

Probing the Pre-BBN Universe with Gravitational Waves

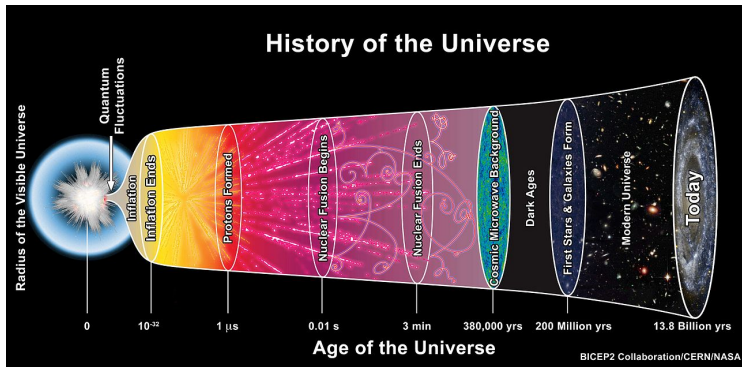
Yong Xu, MITP (JGU Mainz)

Theory Seminar, University of Oslo

January 10, 2025



History of the Universe

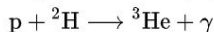
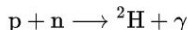


History of the Universe: several key events

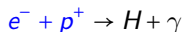
- $t \sim 10^{-6}$ s: **QCD deconfinement transition** with $T \sim 150$ MeV

quark gluon soup $\Rightarrow p, n$

- $t \sim 1$ s – 200 s: **Big Bang Nucleosynthesis** with $\text{MeV} \gtrsim T \gtrsim 50$ keV



- $t \sim 380$ kyrs: **Recombination** $\Rightarrow$ neutral atom with $T \sim 0.3$ eV

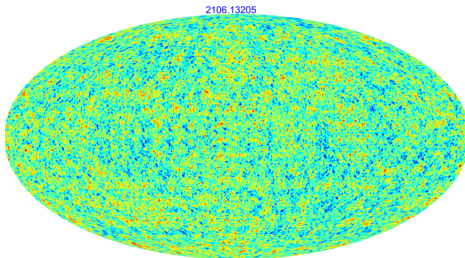


Cosmic Microwave Background (CMB)

- **Before** recombination

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

- **After** recombination: γ became free



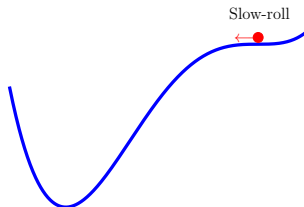
- Questions:

- 1 **Why** looks so **homogeneous**?

Universe so big $\Rightarrow$ causally disconnected $\Rightarrow$ Horizon problem

- 2 What is the **origin of the tiny inhomogeneities**?

Cosmic Inflation



- Cosmic inflation (**exponential expansion**)

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho(\phi)}{3M_P^2} \Rightarrow H = \sqrt{\frac{V(\phi)}{3M_P^2}} \Rightarrow a = e^{Ht} \Rightarrow \text{small to BIG}$$

e.g. Polynomial Inflation $v(\phi) = d[\phi^4 - 8/3\phi_0(1-\beta)\phi^3 + 2\phi_0^2\phi^2]$ [Drees, YX 2104.03977, 2209.07545]

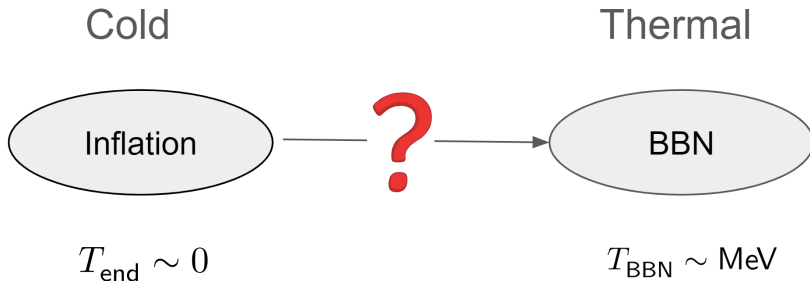
- Quantum fluctuation

$\delta\phi \Rightarrow$ CMB inhomogeneities and seeds for structure

Question

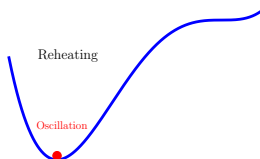
Inflation dilutes everything exponentially

- After inflation: the Universe is **cold and empty**
- However, Big Bang Nucleosynthesis needs a **Thermal** background



Reheating

- A theory describing how inflaton ϕ energy $\implies$ thermal background



- Basic ingredients:
 - Couplings $\mu \phi h^2$, $y \phi \bar{\psi} \psi$
 - Oscillating $\phi \implies$ particles production $\implies$ interaction $\implies$ thermal bath
 $\implies$ define temperature
 - Complex with non-perturbative phenomena [Dolgov, Kirilova '89] [Traschen, Brandenberger '90]
[Kofman, Linde, Starobinsky '97]...

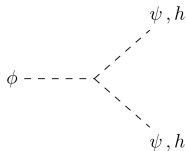
Temperature during reheating is important!

- Enlarge **WIMP** parameter space [Bernal, YX [2209.07546](#)]
- New source of **Leptogenesis** [Drees, YX [2401.02485](#)]
- Double peaked GW from **Phase Transition** [Buen-Abad, Chang, Hook [2305.09712](#)]
- ...

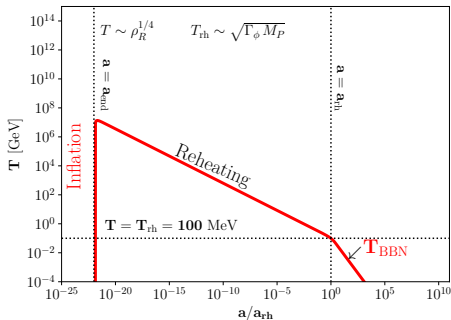
But, what controls the evolution of temperature?

