TWO (MORE) EXCEPTIONS

IN THE CALCULATIONS OF RELIC ABUNDANCES

Andrzej Hryczuk

University of Oslo

based (mostly) on: Beneke, Dighera, AH, 1409.3049

Beneke, Bharucha, Dighera, Hellmann, AH, Recksiegel, Ruiz-Femenia; 1601.04718

Theory seminar, Oslo, 27th January 2016

BEFORE…

U. of Warsaw	SISSA Trieste	TU Munich	NCBJ Warsaw
master	phd	postdoc	(short) postdoc
axions	relic density indirect detection	relic density indirect detection thermal field theory	MSSM scans indirect detection
K. Meissner	P. Ullio, R. Iengo, I. Cholis, …	M. Beneke, F. Dighera, A. Bharucha, …	L. Roszkowski, …

OUTLINE

1. Introduction
    - why Dark Matter is so interesting?*
    - standard approach to thermal relic density
2. Exception IV
    - NLO effects
    - finite temperature effects
3. Exception V
    - velocity dependent annihilation
    - non-perturbative effects
4. Summary

*to have most of seminars about it

Top 3 reasons
why Dark Matter is so fascinating

1. We know it is there waiting for us, but we still don't know what it is

2. It might help us solve some of the mysteries of physics at the fundamental level (Higgs mass stability, baryogengesis, neutrino masses, strong CP, pretty-much-everything, ...)

3. It may be better to spot them first, before they can spot us

vide A. Lipniacka talk last Friday

Relic Density

Standard approach

T ↑ time ↓

$\Gamma_{\rm ann} > H$	DM in equilibrium
$\Gamma_{\rm ann} \sim H$	chemical decoupling
$\Gamma_{\rm ann} < H$	freeze-out

non-equilibrium

time evolution of $f_\chi(p)$ in kinetic theory:

$$\boxed{E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right)} f_\chi = \mathcal{C}[f_\chi] \quad \Rightarrow \quad \frac{dn_\chi}{dt} + 3Hn_\chi = C$$

Liouville operator in FRW background

the collision term

integrated

assumptions for using Boltzmann eq: classical limit, molecular chaos,...

Relic Density

the LO collision term

for $2 \leftrightarrow 2$ CP invariant process:

note: added "by hand"

$$C_{\rm LO} = -h_\chi^2 \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar\chi}}{(2\pi)^3}\, \sigma_{\chi\bar\chi\to ij} v_{\rm rel}\, \left[f_\chi f_{\bar\chi}\boxed{(1\pm f_i)(1\pm f_j)} - f_i f_j \boxed{(1\pm f_\chi)(1\pm f_{\bar\chi})}\right]$$

assuming kinetic equilibrium at chemical decoupling: $f_\chi \sim a(\mu) f_\chi^{\rm eq}$

$$C_{\rm LO} = -\langle \sigma_{\chi\bar\chi\to ij} v_{\rm rel}\rangle^{\rm eq} \left(n_\chi n_{\bar\chi} - n_\chi^{\rm eq} n_{\bar\chi}^{\rm eq}\right)$$

where the thermally averaged cross section:

$$\langle \sigma_{\chi\bar\chi\to ij} v_{\rm rel}\rangle^{\rm eq} = -\frac{h_\chi^2}{n_\chi^{\rm eq} n_{\bar\chi}^{\rm eq}} \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar\chi}}{(2\pi)^3}\, \sigma_{\chi\bar\chi\to ij} v_{\rm rel}\, f_\chi^{\rm eq} f_{\bar\chi}^{\rm eq}$$

crucial point: $p_\chi + p_{\bar\chi} = p_i + p_j \Rightarrow f_\chi^{\rm eq} f_{\bar\chi}^{\rm eq} \approx f_i^{\rm eq} f_j^{\rm eq}$

in Maxwell approx.

RELIC DENSITY
BOLTZMANN EQ.

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}\right)$$

Re-written for the comoving number density:

$$\frac{dY}{dx} = \sqrt{\frac{g_*\pi m_\chi^2}{45G}}\,\frac{\langle \sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}}{x^2}\left(Y^2 - Y_{\rm eq}^2\right)$$

$$\lim_{x\to 0} Y = Y_{\rm eq} \qquad \lim_{x\to\infty} Y = \mathrm{const}$$

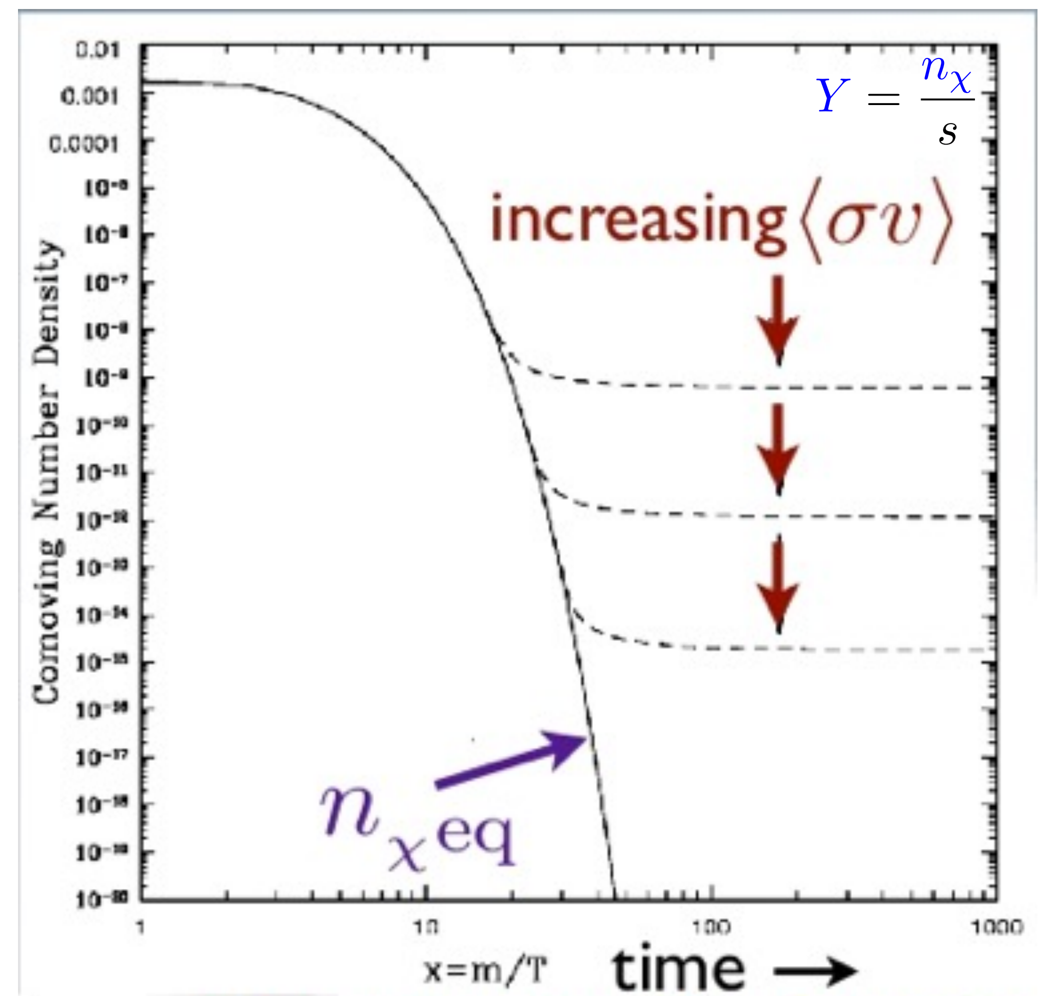


Fig.: Jungman, Kamionkowski & Griest, PR'96

Recipe:
compute LO annihilation cross-section,
take a thermal bath average,
plug in to BE… and voilà

Relic Density

Three Exceptions

Griest & Seckel PRD'91

1. Co-annihilations

if more than one state share a conserved quantum number making DM stable

$$\langle \sigma_{\rm eff} {\rm v} \rangle = \sum_{ij} \langle \sigma_{ij} {\rm v}_{ij} \rangle \frac{n_i^{\rm eq} n_j^{\rm eq}}{n_{\rm eq}^2}$$

with: $\sigma_{ij} = \sum_X \sigma(\chi_i \chi_j \to X)$

e.g., SUSY

2. Annihilation to forbidden channels

if DM is slightly below mass threshold for annihilation $\Longrightarrow$ „forbidden" channel can still be accessible in thermal bath

recent e.g., 1505.07107

3. Annihilation near poles

expansion in velocity (s-wave, p-wave, etc.) not safe

(more historical issue: these days most people use numerical codes)

EXCEPTION IV:

NLO EFFECTS

DARK MATTER AT NLO

Bergstrom '89; Drees et al., 9306325; Ullio & Bergstrom, 9707333 } helicity suppression lifting

⋮

Bergstrom et al., 0507229; Bringmann et al., 0710.3169 } spectral features in indirect searches

⋮

Ciafaloni et al., 1009.0224
Cirelli et al., 1012.4515
Ciafaloni et al., 1202.0692
AH & Iengo, 1111.2916
} large EW corrections

⋮

Chatterjee et al., 1209.2328
Harz et al., 1212.5241
Ciafaloni et al., 1305.6391
Hermann et al., 1404.2931
Boudjema et al., 1403.7459
} ***thermal relic density***

$$\Omega_{DM} h^2 = 0.1187 \pm 0.0017.$$

<1.5% uncertainty!

Planck+WMAP pol.+highL+BAO; 1303.5062

⋮

SloopS, DM@NLO, PPC4DMID } NLO codes

Relic Density at NLO

Recall at LO:

$$C_{\rm LO} = -h_\chi^2 \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar\chi}}{(2\pi)^3}\ \sigma_{\chi\bar\chi\to ij} v_{\rm rel}\ \left[f_\chi f_{\bar\chi}(1\pm f_i)(1\pm f_j) - f_i f_j (1\pm f_\chi)(1\pm f_{\bar\chi})\right]$$

crucial point: $\boxed{p_\chi + p_{\bar\chi} = p_i + p_j \Rightarrow f_\chi^{\rm eq} f_{\bar\chi}^{\rm eq} \approx f_i^{\rm eq} f_j^{\rm eq}}$

in Maxwell approx.

at NLO both virtual one-loop and 3-body processes contribute:

$$C_{1-\rm loop} = -h_\chi^2 \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar\chi}}{(2\pi)^3}\ \sigma^{1-\rm loop}_{\chi\bar\chi\to ij} v_{\rm rel}\ \left[f_\chi f_{\bar\chi}(1\pm f_i)(1\pm f_j) - f_i f_j (1\pm f_\chi)(1\pm f_{\bar\chi})\right]$$

$$C_{\rm real} = -h_\chi^2 \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar\chi}}{(2\pi)^3}\ \sigma_{\chi\bar\chi\to ij\gamma} v_{\rm rel}\ \left[f_\chi f_{\bar\chi}(1\pm f_i)(1\pm f_j)(1+f_\gamma) - f_i f_j f_\gamma (1\pm f_\chi)(1\pm f_{\bar\chi})\right]$$

$$p_\chi + p_{\bar\chi} = p_i + p_j \pm p_\gamma \Rightarrow$$

photon can be arbitrarily soft

$f_\gamma \sim \omega^{-1}$

Maxwell approx. not valid anymore...

