

The Future of de Sitter

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1109.1600, 1104.0002, 1005.1056)

Motivation:

Physics of InfraRed/soft modes in de Sitter

- Are they physical? (is f_{NL}^{local} ?)
- Meaning of Infrared effects
- What happens at equivalent of Page time in dS
- Connection to gravitational memory and asymptotic S.B. Symmetries

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Physics of InfraRed/soft modes in de Sitter

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 - Connection to gravitational memory and asymptotic S.B. Symmetries
- Focus of this talk!*

Taking a Step back: Relation to black holes

- Accelerated obs. in flat space experience temp.

$$T = \frac{a}{2\pi}$$

- Stationary obs. at dist. R from planet of mass M experience acc.

$$a = \frac{G_N M}{R^2}$$

- Schwarzschild radius of black hole

$$R_{bh} = 2 G_N M$$

$\Rightarrow$ Black hole has temp. $T_{bh} = \frac{1}{8\pi G_N M} = \frac{1}{4\pi R_{bh}}$

Relation to black holes

- If black holes has temp. we can associate entropy

- Using $T dS = dU$

(with $U = Mc^2$)
[$c \equiv 1$ everywhere else]

$$\Rightarrow dS = 8\pi G_N M dM$$

$$\Rightarrow S = 4\pi G_N M^2 =$$

$$\Rightarrow \boxed{S = \frac{1}{4} A_{bh}}$$

$$A_{bh} = 4\pi R_{bh}^2$$

Relation to black holes

- Schwarzschild metric

$$ds^2 = \left(1 - \frac{R_{bh}}{r}\right) dt^2 - \left(1 - \frac{R_{bh}}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

- De Sitter metric

$$ds^2 = \left(1 - \frac{r^2}{R_{ds}^2}\right) dt^2 - \left(1 - \frac{r^2}{R_{ds}^2}\right)^{-1} dr^2 - r^2 d\Omega^2$$

BH : $\frac{R_{bh}}{r} = \frac{2GM}{r}$: surface gravity $\chi = \frac{1}{2} \left(\frac{2GM}{r} \right)' \Big|_{r=R_{bh}} = \frac{GM}{R_{bh}^2}$

ds : $\frac{r^2}{R_{ds}^2} = r^2 H^2$: surface gravity $\chi = \frac{1}{2} \left(r^2 H^2 \right)' \Big|_{r=R_{ds}} = H$

$\Rightarrow a_{ds} = H \quad \Rightarrow T_{ds} = \frac{a}{2\pi} = \frac{H}{2\pi} \quad \left(R_{ds} = \frac{1}{H} \right)$

Relation to black holes

- If one collapses pure state of zero entropy to a black hole and wait til it has evaporated

$\Rightarrow$ pure state converted to thermal radiation = mixed state

$\Rightarrow$ violation of Unitarity 

(information loss as mixed state is obtained from pure state by tracing out inaccessible part of the state)

Relation to Black Holes

- Black Hole info. paradox is an indication that a local description must fail on a time scale of order the black hole evaporation time

$$t_{ev} \sim R_{bh} S_{bh} \sim M^3$$

- This is the time scale at which information needs to start to coming out of the black hole
- ➔ Identifying a source for a perturbative breakdown would indicate no information paradox, only information problem
- ✓ A breakdown of the perturbative approach in de Sitter on similar time scale has already been identified

$$t_{ds} \sim R_{ds} S_{ds} \sim 1/H^3$$

Soft hair of Black Holes

- A key assumption of the B.H. info. paradox is the no hair theorem
 - ➔ assumption that the black hole returns to a unique vacuum after evaporation
- It has lately been argued that this assumption is wrong
 - ➔ Vacuum not invariant under addition of soft modes

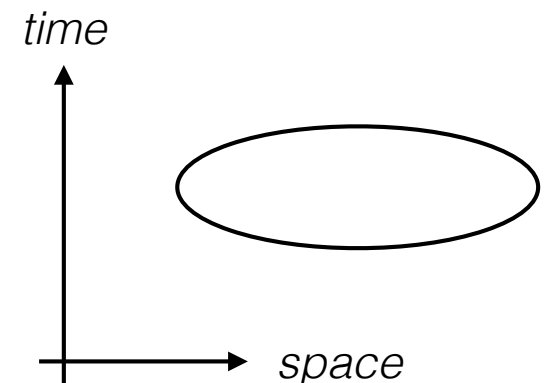
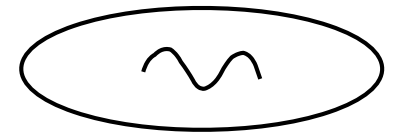
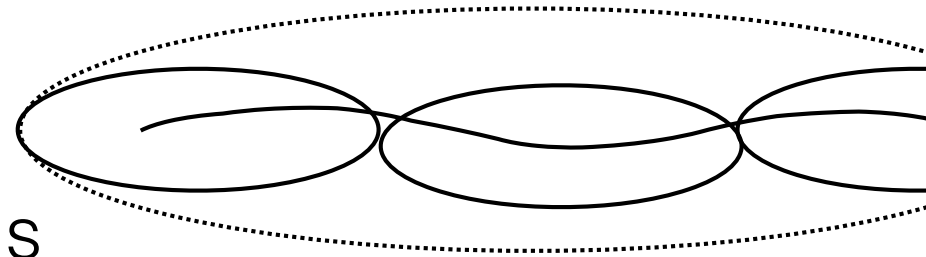
Soft hair of Black Holes

