

Inflation does Not Create Entanglement in Local Observables

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Based on: 2409.16366, 2511.17382

Outline

- 1 Motivation
- 2 Tools
- 3 Entanglement in de Sitter spacetime
- 4 Summary and conclusions

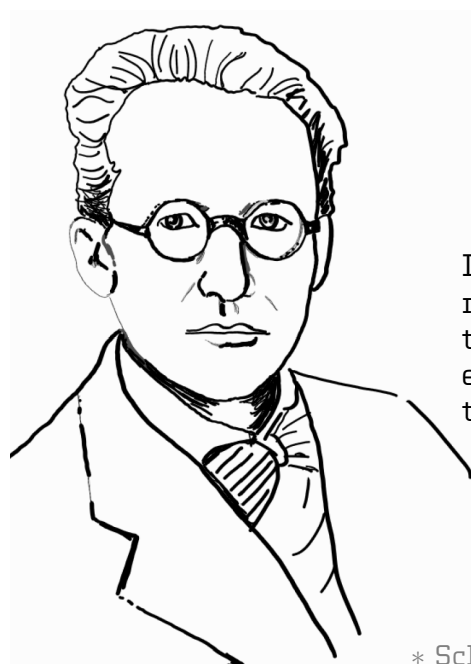
1. MOTIVATION



ENTANGLEMENT

A type of correlation in quantum systems
with **no** classical counterpart





I would not call [entanglement] one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.*

E. Schrödinger

* Schrödinger (1935)



The entanglement content of QFTs is extraordinarily rich* ...

* Reeh and Schlieder (1961)



The entanglement content of QFTs is extraordinarily rich* ...
...but complex, subtle, and not fully understood.

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The entanglement content of QFTs is **extraordinarily rich*** ...

...but **complex, subtle**, and not **fully understood**.

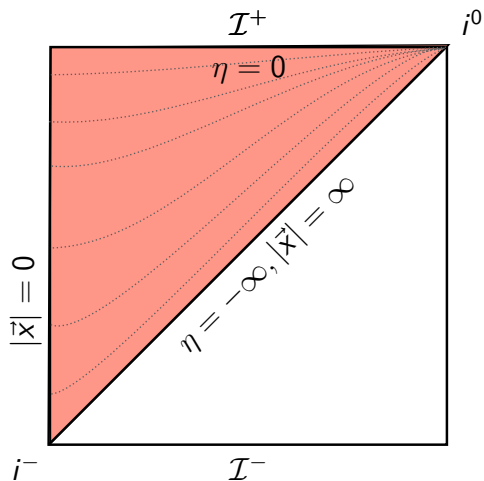
My goal:

Characterizing entanglement in QFTs and elucidating its relationship to spacetime curvature and symmetries

* Reeh and Schlieder (1961)

In this talk: the cosmological patch of de Sitter spacetime

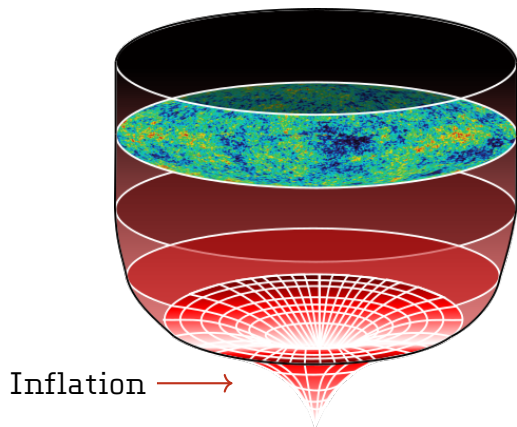
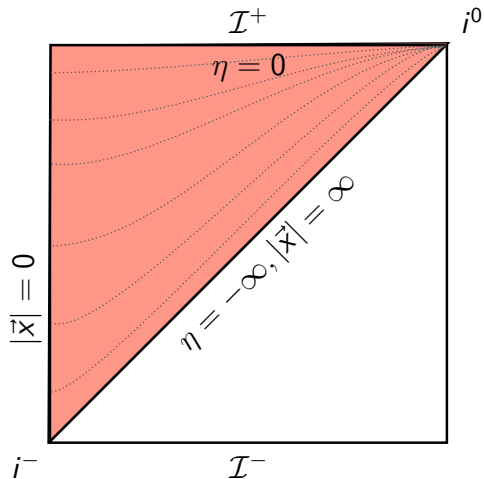
Highly symmetric and constant spacetime curvature.



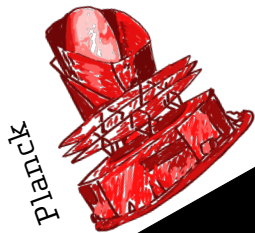
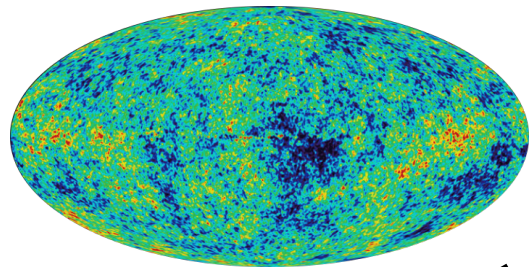
In this talk: the cosmological patch of de Sitter spacetime

Highly symmetric and constant spacetime curvature.

Relevant for cosmology.

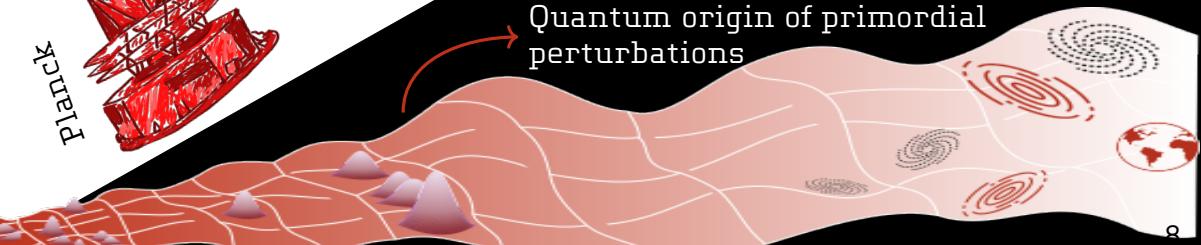


Is entanglement imprinted during Inflation?



Observations
Theory

Quantum origin of primordial perturbations



Some results on entanglement in de Sitter

- Geometric entanglement entropy of a region (UV divergent)

Maldacena, Pimentel (2013)

- Entanglement between $\vec{k}$ and $-\vec{k}$ modes (ambiguous and non-local)*

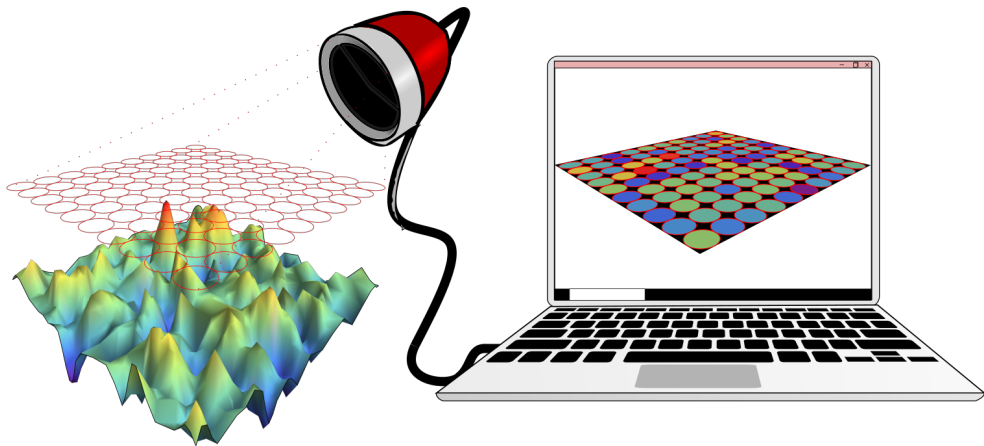
Grishchuk and Sidorov (1990); Albrecht, Ferreira, Joyce, and Prokopec (1994); Lesgourgues, Polarski, and Starobinsky (1997); Kiefer, Lesgourgues, Polarski, Starobinsky (1998); Brahma, Alaryani, Brandenberger (2020); Micheli, Peter (2022); ...

