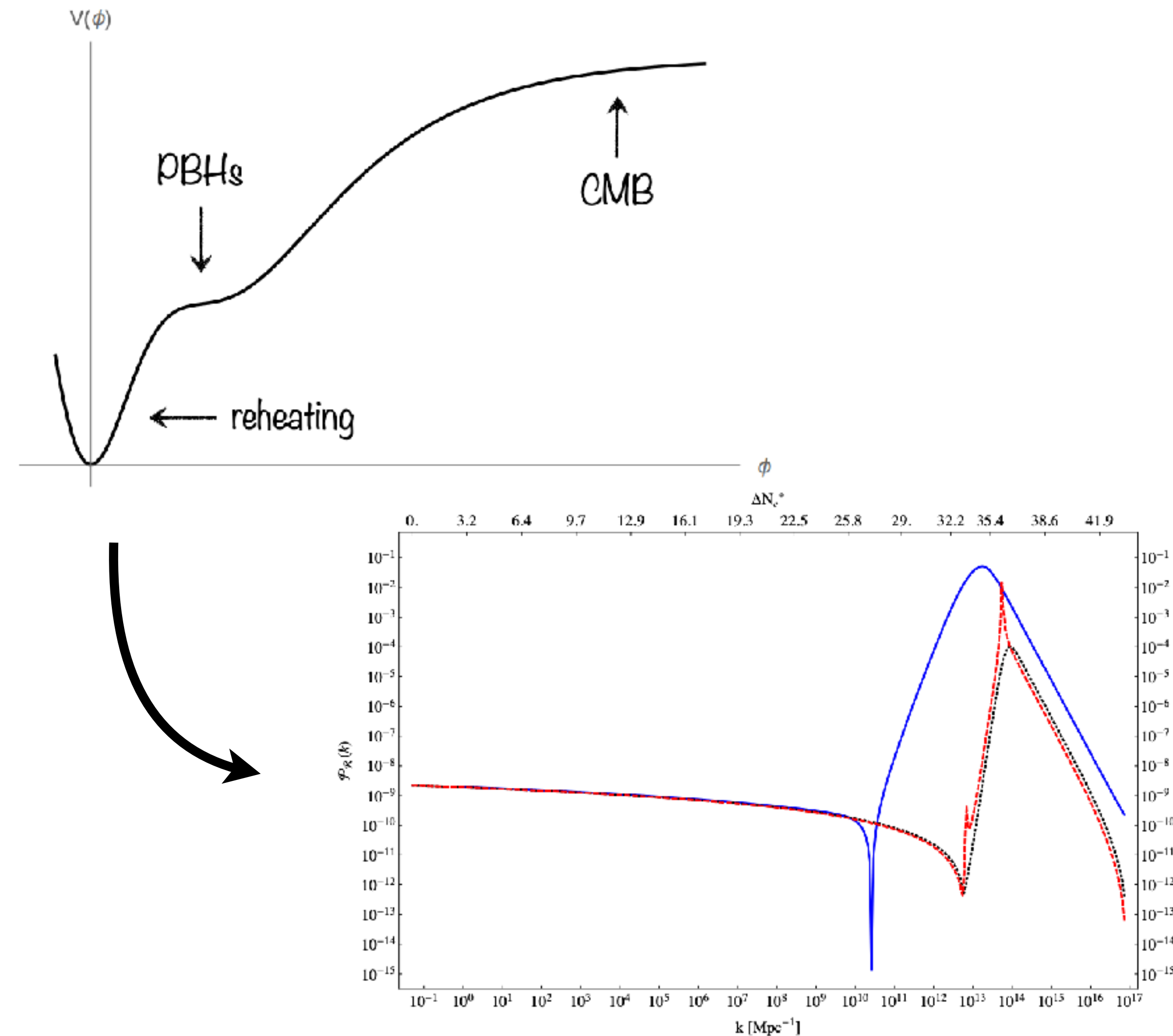
- PBHs are black holes formed in the ***early Universe*** before the formation of stars and galaxies [Zel'dovich, Novikov (1967); Hawking (1971)]
- Can account for some or all of ***dark matter***
- Astrophysical implications:
  - Can account for some LIGO events
  - Can seed supermassive black holes
  - Can account for all or part of *r*-process nucleosynthesis
    - [V. Takhistov, G. M. Fuller, A. Kusenko; *PRL*, 126 (7):071101, 2021]
  - *G* objects
    - [MMF, A. Kusenko, A.M. Ghez, S. Naomi, : *PRD* 108 (2023) 6, L061301]
  - Many more!

# Primordial black holes: An overview

[B. Carr, K. Kohri, Y. Sendouda, J. Yokoyama, *Rept. Prog. Phys.* 84 (2021) 11, 116902]

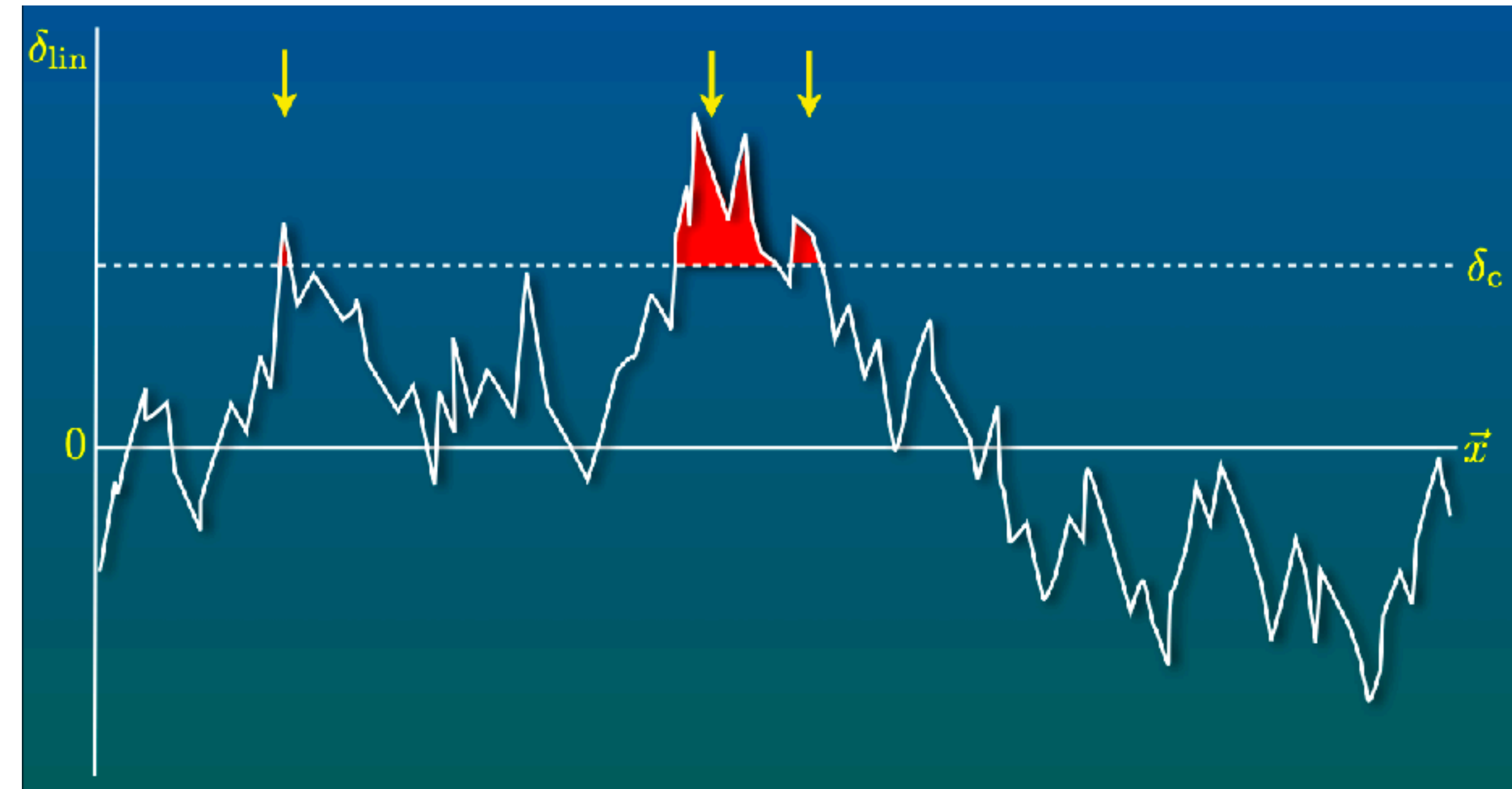


# Primordial black holes; the canonical picture



[Ballesteros, Taoso; 1702.03901]

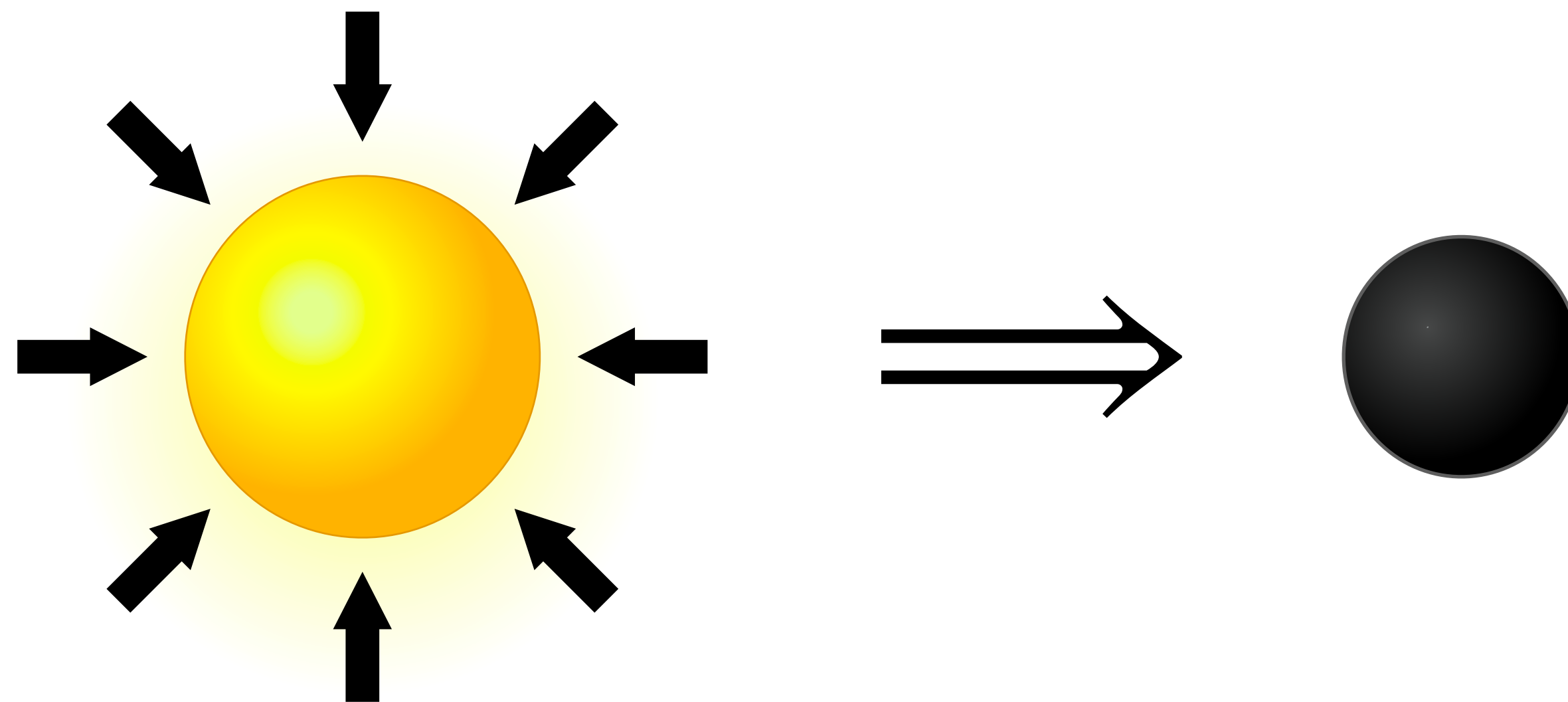
[Credit: Frank van den Bosch, Yale University]



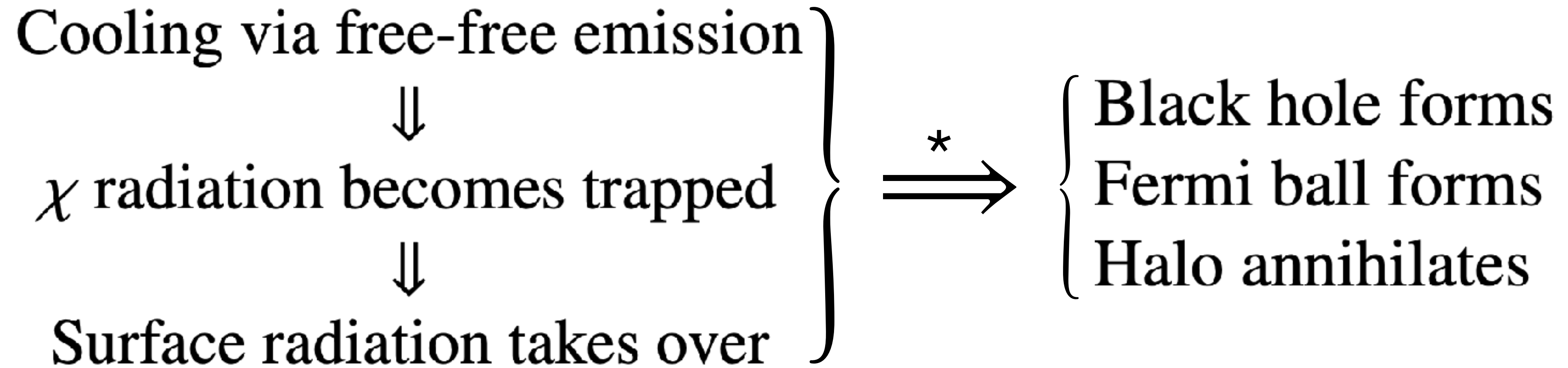
$$\beta(M_H) = \frac{\rho_{\text{PBH}}}{\rho_{\text{tot}}} = \int_{\delta_c}^{\infty} P(\delta) d\delta \sim \sigma(M_H) \exp\left(-\frac{\delta_c}{2\sigma^2(M_H)}\right)$$

$$\sigma^2(M_H) \sim \Delta_{\mathcal{R}}^2(k)$$

# Can you form PBHs in a more “conventional” way?



# PBH Formation: primordial structure formation



To *ensure* that a black hole forms, we will introduce an *asymmetric dark fermion*  $\psi$

$$\mathcal{L} \supset y\chi\bar{\psi}\psi \quad \& \quad \eta_\psi = (n_\psi - n_{\bar{\psi}})/s \neq 0$$

Yukawa Interaction & Scalar Cooling + Fermion Asymmetry  $\Rightarrow$  PBHs

# PBH abundance

- The strength of the long-range force we are considering is likely strong enough to capture all of the dark matter  $\psi$  particles in halos and therefore PBHs.
- Thus, the PBH-dark matter fraction is related to the baryon density:

$$f_{\text{PBH}} = \frac{\Omega_{\text{PBH}}}{\Omega_{\text{DM}}} = 0.2 \frac{m_{\psi}}{m_p} \frac{\eta_{\psi}}{\eta_B} = \left( \frac{m_{\psi}}{5 \text{ GeV}} \right) \left( \frac{\eta_{\psi}}{10^{-10}} \right)$$

- Our mechanism can describe the closeness of  $\Omega_{\text{DM}}$  and  $\Omega_B$ .