- More precisely adding a soft mode corresponds to a gauge transformation that does not fall off at infinity
- ➔ a conserved charge associated with the large gauge transformation has been constructed
- ✓ The charge has been argued to act non-trivially on the final state

$$|X'\rangle = Q |X\rangle \neq |X\rangle$$

Soft hair of de Sitter

- In de Sitter, locally a soft mode is large gauge transformation
- ➔ Construct the corresponding charge
- ➔ Examine the effect on the vacuum
- We expect no local effect, but globally there can be effects



Asymptotic Symmetry Group of de Sitter

- Excited de Sitter spacetimes asymptotically takes the “Fefferman-Graham” (FG) form

$$\frac{ds^2}{\ell^2} = -\frac{d\eta^2}{\eta^2} + \frac{dx^i dx^j}{\eta^2} \left(g_{ij}^{(0)} + \eta^2 g_{ij}^{(2)} + \eta^3 g_{ij}^{(3)} + \dots \right)$$

- The Einstein equation implies the relations

[Starobinsky, 1983]

$$\begin{aligned} g_{ij}^{(2)} &= R_{ij}[g^{(0)}] - \frac{1}{4}R[g^{(0)}]g_{ij}^{(0)} \\ \nabla^j g_{ij}^{(3)} &= \text{tr } g_{ij}^{(3)} = 0 , \end{aligned}$$

and also $g^{(k)}$ for $k > 3$ is determined by $g^{(3)}$ and $g^{(0)}$

- ➡ The spacetime is described by boundary metric $g^{(0)}$ and a traceless conserved tensor $g^{(3)}$.

Asymptotic Symmetry Group of de Sitter

- The asymptotic symmetry group (ASG) can be defined as the quotient group

$$\text{ASG} \equiv \frac{\text{allowed diffs.}}{\text{trivial diffs.}}$$

[Anninos, Ng and Strominger , 2010]

where a trivial diffs. leaves boundary data inv. and allowed diffs. preserves the FG form.

- The most general allowed diff. is generated by

$$\xi = \phi^i(x^k)\partial_i + \eta\delta\sigma(x^k)\partial_\eta + \frac{\eta^2}{2}g^{(0)ij}\partial_j\delta\sigma(x^k)\partial_i + \mathcal{O}(\eta^4)$$

- The $\delta\sigma$ -dependent terms are conformal transformation of the metric, corresponding to FG preserving slice transformations, and must leave physical boundary data inv.

Asymptotic Symmetry Group of de Sitter

➡ In the generally allowed diff.

$$\xi = \phi^i(x^k)\partial_i + \eta\delta\sigma(x^k)\partial_\eta + \frac{\eta^2}{2}g^{(0)ij}\partial_j\delta\sigma(x^k)\partial_i + \mathcal{O}(\eta^4)$$

only the first term corresponds to a non-trivial diff.