- Entanglement harvesting protocol (non-relativistic detectors)

Ver Steeg, Menicucci (2009); Kukita, Nambu (2017)

* Agullo, Bonga, Ribes-Metidieri (2022)

A complementary approach



Entanglement between sets of **finitely** many degrees of freedom **locally** supported in space*

* Bianchi and Satz (2019); Martin and Vennin (2021); Hackl and Bianchi (2021); Agullo, Bonga, and Ribes-Metidieri (2021)

Approach Overview

Features:

Truncation of
degrees of freedom

+

Gaussian formalism
(Gaussian states + free evolution)

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- (1) We avoid UV divergencies (Type I algebras).
- (2) We can compute reduced states.
- (3) We can use results from quantum information.
- (4) We retain features of the full QFT.

Approach Overview

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Limitations:

- Limited to free theories and Gaussian states
- We focus on a finite subset of degrees of freedom (infinitely many of them!)

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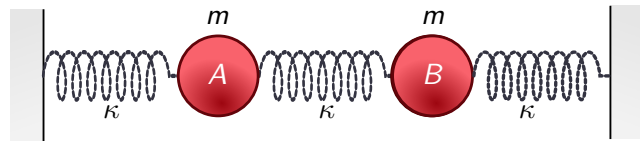


Focus on universal properties!

2. TOOLS

- Local modes
- Gaussian states
- Entropy, correlations, and entanglement

Modes: A warm up example



$$\hat{R}_A := (\hat{q}_A, \hat{p}_A)^\top \quad \hat{R}_B := (\hat{q}_B, \hat{p}_B)^\top$$

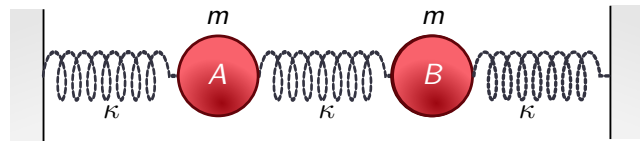
$$\hat{R} = \hat{R}_A \oplus \hat{R}_B$$

$$[\hat{R}^i, \hat{R}^j] = i\Omega^{ij}\hat{\mathbb{I}},$$

$$\Omega^{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Mode: Single-degree of freedom (Algebra of two non-commuting operators)

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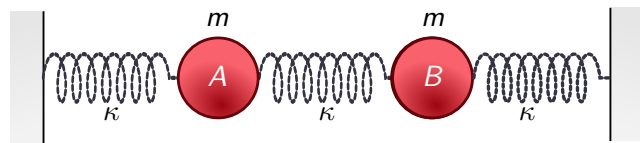
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$$\hat{q}_A, \hat{p}_A$$

(Original mode)

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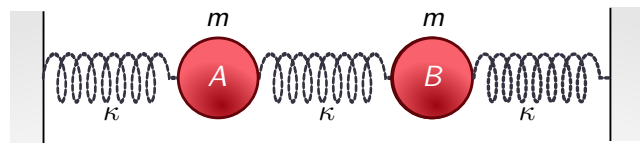
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$\hat{q}_A, \hat{p}_A$
(Original mode)

$\frac{1}{\sqrt{2}}(\hat{q}_A - \hat{q}_B), \frac{1}{\sqrt{2}}(\hat{p}_A - \hat{p}_B)$
(Normal mode)

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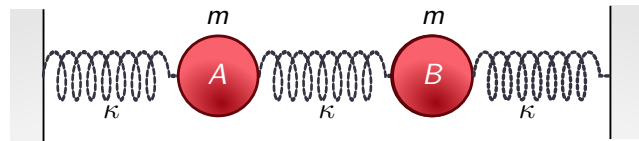
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Mode: Single-degree of freedom (Algebra of two non-commuting operators)

General case:

$$\begin{aligned} \hat{O} &= f_j \hat{q}_j + g_j \hat{p}_j \text{ and } \hat{O}' = f'_j \hat{q}_j + g'_j \hat{p}_j \\ [\hat{O}, \hat{O}'] &= i(f_j g'_j - f'_j g_j). \end{aligned}$$

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$$\hat{O} = f_j \hat{q}_j + g_j \hat{p}_j = \gamma_i \Omega^{ij} \hat{R}_j \text{ and } \hat{O}' = f'_j \hat{q}_j + g'_j \hat{p}_j = \gamma'_i \Omega^{ij} \hat{R}_j, \text{ with } \gamma_i^{(\prime)} = (f_A^{(\prime)} \ g_A^{(\prime)} \ f_B^{(\prime)} \ g_B^{(\prime)})$$

$$[\hat{O}, \hat{O}'] = i(f_j g'_j - f'_j g_j) = i\gamma_i \Omega^{ij} \gamma'_j.$$

Local modes in QFT

Free quantum scalar field $\hat{R} := (\hat{\phi}(\vec{x}), \hat{\pi}(\vec{x}))$ propagating on a **globally hyperbolic** spacetime $(\mathcal{M}, g)$.

$$[\hat{\phi}(\vec{x}), \hat{\pi}(\vec{x}')] = i\delta(\vec{x} - \vec{x}')$$

Mode: “Single-degree of freedom of the field” Requires smearing!

Local modes in QFT

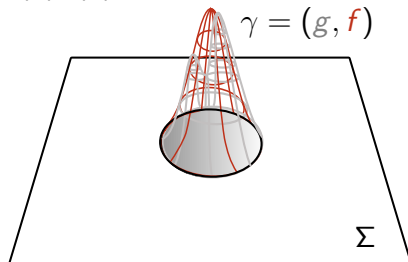
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Example: $\hat{\Phi}[f] := \int_{\Sigma} d^3x f(\vec{x}) \hat{\phi}(\vec{x}), \quad \hat{\Pi}[g] := \int_{\Sigma} d^3x g(\vec{x}) \hat{\pi}(\vec{x})$

s.t. $[\hat{\Phi}[f], \hat{\Pi}[g]] = i$



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General case:

$$\begin{aligned} \hat{O}_1 &= \hat{\Phi}[f] + \hat{\Pi}[g], \quad \hat{O}_2 = \hat{\Phi}[f'] + \hat{\Pi}[g'] \\ \text{s.t. } [\hat{O}_1, \hat{O}_2] &= i \int_{\Sigma} d^3x (f(\vec{x})g'(\vec{x}) - f'(\vec{x})g(\vec{x})) = i \end{aligned}$$

Notation

The classical phase space Γ is a **vector space** equipped with a **symplectic structure**:

$$\Omega(\gamma, \gamma') = \int_{\Sigma} d^3x (f(\vec{x})g'(\vec{x}) - f'(\vec{x})g(\vec{x})), \text{ with } \gamma = (g, f) \in \Gamma, \gamma' = (g', f') \in \Gamma.$$

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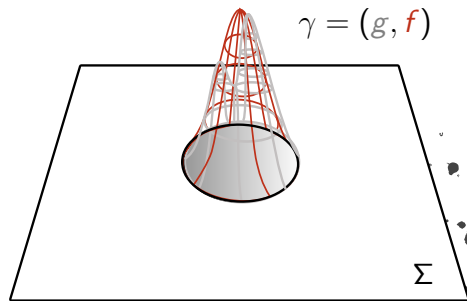
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Convenient way of organizing linear operators:

$$\gamma = (g, f) \in \Gamma_{\mathbb{C}} \rightarrow \hat{O}_{\gamma} = i\Omega(\gamma, \hat{R})$$