Evolution of Background

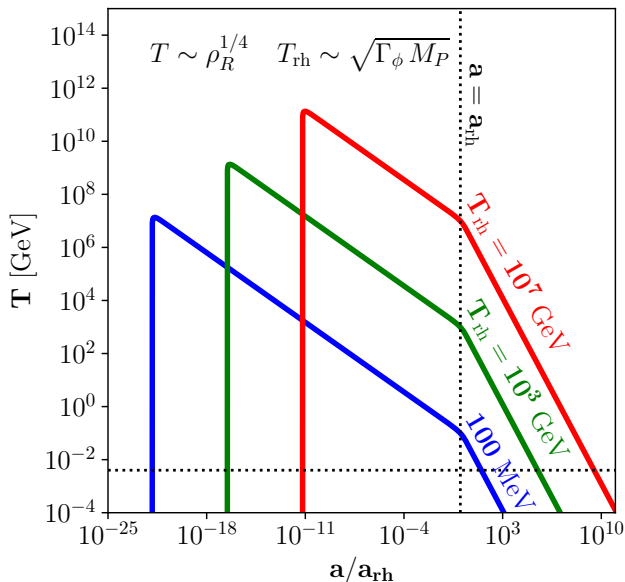
- Shape inflaton potential $V(\phi) \sim m_\phi^2 \phi^2$
- Couplings $\mu \phi h^2, y \phi \bar{\psi} \psi \implies \Gamma_\phi \sim \mu^2/m_\phi, y^2 m_\phi$



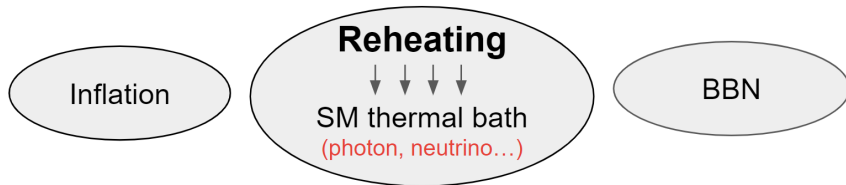
$$\frac{d\rho_R}{dt} + 4H\rho_R = +\Gamma_\phi \rho_\phi$$



Evolution of Background for different T_{rh}



How To Probe Reheating?



transparent to GW

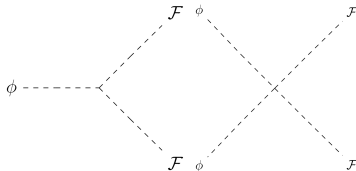
- Probing Reheating with GWs!

Goal of the rest part of this talk

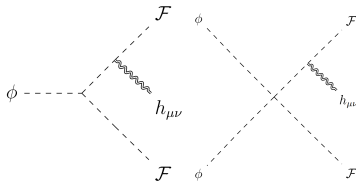
- 1 Show a **simple and unavoidable source of GWs** via graviton Bremsstrahlung
- 2 Demonstrate how such GW spectrum can help to **probe reheating**

Graviton Bremsstrahlung during Reheating

- If, reheating: $\phi\mathcal{F}\mathcal{F}$ (decay) or $\phi^2\mathcal{F}\mathcal{F}$ (annihilation)



- Then, graviton production



gravitons emission $\implies$ propagation $\implies$ SGWB $\implies$ Probe Reheating!

[Nakayama, Tang [1905.08510](#)]
[Barman, Bernal, YX, Zapata [2301.11345](#)]
[Bernal, Cléry, Mambrini, YX [2311.12694](#)]

Graviton Production: a mini review

- The action

$$S \supset \int d^4x \sqrt{-g} (\mathcal{L}_{\text{EH}} + \mathcal{L}_\phi + \mathcal{L}_{\text{int.}})$$

- Inflaton sector

$$\mathcal{L}_\phi = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

- Interaction part

$$\mathcal{L}_{\text{int.}} \supset \mu \phi |\varphi|^2 + \lambda \phi^2 |\varphi|^2 + y \phi \bar{\psi} \psi$$

- Expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + \left(h_{\mu\nu} \frac{2}{M_P} \right) \Leftarrow \begin{cases} (\dots) & \text{pert. in linearized GR} \\ h_{\mu\nu} & \text{a dynamical field in field theory} \end{cases}$$

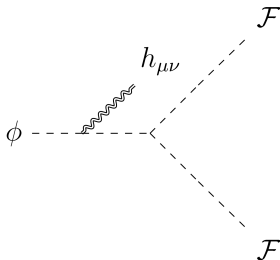
- Gravitational vertex

$$\sqrt{-g} \mathcal{L} \supset -\frac{1}{M_P} \cdot h_{\mu\nu} \cdot \sum_{\phi, \psi, \varphi} T^{\mu\nu}$$

$$\text{e.g. } T_\phi^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - \frac{1}{2} \eta^{\mu\nu} (\partial^\alpha \phi \partial_\alpha \phi - m_\phi^2 \phi^2)$$

Graviton Production: Example with $\mathcal{F} = \varphi$

- For $\phi(\ell) \rightarrow \mathcal{F}(p)\mathcal{F}(q)h_{\mu\nu}(\omega)$



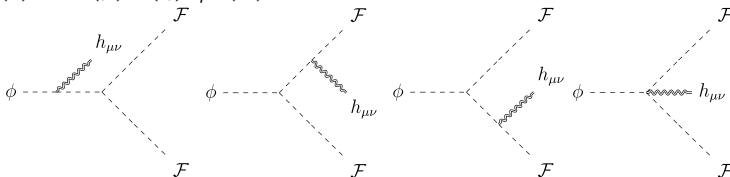
- Matrix element:

$$\begin{aligned} i\mathcal{M}_1 &= (i\mu) \times \frac{i}{(\ell - \omega)^2 - m_\phi^2} \times \frac{-i}{M_P} \left[2\ell_\mu(\ell - \omega)_\nu - \eta_{\mu\nu}(\ell \cdot (\ell - \omega) - m_\phi^2) \right] \times \epsilon^{\mu\nu*} \\ &= -i \frac{\mu}{M_P} \frac{\ell_\mu \ell_\nu}{\ell \cdot \omega} \end{aligned}$$

where $\omega^2 = m_g^2 = 0$

Graviton Production: Example with $\mathcal{F} = \varphi$

- For $\phi(\ell) \rightarrow \mathcal{F}(p)\mathcal{F}(q)h_{\mu\nu}(\omega)$



- Diagram 1:

$$i\mathcal{M}_1 = -i \frac{\mu}{M_P} \frac{\ell_\mu \ell_\nu}{\ell \cdot \omega} \epsilon^{\mu\nu*} \rightarrow 0!$$