# PBH Mass Distribution

- Again, the strength of the scalar interaction will lead to rapid PBH formation. Thus, we expect the mass function of PBHs to represent the structure of the  $\psi$  - fluid at the time of formation.
- We need  $N$ -body simulations to accurately describe the details of  $\psi$  - structure formation.
- In the absence of this, we will approximate the mass function using the Press-Schechter function:

$$M^2 \frac{dN_h}{dM} \propto \frac{1}{\sqrt{\pi}} \left( \frac{M}{M_*} \right)^{1/2} e^{-M/M_*}$$

# Illustrative examples

- PBH dark matter:

$$\left. \begin{array}{l} \eta_{\psi} \sim \eta_B \sim 10^{-10} \\ m_{\psi} = 5 \text{ GeV} \end{array} \right\} f_{\text{PBH}} = 1$$

- Relevant to LIGO

$$\left. \begin{array}{l} \eta_{\psi} \sim 10^{-9} \\ m_{\psi} = 5 \text{ MeV} \end{array} \right\} f_{\text{PBH}} \sim 10^{-3}$$

⇒ Works within the context of SUSY

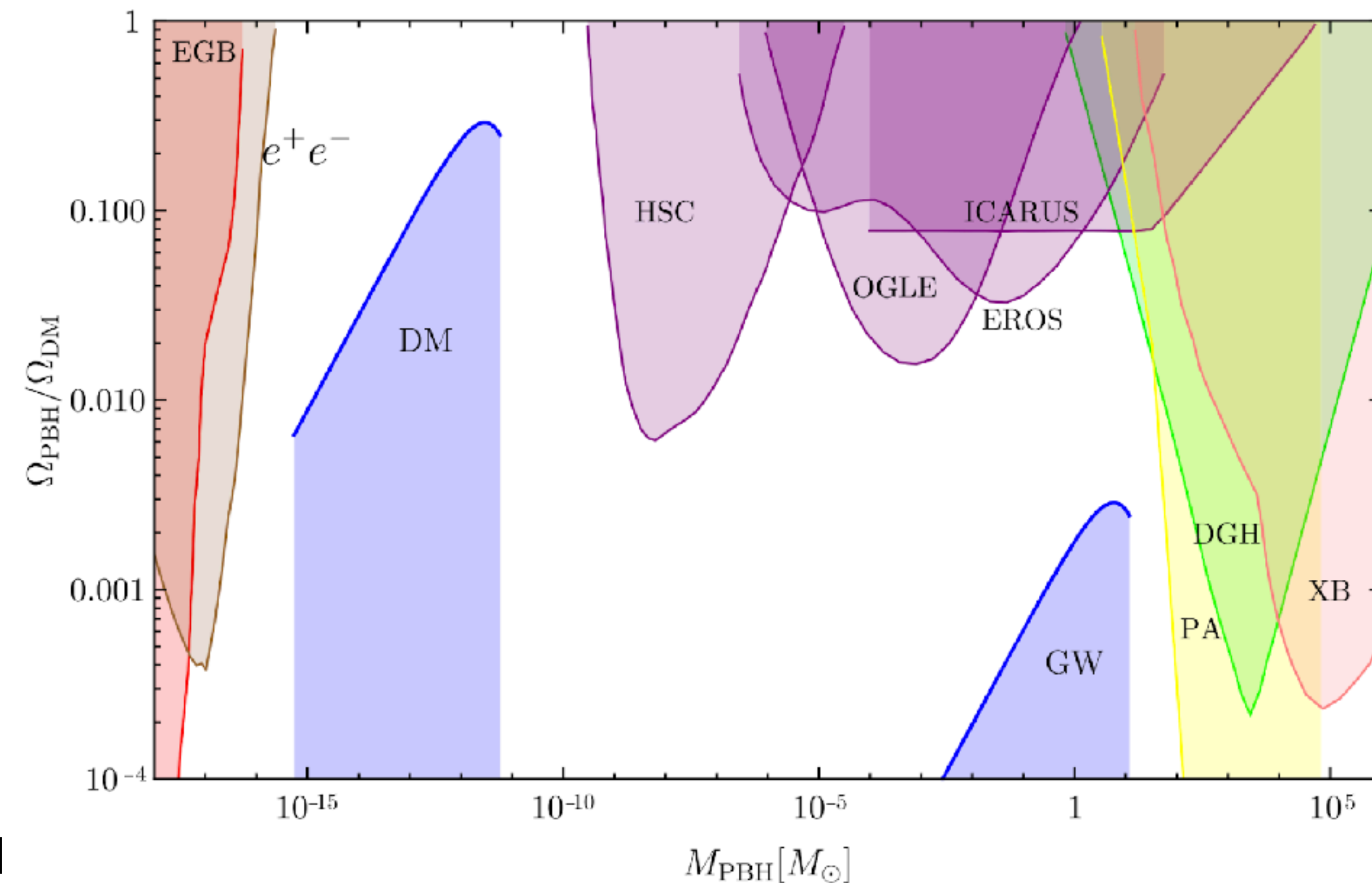
[MMF, A. Kusenko: *JCAP* 05 (2023) 013]

⇒ Can lead to SMBHs

[MMF, Y. Lu, A. Kusenko: *PRD* 108 (2023) 12, 123511]

⇒ PBHs will be spinless

[MMF, A. Kusenko *PRD* 104 (2021) 6, 063008]



[MMF, A. Kusenko: *PRL* 126 (2021) 4, 041101]

# Observational implications of primordial structure formation: gravitational waves

[MMF, A. Kusenko, M. Sasaki; PRL 131 (2023) 1, 1]

# Primordial structure formation & GWs

- Generically, the collapse of dark matter  $\psi$  halos will be *aspherical*  
 $\implies$  Time changing mass quadrupole moment
- It's still not obvious which methodology is best suited to tackling this problem
  - Standard methods, like cosmological perturbation theory do not include forces which couple to charge/number density as a means of generating perturbations
- We utilized the *Zel'dovich approximation* to directly determine the quadrupole moment, allowing for a calculation of the expected GW spectrum

See also:

T. Harada, C. Yoo, K. Kohri, et. al. [1609.01588]

I. Dalianis, C. Kouvaris [2012.09255]

I. Dalianis, C. Kouvaris [2403.15126]

# Zel'dovich Approximation

$\delta$  enters the horizon -  $t_q$



Overdensity increases to maximum size -  $t_{\max}$



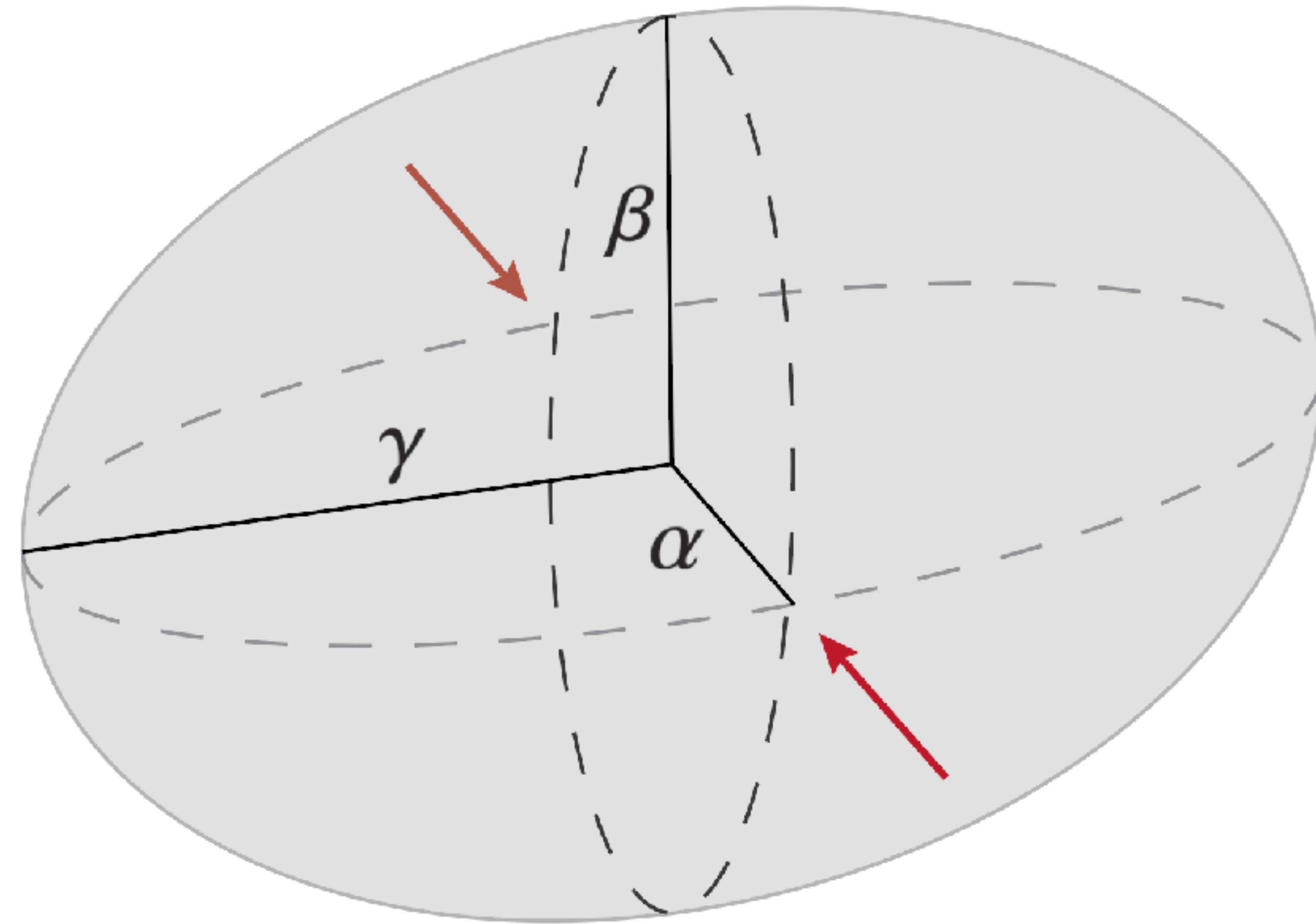
Collapse begins



“Pancake” forms and shell crossing occurs -  $t_{\text{col}}$



BH formation or halo annihilation



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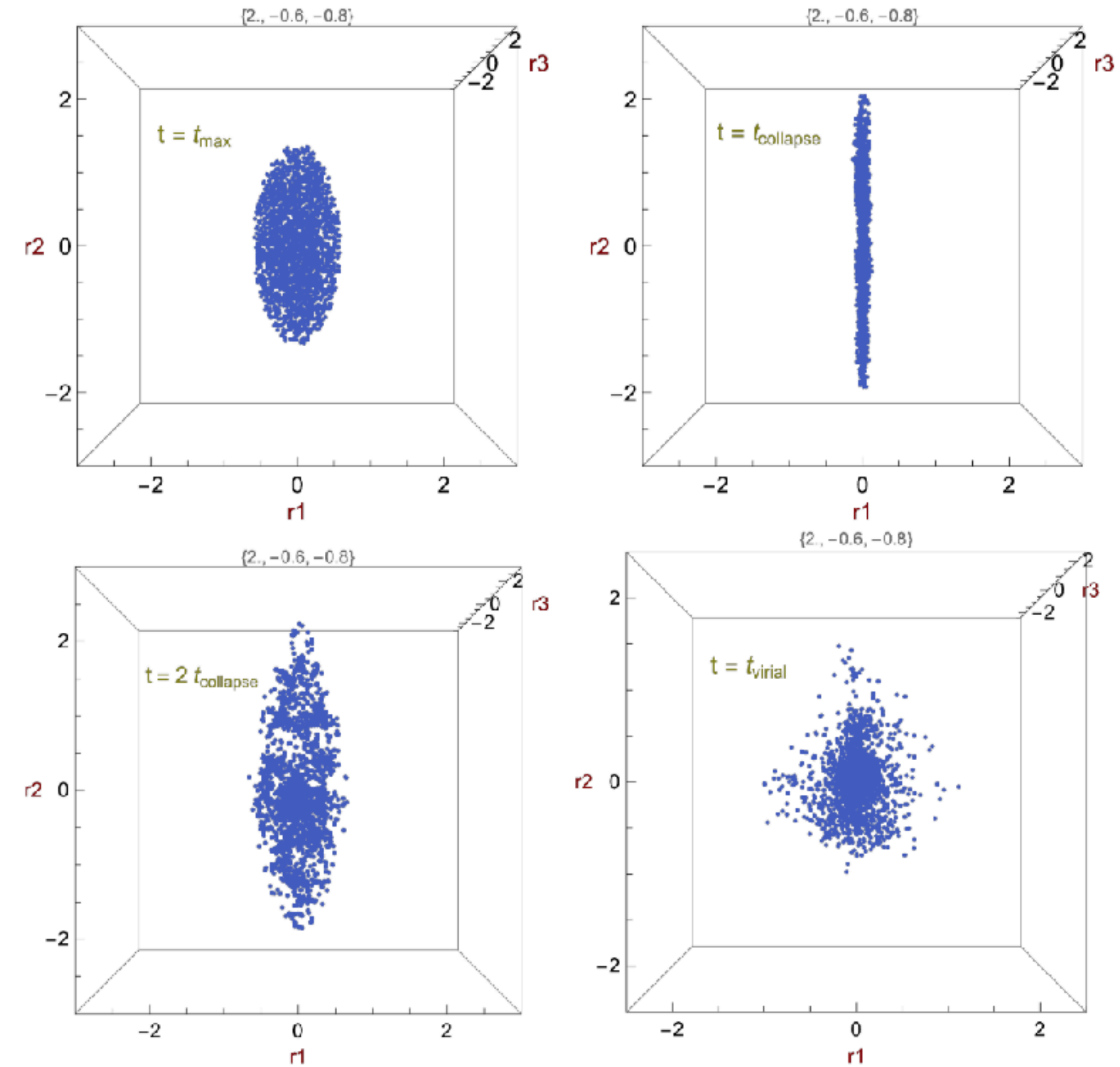
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# Growth of perturbations