RELIC DENSITY AT NLO

...problem: *T*-dependend IR divergence!

it sounds scary - but somehow we all know there has to be a happy-end

Relic Density

what Really happens at NLO?

Beneke, Dighera, AH, 1409.3049

only this used in NLO literature so far

$$C_{\rm NLO} \sim \int d\Pi_{\chi\bar\chi ij} \;\; f_\chi f_{\bar\chi} \;\boxed{\Big\{ |\mathcal{M}^{\rm LO}_{\chi\bar\chi\to ij}|^2 + |\mathcal{M}^{{\rm NLO}\; T=0}_{\chi\bar\chi\to ij}|^2 + \int d\Pi_\gamma |\mathcal{M}_{\chi\bar\chi\to ij\gamma}|^2 +}$$

$$|\mathcal{M}^{{\rm NLO}\; T\neq 0}_{\chi\bar\chi\to ij}|^2 + \int d\Pi_\gamma \Big[f_\gamma \left(|\mathcal{M}_{\chi\bar\chi\to ij\gamma}|^2 + |\mathcal{M}_{\chi\bar\chi\gamma\to ij}|^2 \right)$$

$$- f_i \left(|\mathcal{M}_{\chi\bar\chi\to ij\gamma}|^2 + |\mathcal{M}_{\chi\bar\chi i\to j\gamma}|^2 \right) - f_j \left(|\mathcal{M}_{\chi\bar\chi\to ij\gamma}|^2 + |\mathcal{M}_{\chi\bar\chi j\to i\gamma}|^2 \right) \Big] \Big\}$$

thermal 1-loop

photon absorption

photon emission

SM fermions emission

SM fermions absorption

$$-f_i f_j \Big\{ |\mathcal{M}^{\rm LO}_{ij\to\chi\bar\chi}|^2 + |\mathcal{M}^{{\rm NLO}\; T=0}_{ij\to\chi\bar\chi}|^2 + \int d\Pi_\gamma |\mathcal{M}_{ij\to\chi\bar\chi\gamma}|^2 +$$

$$|\mathcal{M}^{{\rm NLO}\; T\neq 0}_{ij\to\chi\bar\chi}|^2 + \int d\Pi_\gamma \Big[f_\gamma \left(|\mathcal{M}_{ij\to\chi\bar\chi\gamma}|^2 + |\mathcal{M}_{ij\gamma\to\chi\bar\chi}|^2 \right)$$

$$- f_\chi \left(|\mathcal{M}_{ij\to\chi\bar\chi\gamma}|^2 + |\mathcal{M}_{ij\chi\to\chi\gamma}|^2 \right) - f_{\bar\chi} \left(|\mathcal{M}_{ij\to\chi\bar\chi\gamma}|^2 + |\mathcal{M}_{ij\bar\chi\to\bar\chi\gamma}|^2 \right) \Big] \Big\} .$$

QUESTIONS:

1. how the (soft and collinear) IR divergence cancellation happen?

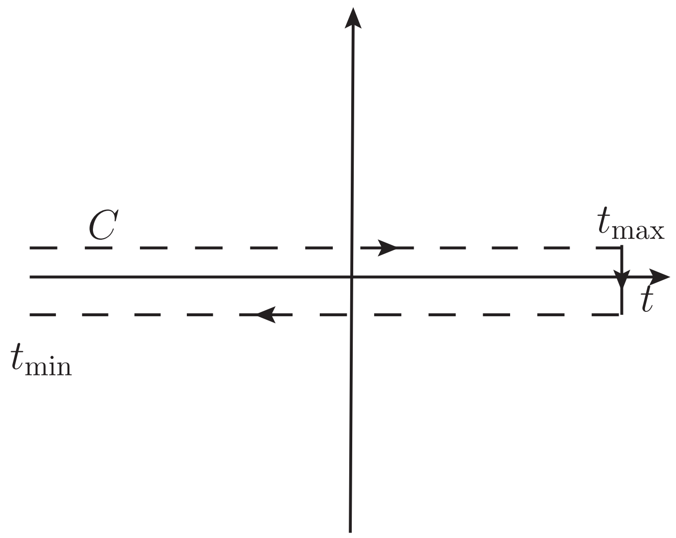
2. does Boltzmann equation itself receive quantum corrections?

3. how large are the remaining finite T corrections?

Program: develop a method for relic density calculation directly from QFT and free from IR problems

framework exists: non-equilibrium thermal field theory

CLOSED TIME PATH
FORMALISM



$$i\Delta(x,y) = \langle T_C \phi(x)\phi^\dagger(y)\rangle,$$
$$iS_{\alpha\beta}(x,y) = \langle T_C \psi_\alpha(x)\bar{\psi}_\beta(y)\rangle,$$

contour Green's functions obey Dyson-Schwinger eqs:

$$\Delta(x,y) = \Delta_0(x,y) - \int_C d^4z \int_C d^4z' \Delta_0(x,z)\Pi(z,z')\Delta(z',y),$$
$$S_{\alpha\beta}(x,y) = S^0_{\alpha\beta}(x,y) - \int_C d^4z \int_C d^4z' S^0_{\alpha\gamma}(x,z)\Sigma_{\gamma\rho}(z,z')S_{\rho\beta}(z',y),$$

which can be rewritten in the form of Kadanoff-Baym eqs:

$$(-\partial^2 - m_\phi^2)\boxed{\Delta^{\lessgtr}(x,y)} - \int d^4z \left(\boxed{\Pi_h(x,z)}\boxed{\Delta^{\lessgtr}(z,y)} - \boxed{\Pi^{\lessgtr}(x,z)}\boxed{\Delta_h(z,y)}\right) = \boxed{\mathcal{C}_\phi},$$
$$(i\partial\!\!\!/ - m_\chi)\boxed{S^{\lessgtr}(x,y)} - \int d^4z \left(\boxed{\Sigma_h(x,z)}\boxed{S^{\lessgtr}(z,y)} - \boxed{\Sigma^{\lessgtr}(x,z)}\boxed{S_h(z,y)}\right) = \boxed{\mathcal{C}_\chi},$$

Closed Time Path

path to Boltzmann equation

Kadanoff-Baym $\Longrightarrow$ Boltzmann

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f = \mathcal{C}[f].$$

collision term **derived** from thermal QFT

Assumptions:

weak inhomogeneity

gradient expansion

quasi-particle approx.

weak interactions

Justification:

inhomogeneity

plasma excitation momenta

$$\partial \ll k$$

freeze-out happens close to equilibrium

Closed Time Path

FORMALISM: COLLISION TERM

the fermion collision terms is defined as:

propagators

$$\mathcal{C}_\chi = \frac{1}{2}\int d^4 z \left(\Sigma^{>}(x,z)\, S^{<}(z,y) - \Sigma^{<}(x,z)\, S^{>}(z,y)\right)$$

self-energies

where the propagators:

thermal part

$$iS^c(p) = \frac{i\left(\not p + m\right)}{p^2 - m^2 + i\eta} - \boxed{2\pi\left(\not p + m\right)\delta\left(p^2 - m^2\right) f\left(p^0\right)}$$

$$iS^a(p) = -\frac{i\left(\not p + m\right)}{p^2 - m^2 + i\eta} + 2\pi\left(\not p + m\right)\delta\left(p^2 - m^2\right)\left(1 - f\left(p^0\right)\right)$$

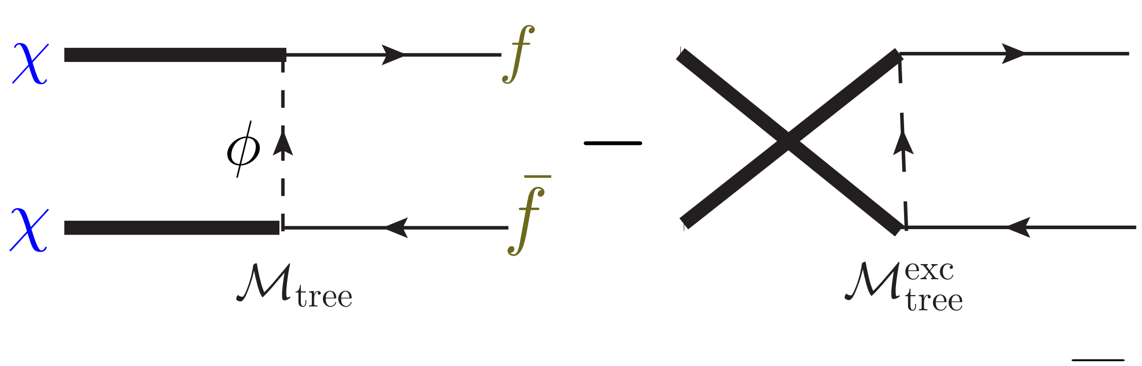
$$\left.\begin{aligned} iS^{>}(p) &= 2\pi\left(\not p + m\right)\delta\left(p^2 - m^2\right)\left(1 - f\left(p^0\right)\right) \\ iS^{<}(p) &= -2\pi\left(\not p + m\right)\delta\left(p^2 - m^2\right) f\left(p^0\right) \end{aligned}\right\} \text{"cut" propagators}$$

the presence of distribution functions inside propagators $\Rightarrow$ known collision term structure

