Towards a more precise prediction of the dark matter relic density

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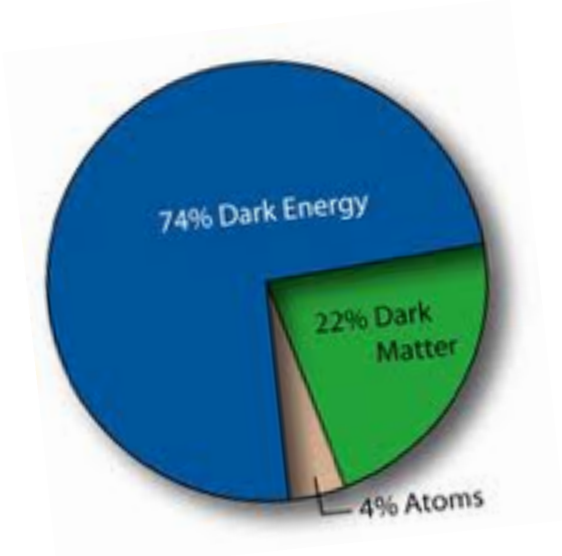
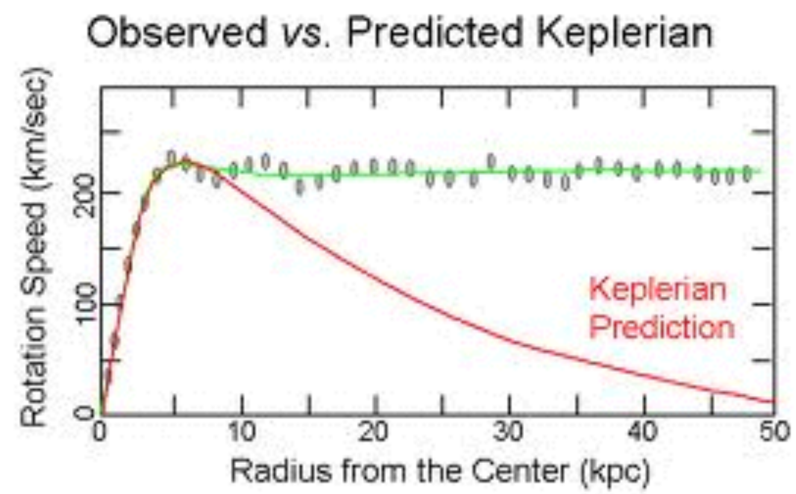
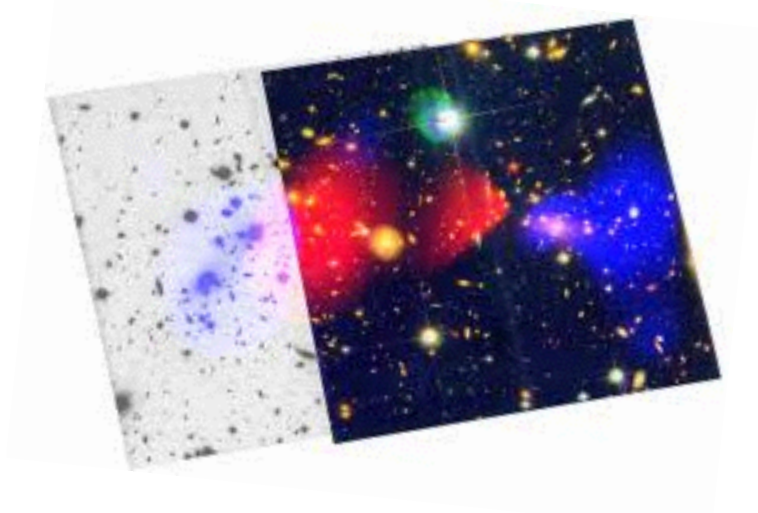
Theory Seminar — Fysisk Institut — Universitetet i Oslo
18 November 2015

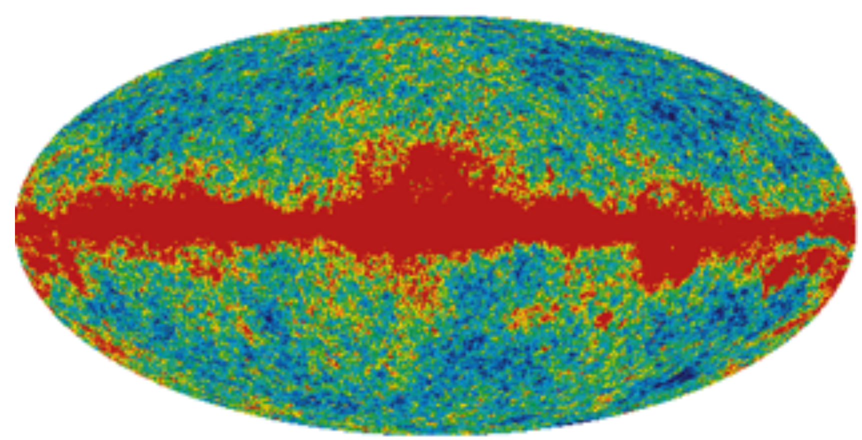
Dark matter…

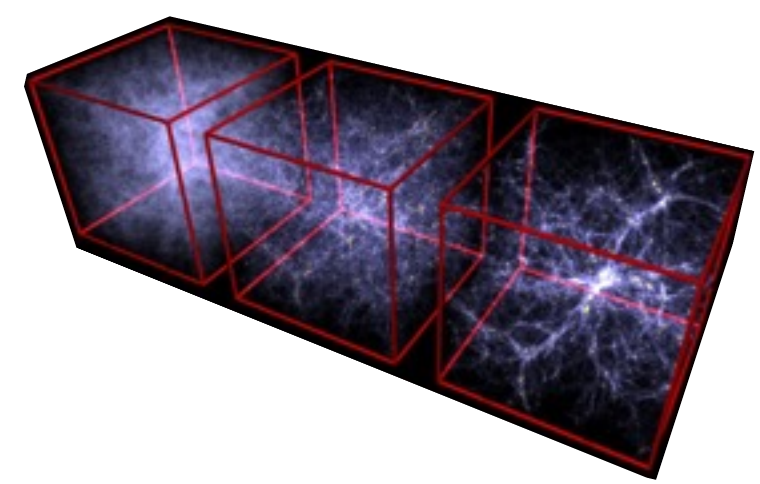




Fritz Zwicky 1933







… accounts for about 22% of the energy content of our Universe

… is supposed to be a "**weakly interacting massive particle**" (WIMP)

… is not explained within the Standard Model

… **therefore strongly hints towards New Physics** — here: Supersymmetry / MSSM

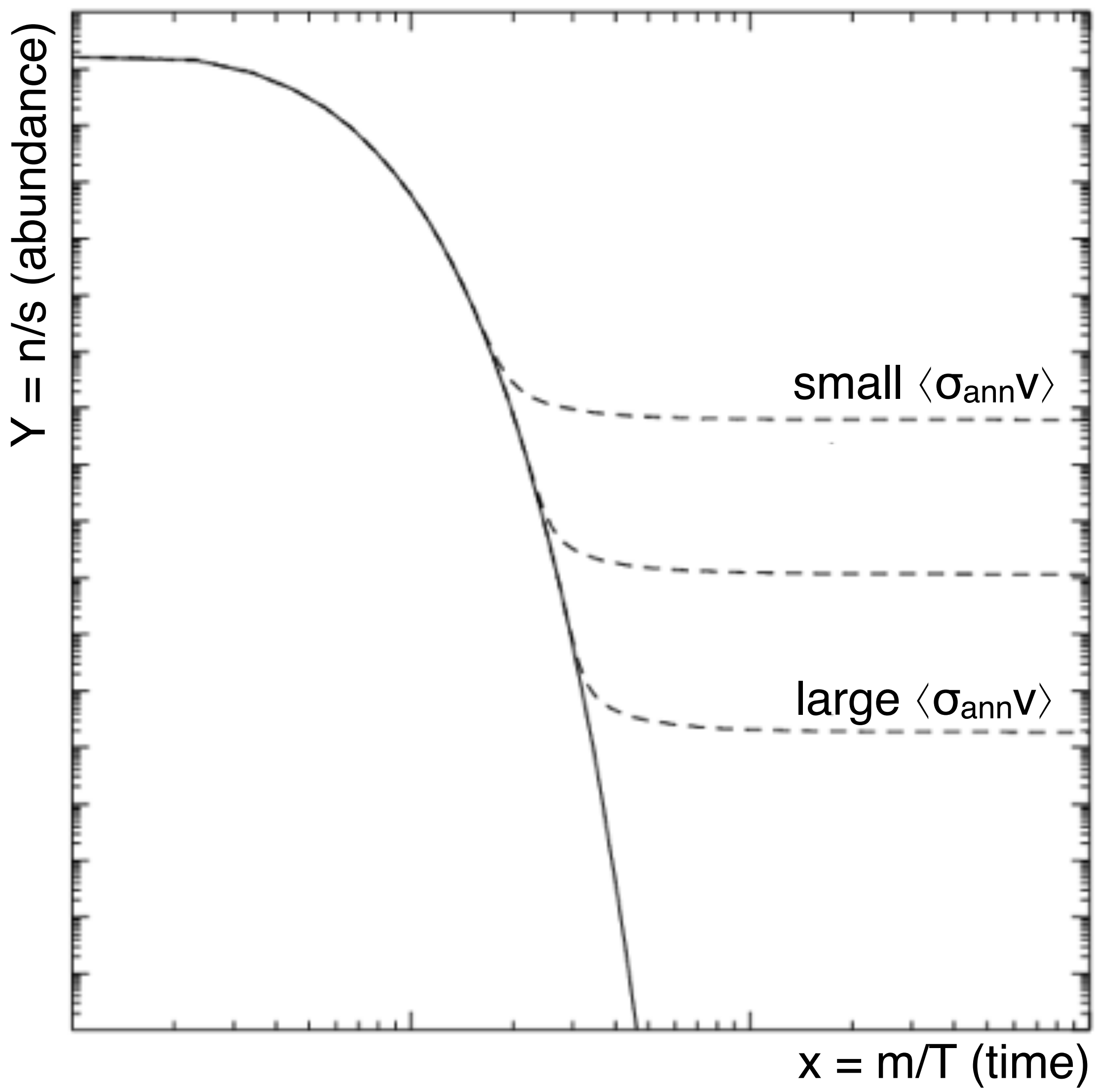
The Minimal Supersymmetric Standard Model (MSSM)

SM Particles		Spin		Spin	Superpartners	
Quarks	$(u_L \;\; d_L)$	$1/2$	Q	0	$(\tilde{u}_L \;\; \tilde{d}_L)$	Squarks
	$u_R^\dagger$	$1/2$	$\bar{u}$	0	$\tilde{u}_R^*$	
	$d_R^\dagger$	$1/2$	$\bar{d}$	0	$\tilde{d}_R^*$	
Leptons	$(\nu \;\; e_L)$	$1/2$	L	0	$(\tilde{\nu} \;\; \tilde{e}_L)$	Sleptons
	$e_R^\dagger$	$1/2$	$\bar{e}$	0	$\tilde{e}_R^*$	
Higgs	$(H_u^+ \;\; H_u^0)$	0	H_u			
	$(H_d^0 \;\; H_d^-)$	0	H_d	$1/2$	$\tilde{\chi}^0_{1,2,3,4}$	Neutralinos
W bosons	$W^0, W^\pm$	1		$1/2$	$\tilde{\chi}^\pm_{1,2}$	Charginos
B boson	B^0	1				
Gluon	g	1		$1/2$	$\tilde{g}$	Gluino
Graviton	G	2		$3/2$	$\tilde{G}$	Gravitino

Lightest neutralino is dark matter (WIMP) candidate "par excellence"

$$\tilde{\chi}^0_1 \;=\; Z_{1\tilde{B}}\tilde{B} + Z_{1\tilde{W}}\tilde{W} + Z_{1\tilde{H}_1}\tilde{H}_1 + Z_{1\tilde{H}_2}\tilde{H}_2$$

Dark matter relic abundance — freeze-out picture



Time evolution of number density of the relic particle described by Boltzmann equation

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3Hn - \langle \sigma_{\mathrm{ann}} v \rangle \left(n^2 - n_{\mathrm{eq}}^2\right)$$

Prediction of dark matter relic density (if masses and interactions are known)

$$\Omega_\chi h^2 = \frac{m_\chi n_\chi}{\rho_{\mathrm{crit}}} \sim \frac{1}{\langle \sigma_{\mathrm{ann}} v \rangle}$$

(dis)favoured parameter regions…?

$$\Omega_{\mathrm{CDM}} h^2 = 0.1199 \pm 0.0022$$

Dark matter relic abundance very precisely known
Planck collaboration 2015

Computational tools allow an efficient calculation of the (neutralino) relic density:
DarkSUSY Bergström, Edsjö, Gondolo *et al.* 2004-2015, **micrOMEGAs** Bélanger, Boudjema, Pukhov *et al.* 2003-2015, **SuperIsoRelic** Arbey, Mahmoudi 2008, …

