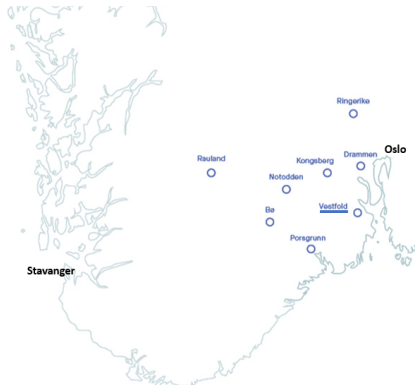


Noncommutative geometry and quantum gravity

Anna Pachol

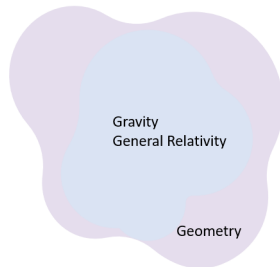
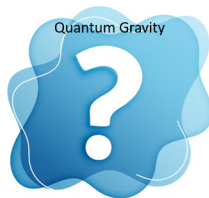
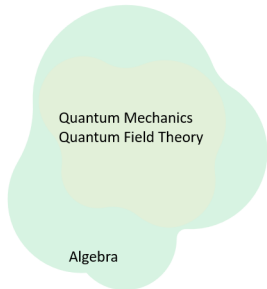
University of South-Eastern Norway

anna.pachol@usn.no



Plan of the talk:

- ☐ Motivation, NCG vs QG, Planck scale
- ☐ Symmetries & Quantum Symmetries (Quantum Groups)
- ☐ Quantum space-times
- ☐ Quantum gravity phenomenology





There is no clear and well-understood passage from the classical geometric world into the quantum world.

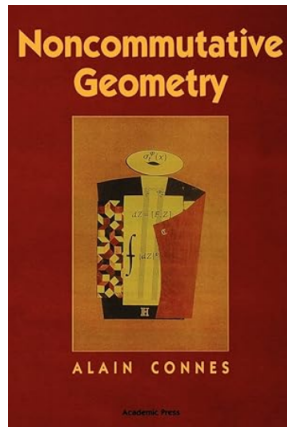
Continuum differential geometry – cannot be the geometry when both quantum and gravitational effects are present.

Need to look for a *more general* mathematical language, which would imply both geometry as well a quantum theory.

NONCOMMUTATIVE GEOMETRY*
is a proposition in this direction.

* NONCOMMUTATIVE GEOMETRY in pure mathematics

- Gelfand-Neimark'43: "Every compact, Hausdorff space, is equivalent to the spectrum of the C^* algebra of continuous, complex valued functions on that space"



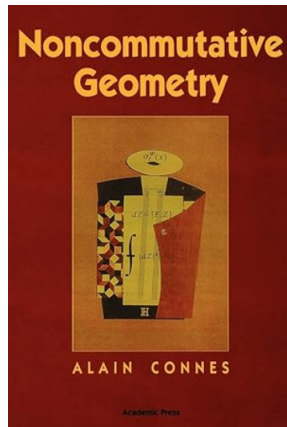
* NONCOMMUTATIVE GEOMETRY in pure mathematics

- Gelfand-Neimark'43: "Every compact, Hausdorff space, is equivalent to the spectrum of the C^* algebra of continuous, complex valued functions on that space"
- Alain Connes'80: "certain commutative algebras with some additional structure (the so-called spectral triples) are in one-to-one relation with compact Riemannian spin manifolds."

→ geometry "written" in the algebra language.

→ non-commutative geometry, i.e.

non-commutative algebras as
"non-commutative geometric spaces"



Classical Geometry

M - manifold,

$C^\infty(M)$ - alg. of functions

$$f \cdot g = g \cdot f$$

Ω^1 space of 1-forms, e.g.

differentials: $df = \sum_\mu \frac{\partial f}{\partial x^\mu} dx^\mu$

$$g \cdot df = (df) \cdot g \quad \forall f, g \in C^\infty(M)$$

order does not matter

Classical Geometry

M - manifold,

$C^\infty(M)$ - alg. of functions

$$f \cdot g = g \cdot f$$

Ω^1 space of 1-forms, e.g.

differentials: $df = \sum_\mu \frac{\partial f}{\partial x^\mu} dx^\mu$

$$g \cdot df = (df) \cdot g \quad \forall f, g \in C^\infty(M)$$

order does not matter



Noncommutative Geometry

- generalization of classical differential geometry,

- encoding geometry of "quantum-like counterparts of spaces": algebras

$$f \star g \neq g \star f$$

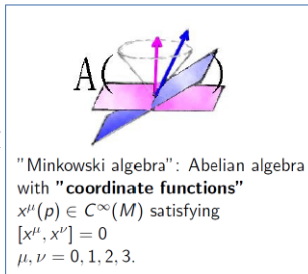
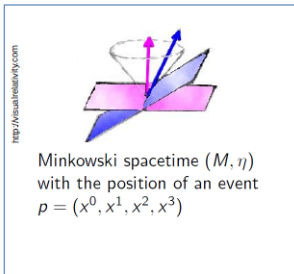
$$g \star (df) \neq (df) \star g$$

order does matter

Take M - manifold as Minkowski space-time
 for each point in the Minkowski space-time manifold $p \in M$ we assign a
 coordinate function $f(p) = x^\mu(p)$

points on manifold M

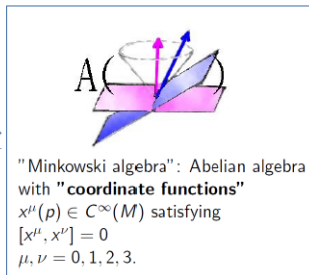
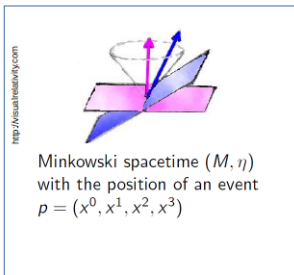
algebra of functions on M



Take M - manifold as Minkowski space-time
 for each point in the Minkowski space-time manifold $p \in M$ we assign a
 coordinate function $f(p) = x^\mu(p)$

points on manifold M

algebra of functions on M



$$x^\mu \star x^\nu \neq x^\nu \star x^\mu$$

$$x^\mu \star x^\nu - x^\nu \star x^\mu \neq 0 \quad \rightarrow \quad [x^\mu, x^\nu] \neq 0$$

Classical Geometry
(General Relativity)

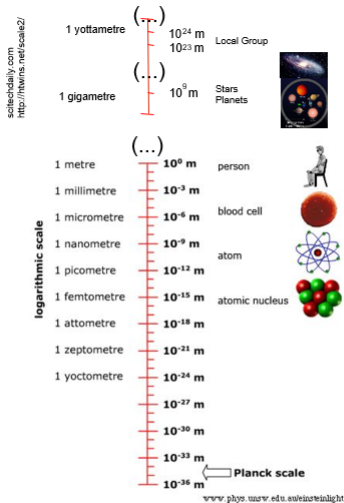


Noncommutative Geometry



Quantum Gravity

Planck scale - fundamental scale of quantum gravity, when both quantum effects and strong-field gravity are equally important



General Relativity

Quantum Mechanics

$$L_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35} \text{ m} ;$$

$$M_P = \sqrt{\frac{\hbar c}{G}} \simeq 1.22 \times 10^{19} \text{ GeV}/c^2$$

From a theoretical point of view:

- the **Planck length** is the size of a black hole at which its **Schwarzschild radius** is the same as its **Compton wavelength**:

The higher the energy the higher the precision of the measurement



Each mass or energy creates the curvature of the space. The highest precision cannot exceed the Schwarzschild radius.