➡ The ASG is generated by

$$\xi^{\text{ASG}} = \phi^i(x^k)\partial_i$$

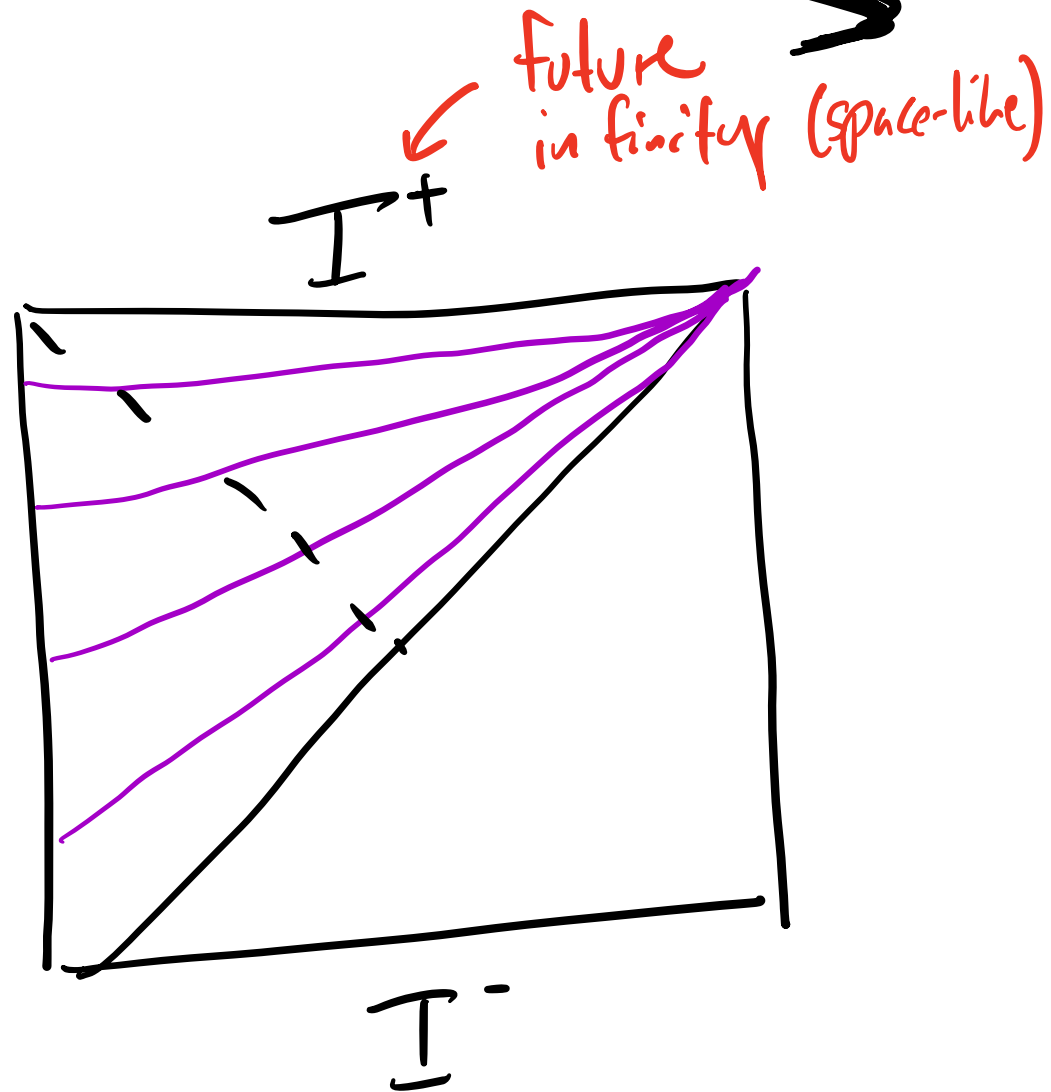
➡ The ASG is diffs. on $\mathbb{R}^3$

[Anninos, Ng and Strominger , 2010]

Asymptotic symmetries of dS

- At future infinity, I^+ , the $SO(4,1)$ inv. of dS enhanced to diffeomorphism inv.

→ Asymptotic sym. group of dS is diffeomorphisms of $\mathbb{R}^3$ (diffeo in 3 spatial dim.)



The asymptotic symmetry group

- The asymptotic symm group (ASG) is diff. in 3 dim.
- General covariance imply that the spatial indices i, j transforms as covariant & contra variant vectors at linearized level by multiplication of matrices

$$\frac{\partial x^i}{\partial \tilde{x}^j} \quad , \quad \frac{\partial \tilde{x}^j}{\partial x^i}$$

belonging to

$$GL(3, \mathbb{R}) = SL(3, \mathbb{R}) \times \mathbb{R}$$

$GL(n, \mathbb{R})$ Subgroups

Euclidean case
Lorentzian case
 $SO(n-1, 1)$

Dimension of $GL(n, \mathbb{R})$ is n^2
which can be divided into

Rotations: $n(n-1)/2$ antisymmetric generators J_{ij}
of $SO(n)$ ← Euclidean case
Lorentzian case: $SO(n-1, 1)$

Shear transformations: $n(n+1)/2 - 1$ traceless
symmetric generators S_{ij}

Dilatations: 1 generator D

Asymptotic symmetry transformation

- In the TT gauge the relevant d.o.f. are contained in γ_{ij} extracted from metric in Planar coord. (Lapse & shift decays and can be ignored here)

$$ds^2 = -dt^2 + a^2(t) [e^\gamma]_{ij} dx^i dx^j$$

- The asymptotic symmetry transf. acts on it as

$$\delta([e^\gamma]_{ij}) = \mathcal{L}_\xi([e^\gamma]_{ij})$$

$$\Rightarrow \delta \gamma_{ij} = \partial_i \xi_j + \partial_j \xi_i - \frac{2}{3} \partial \cdot \xi \delta_{ij} + \mathcal{O}(\gamma)$$

Tensor adiabatic mode

$$\delta\gamma_{ij} = \partial_i \xi_j + \partial_j \xi_i - \frac{2}{3} \partial \cdot \xi \delta_{ij} + \mathcal{O}(\gamma)$$

