Towards a precise prediction of the dark matter relic density within the MSSM

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J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D 93 114023 (2016)
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and M. Meinecke, Phys. Rev D 91 034012 (2015)
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, Phys. Rev. D 91, 034028 (2015)
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I. Motivation & Introduction

II. Processes & Technicalities

III. Results & Phenomenology

I. Motivation & Introduction

What is Dark Matter?

There are ample hints for the existence of dark matter...

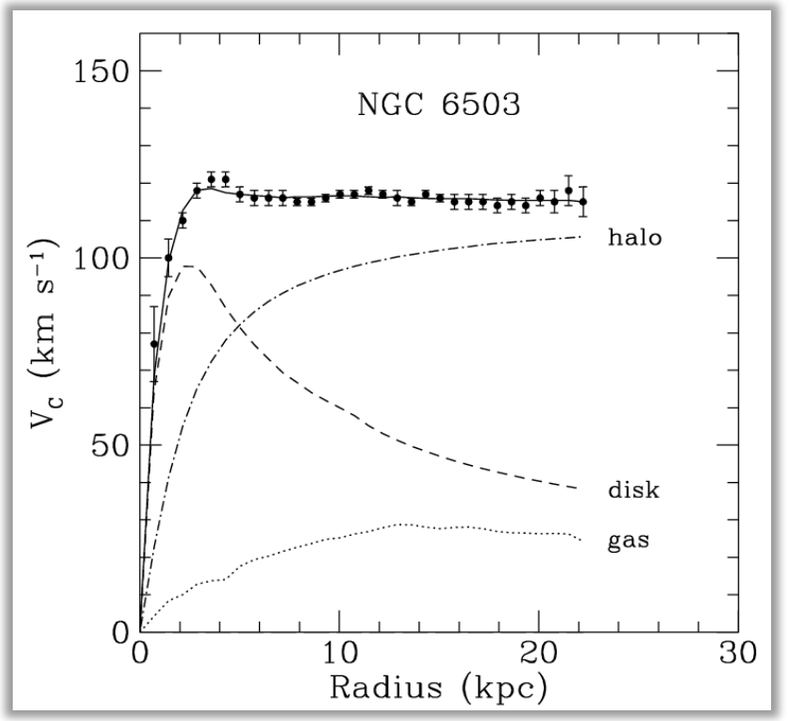
rotation curves of spiral galaxies

circular velocity: $v_c(r) = \sqrt{\frac{GM(r)}{r}}$

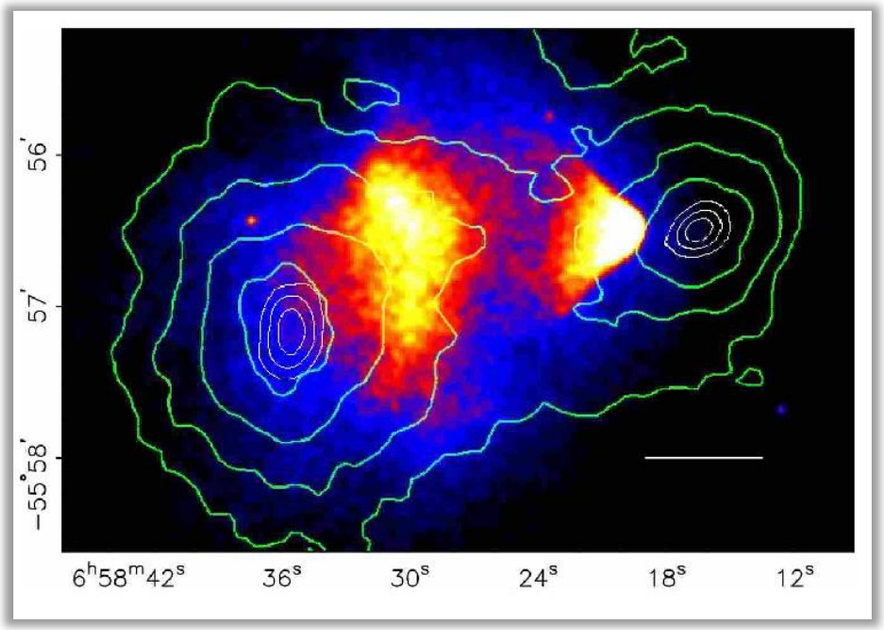
with $M(r) = 4\pi \int_0^r \mathrm{d}r' r'^2 \rho(r')$

expectation: $v_c(r) \propto 1/\sqrt{r}$

observation: $v_c(r) \propto const.$



Begeman, Broeils, Sanders, MNRAS 249 (1991)



Clowe, Bradac, et al., The Astrophysical Journal 648 (2006)

weak gravitational lensing effects of the Bullet Cluster

X-ray emission of colliding shock cones (yellow/red)

↕ 8 σ spatial offset

gravitational potential (green)

What is Dark Matter?

... which give us an idea of its characteristics and the possible theoretical model...

DM characteristics

- non-baryonic
- stable
- neutral
- cold
- consistent with BBN
- consistent with stellar evolution

Minimal Supersymmetric Standard Model (MSSM)

SM Particles		Spin		Spin		Superpartners
Quarks	$(u_L \;\; d_L)$	1/2	Q	0	$(\tilde{u}_L \;\; \tilde{d}_L)$	Squarks
	$u_R^\dagger$	1/2	$\bar{u}$	0	$\tilde{u}_R^*$	
	$d_R^\dagger$	1/2	$\bar{d}$	0	$\tilde{d}_R^*$	
Leptons	$(\nu \;\; e_L)$	1/2	L	0	$(\tilde{\nu} \;\; \tilde{e}_L)$	Sleptons
	$e_R^\dagger$	1/2	$\bar{e}$	0	$\tilde{e}_R^*$	
Higgs	$(H_u^+ \;\; H_u^0)$	0	H_u	1/2	$(\tilde{H}_u^+ \;\; \tilde{H}_u^0)$	Higgsinos
	$(H_d^0 \;\; H_d^-)$	0	H_d	1/2	$(\tilde{H}_d^0 \;\; \tilde{H}_d^-)$	
Gluon	g	1		1/2	$\tilde{g}$	Gluino
W bosons	$W^0, W^\pm$	1		1/2	$\tilde{W}^0, \tilde{W}^\pm$	Winos
B boson	B^0	1		1/2	$\tilde{B}^0$	Bino
Graviton	G	2		3/2	$\tilde{G}$	Gravitino

assuming conserved R-parity:
lightest supersymmetric particle (LSP) stable

$$\mathcal{L} = -\frac{1}{2}(\psi^0)^T \mathcal{M}_{\tilde{\chi}^0} \psi^0 + h.c.$$

$$\psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)$$

$$\mathcal{M}_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix}$$

accommodates naturally a good cold dark matter candidate: the lightest neutralino

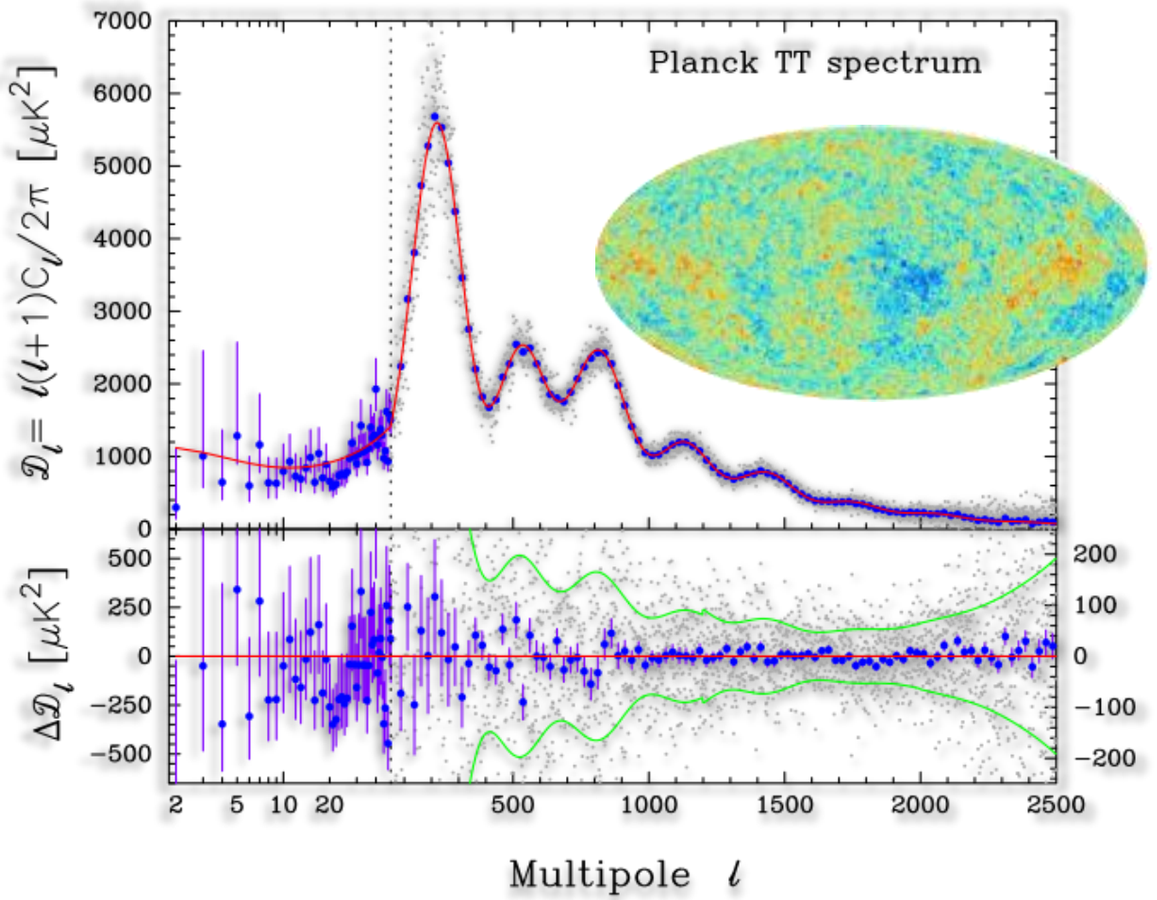
Experimental Determination of the Relic Density

… also the amount of dark matter can be precisely determined…

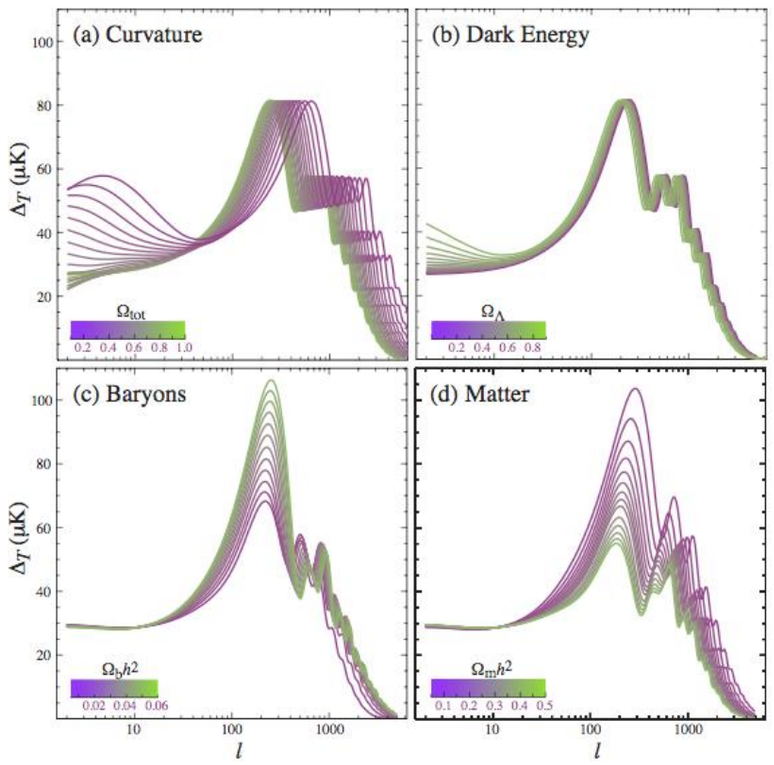
- temperature fluctuations in the CMB can be described by 2-dimensional power spectrum

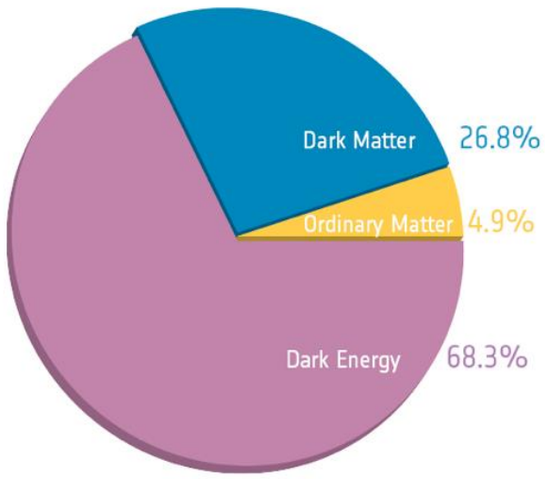
$$\frac{\delta T}{T}(\theta,\phi) = \sum_{l=2}^{+\infty} \sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta,\phi)$$

- assumption of a cosmological model (ΛCDM)
- fit of power spectrum possible



Planck Collaboration, arXiv:1303.5076





- baryonic density obtained to

$$\Omega_b h^2 = 0.02222 \pm 0.00023$$

- dark matter density determined to

$$\Omega_{\mathrm{CDM}} h^2 = 0.1199 \pm 0.0022$$

P. A. R. Ade et al. [Planck Collaboration], arXiv:1502.01589 [astro-ph.CO]

Theoretical Prediction of the Relic Density

- number density of DM in the early universe can be described by the Boltzmann equation

$$\dot{n} + 3Hn = -\langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2)$$

1) Thermal equilibrium regime (T>>m)

annihilation and production of DM in thermal equilibrium

2) Annihilation regime (T ~ m/10)

SM particles not energetic enough to create DM particles

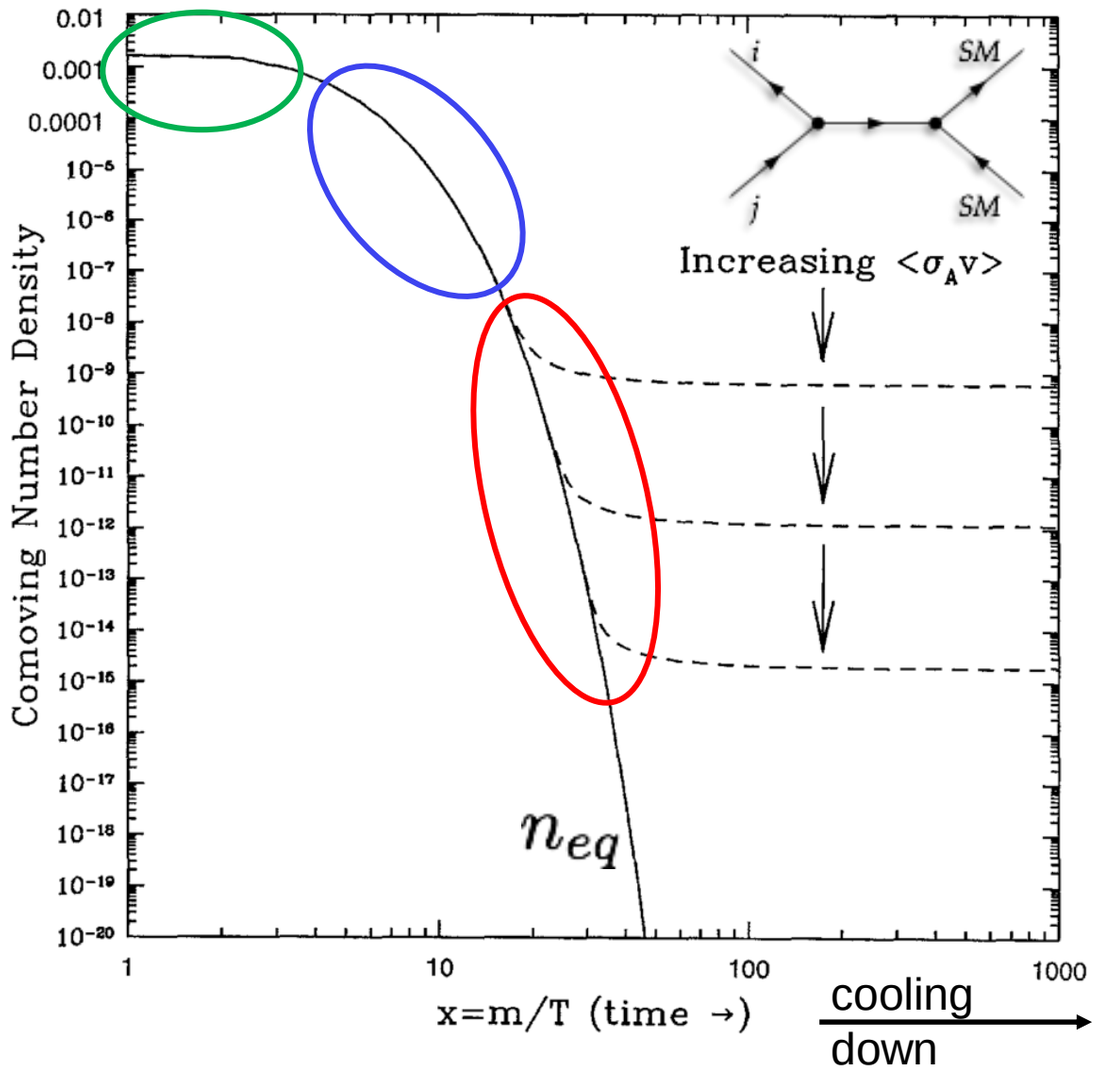
3) Freeze-out (T~m/30)

Annihilation rate falls behind expansion rate → DM abundance

- relic density proportional to cross section

$$\Omega_\chi h^2 = \frac{n_\chi m_\chi}{\rho_{\text{crit}}} \propto \frac{1}{<\sigma_{\text{eff}} v>}$$

Jungman, Kamionkowski, Griest, Phys. Reports 267 (1995)



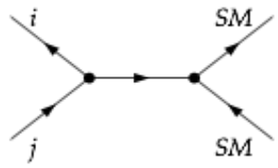
$\chi\ \chi \leftrightarrow \text{SM SM}$ $\chi\ \chi \rightarrow \text{SM SM}$ $\chi\ \chi \not\rightarrow \text{SM SM}$

Theoretical Prediction of the Relic Density

- number density of DM in the early universe can be described by the Boltzmann equation

$$\dot{n} + 3Hn = -\langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2)$$

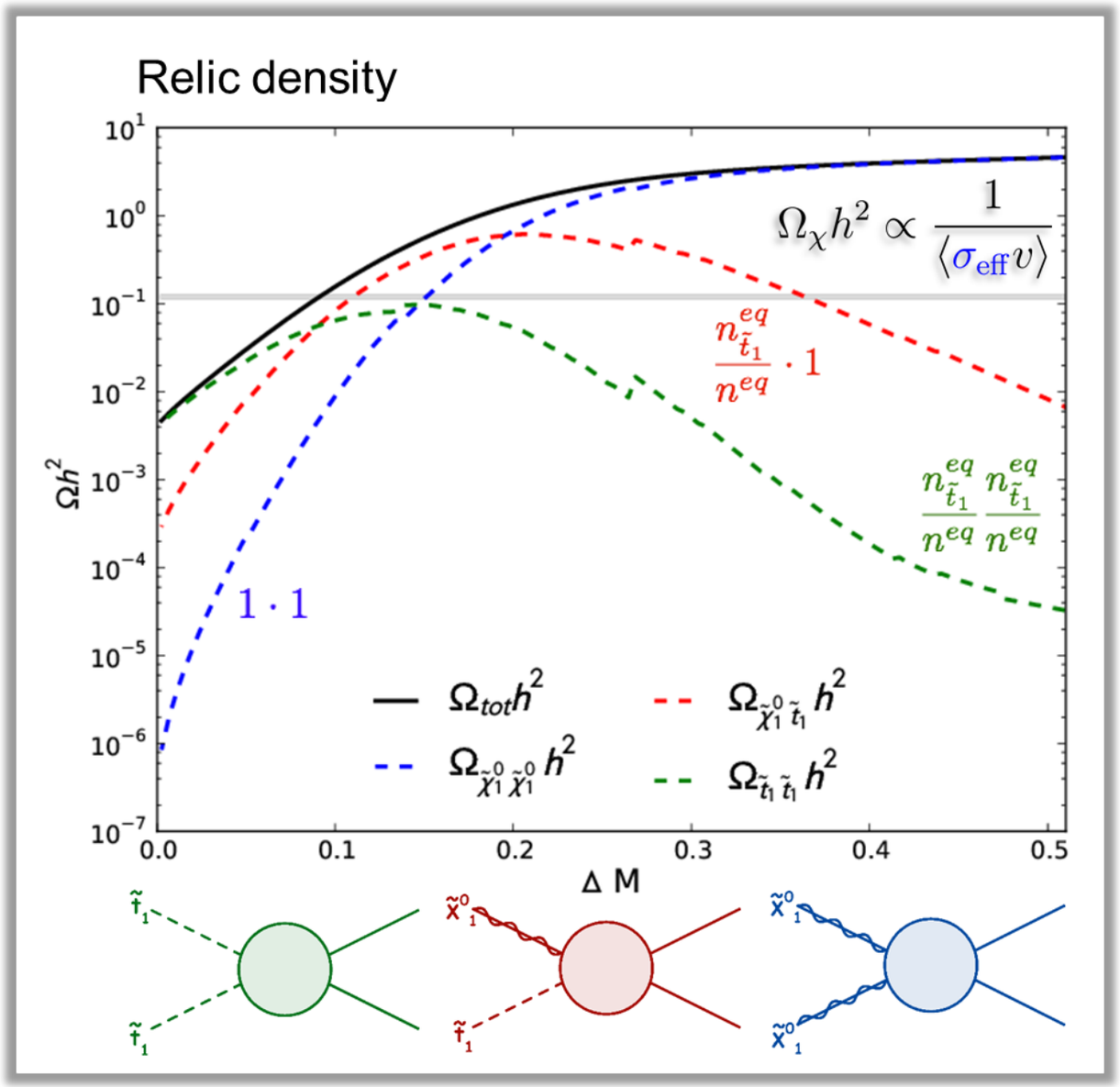
$$\langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq}}{n^{eq}} \frac{n_j^{eq}}{n^{eq}}$$



with $\frac{n_i^{eq}}{n^{eq}} \propto \exp^{\frac{-(m_i - m_\chi)}{T}} = \exp^{\frac{-(m_i - m_\chi)}{x \; m_\chi}}$

- sizeable contributions of AB → SM when particle A is almost degenerate in mass with particle B (DM candidate)

- dependent on mass difference various different processes can be important in the MSSM parameter space:
  → pair annihilation of neutralinos
  → coannihilation with other neutralinos, light stops, taus, etc…



we define $\Delta M = \frac{m_{\tilde{t}_1} - m_{\tilde{\chi}_1}}{m_{\tilde{\chi}_1}}$

Theoretical Prediction of the Relic Density

$$\dot{n} + 3Hn = -\langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2) \quad \Rightarrow \quad \Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\text{eff}} v \rangle}$$

different relic density tools exist:

MicrOMEGAs
Belanger, Boudjema, et al. , CPC (2002)

DarkSUSY
Gondolo, Edsjö, et al. , JCAP (2004)

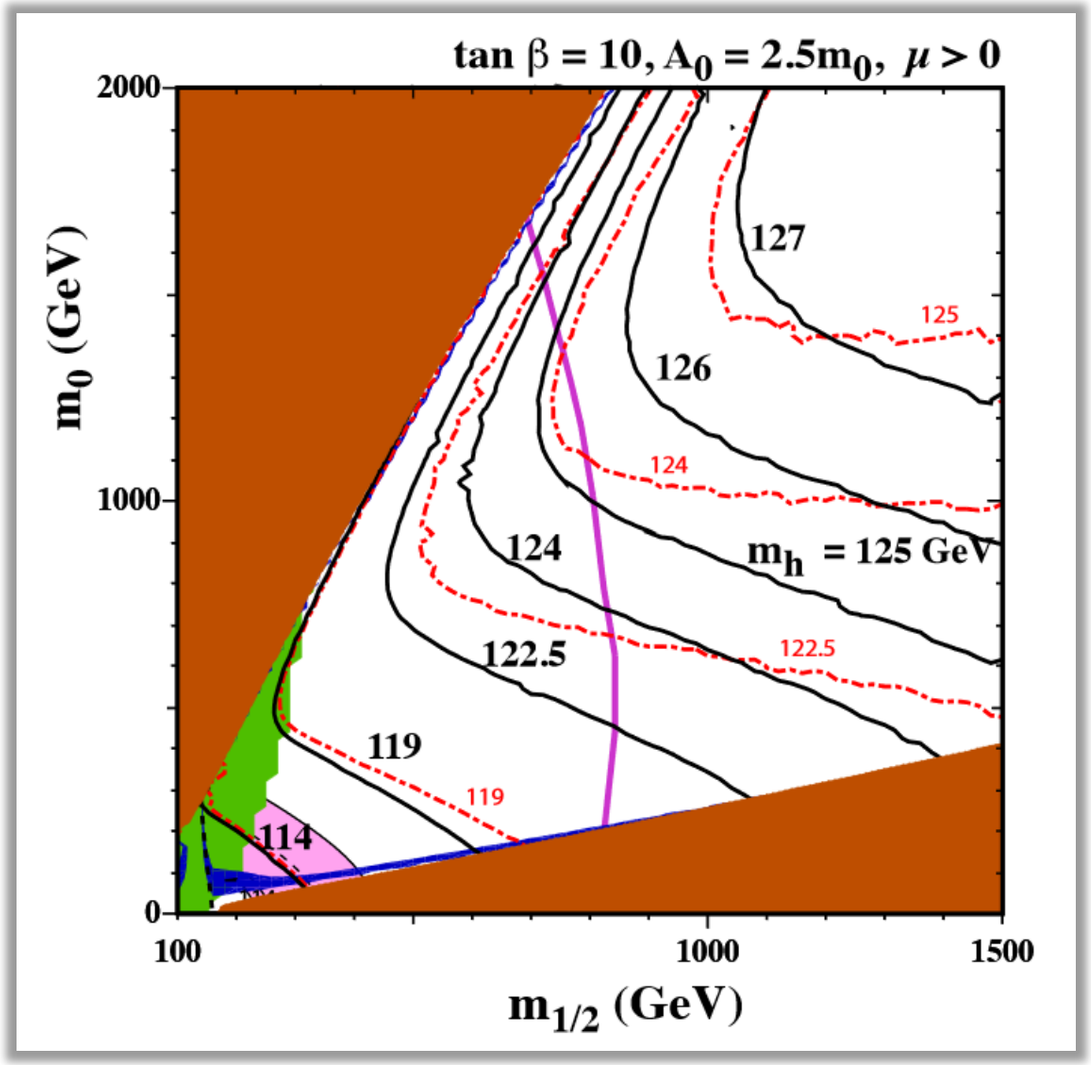
SuperIso Relic
Arbey, Mamoudi, et al. , CPC (2010)