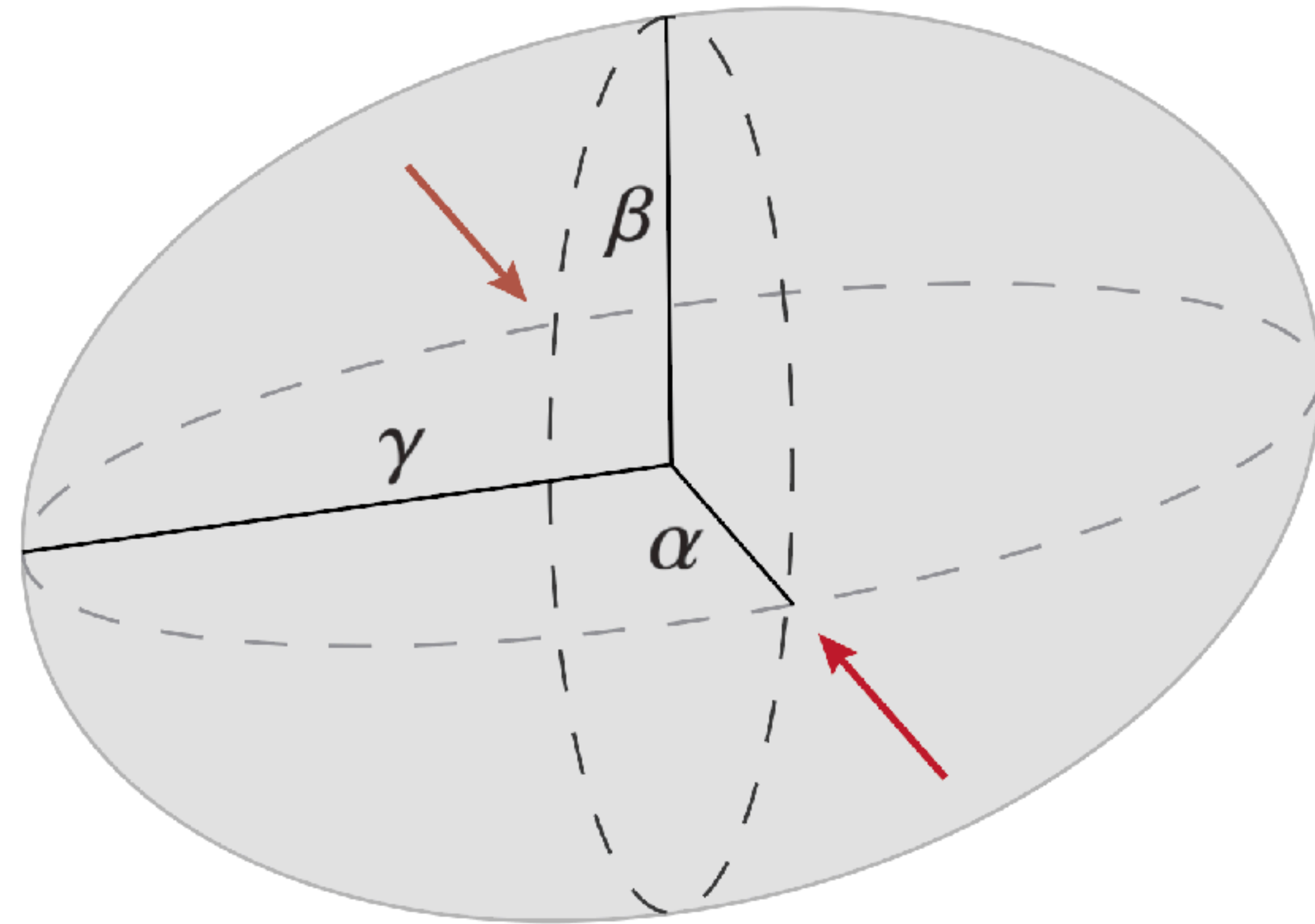
A reminder of the traditional result:

$$\delta \propto \begin{cases} \ln a & \text{for (RD)} \\ a & \text{for (MD)} \end{cases}$$

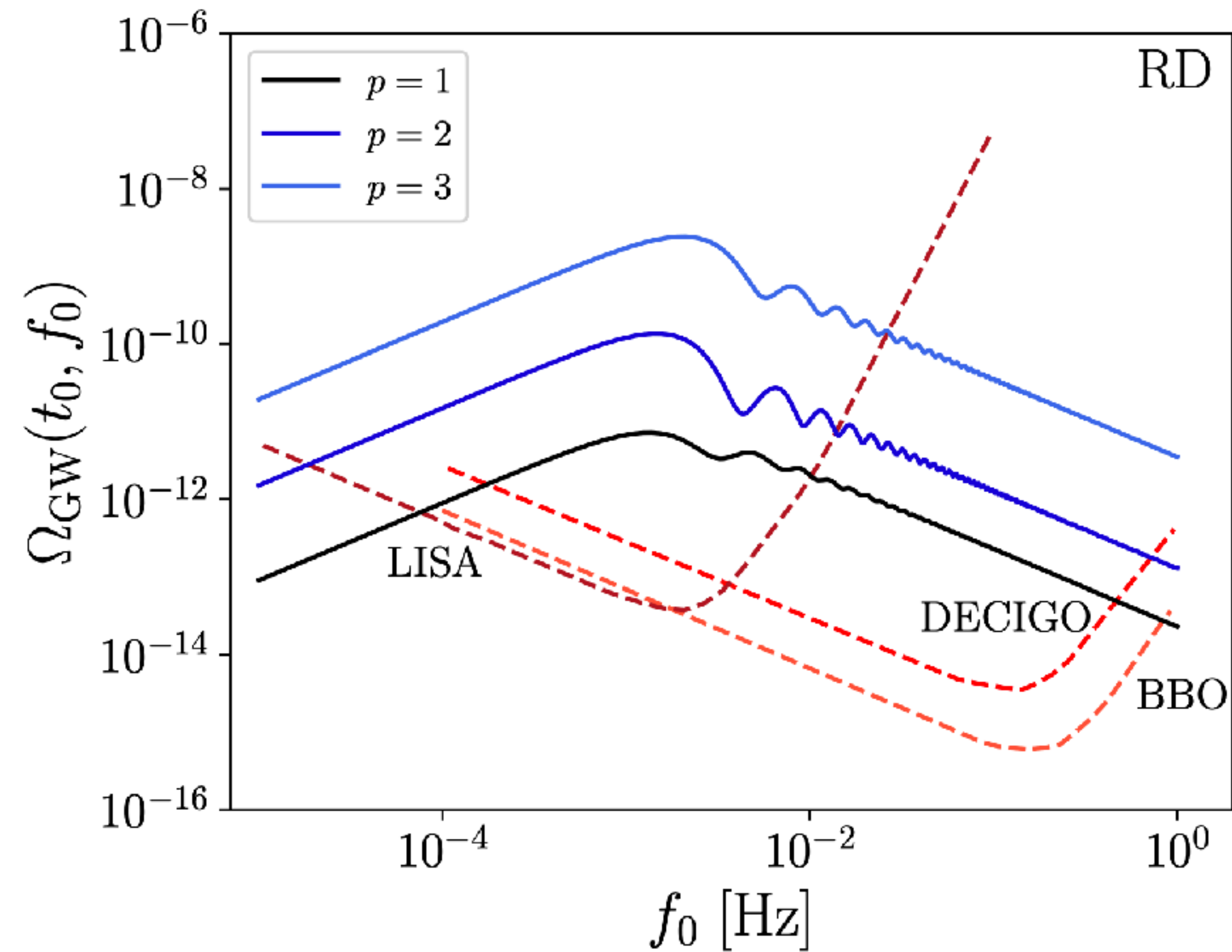
Here, **our fundamental assumption will be that**

$$\delta \propto a^p, \quad p \geq 1$$

As before  $p$  characterizes the strength of the force.



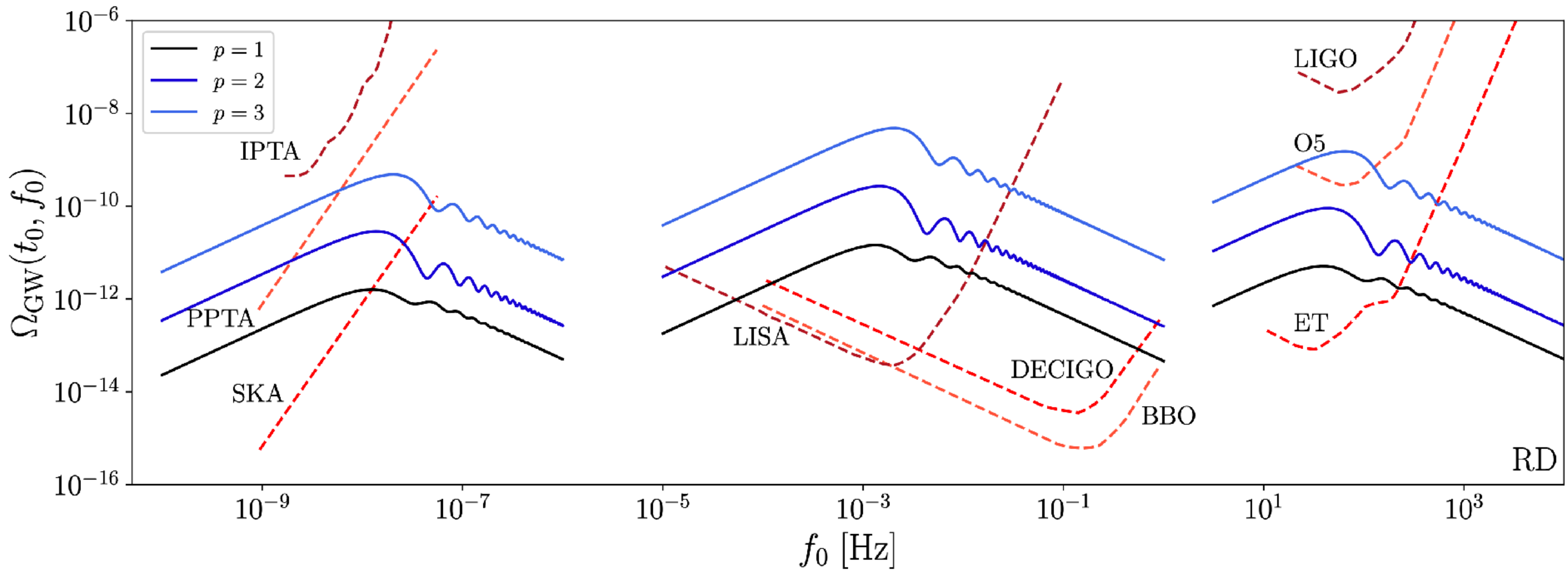
# Gravitational Wave Spectrum



[MMF, A. Kusenko, M. Sasaki; PRL 131 (2023) 1, 1]

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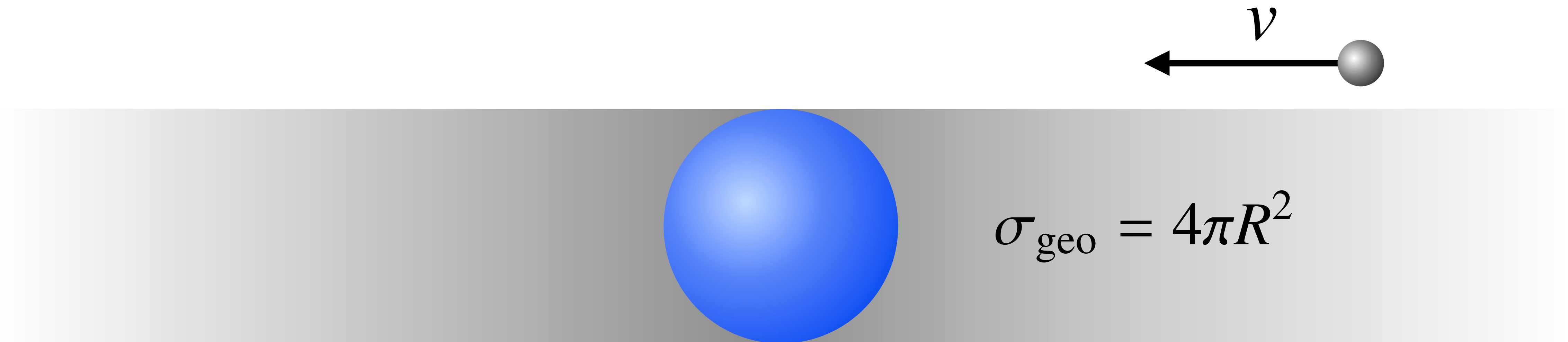
# Long-range interactions & partial wave unitarity limits