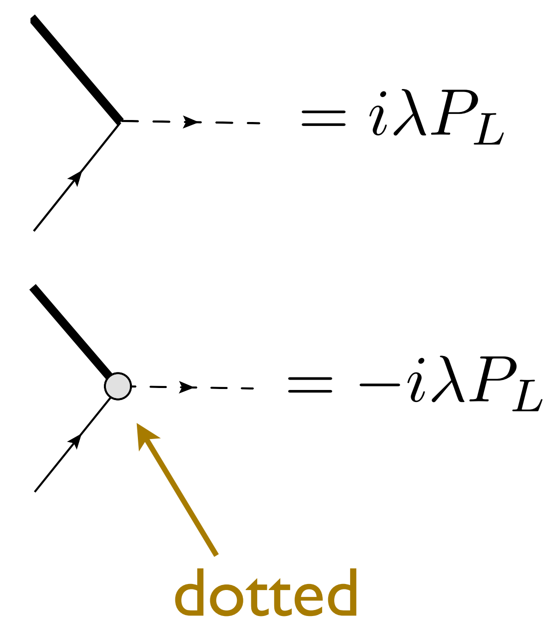
COLLISION TERM
EXAMPLE

Bino-like DM: χ Majorana fermion, SM singlet

annihilation process at tree level:



vertices (2 types):



scale hierarchy: $\boxed{m_\phi \gtrsim m_\chi} \gg T \gg \boxed{m_f}$

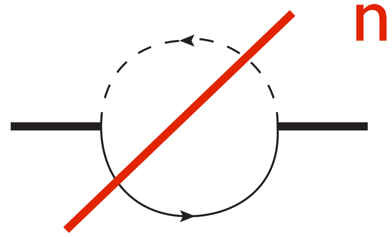
no thermal contributions

effectively massless

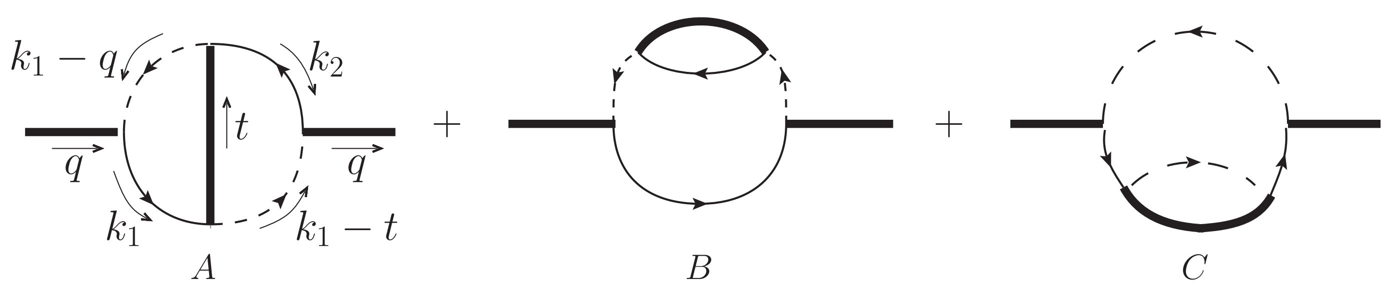
rescaled variables: $\tau = \dfrac{T}{m_\chi} \ll 1$ $\qquad \epsilon = \dfrac{m_f}{2m_\chi} \ll \tau$ $\qquad \xi = \dfrac{m_\phi}{m_\chi} \gtrsim 1$

Collision Term
computation

no # changing processes

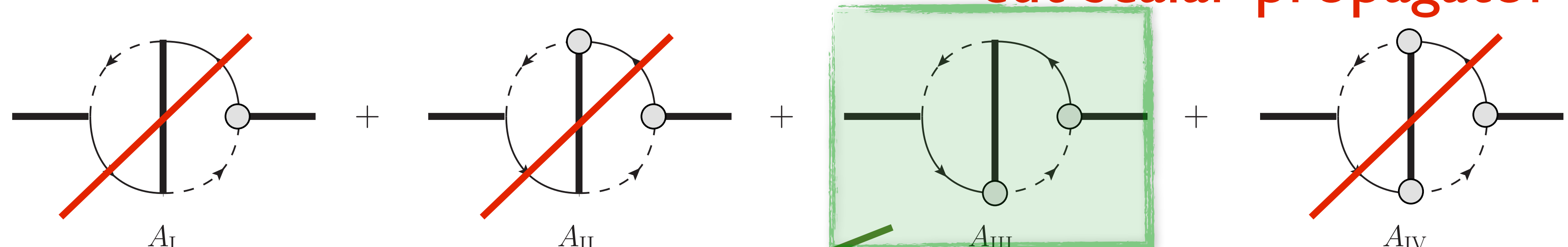
$i\Sigma_1 =$ 

summed over dotted and undotted indices

$i\Sigma_2 =$ 

$\Downarrow$

cut scalar propagator

$i\Sigma^{>}_{A} =$ 

$$\Sigma^{>}_{A_{\mathrm{III}}}(q)\, S^{<}(q) = -\lambda^4 \int \frac{d^4 t}{(2\pi)^4} \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} (2\pi)^4 \delta(q + t - k_1 - k_2)$$
$$\underbrace{i\Delta^{11}(k_1 - q)\, i\Delta^{22}(k_1 - t)}_{\equiv \mathcal{S}} \underbrace{P_{\mathrm{R}} iS^{21}(k_2)\, P_{\mathrm{L}} iS^{12}(t)\, P_{\mathrm{L}} iS^{21}(k_1)\, P_{\mathrm{R}} iS^{12}(q)}_{\equiv \mathcal{F}}$$

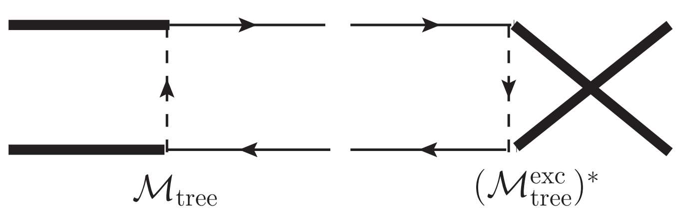
COLLISION TERM
MATCHING

after inserting the propagators:

$$\Sigma^{>}_{A_{\mathrm{III}}}(q)\, S^{<}(q) = \frac{1}{2E_{\chi_1}}(2\pi)\,\delta\left(q^0 - E_{\chi_1}\right)\int \frac{d^4 t}{(2\pi)^3\, 2E_{\chi_2}}\delta\left(t^0 - E_{\chi_2}\right)\times$$

$$\int \frac{d^3\vec{k}_1}{(2\pi)^3\, 2E_{f_1}}\frac{d^3\vec{k}_2}{(2\pi)^3\, 2E_{f_2}}(2\pi)^4\,\delta\left(q+t-k_1-k_2\right)|\mathcal{M}_A|^2\left[f_\chi(q)\, f_\chi(t)\left(1-f_f^{\mathrm{eq}}\left(k_1^0\right)\right)\left(1-f_f^{\mathrm{eq}}\left(k_2^0\right)\right)\right]$$

$\Rightarrow$ one indeed recovers the known collision term and

$|\mathcal{M}_A|^2 =$ 

(part of) tree level $|\mathcal{M}|^2$

repeating the same for B type diagrams the bottom line:

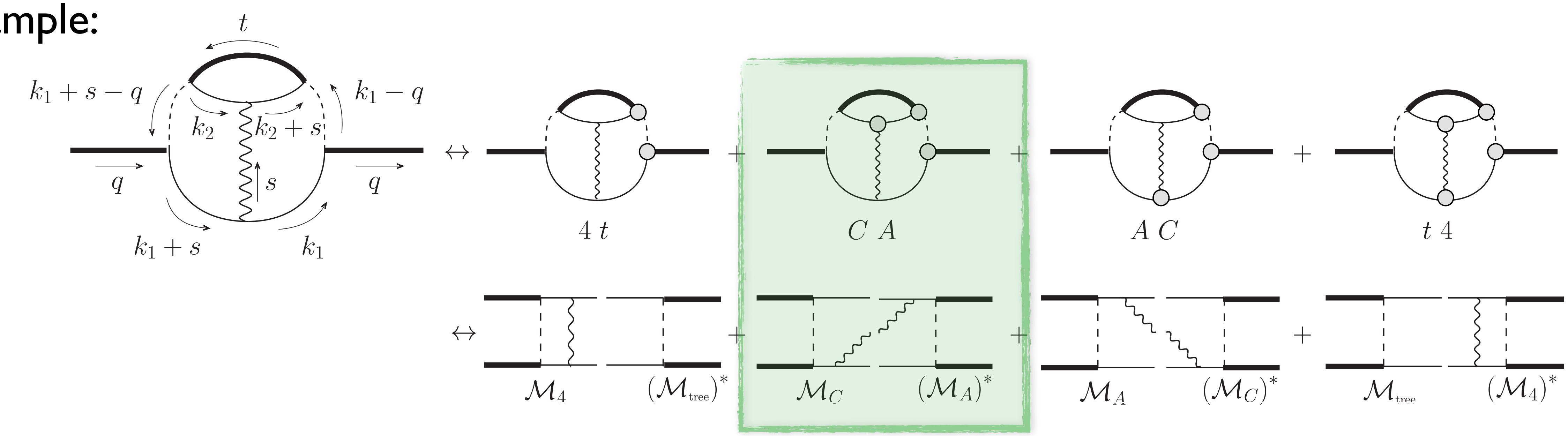
$i\Sigma^{>} \leftrightarrow$ tree level annihilation contribution to the collision term

COLLISION TERM

MATCHING AT NLO

$i\Sigma_3 =$ 20 self-energy diagrams

example:



$$\Sigma^{>}_{\text{CA}}(q)\,S^{<}(q) = \frac{1}{2E_{\chi_1}}(2\pi)\,\delta\left(q^0 - E_{\chi_1}\right)\int\frac{d^4t}{(2\pi)^3\,2E_{\chi_2}}\delta\left(t^0 - E_{\chi_2}\right)$$