A closer look on the (co)annihilation cross-section

Time evolution of relic particle described by Boltzmann equation

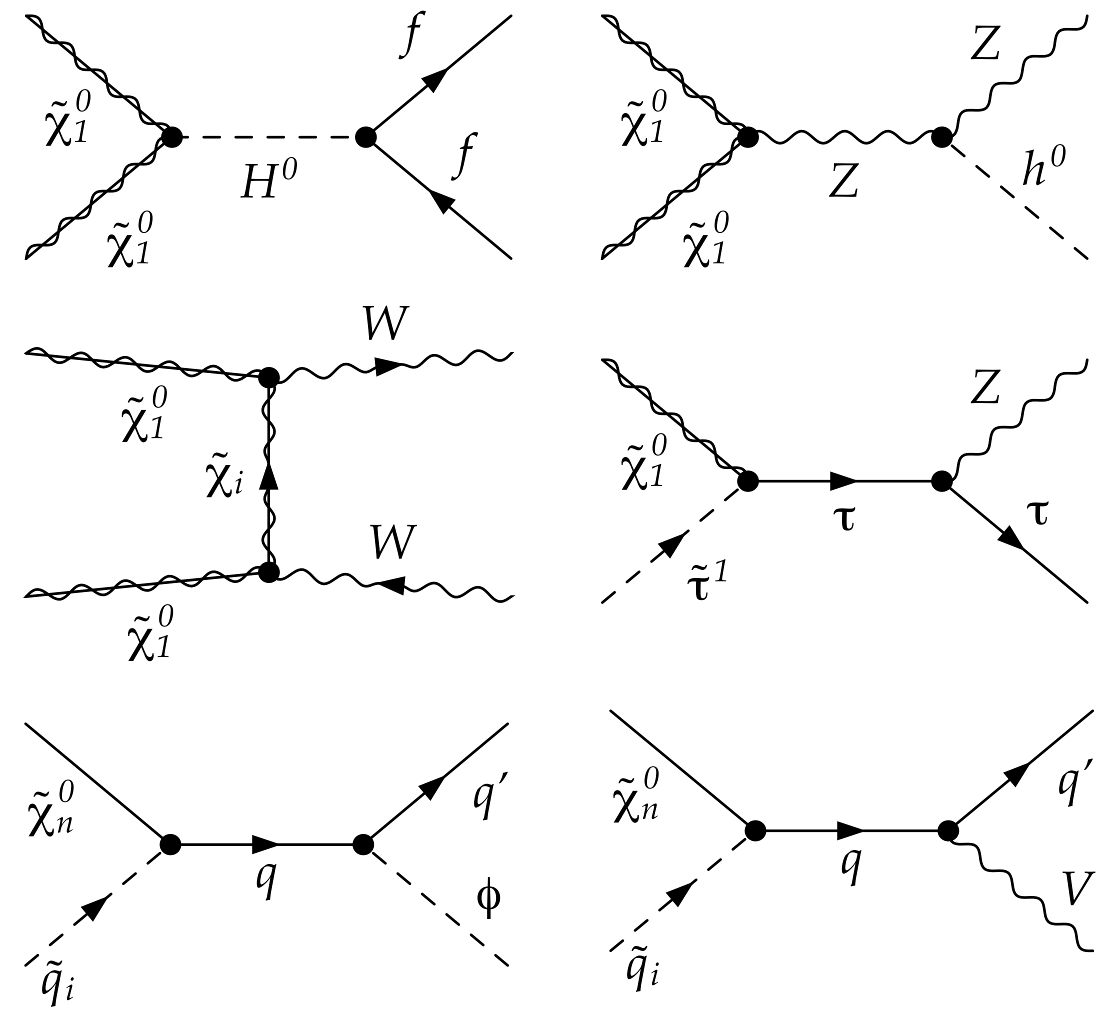
$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3Hn - \langle \sigma_{\mathrm{ann}} v \rangle \left(n^2 - n_{\mathrm{eq}}^2\right)$$

↑

$$\langle \sigma_{\mathrm{ann}} v \rangle = \sum_{i,j} \sigma_{ij} v_{ij} \frac{n_i^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}} \frac{n_j^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}}$$

↑

$$\frac{n_i^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}} \sim \exp\left\{-\frac{m_i - m_\chi}{T}\right\}$$



Only co-annihilations with almost mass-degenerate particles are numerical relevant

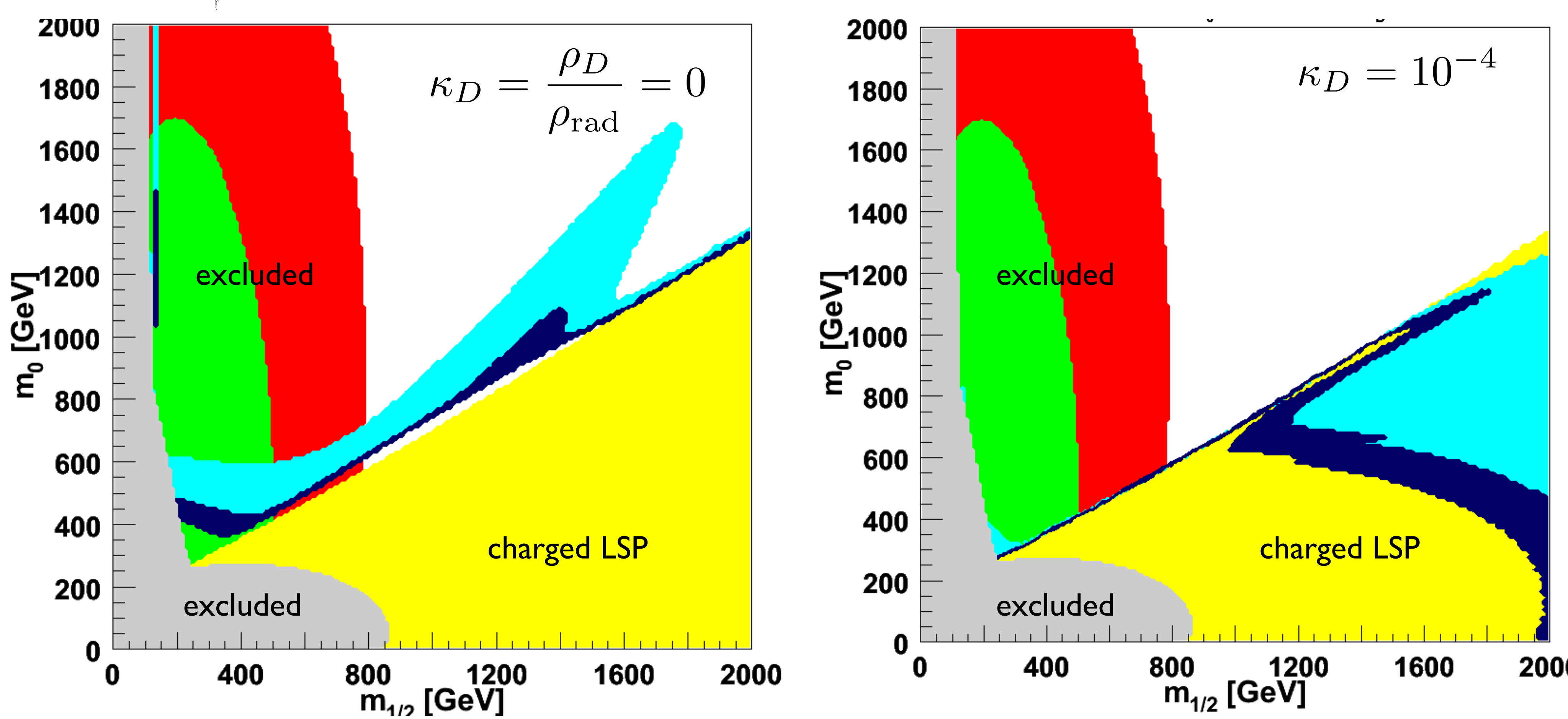
Typical examples in MSSM: other neutralinos, charginos, stau, stop

Uncertainties — Cosmology…

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3Hn - \langle \sigma_{\mathrm{ann}} v \rangle \left(n^2 - n_{\mathrm{eq}}^2 \right)$$

$$\Omega_\chi h^2 = \frac{m_\chi n_\chi}{\rho_{\mathrm{crit}}} \sim \frac{1}{\langle \sigma_{\mathrm{ann}} v \rangle}$$

Hubble expansion rate might be modified by presence of e.g. dark radiation in early universe… — **SuperIso Relic**



Arbey, Mahmoudi — Phys.Lett. B669 (2008) 46-51 — arXiv:0803.0741 [hep-ph]

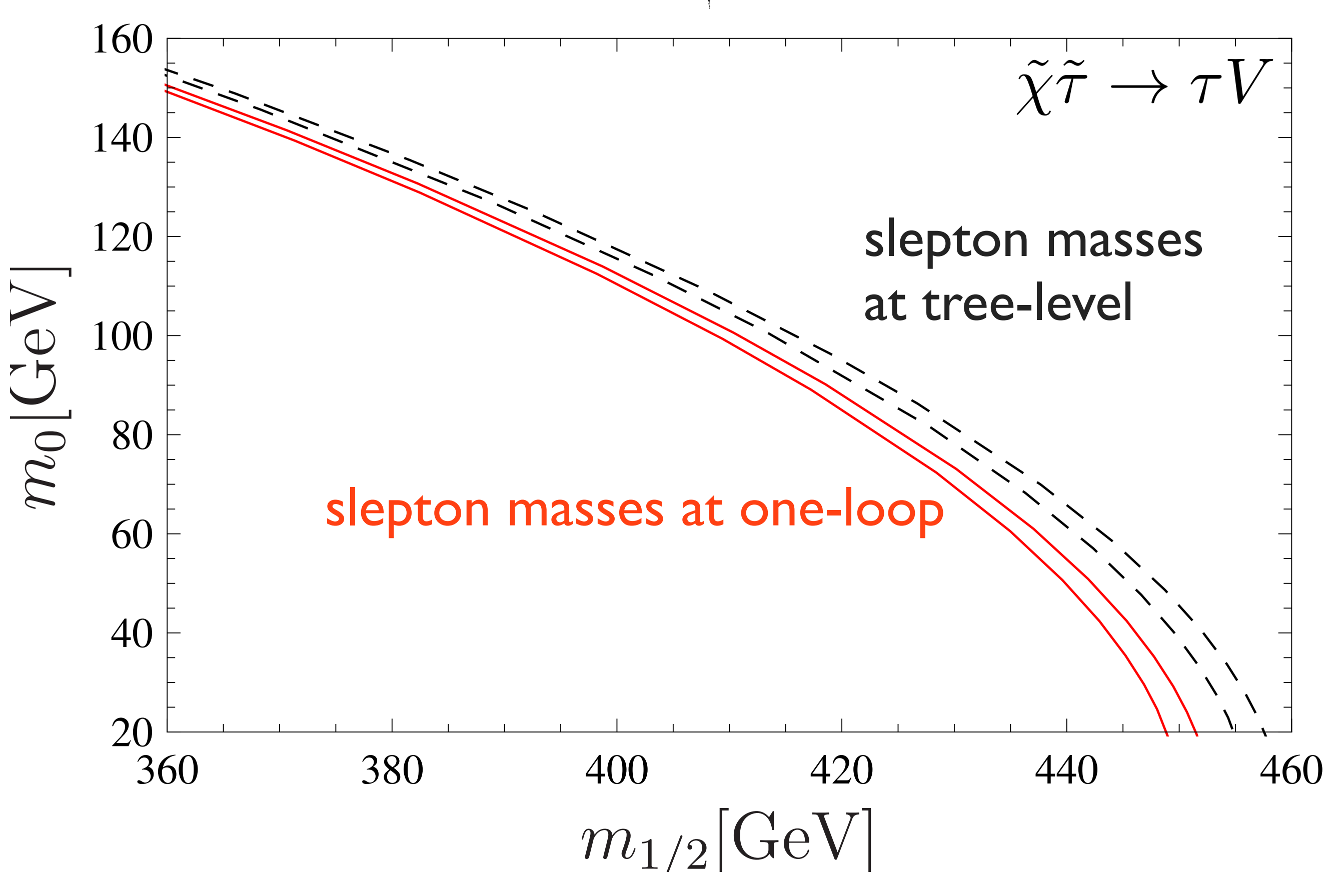
Uncertainties — Particle physics…

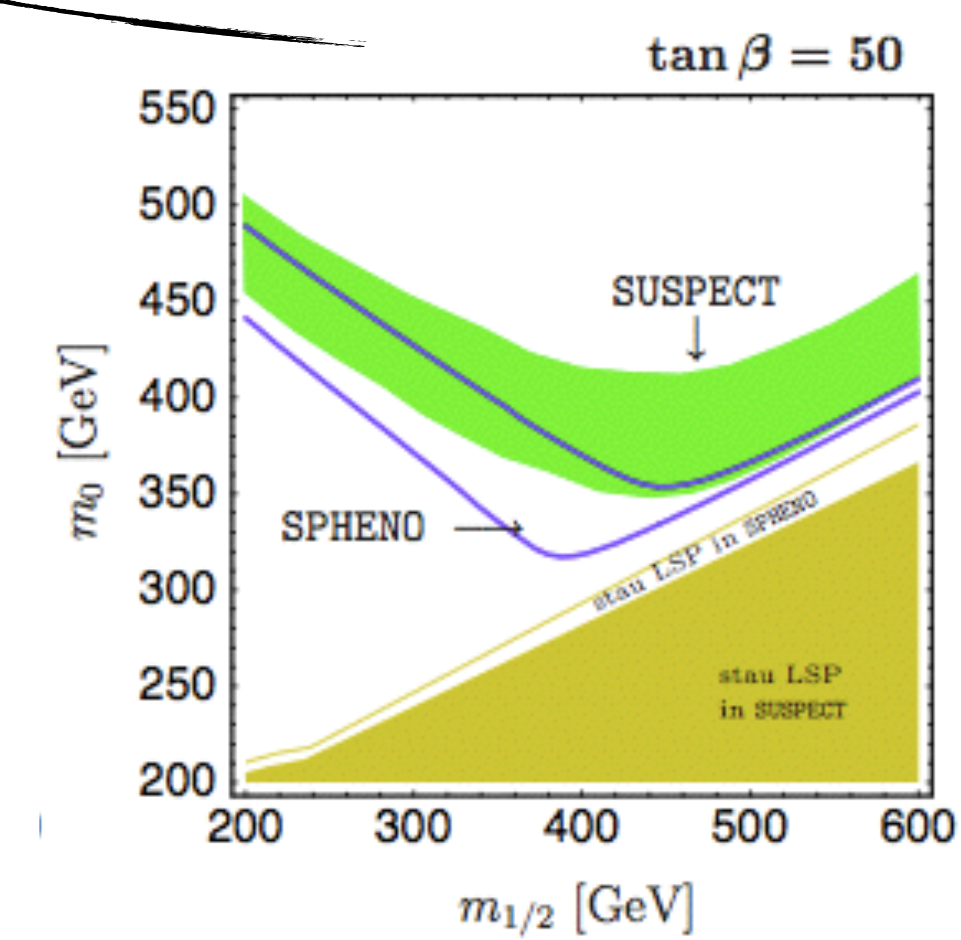
$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3Hn - \langle \sigma_{\mathrm{ann}} v \rangle \left(n^2 - n_{\mathrm{eq}}^2 \right)$$

$$\Omega_\chi h^2 = \frac{m_\chi n_\chi}{\rho_{\mathrm{crit}}} \sim \frac{1}{\langle \sigma_{\mathrm{ann}} v \rangle}$$