$\Rightarrow$ Diffs that satisfy

$$\partial_i \xi_j + \partial_j \xi_i - \frac{2}{3} \partial \cdot \xi \delta_{ij} = 0$$

cannot generate const. adiabatic mode
of $\delta\gamma$

- This eq. is solved by the conformal Killing vector on $\mathbb{R}^3$

$$\xi_i = c_i + \omega_{ij} x^j + c x_i + (x^2 a_i - 2x_i (\bar{x} \cdot \bar{a}))$$

$\nearrow$ translations $\nearrow$ $\omega_{ij} = -\omega_{ji}$ rotations $\nearrow$ dilations $\nearrow$ conformal boost

Tensor adiabatic mode

$\Rightarrow$ out of all generators of $GL(3)$ only symmetric ones can generate $\delta\gamma$

$$S_{ij} = x_i \partial_j + x_j \partial_i - \frac{2}{3} \delta_{ij} \partial \cdot x$$

$$\Rightarrow \quad \delta\gamma_{ij} = S_{ij} \quad \text{for} \quad \xi_i = S_{ij} x^j \quad (S_{ij} = S_{ji})$$

We will think of S_{ij} as the long wavelength adiabatic mode

$$\delta\gamma_{ij} = \gamma_{ij}^L = \langle \gamma_{ij}(x) \rangle, \quad S_{ij} \equiv \gamma_{ij}^L, \quad \xi_i = \gamma_{ij}^L x^j$$

[Ferreira, Sandora, Slot 16]

[Slot, 2025]

Symmetry breaking Pattern

##

- We have the breaking

$$\begin{array}{ccccc}
 & & GL(n) & \supset & SO(n) \times D \\
 & \nearrow & & & \nwarrow \\
 n^2 & & & & n(n-1)/2
 \end{array}
 \begin{array}{l}
 (n=3 \Rightarrow n^2=9) \\
 (n=3 \Rightarrow n(n-1)/2 = 3)
 \end{array}$$

$\Rightarrow$ we expect

$$\begin{array}{ccccc}
 GL(n) & SO(n) & & D & \\
 n^2 - (n(n-1)/2) - 1 & = & n(n+1)/2 - 1 & &
 \end{array}$$

$$n=3: \quad 9 - 3 - 1 = 5$$

Inverse Higgs constraint

#####

- Out of the $n(n+1)/2 - 1$ modes, n can be removed by local translations

$$\xi_i = C_i(x) \text{ with } \partial_i C_j(x) + \partial_j C_i(x) - \frac{2}{3} \delta_{ij} \partial \cdot C = -S_{ij}$$

leaving

$$-\gamma_{ij}^L = -\delta \gamma_{ij}^L$$

$$n(n+1)/2 - 1 - n = n(n-1)/2 - 1$$

independent modes

$\Rightarrow$ # Goldstones = # Graviton d.o.f
in $D=1+n$ dim. !

Symmetry breaking Pattern

- Order parameter = traceless symmetric tensor

(like a nematic Liquid Crystal but symmetry breaking pattern different)

- Long range fluctuations of order parameter = Goldstone mode
= Adiabatic/soft graviton

⇒ Adiabatic/soft mode is the Goldstone mode of the spontaneously broken asymptotic symmetries!!!

Soft Charges

- The Noether charge of the asymptotic S.B. sym. is

$$Q \propto \int d^3x \dot{\gamma}_{ij} \delta \gamma_{ij}$$

$$\Rightarrow [Q, \gamma_{ij}] = \delta \gamma_{ij}$$

$$\delta \gamma_{ij} = -\gamma_{ij}^L - \frac{1}{2} \gamma_{ab}^L x^a \partial^b \gamma_{ij} + \text{h.o.}$$

$$\Rightarrow \langle 0 | [Q, \gamma_{ij}] | 0 \rangle = -\gamma_{ij}^L + \text{h.o.}$$

$$\Rightarrow |0'\rangle = e^{iQ} |0\rangle \text{ is vacuum in presence of the long mode } \gamma_{ij}^L$$

[Ferreira, Souda, Sloth: '16]

From here we have as
a self-consistency check

$$\langle 0' | \chi_{ij} | 0' \rangle = \langle 0 | e^{-iQ} \chi_{ij} e^{iQ} | 0 \rangle$$

$$= \langle 0 | \cancel{\chi_{ij}} | 0 \rangle + \langle 0 | [Q, \chi_{ij}] | 0 \rangle$$

$$= \langle 0 | [Q, \chi_{ij}] | 0 \rangle = \delta \chi_{ij} = -\chi_{ij}^2$$

$\Rightarrow$

$$\langle 0' | \chi_{ij} | 0' \rangle = -\chi_{ij}^2$$

Goldstone Lagrangian

- We have seen that the soft gravitons, γ^{μ} , are Goldstone modes of spontaneously broken asymptotic symmetries