[MMF, K. Petraki; *PLB* 858 (2024) 139022]

+ ongoing work

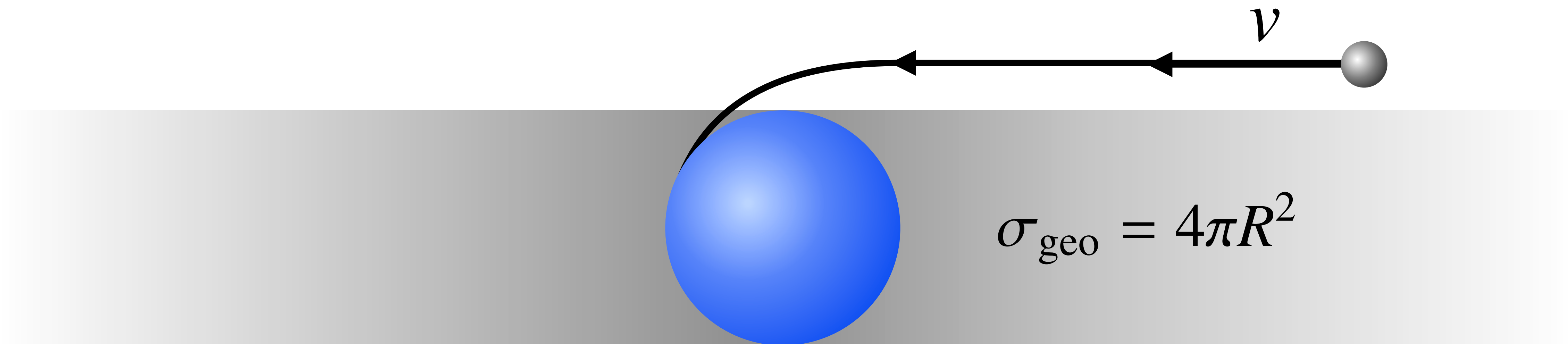


# Long-range interactions & cosmology



[Arkani-Hamed, et. al.; 0810.0713]

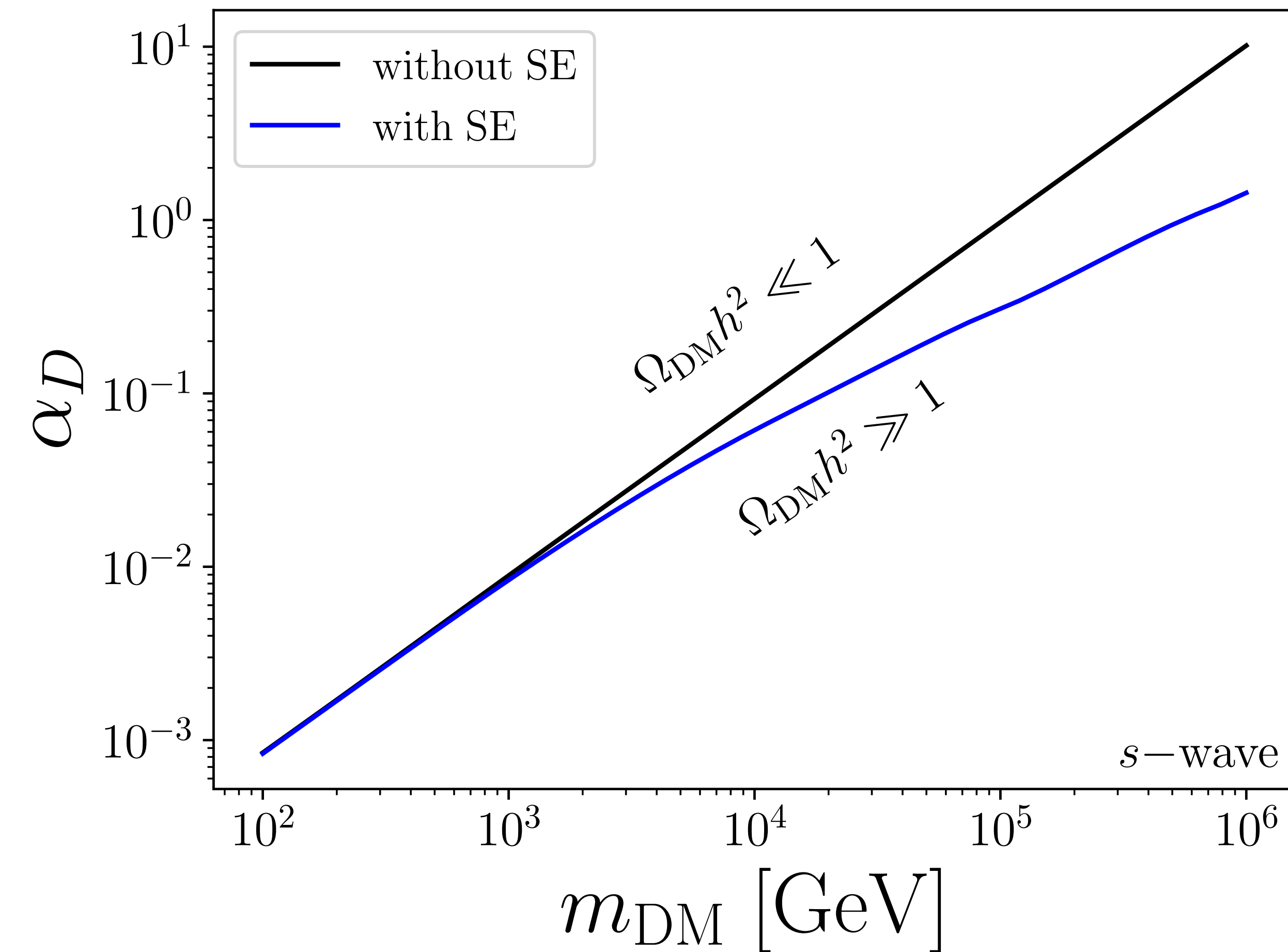
# Long-range interactions & cosmology



$$\sigma_{\text{eff}} = \sigma_{\text{geo}} \times \left[ 1 + \left( \frac{v_{\text{esc}}}{v} \right)^2 \right]$$

[Arkani-Hamed, et. al.; 0810.0713]

# Long-range interactions & dark matter



$$\sigma_{\text{ann}}^{\ell=0} v_{\text{rel}} \simeq \frac{\pi \alpha_D^2}{M_{\text{DM}}^2} \times \underbrace{\frac{2\pi \alpha_D / v_{\text{rel}}}{1 - \exp(-2\pi \alpha_D / v_{\text{rel}})}}_{\text{Sommerfeld effect}}$$

**Crucially:**

$$\Omega_{\text{DM}} h^2 \propto 1 / \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle = 0.120 \pm 0.001$$

$\Rightarrow$  Implications for unitarity bounds

[Arkani-Hamed, et. al.; 0810.0713]

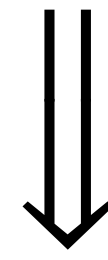
[Petraki, Postma, Wiechers; 1505.00109]

# Unitarity review

“probabilities sum to 1”

$$\underbrace{\mathbb{1} = S S^\dagger = S^\dagger S, \quad S = \mathbb{1} + \mathrm{i}T}$$

$$-\mathrm{i}(T - T^\dagger) = T T^\dagger = T^\dagger T.$$



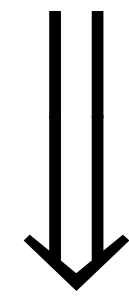
$$-\mathrm{i}(\mathcal{M}^{ab} - \mathcal{M}^{ba*}) = \sum_c \int \mathrm{d}\Pi_c \mathcal{M}^{ac} \mathcal{M}^{bc*} (2\pi)^4 \delta^4(\{k^a\} - \{k^c\})$$

$$\Downarrow \sigma \propto |\mathcal{M}|^2$$

$$\sigma_\ell^{\text{inel}} / \sigma_\ell^U \leq \sqrt{\sigma_\ell^{\text{elas}} / \sigma_\ell^U} \left( 1 - \sqrt{\sigma_\ell^{\text{elas}} / \sigma_\ell^U} \right) \quad \text{with} \quad \sigma_\ell^U = \frac{2^\delta 4\pi(2\ell + 1)}{\mathbf{k}^2}$$

# Unitarity review

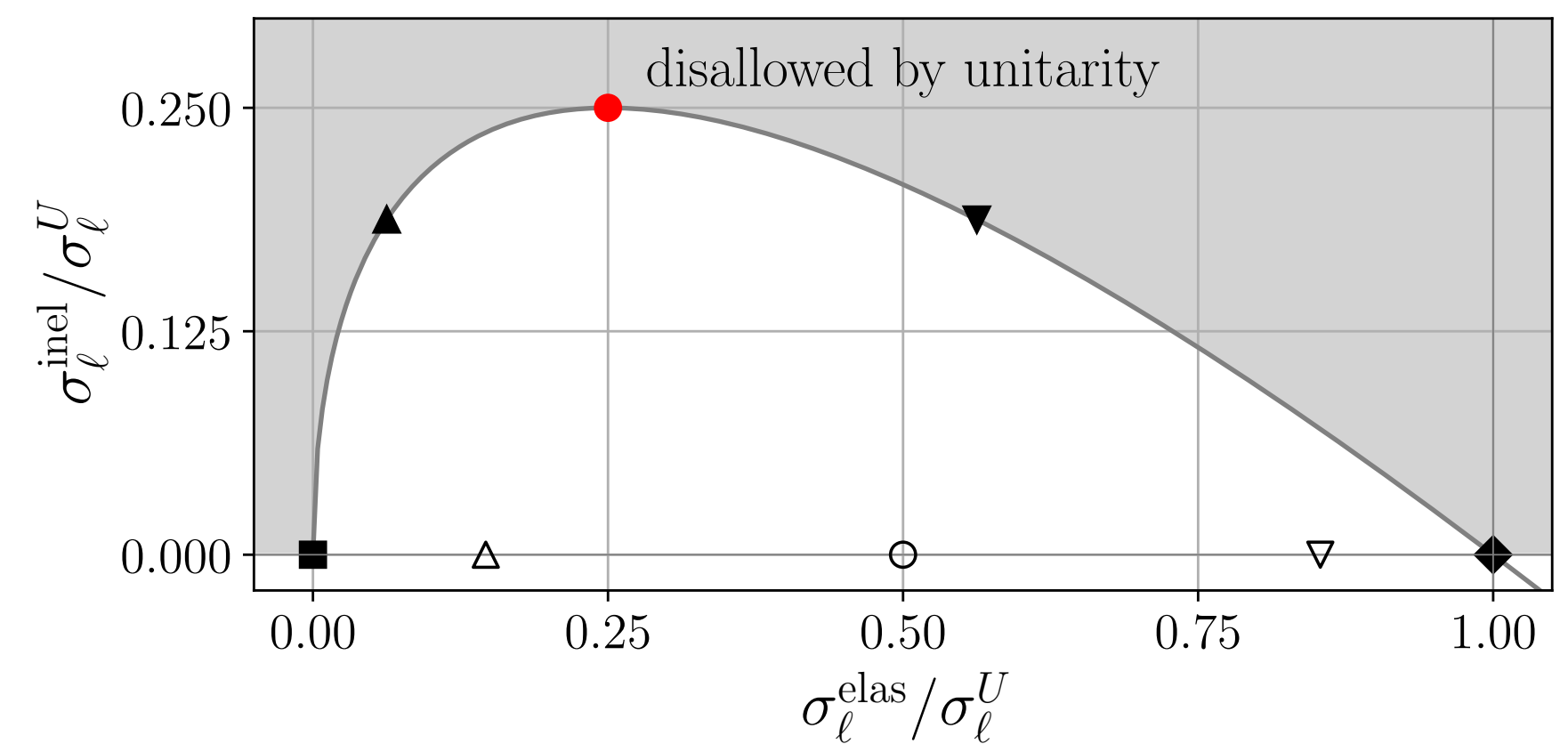
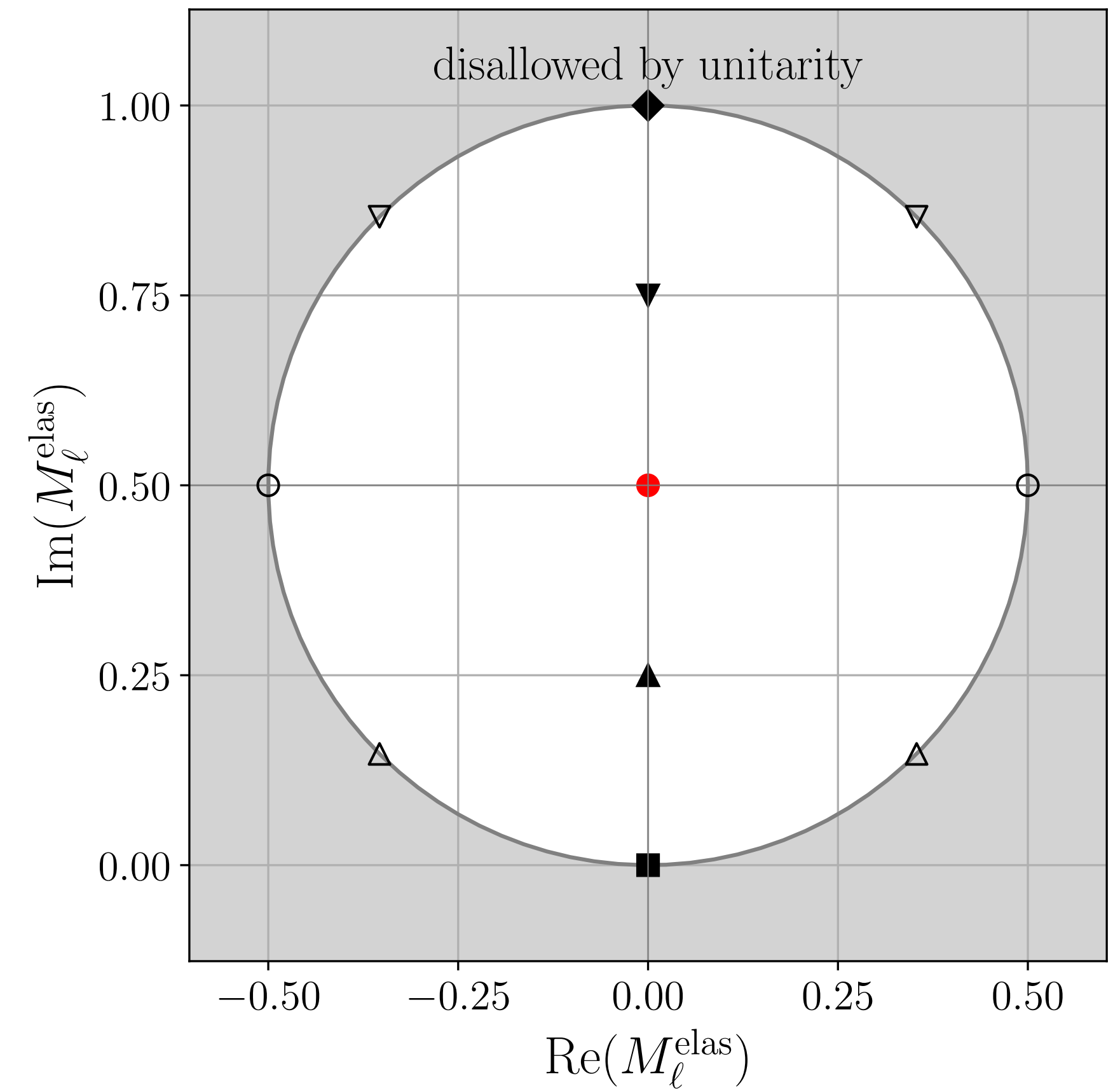
$$\sigma_{\ell}^{\text{inel}} / \sigma_{\ell}^U \leq \sqrt{\sigma_{\ell}^{\text{elas}} / \sigma_{\ell}^U} \left( 1 - \sqrt{\sigma_{\ell}^{\text{elas}} / \sigma_{\ell}^U} \right)$$