MadDM
Backovic, Kong, et al. , (2013)

allows for constraining strongly the MSSM parameter space

Precision data from CMB measurements

PLANCK: ~ 1.5% uncertainty



J. Ellis, K. Olive, J. Zheng, Eur. Phys. J. C74 (2014) 2947

Motivation to include Higher Order Corrections

1. Increased precision of relic density prediction by going to one-loop

2. Inclusion of significant Coulomb enhancement (resummation)

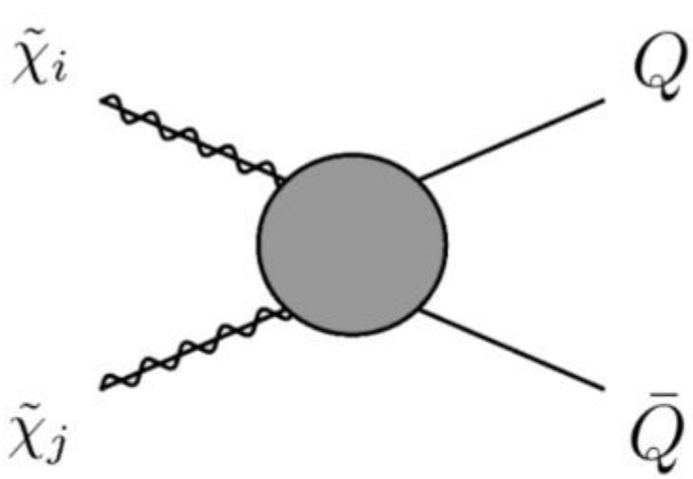
3. First study (at all!) of uncertainties coming from scale variation and

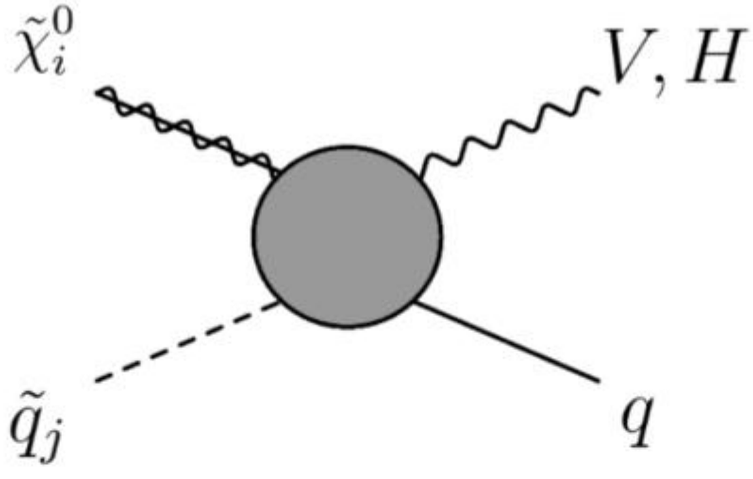
4. Scheme variation to the (co)annihilation cross section and the relic density

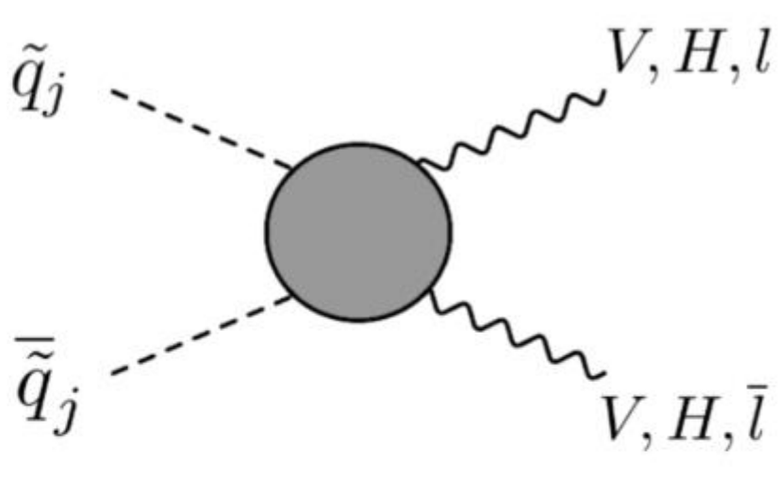
Tools do not take into account those three issues at all!

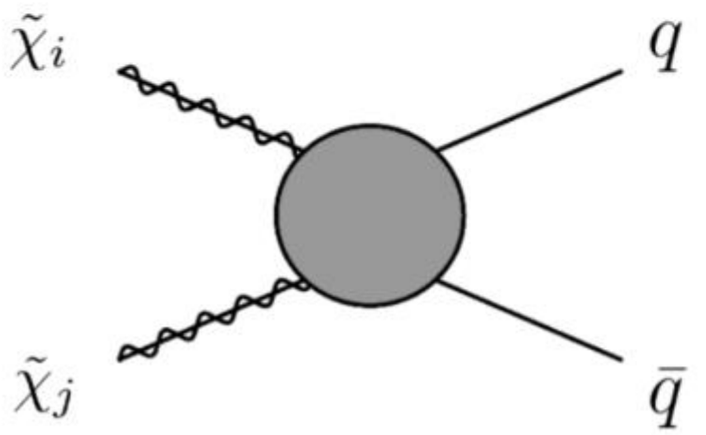
II. Processes & Technicalities

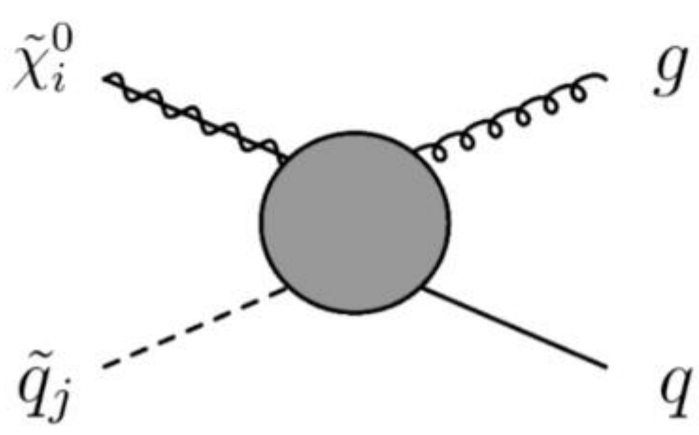
Calculation of SUSY-QCD NLO Corrections

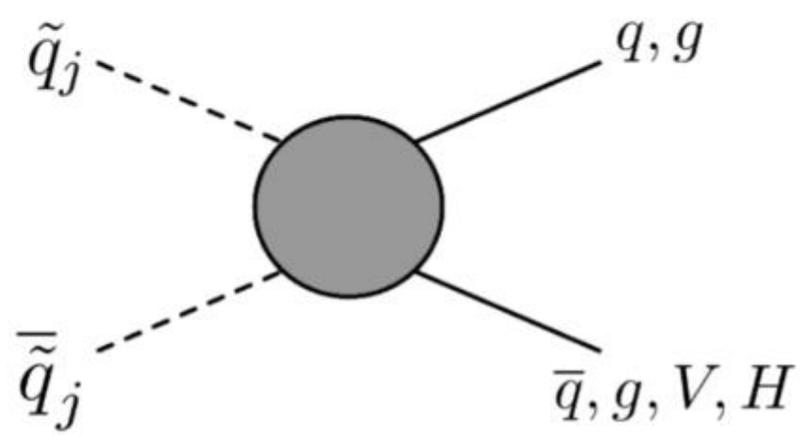












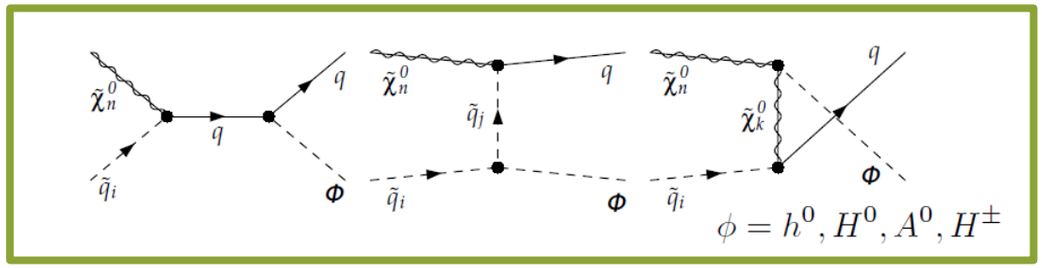
decreasing mass difference between LSP and NLSP

… and provide the results to public tools like micrOMEGAs

→ Aim 1 & 2: Increase precision!

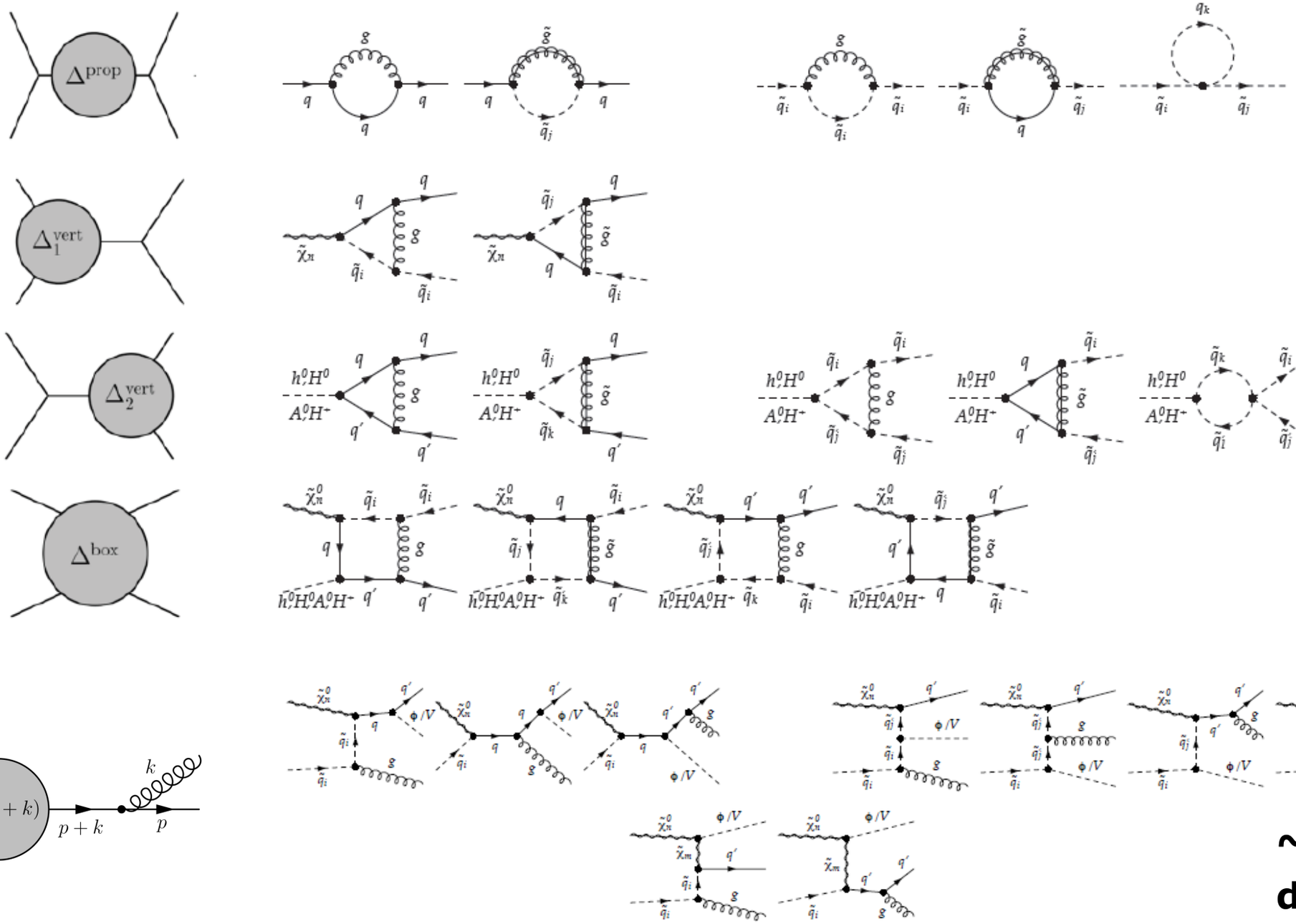
Recap: Loop Calculus in a Nutshell (I)

Tree level:



NLO:

virtual correction



real correction

~ 200 different diagrams

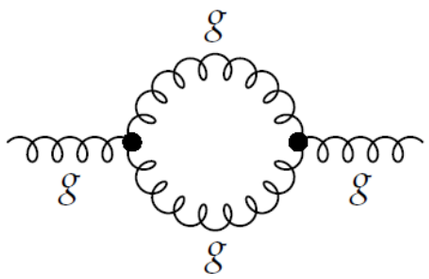
Recap: Loop Calculus in a Nutshell (II)

Problem: treatment of divergences

$$\sigma^{\mathrm{NLO}} = \int_{2\to2} d\sigma^{\mathrm{virtual}} + \int_{2\to3} d\sigma^{\mathrm{real}} = \mathrm{finite}$$

Kinoshita-Lee-Lauenberg Theorem

Dimensional regularisation:



$$\int_a^\infty \frac{1}{r^2}\,\mathrm{d}^3r \;-\; \text{linear divergence} \qquad \int_a^\infty \frac{1}{r^2}\,\mathrm{d}^2r \;-\; \text{logarithmic divergence}$$

$$\int_a^\infty \frac{1}{r^2}\,\mathrm{d}r \;-\; \text{converges}$$

$$\int d^Dq \frac{1}{(q^2)^2} = i(-1)^2 \frac{\pi^{D/2}}{\Gamma(\frac{D}{2})} \left(\int_0^{\mu^2} dq_E^2 (q_E^2)^{D/2-3} + \int_{\mu^2}^\infty dq_E^2 (q_E^2)^{D/2-3} \right)$$

$$= i(-1)^2 \frac{\pi^{D/2}}{\Gamma(\frac{D}{2})} \left(\frac{\mu^{D'-4}}{D'-4} - \frac{\mu^{D-4}}{D-4} \right) \qquad \text{with} \quad D' > 4 \quad \text{and} \quad D < 4$$

$$B_0(0,0,0) = \frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}} \qquad \text{with} \quad D = 4 - 2\epsilon$$

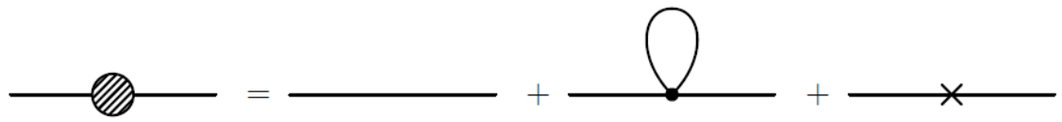
renormalisation

cancellation with real corrections

SUSY → Dimensional Reduction

Recap: Loop Calculus in a Nutshell (III)

UV- divergences → Renormalisation schemes



$$\frac{i}{p^2-m^2}+\frac{i}{p^2-m^2}\left(-i\Sigma(p^2)\right)\frac{i}{p^2-m^2}=\frac{i}{p^2-m^2}\left(1+\frac{\Sigma(p^2)}{p^2-m^2}\right)$$

$$-i\Sigma(p^2)=-i\frac{\lambda}{2}\frac{1}{16\pi^2}A_0(m^2)+i\Big[-m^2\delta Z_m\Big]$$

$$-i\Sigma(p^2)=-i\frac{\lambda}{2}\frac{m^2}{16\pi^2}\left[\frac{1}{\varepsilon}-\gamma_E+\ln 4\pi-\ln\left(\frac{m^2-i\epsilon}{\mu^2}+1\right)\right]-im^2\delta Z_m.$$

OS - scheme

$$\left.\mathrm{Re}\,\Sigma(p^2)\right|_{p^2=m^2}=0$$

$$\delta Z_m^{OS}=-\frac{\lambda}{32\pi^2}\left[\Delta+1-\ln\left(\frac{m^2-i\epsilon}{\mu^2}\right)\right]$$

$$\Sigma(p^2)=0.$$

renormalised parameter fixed

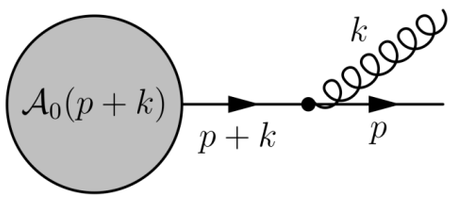
DRbar - scheme

$$\delta Z_m^{\overline{\mathrm{DR}}}=-\frac{\lambda}{32\pi^2}\Delta$$

$$\Sigma(p^2)=\frac{\lambda m^2}{32\pi^2}\left[1-\ln\left(\frac{m^2-i\epsilon}{\mu^2}\right)\right]$$

renormalised parameter scale dependent

IR-divergences → soft & collinear divergences



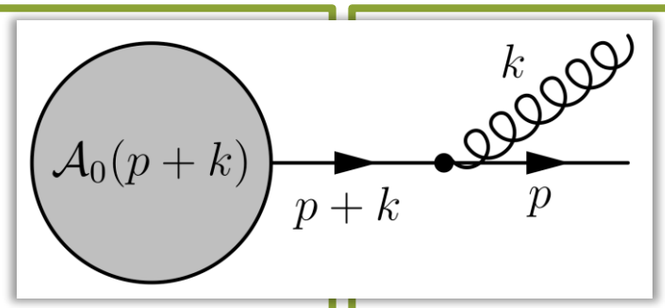
$$\frac{1}{(p+k)^2-m^2}=\frac{1}{2\,p\cdot k}=\frac{1}{\omega(E_p-|\vec{p}|\cos\theta)}\xrightarrow[m_p=0\,\wedge\,\theta\to 0]{\omega\to 0\,\vee}\infty$$

soft

collinear

Infrared Treatment

Dipole subtraction



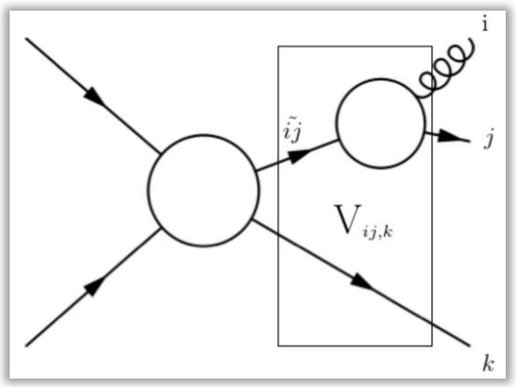
- auxiliary cross section $\mathrm{d}\sigma^A$ is introduced = local counter term
- three-particle and two-particle phase space separately finite

$$\sigma_{\mathsf{NLO}} = \int_3 \left[\mathrm{d}\sigma^{\mathsf{R}} \Big|_{\epsilon=0} - \mathrm{d}\sigma^{\mathsf{A}} \Big|_{\epsilon=0} \right] + \int_2 \left[\mathrm{d}\sigma^{\mathsf{V}} + \int_1 \mathrm{d}\sigma^{\mathsf{A}} \right]_{\epsilon=0}$$

- general structure of $\mathrm{d}\sigma^A$ given as

$$\begin{aligned} \left|M^{\mathsf{A}}\right|^2 &= \sum_{i,j} \sum_{k \neq i,j} \mathcal{D}_{ijk} \\ &= \sum_{i,j} \sum_{k \neq i,j} V_{ij,k}(p_i, \tilde{p}_{ij}, \tilde{p}_k) \otimes \left|M^{\mathsf{B}}(\tilde{p}_{ij}, \tilde{p}_k)\right|^2 \end{aligned}$$

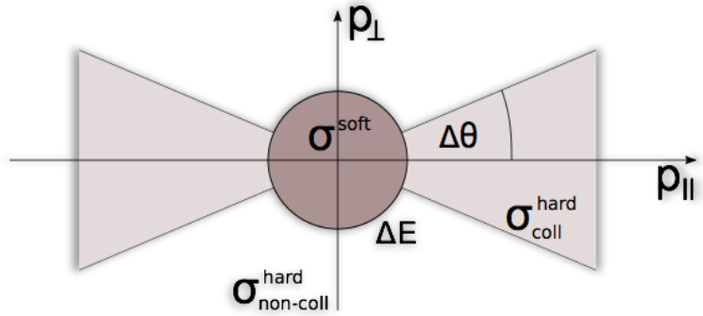
- $V_{ij,k}$ becomes proportional to Altarelli-Parisi splitting functions in collinear limit and to eikonal factors in the soft limit



S. Catani et al. Nucl. Phys B 627 (2002), S. Catani, M.H. Seymour, Phys. Lett. B 378 (1996)
S. Catani, M.H. Seymour Nucl. Phys. B 485 (1997), S. Dittmaier, Nucl. Phys. B 565 (2000)

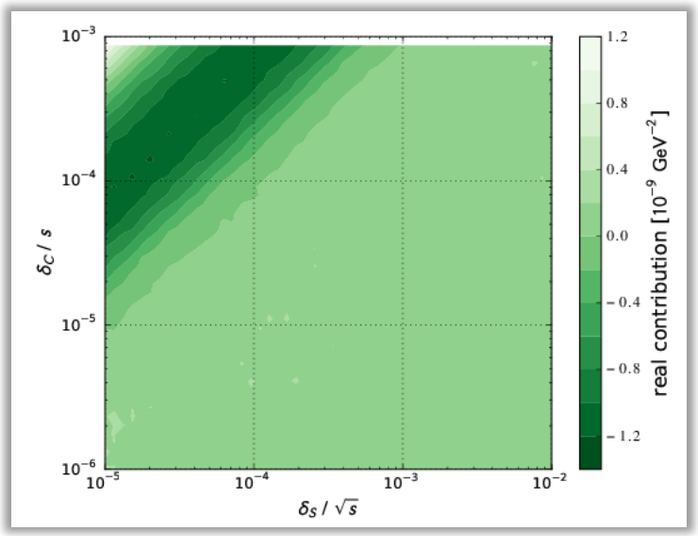
Phase space slicing

- phase space is sliced into regions of different divergent classes
- approximations allow for analytical integration

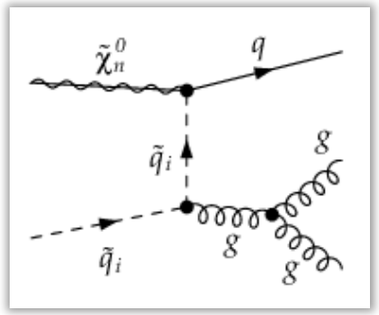


$$\sigma^{real} = \sigma^{\mathrm{soft}}(\Delta E) + \sigma^{\mathrm{hard}}_{\mathrm{coll}}(\Delta E, \Delta\theta) + \sigma^{\mathrm{hard}}_{\mathrm{non-coll}}(\Delta E, \Delta\theta)$$

eikonal approximation — hard-collinear approximation — pure 2 → 3 processes

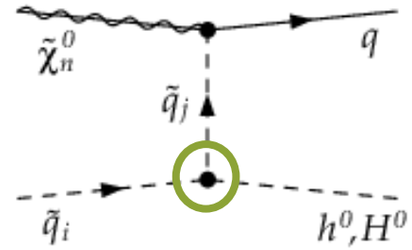


B. W. Harris, J. F. Owens, Phys, Rev. D65 (2002)



Renormalisation I

Renormalisation scheme has to be valid over a wide parameter space for all (co)annihilation processes

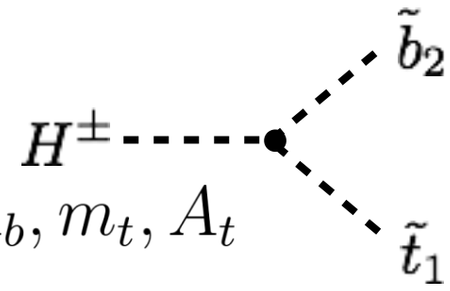


- relevant parameters: $m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_t, m_b, A_t, A_b, \theta_{\tilde{t}}, \theta_{\tilde{b}}$

known masses

relevant as incoming (co)annihilating particles

crucial for coannihilating scenarios; as dependent parameter problem of large effects for large tanbeta



- hybrid on-shell / $\overline{\text{DR}}$ renormalisation scheme

$m_{\tilde{t}_1}^{\text{OS}}, m_{\tilde{b}_1}^{\text{OS}}, m_{\tilde{b}_2}^{\text{OS}} \quad A_t^{\overline{\text{DR}}}, A_b^{\overline{\text{DR}}} \quad m_t^{\text{OS}}, m_b^{\overline{\text{DR}}}$ ⇒ $m_{\tilde{t}_2}, \theta_{\tilde{t}}, \theta_{\tilde{b}}$

input parameters → dependent parameters

$$\begin{pmatrix} m^2_{\tilde{q}_1} & 0 \\ 0 & m^2_{\tilde{q}_2} \end{pmatrix} = U^{\tilde{q}} \begin{pmatrix} M^2_{\tilde{Q}} + (I_q^{3L} - e_q\, s_W^2)\cos 2\beta\, m_Z^2 + m_q^2 & m_q\big(A_q - \mu\,(\tan\beta)^{-2I_q^{3L}}\big) \\ m_q\big(A_q - \mu\,(\tan\beta)^{-2I_q^{3L}}\big) & M^2_{\{\tilde{U},\,\tilde{D}\}} + e_q\, s_W^2 \cos 2\beta\, m_Z^2 + m_q^2 \end{pmatrix} (U^{\tilde{q}})^\dagger$$

→ Aim 4: Study dependence on renormalisation scheme!