Particle mass calculation

Spectrum generator

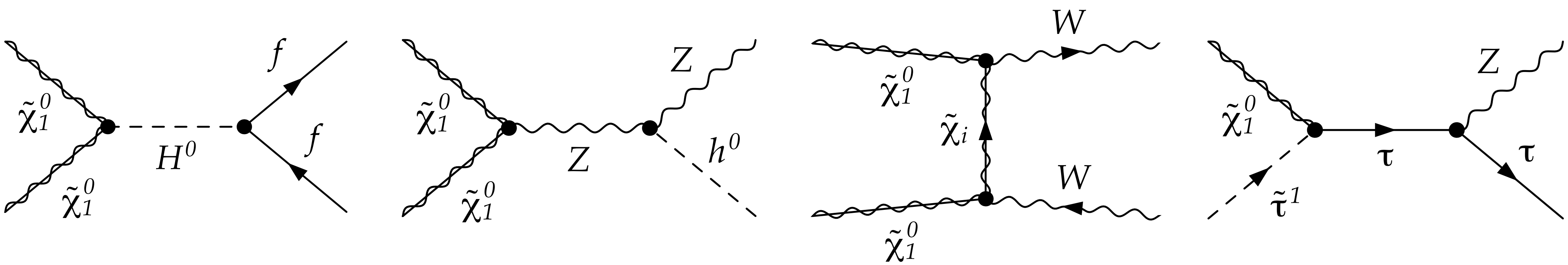




Staub, Herrmann, Porod — JHEP 1010: 040 (2010) — arXiv:1007.4049 [hep-ph]
Bélanger, Kraml, Pukhov — Phys.Rev. D72 (2005) 015003 — arXiv:hep-ph/0502079

Motivation for higher order corrections

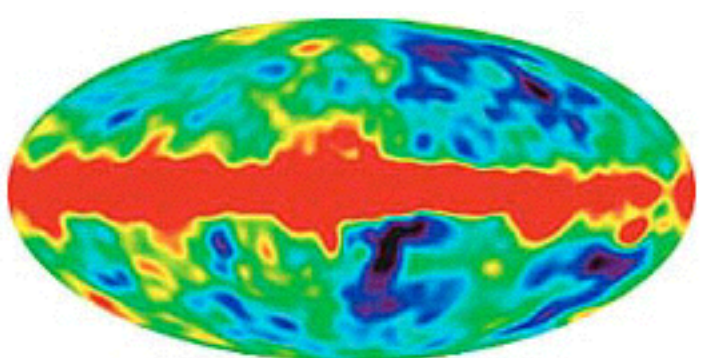
All processes implemented in public codes — **but only at the (effective) tree-level**



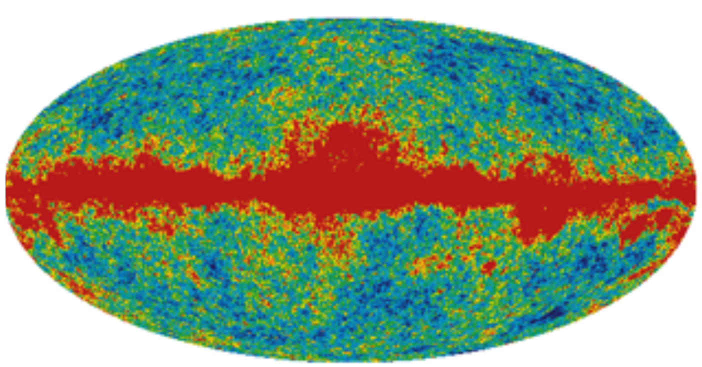
Higher-order loop corrections can give important contributions to cross-sections

In particular, sizeable impact from QCD corrections due to strong coupling constant

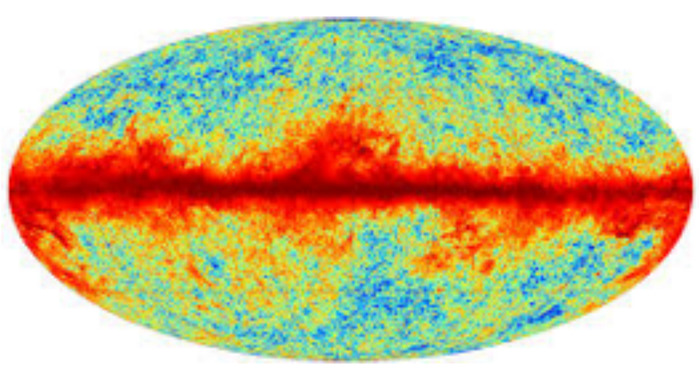
More precise theoretical predictions needed to keep up with experimental improvements



COBE 1989



WMAP 2002



Planck 2013

DM@NLO project — **Provide calculation of σ_{ann} including QCD corrections**

— Extension to public codes (e.g. micrOMEGAs, DarkSUSY)…

DM@NLO — Collaboration

Universität Münster

Sonja Esch, Oleh Fedkevych,
Karol Kovarik, Michael Klasen,
Saskia Schmiemann, Patrick Steppeler

Westfälische Wilhelms-Universität Münster

Graduiertenkolleg 2149
Research Training Group

LAPTh Annecy-le-Vieux

Björn Herrmann

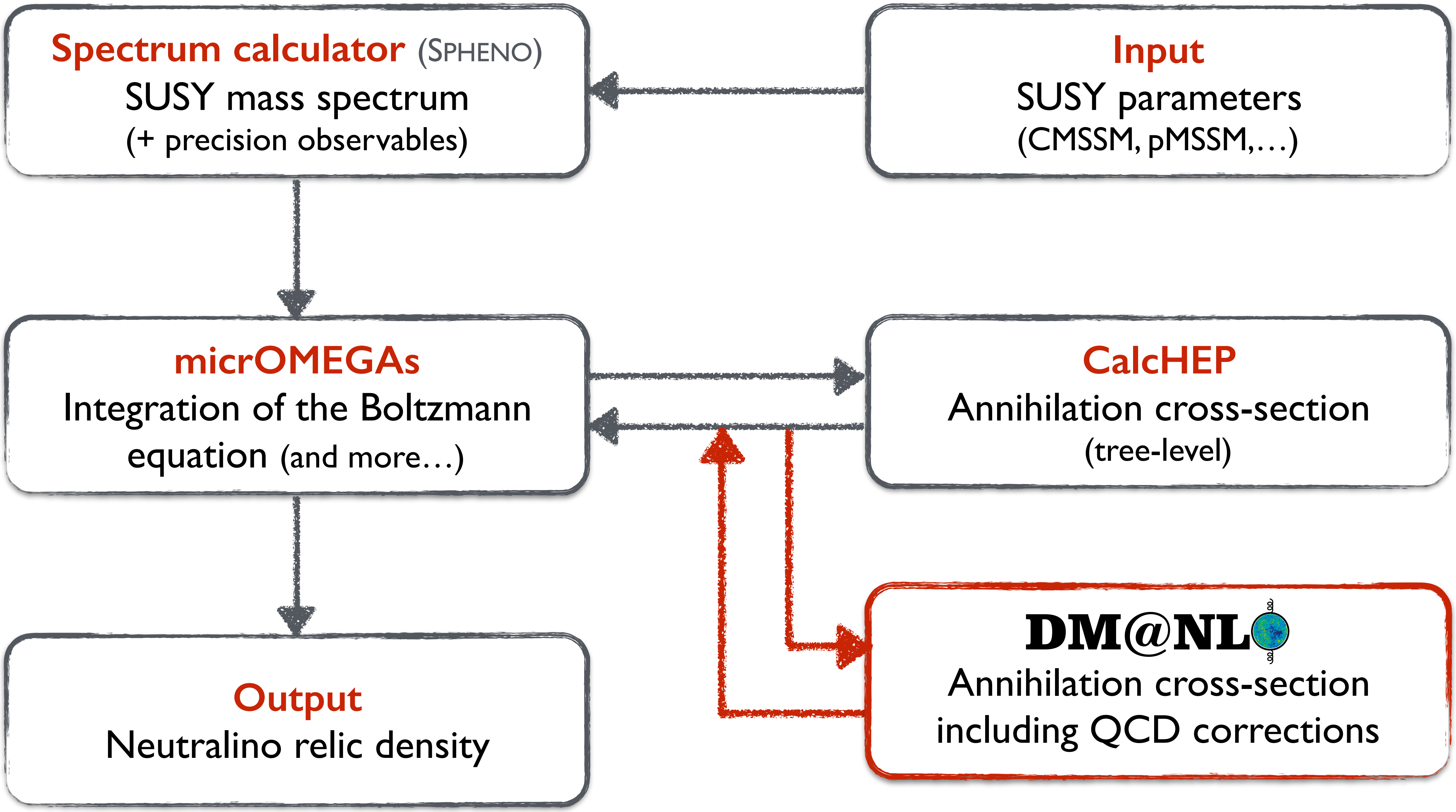
Université Savoie Mont Blanc

LPTHE Paris

Julia Harz

http://dmnlo.hepforge.org

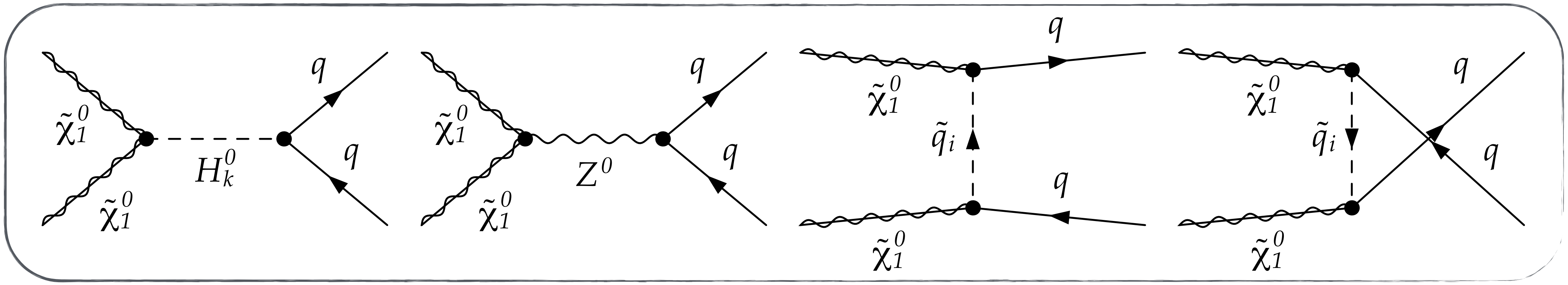
DM@NLO — Setup



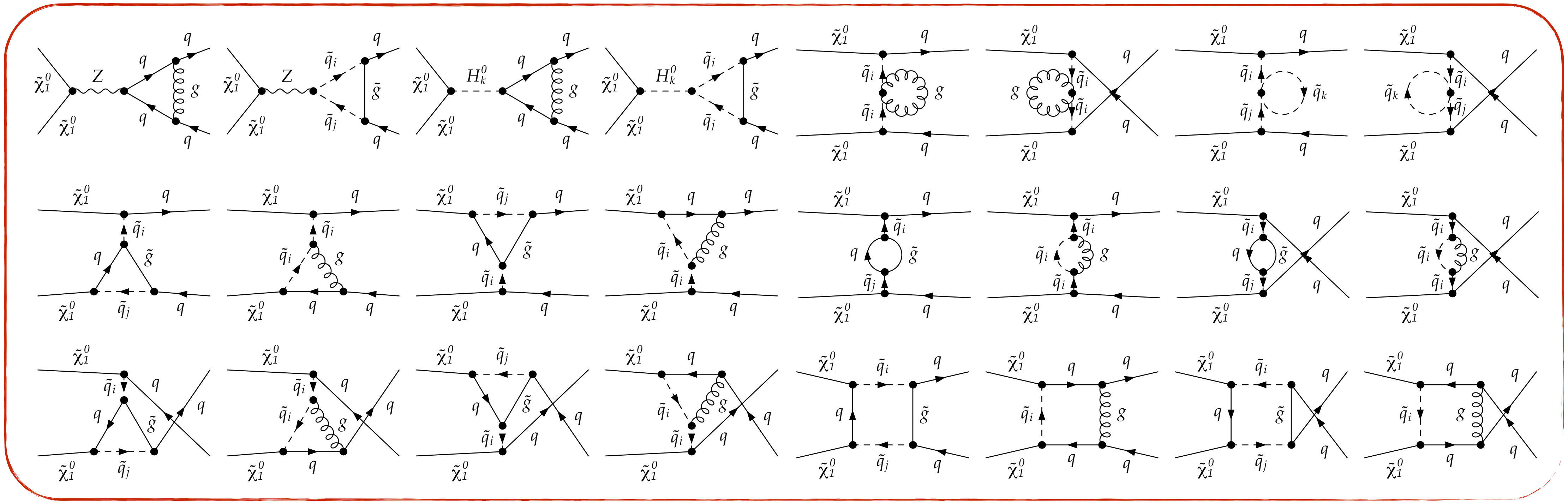
Similar setup for use with DarkSUSY in development…

J. Edsjö, B. Herrmann, C. Niblaeus — *in progress…*

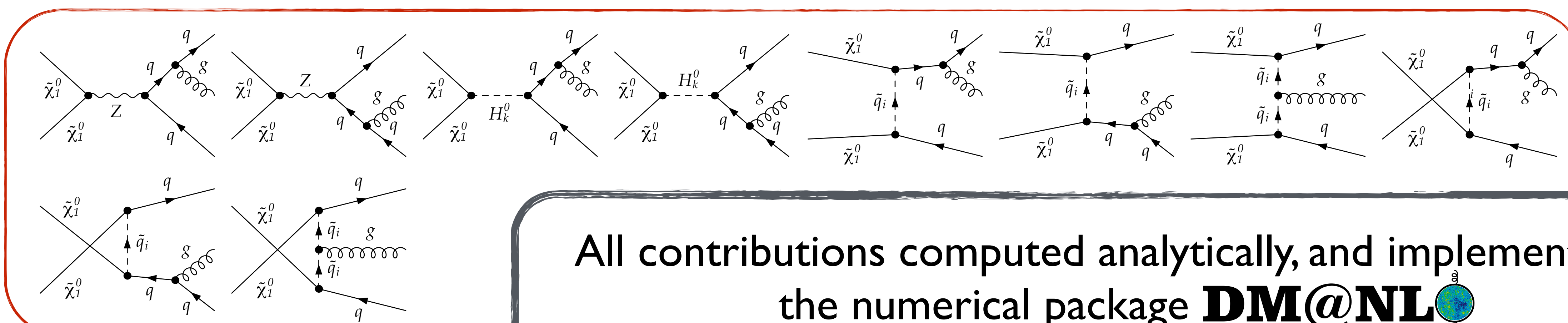
Neutralino pair annihilation into quarks



Loop corrections



Real emission



Herrmann, Klasen (2007); Herrmann, Klasen, Kovarik (2009),
Herrmann, Klasen, Kovarik, Meinecke, Steppeler (2013)

All contributions computed analytically, and implemented in the numerical package **DM@NLO**
(goal: extension to existing dark matter codes)