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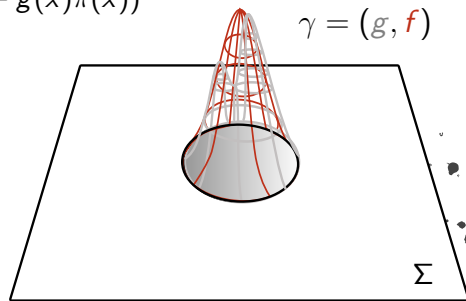
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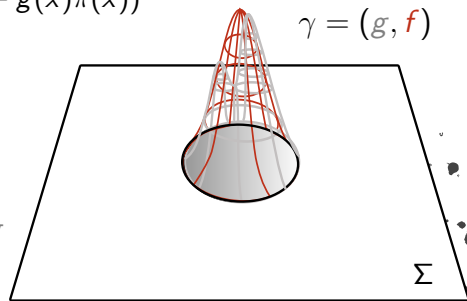
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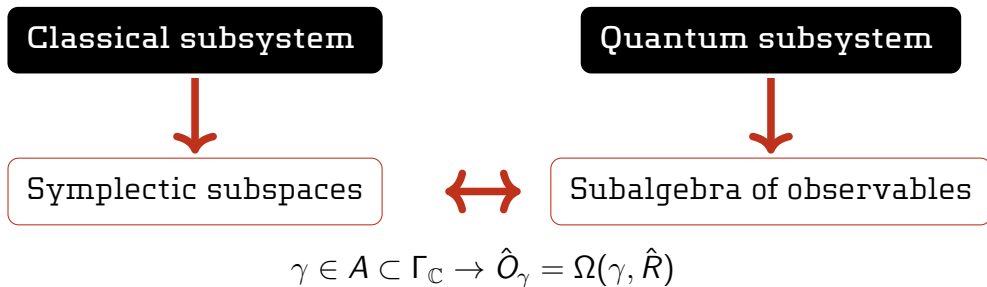
where $\hat{R} = (\hat{\phi}(\vec{x}), \hat{\pi}(\vec{x}))$.

Commutation relations: $[\hat{O}_{\gamma}, \hat{O}_{\gamma'}] = -i\Omega(\gamma, \gamma')$

Commutativity = symplectic orthogonality



Subsystems



Subsystems

Classical subsystem



Symplectic subspaces

Quantum subsystem

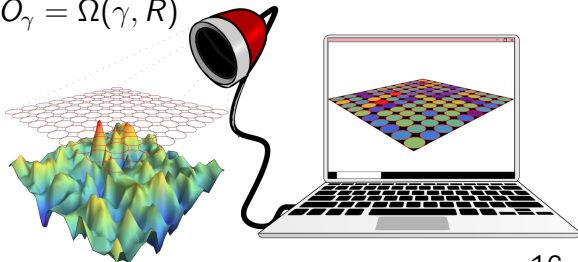


Subalgebra of observables



$$\gamma \in A \subset \Gamma_{\mathbb{C}} \rightarrow \hat{O}_{\gamma} = \Omega(\gamma, \hat{R})$$

Focus on **finite-dimensional**
subsystems



Gaussian States

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• $\blacksquare \mu : \Gamma \rightarrow \mathbb{R}, \gamma \mapsto \mu(\gamma) := \langle \hat{O}_{\gamma}^{\dagger} \rangle,$

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Example:

$A = \text{span}\{\gamma_1, \gamma_2\}$, with

$$\gamma_1 = \begin{pmatrix} 0 \\ f(\vec{x}) \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} -g(\vec{x}) \\ 0 \end{pmatrix}$$

$$\hat{O}_{\gamma_1} = \hat{\Phi}[f], \quad \hat{O}_{\gamma_2} = \hat{\Pi}[g],$$

$$\sigma_A = \begin{pmatrix} \langle \{ \hat{\Phi}[f], \hat{\Phi}[f] \} \rangle & \langle \{ \hat{\Phi}[f], \hat{\Pi}[g] \} \rangle \\ \langle \{ \hat{\Phi}[f], \hat{\Pi}[g] \} \rangle & \langle \{ \hat{\Pi}[g], \hat{\Pi}[g] \} \rangle \end{pmatrix}$$

→ **Covariance matrix**

Entropy, correlations, and entanglement

Consider two single-mode subsystems,
 A and B ,

$$\sigma_{AB} = \begin{pmatrix} \sigma_A & C \\ C^\top & \sigma_B \end{pmatrix}$$

Local **symplectic invariants** of σ_{AB} :

$$\nu_I = \sqrt{\det \sigma_I}, \quad I = A, B$$

$$\nu_{\pm} = \frac{\Delta \pm \sqrt{\Delta^2 - 4 \det \sigma_{AB}}}{2}$$

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where

$$\Delta = \det \sigma_A + \det \sigma_B + 2 \det C$$

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■ Von Neumann entropy:

$$S(\nu_I) = \left(\frac{\nu_I + 1}{2} \right) \log_2 \left(\frac{\nu_I + 1}{2} \right) - \left(\frac{\nu_I - 1}{2} \right) \log_2 \left(\frac{\nu_I - 1}{2} \right)$$

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■ **Mutual information:**

$$\mathcal{I}(A, B) = S(\nu_A) + S(\nu_B) - S(\nu_+) - S(\nu_-)$$

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$$\mathcal{I}(A, B) = S(\nu_A) + S(\nu_B) - S(\nu_+) - S(\nu_-)$$

■ **Logarithmic negativity:**

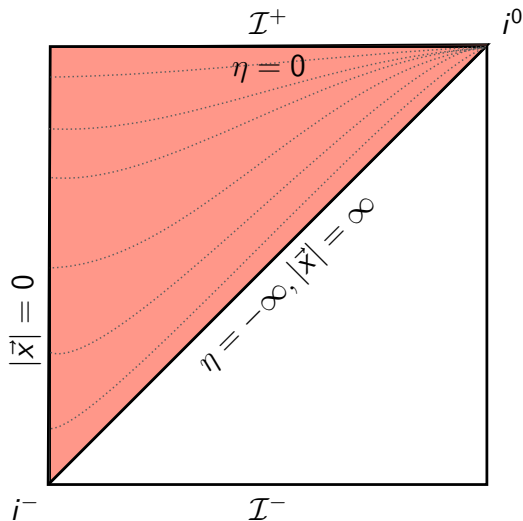
$$\text{LN} = \max\{0, -\log_2 \tilde{\nu}_-\}$$

3. ENTANGLEMENT IN DE SITTER SPACETIME

- Covariance matrix
- Entropy
- Mutual Information
- Entanglement
- Partner modes and entanglement distribution

de Sitter spacetime (Cosmological patch)

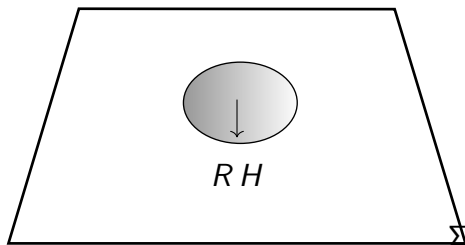
$$ds^2 = a(\eta)^2(-d\eta^2 + d\vec{x}^2), \quad a(\eta) = -\frac{1}{H\eta}, \quad \text{and } \eta \in (-\infty, 0).$$



Setup: Local observables

Free, light scalar field in de Sitter spacetime in the Bunch-Davies vacuum

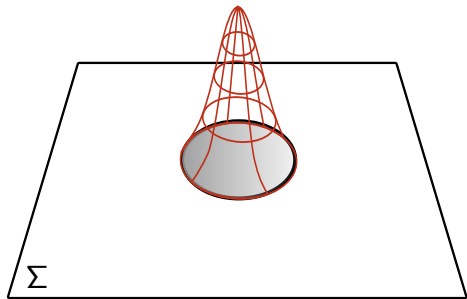
$$\mu^2 := \frac{3}{2} - \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \ll 1, RH \gg 1$$