- Diagram 2:

$$i\mathcal{M}_2 = i \frac{\mu}{M_P} \frac{p_\mu p_\nu}{p \cdot \omega} \epsilon^{\mu\nu*}$$

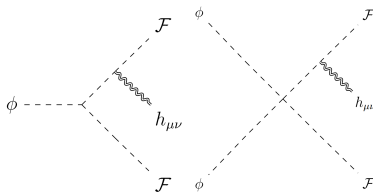
- Diagram 3:

$$i\mathcal{M}_3 = i \frac{\mu}{M_P} \frac{q_\mu q_\nu}{q \cdot \omega} \epsilon^{\mu\nu*}$$

- Diagram 4:

$$i\mathcal{M}_4 \propto \eta_{\mu\nu} \epsilon^{\mu\nu*} = \epsilon_\mu^\mu \rightarrow 0$$

Graviton Production Rate



$$\frac{d\Gamma}{dE_g} \equiv \frac{\int dE |\mathcal{M}|^2}{64\pi^3 m_\phi} \simeq \begin{cases} \frac{1}{64\pi^3} \left(\frac{\mu}{M_P}\right)^2 \frac{(1-2x)^2}{x} & \text{bosonic decay} \\ \frac{y^2}{64\pi^3} \left(\frac{m_\phi}{M_P}\right)^2 \frac{(1-2x)[2x(x-1)+1]}{x} & \text{fermionic decay} \\ \frac{\lambda^2}{8\pi^3} \frac{\rho_\phi}{m_\phi^2 M_P^2} \frac{(1-x)^2}{x} & \text{bosonic annihilation} \end{cases}$$

• $x = \frac{E_g}{m_\phi}$: $x \rightarrow 0 \implies \frac{d\Gamma}{dE_g} \rightarrow \infty$ (soft divergence)

[Nakayama, Tang [1810.04975](#)]
 [Barman, Bernal, **YX**, Zapata [2301.11345](#)]
 [Bernal, Cléry, Mambrini, **YX** [2311.12694](#)]

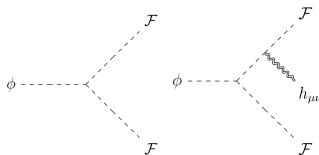
GW Spectrum

- GW amplitude:

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d\ln f} = \Omega_R^0 \frac{d(\rho_{\text{GW}}/\rho_R)}{d\ln f} \sim \Omega_R^0 \frac{d(\rho_{\text{GW}}/\rho_R)}{d\ln E_g}$$

- Differential spectrum

$$\frac{d(\rho_{\text{GW}}/\rho_R)}{dE_g} \propto \left(\frac{d\Gamma}{dE_g} \frac{1}{\Gamma_\phi} \right) \times \left(\frac{E_g}{m_\phi} \right) \sim (\text{differential BR}) \times (\text{energy fraction})$$



- Recall that

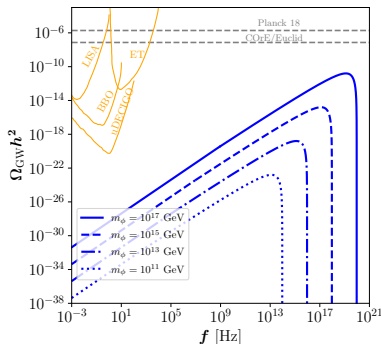
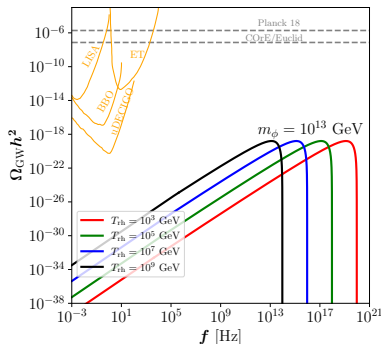
$$\Gamma_\phi \propto \frac{\mu^2}{m_\phi} \quad \text{and} \quad \frac{d\Gamma}{dE_g} \propto \frac{\mu^2}{M_p^2} \frac{m_\phi}{E_g}$$

- Expect

$$\Omega_{\text{GW}} \sim \frac{d(\rho_{\text{GW}}/\rho_R)}{d\ln E_g} \propto \frac{(E_g \times m_\phi)}{M_p^2}$$

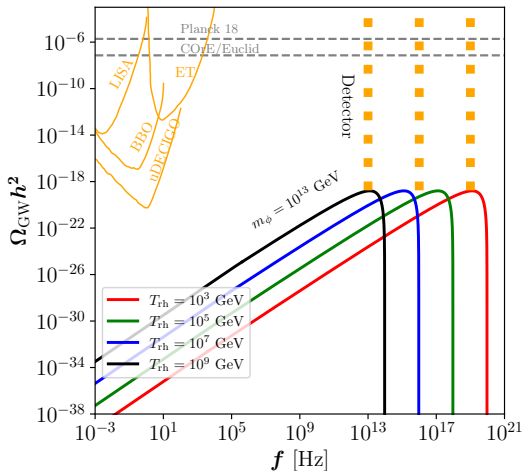
Bremsstrahlung GWs

$$\Omega_{\text{GW}} h^2 \sim 10^{-18} \left(\frac{m_\phi}{10^{13} \text{ GeV}} \right) \left(\frac{T_{\text{rh}}}{10^{13} \text{ GeV}} \right) \left(\frac{f}{10^9 \text{ Hz}} \right)$$