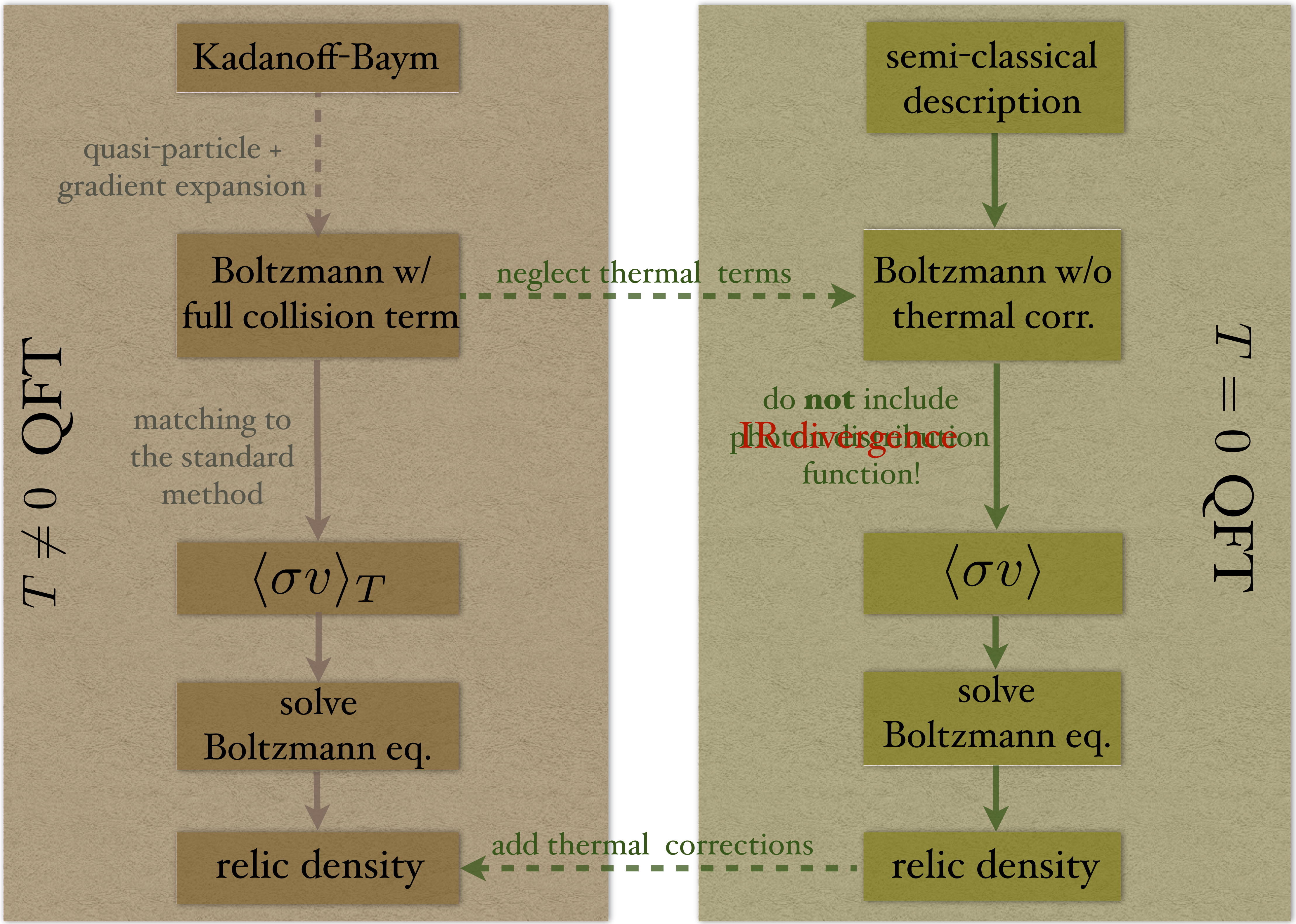
$$\int\frac{d^3\vec{k}_1}{(2\pi)^3\,2E_{f_1}}\frac{d^3\vec{k}_2}{(2\pi)^3\,2E_{f_2}}\frac{d^3\vec{s}}{(2\pi)^3\,2E_{\gamma}}(2\pi)^4\,\delta\left(q + t - k_1 - k_2 - s\right)$$

$$\mathcal{M}_C\left(\mathcal{M}_A\right)^*\left[f_\chi(q)\,f_\chi(t)\left(1 - f_f^{\text{eq}}\left(k_1^0\right)\right)\left(1 - f_f^{\text{eq}}\left(k_2^0\right)\right)\left(1 + f_\gamma^{\text{eq}}\left(s^0\right)\right)\right]$$

$\Rightarrow$ at NLO thermal effects do **not** change the collision therm structure

COLLISION TERM

METHOD SUMMARY



Results

coming back to our example...

every contribution can be written in a form:

$$\int_0^\infty d\omega\, f_\gamma(\omega) \boxed{S(\omega, e_\chi, \epsilon, \xi)} \qquad f_\gamma(\omega) = \frac{1}{1 - e^{\omega/T}}$$

photon energy

expand in ω

$$S = \sum_{i=-1}^{\infty} s_n \omega^n$$

note:

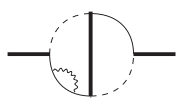
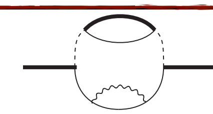
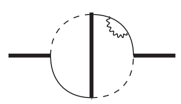
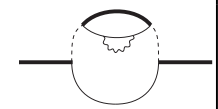
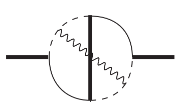
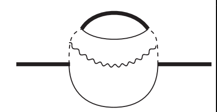
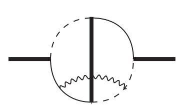
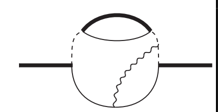
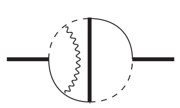
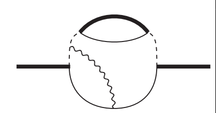
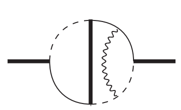
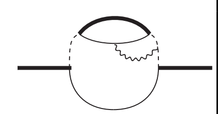
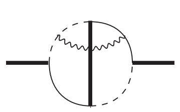
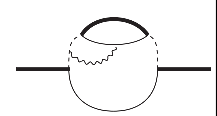
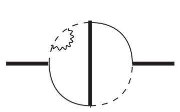
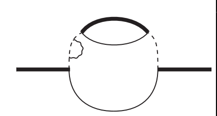
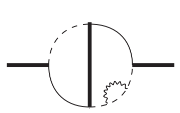
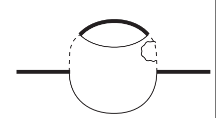
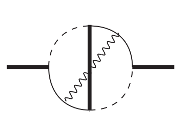
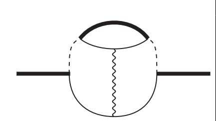
$$J_n \equiv \int_0^\infty f_B(\omega)\omega^n d\omega = \begin{cases} \text{div} & n \leq 0 \\ \sim \tau^{n+1} & n > 0 \end{cases}$$

IR divergence in separate terms: $\begin{array}{rl} J_{-1} \leftrightarrow & T = 0 \text{ soft div} \\ J_0 \leftrightarrow & T = 0 \text{ soft eikonal} \end{array}$

finite T corrections: $J_1 \leftrightarrow \mathcal{O}(\tau^2) \quad \ldots$

Results

IR divergence cancellation: S-wave

The divergent part J_{-1}							
Type A	Real	Virtual	External	Type B	Real	Virtual	External
	$\frac{\alpha(1-2\epsilon^2)}{\pi\epsilon^2}$		$-\frac{\alpha(1-2\epsilon^2)}{\pi\epsilon^2}$		$-\frac{\alpha}{\pi\epsilon^2}$		$\frac{\alpha}{\pi\epsilon^2}$
	$\frac{\alpha(1-2\epsilon^2)}{\pi\epsilon^2}$		$-\frac{\alpha(1-2\epsilon^2)}{\pi\epsilon^2}$		$-\frac{\alpha}{\pi\epsilon^2}$		$\frac{\alpha}{\pi\epsilon^2}$
	0				0		
	0	0			0	0	
	0	0			0	0	
	0	0			0	0	
	0	0			0	0	
		0				0	
		0				0	
	$\frac{2\alpha(1-2\epsilon^2)^2}{\pi\epsilon^2\sqrt{1-4\epsilon^2}}L$	$-\frac{2\alpha(1-2\epsilon^2)^2}{\pi\epsilon^2\sqrt{1-4\epsilon^2}}L$			$\frac{2\alpha(1-2\epsilon^2)}{\pi\epsilon^2\sqrt{1-4\epsilon^2}}L$	$-\frac{2\alpha(1-2\epsilon^2)}{\pi\epsilon^2\sqrt{1-4\epsilon^2}}L$	

cancels in **every** row separately

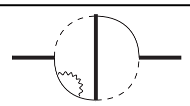
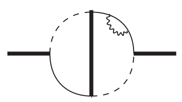
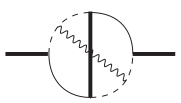
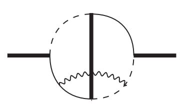
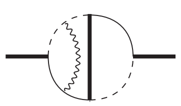
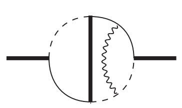
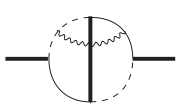
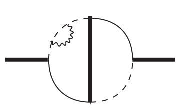
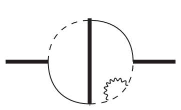
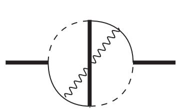
$\Longrightarrow$ every CTP self-energy is IR finite

RESULTS

FINITE T CORRECTION: S-WAVE

factorized $\frac{\pi}{6}\alpha\tau^2\frac{a_{\rm tree}}{\epsilon^2}$

The finite part J_1

Type A	Real	Virtual	External
	$\frac{2(1-\xi^2)}{D^2D_\xi^2}+\frac{(1-2\epsilon^2)p_1(\epsilon,\xi)}{2D^2D_\xi^2}\boxed{+\frac{1}{2\sqrt{D}}L}$		$\frac{(1-2\epsilon^2)(\xi^2-3D)}{2DD_\xi}\boxed{-\frac{1}{2\sqrt{D}}L}$
	— " —		— " —
	$-\frac{4(1-2\epsilon^2)D}{D_\xi^2}$		
	$-\frac{2(1-2\epsilon^2)\xi^2}{D_\xi^2}-\frac{f_1(\epsilon,\xi)}{\sqrt{D}D_\xi^2}L$	$\frac{2(1-2\epsilon^2)(D-\xi^2)}{D_\xi^2}+\frac{f_1(\epsilon,\xi)}{\sqrt{D}D_\xi^2}L$	
	— " —	— " —	
	— " —	— " —	
	— " —	— " —	
		$-\frac{4(1-2\epsilon^2)D}{D_\xi^2}$	
		— " —	
	$\frac{2(1-2\epsilon^2)p_2(\epsilon,\xi)+(1-\xi^2)^2}{D^2D_\xi^2}+\frac{4f_2(\epsilon,\xi)}{\sqrt{D}D_\xi^2}L$	$\frac{16\epsilon^2(2-3\epsilon^2)-(3-\xi^2)^2}{D_\xi^2}-\frac{4f_2(\epsilon,\xi)}{\sqrt{D}D_\xi^2}L$	

Log terms cancels in **every** row separately

$\Downarrow$

no collinear divergence!