$$\sigma_{\ell}^{\text{inel}} / \sigma_{\ell}^U \leq \frac{1}{4} \quad \& \quad \sigma_{\ell}^{\text{elas}} / \sigma_{\ell}^U \leq 1$$

$$\sigma_{\ell}^U = \frac{2^{\delta} 4\pi(2\ell + 1)}{\mathbf{k}^2}$$

“unitarity limit”



# Long-range interactions & unitarity

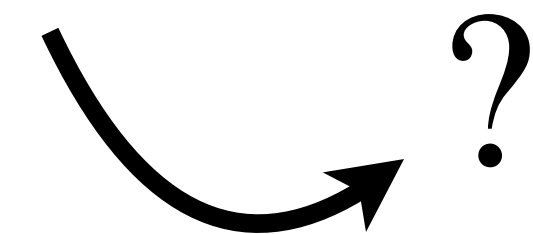
- **Violation of partial wave unitarity** is intimately related to the emergence of long-range physics.
- Unitarity puts a limit on partial-wave cross sections:

$$\sigma_{\text{inel}}^{\ell} v_{\text{rel}} \leq \sigma_{\text{uni}}^{\ell} v_{\text{rel}} = \frac{4\pi(2\ell + 1)}{M_{\text{DM}}^2 v_{\text{rel}}}$$

- Historically, unitarity has been used to place limits on the DM mass

$$\sigma_{\text{can}}^{\ell=0} v_{\text{rel}} = 2.2 \times 10^{-26} \text{ cm}^3/\text{s} \leq \frac{4\pi}{m^2 v_{\text{rel}}}$$

$$(|\mathbf{k}| = mv_{\text{rel}}/2)$$



# Long-range interactions & unitarity

- The mismatch in the hints for the need for new physics

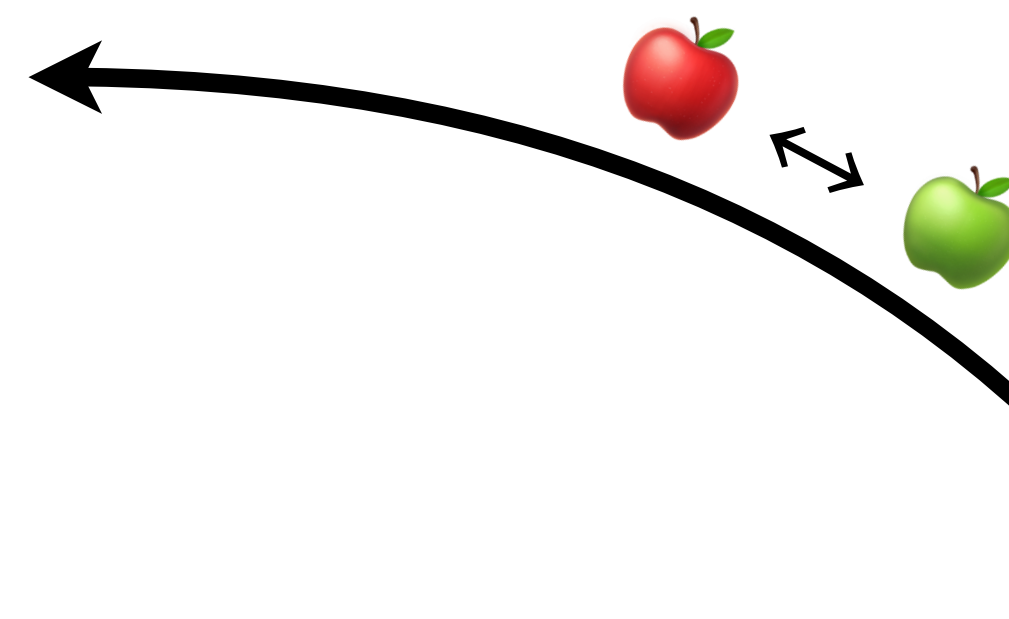
$$\sigma_{\text{inel}}^{\ell} v_{\text{rel}} \leq \sigma_{\text{uni}}^{\ell} v_{\text{rel}} = \frac{4\pi(2\ell + 1)}{M_{\text{DM}}^2 v_{\text{rel}}}$$

- Including resummation of 2PI diagrams:

$$\sigma_{\text{ann}}^{\ell=0} v_{\text{rel}} \simeq \frac{\pi\alpha_D^2}{M_{\text{DM}}^2} \times \frac{2\pi\alpha_D/v_{\text{rel}}}{1 - \exp(-2\pi\alpha_D/v_{\text{rel}})} \xrightarrow{\alpha_D \gg v_{\text{rel}}} \frac{2\pi^3\alpha_D^3}{M_{\text{DM}}^2 v_{\text{rel}}}$$

$\Rightarrow$  true for all partial waves

$\Rightarrow$  still not the whole story



# Inelastic unitary violation at low energies

- Attractive Coulomb(-like) potentials violate unitarity at **large couplings.**

[Petraki, von Harling: 1407.7874]

[Baldes, Petraki: 1703.00478]

- **Radiative bound-state formation, via emission of charged bosons**

⇒ **Unitarity violation at low couplings**

[Onkala, Petraki: 2101.08666, 2101.08667]

[Binder et al.: 2308.01336]

- Potentials featuring the exchange of light, but massive mediators leading to **parametric resonances at the thresholds for bound state formation.**

[e.g. Cassel: 0903.5307]

# Inelastic unitary violation at low energies

- Attractive Coulomb(-like) potentials violate unitarity at **large couplings**.

[Petraki, von Harling: 1407.7874]

[Baldes, Petraki: 1703.00478]

- **Radiative bound-state formation of bosons**

⇒ Unitarity violation at low energies, via emission of charged bosons

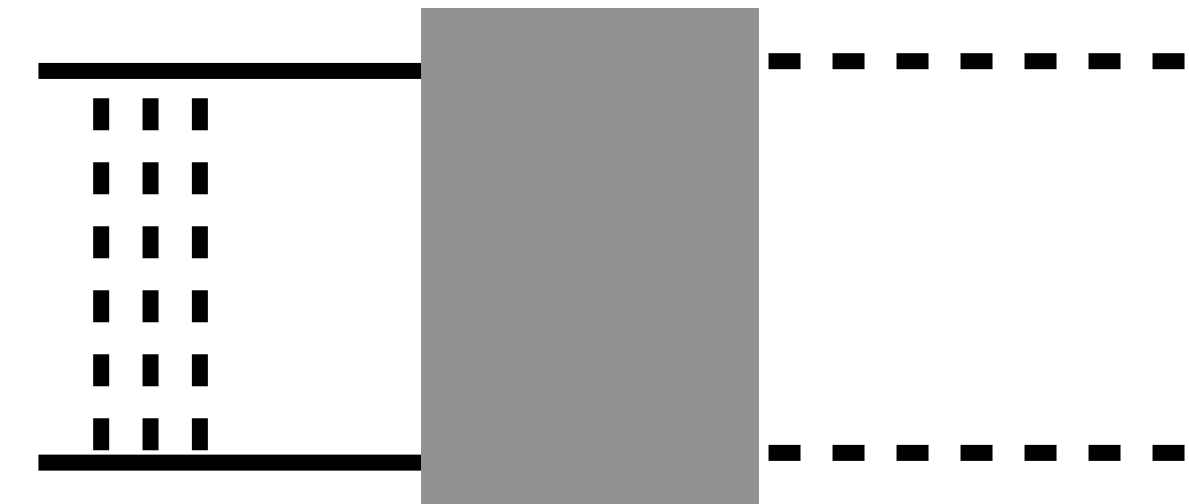
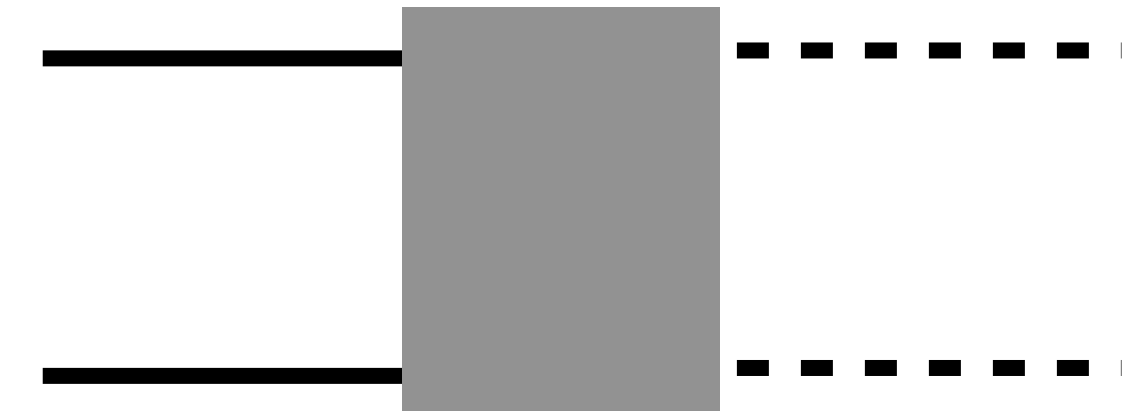
How can we improve (perturbative) calculations to ensure consistency with unitarity?

- Potentials featuring the exchange of light, but massive mediators leading to **parametric resonances at the thresholds for bound state formation**.

[e.g. Cassel: 0903.5307]

# Long-range interactions & resummation

- Traditionally, we consider incoming and outgoing states which are asymptotically free.
- However, in the presence of long-range forces, you need to accommodate the fact that particles are interacting at infinity.
- As we will see, this will require the resummation of two-particle irreducible diagrams.
- This resummation process
  - leads to Sommerfeld effect
  - accommodates formation of bound states



# Resummation review

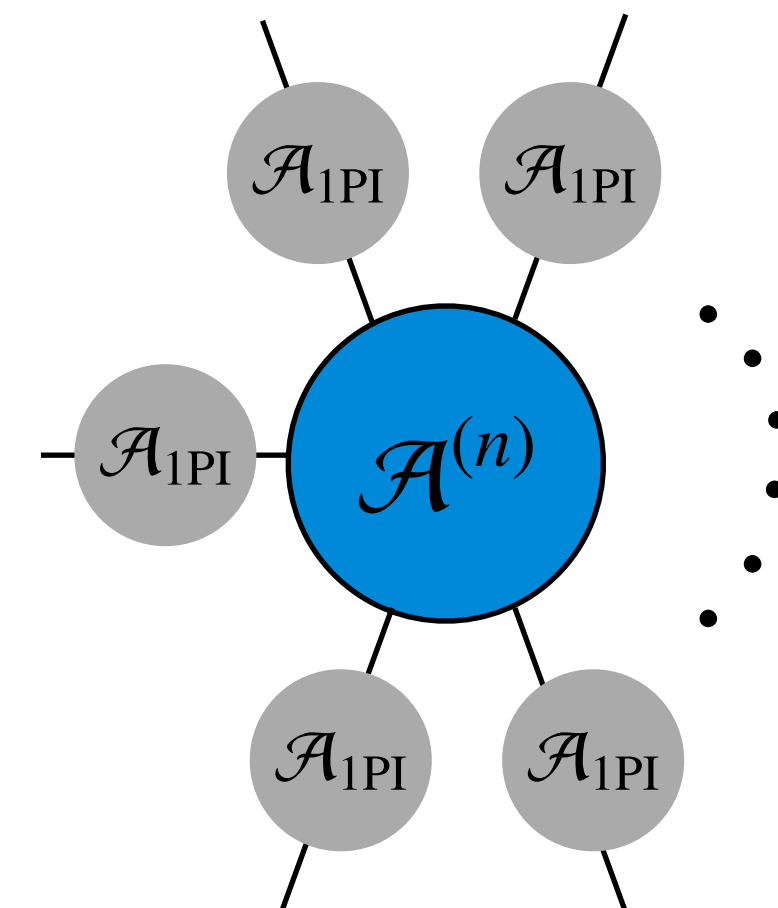
- Recall the standard procedure for determining the exact propagator:

$$\begin{aligned}
 \text{---} \mathcal{A}_{1\text{PI}} \text{---} &= \text{---} + \text{---} \text{1PI} \text{---} + \text{---} \text{1PI} \text{---} \text{1PI} \text{---} + \dots \\
 &= \text{---} + \text{---} \text{1PI} \text{---} \mathcal{A}_{1\text{PI}} \text{---} \\
 &= \frac{iZ}{p^2 - m^2}
 \end{aligned}$$

where, e.g.,

$$\text{---} \text{1PI} \text{---} = \text{---} \text{---} \text{---}$$

$$\Rightarrow \mathcal{M} = \left( \prod_{j=1}^n \sqrt{Z_j} \right) \mathcal{A}$$