Renormalisation II

- bottom quark mass

$$m_b^{\mathrm{SM},\overline{\mathrm{MS}}}(m_b) \xrightarrow{(1)} m_b^{\mathrm{SM},\overline{\mathrm{MS}}}(Q) \xrightarrow{(2)} m_b^{\mathrm{SM},\overline{\mathrm{DR}}}(Q) \xrightarrow{(3)} m_b^{\mathrm{MSSM},\overline{\mathrm{DR}}}(Q)$$

3-loop RGEs — 2-loop relation — 2-loop threshold corrections

K.G Chetyrkin, Phys. Lett. B 404, 161 (1997)

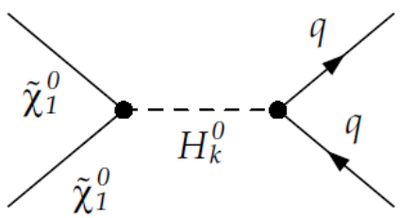
R. Harlander, et al. JHEP 0609, 053 (2006)
A. Bauer et al., JEP 0902, 03 (2009)

A. Bauer et al., JEP 0902, 03 (2009)

- bottom Yukawa coupling

$$\left[\left(h_b^{\overline{\mathrm{MS}},\mathrm{QCD},\Phi}\right)(Q)\right]^2 = \left[\left(h_b^{\overline{\mathrm{MS}},\Phi}\right)(Q)\right]^2 \left[1 + \Delta_{\mathrm{QCD}} + \Delta_t^{\Phi}\right]$$

effective Yukawa couplings with QCD corrections and top-quark induced contributions…



K.G Chetyrkin, Phys. Lett. B 390, 309 (1997)
P.A.Baikov et al. Phys. Rev. Lett. 96, 012003 (2006)
K.G. Chetyrkin et al. Nucl. Phys. B 461, 3 (1996)

$$h_b^{\mathrm{MSSM},h}(Q) = \frac{h_b^{\overline{\mathrm{MS}},\mathrm{QCD},h}(Q)}{1+\Delta_b}\left[1 - \frac{\Delta_b}{\tan\alpha \tan\beta}\right],$$

$$h_b^{\mathrm{MSSM},H}(Q) = \frac{h_b^{\overline{\mathrm{MS}},\mathrm{QCD},H}(Q)}{1+\Delta_b}\left[1 + \Delta_b \frac{\tan\alpha}{\tan\beta}\right],$$

$$h_b^{\mathrm{MSSM},A}(Q) = \frac{h_b^{\overline{\mathrm{MS}},\mathrm{QCD},A}(Q)}{1+\Delta_b}\left[1 - \frac{\Delta_b}{\tan^2\beta}\right]$$

…and resummed Δ_b important for large $\tan\beta$ effects

M. Carena et al., Nucl. Phys. B 577, 88 (2000)
J. Guasch et al., Phys. Rev. D 68 115001 (2003)
D. Noth and M. Spira, JHEP 1106, 084 (2011)

- strong coupling constant

$$\alpha_s^{\overline{\mathrm{MS}},\mathrm{SM},\mathrm{n_q}=5}(m_Z^2) \xrightarrow{(1)} \alpha_s^{\overline{\mathrm{MS}},\mathrm{SM},\mathrm{n_q}=5}(Q^2) \xrightarrow{(2)} \alpha_s^{\overline{\mathrm{DR}},\mathrm{SM},\mathrm{n_q}=5}(Q^2) \xrightarrow{(3)} \alpha_s^{\overline{\mathrm{DR}},\mathrm{MSSM},\mathrm{n_q}=6}(Q^2)$$

3-loop RGEs — 2-loop relation — 2-loop threshold corrections

J. Vermaseren et al., Phys. Lett. B 405, 327 (1997)

R. Harlander, et al. Phys. Rev. D 72 095009 (2005)

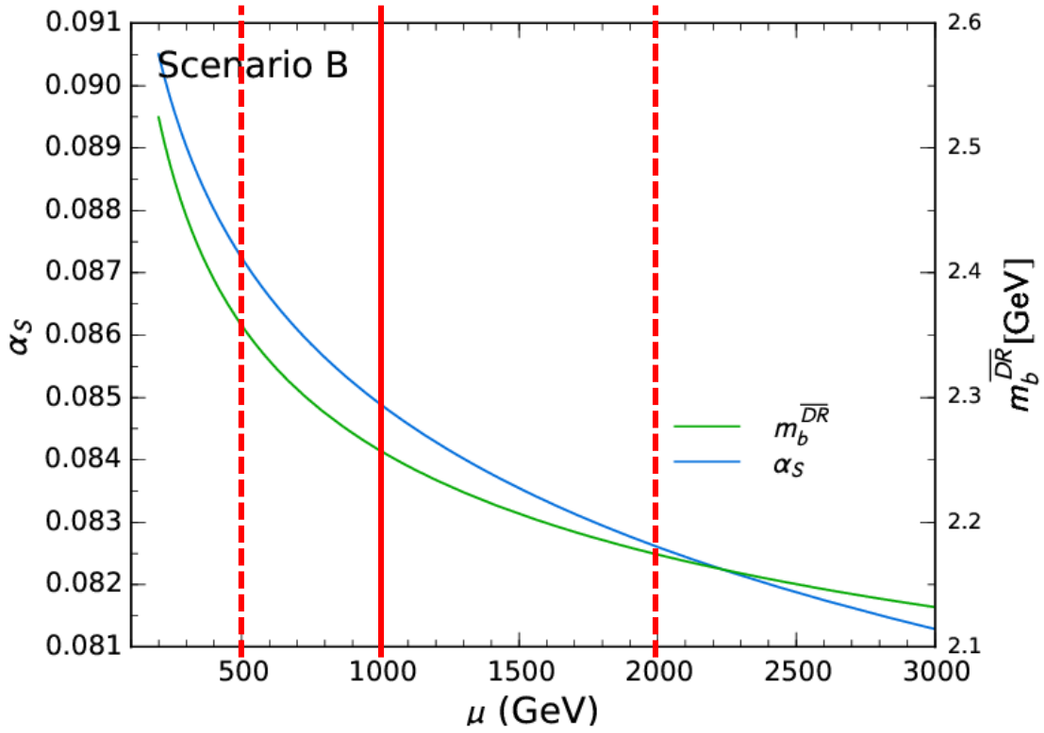
A. Bauer et al., JEP 0902, 03 (2009)

Theoretical Uncertainty from Scale Variation

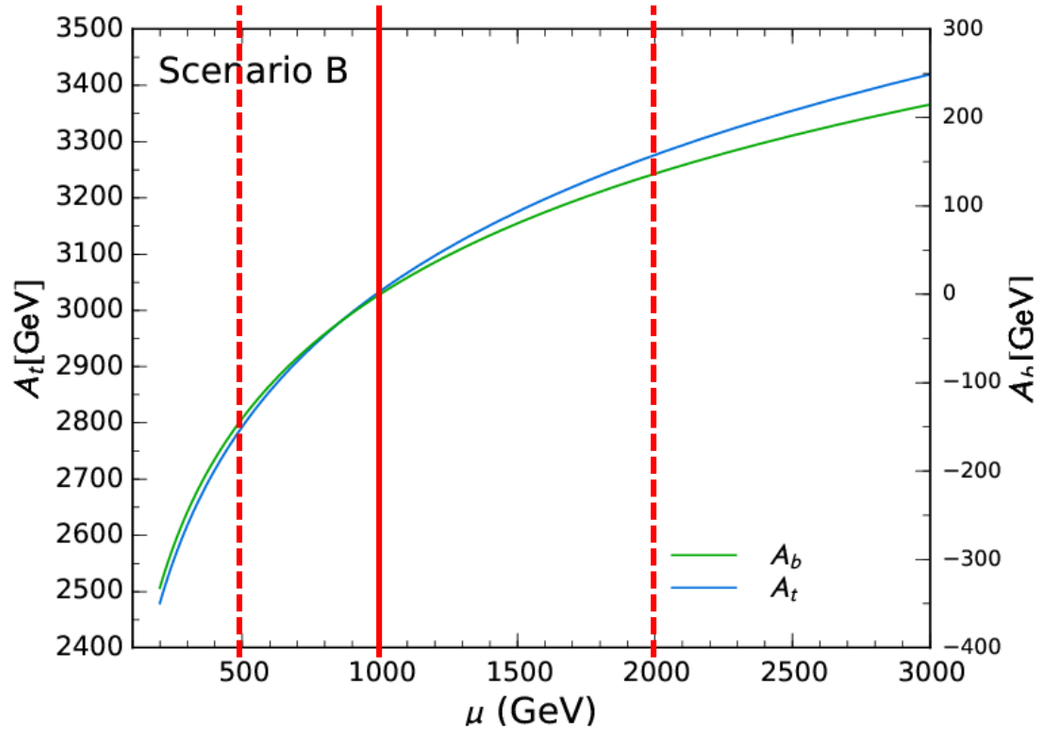
- loop calculation contains explicit uncancelled logs of the renormalisation scale
- is implicitly dependent on scale dependent parameters $\alpha_s, \theta_{\tilde{t}}, \theta_{\tilde{b}}, A_t, A_b, m_b, m_{\tilde{t}_2}$

→ scale variation gives an estimate of the uncertainty of the calculation

$$\frac{1}{2}\mu_R < \mu < 2\mu_R$$



α_s less scale dependent as in TeV range



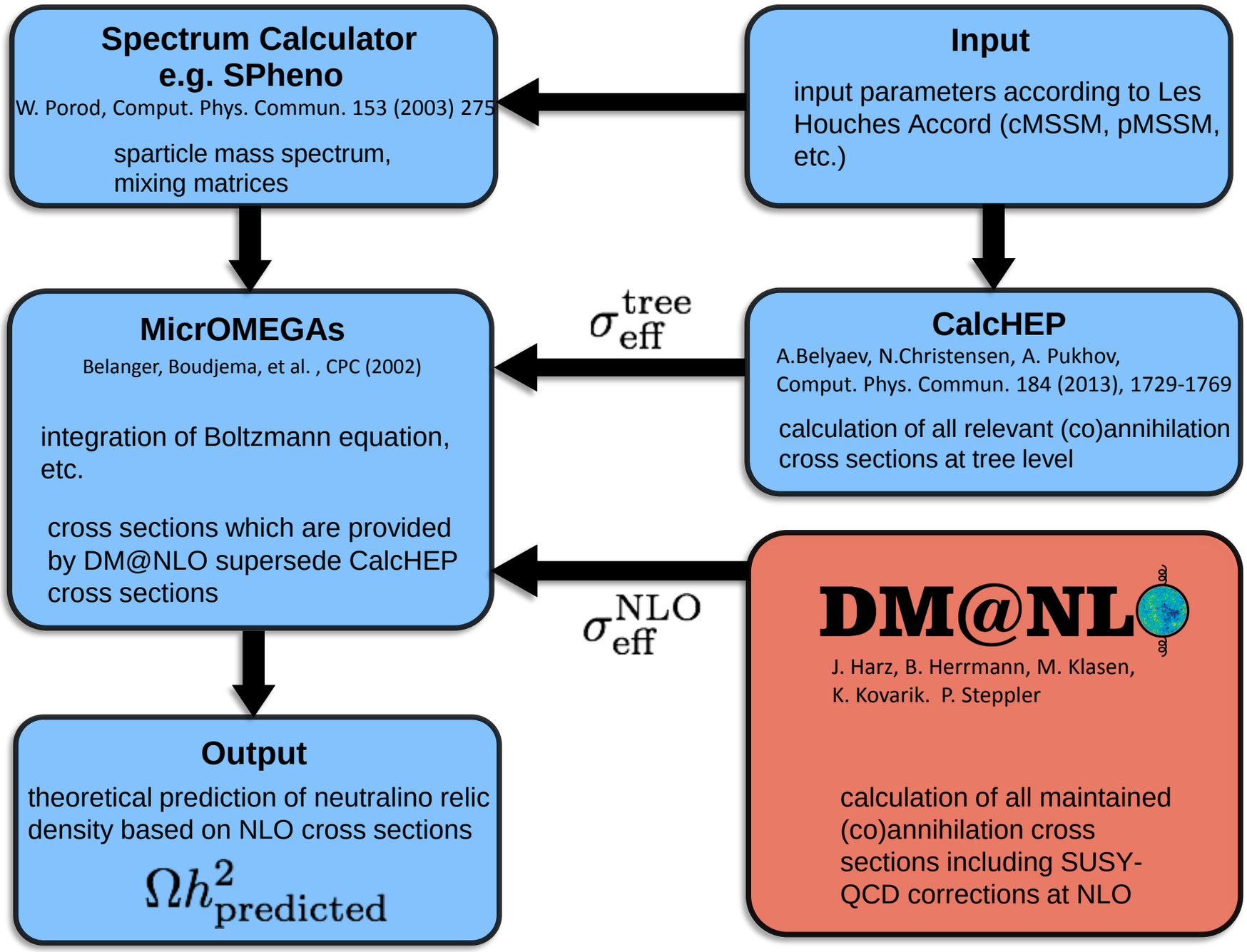
trilinear couplings more scale dependent

→Aim 3: Study dependence on renormalisation scale!

→ first study in literature!

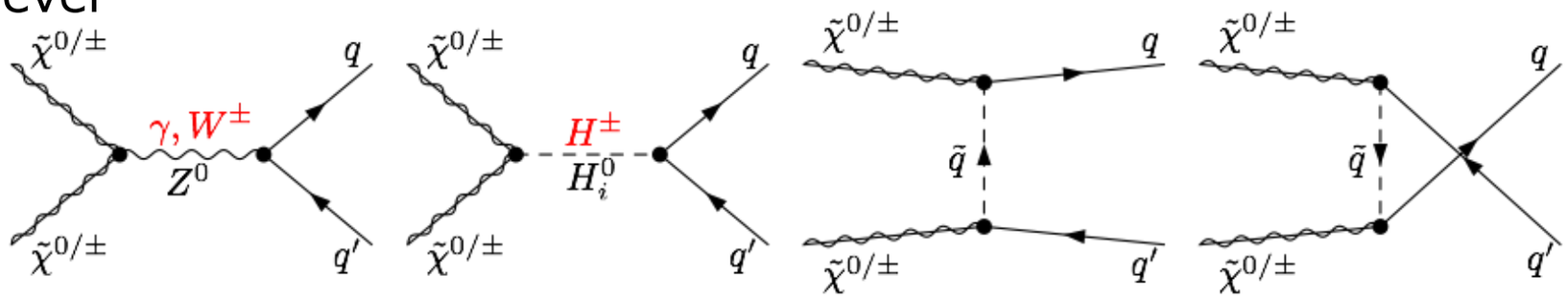
III. Results & Phenomenology

Performing Parameter Studies

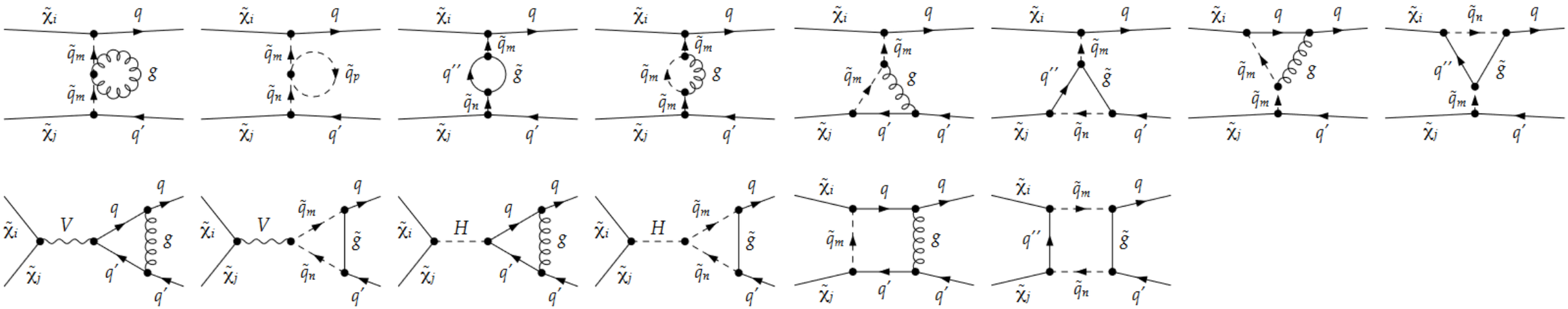


Gaugino Annihilation

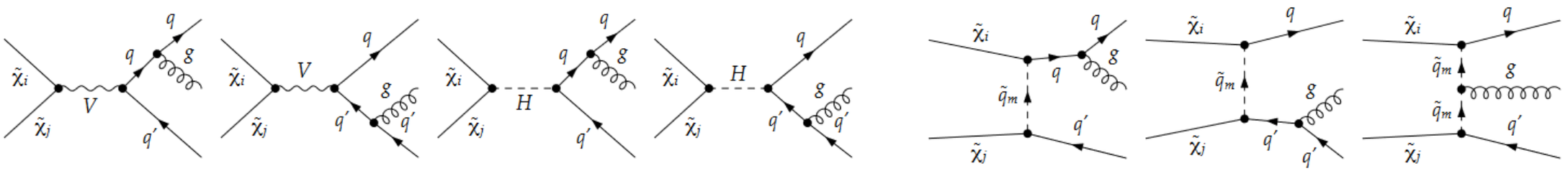
Tree level



Virtual one-loop corrections

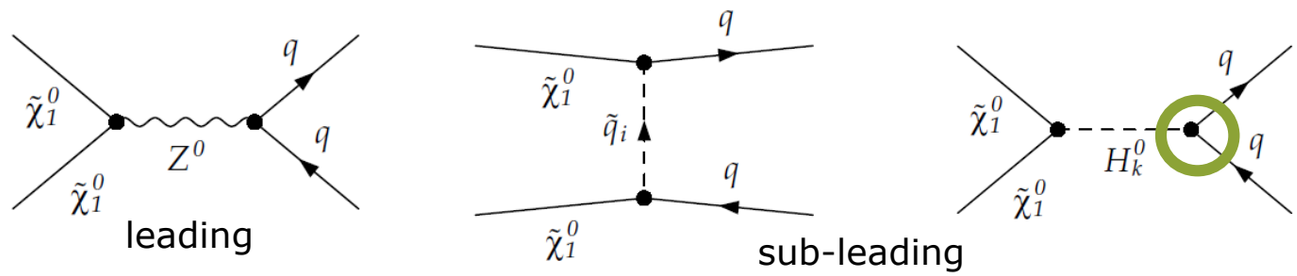


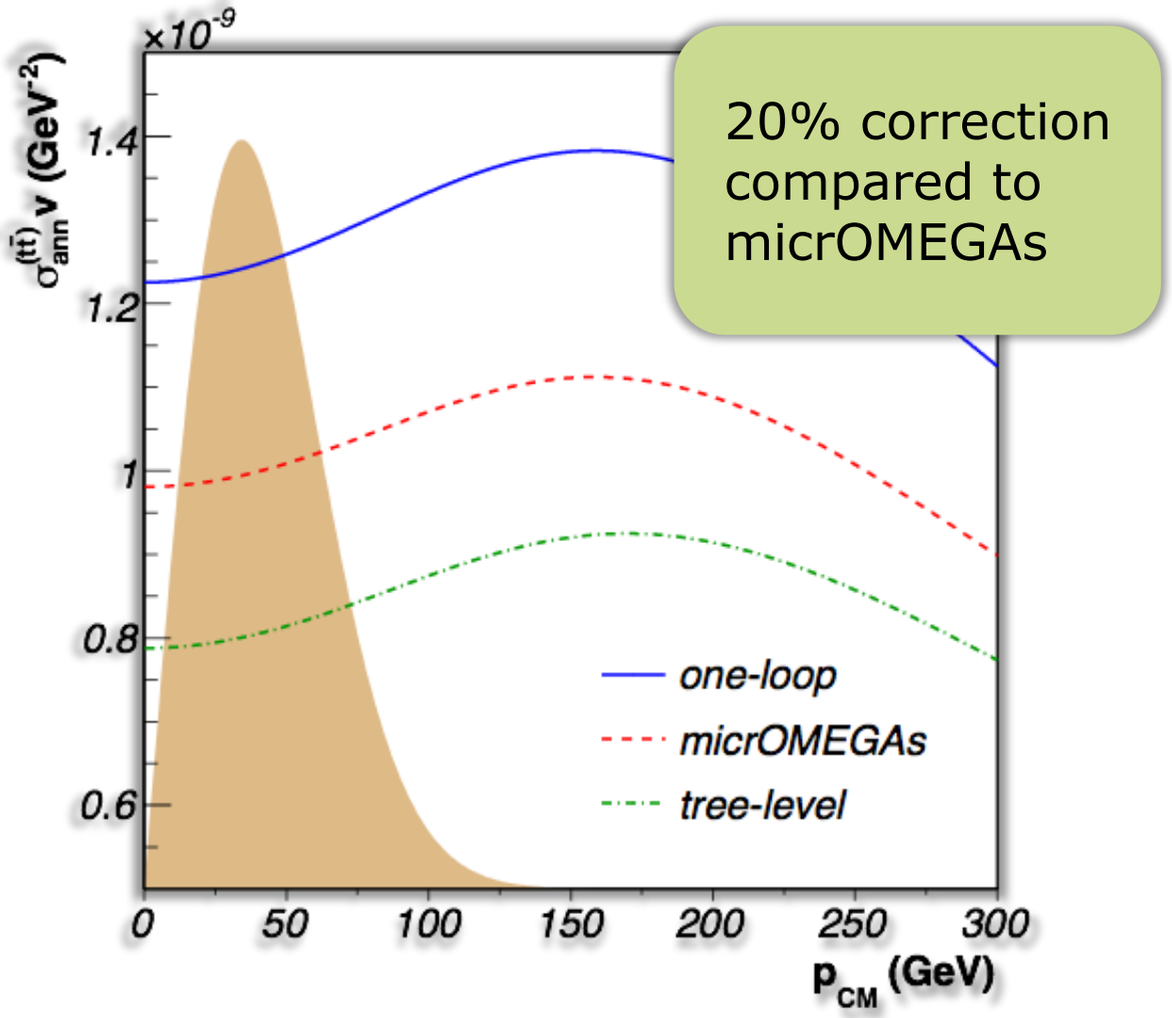
Real gluon emission processes



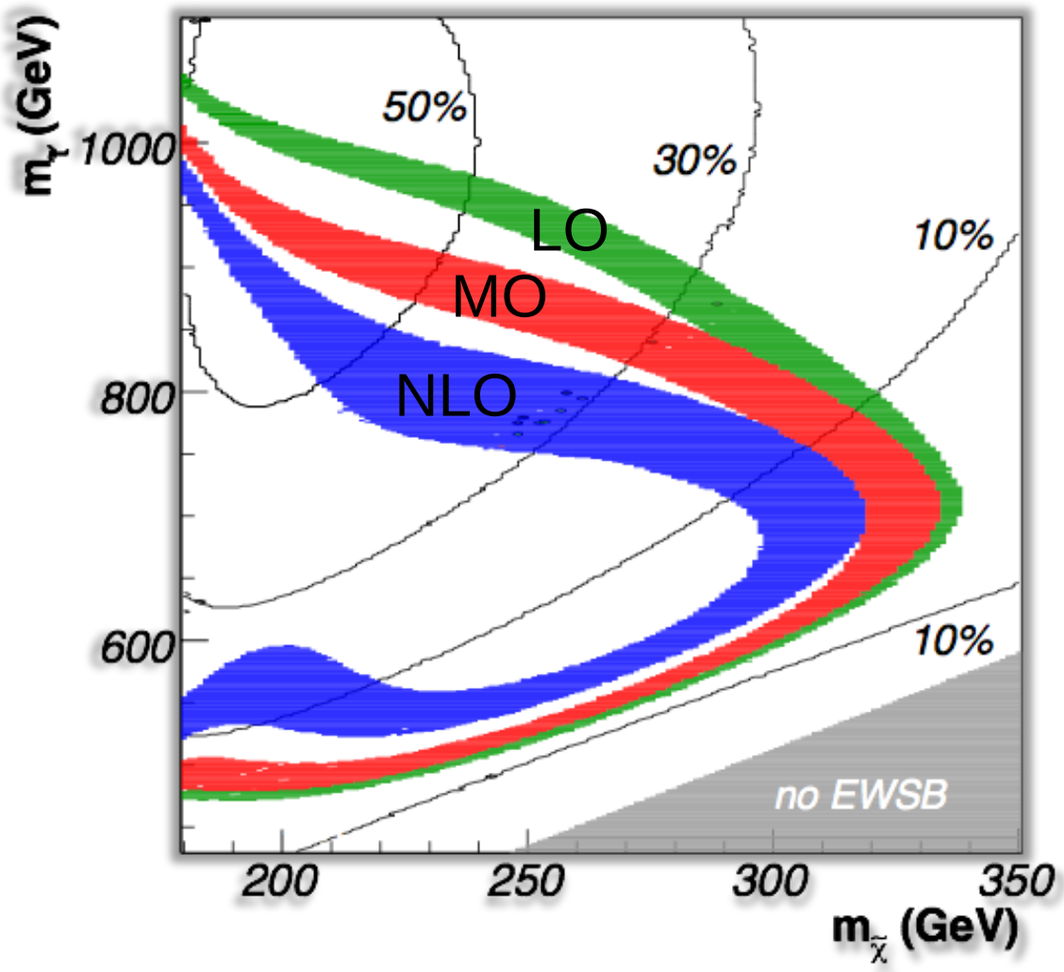
Neutralino Annihilation

- Example: Dominant Z-exchange





20% correction compared to micrOMEGAs



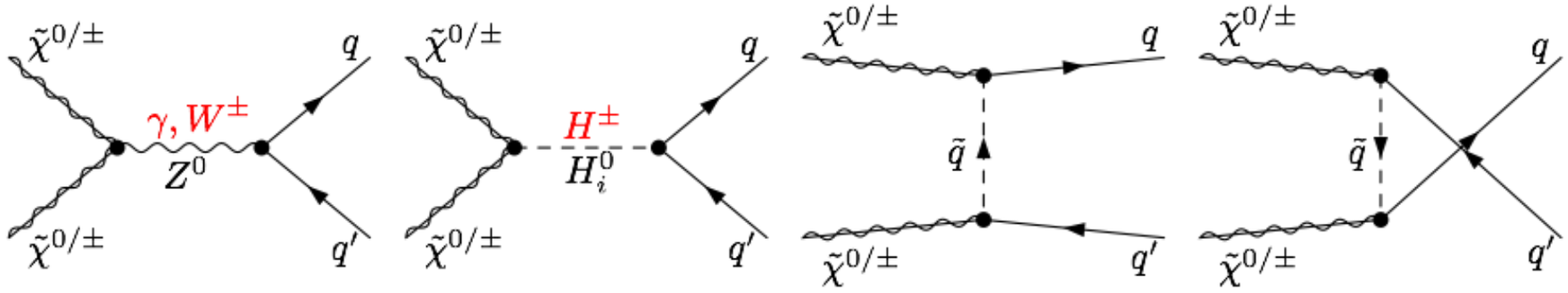
- effective Yukawa couplings not always sufficient, e.g. for dominant Z-exchange