From a theoretical point of view:

- the **Planck length** is the size of a black hole at which its **Schwarzschild radius** is the same as its **Compton wavelength**:
The higher the energy the higher the precision of the measurement



Each mass or energy creates the curvature of the space. The highest precision cannot exceed the Schwarzschild radius.

- may be viewed as the shortest possible length scale which can be probed by particle collisions, since probing scales shorter than this, via higher-energy collisions, may result in the formation of black holes

→ Classical general relativity breaks down at Planck scale

→ At short distances ($\sim L_P$) the geometry of space-time has to be described by a different theory because its points become no longer localizable

The space becomes non-local at the Planck scale

$$\Delta x^\mu \Delta x^\nu \geq (L_p)^2$$

Meaning that the coordinates are non-commutative

$$[x^\mu, x^\nu] = i\Theta^{\mu\nu}(x)$$

The space becomes non-local at the Planck scale

$$\Delta x^\mu \Delta x^\nu \geq (L_p)^2$$

Meaning that the coordinates are non-commutative

$$[x^\mu, x^\nu] = i\Theta^{\mu\nu}(x)$$

Quantum geometry:

- Apply principles of quantum mechanics to space-time itself
- Since gravity affects space-time geometry, **quantum** gravity should **quantize** space-time (in some way):

$$[x^\mu, x^\nu] = i\Theta^{\mu\nu}(x) \quad \leftrightarrow \quad \Delta x^\mu \Delta x^\nu \geq \frac{1}{2} |\Theta^{\mu\nu}|$$

Fundamental Uncertainties are common in many models:

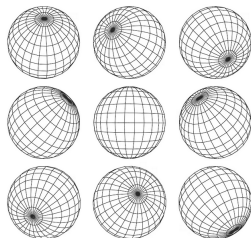
- Loop Quantum Gravity (minimal area/volume)
- String Theory (minimal length uncertainty relations from high-energy scattering)
- Spin Foam Models (effective noncommutative QFTs)
- Group Field Theory, Causal Dynamical Triangulation

- L. J. Garay, "Quantum gravity and minimum length," Int. J. Mod. Phys. A 10 (1995), arXiv:gr-qc/9403008.
- S. Hossenfelder, "Minimal Length Scale Scenarios for Quantum Gravity," Living Rev. Rel. 16 (2013), arXiv:1203.6191 [gr-qc].

Symmetries in physics: Invariance

Example: if the area of a surface remains unchanged under rotation in space $\longrightarrow$ **rotational invariance**.

Applies more generally to “physical laws”, e.g. laws of physics are $SO(3)$ -invariant if they do not distinguish different directions in space

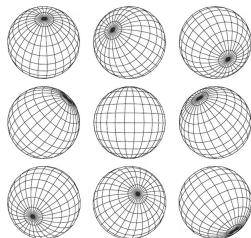


3D rotation symmetry group - $SO(3)$

Symmetries in physics: Invariance

Example: if the area of a surface remains unchanged under rotation in space $\rightarrow$ **rotational invariance**.

Applies more generally to “physical laws”, e.g. laws of physics are $SO(3)$ -invariant if they do not distinguish different directions in space



3D rotation symmetry group - $SO(3)$

- Symmetries simplify solving EoMs: Einstein equations analytically solvable only for systems with very high amount of symmetry.
- Internal symmetries ($U(1)$, $SU(2)$, $SU(3)$...)

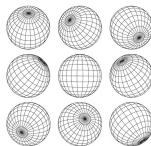
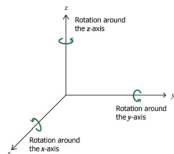
Group theory is the natural (mathematical) language to describe the symmetries of a physical system.

Symmetries in physics: Covariance

- Physical theories – as general descriptions of nature understandable to different observers – use "covariant" quantities.
- Mathematically, the physical quantities must transform covariantly (under a certain representation of the group of coordinate transformations)
- In relativistic physics: Lorentz covariance is a property of the underlying space-time.

Groups and symmetries

Example: Rotations - in 3 dimensions



Symmetry group $G = SO(3)$ $\longrightarrow$
 $R^T R = I, \quad \det R = 1$

$\mathfrak{g} = \mathfrak{so}(3)$ - Lie algebra
 $[J_i, J_j] = i\epsilon_{ijk} J_k$

Poincaré symmetry (of special relativity)

- isometry group of 4-dim Minkowski space-time, $x^\mu = (ct, x, y, z)$

Poincare group $G = ISO(1, 3) = \mathbb{R}^{(1,3)} \rtimes SO(1, 3)$ - semidirect product of the translations and the Lorentz transformations (rotations and boosts)

- The Lorentz group $SO(1, 3)$ is the group of linear transformations $x'^\mu = \Lambda^\mu_\nu x^\nu$ such that $x^2 = x_\mu x^\mu = x'_\mu x'^\mu$ is invariant.
- Translational invariance: $x^\mu \rightarrow x'^\mu = x^\mu + a^\mu$,
 $a^\mu = \text{const}$ - conservation law of momenta P^μ

Poincaré symmetry (of special relativity)

- isometry group of 4-dim Minkowski space-time, $x^\mu = (ct, x, y, x)$

Poincare group $G = ISO(1, 3) = \mathbb{R}^{(1,3)} \rtimes SO(1, 3)$ - semidirect product of the translations and the Lorentz transformations (rotations and boosts)

- The Lorentz group $SO(1, 3)$ is the group of linear transformations $x'^\mu = \Lambda^\mu_\nu x^\nu$ such that $x^2 = x_\mu x^\mu = x'_\mu x'^\mu$ is invariant.
- Translational invariance: $x^\mu \rightarrow x'^\mu = x^\mu + a^\mu$,
 $a^\mu = \text{const}$ - conservation law of momenta P^μ

Poincaré-Lie algebra $\mathfrak{g}$:

$$\left. \begin{array}{l} \text{- rotations} \\ \text{- boosts} \end{array} \right\} \begin{array}{l} J_i \\ N_i \end{array} = M_{\mu\nu}$$

$$\text{- translations} \quad P_\mu$$

$$[M_{\mu\nu}, M_{\lambda\rho}] = i (M_{\mu\lambda}\eta_{\nu\rho} - M_{\mu\rho}\eta_{\nu\lambda} - M_{\nu\lambda}\eta_{\mu\rho} + M_{\nu\rho}\eta_{\mu\lambda}),$$

$$[M_{\mu\nu}, P_\lambda] = i (P_\nu\eta_{\mu\lambda} - P_\mu\eta_{\nu\lambda}),$$

$$[P_\mu, P_\nu] = 0,$$

where $\eta_{\mu\nu}$ is the metric tensor with Lorentzian signature $\square$

Now that 'space' is quantum (noncommutative):

$$[x^\mu, x^\nu] \neq 0$$

we need

Quantum Group as (generalized) symmetry.

Now that 'space' is quantum (noncommutative):

$$[x^\mu, x^\nu] \neq 0$$

we need

Quantum Group as (generalized) symmetry.

- One can obtain them by **deformation procedure**.

Deformation = Quantization

**Classical
Mechanics**

$\hbar$

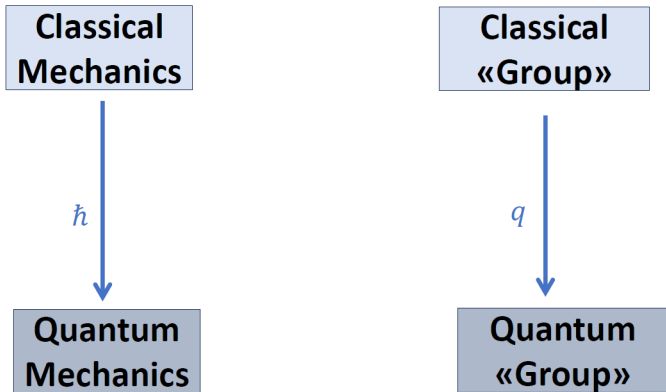
**Quantum
Mechanics**

A **new physical theory** can be seen as a *deformation* (generalization) of the previous existing one.