$\xi=\frac{m_\phi}{m_\chi}\gtrsim 1$

$\tau=\frac{T}{m_\chi}\ll 1$

$\epsilon=\frac{m_f}{2m_\chi}\ll\tau$

separate contributions complicated, but when summed up...

$$\Delta a_{\tau^4}^{\epsilon=0}=\frac{8\pi^2\lambda^4\alpha\tau^4}{45}\frac{1}{(1+\xi^2)^4}=\frac{4\pi}{45}\alpha\tau^4\frac{1}{(1+\xi^2)^2}\frac{a_{\rm tree}}{\epsilon^2}\bigg|_{\epsilon=0}$$

strongly suppressed as at kinetic equilibrium $\tau\sim v^2$

SUMMARY: PART I

1. how the (soft and collinear) IR divergence cancellation happen?
   automatic in thermal QFT formalism, cancellation at the level of every CTP self-energy

2. does Boltzmann equation itself receive quantum corrections?
   no, not at NLO

3. how large are the remaining finite T corrections?
   strongly suppressed, of order $\mathcal{O}(\alpha T^4)$

Exception IV:
LO sometimes is not enough
(and then in principle $T \neq 0$ QFT needed)

...but in practice one can safely use BE with NLO cross-section

Exception V:

v-dependent interactions and non-perturbative effects

VELOCITY-DEPENDENT σv

(Note: the 3rd exception from Griest&Seckel is actually of this type as well)

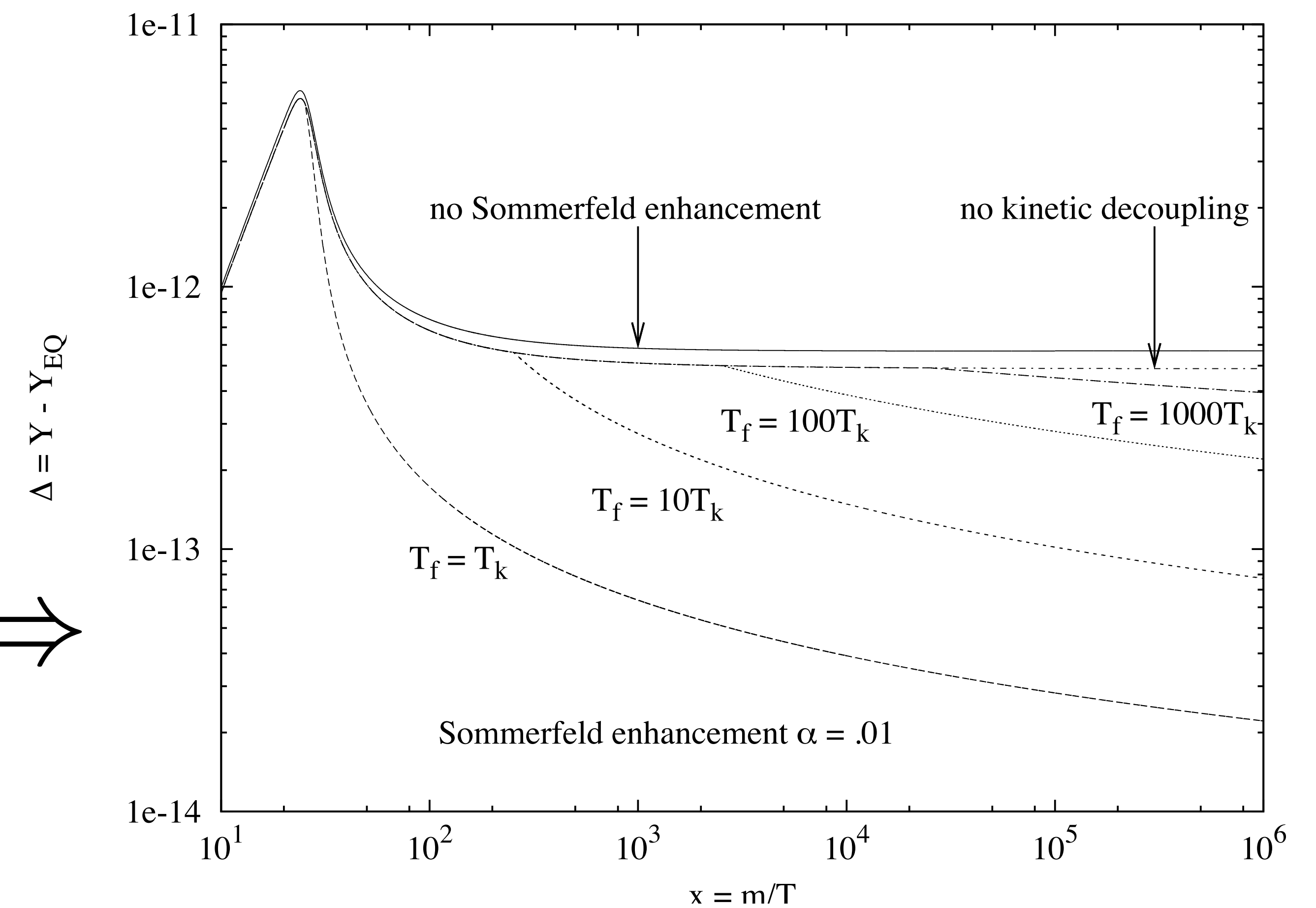
The annihilation cross-section is always velocity dependent… but typically $\sigma v \approx a + bv^2$

O(few %)

What if for a given model $\sigma v \propto v^{-n} \quad n > 0$?

well… not much as long as DM is in kinetic equilibrium

but if it the kinetic decoupling (KD) happens relatively **early** then $\Rightarrow$



Dent, Dutta, Scherrer '10

Are there any real physical situations in which this can happen?

The Sommerfeld effect

re-summation

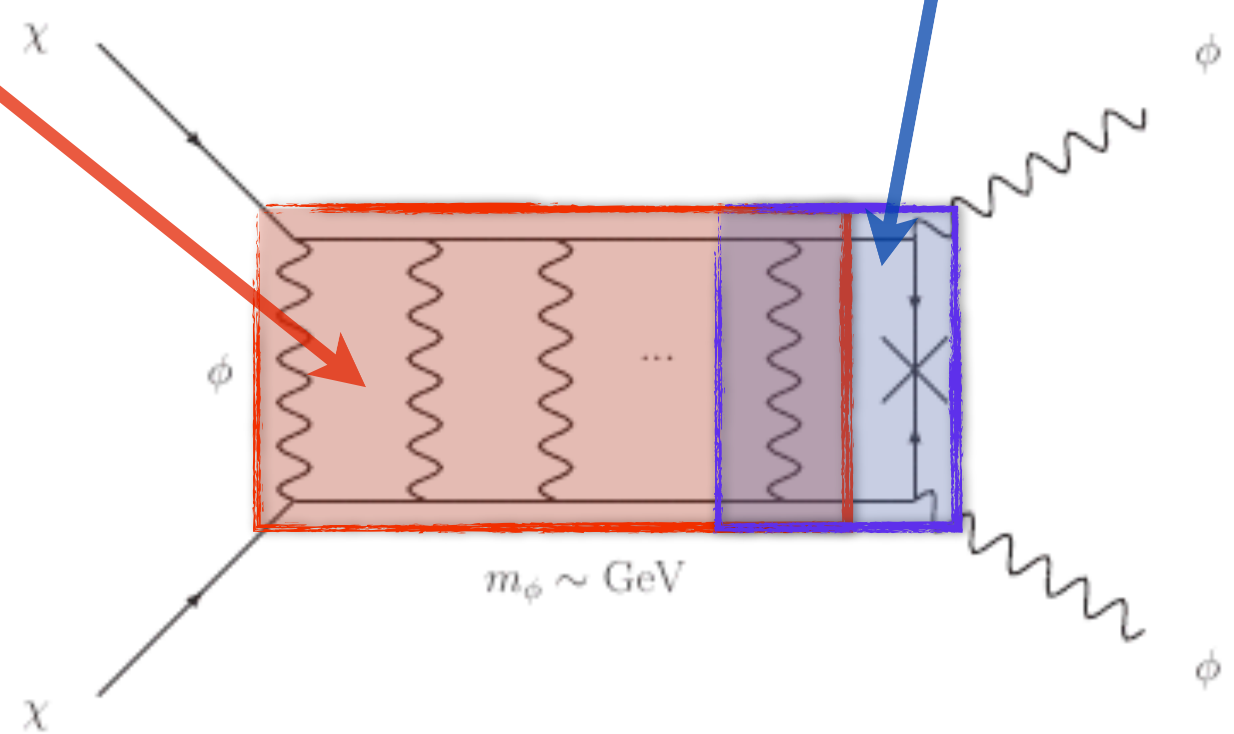
one-loop $\propto \alpha \frac{m_\chi}{m_\phi}$

$$\frac{1}{m_\phi} \gtrsim \frac{1}{\alpha m_\chi}$$

force range | Bohr radius

$$m_\chi v^2 \lesssim \alpha^2 m_\chi$$

kinetic energy | Bohr energy



$$\boxed{\sigma_{\rm SE} = S(v)\,\sigma_0}$$

Arkani-Hamed *et al.* '09

→ in a special case of Coulomb force: $S(v) = \dfrac{\pi\alpha/v}{1-e^{-\pi\alpha/v}} \approx \pi\dfrac{\alpha}{v}$