Setup: Local observables

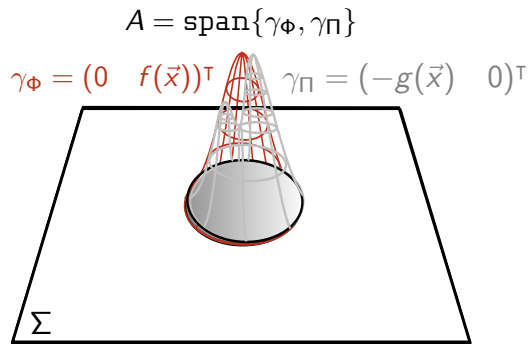
Free, light scalar field in de Sitter spacetime in the Bunch-Davies vacuum

$$A = \text{span}\{\gamma_\Phi, \gamma_\Pi\}$$



Setup: Local observables

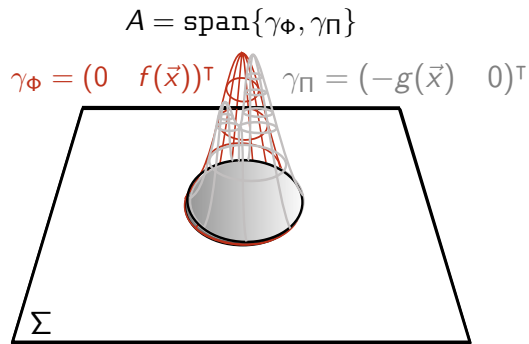
Free, light scalar field in de Sitter spacetime in the Bunch-Davies vacuum



$$\begin{aligned} \hat{O}_{\gamma_\Phi} &:= \hat{\Phi}[f], & \hat{O}_{\gamma_\Pi} &:= \hat{\Pi}[g], \\ \text{s.t. } [\hat{\Phi}[f], \hat{\Pi}[g]] &= i \end{aligned}$$

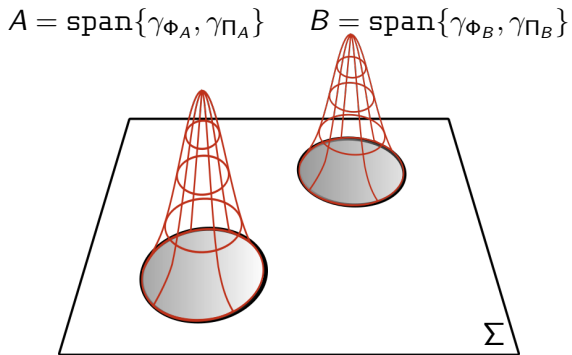
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Free, light scalar field in de Sitter spacetime in the **Bunch-Davies vacuum**



$$\hat{O}_{\gamma_\Phi} := \hat{\Phi}[f], \quad \hat{O}_{\gamma_\Pi} := \hat{\Pi}[g],$$

$$\text{s.t. } [\hat{\Phi}[f], \hat{\Pi}[g]] = i$$



$$\{\hat{\Phi}[f_A], \hat{\Pi}[g_A], \hat{\Phi}[f_B], \hat{\Pi}[g_B]\}$$

$$\text{s.t. } [\hat{\Phi}[f_A], \hat{\Pi}[g_A]] = i, [\hat{\Phi}[f_B], \hat{\Pi}[g_B]] = i$$

Covariance matrix

$$\langle \{ \hat{\Phi}[f_I], \hat{\Phi}[f_J] \} \rangle =$$

$$\langle \{ \hat{\Pi}[g_I], \hat{\Pi}[g_J] \} \rangle =$$

$$\langle \{ \hat{\Phi}[f_I], \hat{\Pi}[g_J] \} \rangle =$$

$$\mu^2 \ll 1, \quad RH \gg 1, \quad I, J = A, B.$$

Covariance matrix

Minkowski

$$\langle \{\hat{\Phi}[f_I], \hat{\Phi}[f_J]\} \rangle = \text{Re} \left[(f_I | f_J)_{-\frac{1}{2}} \right] + \frac{2^{2-2\mu^2} \pi (RH)^{2-2\mu^2}}{\cos^2(\pi\mu^2) \Gamma(-\frac{1}{2} + \mu^2)^2} \text{Re} \left[(f_I | f_J)_{-\frac{3}{2} + \mu^2} \right] + \mathcal{O}(\mu^2 \log(RH)),$$

$$\langle \{\hat{\Pi}[g_I], \hat{\Pi}[g_J]\} \rangle =$$

$$\langle \{\hat{\Phi}[f_I], \hat{\Pi}[g_J]\} \rangle =$$

$$(f|g)_s := \int_{\Sigma} \frac{d^3q}{(2\pi)^3} |\vec{q}|^{2s} \tilde{f}(\vec{q}) \tilde{g}^*(\vec{q}), \quad \text{Sobolev inner product}$$

Covariance matrix

Curvature

$$\langle \{\hat{\Phi}[f_I], \hat{\Phi}[f_J]\} \rangle = \text{Re} \left[(f_I | f_J)_{-\frac{1}{2}} \right] + \frac{2^{2-2\mu^2} \pi (RH)^{2-2\mu^2}}{\cos^2(\pi\mu^2) \Gamma(-\frac{1}{2} + \mu^2)^2} \text{Re} \left[(f_I | f_J)_{-\frac{3}{2} + \mu^2} \right] + \mathcal{O}(\mu^2 \log(RH)),$$

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$$\langle \{\hat{\Pi}[g_I], \hat{\Pi}[g_J]\} \rangle = \operatorname{Re} \left[(g_I | g_J)_{\frac{1}{2}} \right] + \mathcal{O}(\mu^2 \log(RH)),$$

$$\langle \{\hat{\Phi}[f_I], \hat{\Pi}[g_J]\} \rangle = \frac{2^{1-\mu^2} \sqrt{\pi} (RH)^{1-\mu^2}}{\cos(\pi \mu^2) \Gamma(-\frac{1}{2} + \mu^2)} \operatorname{Re} \left[(f_I | g_J)_{-\frac{1-\mu^2}{2}} \right] + \mathcal{O}(\mu^2 (RH)^{1-\mu^2}),$$

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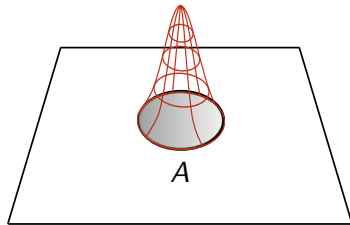
Key ingredient:

Well-known relations between Sobolev norms



Entropy of a single-mode

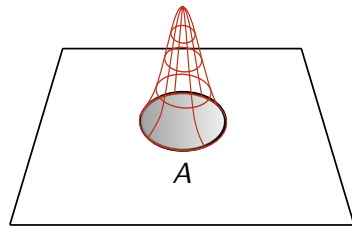
$$\nu_A^2 \approx (\nu_A^{\text{Mink}})^2 + \mathcal{F}_A + \text{subleading contributions}$$



Entropy of a single-mode

$$\nu_A^2 \approx (\nu_A^{\text{Mink}})^2 + \mathcal{F}_A + \text{subleading contributions}$$

Proposition: $\mathcal{F}_A \geq 0$, when $RH \gg 1$, $\mu^2 \ll 1$.*



*Ribes-Metidieri, Agullo, Bonga (2025)

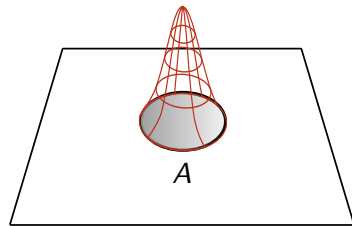
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Sketch of the proof: Hölder and Cauchy-Schwartz inequalities.

$S(\nu_A)$ is monotonically increasing function of ν_A ...