- The existing theory *is recovered as a limiting case* in which some parameters of the new theory become negligible.

- Quantum Mechanics is the deformation of Classical Mechanics with the **parameter $\hbar$**
- For $\hbar \rightarrow 0$ we recover Classical Mechanics.

Deformation = Quantization



Classical Algebra

Classical (Hopf) algebra

q

Quantum Algebra

Quantized (Hopf) algebra
(Quantum Group)

- The **quantization** is accompanied by the **introduction of an additional (deformation) parameter q**

or $(\hbar, \kappa, \theta)$

- In the limit $q \rightarrow 1$
or $(\hbar, \theta) \rightarrow 0, (\kappa \rightarrow \infty)$:
quantum (Hopf) algebra $\rightarrow$
classical (Hopf) algebra

**Classical
Algebra**

Classical (Hopf) algebra

$F(G),$
 $U(g)$

q



**Quantum
Algebra**

Quantized (Hopf) algebra
(Quantum Group)

$F_q(G),$
 $U_q(g)$

Quantum Groups
deformations of (Hopf) algebras,
but still
"symmetry groups"

↓

Quantum relativistic symmetries

**Classical
Symmetry**

Classical (Hopf) algebra

$F(G),$
 $U(g)$

q



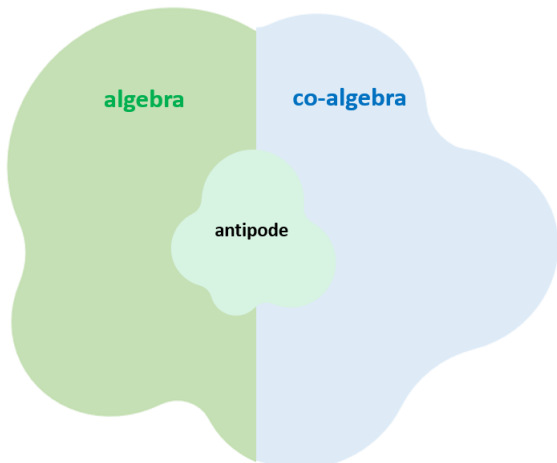
**Quantum
Symmetry**

Quantized (Hopf) algebra
(Quantum Group)

$F_q(G),$
 $U_q(g)$

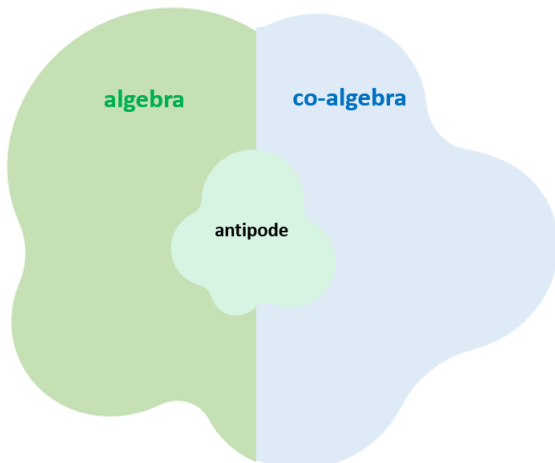
Hopf algebra (Quantum Group) is a **bialgebra**:

- 1) an algebra (unital, associative)
- 2) a co-algebra (co-unital, co-associative)



Hopf algebra (Quantum Group) is a **bialgebra**:

- 1) an algebra (unital, associative)
- 2) a co-algebra (co-unital, co-associative)



- 3) with an additional map S called the antipode

Algebra = $(A, \cdot)$

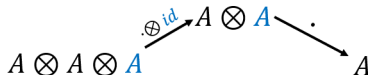
Has an operation $\cdot$



$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

which is **associative**

$$(\blacksquare \cdot \blacksquare) \cdot \blacksquare = \blacksquare = \blacksquare$$



Algebra= $(A, \cdot)$

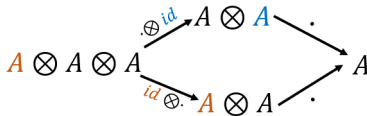
Has an operation $\cdot$



$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

which is **associative**

$$(\blacksquare \cdot \blacksquare) \cdot \blacksquare = \blacksquare \cdot (\blacksquare \cdot \blacksquare) = \blacksquare$$

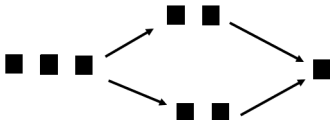


Algebra= $(A, \cdot)$

Has an operation $\cdot$



which is **associative**

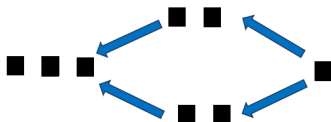


co-algebra

Has an operation



which is **co-associative**

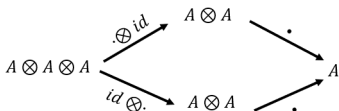


algebra

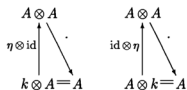
(unital, associative)

with product $\cdot : A \otimes A \rightarrow A$

Axiom A1: associativity $\cdot (\cdot \otimes id) = \cdot (id \otimes \cdot)$



Axiom A2: unit (map)



$\eta: k \rightarrow A$

$$\eta(1_k) = 1_A$$

$$\cdot (\eta \otimes id) = id = \cdot (id \otimes \eta)$$

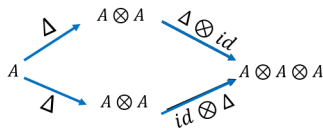
and

co-algebra

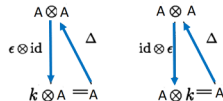
(co-unital, co-associative)

with co-product $\Delta : A \rightarrow A \otimes A$

Axiom C1: co-associativity $(\Delta \otimes id)\Delta = (id \otimes \Delta)\Delta$



Axiom C2: co-unit



$\epsilon: A \rightarrow k$

$$(\epsilon \otimes id)\Delta = id = (id \otimes \epsilon)\Delta$$

- Hopf algebra $A(\cdot, \eta, \Delta, \epsilon, S)$ is a structure composed by

- ① a (unital associative) algebra $(A, \cdot, \eta)$
- ② a (counital coassociative) coalgebra (A, Δ, ϵ)
in a compatible way:
 Δ, ϵ – are algebra homomorphisms

$$\Delta(XY) = \Delta(X)\Delta(Y), \quad \epsilon(XY) = \epsilon(X)\epsilon(Y)$$

And

- ③ $S : A \rightarrow A$ is the antipode

$$\cdot(S \otimes id) \circ \Delta = \eta \circ \epsilon = \cdot(id \otimes S) \circ \Delta$$

- Hopf algebra $A(\cdot, \eta, \Delta, \epsilon, S)$ is a structure composed by

- ① a (unital associative) algebra $(A, \cdot, \eta)$
- ② a (counital coassociative) coalgebra (A, Δ, ϵ)
in a compatible way:
 Δ, ϵ – are algebra homomorphisms

$$\Delta(XY) = \Delta(X)\Delta(Y), \quad \epsilon(XY) = \epsilon(X)\epsilon(Y)$$

And

- ③ $S : A \rightarrow A$ is the antipode

$$\cdot(S \otimes id) \circ \Delta = \eta \circ \epsilon = \cdot(id \otimes S) \circ \Delta$$

From any Lie algebra g one can make a Hopf algebra!