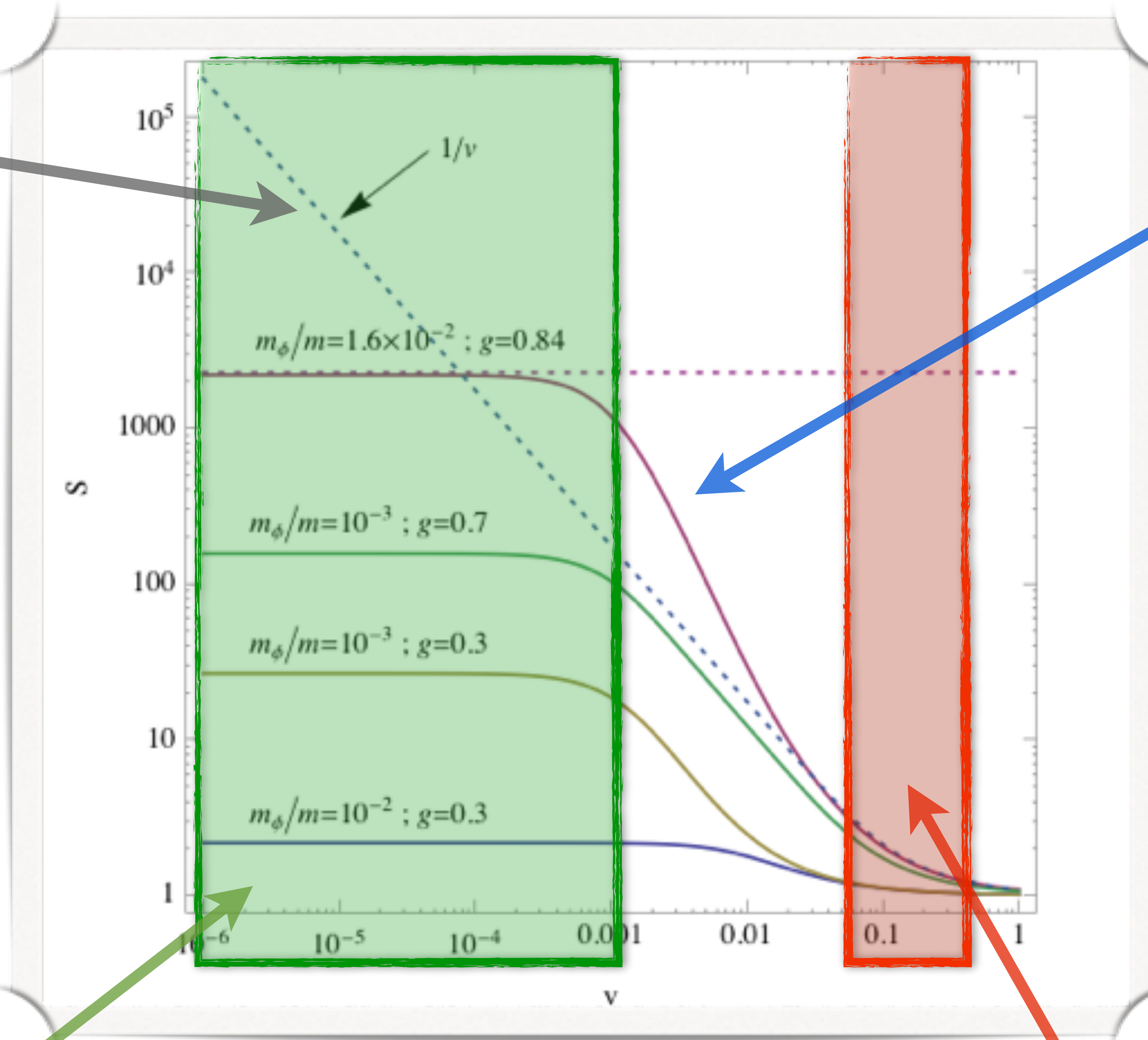
THE SOMMERFELD EFFECT

WITH A DARK FORCE

Coulomb

resonance

$$\frac{1}{m_\phi} \approx \frac{1}{\alpha m_\chi}$$



present day: indirect detection

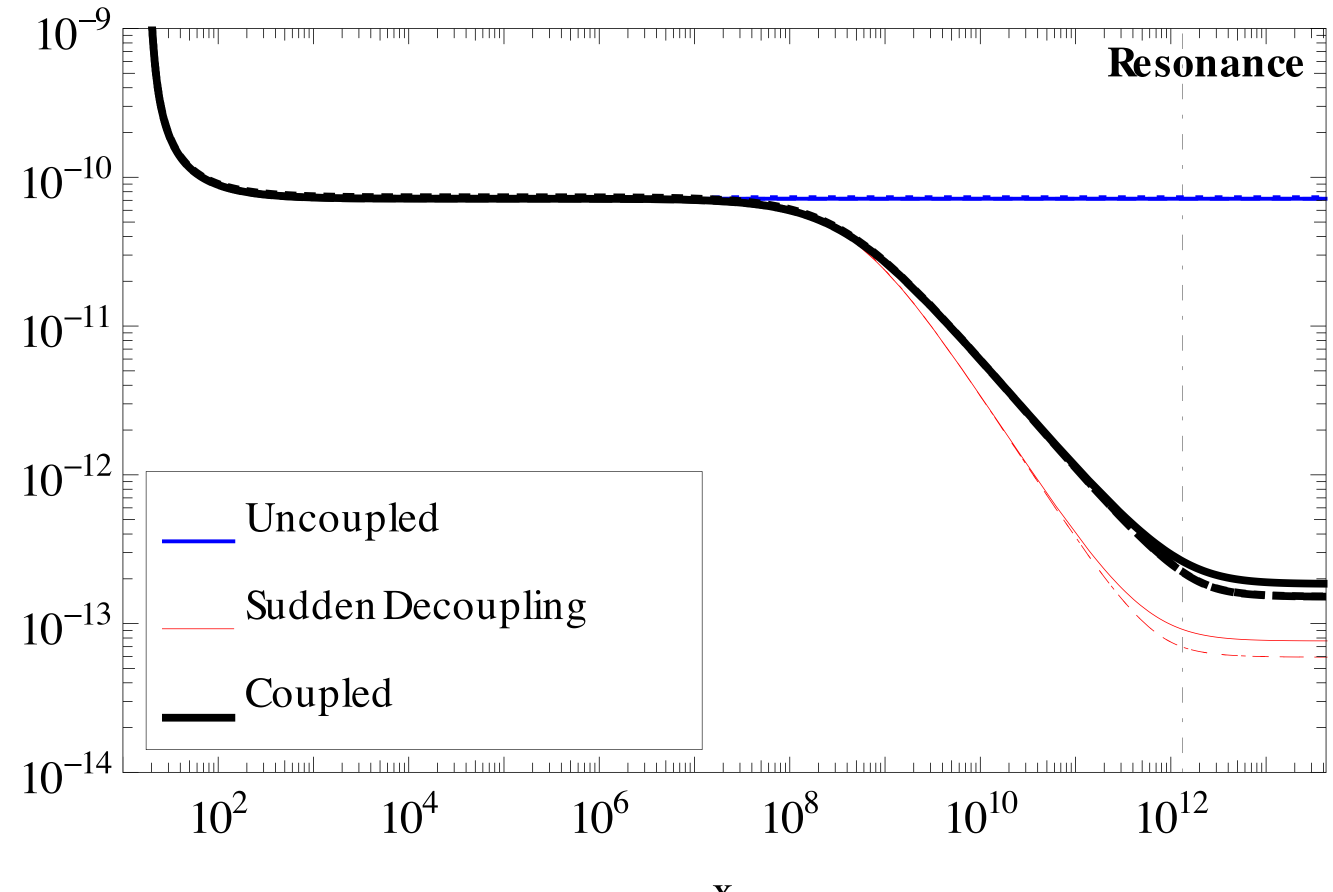
freeze-out: relic density

SOMMERFELD EFFECT AND KD

If on the dark side of the Universe a „dark force" awakens…

$$\frac{Y'}{Y} = -\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{Hx}\, sY\, \langle\sigma v_{\rm rel}\rangle\big|_{x=m_\chi^2/(s^{2/3}y)}$$

$$\frac{y'}{y} = -\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{Hx}\left[2m_\chi c(T)\left(1-\frac{y_{\rm eq}}{y}\right) - sY\Big(\langle\sigma v_{\rm rel}\rangle - \langle\sigma v_{\rm rel}\rangle_2\Big)_{x=m_\chi^2/(s^{2/3}y)}\right].$$



van den Aarssen, Bringmann, Goedecke '12

… one has to be prepared with a more sophisticated formalism

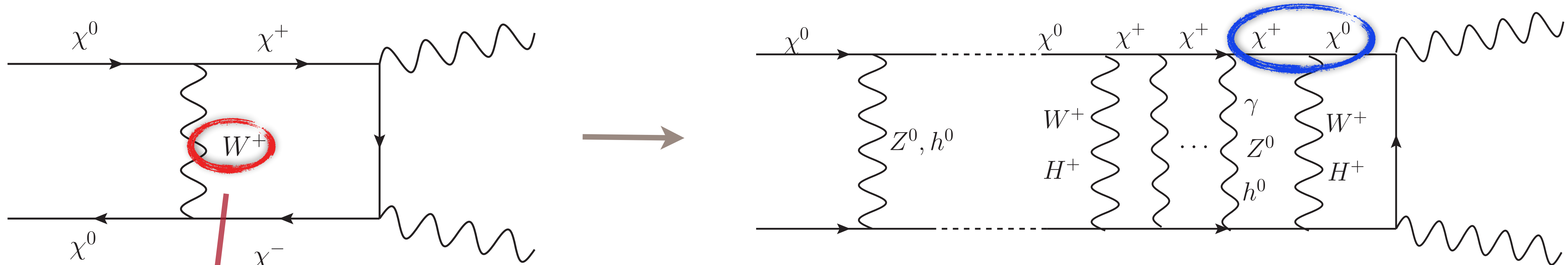
The Sommerfeld effect
from EW interactions

force carriers in the MSSM:

$\cancel{\gamma}$, $W^{\pm}$, Z^0, h_1^0, h_2^0, $H^{\pm}$

Hisano *et al.* '04,'06

$\delta m \ll m_\chi$



$m_\chi \gg m_W$

at TeV scale $\Rightarrow$ generically effect of $\mathcal{O}(1-100\%)$

on top of that resonance structure

$\hookrightarrow$ effect of $\mathcal{O}(\text{few})$

for the relic density

Note: for ID the enhancement is significantly stronger!

What is known
with the Sommerfeld enhancement

- pure wino, pure higgsino
  Hisano *et al.* ’04,’06
- mixed wino-higgsino (with everything else decoupled)
  AH, Iengo, Ullio, ’11, Beneke *et al.* ’14
- stop and stau co-annihilations
  Freitas ’07, AH ’11, Klasen *et al.* ’14
- gluino co-annihilation
  Ellis *et al.* ’15
- Minimal DM model
  Cirelli *et al.* ’07,’08,’09

Currently only available tool for the MSSM:

DarkSE package extending the relic density by SE in DarkSUSY

AH, ’11

…AND WHAT WAS IMPROVED

Based on a framework by Beneke,Hellmann,Ruiz-Femenia ’12, ’13, ’14:

1. the Sommerfeld effect for P- and O(v^2) S-wave
2. off-diagonal annihilation matrices

not present in DarkSE

total effect up to O(10%)

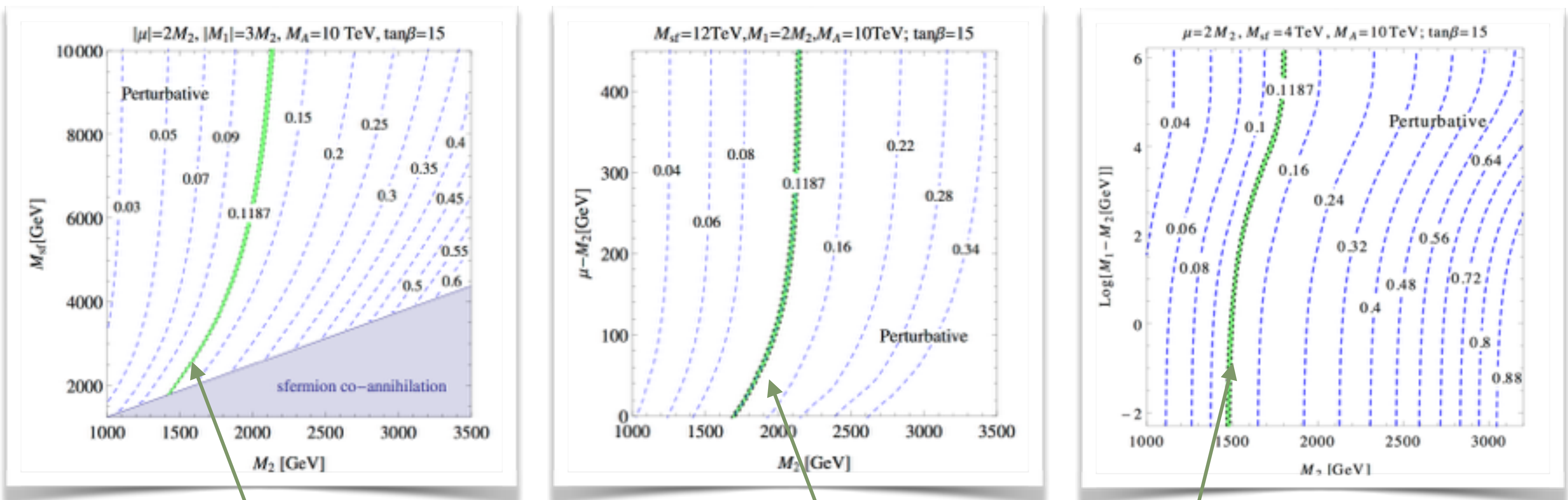
New code (to be public):