- Peak frequency: $f_{\text{peak}} \simeq \frac{m_\phi}{2} \frac{1}{2\pi} \frac{a_{\text{rh}}}{a_0} \simeq 10^{10} \left(\frac{m_\phi}{10^{13} \text{ GeV}} \right) \left(\frac{10^{13} \text{ GeV}}{T_{\text{rh}}} \right) \text{ Hz}$
- At the peak: $\Omega_{\text{GW}} \propto \left(\frac{m_\phi}{10^{13} \text{ GeV}} \right)^2$

Probing Reheating (m_ϕ and T_{rh}) with Bremsstrahlung GWs

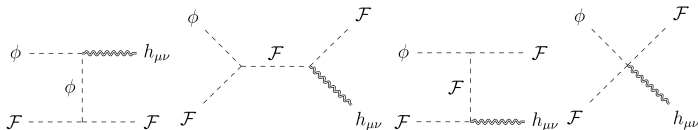


- If null signal at a frequency $\implies m_\phi \gtrsim 10^{13}$ GeV with a specific T_{rh} ruled out

[YX, 2407.03256 JHEP 10 (2024) 174]

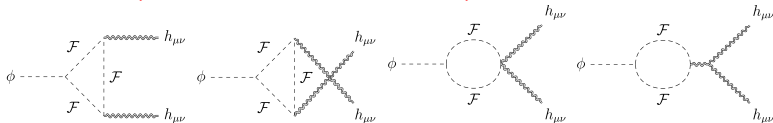
Inflaton Scattering and Decay

- So far: $\phi \rightarrow \mathcal{F}\mathcal{F}h_{\mu\nu}$
- Inflaton scattering with decay product



$$\Gamma_g^{2 \rightarrow 2} \sim \Gamma_g^{1 \rightarrow 3} \times \left(\frac{T}{m_\phi} \right)^n$$

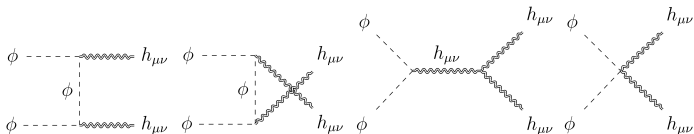
- Inflaton decays (possible only at 1-loop level!) in Einstein-Hilbert framework



$$\Gamma_g^{1 \rightarrow 2} \propto \frac{m_\phi^3 \mu^2}{M_P^4}$$

Pure Inflaton Scattering

- Inflaton Inflaton

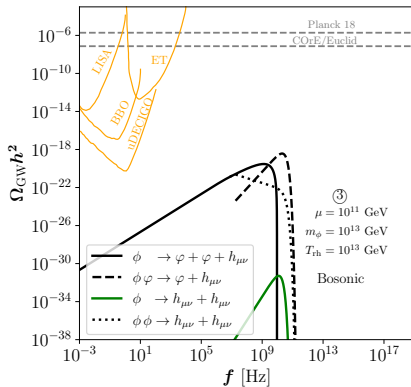
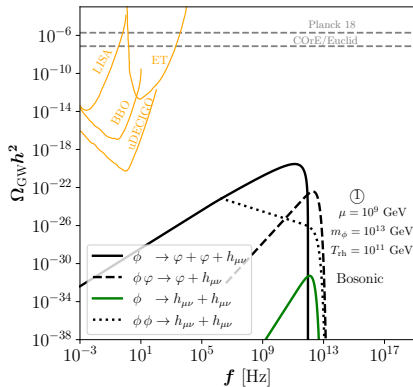


$$\Gamma_g^{2 \rightarrow 2} = \frac{\rho_\phi}{m_\phi} \frac{m_\phi^2}{32\pi M_P^4} = \frac{\rho_\phi m_\phi}{32\pi M_P^4}$$

[Ema, Jinno, Nakayama [2006.09972](#)]

[Choi, Ke, Olive, [2402.04310](#)]

Comparison

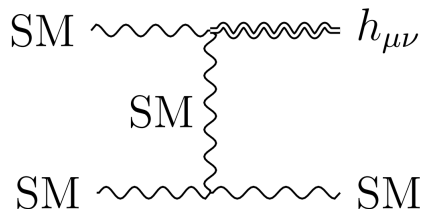


- Left $m_\phi \gg T_{\text{rh}}$: 1 $\rightarrow$ 3 dominates
- Right $m_\phi \lesssim T_{\text{rh}}$: 2 $\rightarrow$ 2 dominate

[YX, 2407.03256 JHEP 10 (2024) 174]

Pure Thermal Scatterings

- Graviton from SM thermal plasma



- Graviton production rate

$$\Gamma_g \sim n \times \sigma \sim \frac{T^3}{M_P^2}$$

- Graviton energy:

$$E_g \sim T \implies E_0 = T \frac{a}{a_0} \sim T_0 \sim \mathcal{O}(10)\text{GHz}$$

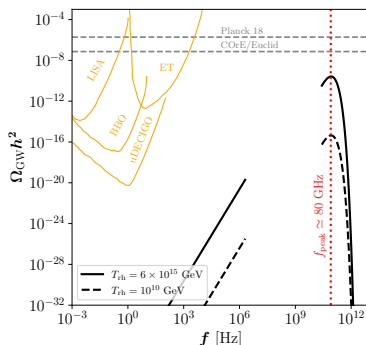
[Ghiglieri, Laine [1504.02569](#)]

[Ghiglieri, Jackson, Laine, Zhu, [2004.11392](#)]