# Resummation of long-range interactions

- In order to account for the fact that the incoming states are no longer asymptotically free at infinity, we now have to sum the *two-particle irreducible diagrams*

$$\begin{aligned}
 \text{Diagram with } \mathcal{A}_{2PI} &= \text{Diagram with no box} + \text{Diagram with } 2PI \text{ box} + \text{Diagram with } 2PI \text{ box and } 2PI \text{ box} + \dots \\
 &= \text{Diagram with no box} + \text{Diagram with } 2PI \text{ box and } \mathcal{A}_{2PI} \text{ box}
 \end{aligned}$$

where, e.g.,

$$\text{Diagram with } 2PI \text{ box} = \text{Diagram with a vertical dashed line}$$

$$\Rightarrow \mathcal{M}(\mathbf{k}) = \int \frac{d^3 q}{(2\pi)^3} \tilde{\psi}_{\mathbf{k}}(\mathbf{q}) \mathcal{A}(\mathbf{q}) \quad \psi_{\mathbf{k}}(\mathbf{x}): \text{ satisfies the Schrödinger equation!}$$

# Long-range interactions & resummation

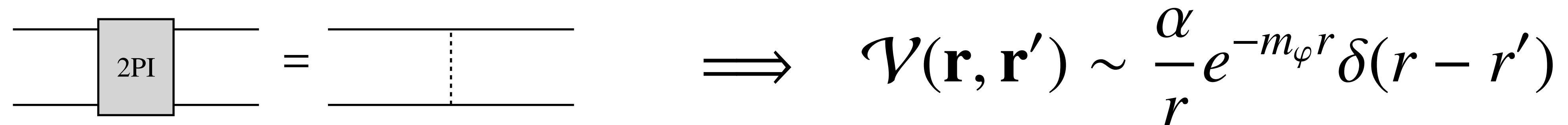
- The wave function satisfies the (momentum space) Schrödinger equation:

$$\left(\frac{\mathbf{p}^2}{2\mu} - \mathcal{E}_{\mathbf{k}}\right) \tilde{\psi}_{\mathbf{k}}(\mathbf{p}) = \frac{1}{4m_t\mu} \int \frac{d^3 p'}{(2\pi)^3} \mathcal{A}^{2\text{PI}}(s, \mathbf{p}, \mathbf{p}') \tilde{\psi}_{\mathbf{k}}(\mathbf{p}')$$

- We define the (non-local) potential as

$$\mathcal{V}(\mathbf{r}, \mathbf{r}') = \frac{-1}{4m_t\mu} \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} \mathcal{A}^{2\text{PI}}(s, \mathbf{p}, \mathbf{p}') e^{-i\mathbf{p}'\cdot\mathbf{r}'}$$

- If one only considers the one-boson-exchange (OBE):



The diagram shows a box labeled '2PI' with two horizontal lines entering from the left and two exiting to the right. This is followed by an equals sign and a dashed vertical line between two horizontal lines. To the right of this is a double-lined arrow pointing to the right, followed by the equation:  $\mathcal{V}(\mathbf{r}, \mathbf{r}') \sim \frac{\alpha}{r} e^{-m_\phi r} \delta(r - r')$

# Inelastic unitary violation at low energies

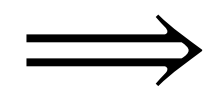
Optical theorem:

$$\text{Im} \left( \text{Diagram: circle with } M_\ell^{aa} \text{ and four solid lines} \right) = \sum_b \left( \text{Diagram: circle with } M_\ell^{ab*} \text{ and two solid, two dashed lines} \right) \left( \text{Diagram: circle with } M_\ell^{ab} \text{ and two dashed, two solid lines} \right)$$

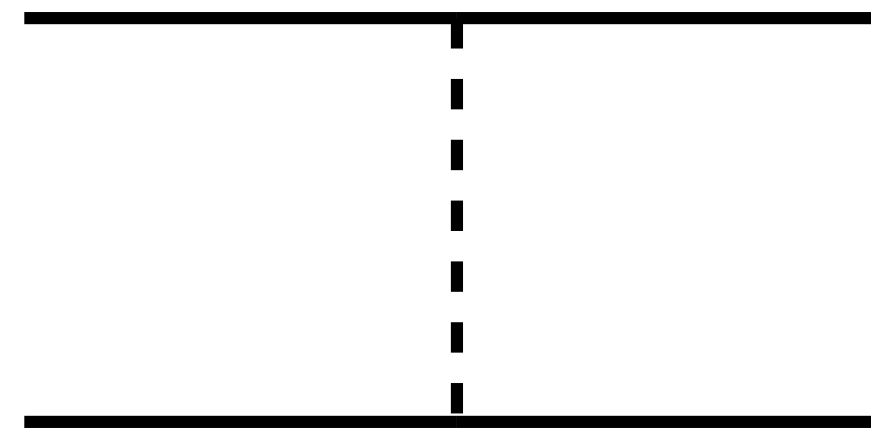
$O(\alpha)$  vs.  $O(\alpha^2)$

**Key:** Resummation is required

Purely elastic  
kernel

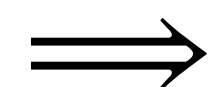


Real potential

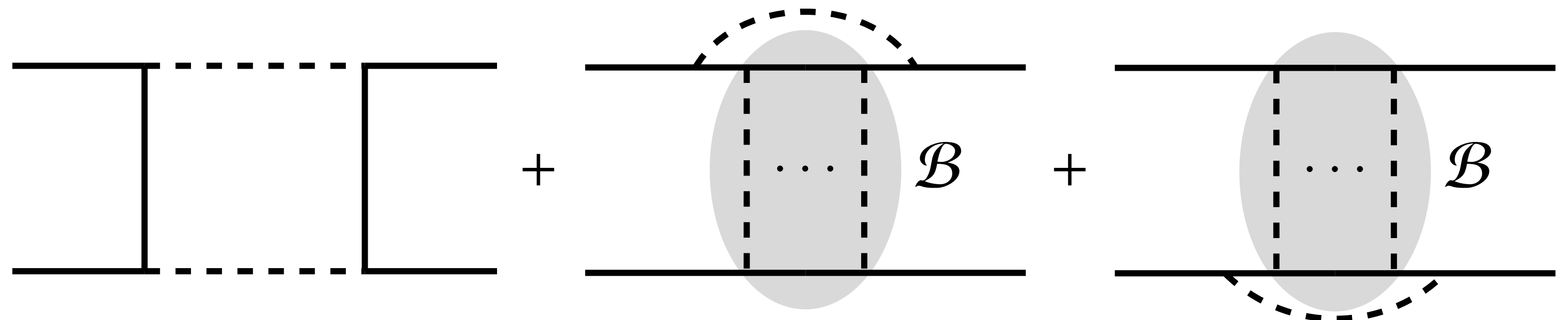


$\Rightarrow$  Leads to Sommerfeld effect  
 $\Rightarrow$  Unitarizes elastic cross sections

Squared inelastic  
kernel



Imaginary  
potential



# Resummation and inelastic processes

- Resummation of inelastic processes is equivalent to solving the following Schrödinger equation:

$$[\mathcal{S}_\ell(r) - \mathcal{E}_\mathbf{k}]u_{k,\ell}(r) = i \sum_j r v_\ell^{j*}(r) \int_0^\infty dr' r' v_\ell^j(r') u_{k,\ell}(r')$$

where

---

new ingredient  $\text{Im} \text{---} \boxed{\mathcal{A}_{2PI}} \text{---} \sim \sum_c \left| \text{---} \boxed{\text{---}} \text{---} \right|^2$

$$\mathcal{S}_\ell(r) \equiv -\frac{1}{2\mu} \frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{2\mu r^2} + V(r), \quad u_{k,\ell}(r) \equiv kr\psi_{k,\ell}(r)$$

- We solve the above equations subject to the boundary conditions:

$$\psi_\mathbf{k}(\mathbf{r}) \stackrel{r \rightarrow \infty}{\simeq} e^{i\mathbf{k} \cdot \mathbf{r}} + f_\mathbf{k}(\Omega_\mathbf{r}) \frac{e^{ikr}}{r}$$

- The equation above can actually be **solved analytically**.

# Regulated & Unregulated Amplitude Relation

- Using our solution to Schrödinger's Equation with imaginary potential:

$$M_{\ell,\text{reg}}^{\text{inel},i}(k) = \sum_j [\mathbb{N}_{\ell}^{-1}(k)]^{ij} M_{\ell,\text{unreg}}^{\text{inel},j}(k).$$

with

$$[\mathbb{N}_{\ell}(k)]^{ij} = \delta^{ij} - \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{M_{\ell,\text{unreg}}^{\text{inel},i}(q) M_{\ell,\text{unreg}}^{\text{inel},j*}(q)}{q^2 - k^2 - i\epsilon} q dq$$

- If the amplitudes are well-behaved at complex infinity, the contour integration can be completed and we obtain a simple relationship:

$$M_{\ell,\text{reg}}^{\text{inel},i} = \frac{M_{\ell,\text{unreg}}^{\text{inel},i}}{1 + \sigma_{\ell,\text{unreg}}^{\text{inel}} / (4\sigma_{\ell}^U)}$$

# Unitarity restoration: resummation of inelastic interactions

[MMF, K. Petraki; *PLB* 858 (2024) 139022]

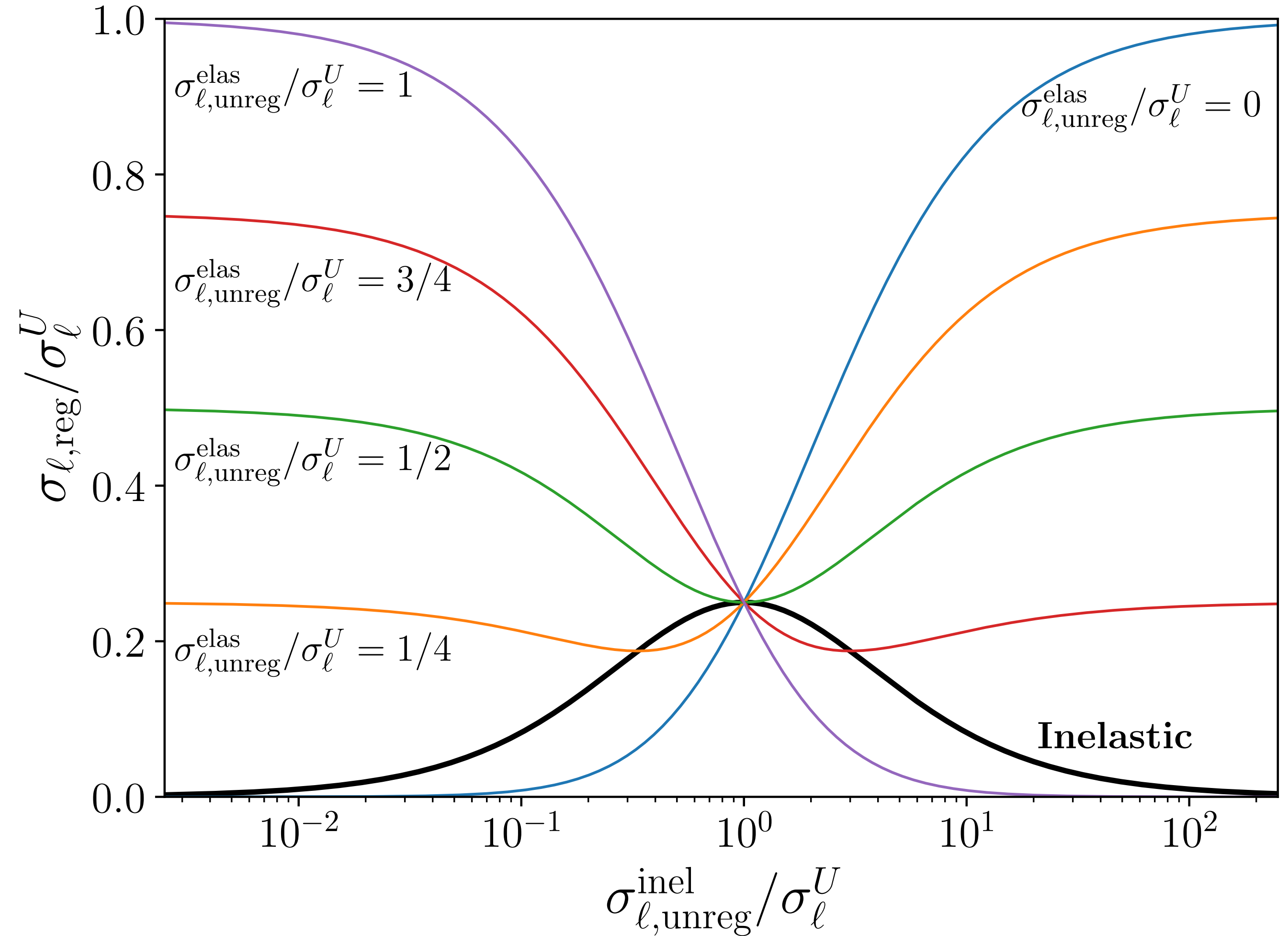
$$x_{\ell,(\text{un})\text{reg}} \equiv \sigma_{\ell,(\text{un})\text{reg}}^{\text{elas}} / \sigma_{\ell}^U$$

$$y_{\ell,(\text{un})\text{reg}}^j \equiv \sigma_{\ell,(\text{un})\text{reg}}^{\text{inel},j} / \sigma_{\ell}^U$$

$$y_{\ell,(\text{un})\text{reg}} \equiv \sum_j y_{\ell,(\text{un})\text{reg}}^j$$

$$x_{\ell,\text{reg}} = \frac{x_{\ell,\text{unreg}} + (1 - x_{\ell,\text{unreg}})y_{\ell,\text{unreg}}^2}{(1 + y_{\ell,\text{unreg}})^2}$$

$$y_{\ell,\text{reg}}^j = \frac{y_{\ell,\text{unreg}}^j}{(1 + y_{\ell,\text{unreg}})^2}$$