- suitable for full MSSM
- using EFT computation of annihilation matrices
- one-loop on-shell mass splittings and running couplings
- possibility of including thermal corrections
- present day annihilation in the halo (for ID)
- accuracy at O(%), dominated by theoretical uncertinities of EFT

caveat: still no NLO effects…

RESULTS
AT THE BORN LEVEL

Beneke, Bharucha, Dighera, Hellmann, AH, Recksiegel, Ruiz-Femenia; 1601.04718



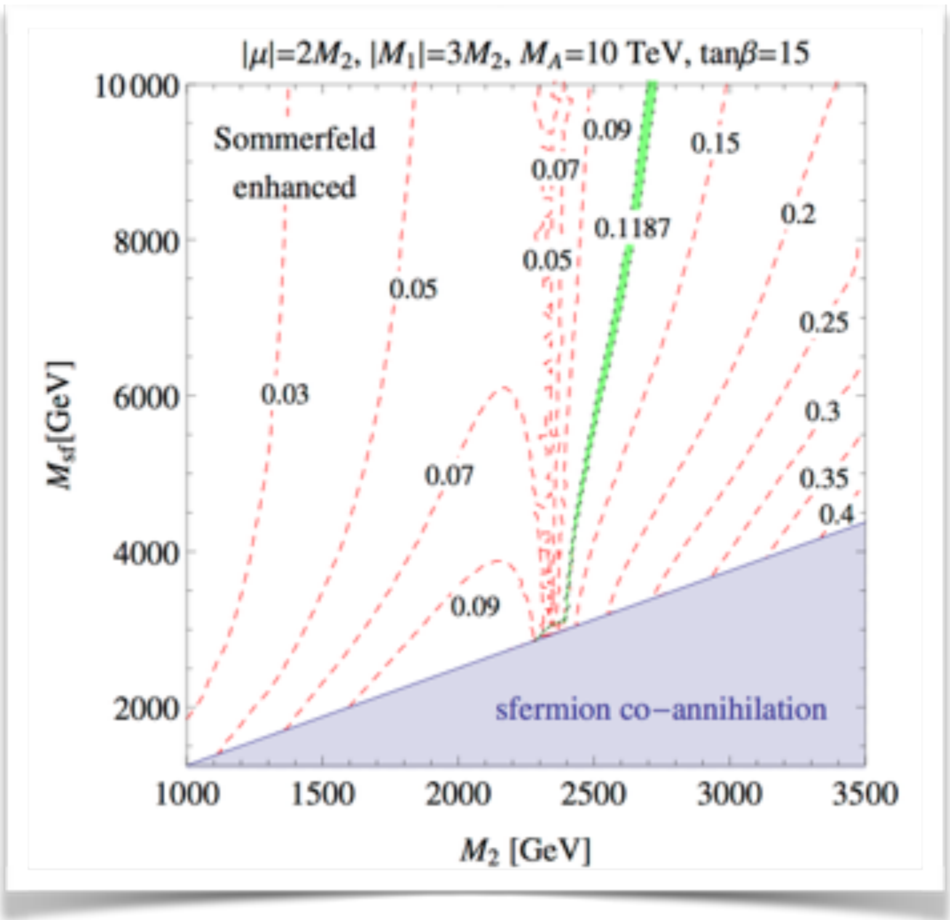
As the sfermion mass decreases the effective annihilation rate is suppressed due to t-channel interference - the correct relic abundance is obtained for masses of around 1.4 TeV*

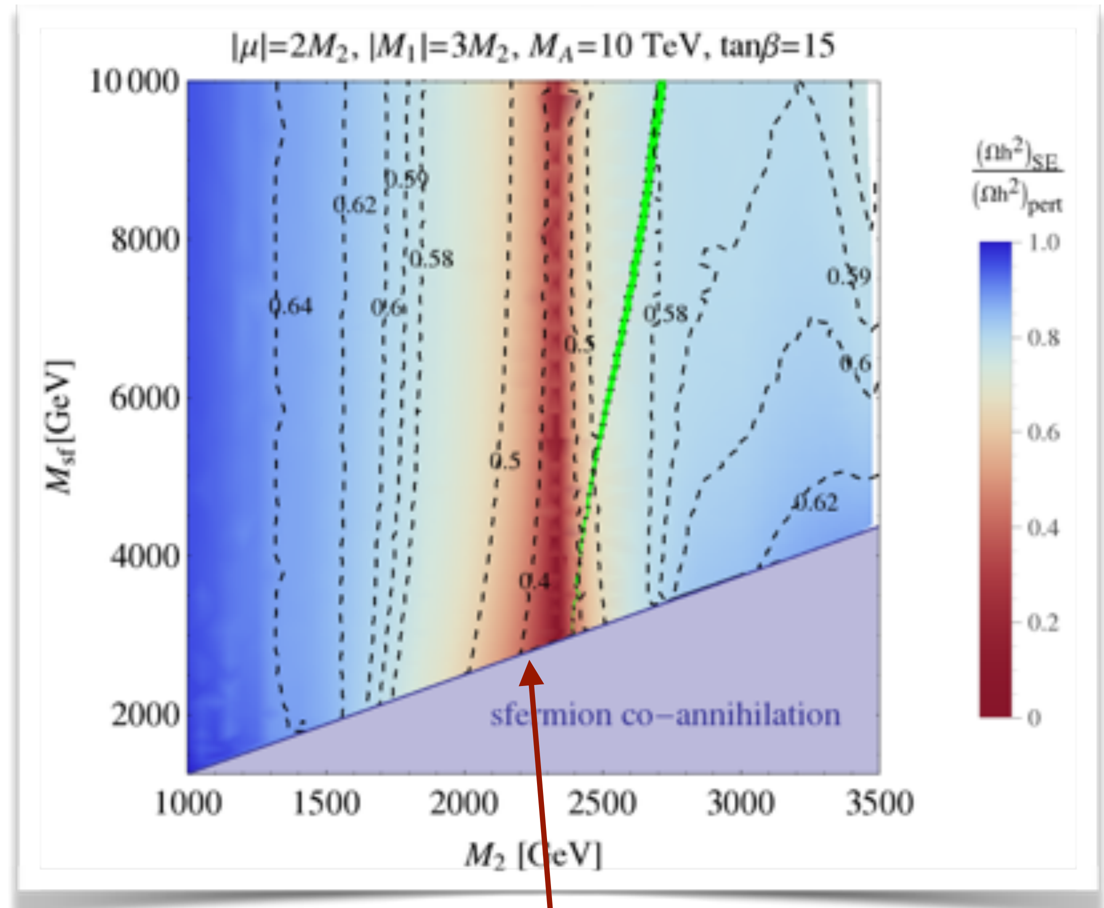
Higgsino and bino annihilate less strongly - dilute the wino annihilation and reduce the mass to 1.7 and 1.5 TeV respectively*

*for the chosen set of parameters

RESULTS

PURE WINO WITH NON-DECOUPLED SFERMIONS





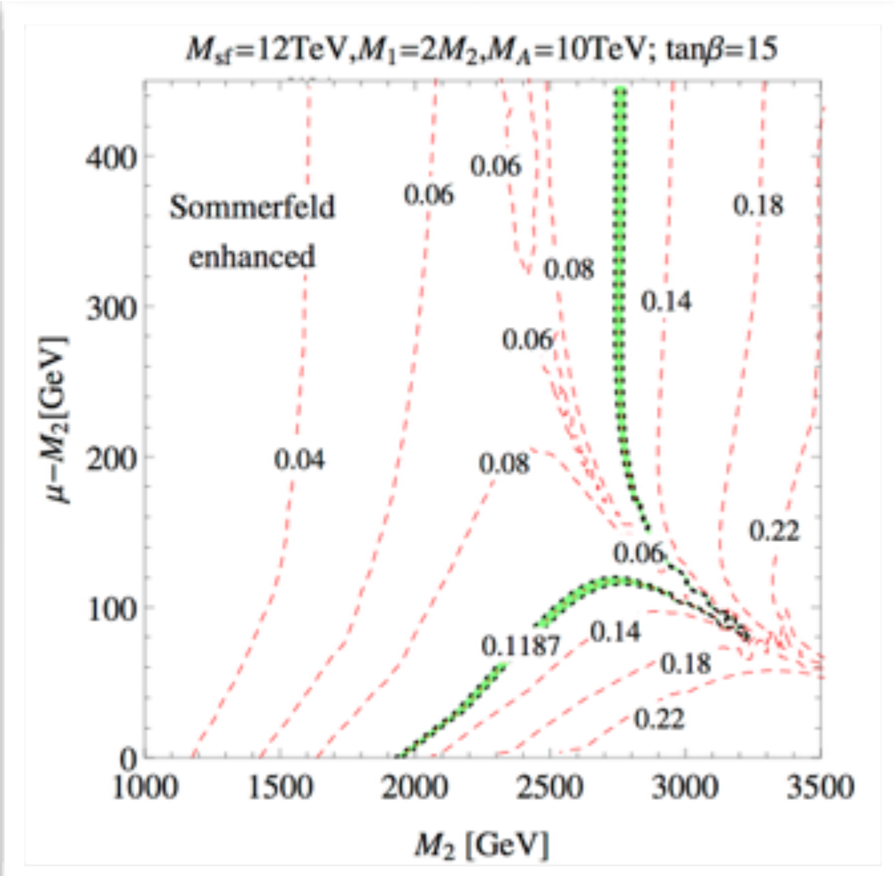
The correct relic density is moved from 1.5-2.1 TeV up to 2.4-2.8 TeV