- Lagrangian of the Goldstone modes are of the General form

$$\mathcal{L}(\pi) \propto g_{ab} \partial_{\mu} \pi^a \partial^{\mu} \pi^b + \mathcal{O}(\partial^4)$$

where g most general metric inv. under the isometries of broken group G
[Naegels, 2021]

- In our case $G = \text{diffeos of } \mathbb{R}^3 \Rightarrow g_{ab} \sim \delta_{ij}$

Goldstone Lagrangian

$\Rightarrow$ Lagrangian of soft gravitons in de Sitter

$$\mathcal{L}(\gamma) \propto -\dot{\gamma}_{ij}(x)\dot{\gamma}_{ij}(x) + a^{-2}\partial_\ell\gamma_{ij}(x)\partial_\ell\gamma_{ij}(x)$$

$\Rightarrow$ A different way of understanding the quadratic action of metric perturbations in de Sitter

Symmetry breaking:

Local vs. global observers

- We can now define the symmetric unbroken state as

$$|\varphi_{\text{sym}}\rangle = \frac{1}{\mathcal{N}} \int \mathcal{D}\gamma_{ij}^L \varphi[\gamma_{ij}^L] |\gamma^L\rangle \quad (|\gamma^L\rangle \equiv |0^L\rangle)$$

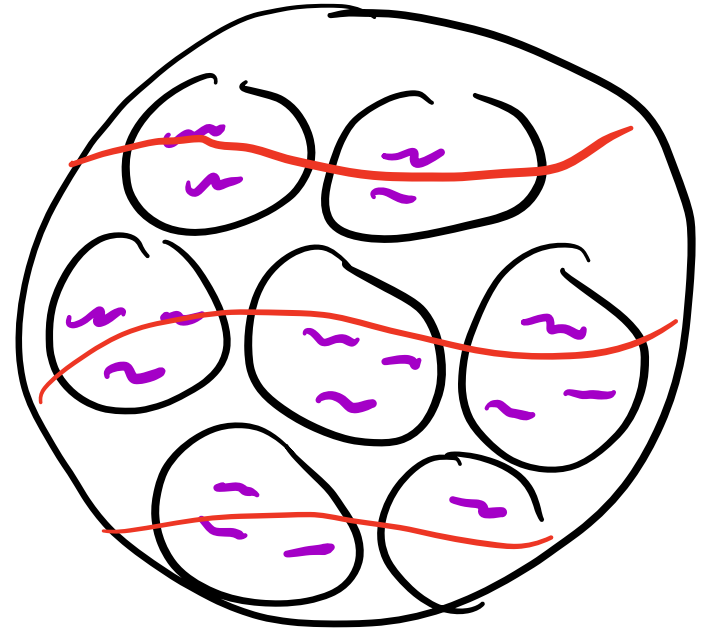
if $\varphi[\gamma_{ij}^L]$ even in $\gamma_{ij}^L \rightarrow -\gamma_{ij}^L$

$$\Rightarrow \langle \varphi_{\text{sym}} | \gamma_{ij}(x) | \varphi_{\text{sym}} \rangle = 0$$

$$\langle \gamma^L | \varphi_{\text{sym}} \rangle = \varphi[\gamma_{ij}^L] \quad \text{soft part of} \\ \text{wave function of the} \\ \text{universe}$$

Local vs. global observers

- Now imagine dividing into many uncorrelated patches except for along the long range order parameter



$$\Rightarrow \Psi_{\gamma_L}(x_1, x_2, \dots) \approx \prod_{i=1}^N \psi_{\gamma_L}(x_i)$$

- using $|\langle \psi_{\gamma_L}(x_i) | \psi_{\gamma'_L}(x_i) \rangle| = |\epsilon_{\gamma_L \gamma'_L}| < 1$
and patches highly uncorr. except for order par.

$$\Rightarrow \langle \Psi_{\gamma_L} | \Psi_{\gamma'_L} \rangle \approx \prod_{i=1}^N \langle \psi_{\gamma_L}(x_i) | \psi_{\gamma'_L}(x_i) \rangle \propto (\epsilon_{\gamma_L \gamma'_L})^N \rightarrow 0$$

for $N \rightarrow \infty$

Local vs. global observers

- Similarly for any finite combination of local observers of the form $\mathcal{O}(x_i)$, we have

$$\langle \Psi_{x'_L} | \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \dots | \Psi_{x_L} \rangle \propto (\epsilon_{x'_L x_L})^{N-n} \prod_{i=1}^n \langle \mathcal{O}_i \rangle \rightarrow 0$$

for $x'_L \neq x_L$ and $N \rightarrow \infty$

- $\Rightarrow$ Local operators approx. diag. in $|\Psi_{x_L}\rangle$ basis
- $\Rightarrow$ Local measurement in $|\Psi_{\text{sym}}\rangle$ state will project onto a $|\Psi_{x_L}\rangle$
- $\Rightarrow$ Local measurements (incl. environment effects) spontaneously break asymptotic sym.