*Ribes-Metidieri, Agullo, Bonga (2025)

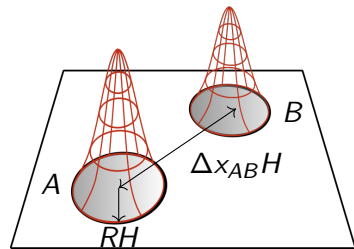
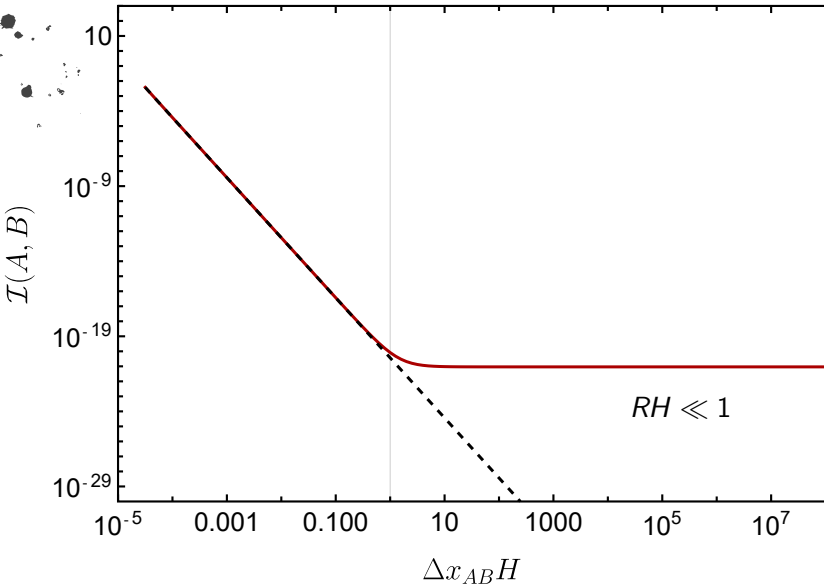


Entropy increases with H

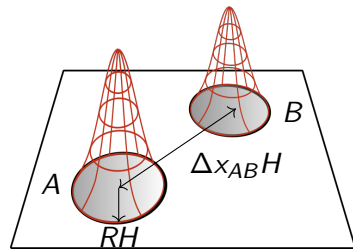
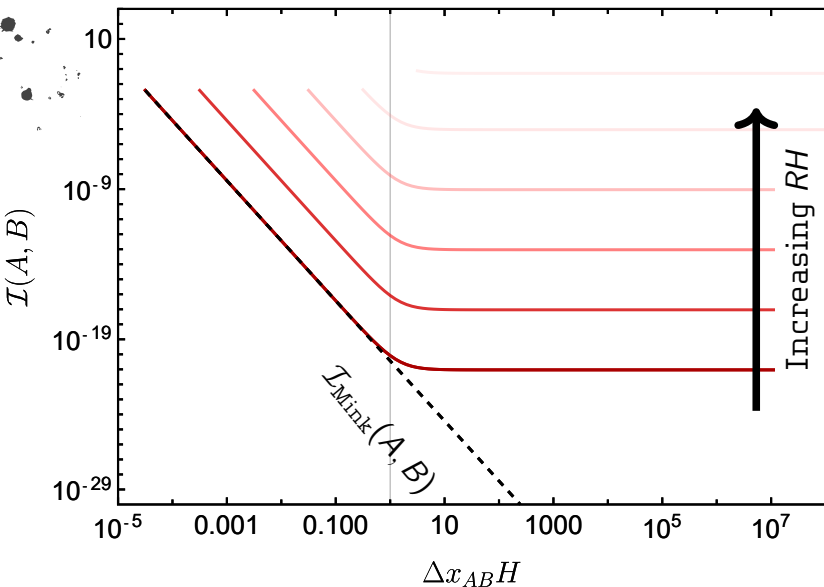
At least as much entanglement as in
Minkowski spacetime



Mutual information



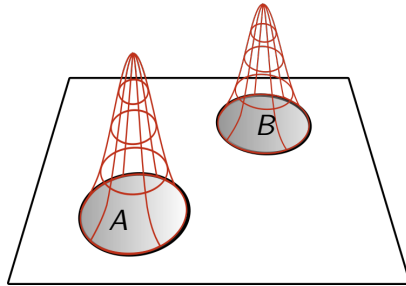
Mutual information



Total correlations
increase with H

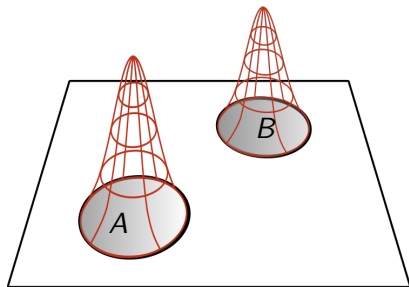
Entanglement between A and B

$$RH \gg 1 \text{ and } 0 < \mu^2 \ll 1$$



Entanglement between A and B

$$RH \gg 1 \text{ and } 0 < \mu^2 \ll 1$$



Proposition: The entanglement between **any pair** of non-overlapping, local modes with $RH \gg 1$, **decreases** with H .

Proof: Based on Cauchy-Schwartz, Hölder, and Fischer inequalities.*

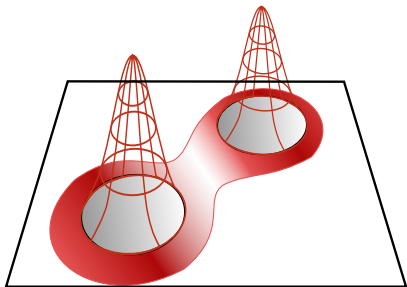
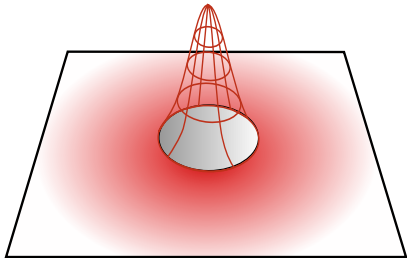
* Ribes-Metidieri, Agullo, Bonga (2025)



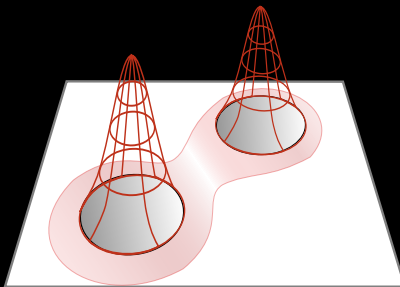
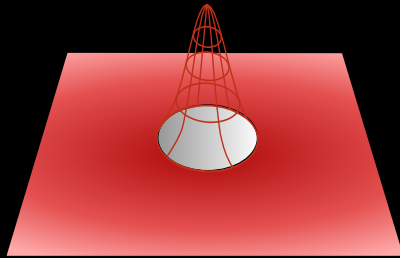
Less entanglement between local
observables

Summary

Minkowski

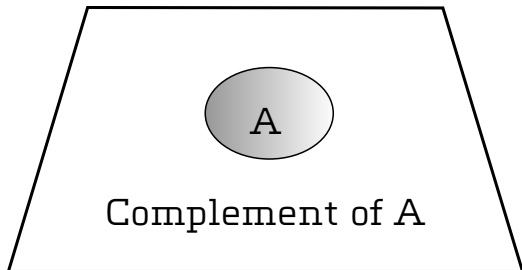


de Sitter



Entanglement in de Sitter

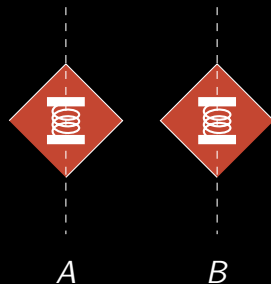
Entanglement Entropy



Larger in de Sitter
than in Minkowski

Maldacena and Pimentel (2013)

Harvesting



Smaller in de Sitter
than in Minkowski

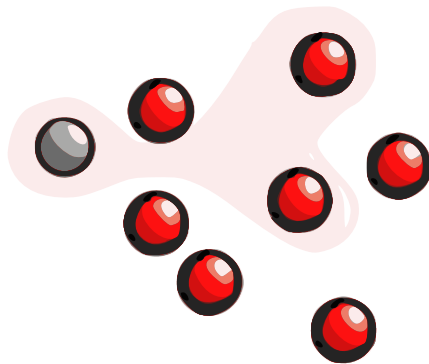
Ver Steeg and Menicucci (2009)



Where is the entanglement?