Interlude — a few technical details

Loop diagrams include UV-divergent integrals → **Renormalization!**

Hybrid on-shell/$\overline{\text{DR}}$ renormalization scheme for the squark sector (3rd generation), which is applicable to all (co)annihilation processes

$$A_t, A_b \quad m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{b}_2} \longrightarrow M^2_{\tilde{Q}}, M^2_{\tilde{U}}, M^2_{\tilde{D}} \quad m_{\tilde{t}_2}, \theta_{\tilde{t}}, \theta_{\tilde{b}}$$

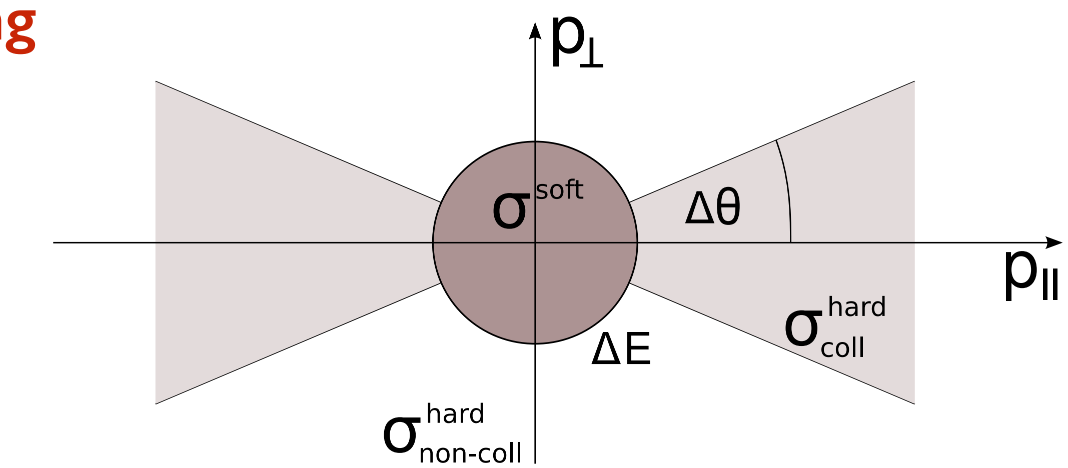
Independent "input" parameters → Dependent parameters

Loop diagrams contain **IR-divergencies** (soft and/or collinear), which vanish when taking into account the real emission of a gluon (2→3 processes)

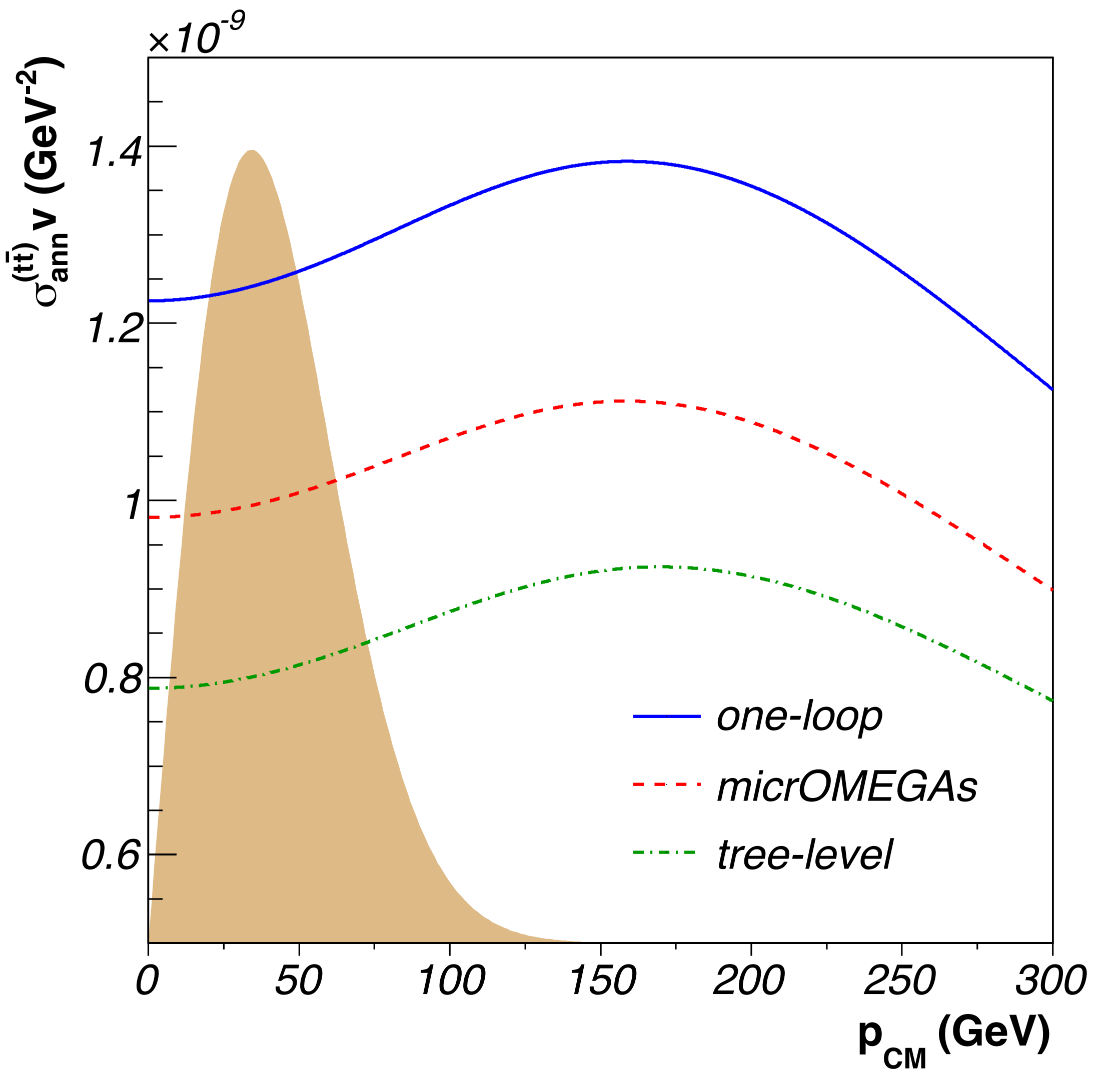
Dipole Subtraction Method and **Phase Space Slicing**

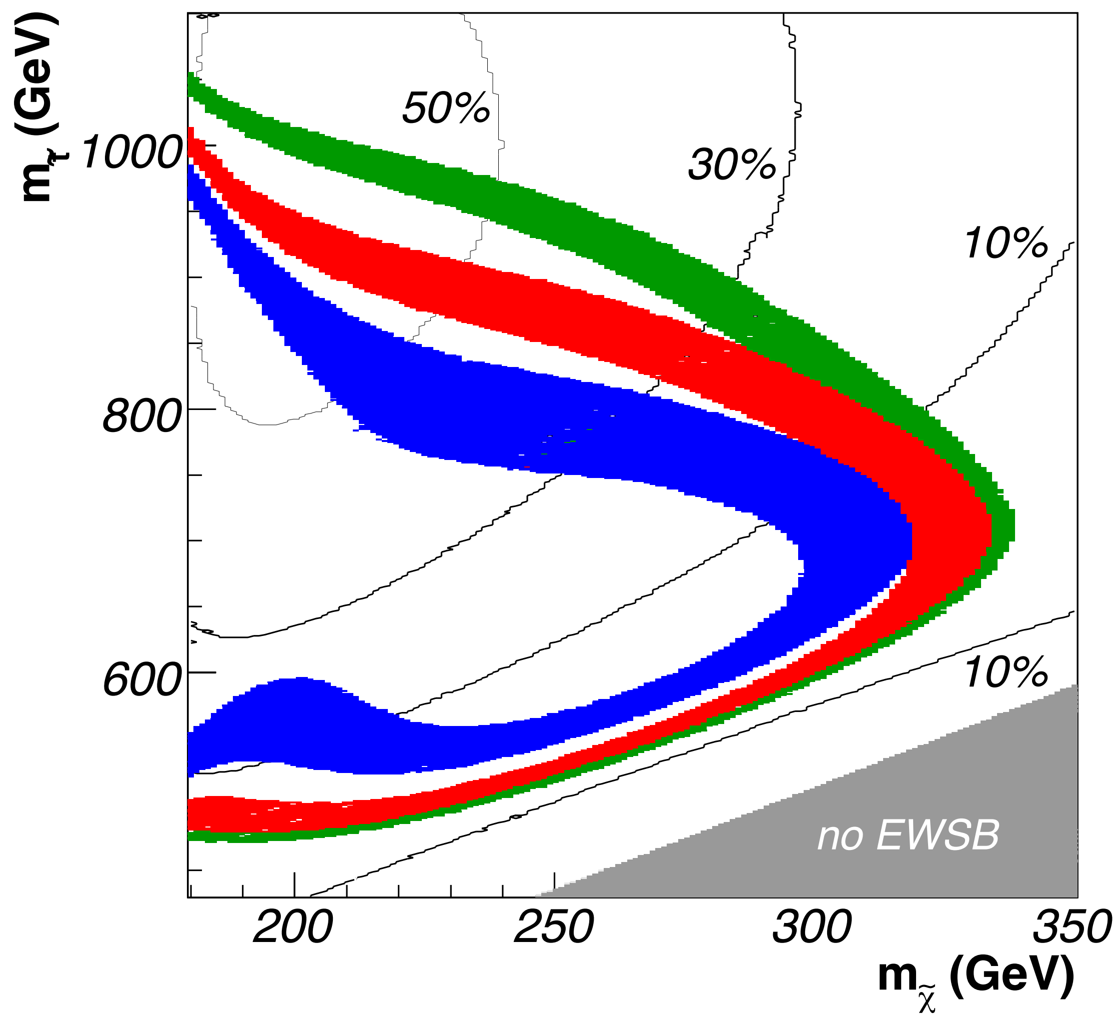
Catani, Seymour (2001)

$$\sigma_{\rm NLO} = \int_3 \left[{\rm d}\sigma^{\rm R}\Big|_{\epsilon=0} - {\rm d}\sigma^{\rm A}\Big|_{\epsilon=0} \right] + \int_2 \left[{\rm d}\sigma^{\rm V} + \int_1 {\rm d}\sigma^{\rm A} \right]_{\epsilon=0}$$



Neutralino pair annihilation into top quarks





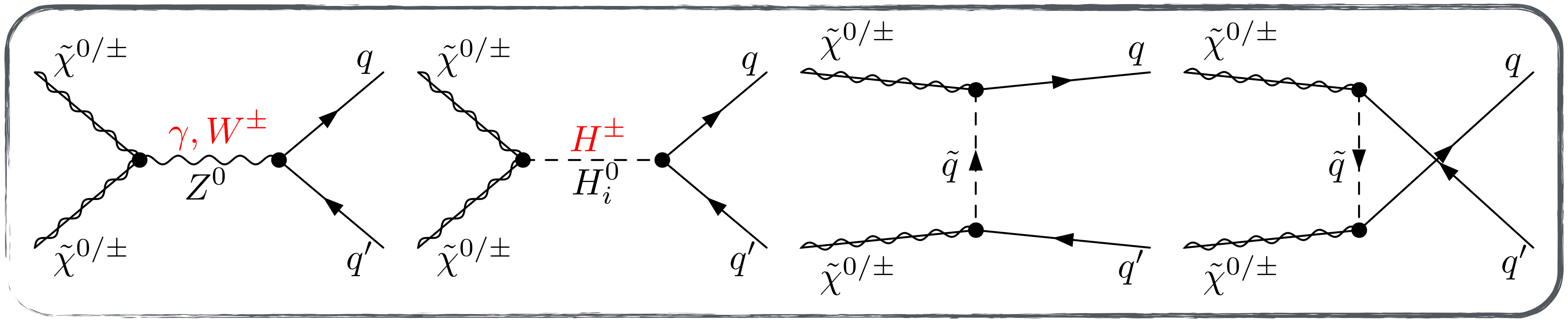
Annihilation cross-section enhanced by up to 50% by radiative corrections

Corrections can lead to **important shifts for preferred regions** (e.g. ~200 GeV for m_{stop})

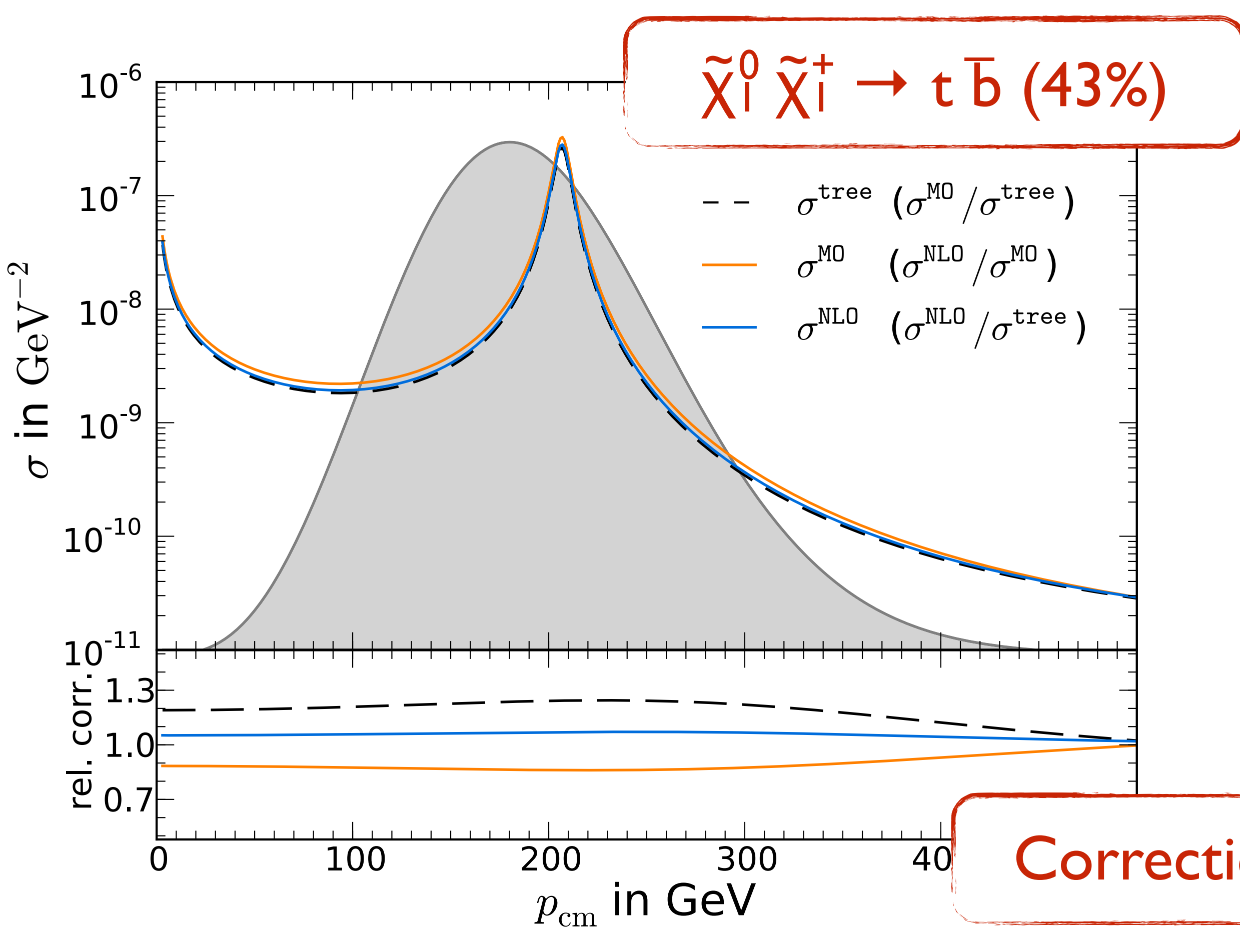
Effective Yukawa couplings (as e.g. in micrOMEGAs) very good approximation around Higgs-resonances, **but other sub-channels can be dominant** (here: Z/squark-exchange)