Local vs. global observers

- If one compute the ensemble average of γ_L in the local states one will find that it vanish $\langle \gamma_L \rangle_{\text{ens.}} = 0$

$\Rightarrow$ local state well described by the symmetric state $|\Psi_{\text{sym}}\rangle$

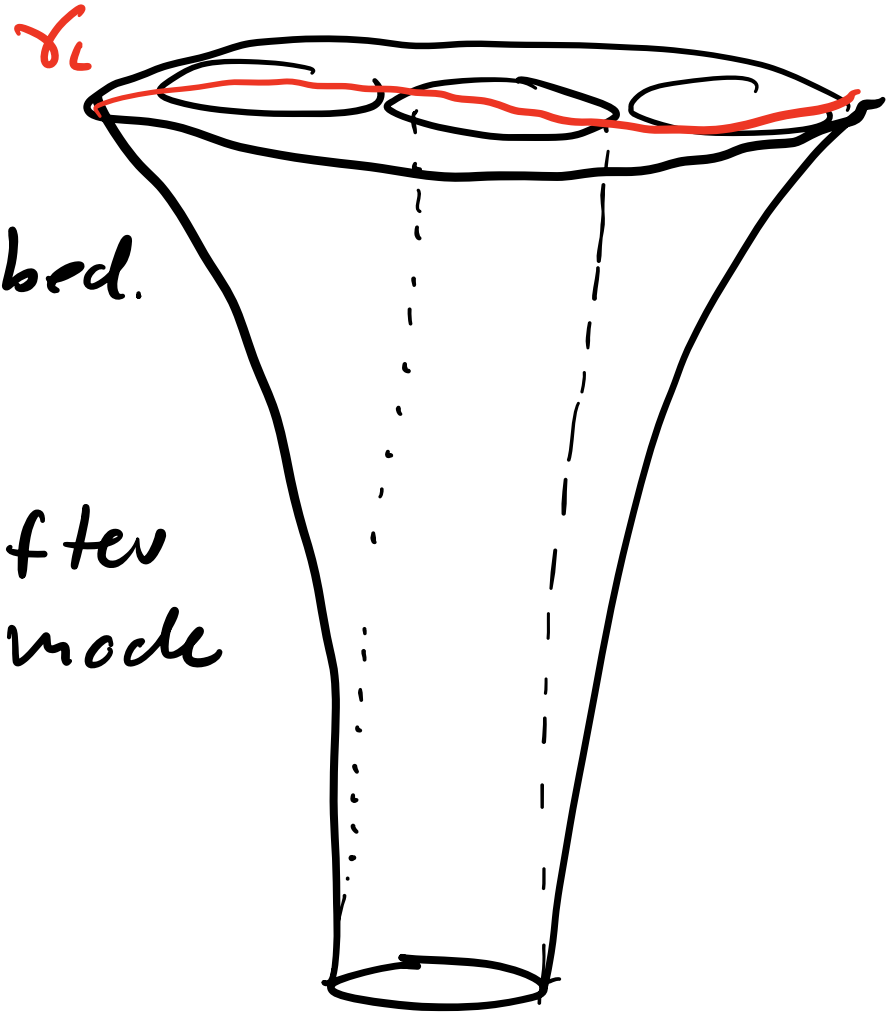
$\Rightarrow$ Local state do not depend on γ_L and it has no physical meaning locally

- But globally system is in broken state $|\Psi_{\gamma_L(x)}\rangle$ with long range correlation!

Local vs. global observers

- When comparing different patches physical effect is probed.

(like in late universe after reheating when long mode enter horizon)



inflation

and spontaneously broken dilatation

- In case of inflation, also time-translation is broken
- $\Rightarrow$ Scalar curvature perturbation, $\mathcal{S}$, in addition to two polarizations of grav. (trace part of metric)
- $\Rightarrow \delta(e^{2\mathcal{S}}[e^\gamma]_{ij}) = \mathcal{L}_\xi(e^{2\mathcal{S}}[e^\gamma]_{ij})$
- $\Rightarrow \delta\mathcal{S} = \frac{1}{3}\partial \cdot \xi + \text{h.o.}$
- $\Rightarrow$ dilatation $\xi = c x^i$ induce constant shift $\delta\mathcal{S} = c$

inflation

and spontaneously broken dilatation

$\Rightarrow$ we can think of c as soft mode of the curvature pert. and take

$$\xi_L^i = \xi_L x^i$$

so that $\delta \xi = \xi_L$

$\Rightarrow$ soft ξ modes are Goldstone modes of spontaneously broken Dilatation Sym.

Goldstone Lagrangian

$\Rightarrow$ Just like before, we can write down the Goldstone Lagrangian

$$\mathcal{L}(\zeta) \propto -\dot{\zeta}^2 + a^{-2}(\partial\zeta)^2$$

which agrees with the quadratic action for ζ in cosmology.