→ shift of about 100 GeV in physical mass plane

B. Herrmann, M. Klasen and K. Kovařík, Phys. Rev. D 79: 061701 (2009), arXiv:0901.0481 [hep-ph]
B. Herrmann, M. Klasen and K. Kovařík, Phys. Rev. D 80: 085025 (2009), arXiv:0907.0030 [hep-ph]

Gaugino Annihilation

Tree level

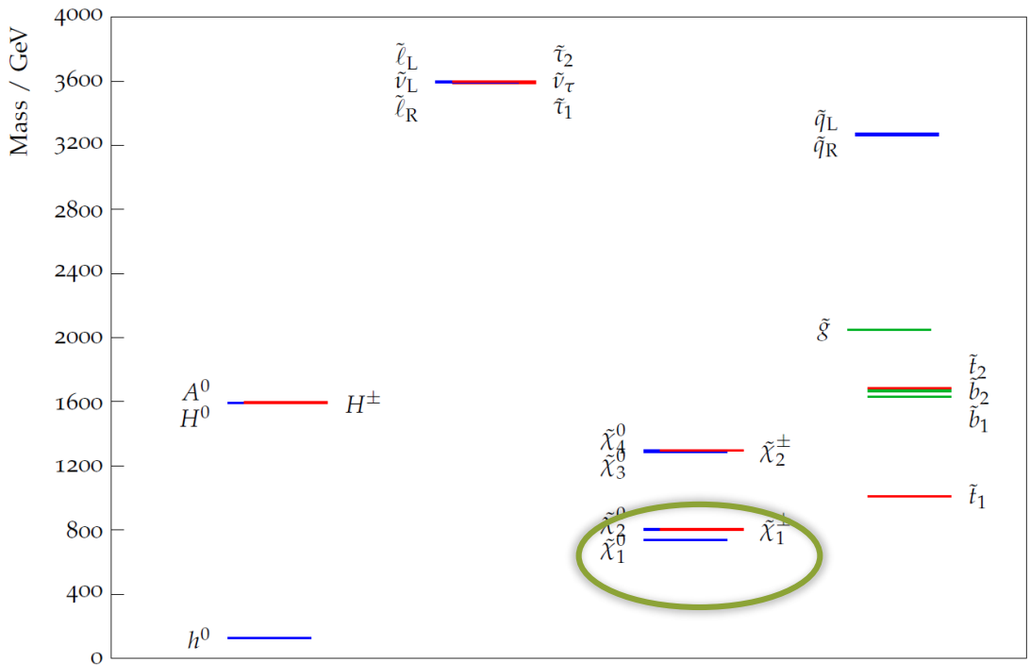


Input parameter pMSSM11

$\tan\beta$	μ	m_A	M_1	M_2	M_3	$M_{\tilde{q}_{1,2}}$	$M_{\tilde{q}_3}$	$M_{\tilde{u}_3}$	$M_{\tilde{\ell}}$	A_t
13.4	1286.3	1592.9	731.0	766.0	1906.3	3252.6	1634.3	1054.4	3589.6	-2792.3

Masses, mixing & observables

	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$Z_{1\tilde{B}}$	$Z_{1\tilde{W}}$	$Z_{1\tilde{H}_1}$	$Z_{1\tilde{H}_2}$	m_{h^0}	$\Omega_{\tilde{\chi}_1^0}h^2$	$\mathrm{BR}(b\to s\gamma)$
A	738.1	802.5	802.4	1295.3	1032.1	1682.0	- 0.996	0.049	-0.059	0.037	126.5	0.1248	$3.0\cdot10^{-4}$



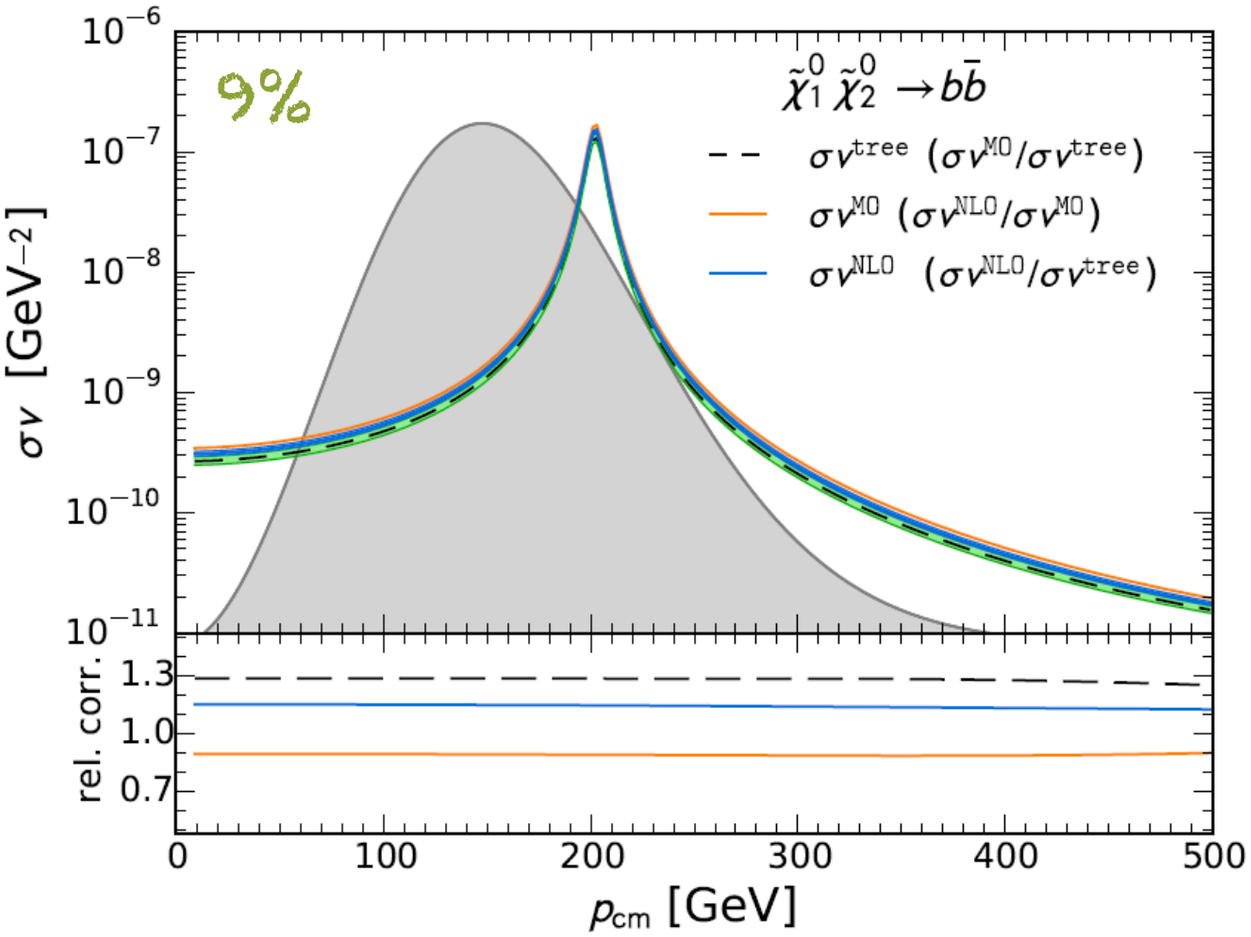
		A
$\tilde{\chi}_1^0\tilde{\chi}_1^0 \to$	$t\bar{t}$	2%
	$b\bar{b}$	9%
$\tilde{\chi}_1^0\tilde{\chi}_2^0 \to$	$t\bar{t}$	3%
	$b\bar{b}$	23%
$\tilde{\chi}_1^0\tilde{\chi}_1^\pm \to$	$t\bar{b}, \bar{t}b$	43%
Total		80%

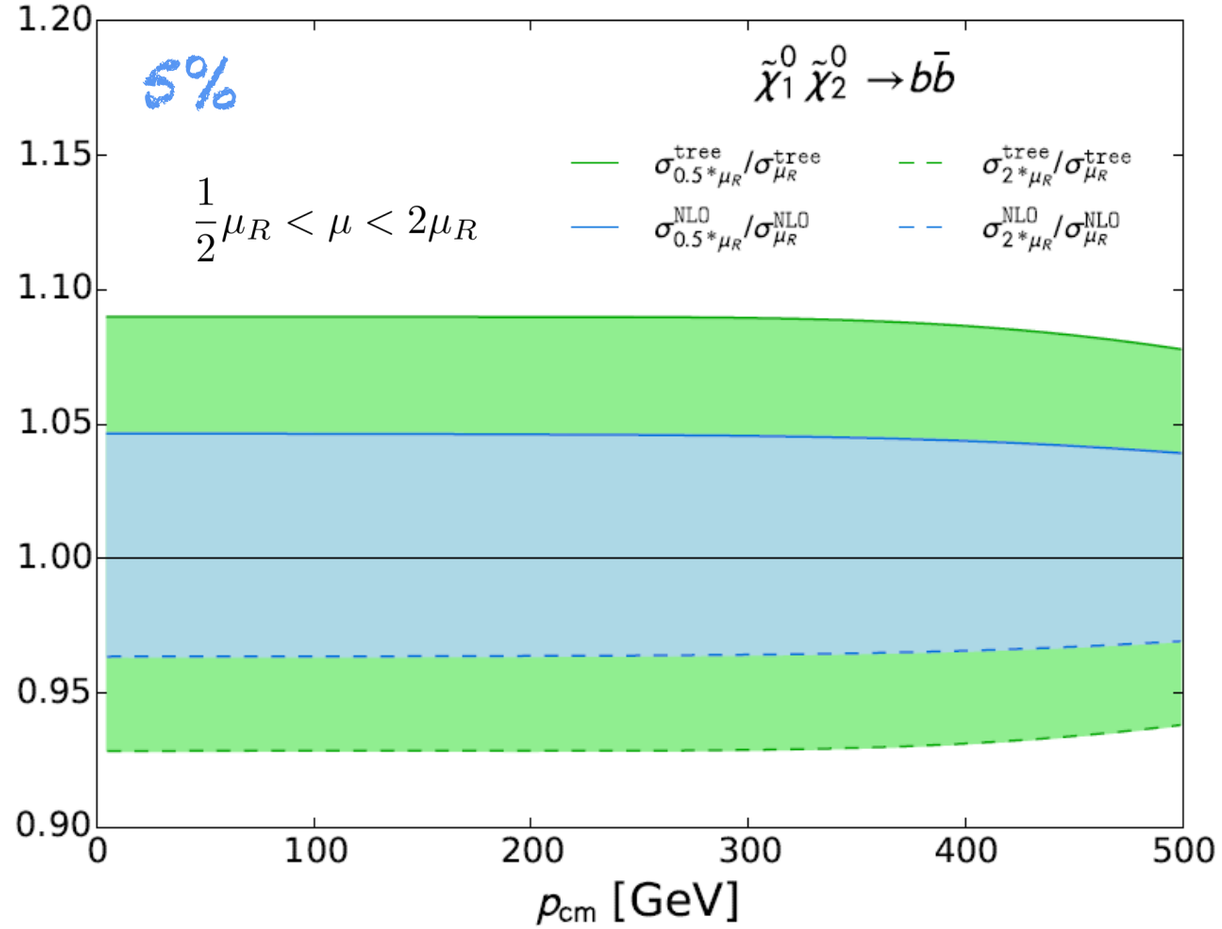
$$m_{\tilde{\chi}_1^0} + m_{\tilde{\chi}_1^+} = 1540.5\,\mathrm{GeV}$$
$$m_{\tilde{\chi}_1^0} + m_{\tilde{\chi}_2^0} = 1540.6\,\mathrm{GeV}$$
$$2m_{\tilde{\chi}_1^0} = 1476.4\,\mathrm{GeV}$$
$$2m_{\tilde{\chi}_2^0} \approx 2m_{\tilde{\chi}_1^+} \approx 1600\ \mathrm{GeV}$$

$$m_{H^\pm} = 1595.1\,\mathrm{GeV}$$
$$m_{A^0} = 1592.9\,\mathrm{GeV}$$

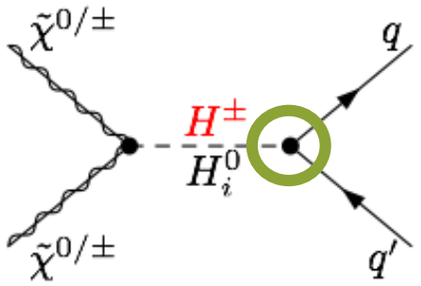
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, arXiv:1602.08103 [hep-ph]
B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke and P. Steppeler, Phys. Rev. D 89, 114012 (2014), arXiv:1404.2931 [hep-ph]

Gaugino Annihilation





- NLO value does not lie within LO uncertainty band ← pure electroweak process
- **Larger** LO uncertainty band ← scale dependent $m_b^{\overline{DR}}$
- **Smaller** NLO uncertainty band ← scale dependence decreased by virtual corrections to $A^0 b\bar{b}$ coupling

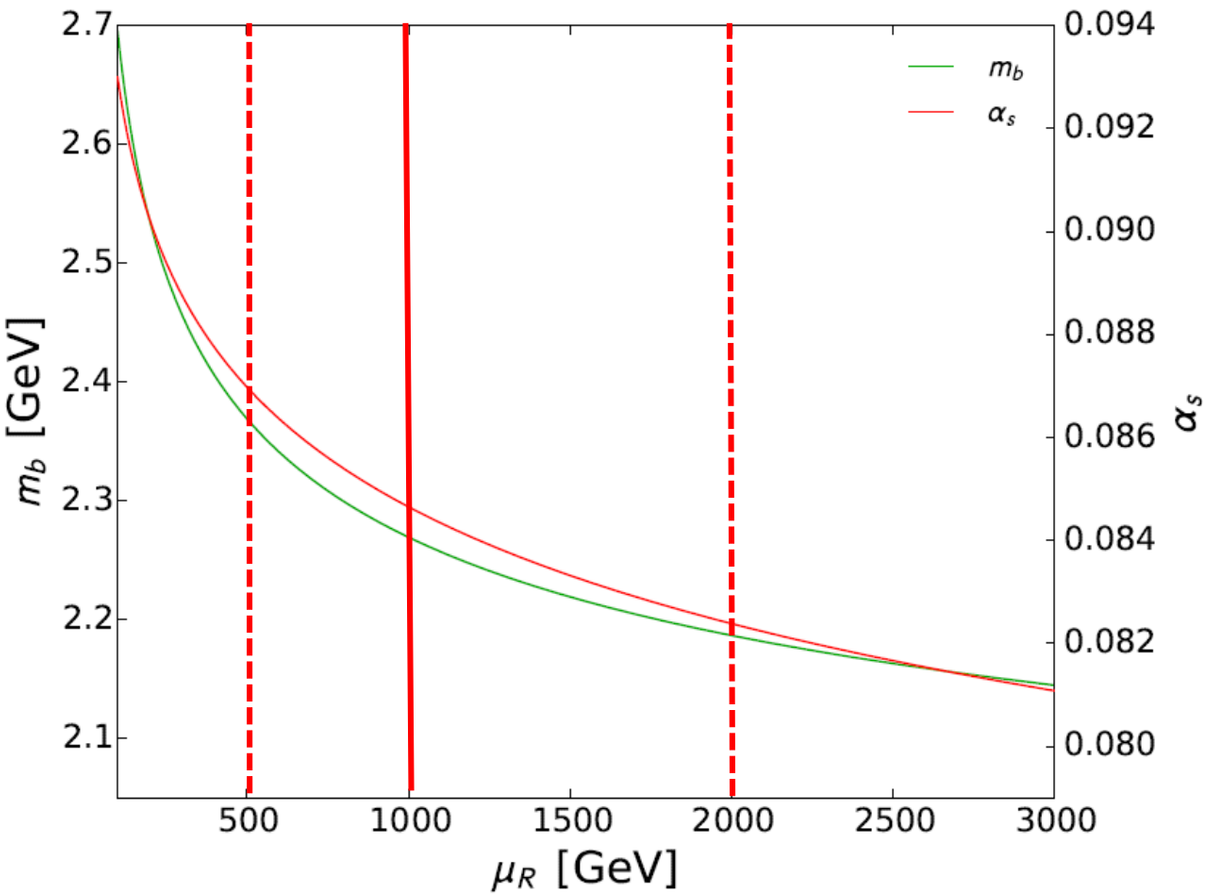


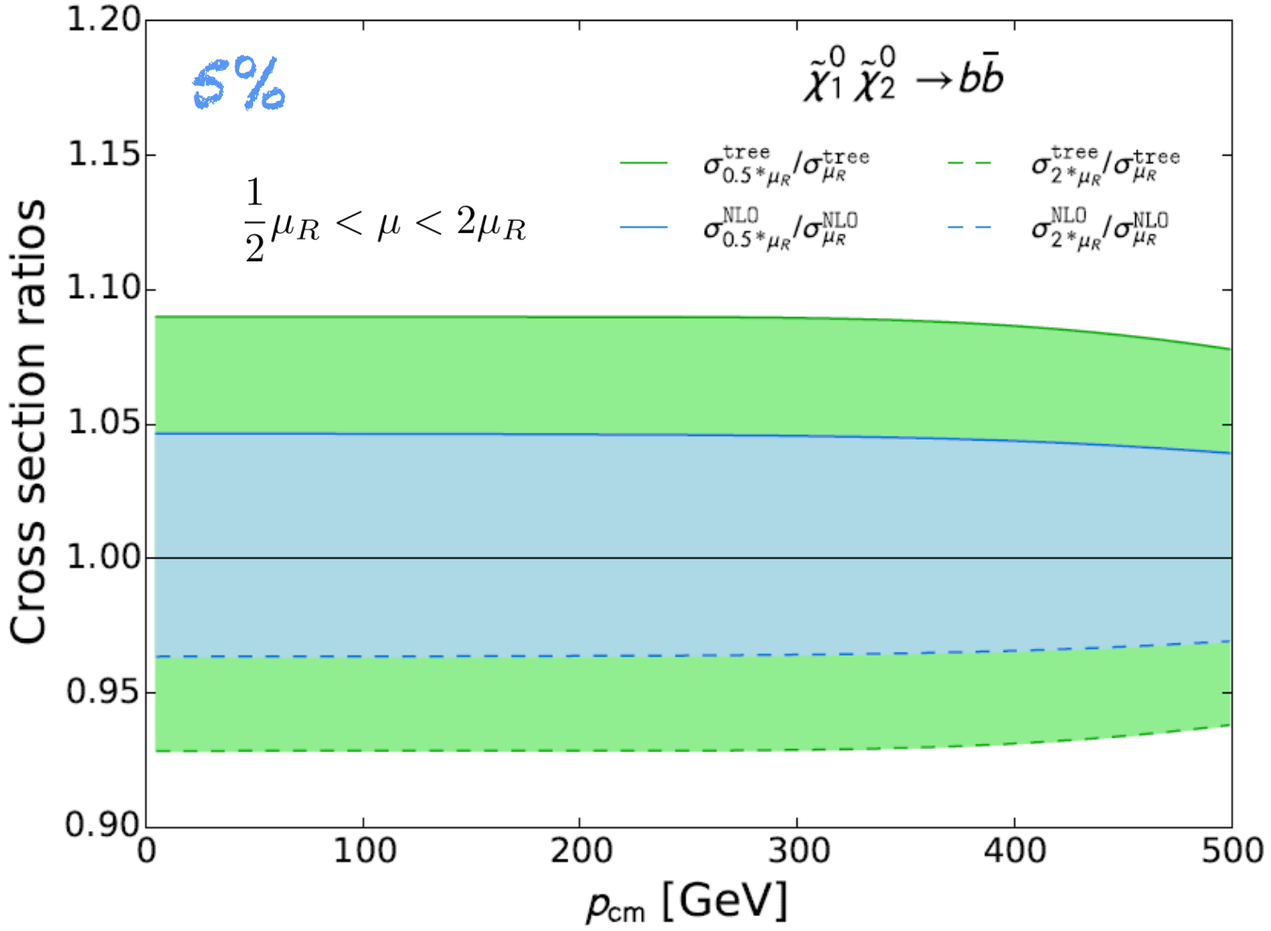
→ NLO calculation shows 10-15% correction with respect to default MicrOMEGAs value

→ 5 % uncertainty on the NLO cross section

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys.Rev. D 93 114023 (2016), arXiv:1602.08103 [hep-ph]
B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke and P. Steppeler, Phys. Rev. D 89, 114012 (2014), arXiv:1404.2931 [hep-ph]

Gaugino Annihilation





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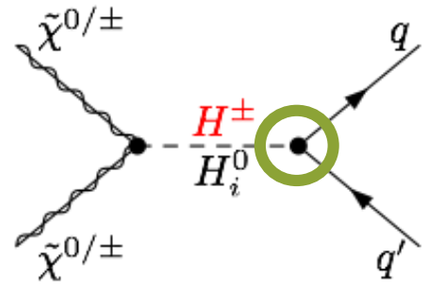