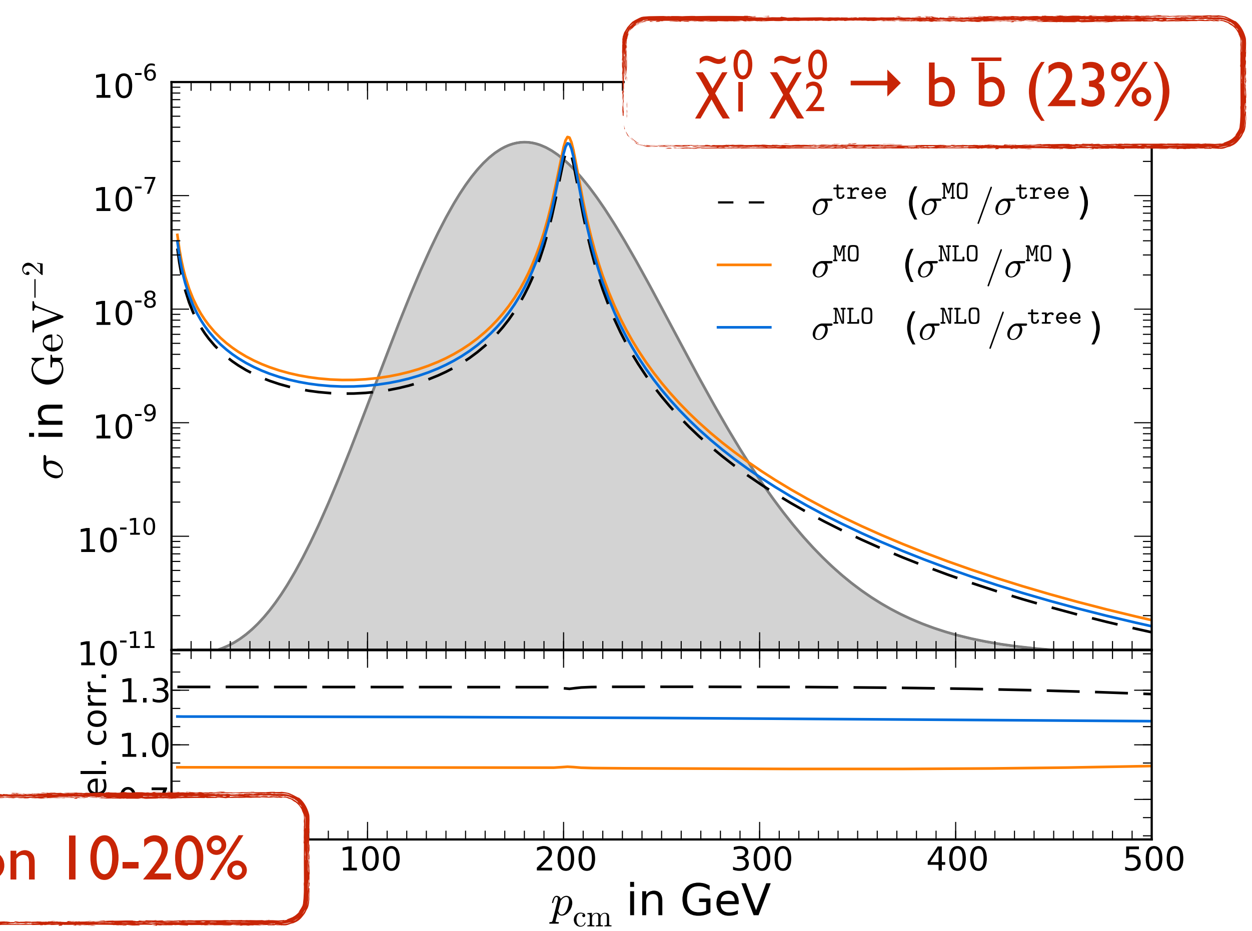
Herrmann, Klasen, Kovarik (2009); Herrmann, Klasen, Kovarik, Meinecke, Steppeler (2013)

Generalisation to gaugino pair-annihilation



$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{\chi}_4^0}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$Z_{1\tilde{B}}$	$Z_{1\tilde{W}}$	$Z_{1\tilde{H}_1}$	$Z_{1\tilde{H}_2}$	m_{h^0}	$\Omega_{\tilde{\chi}_1^0}h^2$	$\mathrm{BR}(b \to s\gamma)$
738.2	802.4	1288.4	1294.5	802.3	1295.1	-0.996	0.049	-0.059	0.037	126.3	0.1243	$3.0 \cdot 10^{-4}$

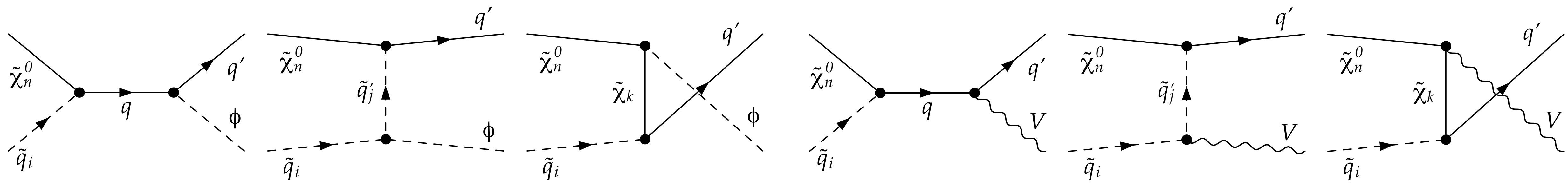


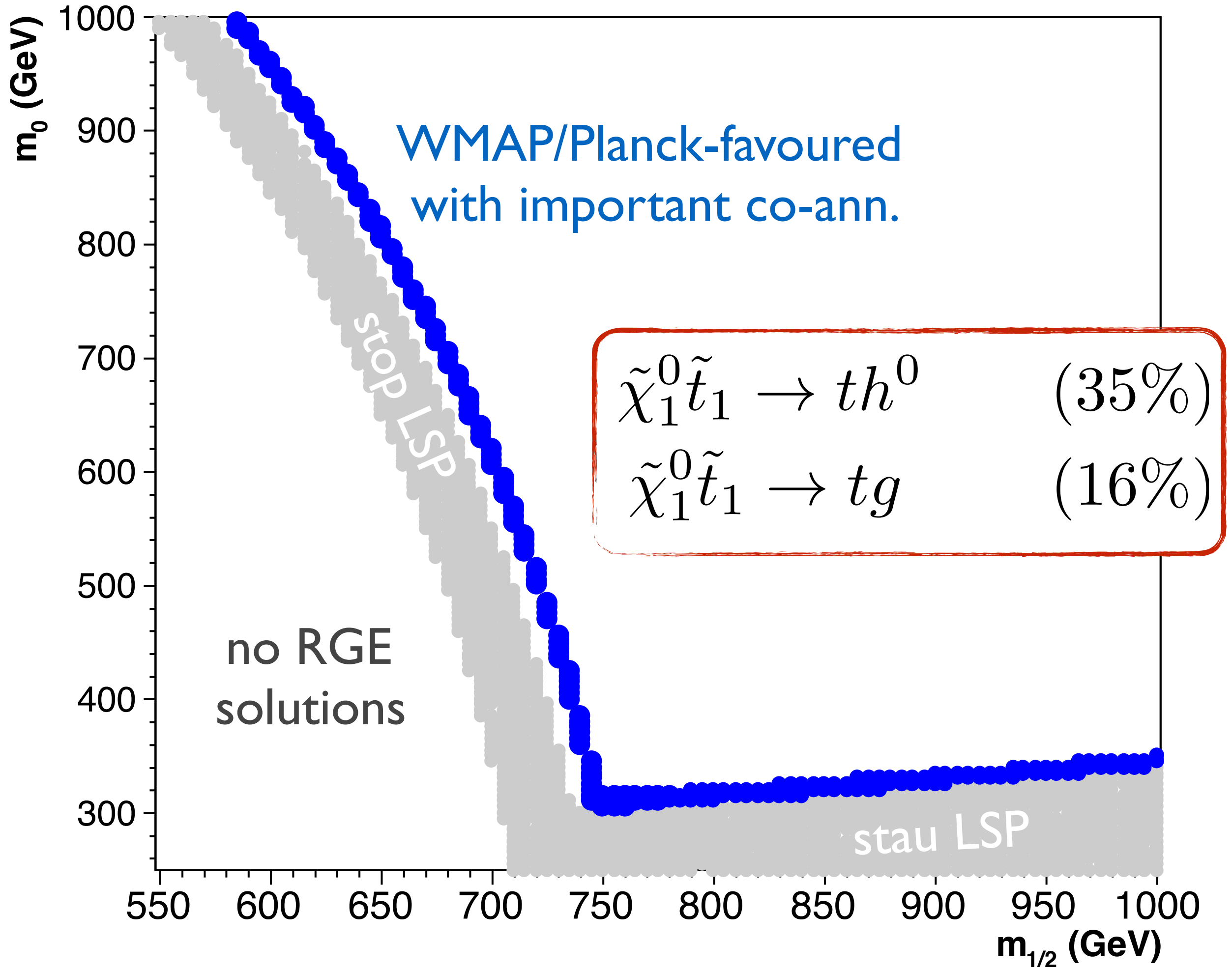


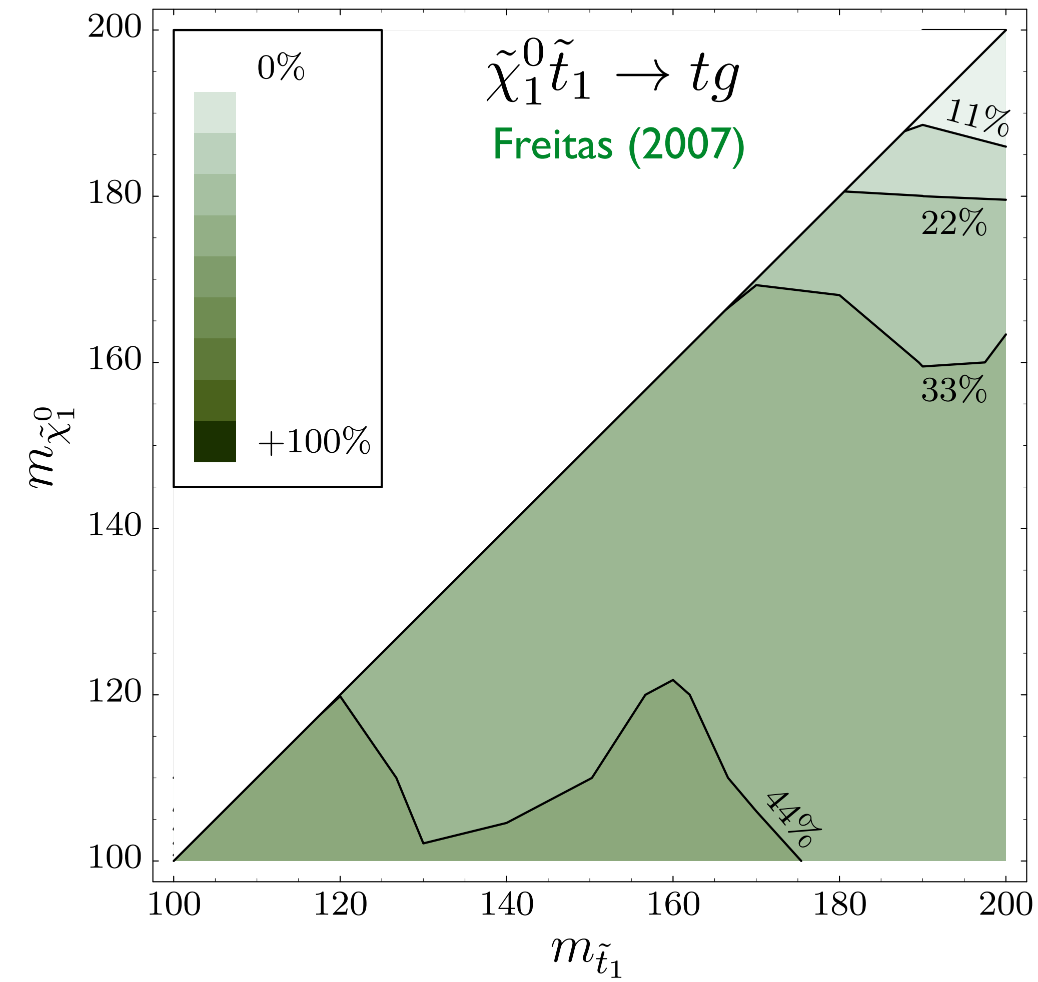
Correction 10-20%

[Herrmann, Klasen, Kovarik, Meinecke, Steppeler (2014)]