$\Rightarrow$ explains why action of cosmological perturbations looks particularly simple in comoving gauge.

Checks

The 3pt function

- To check that our explicit construction of the Hubble charge makes sense, let's consider

$$\begin{aligned}\langle 0' | \gamma_L \gamma_S \gamma_S | 0' \rangle &= \langle 0 | e^{-iQ_H} \gamma_L \gamma_S \gamma_S e^{iQ_H} | 0 \rangle \\ &= \langle 0 | \gamma_L \gamma_S \gamma_S | 0 \rangle - i \langle 0 | [Q_H, \gamma_L \gamma_S \gamma_S] | 0 \rangle + \dots\end{aligned}$$

- ➔ An explicit evaluation of the commutator reproduces the Maldacena consistency relation (2001)

$$\langle 0 | [Q_H, \gamma_L \gamma_S \gamma_S] | 0 \rangle = \langle \gamma_L \gamma_L \rangle k^2 \frac{\partial}{\partial k^2} \langle \gamma_S \gamma_S \rangle$$

Note

γ_L = "Long" or "soft" mode

γ_S = "Short" or "UV" mode

[Ferreira, Sandora and MSS, 2016]

Q_H : Hubble charge $\leftarrow$ UV cutoff Hubble scale

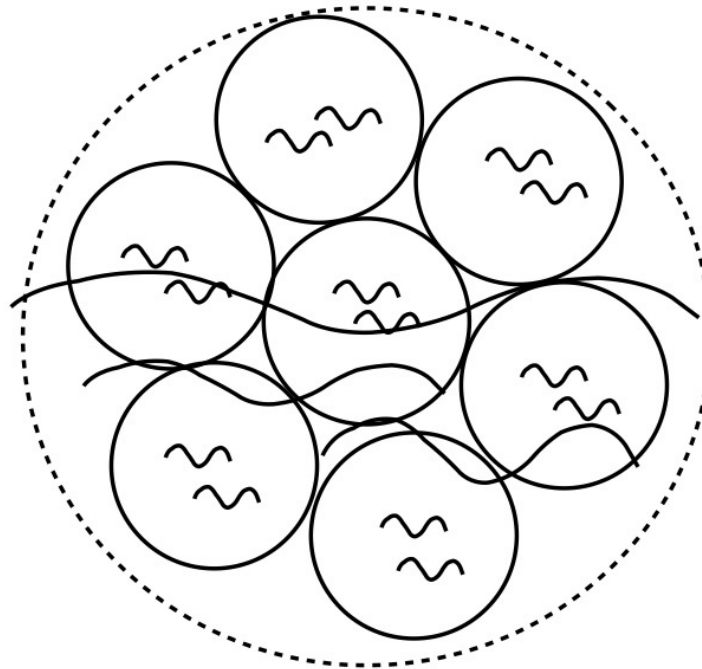
Checks

One-loop

- We can also check the IR one-loop corrections

$$\langle 0' | \gamma_S \gamma_S | 0' \rangle = \langle 0 | e^{-iQ_H} \gamma_S \gamma_S e^{iQ_H} | 0 \rangle$$

- ➔ An explicit evaluation and averaging over soft modes reproduces the Giddings & MSS one-loop result (2012)



[Ferreira, Sandora and MSS, 2016]

The overlap

- One diagnostic of a perturbative breakdown in Black Holes, is the overlap

$$\langle \psi(t) | \psi \rangle_0$$

where $|\psi\rangle_0$ is the unperturbed state, and the time-dependent perturbed state is