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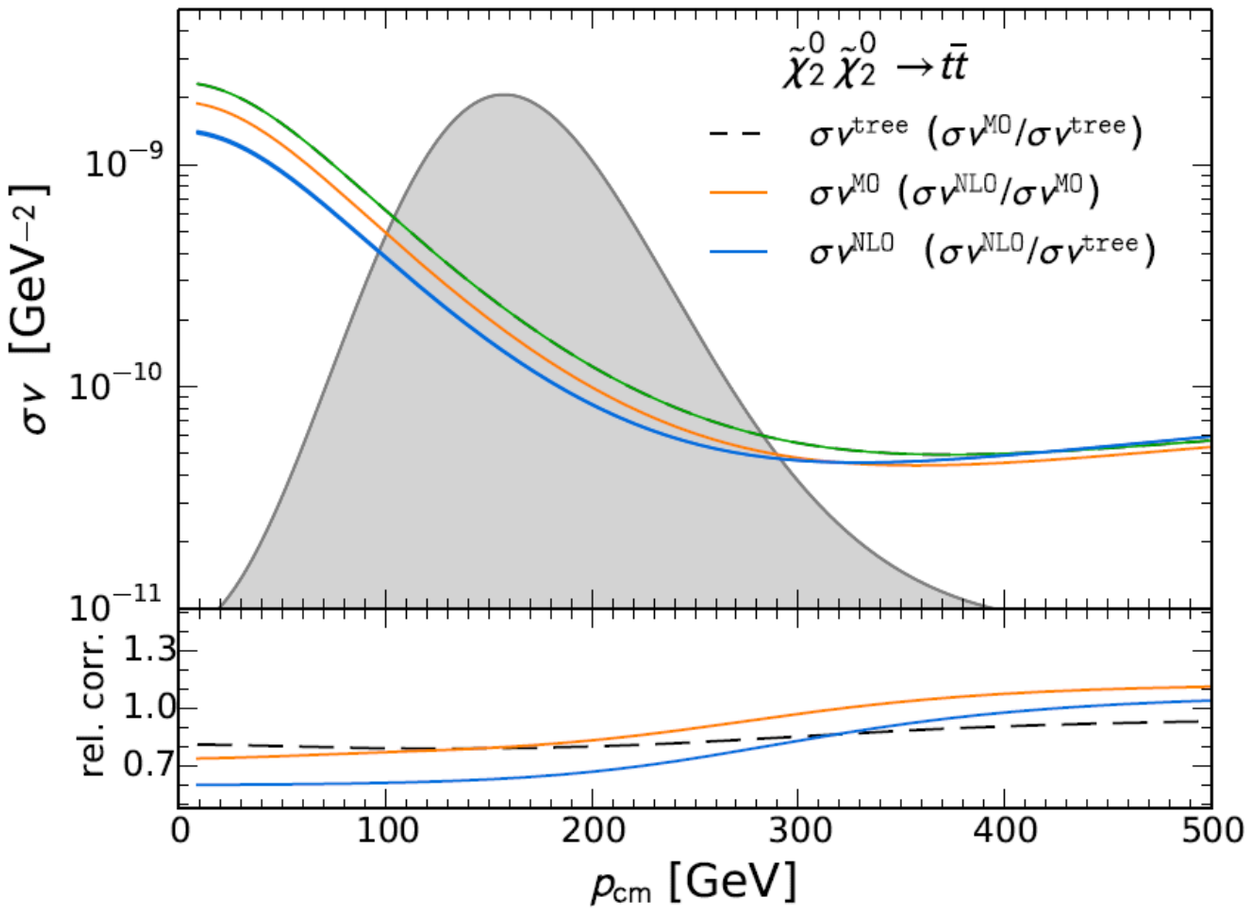
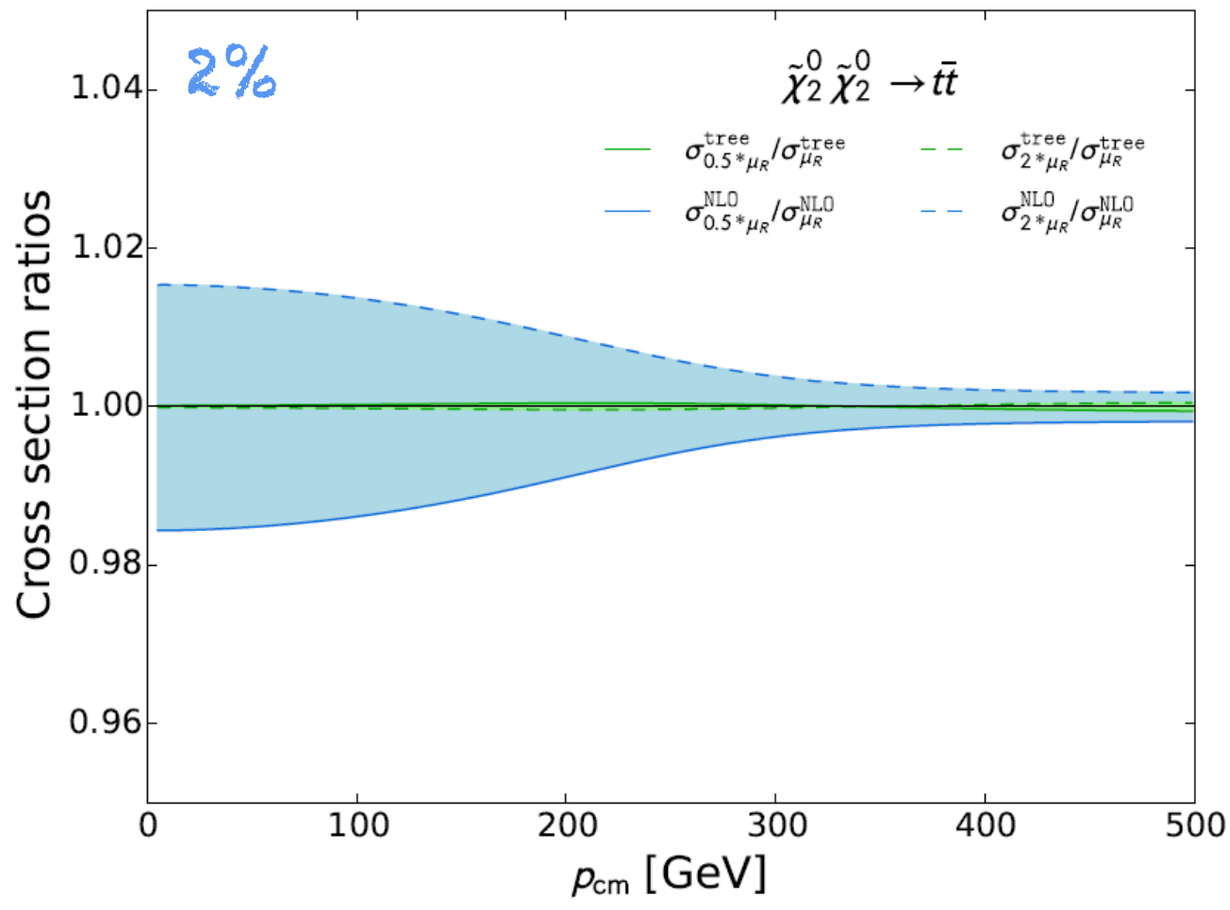
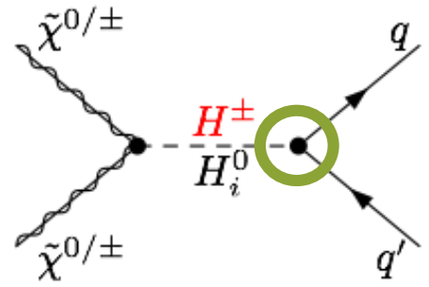
B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke and P. Steppeler, Phys. Rev. D 89, 114012 (2014), arXiv:1404.2931 [hep-ph]

Gaugino Annihilation





- Tiny scale dependence at LO and NLO
- **Smaller** LO uncertainty band ← scale independent m_t^{OS}
- **Larger** NLO uncertainty band ← first appearance of scale dependent α_s

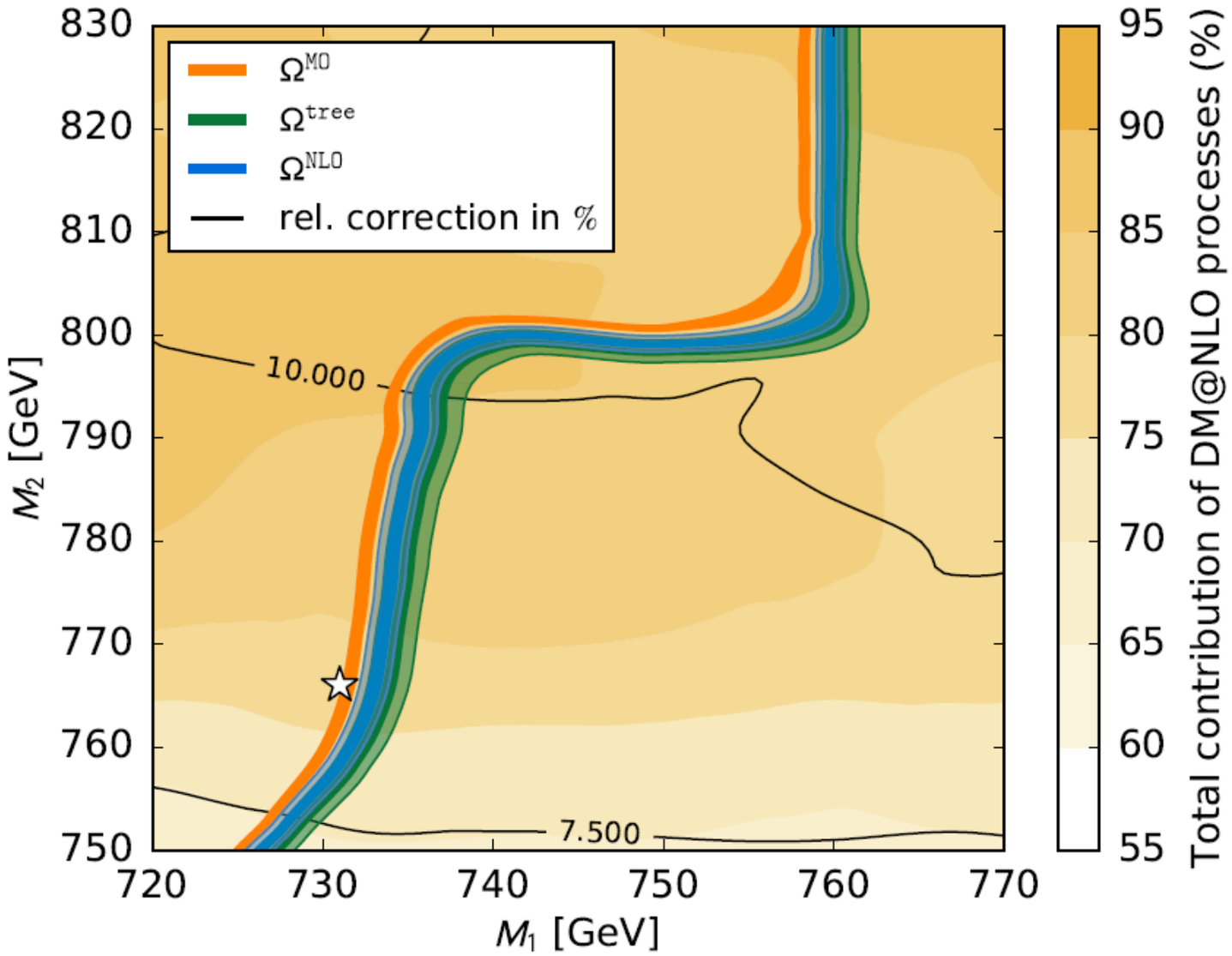


→ size of theoretical uncertainty is process dependent

→ theoretical uncertainty becomes possible to quantify only at NLO for the first time

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys.Rev. D 93 114023 (2016), arXiv:1602.08103 [hep-ph]
B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke and P. Steppeler, Phys. Rev. D 89, 114012 (2014), arXiv:1404.2931 [hep-ph]

Gaugino Annihilation



→ scale dependence at LO mainly triggered by $m_b^{\overline{DR}}$

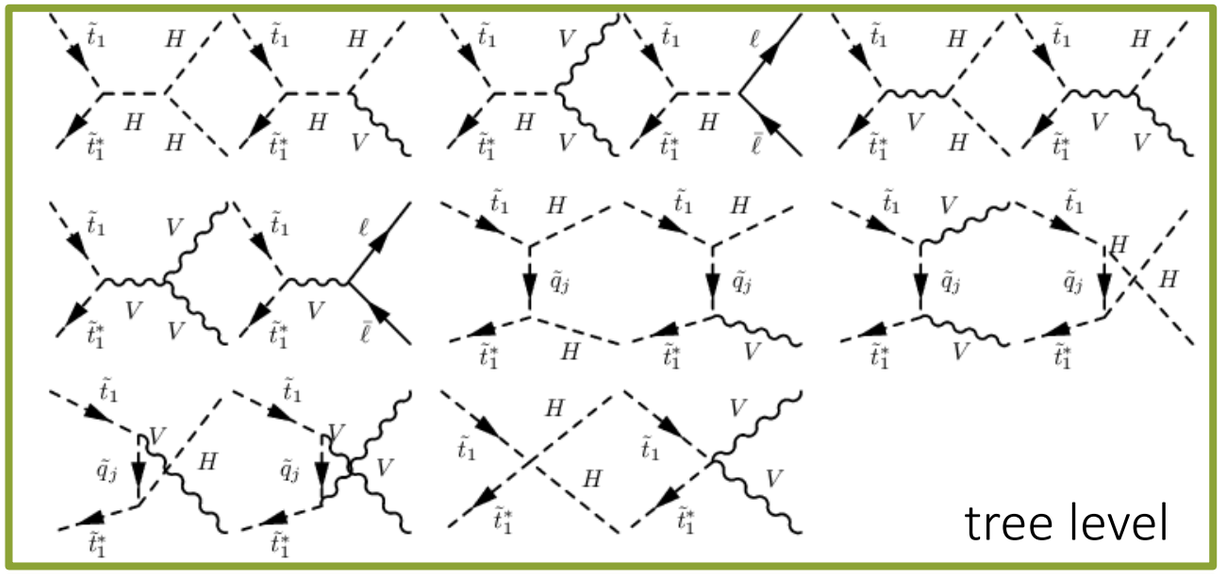
→ decreased scale dependence at NLO

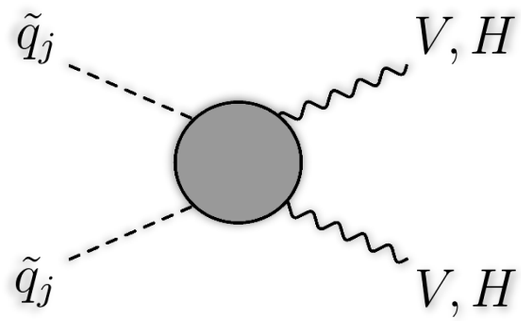
- NLO calculation shows 5-10% relative correction with respect to default MO value
- shift in the parameter space with respect to MO value is larger than experimental uncertainty, even when including theoretical uncertainty
- error 2 times larger than naively expected

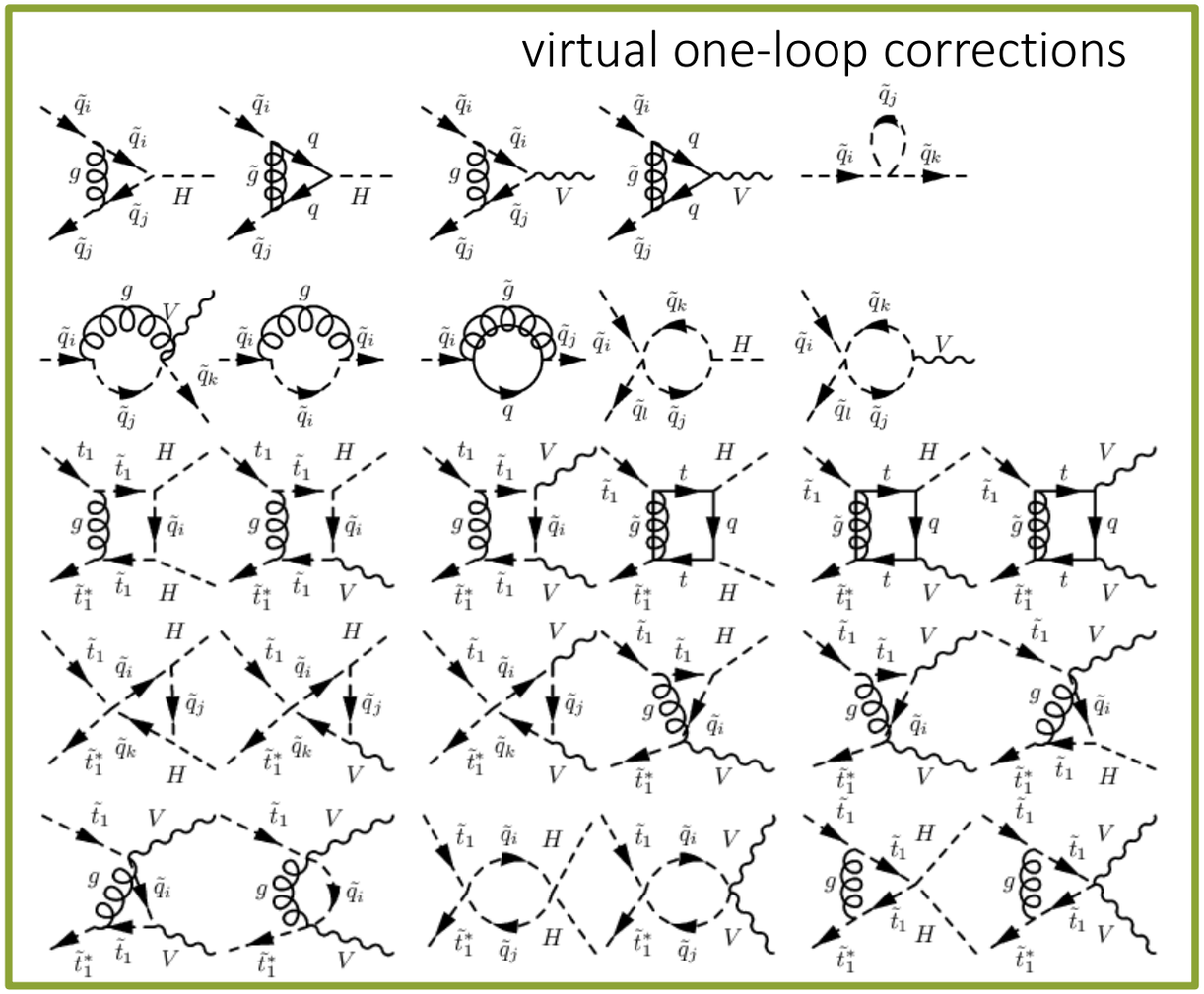
→ first quantitative estimation of theoretical uncertainty!

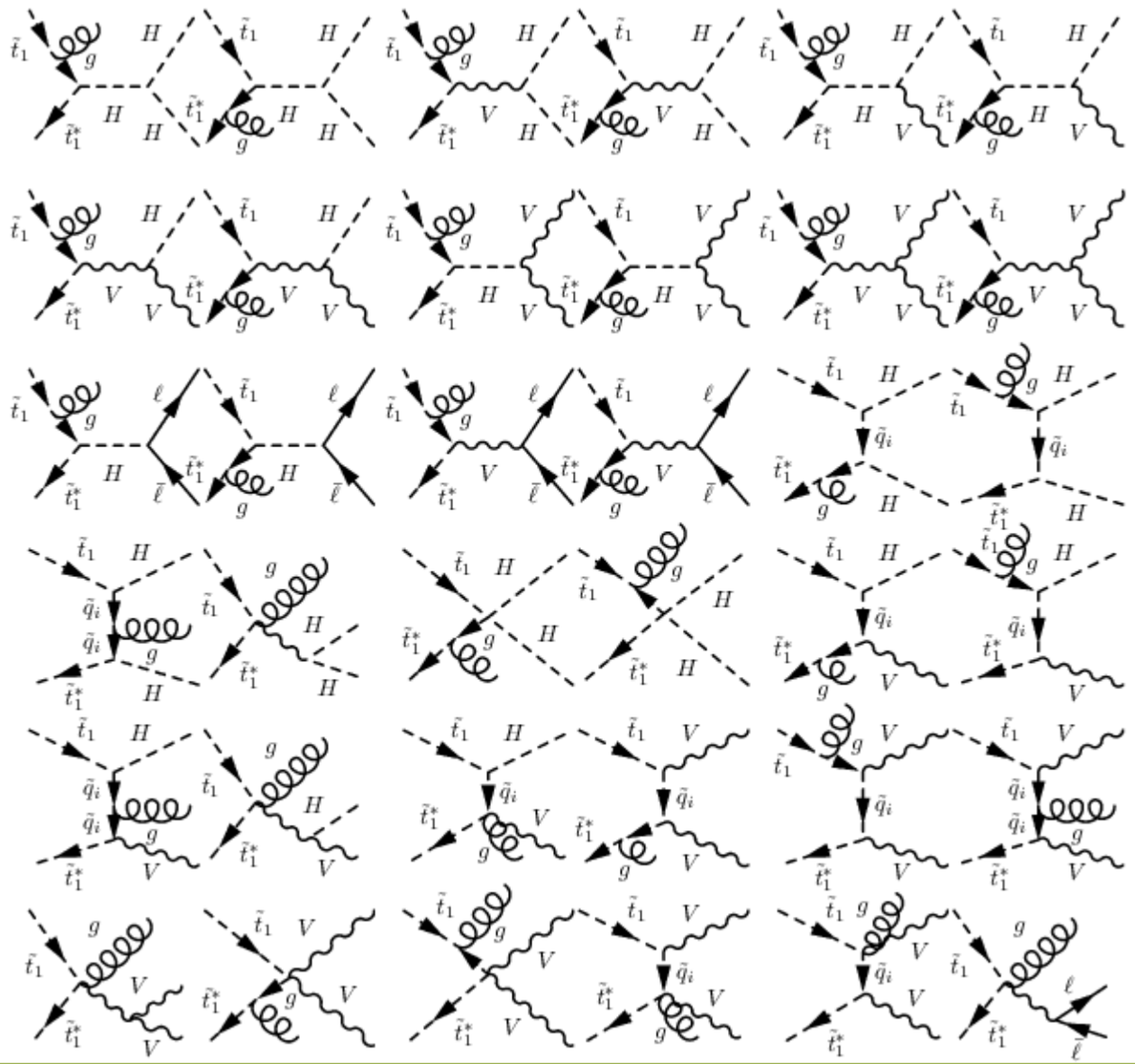
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys.Rev. D 93 114023 (2016), arXiv:1602.08103 [hep-ph]

Stop-Antistop Annihilation





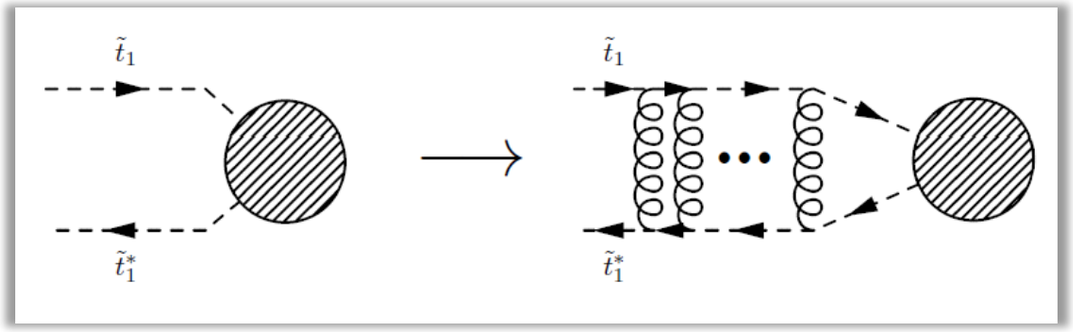




J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and M. Meinecke, Phys. Rev D 91 034012 (2015), arXiv:1410.8063 [hep-ph]

Stop-Antistop Annihilation

+ Coulomb enhancement



- during freeze-out stop-antistop pair is moving slowly
- exchange of *n* gluons lead to a correction factor proportional to $(\frac{\alpha_s}{v})^n$
- with $\alpha_s/v > \mathcal{O}(1)$ Coulomb corrections can be become sizeable

→ resummation to all orders within framework of nonrelativistic QCD

M.J. Strassler and M.E.Peskin, Phys. Rev. D 43 1500 (1991)
Y. Kiyo et al. Eur. Phys. J. C. 60 375 (2009)

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and M. Meinecke, Phys. Rev D 91 034012 (2015), arXiv:1410.8063 [hep-ph]

Stop-Antistop Annihilation

- Coulomb enhanced cross section factorises in LO cross section and Greens function:

$$\sigma^{\text{Coul.}}\left(\tilde{t}_1\tilde{t}_1^* \to \text{EW}\right) = \frac{4\pi}{v m_{\tilde{t}_1}^2}\Im\left\{G^{[1]}\left(\mathbf{r}=0;\sqrt{s}+i\Gamma_{\tilde{t}_1},\mu_C\right)\right\} \times \sigma^{\text{LO}}\left(\tilde{t}_1\tilde{t}_1^* \to \text{EW}\right)$$

- Greens function is determined as solution of the would-be stoponium Schroedinger equation

$$\left[H^{[1]} - \left(\sqrt{s}+i\Gamma_{\tilde{t}_1}\right)\right]G^{[1]}\left(\mathbf{r};\sqrt{s}+i\Gamma_{\tilde{t}_1},\mu_C\right) = \delta^{(3)}(\mathbf{r})$$

M.J. Strassler and M.E.Peskin, Phys. Rev. D 43 1500 (1991)
Y. Kiyo et al. Eur. Phys. J. C. 60 375 (2009)

$$H^{[1]} = -\frac{1}{m_{\tilde{t}_1}}\Delta + 2m_{\tilde{t}_1} + V^{[1]}(\mathbf{r}) \qquad \tilde{V}^{[1]}(\mathbf{q}) = -\frac{4\pi\alpha_s(\mu_C)C^{[1]}}{\mathbf{q}^2} \times \left[1 + \frac{\alpha_s(\mu_C)}{4\pi}\left(\beta_0 \ln\frac{\mu_C^2}{\mathbf{q}^2} + a_1\right)\right]$$

Coulomb potential
@ NLO $\mathcal{O}((\alpha_s^2/v)^n)$

- Greens function carries scale as it has to be regularised at r = 0

$$\mu_C^{\text{central}} = \max\{\mu_B, 2m_{\tilde{t}_1}v\}$$

$$\mu_B = C_F\alpha_s m_{\tilde{t}_1}$$

- Scale is independent of renormalisation scale, chosen to be close to inverse Bohr radius (characteristic scale of stoponium bound state)

- Explicit logarithmic scale dependence kicks in only at NNLO:

$$\Im\left\{G^{[1]}\left(0;\sqrt{s}+i\Gamma_{\tilde{t}_1},\mu_C\right)\right\} = m_{\tilde{t}_1}^2\,\Im\left\{\frac{v}{4\pi}\left[i + \frac{\alpha_s(\mu_C)C_F}{v}\left(\frac{i\pi}{2} + \ln\frac{\mu_C}{2m_{\tilde{t}_1}v}\right) + \mathcal{O}(\alpha_s^2)\right]\right\}$$

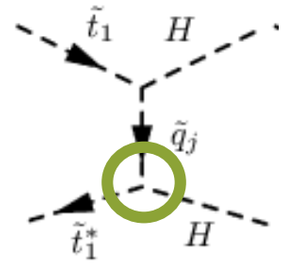
→ In the following: vary Coulomb scale as well around its central value $\frac{1}{2}\mu_C < \mu < 2\mu_C$