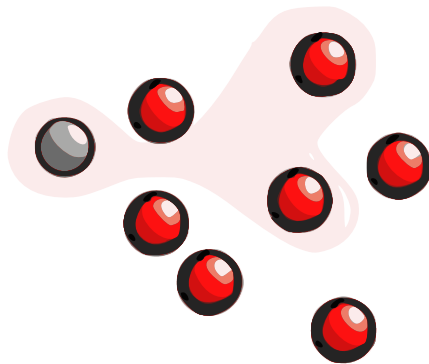
Partner modes and entanglement distribution

■ System A ■ System B



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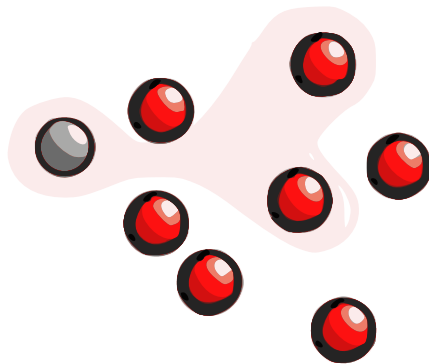


Partner modes and entanglement distribution

The **partner** of A in the pure state $\hat{\rho}$, A_p , encodes all entanglement with A .*

■ System A

■ System B



*Wald (1975); Botero and Reznik (2003); Hotta, Schützhold, and Unruh (2015); Trevison, Yamaguchi, and Hotta (2019); ** Agullo, Martin-Martinez, Nadal-Gisbert, Ribes-Metidieri, and Yamaguchi (2025); *** Agullo, Martin-Martinez, Nadal-Gisbert, and Yamaguchi (2025)

Partner modes and entanglement distribution

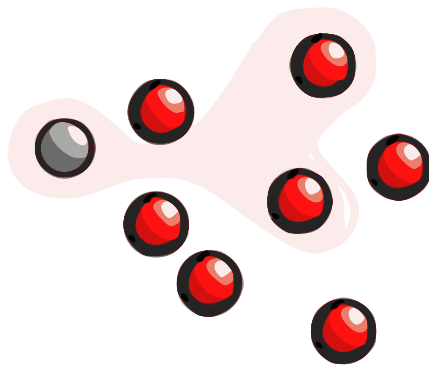
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■ For pure Gaussian states, the partner exists and is unique

$$A_P = \Pi_A^\perp(JA), \text{ where } J_\beta^\alpha = -\Omega^{\alpha\gamma}\sigma_{\gamma\beta}.^{**}$$

■ System A

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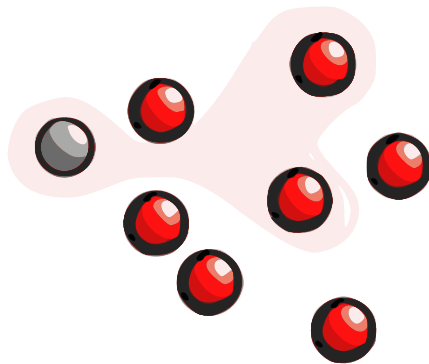
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■ The partner encodes the spatial distribution of entanglement.***

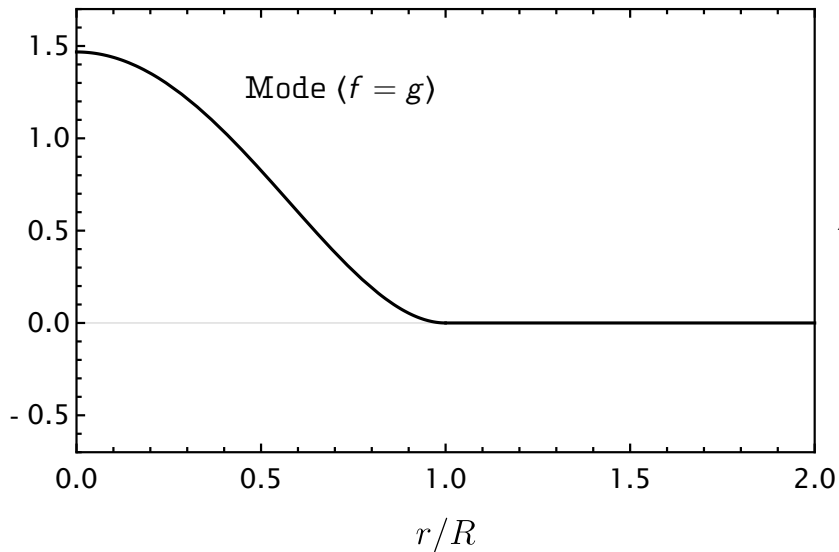
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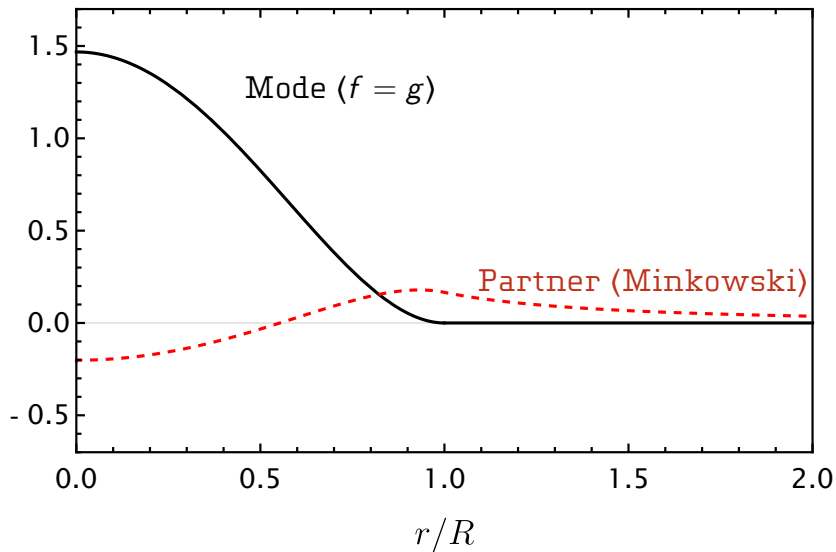
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Example: Partner of a single-mode in de Sitter



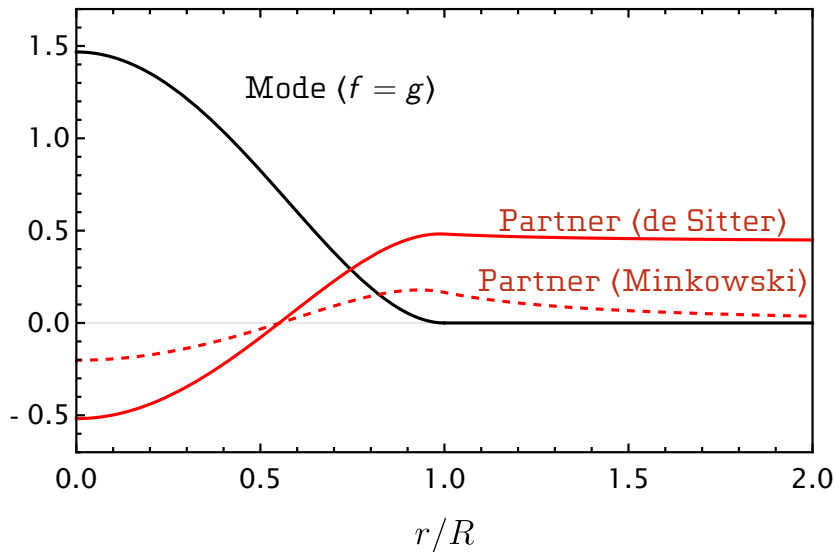
$$A = \text{span} \left\{ \begin{pmatrix} 0 \\ f \end{pmatrix}, \begin{pmatrix} -g \\ 0 \end{pmatrix} \right\}$$