Neutralino-stop co-annihilation





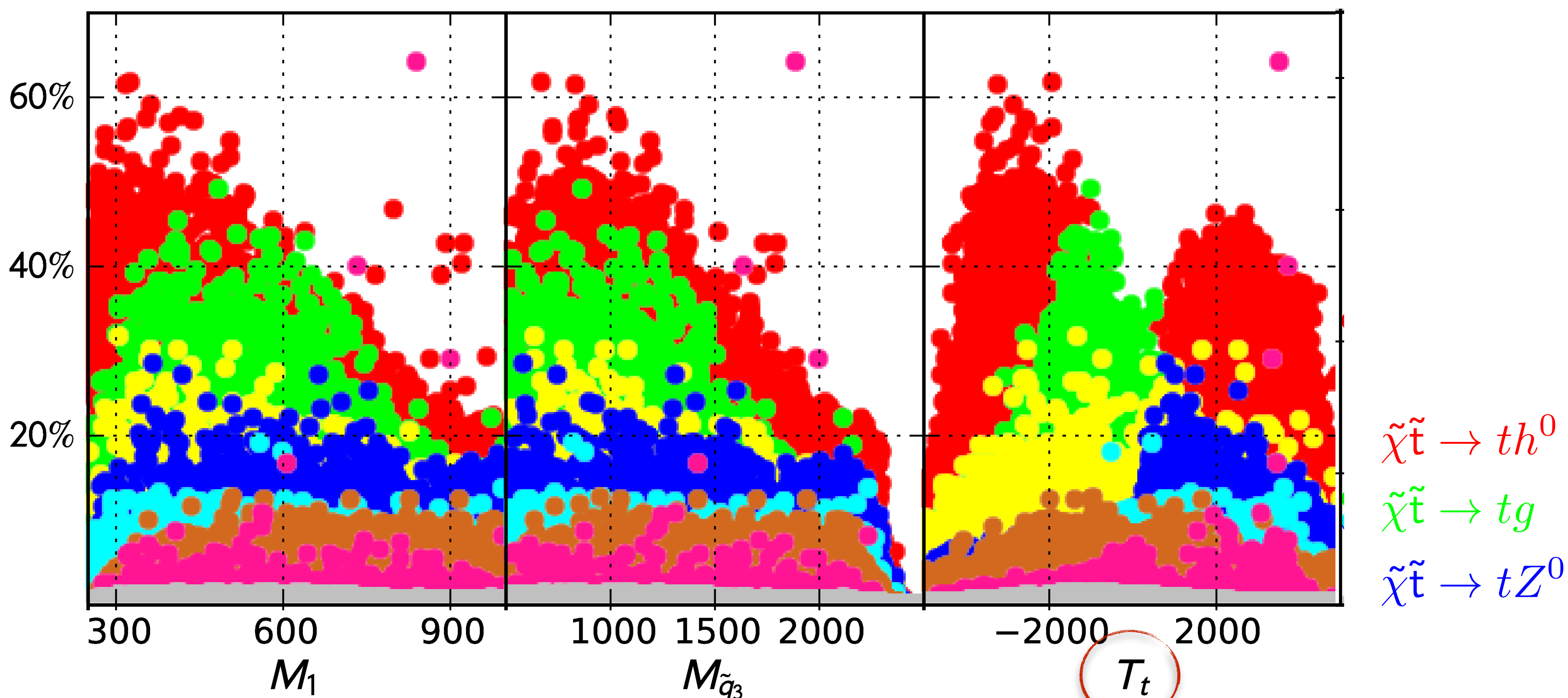


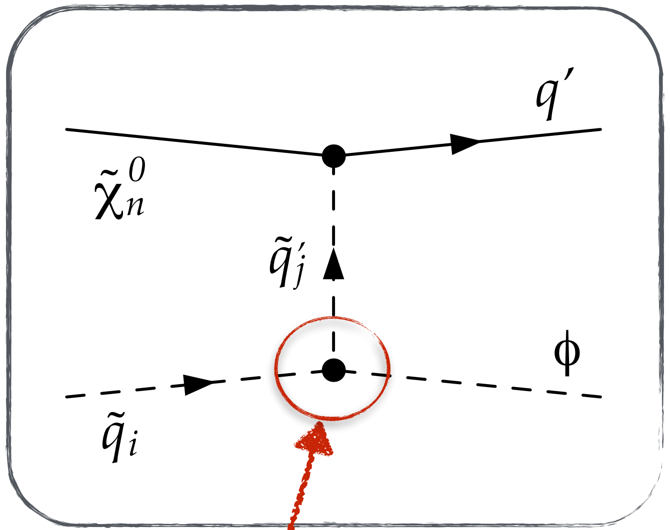
Radiative corrections to neutralino-stop co-annihilation into top+gluon have shown to be important, but only for a rather restricted setup ($\tilde{B}\tilde{t}_R \to tg$ and $\tilde{B}\tilde{t}_R \to bW^+$)

Other final states, in particular top+Higgs, can be equally important…

Harz, Herrmann, Klasen, Kovařík, Le Boulc'h — Phys. Rev. D 87: 054031 (2013) — arXiv:1212.5241 [hep-ph]

Neutralino-stop co-annihilation





Contribution from $\tilde{\chi}_1\tilde{t}_1 \to th^0$ via t-channel stop-exchange increased

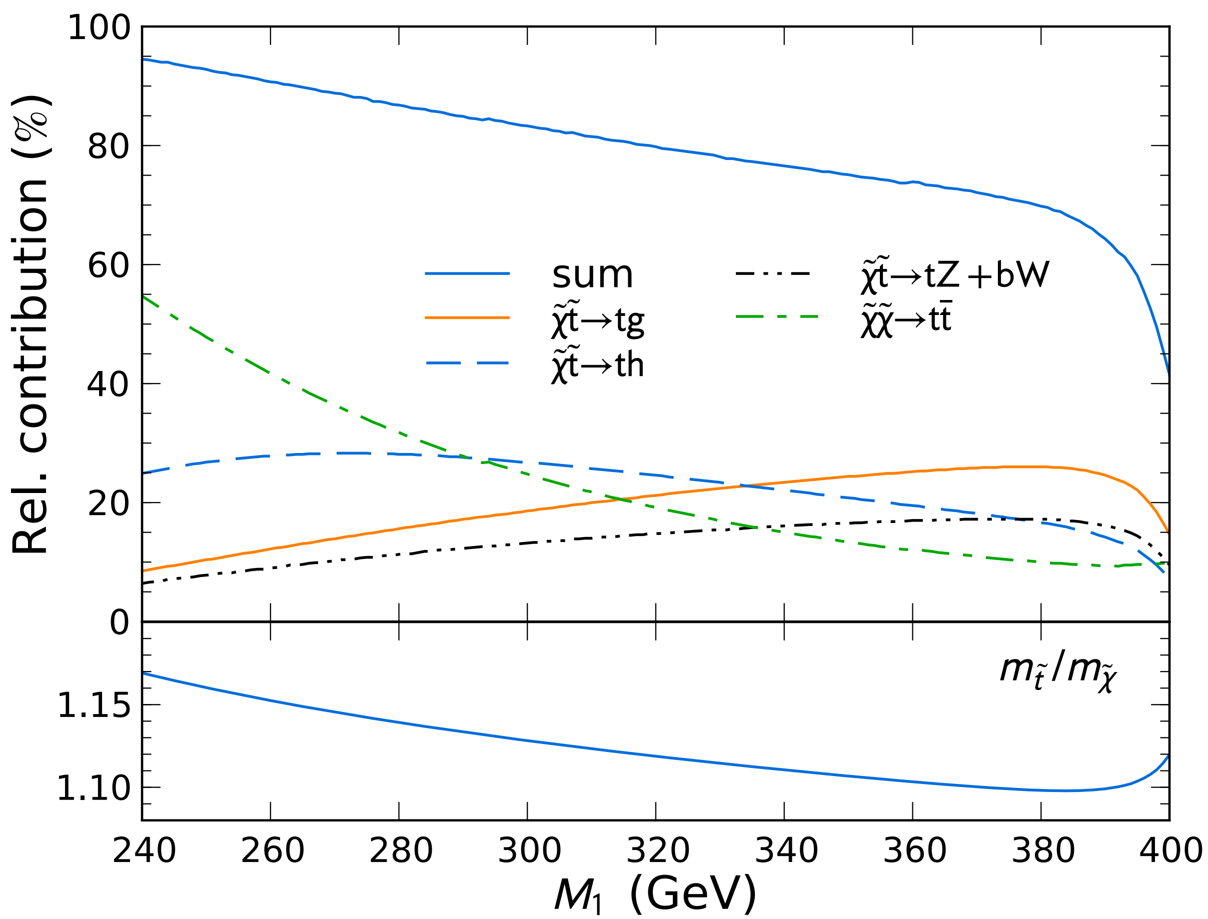
The observed Higgs mass favours large mass splitting of the stops, typically achieved through large trilinear coupling A_t

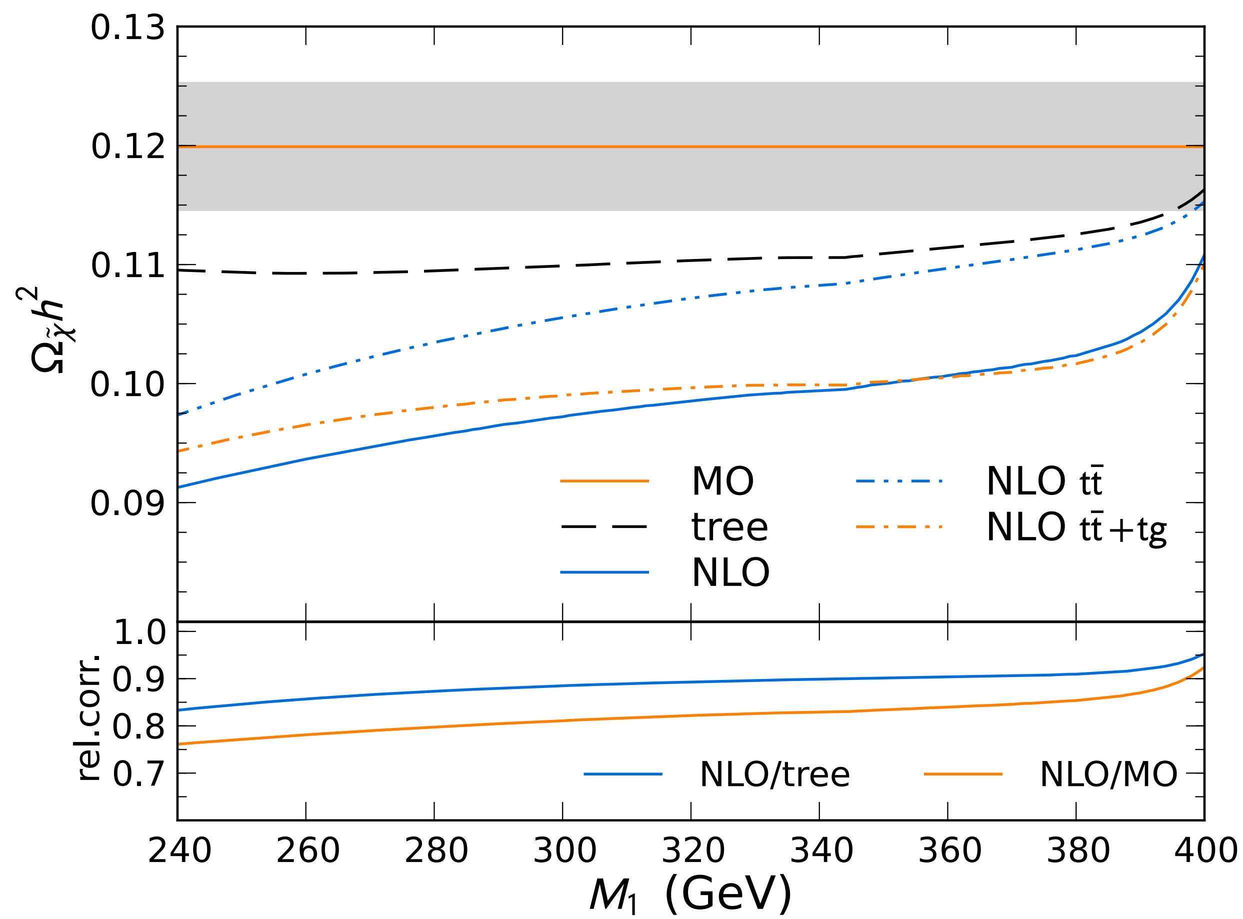
$$m_{h^0} = m_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\log\frac{M^2}{m_t^2} + \frac{X_t^2}{M^2}\left(1 - \frac{X_t^2}{12M^2}\right)\right]$$

$$X_t = A_t - \mu\tan\beta \approx \sqrt{6}M = \sqrt{6}\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$$

Harz, Herrmann, Klasen, Kovařík, Le Boulc'h — Phys. Rev. D 87: 054031 (2013) — arXiv:1212.5241 [hep-ph]

Neutralino-stop co-annihilation





Relative corrections of 40-50% observed for the co-annihilation cross-section, leading to an **important shift** (up to almost 25% — more than Planck uncertainty!) for the predicted **neutralino relic density**

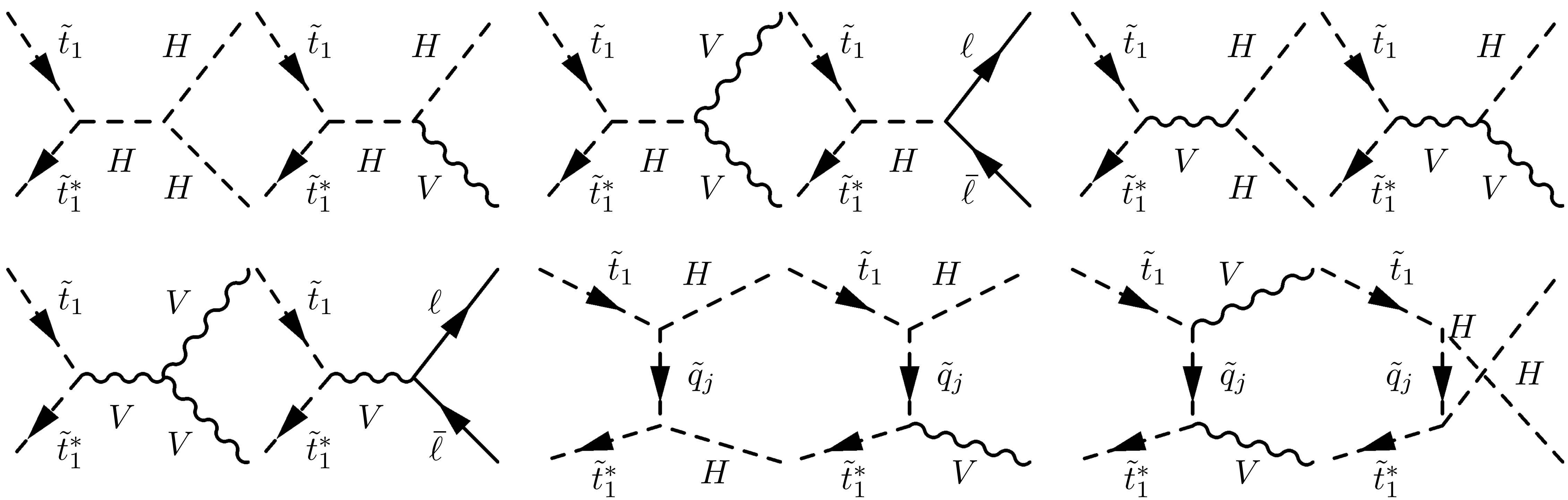
Co-annihilation into SM-like Higgs and gluon most important (other final states generally subdominant)