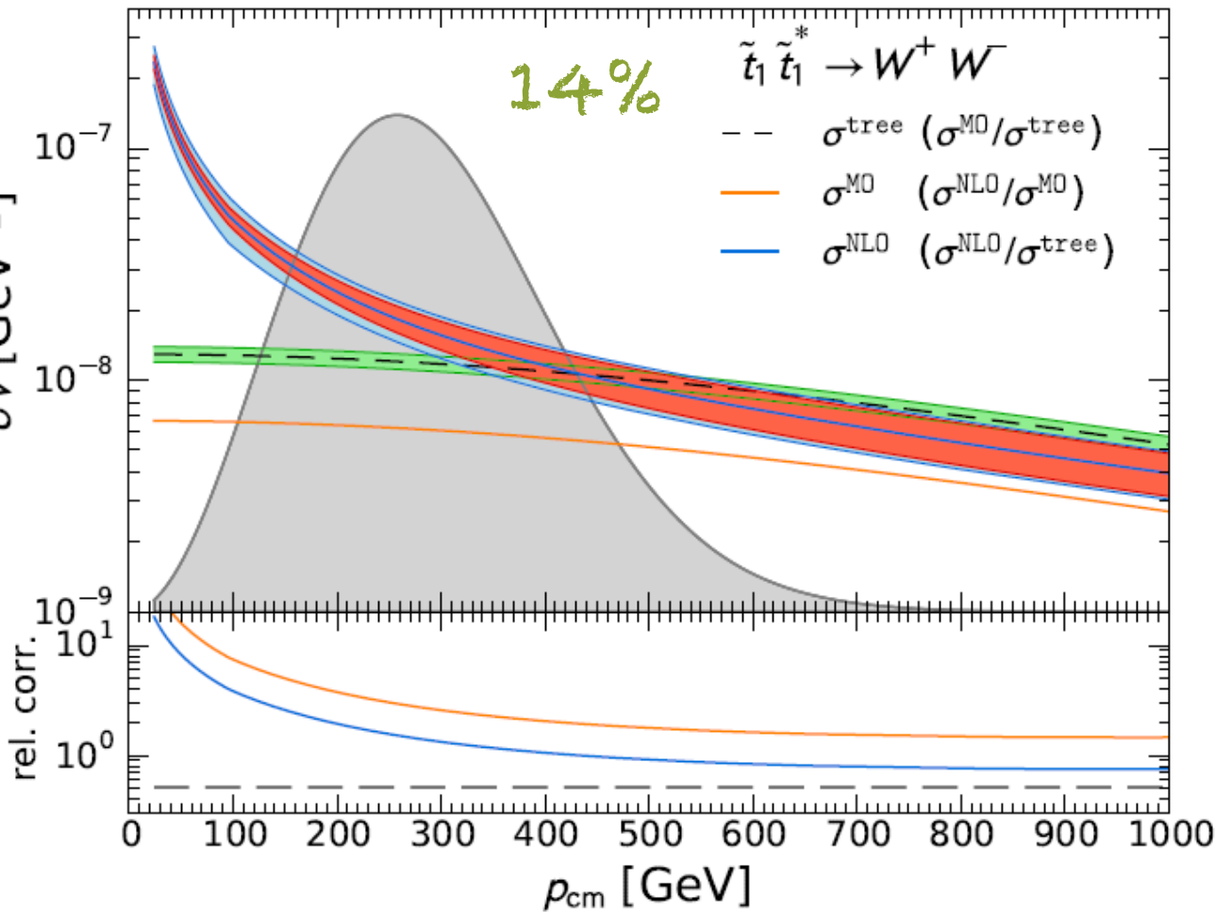
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and M. Meinecke, Phys. Rev D 91 034012 (2015), arXiv:1410.8063 [hep-ph]

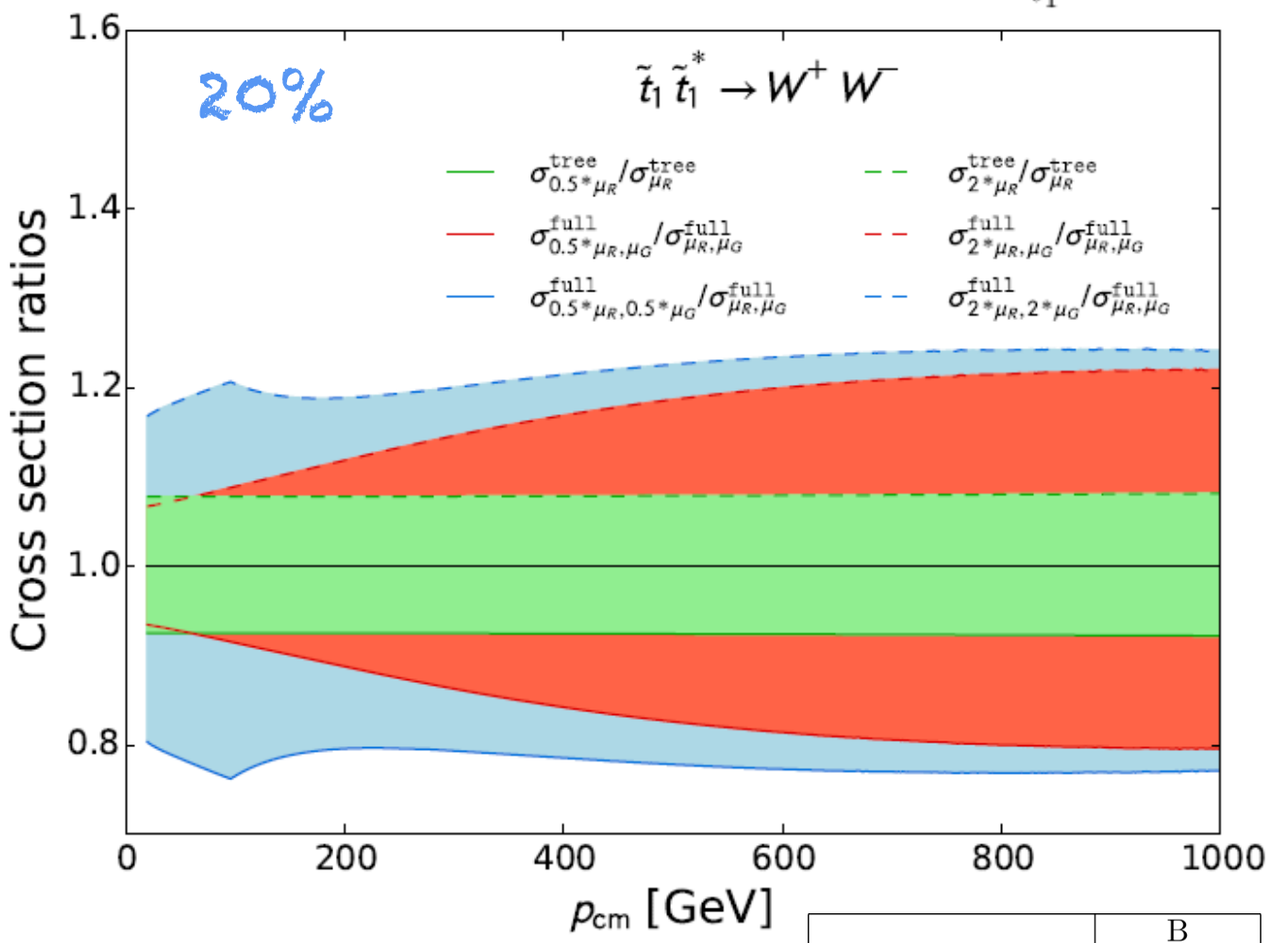
Stop-Antistop Annihilation

Scenario with a small mass gap between LSP and NLSP

	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$Z_{1\tilde{B}}$	$Z_{1\tilde{W}}$	$Z_{1\tilde{H}_1}$	$Z_{1\tilde{H}_2}$	m_{h^0}	$\Omega_{\tilde{\chi}_1^0}h^2$	$\mathrm{BR}(b \to s\gamma)$
B	1306.3	1827.0	1827.2	2640.0	1361.7	2157.3	-1.000	0.002	-0.024	0.013	123.7	0.1134	$3.1 \cdot 10^{-4}$







- Scale dependence at LO triggered by trilinear coupling A_t
- Scale dependence at NLO due to explicit logarithms and implicit ($A_t\,\alpha_s$)
- Coulomb-scale leads to 20% uncertainty

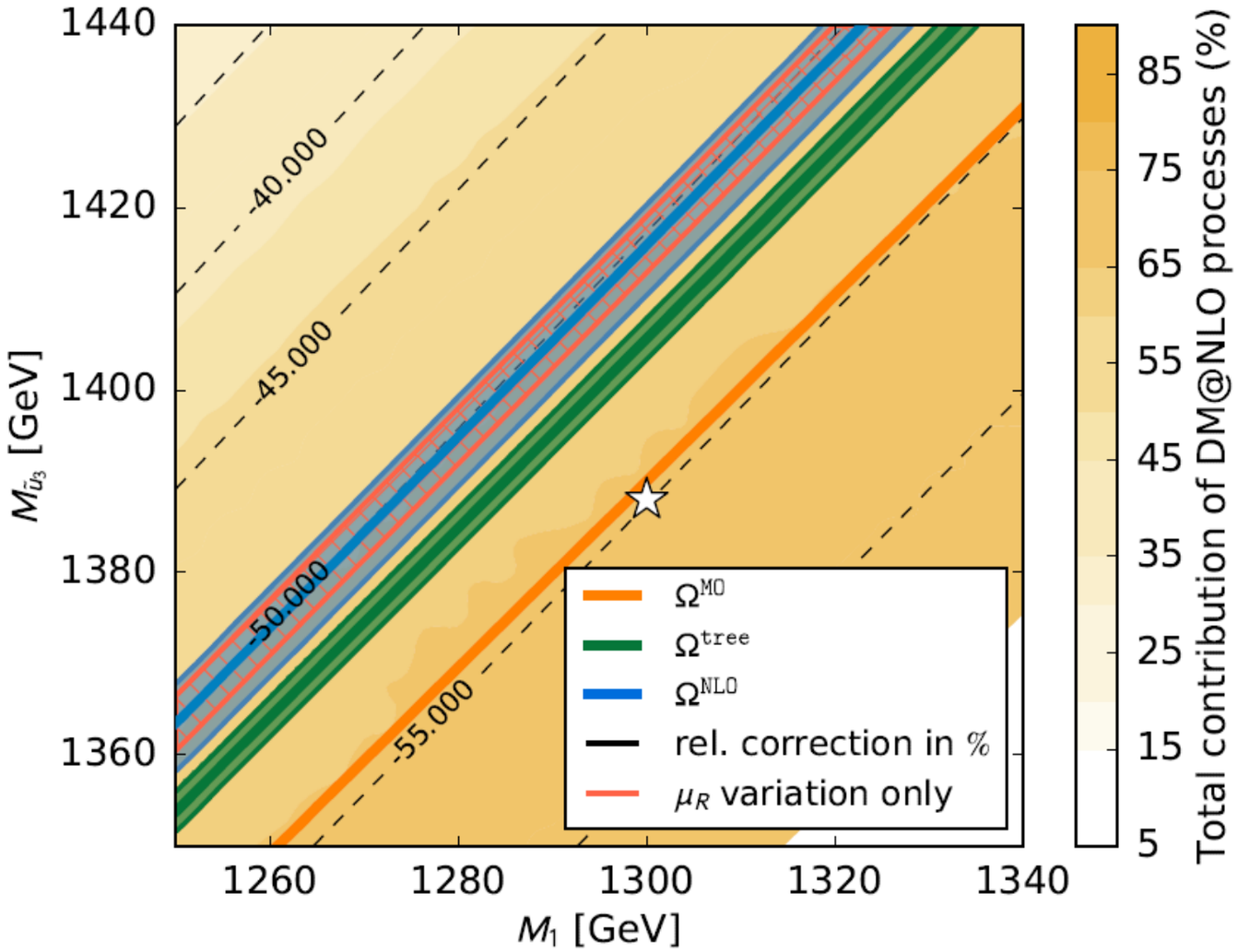
→ large correction (K-factor of 1-9) in relevant region, 20 % theoretical error

		B
$\tilde{\chi}_1^0\tilde{t}_1 \to$	th^0	1%
	tg	6%
$\tilde{t}_1\tilde{t}_1^* \to$	h^0h^0	12%
	h^0H^0	11%
	Z^0A^0	7%
	$W^\pm H^\mp$	13%
	Z^0Z^0	8%
	W^+W^-	14%
Total		72%

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys.Rev. D 93 114023 (2016), arXiv:1602.08103 [hep-ph]

Stop-Antistop Annihilation

Scenario with a small mass gap between LSP and NLSP

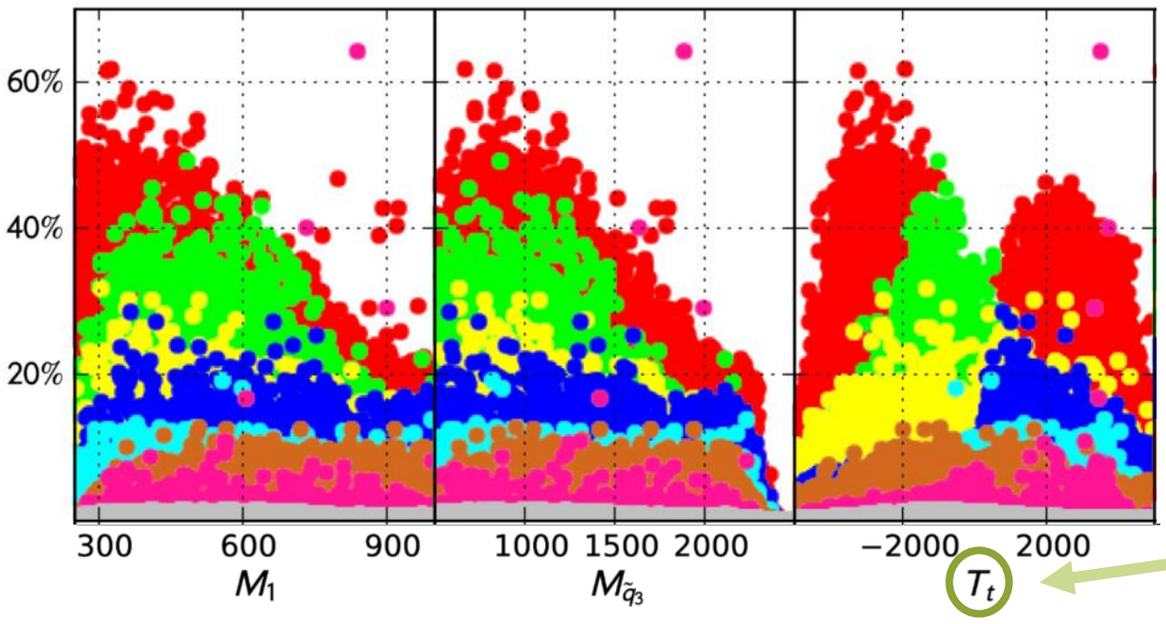


		B
$\tilde{\chi}_1^0\tilde{t}_1 \to$	th^0	1%
	tg	6%
$\tilde{t}_1\tilde{t}_1^* \to$	h^0h^0	12%
	h^0H^0	11%
	Z^0A^0	7%
	$W^\pm H^\mp$	13%
	Z^0Z^0	8%
	W^+W^-	14%
Total		72%

- NLO calculation shows 50-60% relative difference with respect to default MO value
- even including theoretical uncertainty, significant shift in the parameter space with respect to the MO value
- mass shift of 30 GeV with an accuracy of 0.5%
- error 5 times larger than naively expected

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys.Rev. D 93 114023 (2016), arXiv:1602.08103 [hep-ph]

Stop Coannihilation

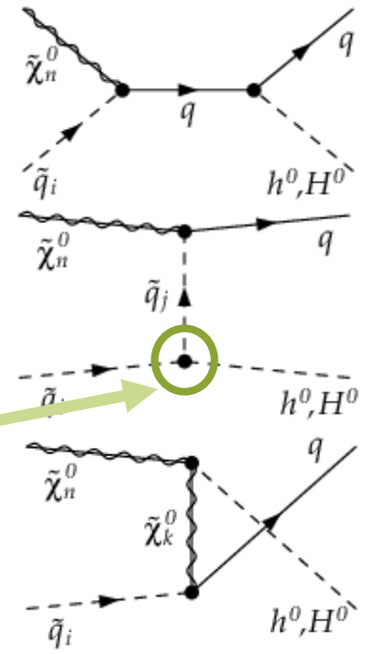


$\tilde{\chi}\tilde{\mathrm{t}} \to t h^0$

$\tilde{\chi}\tilde{\mathrm{t}} \to t g$

$\tilde{\chi}\tilde{\mathrm{t}} \to t Z^0$

large trilinear coupling enhances $\tilde{\chi}_1^0 \tilde{t}_1 \to th$ via t-channel stop-exchange



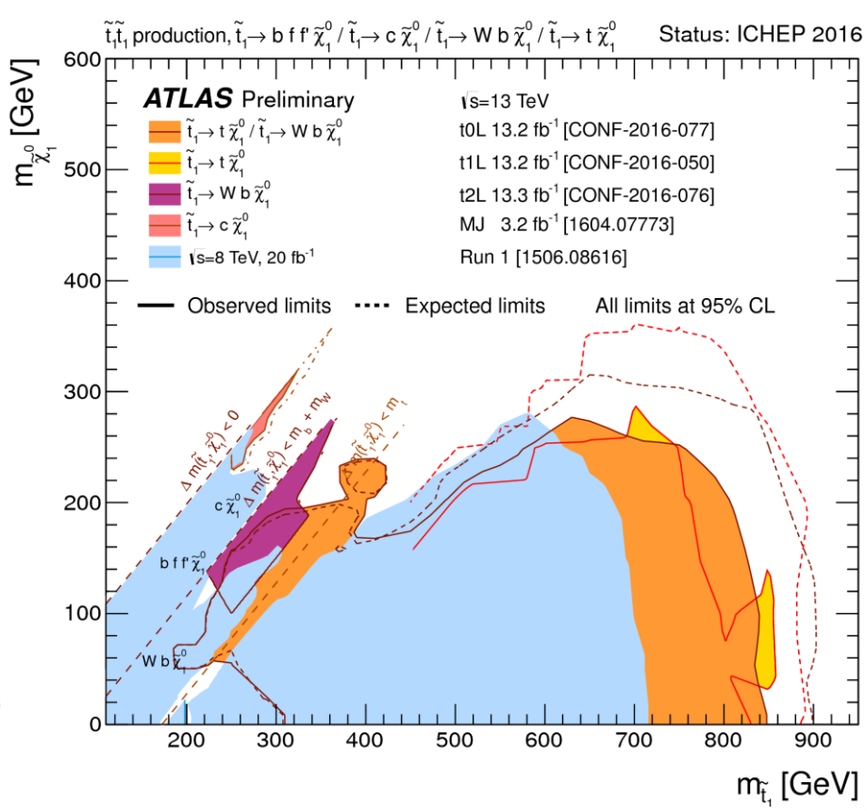
observed Higgs mass of 125 GeV suggests large stop mass splitting, e.g. triggered by large trilinear coupling A_t

$$m_{h_0}^2 \approx m_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\frac{M_{\mathrm{SUSY}}^2}{m_t^2} + \frac{X_t^2}{M_{\mathrm{SUSY}}^2}\left(1 - \frac{X_t^2}{12 M_{\mathrm{SUSY}}^2}\right)\right]$$

$$X_t = A_t - \mu/\tan\beta$$

$$M_{\mathrm{SUSY}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$$

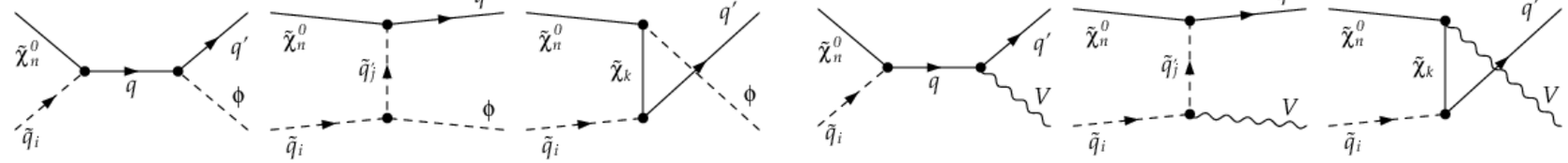
$$\begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{t}_2}^2 \end{pmatrix} = U^{\tilde{q}} \begin{pmatrix} M_{\tilde{Q}}^2 + m_t^2 + (I_q^{3L} - e_q \sin_W^2)\cos 2\beta m_Z^2 & m_t X_t \\ m_t X_t & M_{\tilde{U}}^2 + m_t^2 + e_q \sin_W^2 \cos 2\beta m_Z^2 \end{pmatrix} (U^{\tilde{q}})$$

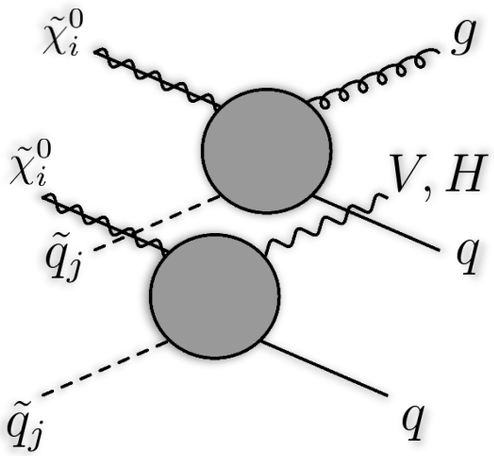


J. Harz, B. Herrmann, M. Klasen, K. Kovařík and Q. Le Boulc'h, Phys. Rev. D 87, 054031 (2013), arXiv:1212.5241 [hep-ph]

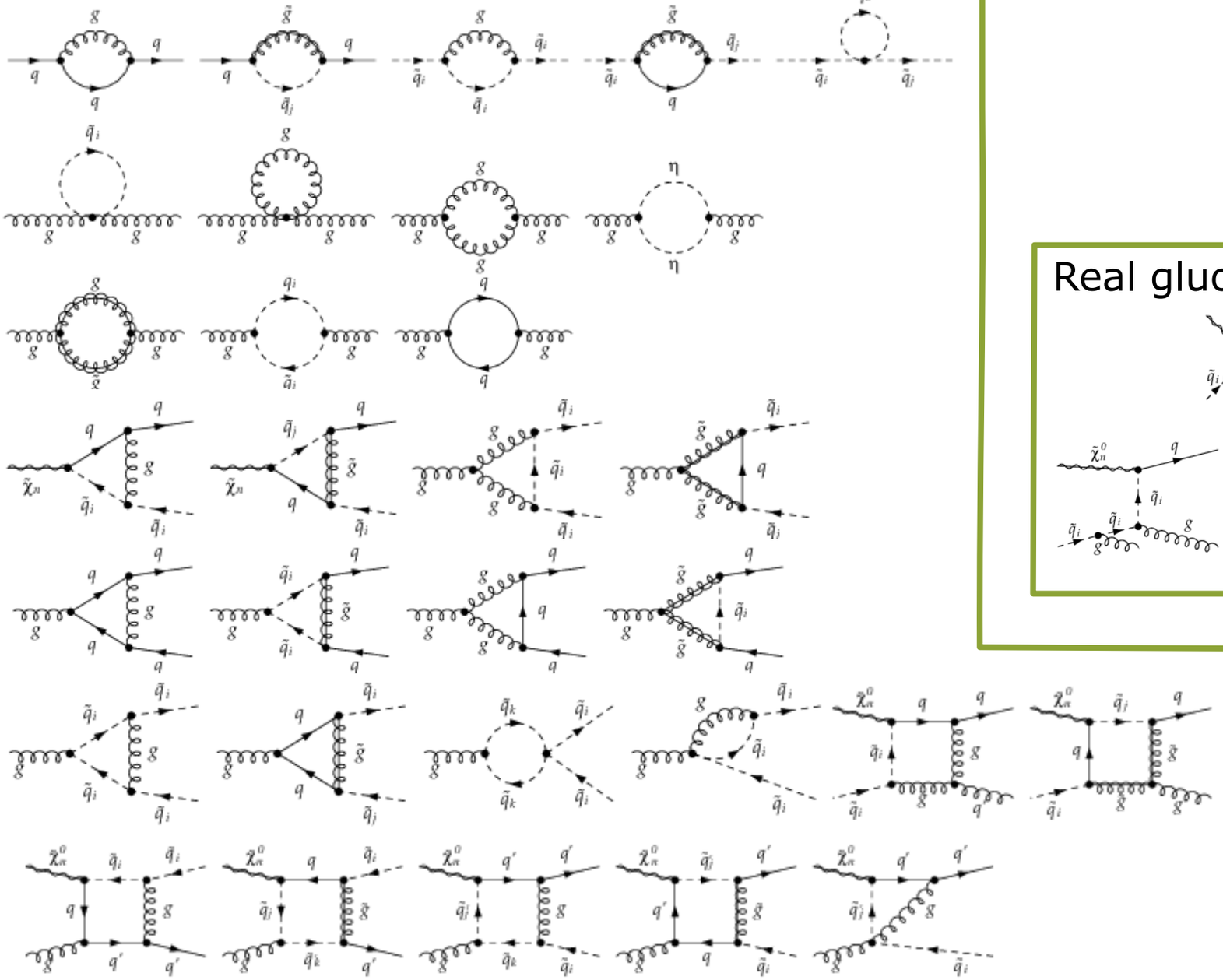
Stop Coannihilation

Tree level

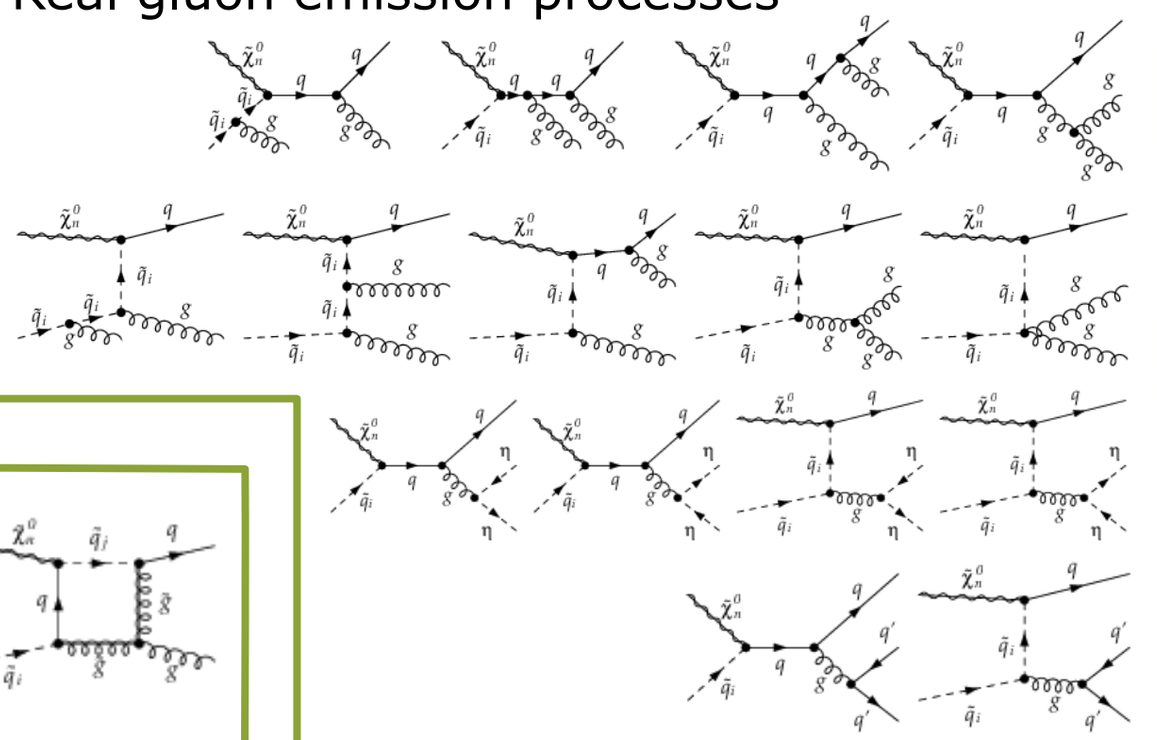




Virtual one-loop corrections

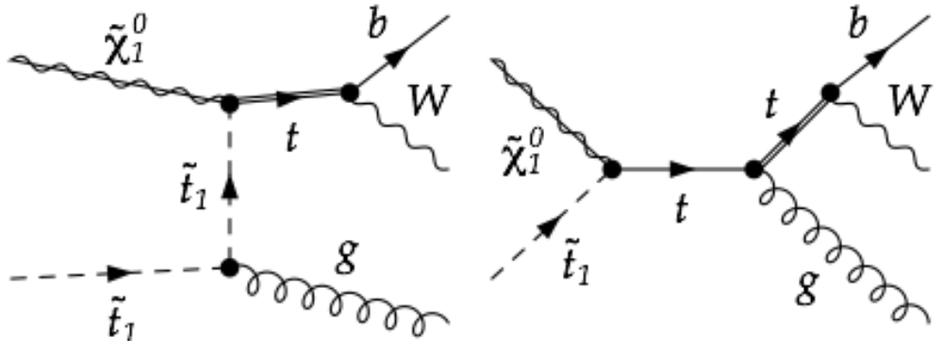


Real gluon emission processes



Stop Coannihilation – intermediate OS states

- with $m_t > m_b + m_W$ an intermediate on-shell state can occur as soon as $\sqrt{s} > m_t$