Hopf from Lie algebra



© Freepik - Flaticon

① g - Lie algebra: $[L_i, L_j] = c_{ij}^k L_k$

Hopf from Lie algebra



© Freepik - Flaticon

- ① g - Lie algebra: $[L_i, L_j] = c_{ij}^k L_k$
- ② $U(g)$ - universal enveloping algebra of g

$$U(g) = T(g)/I$$

i.e. the tensor algebra of g :

$$T(g) = k \oplus g \oplus (g \otimes g) \oplus (g \otimes g \otimes g) \oplus \cdots$$

modulo the two-sided ideal generated by $I = \{L_i L_j - L_j L_i - [L_i, L_j]\}$

Hopf from Lie algebra



© Freepik - Flaticon

- ① g - Lie algebra: $[L_i, L_j] = c_{ij}^k L_k$
- ② $U(g)$ - universal enveloping algebra of g

$$U(g) = T(g)/I$$

i.e. the tensor algebra of g :

$$T(g) = k \oplus g \oplus (g \otimes g) \oplus (g \otimes g \otimes g) \oplus \dots$$

modulo the two-sided ideal generated by $I = \{L_i L_j - L_j L_i - [L_i, L_j]\}$

- ③ then we define Δ_0, ϵ, S on the generators of g .

Hopf from Lie algebra



© Freepik - Flaticon

- ① g - Lie algebra: $[L_i, L_j] = c_{ij}^k L_k$
- ② $U(g)$ - universal enveloping algebra of g

$$U(g) = T(g)/I$$

i.e. the tensor algebra of g :

$$T(g) = k \oplus g \oplus (g \otimes g) \oplus (g \otimes g \otimes g) \oplus \dots$$

modulo the two-sided ideal generated by $I = \{L_i L_j - L_j L_i - [L_i, L_j]\}$

- ③ then we define Δ_0, ϵ, S on the generators of g .

$U(g)$ is a Hopf algebra with:

- coproduct : $\Delta_0(L) = L \otimes 1 + 1 \otimes L$; $\Delta_0(1) = 1 \otimes 1$;
- counit : $\epsilon(L) = 0$; $\epsilon(1) = 1$;
- antipode : $S_0(L) = -L$; $S_0(1) = 1$.

But what does co-algebra bring to physics?

1) How does $P_\mu = -i\partial_\mu$ act on plain waves e^{ikx} ?

$$P_\mu e^{ikx} = k_\mu e^{ikx}$$

2) How does P_μ act on a product of two plane waves $e^{ik_1x} \cdot e^{ik_2x}$?

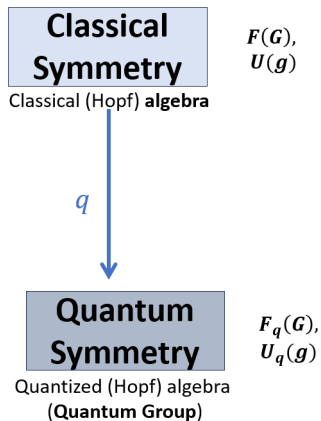
Leibniz rule $\leftrightarrow$ addition of momenta:

$$\begin{aligned} P_\mu(e^{ik_1x} \cdot e^{ik_2x}) &= \cdot(\Delta_0 P_\mu(e^{ik_1x} \otimes e^{ik_2x})) \\ &= \cdot(P_\mu(e^{ik_1x}) \otimes e^{ik_2x} + e^{ik_1x} \otimes P_\mu(e^{ik_2x})) = (k_1 + k_2)_\mu(e^{ik_1x} \cdot e^{ik_2x}) \end{aligned}$$

$$\text{Coproduct } \Delta_0 P_\mu = P_\mu \otimes 1 + 1 \otimes P_\mu$$

Deformation=Quantization

Quantum Groups
deformations of (Hopf) algebras,
but still
"symmetry groups"
↓
Quantum relativistic symmetries



Deformed/Quantum relativistic symmetries

The Poincaré symmetry $U(iso(1,3))$

$$[M_{\mu\nu}, M_{\lambda\rho}] = i (M_{\mu\lambda}\eta_{\nu\rho} - M_{\mu\rho}\eta_{\nu\lambda} - M_{\nu\lambda}\eta_{\mu\rho} + M_{\nu\rho}\eta_{\mu\lambda}),$$

$$[M_{\mu\nu}, P_\lambda] = i (P_\nu\eta_{\mu\lambda} - P_\mu\eta_{\nu\lambda}), \quad [P_\mu, P_\nu] = 0$$

(with $\Delta_0(P_\mu) = P_\mu \otimes 1 + 1 \otimes P_\mu$, $\Delta_0(M_{\mu\nu}) = M_{\mu\nu} \otimes 1 + 1 \otimes M_{\mu\nu}$ etc.)

Deformed/Quantum relativistic symmetries

The Poincaré symmetry $U(iso(1,3))$

$$[M_{\mu\nu}, M_{\lambda\rho}] = i (M_{\mu\lambda}\eta_{\nu\rho} - M_{\mu\rho}\eta_{\nu\lambda} - M_{\nu\lambda}\eta_{\mu\rho} + M_{\nu\rho}\eta_{\mu\lambda}),$$

$$[M_{\mu\nu}, P_\lambda] = i (P_\nu\eta_{\mu\lambda} - P_\mu\eta_{\nu\lambda}), \quad [P_\mu, P_\nu] = 0$$

(with $\Delta_0(P_\mu) = P_\mu \otimes 1 + 1 \otimes P_\mu$, $\Delta_0(M_{\mu\nu}) = M_{\mu\nu} \otimes 1 + 1 \otimes M_{\mu\nu}$ etc.)
can be deformed in various ways:

κ - deformation

contraction of q -deformed $so(2,3)$
(AdS algebra)

$$[L, L] = \text{corrections}$$

$$\Delta(L) = \Delta_0(L) + \text{corrections}$$

$$L = M_{\mu\nu}, P_\mu$$

non-standard deformation

twist deformation

$$[L, L] = \text{unchanged}$$

$$\Delta(L) = \Delta_0(L) + \text{corrections}$$

Deformed/Quantum relativistic symmetries

The Poincaré symmetry $U(iso(1,3))$

$$[M_{\mu\nu}, M_{\lambda\rho}] = i (M_{\mu\lambda}\eta_{\nu\rho} - M_{\mu\rho}\eta_{\nu\lambda} - M_{\nu\lambda}\eta_{\mu\rho} + M_{\nu\rho}\eta_{\mu\lambda}),$$

$$[M_{\mu\nu}, P_\lambda] = i (P_\nu\eta_{\mu\lambda} - P_\mu\eta_{\nu\lambda}), \quad [P_\mu, P_\nu] = 0$$

(with $\Delta_0(P_\mu) = P_\mu \otimes 1 + 1 \otimes P_\mu$, $\Delta_0(M_{\mu\nu}) = M_{\mu\nu} \otimes 1 + 1 \otimes M_{\mu\nu}$ etc.)
can be deformed in various ways:

κ - deformation

contraction of q -deformed $so(2,3)$
(AdS algebra)

$$[L, L] = \text{corrections}$$

$$\Delta(L) = \Delta_0(L) + \text{corrections}$$

$$L = M_{\mu\nu}, P_\mu$$

non-standard deformation

twist deformation

$$[L, L] = \text{unchanged}$$

$$\Delta(L) = \Delta_0(L) + \text{corrections}$$

Quantum Group!

Quantum Group!