Harz, Herrmann, Klasen, Kovařík, Le Boulc'h — Phys. Rev. D 87: 054031 (2013) — arXiv:1212.5241 [hep-ph]
Harz, Herrmann, Klasen, Kovařík — Phys. Rev. D 91: 034028 (2015) — arXiv:1409.2898 [hep-ph]

Stop pair annihilation

Numerically important or even dominant when stop mass very close to neutralino mass

$$\langle \sigma_{\text{ann}} v \rangle = \sum_{i,j} \sigma_{ij} v_{ij} \frac{n_i^{\text{eq}}}{n_\chi^{\text{eq}}} \frac{n_j^{\text{eq}}}{n_\chi^{\text{eq}}} \quad \longrightarrow \quad \frac{n_i^{\text{eq}}}{n_\chi^{\text{eq}}} \sim \exp\left\{ - \frac{m_i - m_\chi}{T} \right\}$$

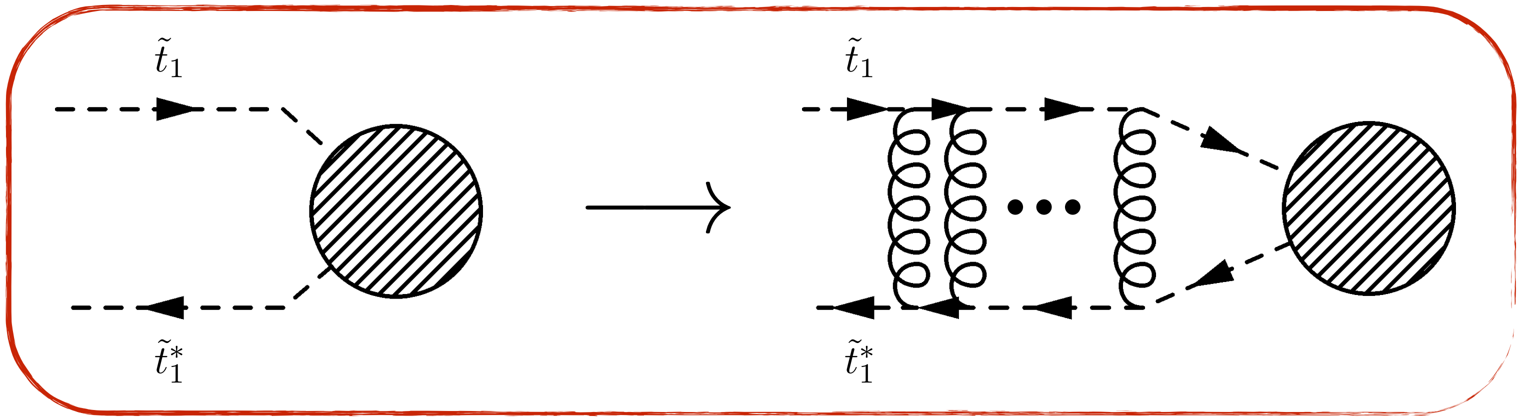


Stop pair annihilation into **electroweak final states** included in **DM@NLO**
— **coloured final states** to be implemented…

Harz, Herrmann, Klasen, Kovařík, Meinecke — Phys. Rev. D 91: 034012 (2015) — arXiv:1410.8063 [hep-ph]
Herrmann, Klasen, Kovařík, Schmiemann — *in progress…*

Stop pair annihilation — Coulomb corrections

Exchange of multiple gluons in the initial state (in addition to one-loop diagrams)
— **resummation to all orders using non-relativistic QCD**



$$\sigma^{\mathrm{Coul}} = \frac{4\pi}{v m_{\tilde{t}}^2} \Im\left\{ G^{[1]}\left(\mathbf{r}=0; \sqrt{s} + i\Gamma_{\tilde{t}}\right)\right\} \sigma^{\mathrm{LO}}$$

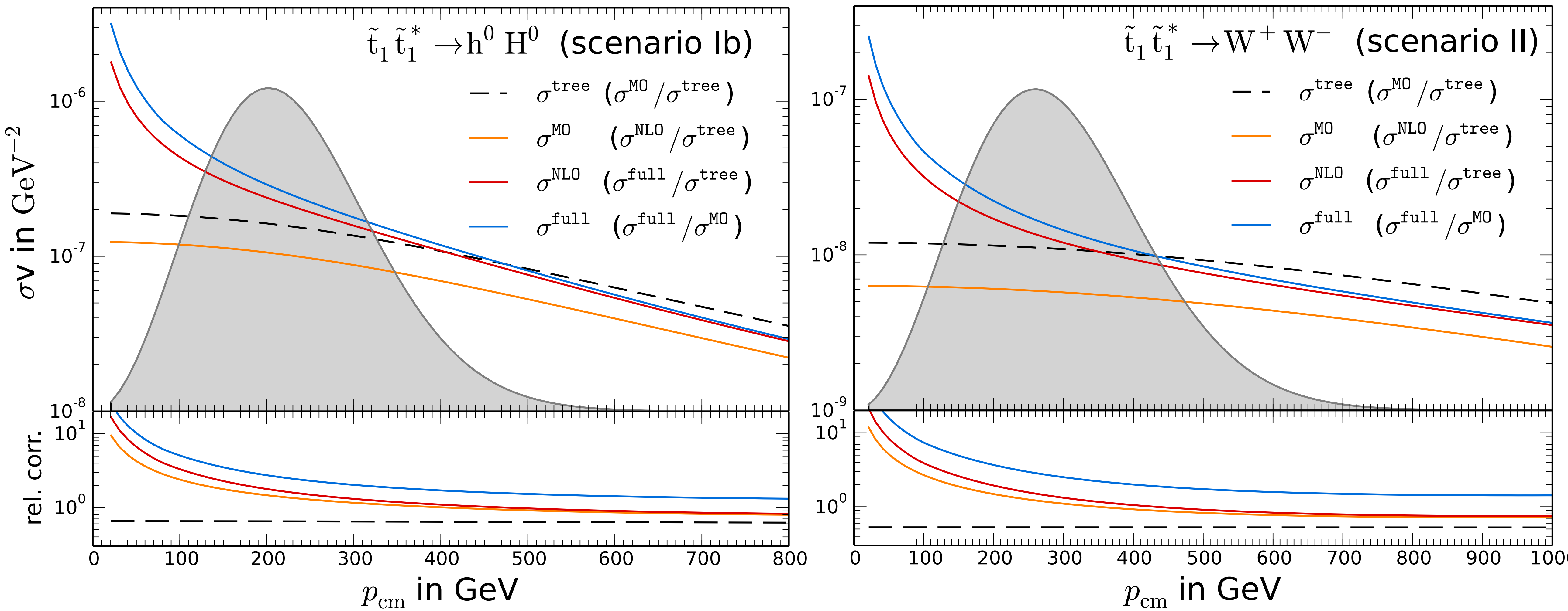
$$\left[H^{[1]} - \left(\sqrt{s} + i\Gamma_{\tilde{t}}\right)\right] G^{[1]} = \delta^{(3)}(\mathbf{r})$$

$$G^{[1]}\left(\mathbf{r}=0; \sqrt{s} + i\Gamma_{\tilde{t}}\right) = \frac{C^{[1]}\alpha_s(\mu_G) m_{\tilde{t}}^2}{4\pi}\left[g_{\mathrm{LO}} + \frac{\alpha_s(\mu_G)}{4\pi} g_{\mathrm{NLO}} + \ldots\right]$$

Avoid double counting of NLO corrections contained in Green's function and one-loop result!

Harz, Herrmann, Klasen, Kovařík, Meinecke — Phys. Rev. D 91: 034012 (2015) — arXiv:1410.8063 [hep-ph]
Herrmann, Klasen, Kovařík, Schmiemann — *in progress…*

Stop pair annihilation — electroweak final states



Coulomb corrections **dominant for small values of p_{cm}** (Coulomb singularity),
while fixed-order corrections dominant for high-momentum region

Resulting relic density receives corrections of up to 40% (more important than Planck uncertainty!)

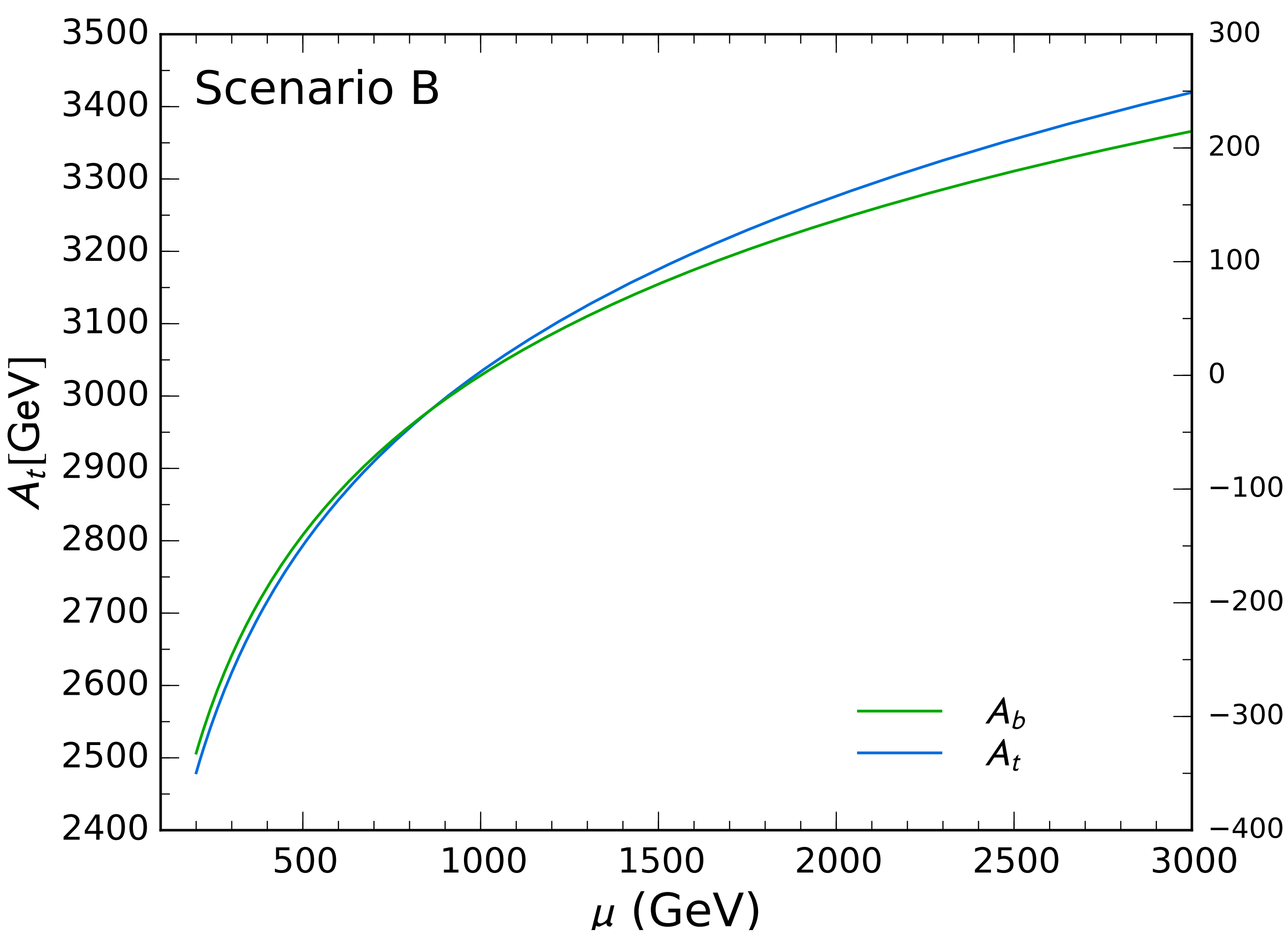
Harz, Herrmann, Klasen, Kovařík, Meinecke — Phys. Rev. D 91: 034012 (2015) — arXiv:1410.8063 [hep-ph]
Herrmann, Klasen, Kovařík, Schmiemann — *in progress…*