- local on-shell subtraction (DS) / "Prospino" scheme

W. Beenakker, R. Hoepker, M. Spira, P.M. Zerwas, Nuclear Physics B 492 (1997)

$$|\mathcal{M}|^2 = |\mathcal{M}_{\rm res}|^2 - |\mathcal{M}_{\rm res}^{\rm sub}|^2 + 2Re(\mathcal{M}_{\rm res}^* \mathcal{M}_{\rm rem}) + |\mathcal{M}_{\rm rem}|^2$$

- "counterterm" consists of Breit-Wigner weighted on-shell squared matrix element

$$|\mathcal{M}_{\rm res}^{\rm sub}|^2 = \frac{m_t^2 \Gamma_t^2}{(p_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} |\mathcal{M}_{\rm res}|^2_{p_t^2 = m_t^2}$$

- resonant propagators are regularized by Breit-Wigner propagator

→ consistent, width independent, gauge invariant treatment retaining interference terms

J. Harz, B. Herrmann, M. Klasen, K. Kovařík and Q. Le Boulc'h, Phys. Rev. D 87, 054031 (2013), arXiv:1212.5241 [hep-ph]

Combined Scenario incl. Stop Coannihilation

Combined scenario features

- gaugino annihilation
- Neutralino-Stop Coannihilation
- Stop-Antistop-Annhilation

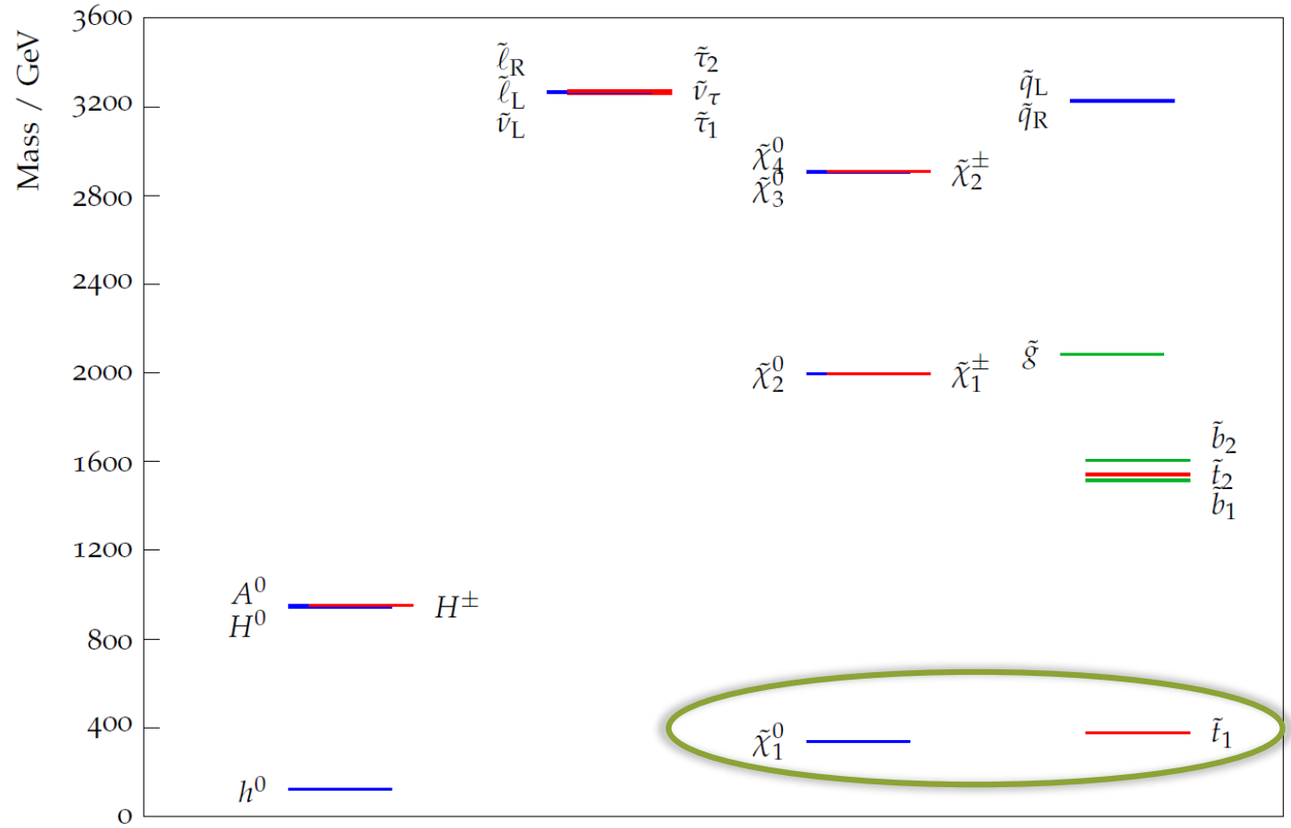
$\tan\beta$	μ	m_A	M_1	M_2	M_3	$M_{\tilde{q}_{1,2}}$	$M_{\tilde{q}_3}$	$M_{\tilde{u}_3}$	$M_{\tilde{\ell}}$	T_t
5.8	2925.8	948.8	335.0	1954.1	1945.6	3215.1	1578.0	609.2	3263.9	2704.1

Input parameter pMSSM11

$m_{\tilde{\chi}_1^0}$	$m_{\tilde{t}_1}$	$m_{\tilde{h}^0}$	$\Omega_{\tilde{\chi}_1^0}h^2$	$\mathrm{BR}(b\to s\gamma)$
338.3GeV	375.6GeV	122.0GeV	0.1135	$3.25\cdot10^{-4}$

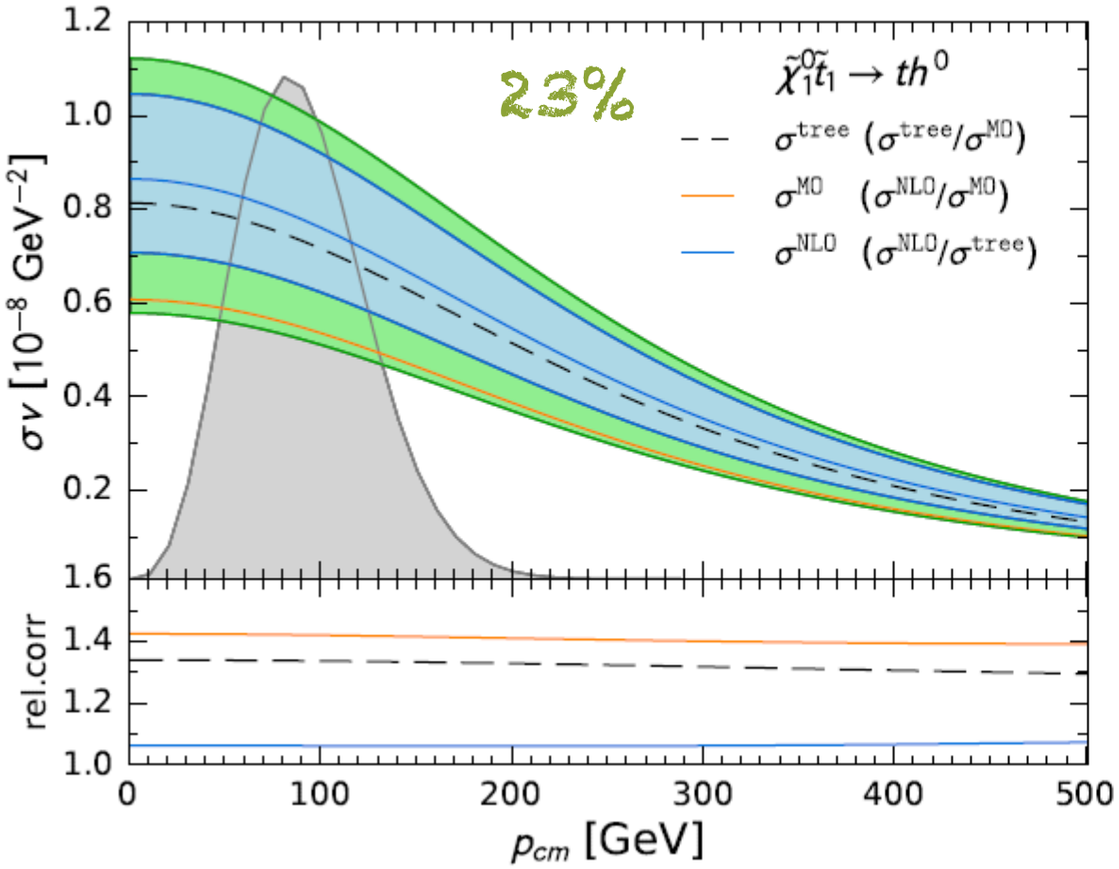
Masses & observables

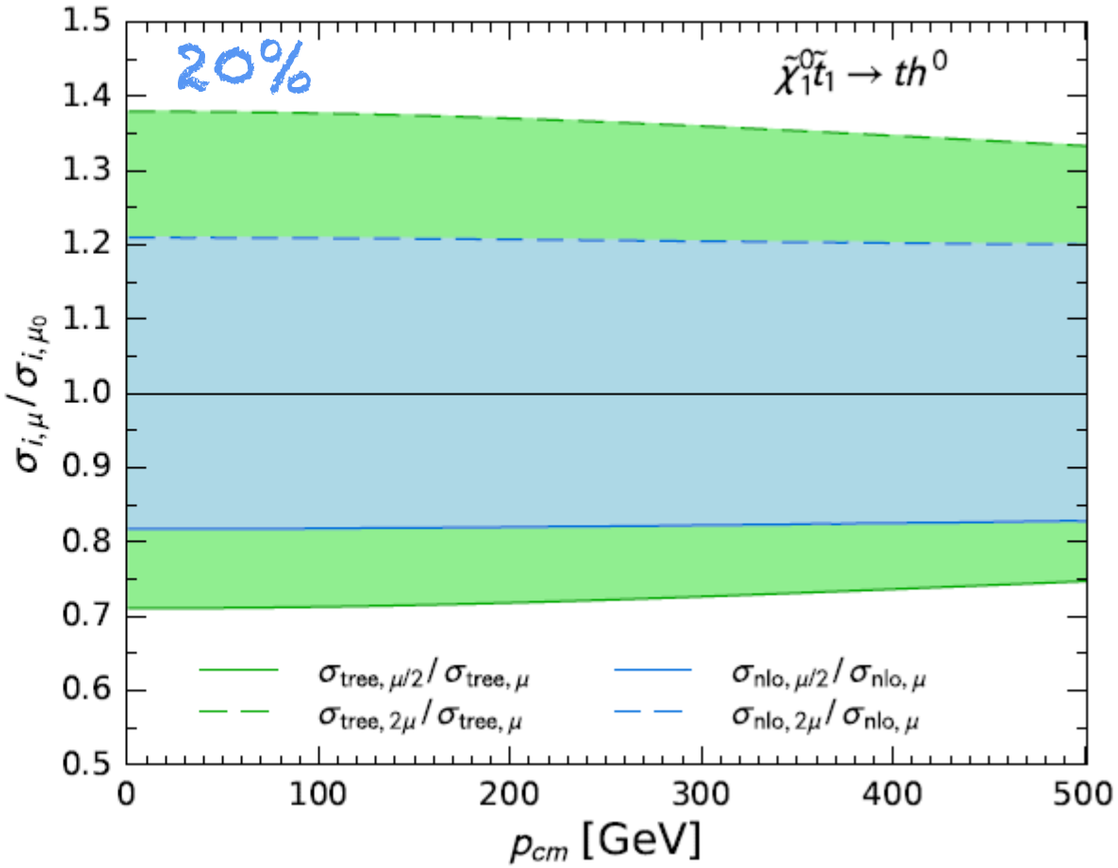
		C
$\tilde{\chi}_1^0\tilde{\chi}_1^0\to$	$t\bar{t}$	16%
$\tilde{\chi}_1^0\tilde{t}_1\to$	th^0	23%
	tg	23%
	tZ^0	5%
	bW^+	11%
$\tilde{t}_1\tilde{t}_1^*\to$	h^0h^0	5%
	Z^0Z^0	2%
	W^+W^-	3%
Total		88%



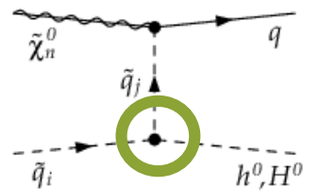
J. Harz, B. Herrmann, M. Klasen, K. Kovařík and Q. Le Boulc'h, Phys. Rev. D 87, 054031 (2013), arXiv:1212.5241 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, and K. Kovařík, Phys. Rev. D 91, 034028 (2015), arXiv:1409.2898 [hep-ph]

Stop Coannihilation





- Scale dependence at LO triggered by trilinear coupling A_t
- Scale dependence at NLO due to explicit logarithms and implicit A_t α_s
- Scale uncertainty of 20%

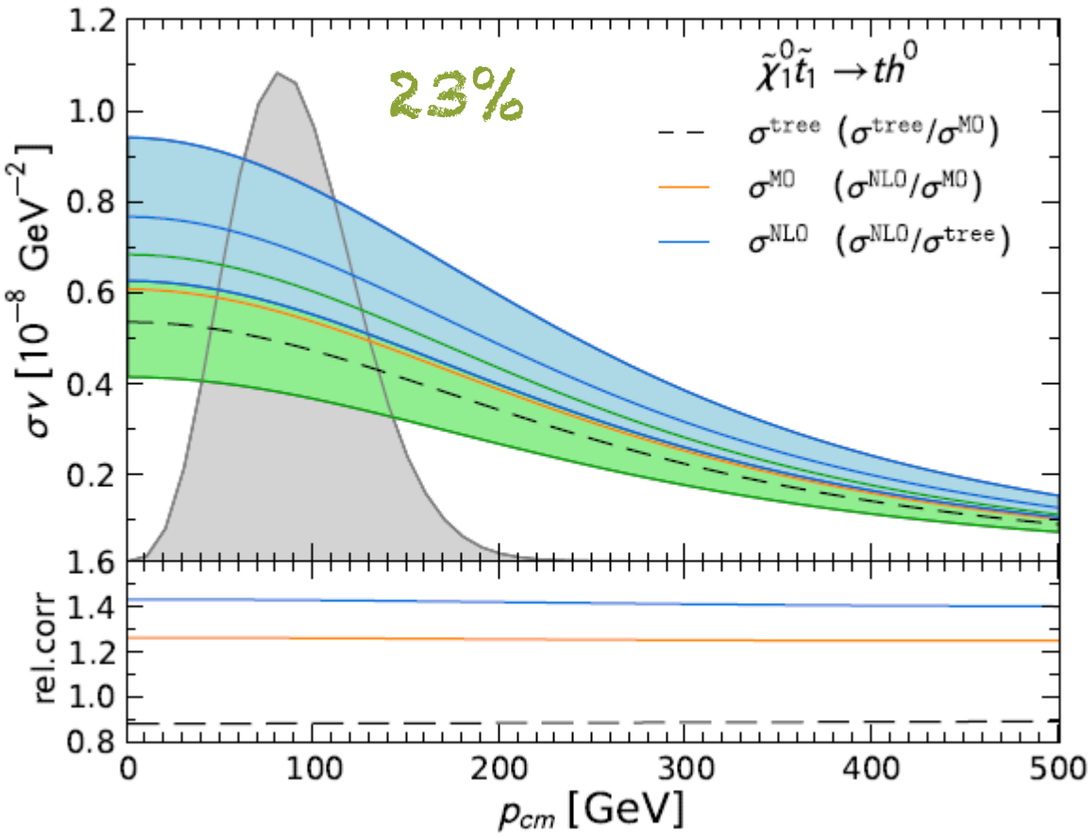


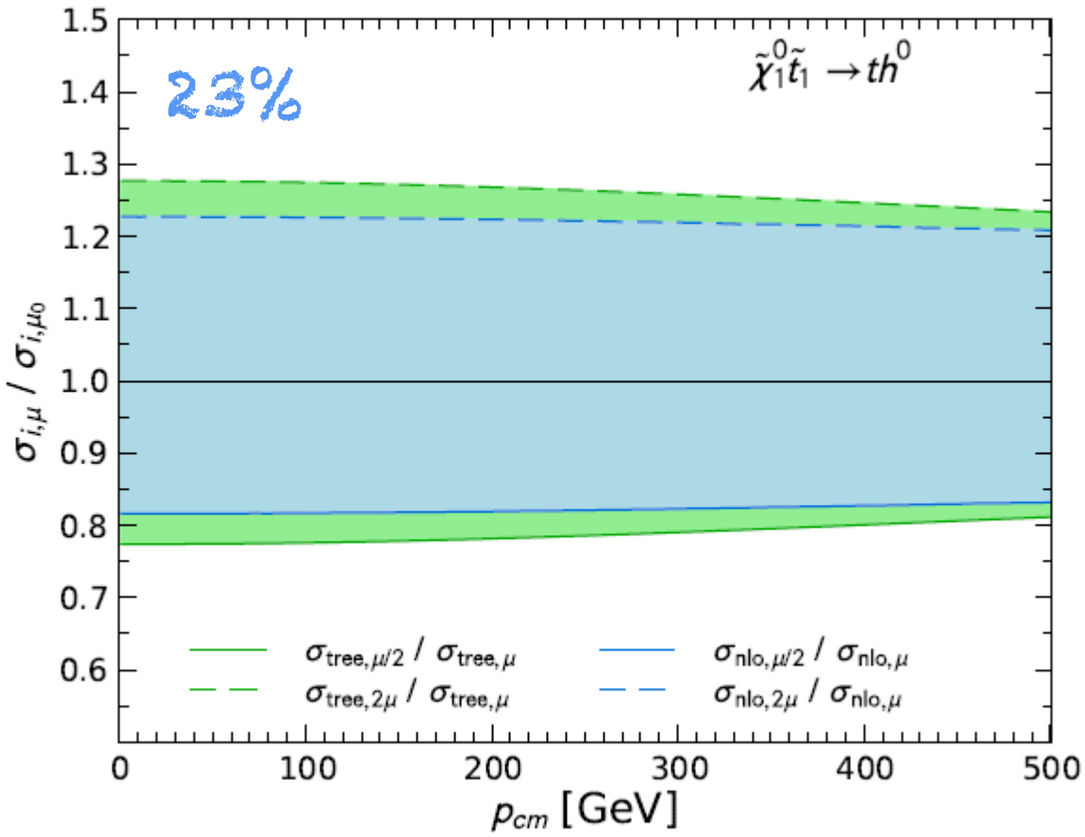
→ K-factor of 1.05, correction of 40% w.r.t. MicrOMEGAs, 20 % theoretical uncertainty

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D 93 114023 (2016), arXiv:1602.08103 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, and K. Kovařík, Phys. Rev. D 91, 034028 (2015), arXiv:1409.2898 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, K. Kovařík and Q. Le Boulc'h, Phys. Rev. D 87, 054031 (2013), arXiv:1212.5241 [hep-ph]

Impact of Scheme variation

Study effect of taking $m_t^{\overline{\mathrm{DR}}}$ instead of m_t^{OS}





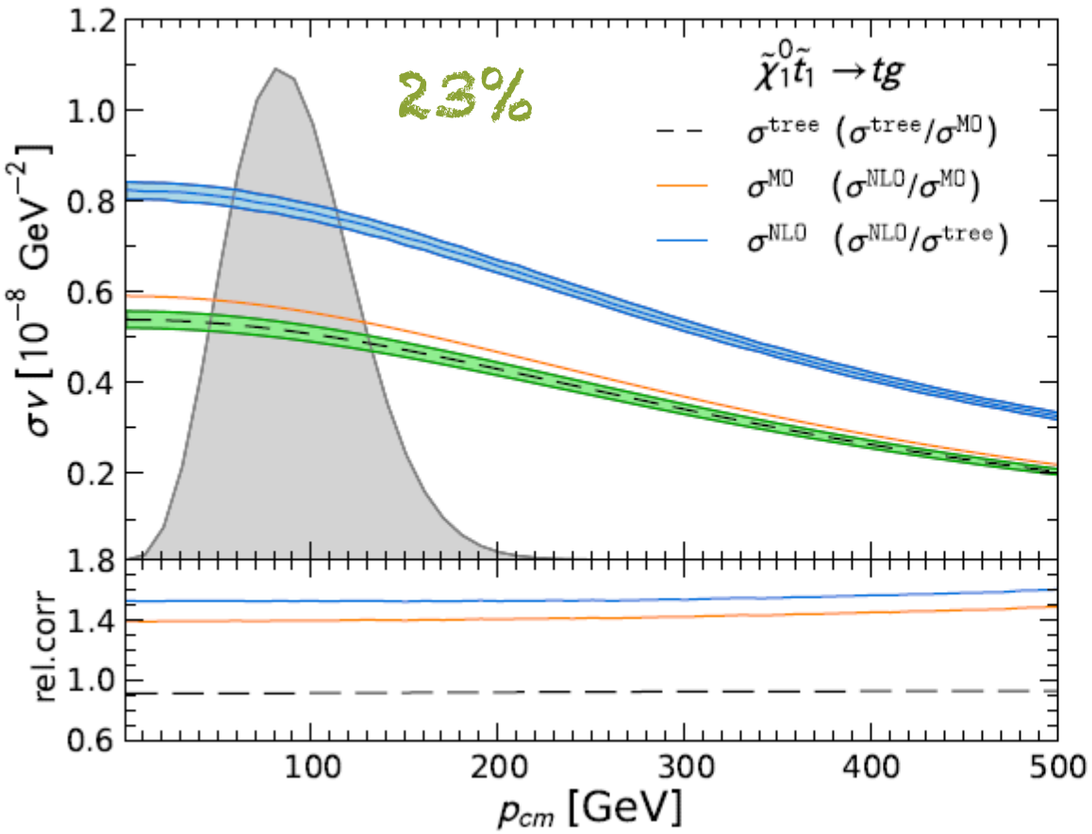
- MO larger than LO cross section as MO uses effective top mass
- Larger K-factor of 1.4 in contrast to our scheme with 1.05

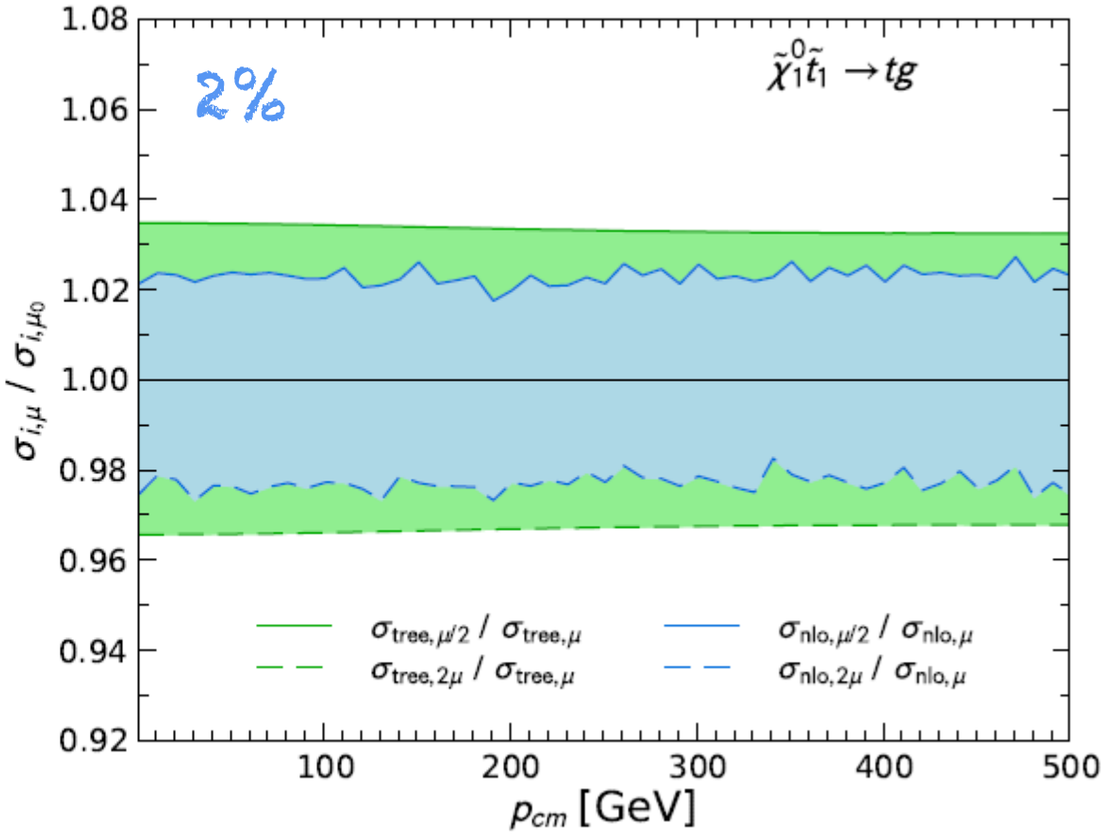
→ confirms our choice of renormalisation scheme