Quantum Poincaré symmetry

Quantum Poincaré symmetry

κ - deformation $U_\kappa(iso(1,3))$

contraction of q -deformed $so(2,3)$
(AdS algebra)

$$[L, L] = \text{corrections}$$

$$\Delta(L) = \Delta_0(L) + \text{corrections}$$

non-standard deformation

twist deformation

$$[L, L] = \text{unchanged}$$

$$\Delta(L) = \Delta_0(L) + \text{corrections}$$

[• Drinfel'd, Jimbo, Reshetikin, Takhtajan, Fadeev, 1980s]

- J. Lukierski, H. Ruegg, A. Nowicki, and V. N. Tolstoi, Phys. Lett. B 264 (1991);
- S. Majid and H. Ruegg, Phys. Lett. B 334 (1994)

[• V. Drinfel'd 1983 - 1985]

- M. Chaichian, P. P. Kulish, K. Nishijima and A. Tureanu Phys. Lett. B 604 (2004);
- A. Borowiec, A.P., Phys. Rev. D 79 (2009)

- A. Borowiec, A. P., Eur. Phys. J. C 74 (2014)

"Unified description for κ -deformations of orthogonal groups"

Consequences:

- for example P_μ acting on a product of two plane waves:

$$\begin{aligned} P_\mu(e^{ik_1x} \star e^{ik_2x}) &= \star(\Delta P_\mu(e^{ik_1x} \otimes e^{ik_2x})) \\ &= \star(P_\mu(e^{ik_1x}) \otimes e^{ik_2x} + e^{ik_1x} \otimes P_\mu(e^{ik_2x}) + \text{corrections}) \\ &= (k_1 + k_2 + \text{corrections})_\mu(e^{ik_1x} \star e^{ik_2x}) \end{aligned}$$

- deformed Casimir $\leftrightarrow$ deformed dispersion relations
- ...

Quantum relativistic symmetries: Symmetries of what?

Classical

Minkowski space-time with $x^\mu = (ct, x, y, z)$
(Commutative algebra of coordinate functions)

$$x^\mu \cdot x^\nu = x^\nu \cdot x^\mu$$

$$[x^\mu, x^\nu] = 0$$

Quantum

Quantum Minkowski space-time

(Non-commutative algebra of coordinate functions)*

$$x^\mu \star x^\nu \neq x^\nu \star x^\mu$$

$$[x^\mu, x^\nu] \neq 0$$

QUANTUM GROUPS -> symmetries of noncommutative "spaces"

*in the realm of non-commutative geometry (A. Connes): any 'space' is determined by the algebra of functions on it.

Quantum relativistic symmetries: Covariance

Classical

Minkowski space-time $x^\mu = (ct, x, y, z)$
(Commutative algebra of coordinate functions)
 $x^\mu \cdot x^\nu = x^\nu \cdot x^\mu$

$$[x^\mu, x^\nu] = 0$$

is **covariant under Poincare symmetry**

Quantum

Quantum Minkowski space-time
(Non-commutative algebra of coordinate functions)*

$$x^\mu \star x^\nu \neq x^\nu \star x^\mu$$

$$[x^\mu, x^\nu] \neq 0$$

is **covariant under quantum Poincare** (Hopf) algebra **symmetry**:

$$\begin{aligned} L \triangleright (x^\mu \star x^\nu) &= \star (\Delta(L)(x^\mu \otimes x^\nu)) \\ &= (L_{(1)} \triangleright x^\mu) \star (L_{(2)} \triangleright x^\nu) \end{aligned}$$

$$L = M_{\mu\nu}, P_\mu$$

determined by co-product
 $\Delta(L) = \Delta_0(L) + \text{deformed}$

Dictionary

Classical

Manifold, functions on a manifold
(space-time, coordinate functions)

Symmetries

(Lie group, Lie algebra)

Differential geometry

Quantum

$\mathcal{M}$ – abstract algebra

(quantum space-time) $x^\mu x^\nu \neq x^\nu x^\mu$
(Hopf module algebra)

Quantum symmetries

(Quantum Groups - Hopf algebras)

Non-commutative differential geometry

-> Twisted diff. geom.

-> Quantum Riemannian geom.

(build on bimodule of 'algebraic' NC
1-forms)

Noncommutative differential and Riemannian geometry

Well established NC differential geometry ingredients

- ① A a NC ('coordinate') algebra (given by generators and relations)
- ② Differential calculus $(\Omega^\bullet, \wedge, d)$
 $\Omega^\bullet = A \oplus \Omega^1 \oplus \Omega^2 \oplus \dots$

Noncommutative differential and Riemannian geometry

Well established NC differential geometry ingredients

- ① A a NC ('coordinate') algebra (given by generators and relations)
- ② Differential calculus $(\Omega^\bullet, \wedge, d)$
 $\Omega^\bullet = A \oplus \Omega^1 \oplus \Omega^2 \oplus \dots$
- ③ Connection $\nabla : \Omega^1 \rightarrow \Omega^1 \otimes_A \Omega^1$ satisfying
 $\nabla(\omega a) = \nabla(\omega)a + \omega \otimes_A da, \quad a \in A, \omega \in \Omega^1$
- ④ Torsion $T_\nabla = \wedge \circ \nabla + d$
- ⑤ Curvature R_∇

Noncommutative differential and Riemannian geometry

Well established NC differential geometry ingredients

- ① A a NC ('coordinate') algebra (given by generators and relations)
- ② Differential calculus $(\Omega^\bullet, \wedge, d)$
 $\Omega^\bullet = A \oplus \Omega^1 \oplus \Omega^2 \oplus \dots$
- ③ Connection $\nabla : \Omega^1 \rightarrow \Omega^1 \otimes_A \Omega^1$ satisfying
 $\nabla(\omega a) = \nabla(\omega)a + \omega \otimes_A da, \quad a \in A, \omega \in \Omega^1$
- ④ Torsion $T_\nabla = \wedge \circ \nabla + d$
- ⑤ Curvature R_∇

...However - different formulations of T and R (forms versus vector fields)?

Noncommutative differential and Riemannian geometry

different formulations

Twisted diff. geom.

quantum Riemannian geom.

Noncommutative differential and Riemannian geometry

different formulations

Twisted diff. geom.

Guiding principle:

NC diff. geom. on M is covariant under $UVec(M)_\star$ (twisted diffeomorphism symmetry).

In practice:

- change all the maps to $\star, \wedge_\star, \star_\star$
- $A = (\text{functions, forms, tensors, ...})$ are $UVec(M)_\star$ -module algebras and for $a, b \in A, \xi \in Vec(M), \xi \triangleright (ab)$
- ξ acts via Lie derivative & Leibniz rule (coproduct).
- Arbitrary metric $g \in \Omega^1 \otimes_A \Omega^1$

quantum Riemannian geom.

- "Noncommutative Geometry and Gravity", P. Aschieri, M. Dimitrijevic, F. Meyer, J. Wess, Class.Quant.Grav. 23 (2006) [arXiv:hep-th/0510059]
- P. Aschieri, A. Borowiec, A. P. "Observables and Dispersion Relations in κ -Minkowski Spacetime", JHEP 2017, 152 (2017) [arXiv:1703.08726]
- Dispersion Relations in κ -Noncommutative Cosmology, JCAP 04 (2021) 025 [arXiv:2009.01051]

Noncommutative differential and Riemannian geometry

different formulations

Twisted diff. geom.

Guiding principle:

NC diff. geom. on M is covariant under $UVec(M)_\star$ (twisted diffeomorphism symmetry).