Example: Partner of a single-mode in de Sitter

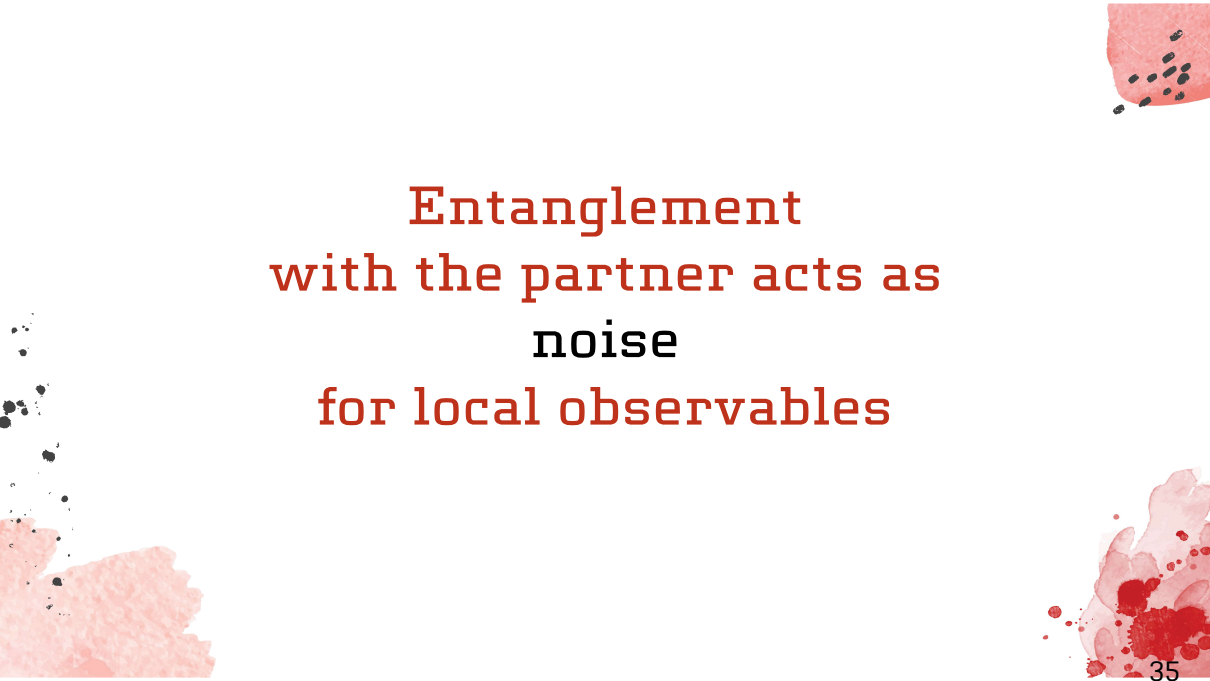


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$$A = \text{span} \left\{ \begin{pmatrix} 0 \\ f \end{pmatrix}, \begin{pmatrix} -g \\ 0 \end{pmatrix} \right\}$$



Entanglement
with the partner acts as
noise
for local observables

4. SUMMARY AND CONCLUSIONS

Summary

QFT vacuum has a surprisingly rich entanglement structure

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What part of this entanglement
can we access?

Summary

QFT vacuum has a surprisingly rich entanglement structure



```
graph TD; A[QFT vacuum has a surprisingly rich entanglement structure] --> B[What part of this entanglement can we access?]; A --> C[How is entanglement distributed?];
```

What part of this entanglement
can we access?

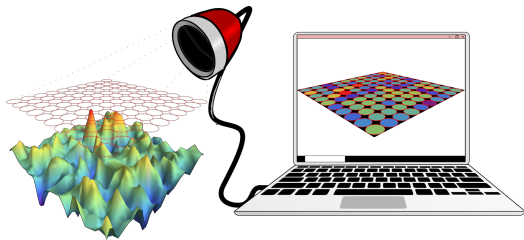
How is entanglement
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Summary

QFT vacuum has a surprisingly rich entanglement structure

What part of this entanglement
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How is entanglement
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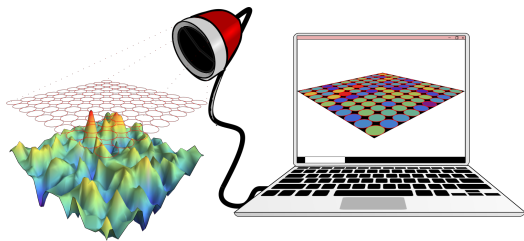


Entanglement between a finite
number of local degrees of freedom

Summary

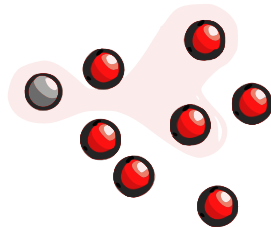
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Entanglement between a finite
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Partner systems

Conclusions

- Curvature in de Sitter enhances total correlations.
- Each mode is more entangled with its partner.
- More entanglement, but not accessible to local observables.
- Any two modes of compact support in de Sitter are less entangled than in Minkowski.



Conclusions

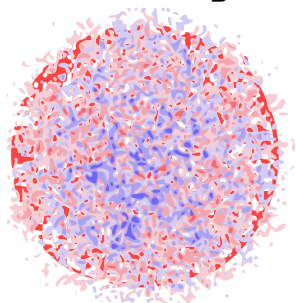
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**Inflation does not generate entanglement
between local observables**



Outlook

A proposal to probe QFT vacuum entanglement



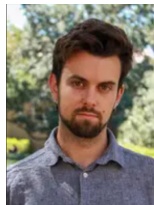
Analogue QFT simulators
(BEC, polaritons, ...)

Similar techniques

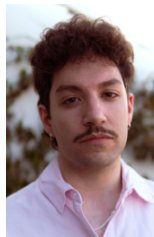
Promising theoretical results!



Ivan Agullo



Adrià Delhom



Álvaro Parra



Thank you for your attention!

Questions?

Contact information:

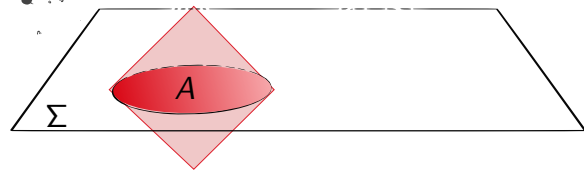
patricia.ribesmetidieri@york.ac.uk



BACKUP

The Reeh-Schlieder Theorem

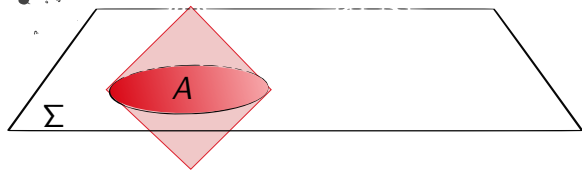
Consider a quantum free scalar field $\hat{\phi}$ in Minkowski space



- Let $|0\rangle$ denote the vacuum.
- Define $\hat{\Phi}_F := \int dV F(x) \hat{\phi}(x)$ such that $\text{supp} F \subset A$.

The Reeh-Schlieder Theorem

Consider a quantum free scalar field $\hat{\phi}$ in Minkowski space



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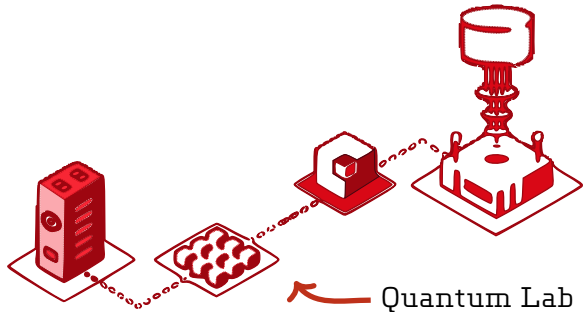
■ Define $\hat{\Phi}_F := \int dV F(x) \hat{\phi}(x)$ such that $\text{supp} F \subset A$.

Theorem: Acting on $|0\rangle$ with operators $\hat{\Phi}_{F_1} \cdots \hat{\Phi}_{F_N}$ localized in **any** open spacetime region A , we can approximate **any** state in the Hilbert space of the theory **arbitrarily well**.*

* Reeh, Schlieder (1961). See Witten (2018) for a pedagogical proof.