$$|\psi(t)\rangle = T \exp \left\{ -i \int^t dt H_I(t) \right\} |\psi\rangle_0$$

- Suppose that at some time

$$\langle \psi(t) | \psi \rangle_0 = 0$$

➡ Perturbation theory has broken down since

$$\langle \delta\psi | \psi \rangle_0 \approx -1$$

- ✓ Effects of perturbations are order unity

The overlap

- We have computed the average overlap

$$\langle 0' | 0 \rangle = \langle 0 | e^{-iQ_H} | 0 \rangle$$

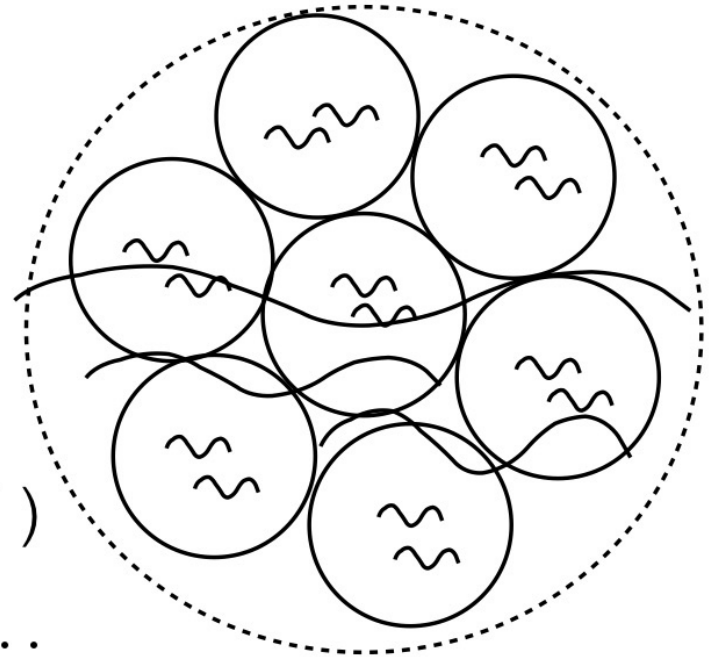
in the larger box

- ➡ The overlap correction grows with the size of the box ($L = aH$)

$$\langle 0' | 0 \rangle = 1 - \alpha H^2 \log(a) + \dots$$

- ✓ The overlap receives order one corrections on the the time scale

$$t_{ds} \sim R_{ds} S_{ds} \sim 1/H^3$$



Page time of de Sitter

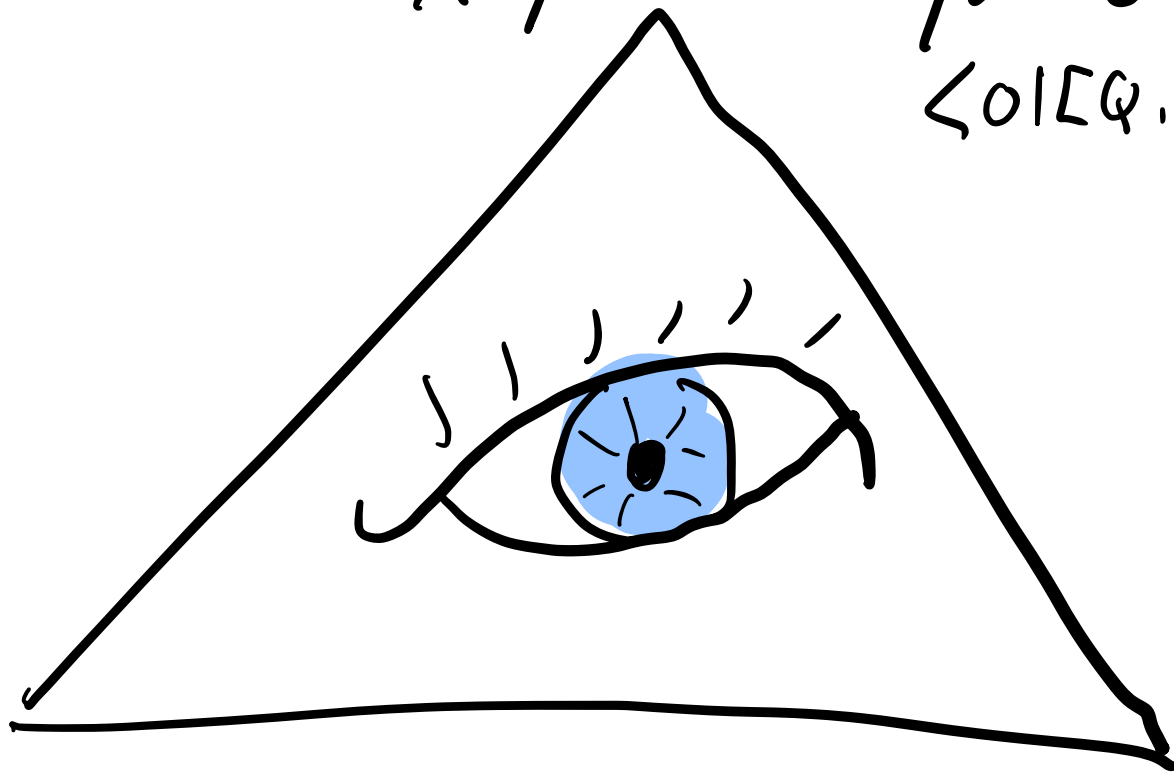
- The Page time of de Sitter is
$$t_{ds} \sim R_{ds} S_{ds} \sim 1/H^3$$

$\Rightarrow$ The Page time of de Sitter is the time scale on which the symmetric state is no longer a good description and instead we are probing the broken state.

Conclusion

spontaneously broken
Asymptotic symmetries

$$\langle 0 | [Q, \gamma_i] | 0 \rangle = \delta \gamma$$



Soft
theorems

$$\langle 0' | \gamma_L \gamma_R | 0' \rangle = \langle 0 | e^{iQ} \gamma_L \gamma_R e^{iQ} | 0 \rangle$$

Gravitational
memory
(long range
order
parameter)

Outlook

- CFT dual description
- Massive gravity
- EFT of tensor pert.
- de Sitter Liquid crystal ... ?