In practice:

- change all the maps to $\star, \wedge_\star, \star_\star$
- $A = (\text{functions, forms, tensors, ...})$ are $UVec(M)_\star$ -module algebras and for $a, b \in A, \xi \in Vec(M), \xi \triangleright (ab)$
- ξ acts via Lie derivative & Leibniz rule (coproduct).
- Arbitrary metric $g \in \Omega^1 \otimes_A \Omega^1$

• "Noncommutative Geometry and Gravity", P. Aschieri, M. Dimitrijevic, F. Meyer, J. Wess, Class.Quant.Grav. 23 (2006) [arXiv:hep-th/0510059]
• P. Aschieri, A. Borowiec, A. P. "Observables and Dispersion Relations in κ -Minkowski Spacetime", JHEP 2017, 152 (2017) [arXiv:1703.08726]
Dispersion Relations in κ -Noncommutative Cosmology, JCAP 04 (2021) 025 [arXiv:2009.01051]

quantum Riemannian geom.

Guiding principle:

start with the NC 'coordinate' algebra A
build up whole differential structure from bottom up.

In practice:

- Ω^1 -bimodule algebra of abstract "algebraic" 1-forms
- bimodule connection ∇ additional requirement
- quantum metric $g \in \Omega^1 \otimes_A \Omega^1$, central in A
- metric compatibility...

• E. J. Beggs and S. Majid, "Quantum Riemannian Geometry", Springer 2020,
• S. Majid, A.P. J. Math. Phys. 59 (2018); arXiv:1701.06919 J. Phys. A: Math. Theor. 53 (2020); arXiv:1807.08492

Different models of quantum space-times

$$x^\mu \star x^\nu \neq x^\nu \star x^\mu \longrightarrow [x^\mu, x^\nu] \neq 0$$

(1) Canonical or theta-non-commutative spacetime:

$$[x^\mu, x^\nu] = i\hbar\theta^{\mu\nu}$$

with deformation parameter $\hbar$ of length² (L_P) dim.

S. Doplicher, K. Fredenhagen, J. E. Roberts, *Commun.Math. Phys.* 172 (1995),
[arXiv:hep-th/0303037].

Different models of quantum space-times

$$x^\mu \star x^\nu \neq x^\nu \star x^\mu \longrightarrow [x^\mu, x^\nu] \neq 0$$

(1) Canonical or theta-non-commutative spacetime:

$$[x^\mu, x^\nu] = i\hbar\theta^{\mu\nu}$$

with deformation parameter $\hbar$ of length² (L_P) dim.

S. Doplicher, K. Fredenhagen, J. E. Roberts, *Commun.Math. Phys.* 172 (1995),
[arXiv:hep-th/0303037].

- The Euclidean version of this non-commutative spacetime is known as the Moyal-Weyl non-commutative spacetime
- field theory has been thoroughly studied (writing the action in terms of the star product)'2001
- Non-commutative versions of gravity'2005

Different models of quantum space-times

(2) Lie-algebraic type space-time:

$$[x^\mu, x^\nu] = i\hbar\theta^{\mu\nu}_\rho x^\rho$$

with deformation parameter $\hbar$ of mass (M_P) dim.

Special case: κ -Minkowski spacetime:

$$[x^\mu, x^\nu] = \frac{i}{\kappa}(x^\mu a^\nu - x^\nu a^\mu)$$

$\kappa \sim \hbar/L_P$ (mass scale), a^μ -const.

S. Majid, H. Ruegg Phys.Lett. B334 (1994)

S. Zakrzewski J. Phys. A 127 (1994).

κ - Minkowski & κ -Poincaré:

- is related with Doubly (or deformed) special relativity (DSR) - a class of models of relativistic kinematics in which a fundamental observer-independent length scale is present.
- contains modified composition laws of energy and momentum (which comes from the co-algebra)
- one can identify the composition of energy and momentum as a translation in a curved momentum space of constant curvature and non-commutative spacetime coordinates are generators of translations in momentum space

This leads to many phenomenological consequences.

Lorentz-invariant quantum space-time models

$$[x_\mu, x_\nu] = i\beta^2 M_{\mu\nu}$$

Lorentz-invariant quantum space-time models

Snyder model

H.S. Snyder, Phys. Rev. 71 38 (1947)

$$[p_\mu, p_\nu] = 0$$

Yang model

C.N. Yang, Phys. Rev. 72 874 (1947)

$$[p_\mu, p_\nu] = i\alpha^2 M_{\mu\nu}$$

$$[x_\mu, x_\nu] = i\beta^2 M_{\mu\nu}$$

$$[M_{\mu\nu}, x_\rho] = i(\eta_{\mu\rho}x_\nu - \eta_{\nu\rho}x_\mu),$$

$$[M_{\mu\nu}, M_{\rho\tau}] = i(\eta_{\mu\rho}M_{\nu\tau} - \eta_{\mu\tau}M_{\nu\rho} + \eta_{\nu\tau}M_{\mu\rho} - \eta_{\nu\rho}M_{\mu\tau}).$$

$$\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1) \text{ and } \mu, \nu = 0, 1, 2, 3$$

Lorentz-invariant quantum space-time models

Snyder model

H.S. Snyder, Phys. Rev. 71 38 (1947)

$$[p_\mu, p_\nu] = 0$$

Yang model

C.N. Yang, Phys. Rev. 72 874 (1947)

$$[p_\mu, p_\nu] = i\alpha^2 M_{\mu\nu}$$

$$[x_\mu, x_\nu] = i\beta^2 M_{\mu\nu}$$

$$[M_{\mu\nu}, x_\rho] = i(\eta_{\mu\rho}x_\nu - \eta_{\nu\rho}x_\mu),$$

$$[M_{\mu\nu}, M_{\rho\tau}] = i(\eta_{\mu\rho}M_{\nu\tau} - \eta_{\mu\tau}M_{\nu\rho} + \eta_{\nu\tau}M_{\mu\rho} - \eta_{\nu\rho}M_{\mu\tau}).$$

$$\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1) \text{ and } \mu, \nu = 0, 1, 2, 3$$

Parameters:

- β related to the Planck (length) scale L_P
- α related with the non-vanishing cosmological constant Λ , (Anti-)de Sitter geometric background $\leftrightarrow \alpha$ linked with the (Anti-)de Sitter radius

In short:

$\beta \sim L_P$ - gravitational modifications at the small distances; while

$\alpha \sim \Lambda$ - modifications at large distances.

The Snyder and Yang models are defined on the full relativistic phase space and are generated by position x^μ , momentum p_μ and Lorentz generators $M_{\mu\nu}$.

- the first specific proposal, aimed to introduce a UV cut-off in momentum space, was the Snyder non-commutative space-time
- By construction, this non-commutative spacetime is Lorentz invariant
- The spectrum of the spatial coordinates operators is discrete and its momentum space is a de Sitter manifold with a non-vanishing constant curvature.

Back to physics: Experimental tests?

If q , θ , $1/\kappa$... (deformation parameters) are so small – does this make models totally untestable?

- There are situations where the quantum-gravity effect can be magnified and (in principle) measurable.
- For example, a small energy-dependence on the speed of light would result in **different arrival times of a few milliseconds for γ -ray bursts** of cosmological origin.
- Other effects could be a phase transition such as ‘Bose-Einstein condensation’ (so far modelled in mathematically analogous situations rather than quantum gravity itself).

Quantum Gravity Phenomenology

- A key question is whether there exists a regime in which phenomena induced by the quantum properties of the space-time geometry could possibly be observed.
- Probing the quantum nature of space-time date originated 2000 and now the study of quantum gravity phenomenology has developed into a full fledged research program,

Recent review:

A. Addazi et al., Quantum gravity phenomenology at the dawn of the multi-messenger era – A review, Prog.Part.Nucl.Phys. 125 (2022), arXiv: 2111.05659 hep-ph, 1500 references!