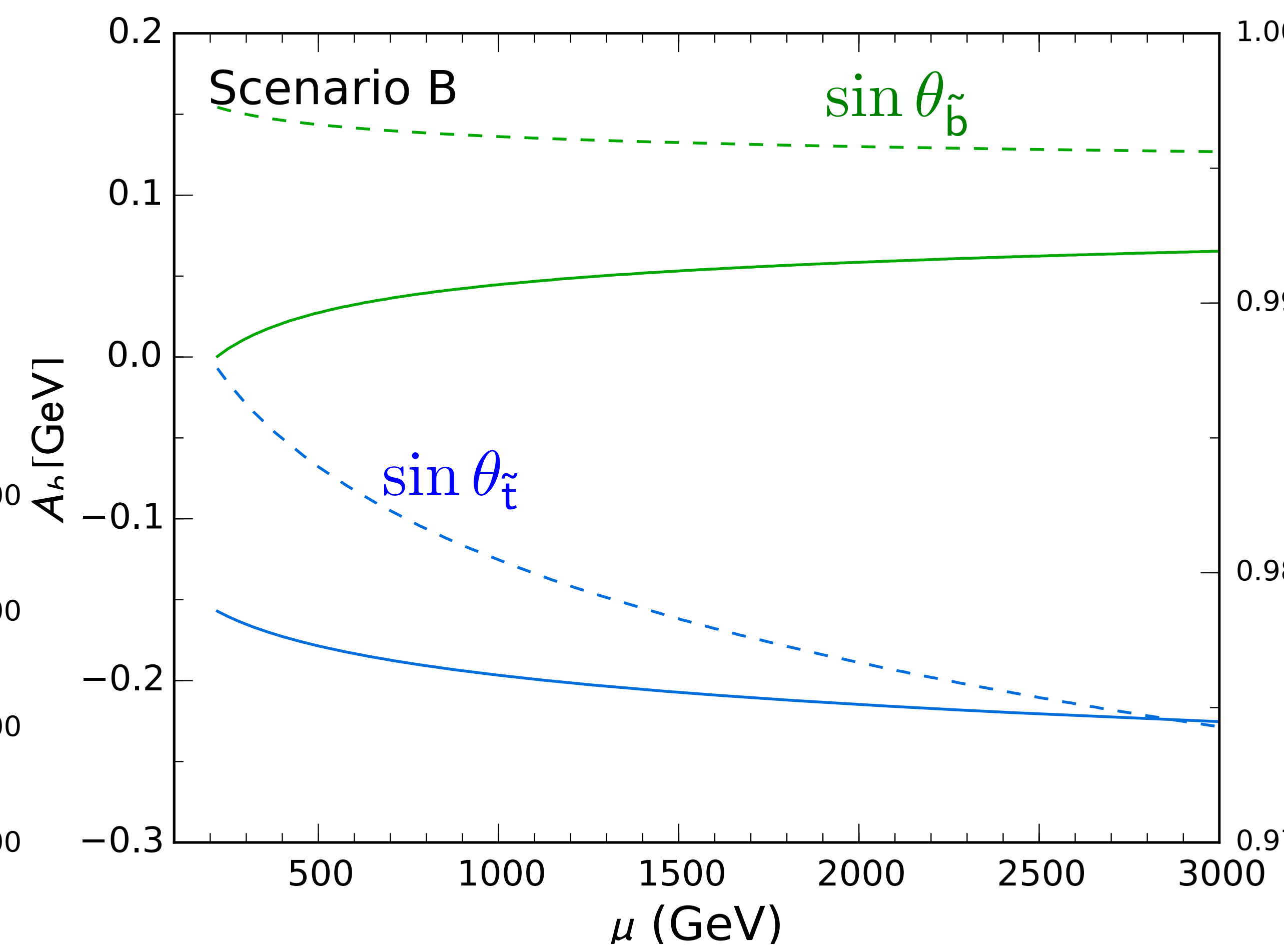
Scale dependence of neutralino (co)annihilation

Loop calculation introduces a dependence on an unphysical parameter: **renormalization scale**
— Evaluation of uncertainty by varying renormalization scale

$\mu_R = 500 \ldots 2000$ GeV → $A_t,\ A_b,\ \theta_{\tilde{t}},\ \theta_{\tilde{t}},\ \alpha_s,\ m_b$

scale-dependent parameters





Harz, Herrmann, Klasen, Kovarik, Steppeler — *to be published…*

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— **Evaluation of uncertainty by varying renormalization scale**

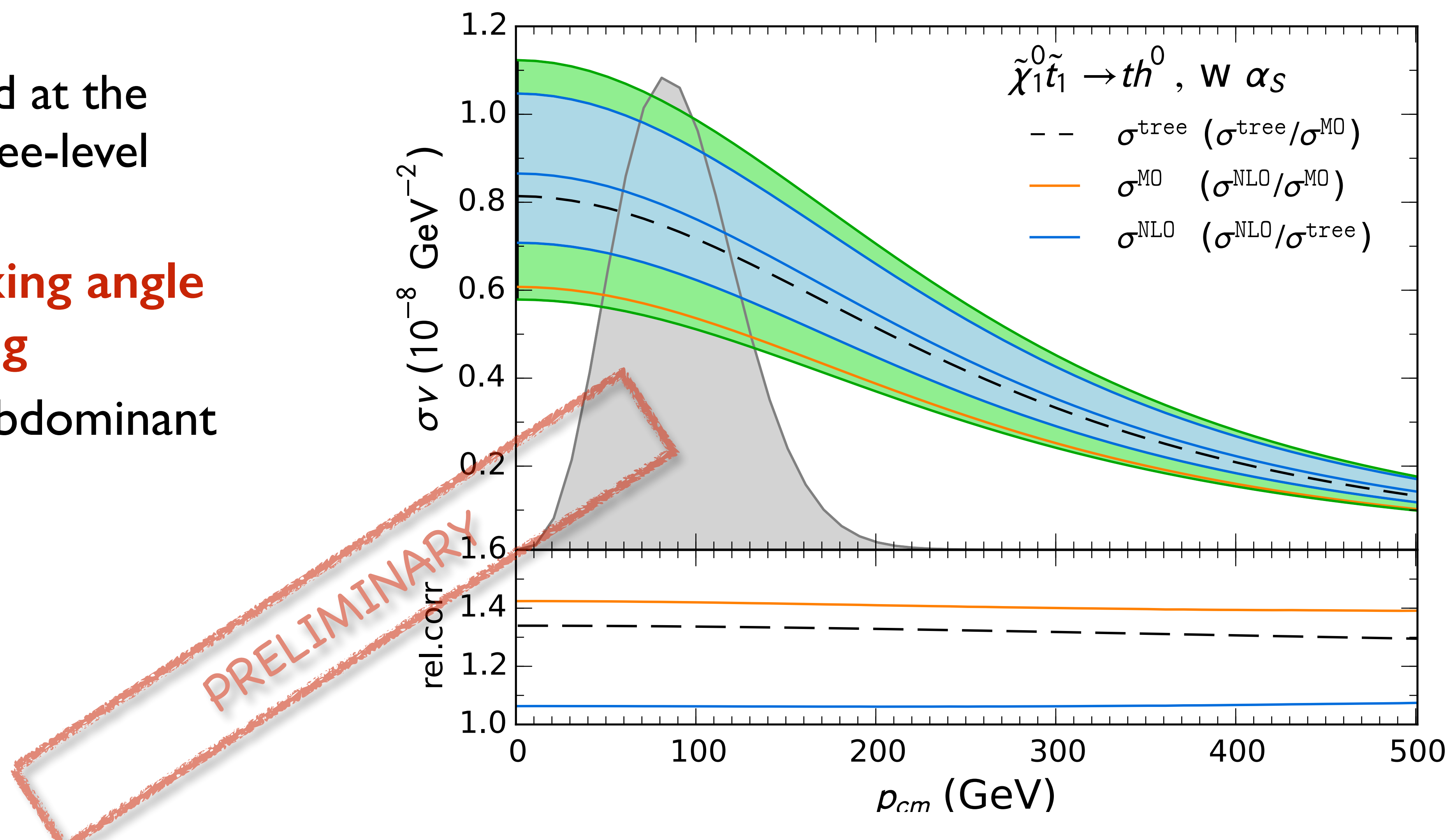
$\mu_R = 500 \ldots 2000$ GeV → A_t, A_b, $\theta_{\tilde{t}}$, $\theta_{\tilde{t}}$, α_s, m_b

scale-dependent parameters

Scale uncertainty reduced at the one-loop level w.r.t. to tree-level result (as expected)

— main effect from **mixing angle** and **trilinear coupling**

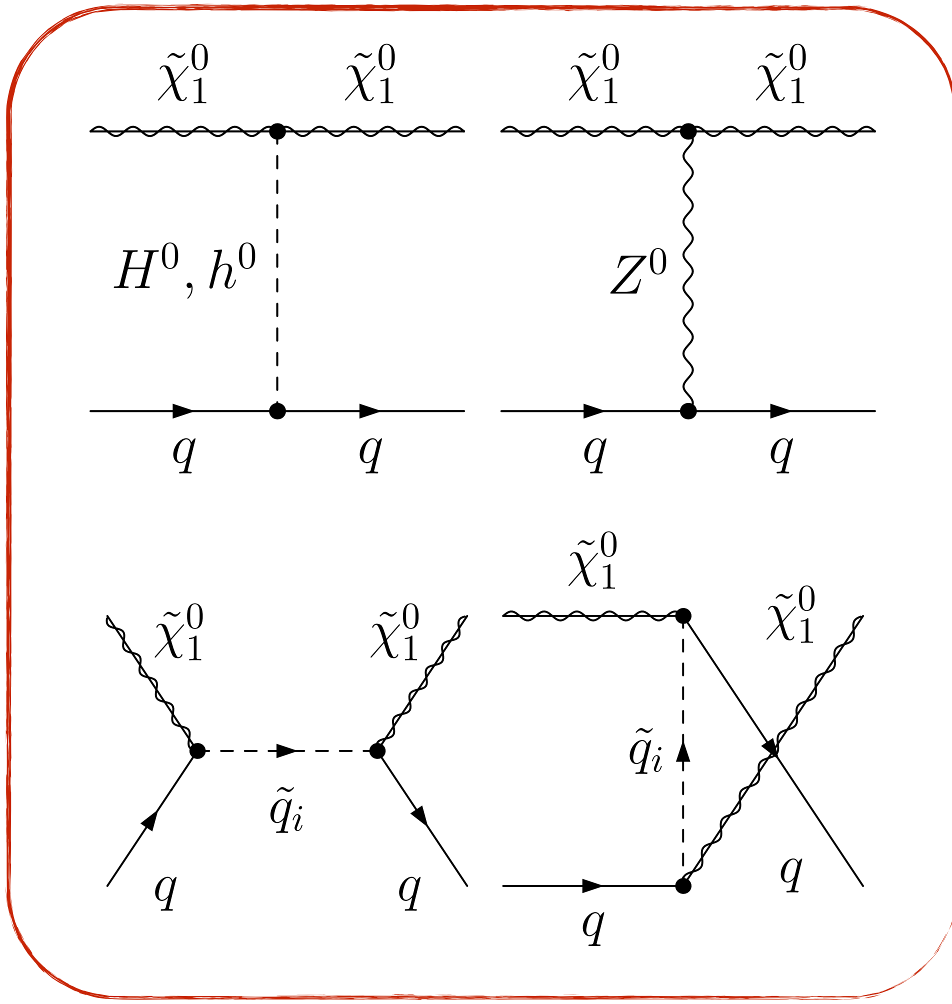
— dependence of α_s subdominant



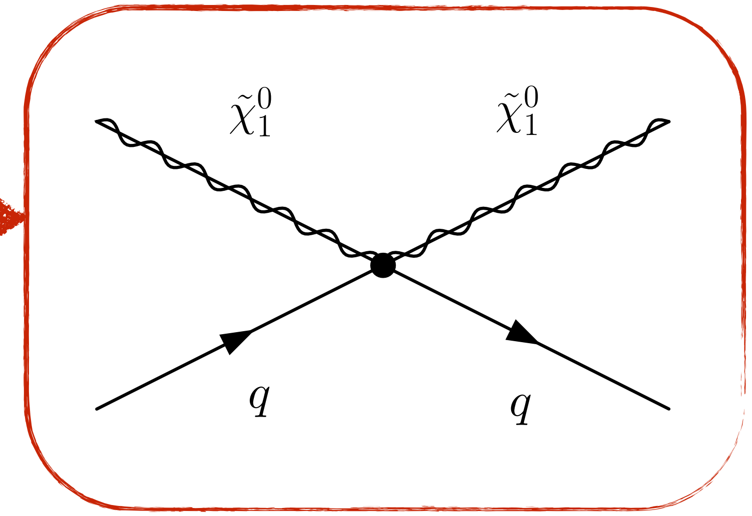
Harz, Herrmann, Klasen, Kovarik, Steppeler — *to be published…*

Direct dark matter detection

Same topologies as neutralino pair annihilation into quarks (*s*-*t* crossing)



Matching procedure
Q ~ 1 TeV



Low-energy effective theory
Q ~ 2-5 GeV
(intermediate particles integrated out)

Calculation carried out at very low energy: p_{cm}~0

— standard **reduction of loop tensor integrals** not applicable

— need to implement specific reduction procedure for threshold…

(also relevant for application to **indirect detection**…)

Klasen, Kovařík, Steppeler — *in progress…*

Electroweak corrections to dark matter annihilation

Smaller coupling constant compensated by large number of diagrams

— **need for automatisation**: **SloopS project**

		Tree	$A_{\tau\tau}$	$\overline{DR}$	MH
$\tilde{\chi}_1^0\tilde{\chi}_1^0 \to W^+W^-$ [44%]	a	+0.81	+7.6%	+12.16%	+29.6%
	b	+1.219	+0.78%	+7.1%	+24.2%
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \to u\bar{d}$ [8%]	a	+15.61	+7.2%	+9.8%	+18.8%
	b	−5.81	+5.7%	+8.3%	+17.4%
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \to Z^0W^+$ [5%]	a	+8.26	+2.9%	+4.4%	+9.7%
	b	+1.42	−7.3%	−3.3%	+10.7%
$\tilde{\chi}_1^0\tilde{\chi}_2^0 \to W^+W^-$ [5%]	a	+17.81	+9.0%	+11.1%	+18.2%
	b	+11.86	+4.8%	+7.3%	+16.1%
$\Omega_\chi h^2$		0.108	0.105	0.102	0.097
$\frac{\delta\Omega_\chi h^2}{\Omega_\chi h^2}$			−2.8%	−5.6%	−10.2%

Impact on cross-section and relic density equally important as for QCD corrections

Renormalisation more involved than for QCD — **important scheme dependence**…

Sommerfeld enhancement numerically not relevant for neutralino masses below 1 TeV…

Baro, Boudjema, Semenov — Phys.Lett. B 660,: 550-560 (2008) — arXiv:0710.1821 [hep-ph]
Baro, Boudjema, Chalons, Hao — Phys.Rev. D81: 015005 (2010) — arXiv:0910.3293 [hep-ph]

Summary

Recent experimental improvements (WMAP, Planck…) require more precise predictions of the dark matter relic density on the theory side…

DM@NLO — calculation of neutralino (co)annihilation including QCD corrections

$$\tilde{\chi}\tilde{\chi}' \to q\bar{q}'$$

$$\tilde{\chi}\tilde{q} \to q'H/q'V$$

$$\tilde{q}\tilde{q}^* \to HH/HV/VV$$

numerically implemented
results published

$$\tilde{\chi}\tilde{\chi}' \to gg/\gamma\gamma$$

$$\tilde{q}\tilde{q}^* \to q\bar{q}'$$

$$\tilde{q}\tilde{q} \to qq$$

$$\tilde{\tau}\tilde{\tau}^* \to qq'$$

work in progress…

Impact of corrections on the relic density more important than current exp. uncertainty

Application to direct / indirect detection…?
Generalisation to other New Physics models…?
Publish the package…!

http://dmnlo.hepforge.org


thank you
gracias
danke
謝謝
merci
obrigado
teşekkür ederim
go raibh maith agat
dziękuję
спасибо
bedankt
sukriya
kop khun krap
terima kasih
감사합니다
ありがとう
arigato
dakujem
grazie
dank je
mochchakkeram
ngiyabonga
tapadh leat
sagolun
takk
hvala
мерси