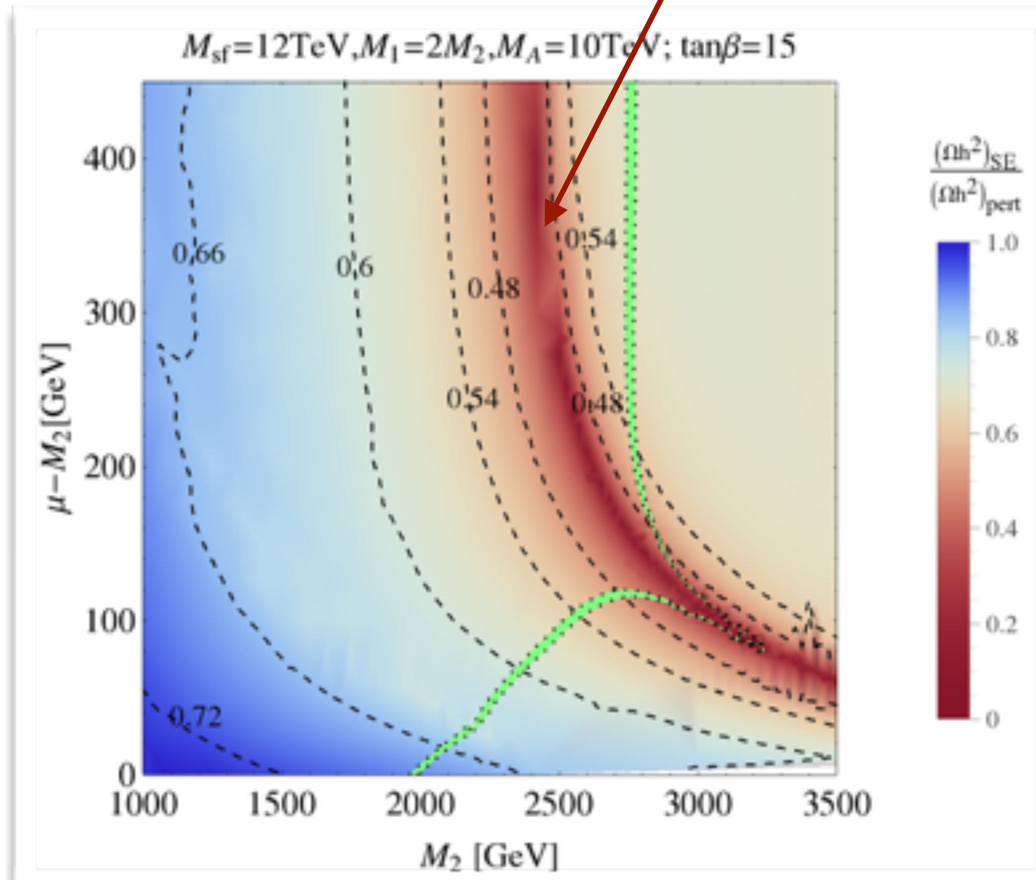
At 2.4 TeV resonance occurs, for low sfermion masses region with correct RD is resonant

Results
WINO-HIGGSINO ADMIXTURE

$$\frac{1}{m_W} \simeq \frac{1}{\alpha m_\chi}$$

force range   Bohr radius

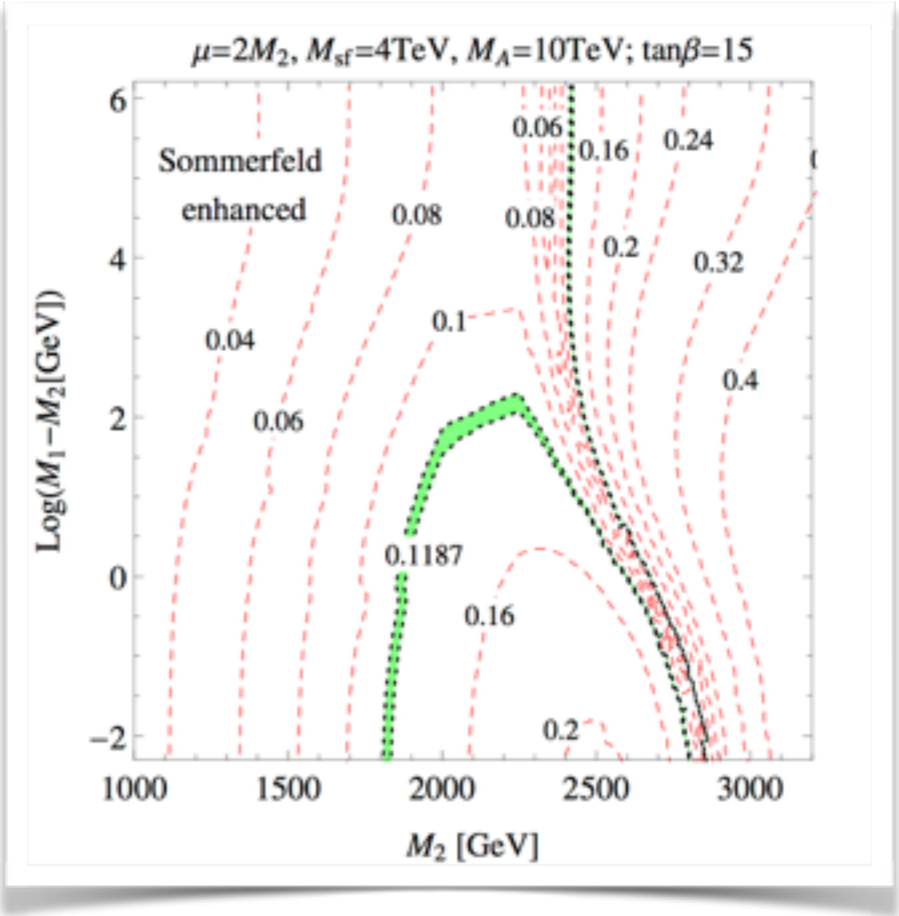




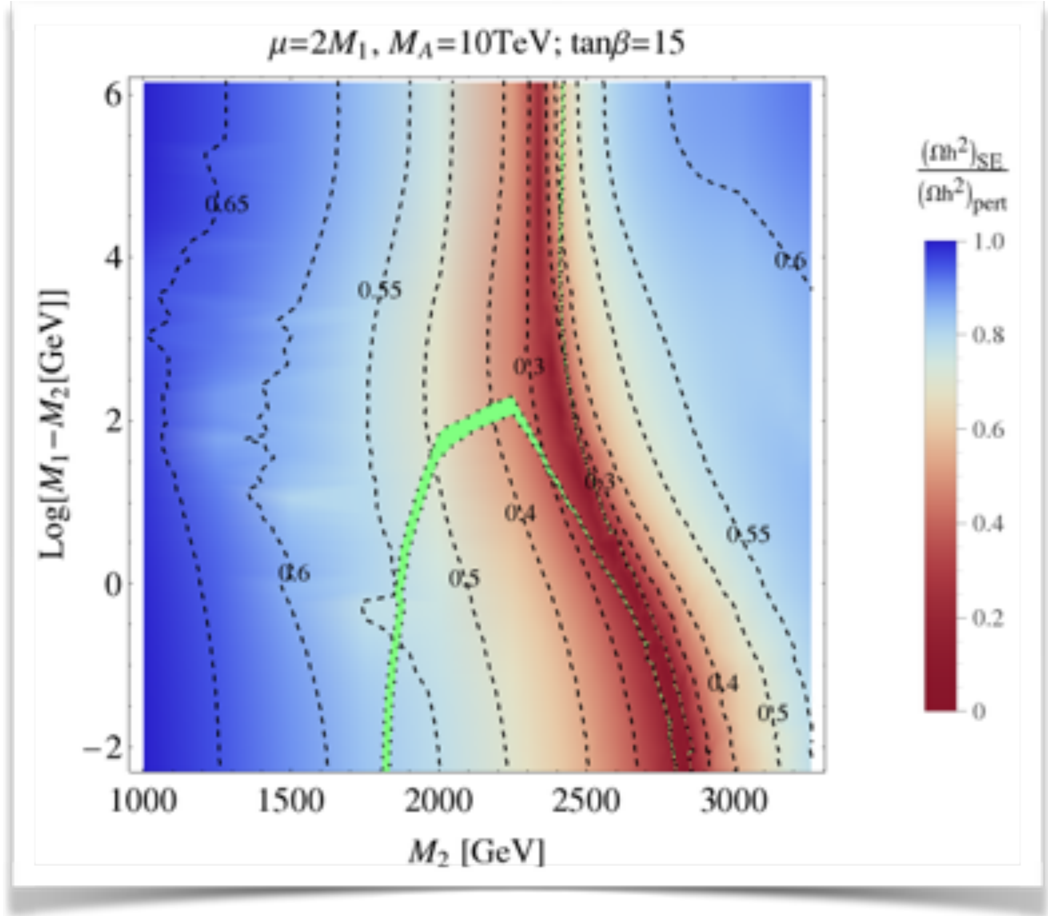
The correct relic density is moved from 1.7-2.2 TeV up to 1.9-3.3 TeV

The position of the resonance is strongly μ dependent

RESULTS
WINO-BINO ADMIXTURE



The correct relic density is moved from 1.5-1.8 TeV up to 1.8-2.9 TeV

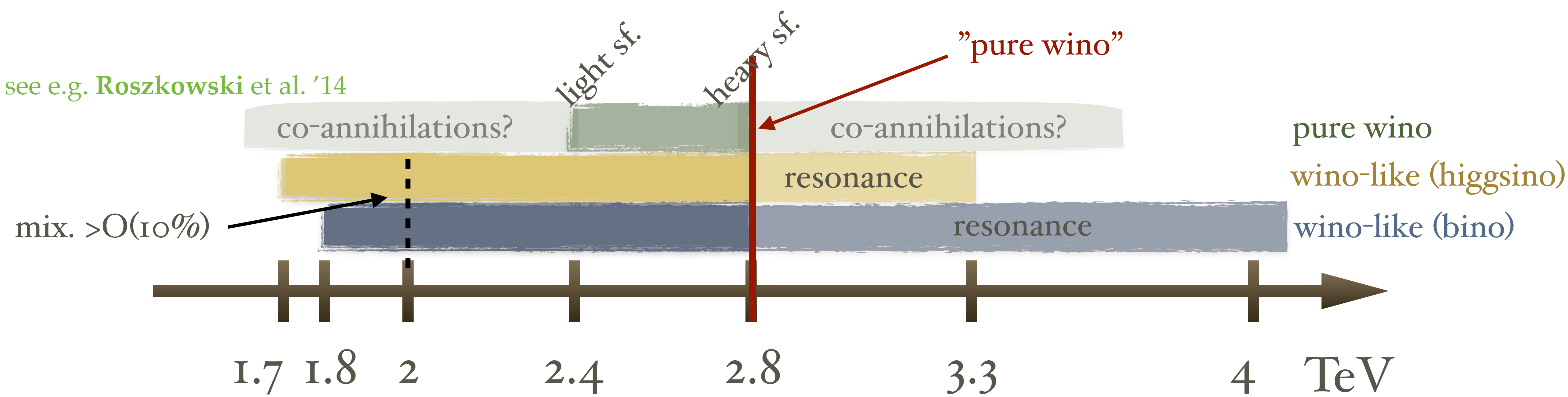


The position of the resonance is strongly M_1 dependent

SUMMARY: PART II

Velocity dependence and non-perturbative effects on the cross-section can lead to significant modification of the relic density

E.g. for the wino-like neutralino in MSSM correct relic density is obtained for wide range of masses:



Public code including full SE in the MSSM with accuracy for relic density O(%) and running time O(min) to become available

TAKEAWAY MESSAGE

We do have the tools to calculate DM relic reliably; it is worth the effort to use them!

"Everything should be made as simple as possible, but no simpler."

attributed to* **Albert Einstein**

*The published quote reads:
"It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience."
„On the Method of Theoretical Physics" ,The Herbert Spencer Lecture, delivered at Oxford (10 June 1933); also published in *Philosophy of Science*, Vol. 1, No. 2 (April 1934), pp. 163-169., p. 165