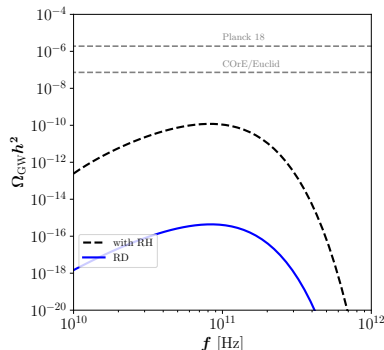
[Ringwald, Schütte-Engel, Tamarit [2011.04731](#)]

Thermal GWs during Reheating

- Left: after reheating



- Right: during reheating [Bernal, YX 2410.21385]



- A very interesting application:

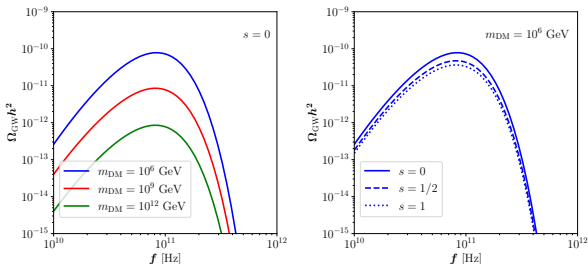
“Probing Gravitational DM with Thermal GWs” [YX, 2412.21137]

Probing “Nightmare” Dark Matter with Thermal GWs

- A minimal scenario: DM with pure gravity interaction $\implies$ Nightmare
- Unavoidableogenesis of DM and GW



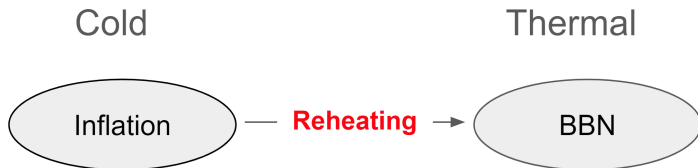
$$\Omega_{\text{GW}} h^2 \sim 8.6 \times 10^{-11} \left(\frac{\Omega_{\text{DM}} h^2}{0.12} \right) \left(\frac{f}{10^{11} \text{ Hz}} \right)^3 \left(\frac{\alpha}{3 \cdot 10^{-3}} \right)^{-1/3} \left(\frac{m_{\text{DM}}}{10^9 \text{ GeV}} \right)^{-1/3}$$



[YX, 2412.21137]

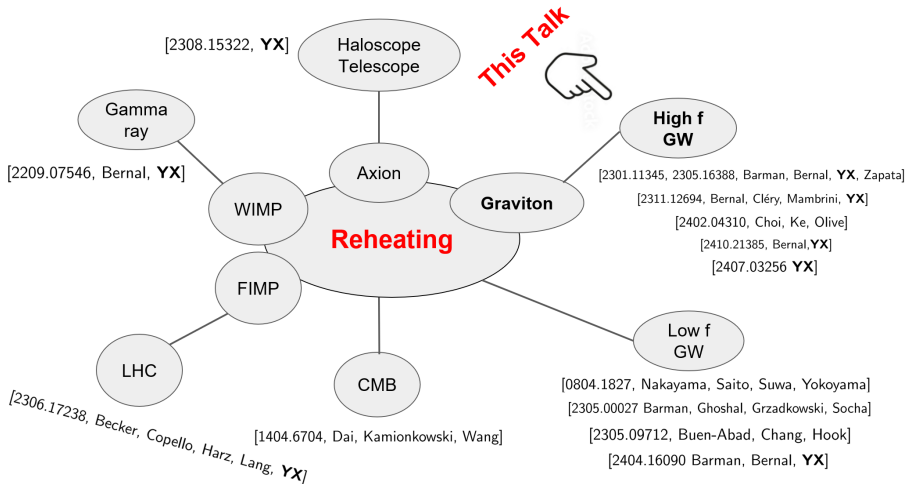
- Probing WIMP with GW from PT [Bringmann et al., 2311.06346]
- Other scenarios also possible depending on the source of GW and DM

Summary



- **Reheating** explains Cold $\rightarrow$ Thermal (**important for many phenomena**)
- Two important parameters: m_ϕ and T_{rh}
- $m_\phi \gg T_{\text{rh}} \implies$ Bremsstrahlung GWs dominate $\implies$ **probe reheating**
- Large $T_{\text{rh}} \implies$ Thermal GWs dominate $\implies$ **probe pure gravitational DM**

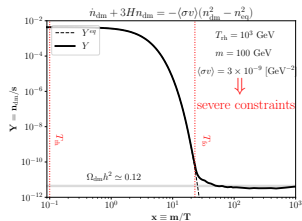
Probing **Pre-BBN**: Astro-Cosmo-Collider Synergy



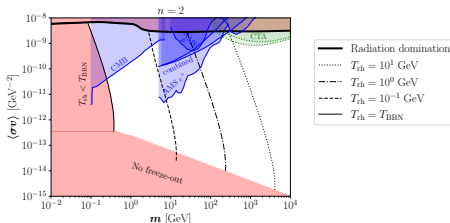
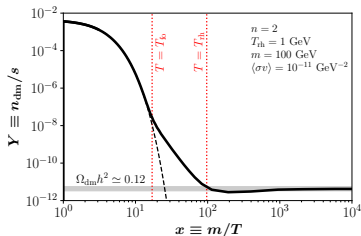
Thanks for your attention!