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D 93 114023 (2016), arXiv:1602.08103 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, and K. Kovařík, Phys. Rev. D 91, 034028 (2015), arXiv:1409.2898 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, K. Kovařík and Q. Le Boulc'h, Phys. Rev. D 87, 054031 (2013), arXiv:1212.5241 [hep-ph]

Stop Coannihilation

Top gluon final state only process with QCD coupling at tree-level





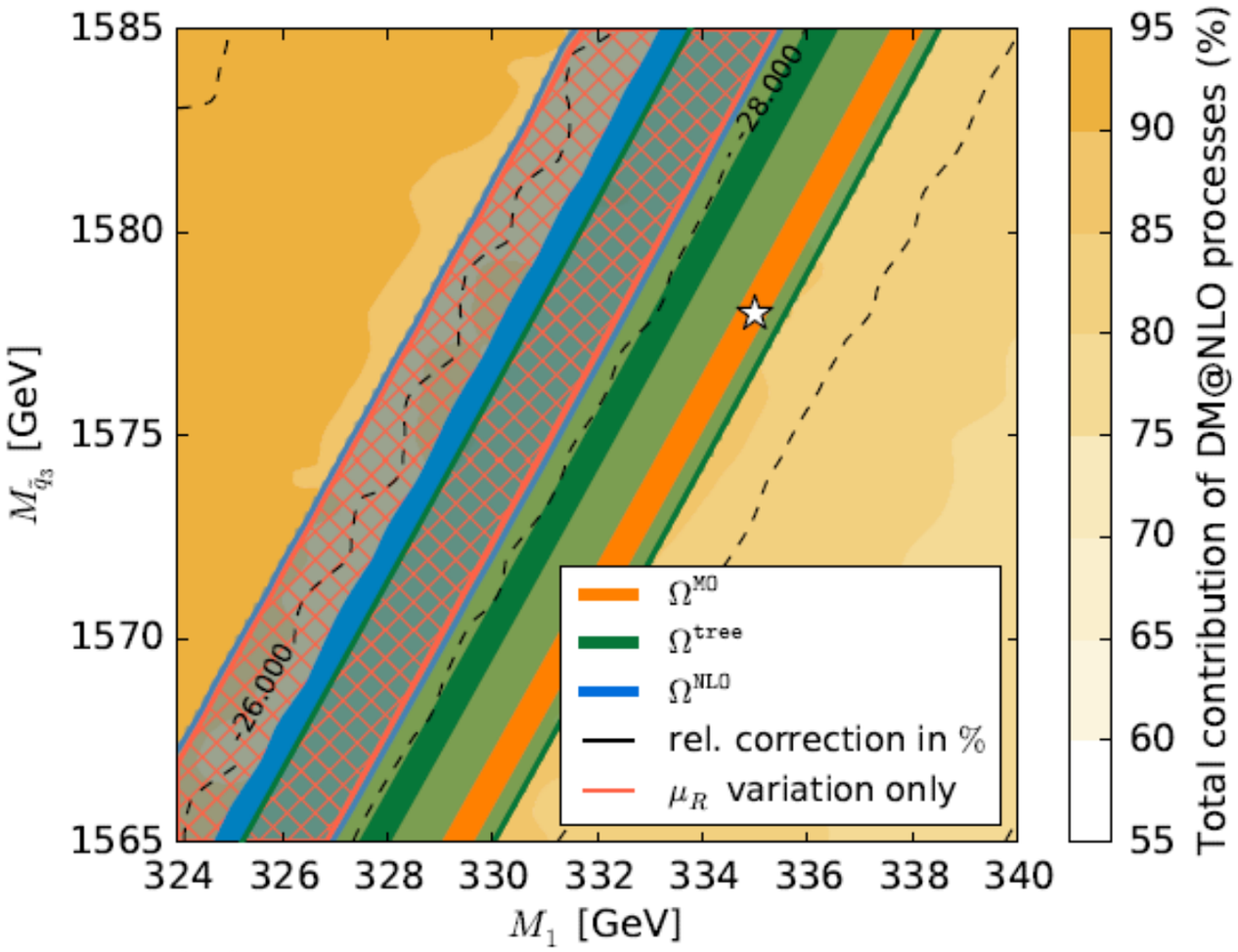
- Scale dependence on LO triggered already by α_s
- Scale dependence at NLO reduced to 2 %

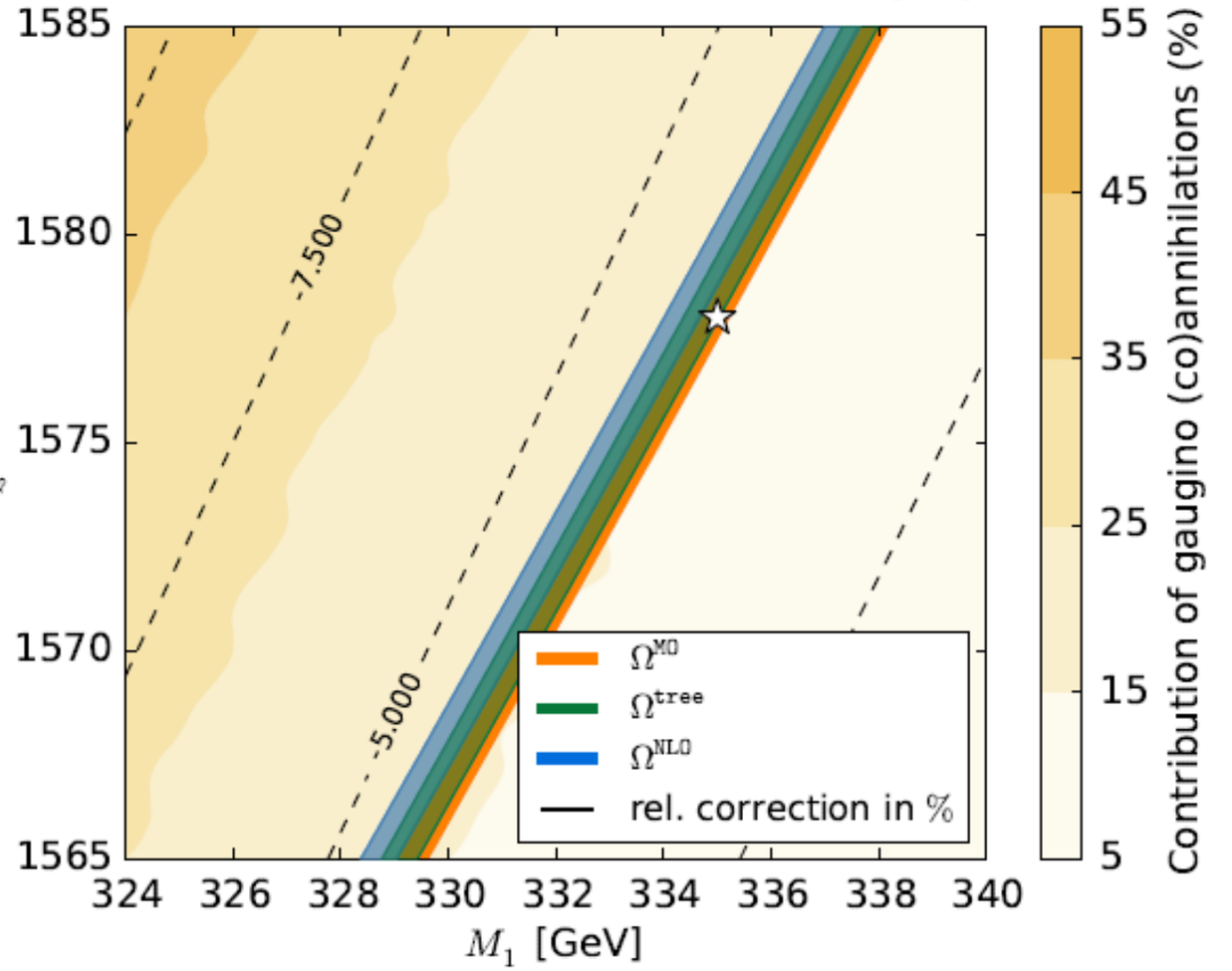
→ K-factor of 1.5, correction of 40% w.r.t. MicrOmegas, 2 % theoretical uncertainty

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D 93 114023 (2016), arXiv:1602.08103 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, and K. Kovařík, Phys. Rev. D 91, 034028 (2015), arXiv:1409.2898 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, K. Kovařík and Q. Le Boulc'h, Phys. Rev. D 87, 054031 (2013), arXiv:1212.5241 [hep-ph]

Combined Scenario

$\tilde{\chi}\tilde{\chi}$





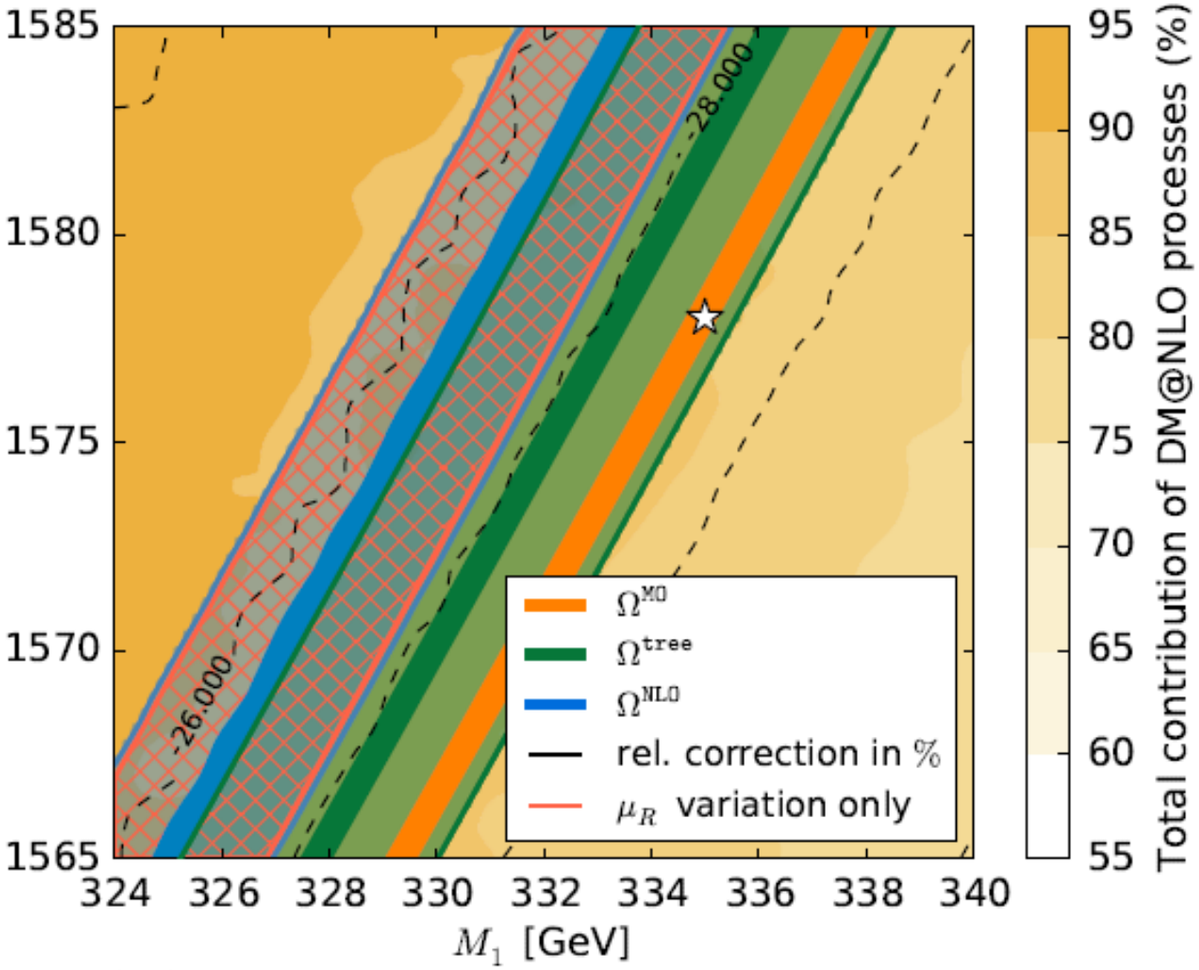
$\tilde{\chi}\tilde{\chi}$

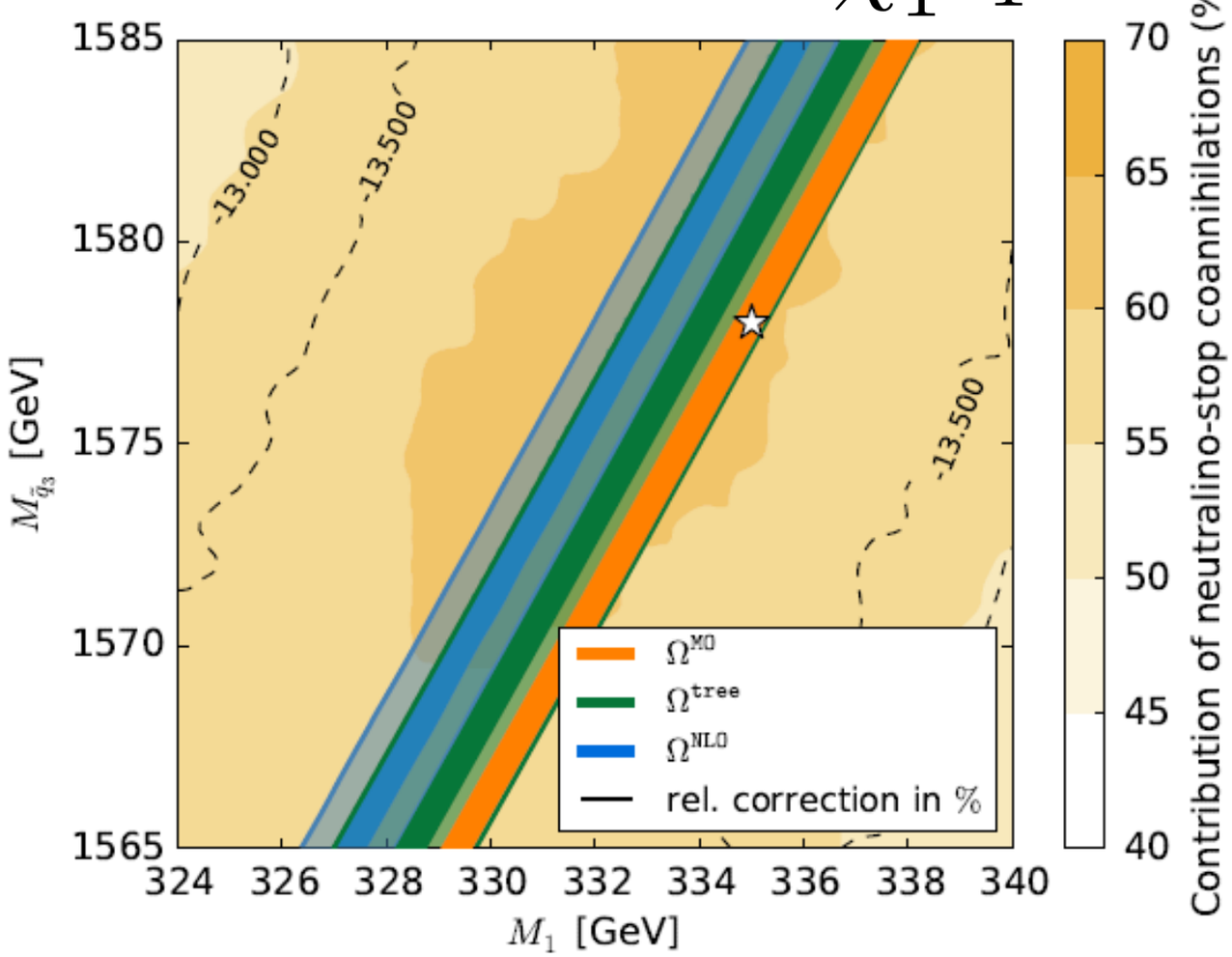
- K-factor of 1.15, 5-10% correction w.r.t. MO, 5% scale uncertainty

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D 93 114023 (2016), arXiv:1602.08103 [hep-ph]

Combined Scenario

$\tilde{\chi}_1^0 \tilde{t}_1$





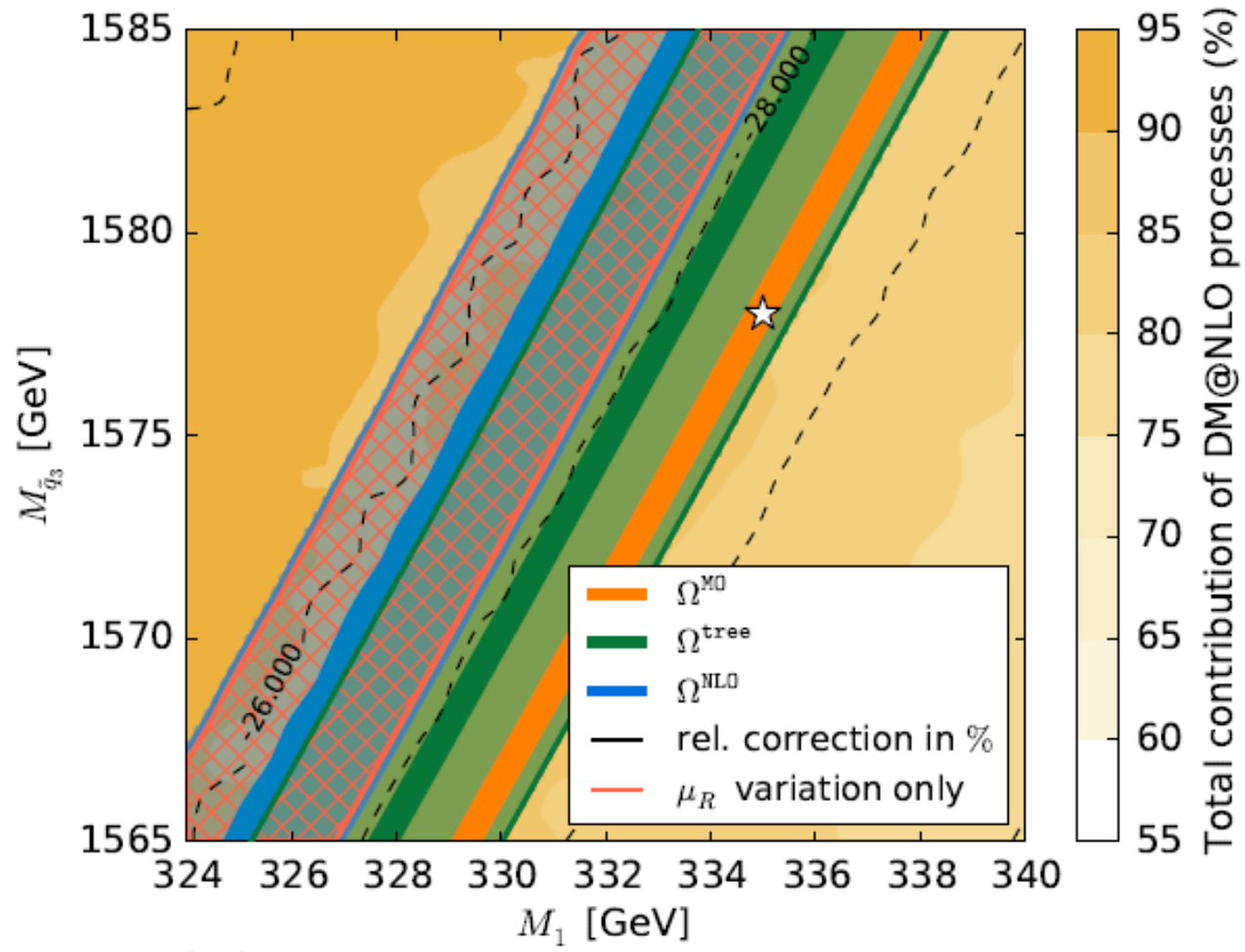
$\tilde{\chi}\tilde{\chi}$
- K-factor of 1.15, 5-10% correction w.r.t. MO, 5% scale uncertainty

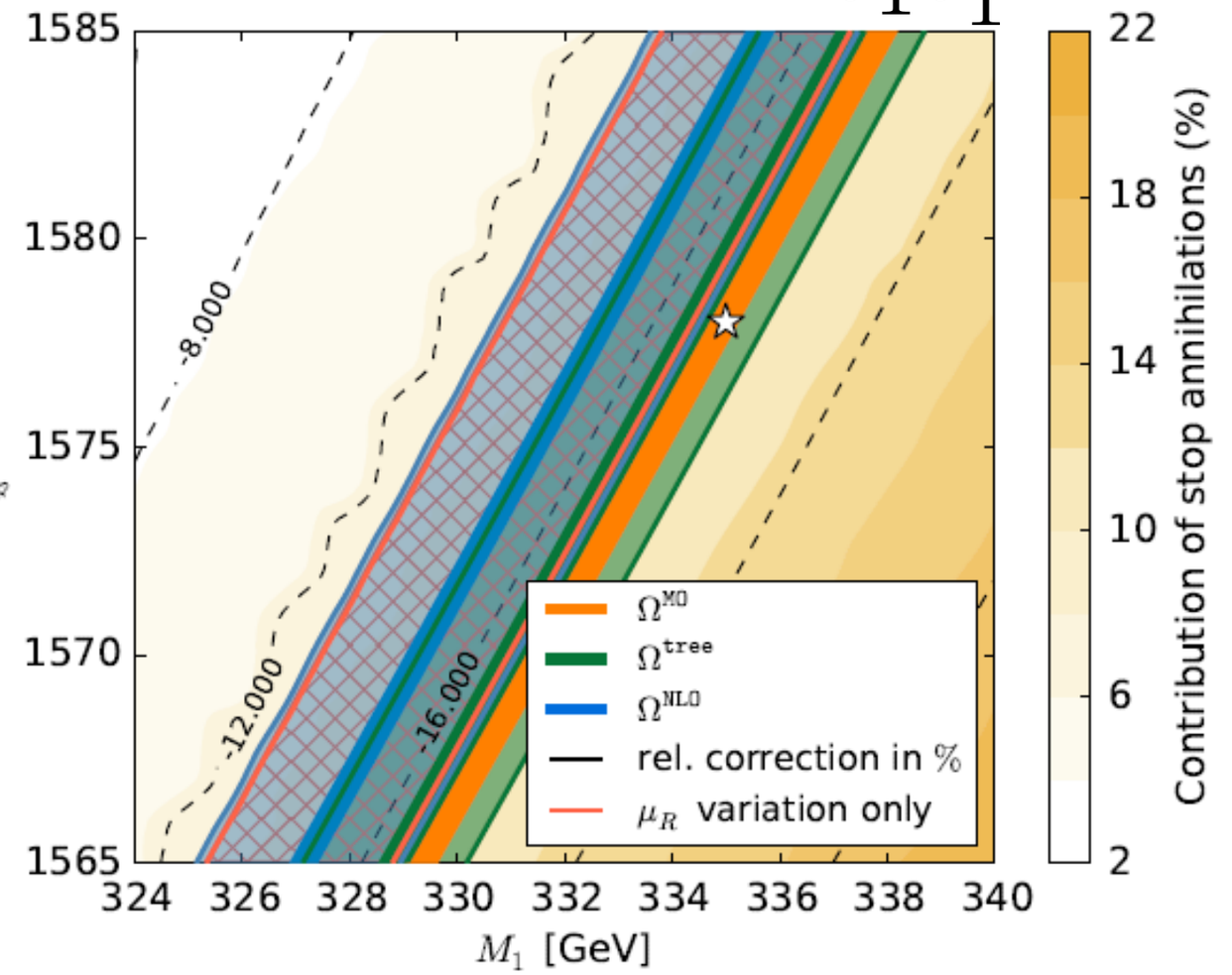
$\tilde{\chi}_1^0 \tilde{t}_1$
- K-factors of 1.05-1.5, 15% correction w.r.t. MO, 20% scale uncertainty

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D 93 114023 (2016), arXiv:1602.08103 [hep-ph]

Combined Scenario

$\tilde{t}_1\tilde{t}_1^*$





- $\tilde{\chi}\tilde{\chi}$ • K-factor of 1.15, 5-10% correction w.r.t. MO, 5% scale uncertainty
- $\tilde{\chi}_1^0\tilde{t}_1$ • K-factors of 1.05-1.5, 15% correction w.r.t. MO, 20% scale uncertainty
- $\tilde{t}_1\tilde{t}_1^*$ • K-factors of 1-9, up to 16% correction w.r.t. MO, 20% scale uncertainty

→ Correction of almost 30 % w.r.t. MO relic density calculation

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, Phys. Rev. D 93 114023 (2016), arXiv:1602.08103 [hep-ph]

Conclusions

- current tools calculate relic density just at an (effective) tree level
- increased precision of theoretical relic density prediction by including SUSY-QCD corrections & Coulomb enhancement effects
- corrections can have significant impact of 10% - 60% depending on scenario
- first estimation of theoretical uncertainty coming from scheme and scale variations (scenario dependent!)
- we provide a library, linkable to MicrOMEGAs, for precise parameter studies and global fits

DM@NLO

Thank you!