$\Rightarrow$  Like the inclusion of the Sommerfeld effect, this procedure changes the sets of parameters consistent with observations

# Unitarity restoration: resummation of inelastic interactions

[MMF, K. Petraki; *PLB* 858 (2024) 139022]

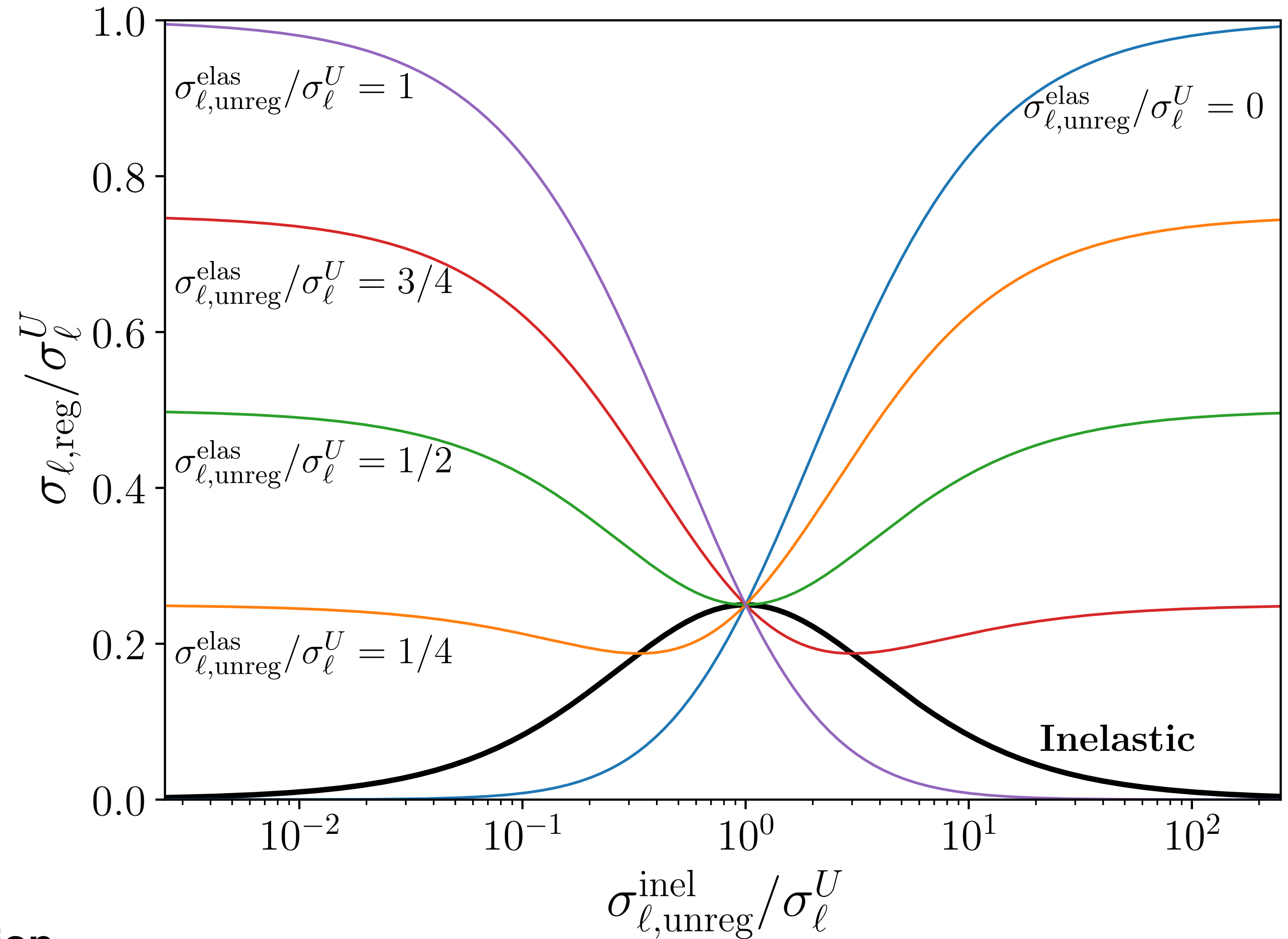
$$x_{\ell,(\text{un})\text{reg}} \equiv \sigma_{\ell,(\text{un})\text{reg}}^{\text{elas}} / \sigma_{\ell}^U$$

$$y_{\ell,(\text{un})\text{reg}}^j \equiv \sigma_{\ell,(\text{un})\text{reg}}^{\text{inel},j} / \sigma_{\ell}^U$$

$$y_{\ell,(\text{un})\text{reg}} \equiv \sum_j y_{\ell,(\text{un})\text{reg}}^j$$

$$x_{\ell,\text{reg}} = \frac{x_{\ell,\text{unreg}} + (1 - x_{\ell,\text{unreg}})y_{\ell,\text{unreg}}^2}{(1 + \underline{y_{\ell,\text{unreg}}})^2}$$

$$y_{\ell,\text{reg}}^j = \frac{y_{\ell,\text{unreg}}^j}{(1 + \underline{y_{\ell,\text{unreg}}})^2}$$



## Inelastic scattering: DM production & indirect detection

- Processes that do not directly impact production or indirect signals (e.g., BSF at high temperatures)  
renormalize all inelastic channels  $\implies$  significant effect

# Unitarity restoration: resummation of inelastic interactions

[MMF, K. Petraki; *PLB* 858 (2024) 139022]

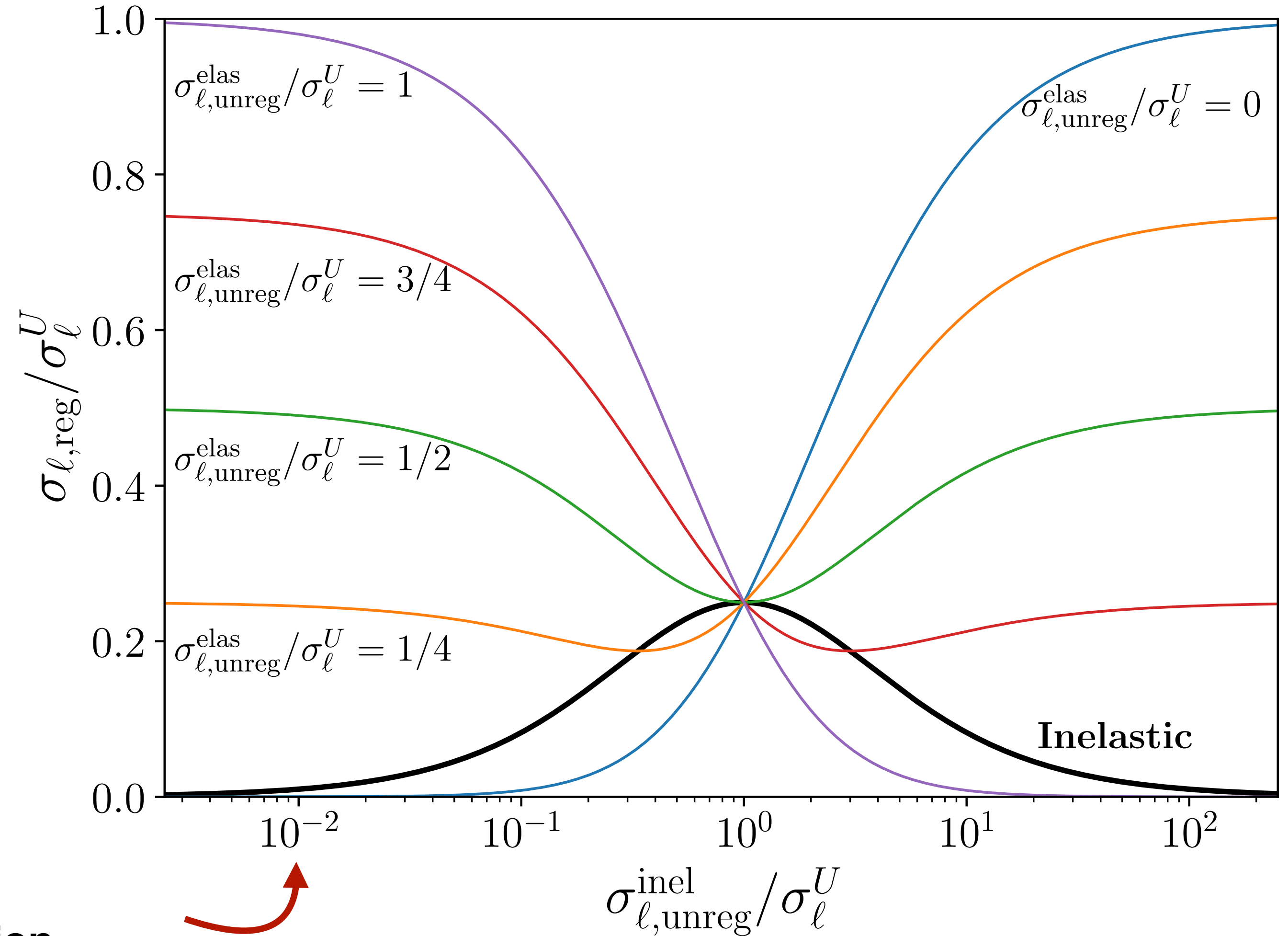
$$x_{\ell,(\text{un})\text{reg}} \equiv \sigma_{\ell,(\text{un})\text{reg}}^{\text{elas}} / \sigma_{\ell}^U$$

$$y_{\ell,(\text{un})\text{reg}}^j \equiv \sigma_{\ell,(\text{un})\text{reg}}^{\text{inel},j} / \sigma_{\ell}^U$$

$$y_{\ell,(\text{un})\text{reg}} \equiv \sum_j y_{\ell,(\text{un})\text{reg}}^j$$

$$x_{\ell,\text{reg}} = \frac{x_{\ell,\text{unreg}} + (1 - x_{\ell,\text{unreg}})y_{\ell,\text{unreg}}^2}{(1 + y_{\ell,\text{unreg}})^2}$$

$$y_{\ell,\text{reg}}^j = \frac{y_{\ell,\text{unreg}}^j}{(1 + y_{\ell,\text{unreg}})^2}$$



## Inelastic scattering: DM production & indirect detection

- When  $\sigma_{\ell,\text{unreg}}^{\text{inel}}/\sigma_{\ell}^U \sim 10^{-2}$  then the impact of the unitarization exceeds the observational precision of the dark matter abundance.

# Unitarity restoration: resummation of inelastic interactions

[MMF, K. Petraki; *PLB* 858 (2024) 139022]

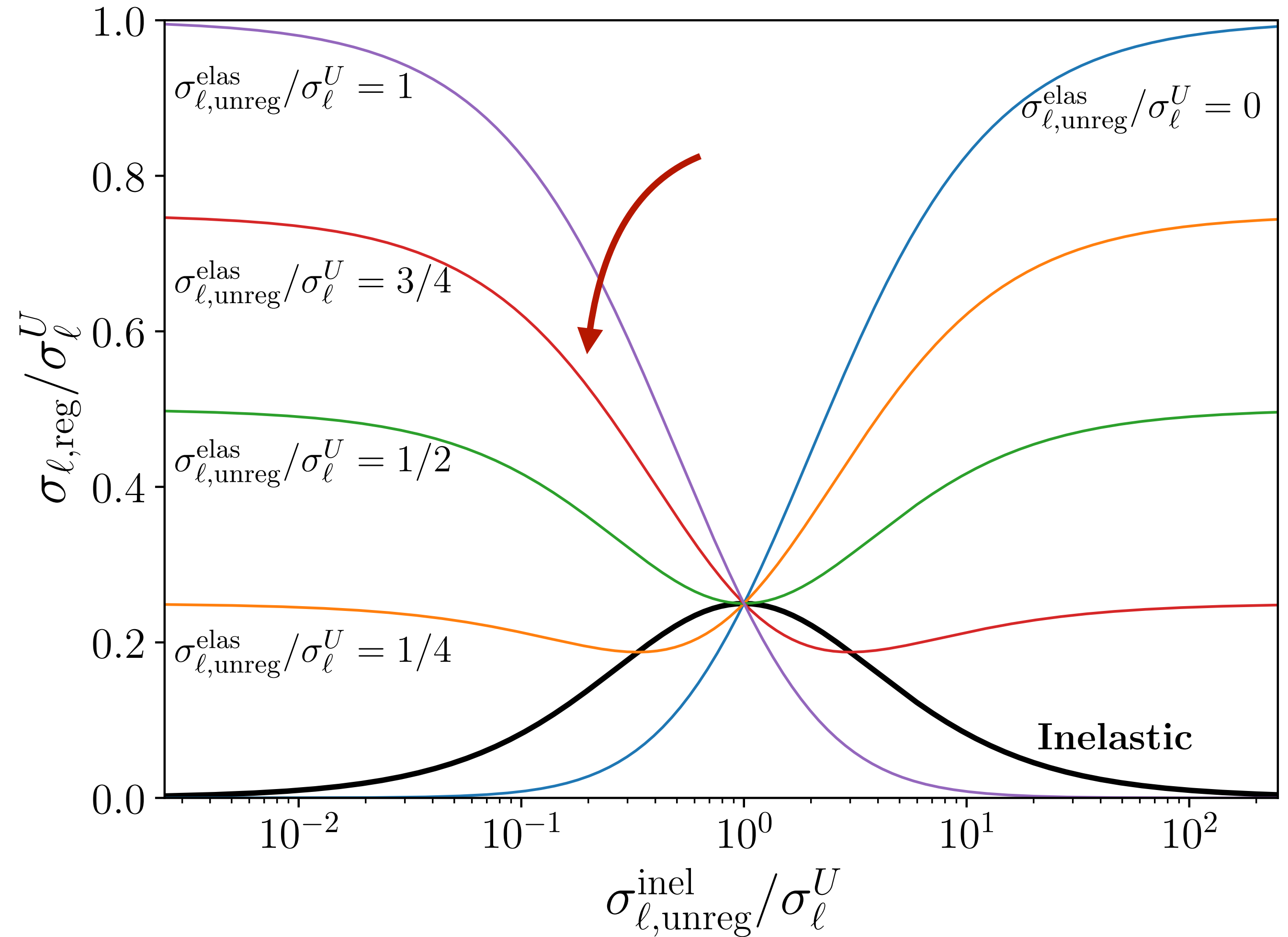
$$x_{\ell,(\text{un})\text{reg}} \equiv \sigma_{\ell,(\text{un})\text{reg}}^{\text{elas}} / \sigma_{\ell}^U$$

$$y_{\ell,(\text{un})\text{reg}}^j \equiv \sigma_{\ell,(\text{un})\text{reg}}^{\text{inel},j} / \sigma_{\ell}^U$$

$$y_{\ell,(\text{un})\text{reg}} \equiv \sum_j y_{\ell,(\text{un})\text{reg}}^j$$

$$x_{\ell,\text{reg}} = \frac{x_{\ell,\text{unreg}} + (1 - x_{\ell,\text{unreg}})y_{\ell,\text{unreg}}^2}{(1 + y_{\ell,\text{unreg}})^2}$$