Quantum Gravity Phenomenology

The phenomenological consequences - reflect the fact that the kinematics of particles and fields propagating on quantum space-time are affected through their interaction with quantum space-time:

- Time delays
- Gravitational redshift
- Non-localizability of events: Limits on the localizability of events and properties of reference frames for the κ -Minkowski spacetime
- Emergence of a cosmological constant
- Threshold anomalies: The deformed energy-momentum dispersion relation and four-momentum conservation laws could affect the kinematics and energy threshold conditions for certain particle production processes
- Muon lifetimes
- Decoherence
- Dark Matter and axion-like backgrounds

Low-energy effects?

Quantum mechanical phase-space
the phase space of classical
mechanics $\{x_i, p_j\}$ is QUANTIZED
(deformed) into a **noncommutative
phase space**

At the Planck scale
space and **space-time** is
QUANTIZED (deformed): i.e.
becomes **noncommutative**

$$[x_i, p_j] = 0$$

↓

$$[x_i, p_j] = i\delta_{ij}\hbar$$

$$[x^\mu, x^\nu] = 0$$

↓

$$[x^\mu, x^\nu] \neq 0$$

Low-energy effects?

Quantum mechanical phase-space
the phase space of classical
mechanics $\{x_i, p_j\}$ is QUANTIZED
(deformed) into a **noncommutative
phase space**

$$[x_i, p_j] = 0$$

↓

$$[x_i, p_j] = i\delta_{ij}\hbar$$

At the Planck scale
space and **space-time** is
QUANTIZED (deformed): i.e.
becomes **noncommutative**

↓

and QM phase space becomes
modified/deformed

$$[x^\mu, x^\nu] = 0$$

↓

$$[x^\mu, x^\nu] \neq 0$$

↓

$$[x^\mu, p_\nu] = i\hbar\delta_\nu^\mu + \text{corrections}$$

Modified (Heisenberg) Uncertainty
Principles

Modified/Deformed Quantum Phase Space

any modifications in the canonical Heisenberg commutation relations between coordinates and momenta will imply changes in the Heisenberg Uncertainty Principle:

$$[x_i, p_j] = i\hbar\delta_{ij} + \text{corrections}$$

↓

$$(\Delta x_i)(\Delta p_i) \geq \frac{\hbar}{2} (1 + \text{corrections})$$

Modified/Deformed Quantum Phase Space

any modifications in the canonical Heisenberg commutation relations between coordinates and momenta will imply changes in the Heisenberg Uncertainty Principle:

$$[x_i, p_j] = i\hbar\delta_{ij} + \text{corrections}$$

↓

$$(\Delta x_i)(\Delta p_i) \geq \frac{\hbar}{2} (1 + \text{corrections})$$

Generalized Uncertainty Principle (GUP): $\text{corrections} \sim f(p)$

Extended (EUP) ($\text{corrections} \sim f(x)$)

and GEUP ($\text{corrections} \sim f(x, p)$)

- providers of phenomenological deformations of QM that could accommodate the presence of a minimal length

Noncommutative geometry and quantum groups origins

- M. Maggiore,
Quantum groups, gravity and the generalized uncertainty principle. Phys. Rev. D (1994) arXiv:hep-th/9305163.
- M. Maggiore,
The Algebraic structure of the generalized uncertainty principle. Phys. Lett. B (1993) arXiv:hep-th/9309034
- A. Kempf,
Uncertainty relation in quantum mechanics with quantum group symmetry. J. Math. Phys. (1994) arXiv:hep-th/9311147.

- originally derived in the context of string theory [D. Amati, M. Ciafaloni, and G. Veneziano, "Can Space-Time Be Probed Below the String Size?," Phys. Lett. B 216 (1989)]

In general, noncommutative QM phase space algebra:

$$[x_i, x_j] = i\hbar a_{ij}(x, p), \quad [p_i, p_j] = i\hbar b_{ij}(x, p),$$

$$[x_i, p_j] = i\hbar c_{ij}(x, p),$$

A.P., Generalized Extended Uncertainty Principles, Liouville theorem and density of states: Snyder-de Sitter and Yang models, Nucl. Phys. B 1010 (2025)
[arXiv:2409.05110]

In general, noncommutative QM phase space algebra:

$$[x_i, x_j] = i\hbar a_{ij}(x, p), \quad [p_i, p_j] = i\hbar b_{ij}(x, p),$$

$$[x_i, p_j] = i\hbar c_{ij}(x, p),$$

leading to:

$$\Delta x_i \Delta p_j \geq \frac{\hbar}{2} |\langle c_{ij}(x, p) \rangle|$$

e.g.

$$\Delta x \Delta p \geq \frac{\hbar}{2} (1 + \alpha^2 (\Delta x)^2 + \beta^2 (\Delta p)^2)$$

A.P., Generalized Extended Uncertainty Principles, Liouville theorem and density of states: Snyder-de Sitter and Yang models, Nucl. Phys. B 1010 (2025)
[arXiv:2409.05110]

Liouville theorem in statistical physics and density of states

$$[x_i, p_j] = i \underbrace{\hbar \delta_{ij} (1 + \text{corrections})}_{\hbar_{\text{eff}}}$$

- new effective value of $\hbar$
- affects the size of the unit cell that each quantum state occupies in the phase space
- has an effect on the density of states and e.g. on thermodynamical properties.

Need to preserve the number of states in the volume element of phase space under the time evolution (analogue of the Liouville theorem)

L.N. Chang, D. Minic, N. Okamura, T. Takeuchi, Phys. Rev. D 65, 125028 (2002)

Modified thermodynamics

The weighted phase space volume ¹ is modified

$$\frac{d^D x d^D p}{1 + \text{corrections}}$$

→ modified Fermi equation of state - from the partition function in the grand-canonical ensemble:

$$\ln \mathcal{Z} = \frac{1}{(2\pi\hbar)^3} \frac{g}{a} \int \ln \left(1 + a z e^{-E/k_B T} \right) \frac{d^3 x d^3 p}{1 + \text{corrections}}$$

where T is the temperature, k_B Boltzmann constant, $z = e^{\mu/k_B T}$, μ chemical potential $a = 1$ fermions ($a = -1$ bosons) and g is a spin of a particle.

Then pressure, number of particles, and internal energy are:

$$P = k_B T \frac{\partial}{\partial V} \ln \mathcal{Z},$$

$$n = k_B T \frac{\partial}{\partial \mu} \ln \mathcal{Z} |_{T, V},$$

$$U = k_B T^2 \frac{\partial}{\partial T} \ln \mathcal{Z} |_{z, V}$$

¹in the classical limit we take $\frac{1}{i\hbar}[\cdot, \cdot] \rightarrow \{\cdot, \cdot\}$

- A.P., Generalized Extended Uncertainty Principles, Liouville theorem and density of states: Snyder-de Sitter and Yang models, Nucl. Phys. B 1010 (2025) [arXiv:2409.05110]

A.P., A. Wojnar:

- **Fermi equation of state** with finite temperature corrections in quantum space-times approach: Snyder model vs GUP case, 2023 Class. Quantum Grav. 40 195021 [arXiv:2304.08215]
- Constraining Snyder and GUP models with **low-mass stars**, Eur. Phys. J. C 83, 1097 (2023) [arXiv:2307.03520]

From experimental point of view

$$\text{Testing: } [x, p] = i\hbar(1 + \beta^2 p^2) \quad \Leftrightarrow \quad \Delta x \Delta p \geq \frac{\hbar}{2}(1 + \beta^2 p^2)$$

- Marin et al., Gravitational bar detectors set limits to Planck-scale physics on macroscopic variables, *Nature Phys.* 9 (2013)
- Bawaj et al., Probing deformed commutators with macroscopic harmonic oscillators, *Nature Commun.* 6, (2015)
- Campbell et al., Improved constraints on minimum length models with a macroscopic low loss phonon cavity, *Phys. Rev. D* (2023)
- Donadi, Fadel, Quantum gravitational decoherence of a mechanical oscillator from spacetime fluctuations, *Phys. Rev. D* (2025)