Temperature is important: WIMP Freeze-out

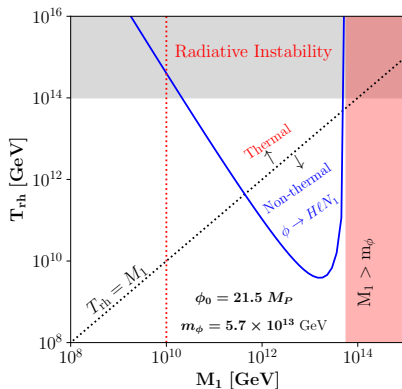
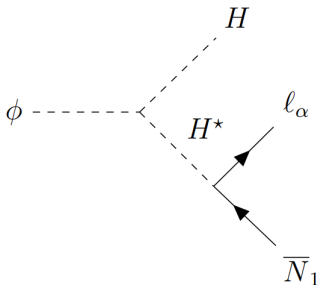
- Freeze-out after reheating $T_{\text{fo}} < T_{\text{rh}} \implies$



- Freeze-out during reheating with $T_{\text{fo}} > T_{\text{rh}} \implies$ **much smaller $\langle \sigma v \rangle$** $\implies$ **evade experimental constraints**



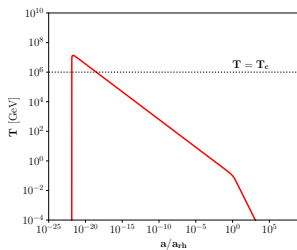
Temperature is important: Leptogenesis



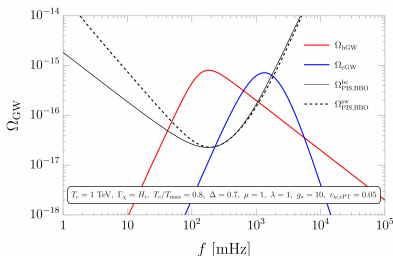
- $T_{\text{rh}} > M_1 \implies$ **thermal** leptogenesis
- $T_{\text{rh}} < M_1 \implies$ **non-thermal** leptogenesis $\phi \rightarrow H\ell N$

Temperature is important: Phase Transition

- Possible to have $T_c < T_{\max}$



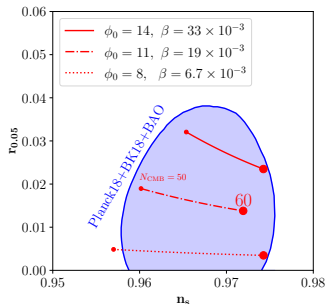
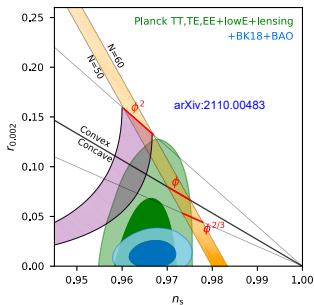
- Double PT $\implies$ double peaked GW [Buen-Abad, Chang, Hook [2305.09712](#)]



Polynomial Inflation

- Monomial Chaotic Inflation $V(\phi) \sim \phi^p$ has been ruled out
- A General and Renormalizable Potential

$$V(\phi) = b\phi^2 + c\phi^3 + d\phi^4 \sim d[\phi^4 - 8/3\phi_0(1-\beta)\phi^3 + 2\phi_0^2\phi^2]$$

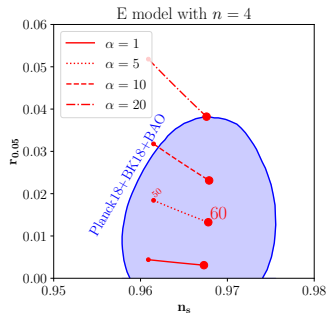
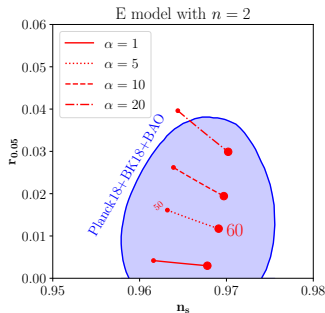


- During reheating: $V(\phi) \sim \phi^2$

Alpha Attractor Inflation

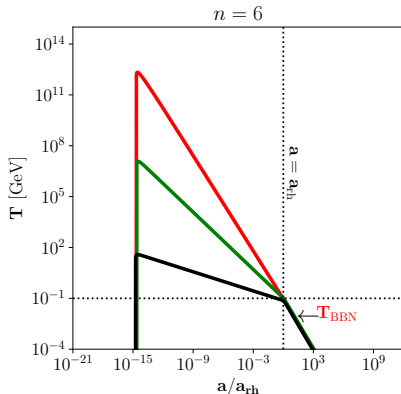
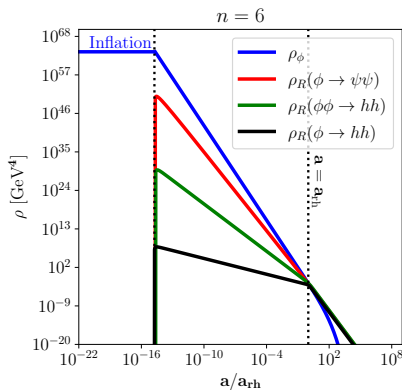
- The E model [Kallosh and Linde '13]

$$V(\phi) = \lambda M_P^4 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_P}}\right)^n$$



- During reheating, i.e. $\phi \ll M_P \implies V(\phi) \sim \phi^n$

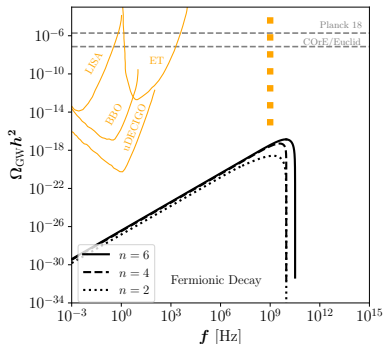
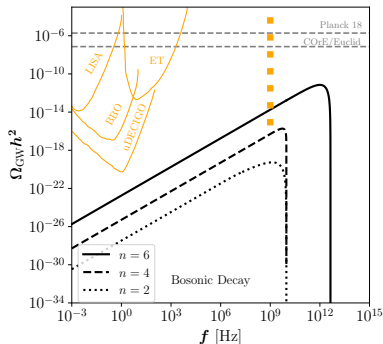
Evolution of Background for $n > 2$