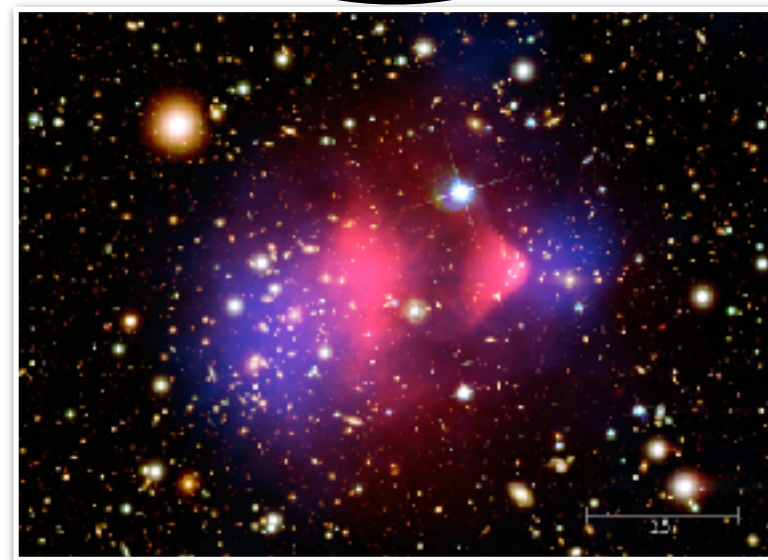
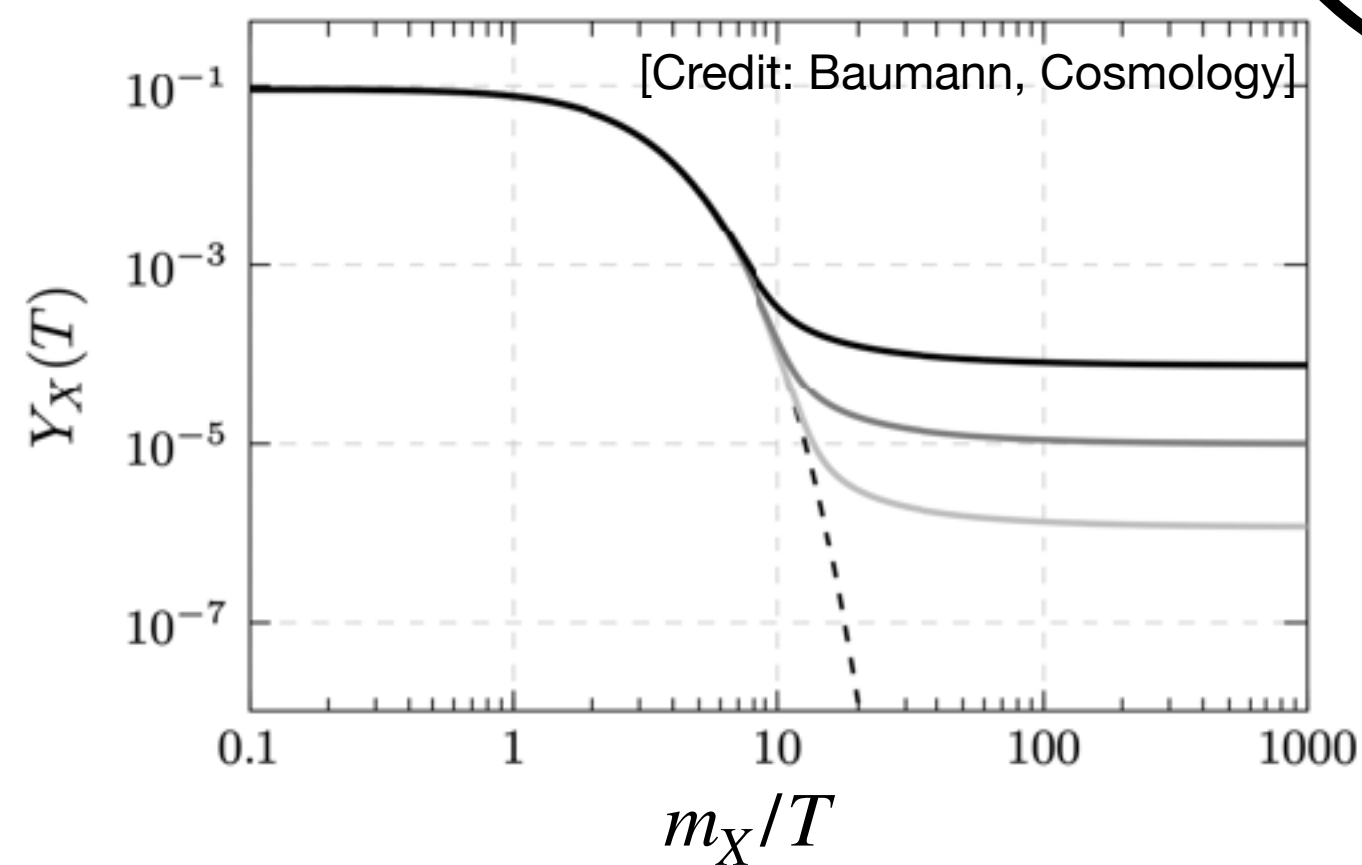
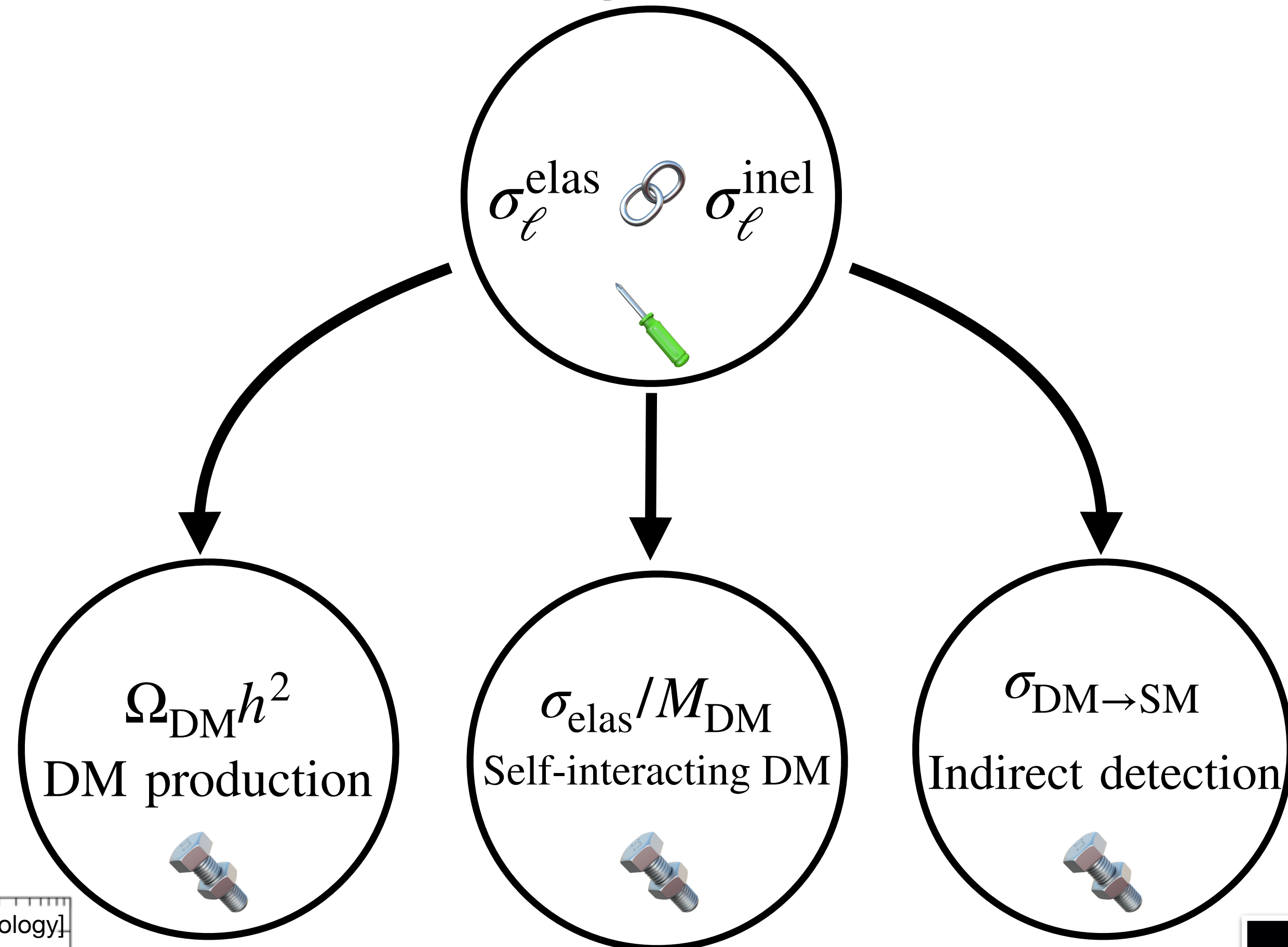
$$y_{\ell,\text{reg}}^j = \frac{y_{\ell,\text{unreg}}^j}{(1 + y_{\ell,\text{unreg}})^2}$$



## Elastic scattering: DM self-interactions

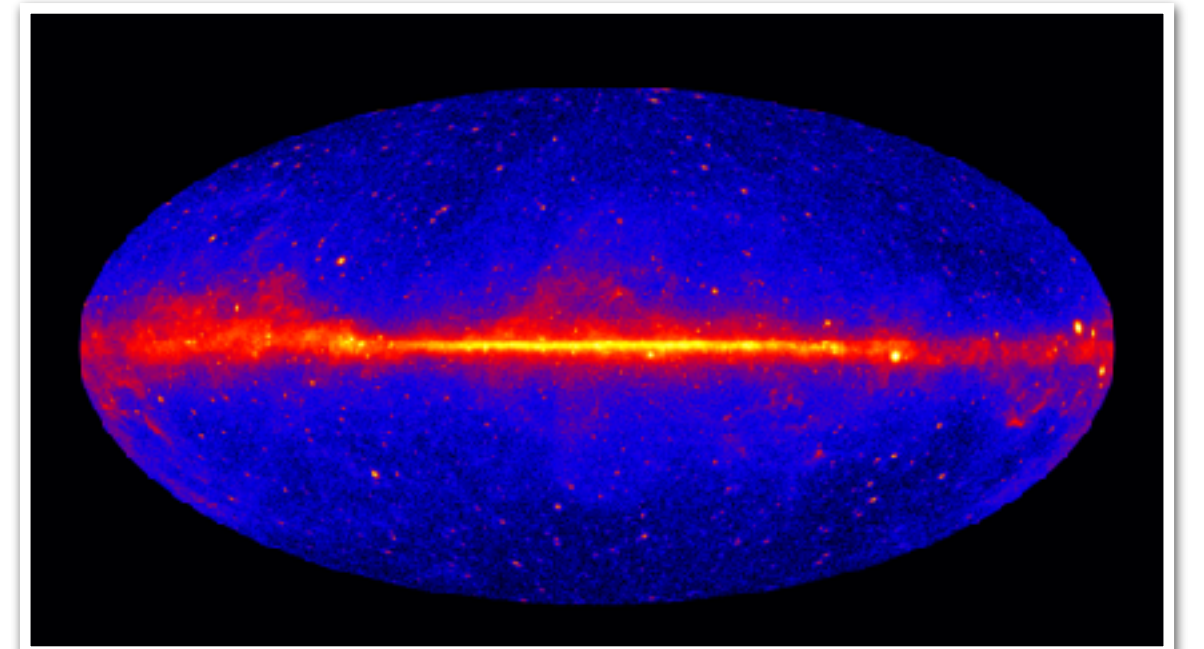
- Inelastic scattering changes elastic scattering, significantly and non-monotonically.
- Changes calculations of DM self-interactions and their effect on galactic structure.

# Ongoing: Phenomenological implications



[Credit: NASA]

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[Credit: NASA, FermiLAT]

# Final thoughts and future directions

- **Primordial structure formation** *can occur* and has many interesting phenomenological implications:
  - Primordial black holes & gravitational waves
- Recent work has focused on **improving non-relativistic perturbative calculations** so that they are consistent with unitarity.
- **Formal aspects:**
  - Generalizations to include processes with **arbitrary number of particles and spin**.
  - Demonstrate result is a **natural consequence** of non-relativistic QFT
- **Phenomenological implications of unitarization:**
  - Regularization of inelastic cross sections is important for **dark matter freeze-out** and **indirect detection** experiments.
  - Regularization of *elastic* cross sections has implications for **self-interacting dark matter**.
- **Other interests**
  - Exploring the implications of long-range forces in **baryogenesis scenarios**.
  - Other topics like **gravitational particle production**, **cosmological phase transitions** and more!