The Reeh-Schlieder Theorem II

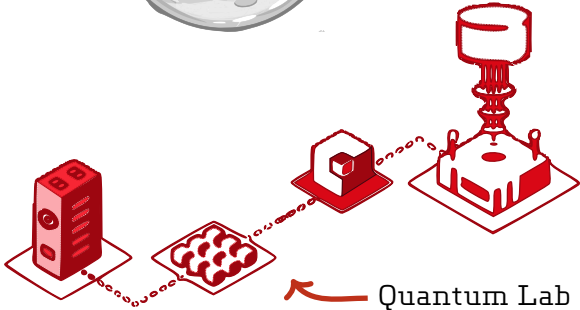
Why surprising?



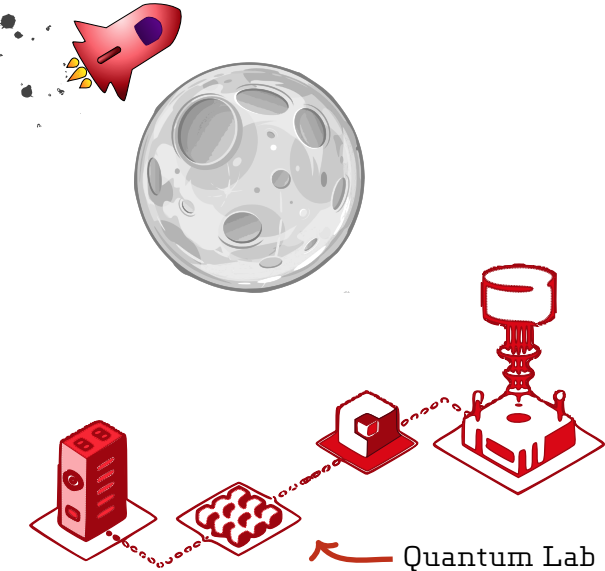
The Reeh-Schlieder Theorem II



Why surprising?



The Reeh-Schlieder Theorem II



Why surprising?

Entanglement is
ubiquitous in the vacuum

Intuitive explanation: Entanglement

This is reminiscent of something well-known in Quantum Mechanics

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Let $\mathcal{H}_A$ and $\mathcal{H}_B$ be Hilbert spaces of two systems, both of dimension n . Any state in $\mathcal{H}_A \otimes \mathcal{H}_B$ can be written as

$$|\psi\rangle = \sum_{i=1}^n c_i |i\rangle_A \otimes |i\rangle_B, \quad (\text{Schmidt form})$$

with $\{|i\rangle_A\}_{i=1}^n$ and $\{|i\rangle_B\}_{i=1}^n$ bases in $\mathcal{H}_A$ and $\mathcal{H}_B$

Intuitive explanation: Entanglement

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If $c_i \neq 0 \quad \forall i \rightarrow |\psi\rangle$ is fully entangled

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If $|\psi\rangle$ is fully entangled, then

Any state in $\mathcal{H}_A \otimes \mathcal{H}_B$ can be written as $\hat{O}_A \otimes \hat{I}_B |\psi\rangle$

Quantifying Entanglement

• **Entanglement monotones:** Functions of a quantum state that

- vanish on separable states
- do not increase under LOCC

Example: Logarithmic negativity

- Measures distillable entanglement
- Valid for mixed Gaussian states
- Faithful for system of 1 vs N modes



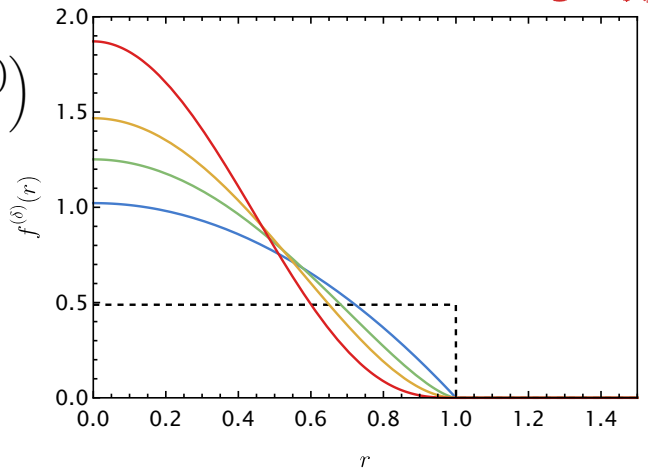
Single-Mode Subsystem in a Massless Scalar Theory

Free, massless scalar field in Minkowski spacetime

$$\gamma_1(\vec{x}) = \begin{pmatrix} 0 \\ f^{(\delta)}(\vec{x}) \end{pmatrix}, \quad \gamma_2(\vec{x}) = \begin{pmatrix} -f^{(\delta)}(\vec{x}) \\ 0 \end{pmatrix}$$

$$f^{(\delta)}(\vec{x}) = A_\delta \left(1 - \frac{|\vec{x}|^2}{R^2}\right)^\delta \Theta(R - r),$$

$$\Gamma_A = \text{span}(\gamma_1, \gamma_2) \subset \bar{\Gamma}_{\mathbb{C}}$$



--- $\delta = 0$ $\delta = 1$ $\delta = 3/2$ $\delta = 2$ $\delta = 3$

Single-Mode Subsystem in a Massless Scalar Theory

The corresponding smeared operators are:

$$\hat{\Phi}_\delta := \hat{O}_{\gamma_1} = \int_\Sigma d^3x f^{(\delta)}(\vec{x}) \hat{\phi}(\vec{x}), \quad \hat{\Pi}_\delta := \hat{O}_{\gamma_2} = \int_\Sigma d^3x f^{(\delta)}(\vec{x}) \hat{\pi}(\vec{x}),$$

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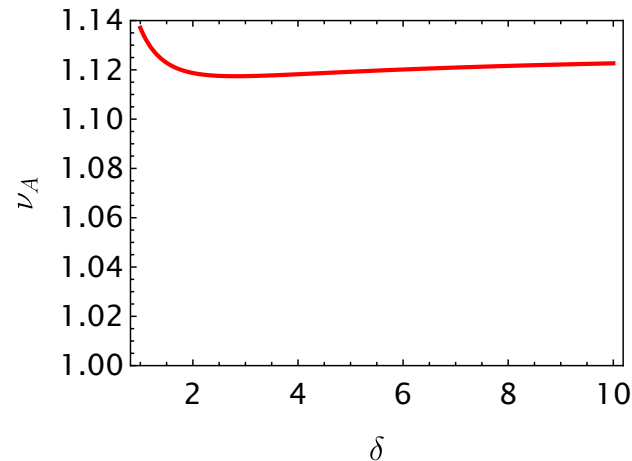
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Single-Mode Subsystem in a Massless Scalar Theory

All symplectic invariant information is contained in the symplectic eigenvalue!

$$\|f^{(\delta)}\|_{1/2}^2 = \frac{4\Gamma(2\delta)\Gamma(\delta+1)^2\Gamma(2\delta+5/2)}{\sqrt{\pi}\Gamma(\delta+1/2)^2\Gamma(2\delta+1)\Gamma(2\delta+2)}, \quad \|f^{(\delta)}\|_{-1/2}^2 = \frac{2\Gamma(\delta+1)^2\Gamma(2\delta+5/2)}{\sqrt{\pi}\Gamma(\delta+1/2)\Gamma(\delta+3/2)\Gamma(2\delta+3)}.$$



$$\nu_A^2 = \text{Det} \sigma_A = \|f^{(\delta)}\|_{-1/2}^2 \|f^{(\delta)}\|_{1/2}^2 > 1$$

for all $\delta \geq 1$

Partner of a System in a Pure State

Proposition: Let A be an N -mode subsystem correlated in the pure Gaussian state J . Then, the single-mode subsystem

$$A_P = \Pi_A^\perp(JA),$$

is the partner of A in J . *

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