- Fermionic decay: $\Gamma_\phi \sim \frac{y^2 m_\phi}{8\pi}$ with $m_\phi \propto V' \propto \phi^{n-2}$
- Annihilation: $\Gamma_\phi \sim \frac{\lambda^2 \rho_\phi}{16\pi m_\phi^3}$ Bosonic decay: $\Gamma_\phi \sim \frac{\mu^2}{8\pi m_\phi}$

n and type inflaton-matter couplings $\implies$ background evolution

GW Spectrum: Bosonic decay vs Fermionic decay

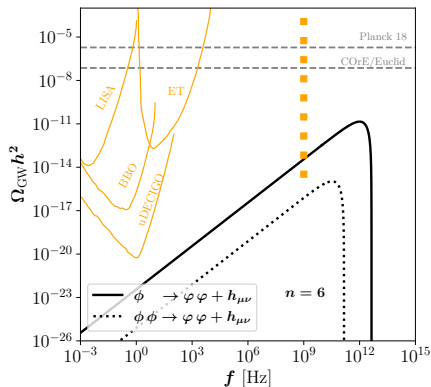
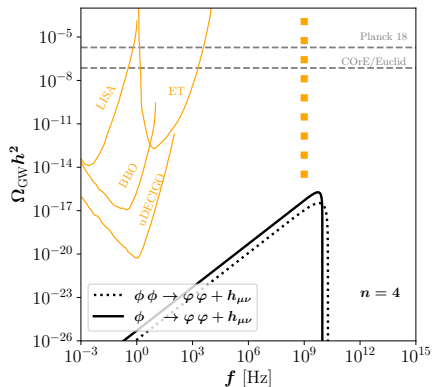


- GW amplitude **larger in bosonic decay** for $n > 2$ with $V(\phi) \sim \phi^n$

$$\Omega_{\text{GW}}(f) \propto \frac{d(\rho_{\text{GW}}/\rho_R)}{d \ln f}$$

- The distinction in GW $\implies$ a novel channel to probe the shape parameter n

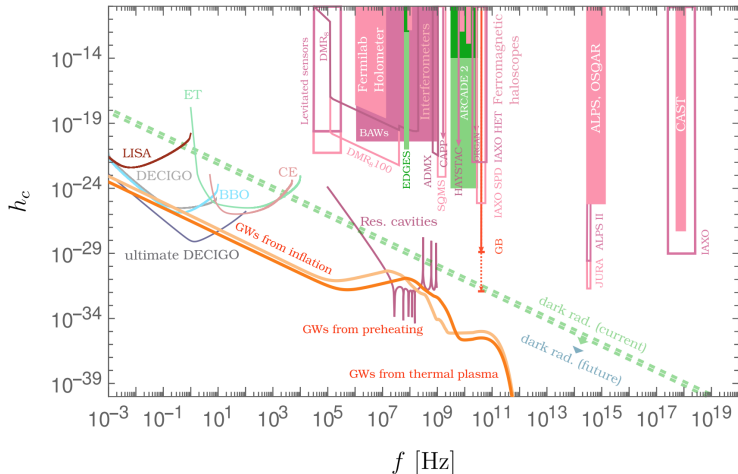
GW Spectrum: Bosonic Decay vs Bosonic Annihilation



Probing inflaton decay or annihilation

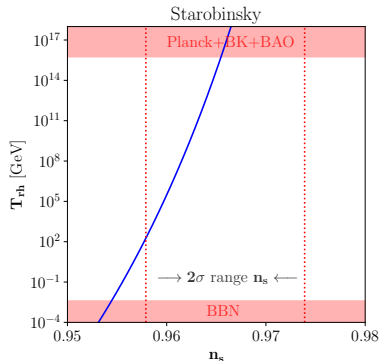
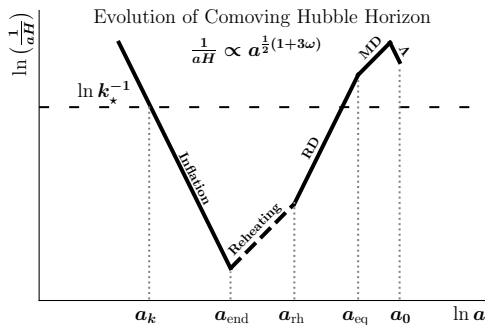
[Bernal, Cléry, Mambrini, **YX** 2311.12694]

Ultra-high frequency GW Detection Prospect



$$h_c(f) \equiv \frac{H_0}{f} \sqrt{\frac{3}{2\pi^2} \Omega_{\text{GW}}(f)} \simeq 1.26 \times 10^{-18} \left(\frac{\text{Hz}}{f} \right) \sqrt{h^2 \Omega_{\text{GW}}(f)},$$

Matching reheating with inflationary predictions



$$\implies T_{\text{rh}} = f(n_s, r)$$

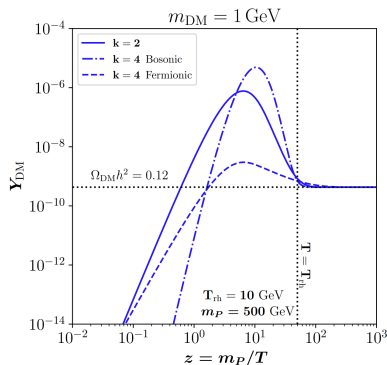
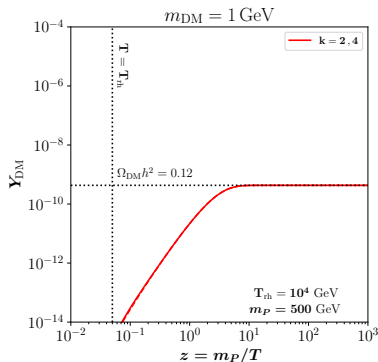
- Low bound on T_{rh} in Starobinsky inflaton: $T_{\text{rh}} \simeq 180 \text{ GeV}$

FIMP from LLP decay

- DM freeze-in from a gauge charged parent particle decay

$$P \rightarrow \text{DM} + f_{\text{SM}}$$

- Left: after reheating ($T_{\text{FI}} < T_{\text{rh}}$) Right: during reheating ($T_{\text{FI}} > T_{\text{rh}}$)