ARTICLE

Received 25 Nov 2024 | Accepted 12 May 2025 | Published 19 Jun 2025

DOI: 10.1038/s41467-025-00890-9

OPEN

Probing deformed commutators with macroscopic harmonic oscillators

Mateusz Bawaj^{1,2}, Ciro Biancofiore^{1,2}, Michele Bonaldi^{3,4}, Federica Bonfigli^{1,2}, Antonio Borrielli^{3,4}, Giovanni Di Giuseppe^{1,2}, Lorenzo Marconi⁵, Francesco Marino^{6,7}, Riccardo Natali^{1,2}, Antonio Portini^{5,6}, Giovanni A. Prodi^{4,8}, Enrico Serra^{4,8,9,10}, David Vitis^{1,2} & Francesco Marin^{5,6,11}

A minimal observable length is a common feature of theories that aim to merge quantum physics and gravity. Quantum mechanically, this concept is associated with a nonzero minimal uncertainty in position measurements, which is encoded in deformed commutation relations. In spite of increasing theoretical interest, the subject suffers from the complete lack of dedicated experiments and bounds to the deformation parameters have just been extrapolated from indirect measurements. As recently proposed, low-energy mechanical oscillators could allow to reveal the effect of a modified commutator. Here we analyze the free evolution of high-quality factor micro- and nano-oscillators, spanning a wide range of masses around the Planck mass m_P ($\approx 22 \mu\text{g}$). The direct check against a model of deformed dynamics substantially lowers the previous limits on the parameters quantifying the commutator deformation.

PHYSICAL REVIEW D **108**, 102006 (2023)

Improved constraints on minimum length models with a macroscopic low loss phonon cavity

William M. Campbell¹, Michael E. Tobar, and Maxim Goryachev¹

Quantum Technologies and Dark Matter Labs, Department of Physics, University of Western Australia
35 Stirling Highway, Crawley, Washington 6009, Australia

Serge Galliou²

SUPMICROTECH-ENSMM, CNRS, Institut FEMTO-ST, 26 Rue de l'Épitaphe 25000 Besançon, France



(Received 3 April 2023; revised 1 August 2023; accepted 31 October 2023; published 21 November 2023)

Many theories that attempt to formulate a quantum description of gravity suggest the existence of a fundamental minimum length scale. A popular method for incorporating this minimum length is through a modification of the Heisenberg uncertainty principle known as the generalized uncertainty principle (GUP). Experimental tests of the GUP applied to composite systems can be performed by searching for the induced frequency perturbations of the modes of mechanical resonators, thus constraining the degree of minimum length in certain scenarios. In this work previous constraints made with mechanical resonator are improved upon by three orders of magnitude, via the utilization of a cryogenic quartz bulk acoustic wave resonator. As well as purely mechanical resonant modes, hybrid electromechanical antiresonant modes are investigated and shown to be sensitive to the same GUP induced effects.

DOI: 10.1103/PhysRevD.108.102006

"indirect" experiments - providing bounds on GUP parameters

P.Bosso, G. Luciano, L.Petruzzello and F.Wagner, 30 years in:
Quo vadis generalized uncertainty principle?, " Class. Quant.
Grav. 40 (2023) no.19, 195014 [arXiv:2305.16193 [gr-qc]].

experiment
perihelion precession (solar system, 1)
time-of-flight measurements
equivalence principle (pendula)
gravitational bar detectors
equivalence principle (atoms)
low-mass stars
LIV in torsion pendulum
perihelion precession (solar system, 2)
perihelion precession (pulsars)
gravitational redshift
black hole quasi normal modes
light deflection
time delay of light
black hole shadow
black hole shadow

experiment
phonon cavity
harmonic oscillators
LIV in hydrogen atom
scanning tunnelling microscope
μ anomalous magnetic moment
Hydrogen atom
lamb shift
^{87}Rb interferometry
Kratzer potential
stimulated emission
Landau levels
quantum noise



CA23130 - Bridging high and low energies in search of quantum gravity (BridgeQG)

Action keywords

Phenomenology of quantum gravity - Gravity in quantum systems -
Multi-messenger astrophysics - Lorentz invariance violation and
deformation - Table-top gravity measurements

<https://www.cost.eu/actions/CA23130/>

- Recent advances in both **high-energy astrophysics** and **high-precision table-top experiments** are pushing our capability to test nature in regimes where gravity meets quantum physics.
- **Astrophysical observations** are now **potentially** sensitive to tiny residual effects of Planck-scale physics, while **table-top experiments** are **reaching the precision** needed to test the interplay between gravity and quantum systems at ultra-low energies.
- Investigations of these regimes, in particular once they are combined, will provide important clues towards the understanding of the full-fledged theory of quantum gravity.
- Current focus: investigations at the interface between high-energy quantum gravity and quantum aspects of gravity in the weak-field regime.

Noncommutative Geometry (NCG) as an intermediate step

Two alternatives:

Quantum Mechanics

Quantum Field Theory

+ NCG ("quantum gravity")
corrections

Gravity

(General and Special Relativity)

+ NCG (quantum) corrections

Summary

Noncommutative Geometry

- generalization of classical differential geometry,
- encodes geometry of "quantum-like counterparts of spaces": algebras

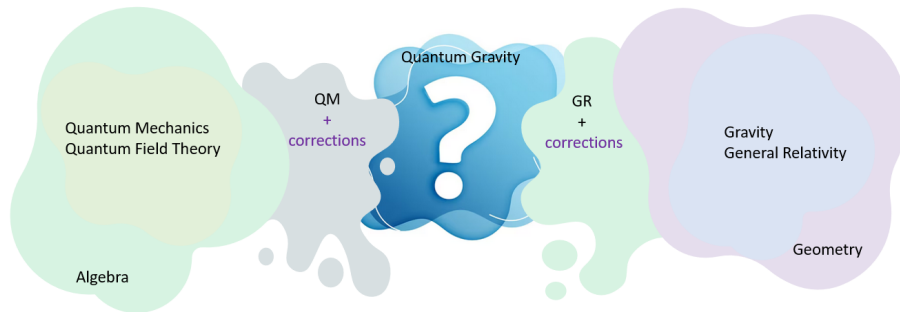
Summary

Noncommutative Geometry

- generalization of classical differential geometry,
- encodes geometry of "quantum-like counterparts of spaces": algebras - the noncommutative (quantum) nature of space-time $\rightarrow$ modifications (deformations) of
 - quantum mechanical phase spaces
 - effects on physical systems
 - corrections to classical solutions

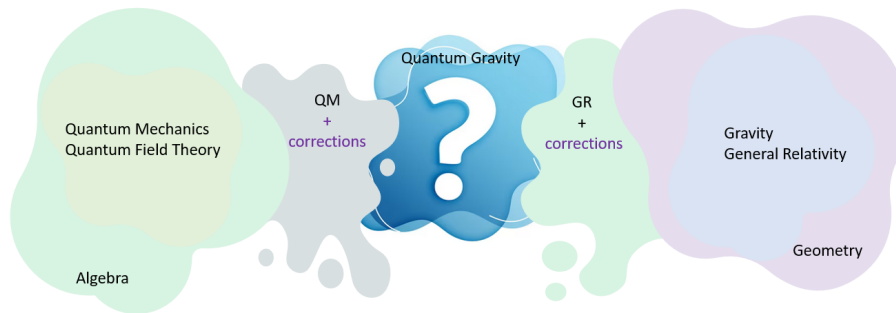
Summary

Noncommutative Geometry as an intermediate step



Summary

Noncommutative Geometry as an intermediate step



Thank you for your attention!