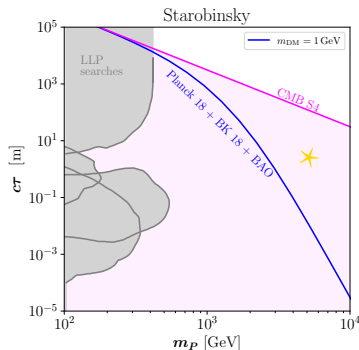


FIMP from LLP decay

- DM freeze-in from a gauge charged parent particle decay

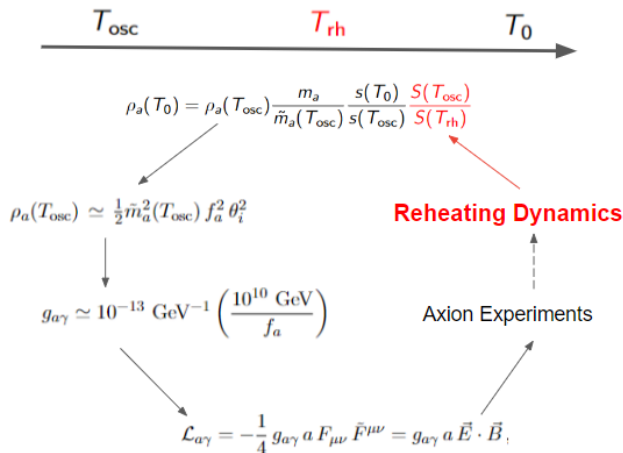
$$P \rightarrow \text{DM} + f_{\text{SM}}$$

- $T_{\text{FI}} < T_{\text{rh}} \implies c\tau \sim m_P^{-2}$ $T_{\text{FI}} > T_{\text{rh}} \implies c\tau \sim m_P^{-9}$



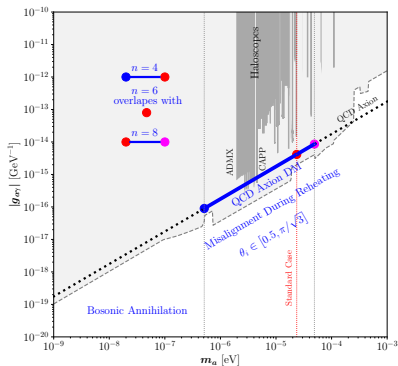
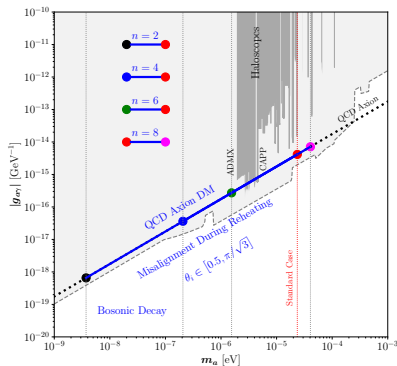
- In Starobinsky inflation with $m_{\text{DM}} = 1 \text{ GeV}$: if an event (e.g. yellow star) at collider $\implies T_{\text{rh}} \gg 180 \text{ GeV}$!

Axion Production during reheating



- **Reheating Dynamics:** n and $\Gamma \implies$ different $T \implies$ different $m_a \implies$ experimental implications

Probing Reheating via Axion experiments: QCD Axion

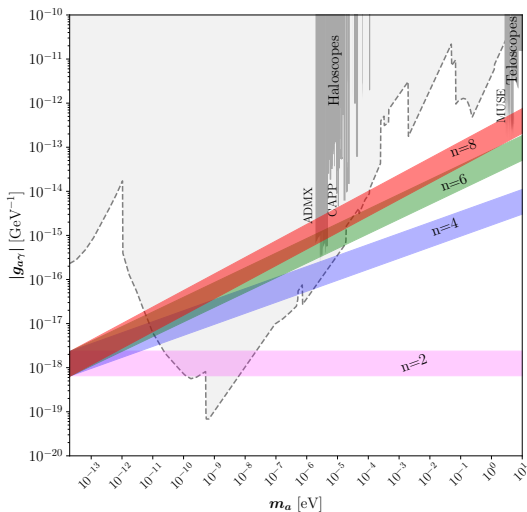


- Assume QCD axion makes up all DM. If a positive signal with

$$4 \cdot 10^{-9} \text{ eV} \lesssim m_a \lesssim 5 \cdot 10^{-7} \text{ eV}$$

⇒ **bosonic annihilation less likely**

Probing Reheating via Axion experiments: ALP



$$\Omega_a h^2 \simeq \left(\frac{\theta_i}{1.0}\right)^2 \left(\frac{f_a}{1.3 \times 10^{13} \times \exp(8.1/n) \text{ GeV}}\right)^2 \left(\frac{T_{\text{rh}}}{4 \text{ MeV}}\right)^{\frac{4-n}{n}} \left(\frac{m_a}{7.4 \times 10^{-11} \text{ eV}}\right)^{\frac{n-2}{n}}$$

GR as a Low Energy EFT

- Einstein-Hilbert action:

$$S_{\text{EH}} = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R \right)$$

- Recall the definition: $R_{\mu\nu\kappa}^\lambda \sim \partial_\nu \Gamma_{\nu\kappa}^\lambda + \dots$ and $\Gamma_{\mu\nu}^\lambda \sim g^{\lambda\rho} \partial_\nu g_{\rho\mu} \implies R \sim \partial g \partial g$
- Consider the expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + \left(h_{\mu\nu} \frac{2}{M_P} \right)$$

- Effective terms in the action

$$S_{\text{EH}} \supset \int d^4x \sqrt{-g} M_P^2 \left(\partial h \partial h + \frac{h}{M_P} \partial h \partial h + \frac{h^2}{M_P^2} \partial h \partial h + \dots \right)$$

[A. Zee Quantum Field Theory in a Nutshell: Second Edition]