Self-organized topological superconductivity in Yu-Shiba-Rusinov chains

...and frustrated magnetism in YSR lattices

Jens Paaske
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Outline

1. From single Yu-Shiba-Rusinov states to YSR chains and Top-SC
2. Magnetic order of 1D chains in 3D SC
3. Interplay between magnetic order and Top-SC
4. Magnetic order of 1D chains in 2D SC *(Briefly)*
5. Magnetic order of 2D lattice in 3D SC

Collaborators

-Michael Schecter
-Morten Holm Christensen
-Karsten Flensberg
-Brian Møller Andersen
-Olav Fredrik Syljuåsen

Papers

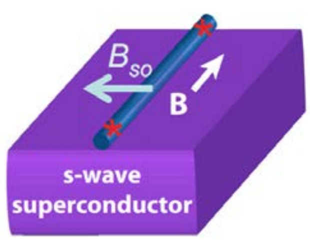
Schecter *et al.*, PRB **93**, 140503(R) (2016)

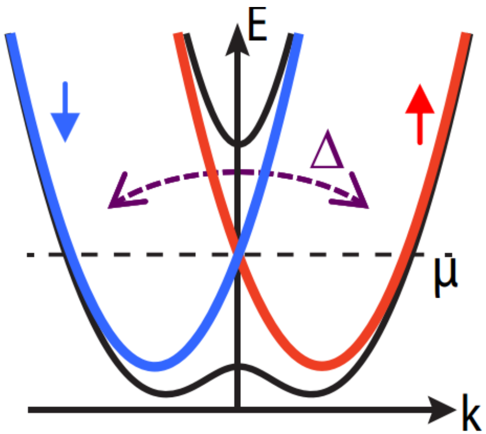
Christensen *et al.*, arXiv:1607.08190

The 1D topological superconductor

- Superconductor with localized, zero-energy q.p. excitations (Majorana)
- Simplest example: 1d spinless electrons with pairing
- InSb, InAs in proximity to Nb, or Al

Spin-orbit coupling
+
Orthogonal Zeeman field





$$H = \left(\frac{k^2}{2m} - \mu + \alpha k \sigma_z \right) \tau_z + \Delta \tau_x + B \sigma_x$$

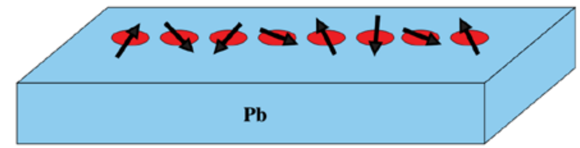
R. M. Lutchyn et al., PRL **105**, 077001 (2010)
Y. Oreg et al., PRL **105**, 177002 (2010)
V. Mourik et al., Science **336**, 1003 (2012)

Replacing SOI with spiraling B-fields or spins

PHYSICAL REVIEW B **84**, 195442 (2011)

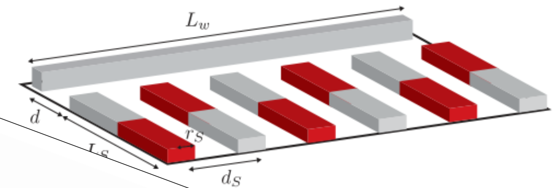
Majoran... ...ing from magnetic nanoparticles on a superconductor without spin-orbit coupling

...erov, and C. W. J. Beenakker



Majorana fermions in superc... ...ling

Morten Kjaergaard,[1] Konrad Wöl...



PHYSICAL REVIEW B 88, 020407(R) (2013)

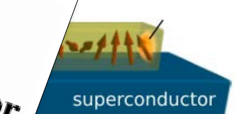
Proposal for realizing Majorana fermions in chains of magnetic atoms on a superconductor

S. Nadj-Perge, I. K. Drozdov, B. A. Bernevig, and Ali Yazdani*

PRL **111**, 147202 (2013) PHYSICAL REVIEW LETTERS

Interplay between Classical Magnetic Moments and Superconductivity in Quan... One-Dimensional Conductors: Toward a Self-Sustained Topol... ...rana Phase

Bernd Braunecker[1]...



PHYSICAL REVIEW B **88**, 155420 (2013)

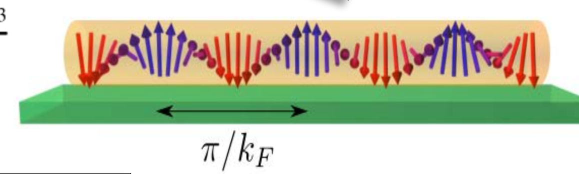
Topological superconducting phase in helical Shiba chains

Falko Pientka,[1] Leonid I. Glazman,[2] and Felix von Oppen[1]

PRL **111**, 186805 (201... week ending ...NOVEMBER 2013

... Systems

...zdani,[4] and Daniel Loss[1]

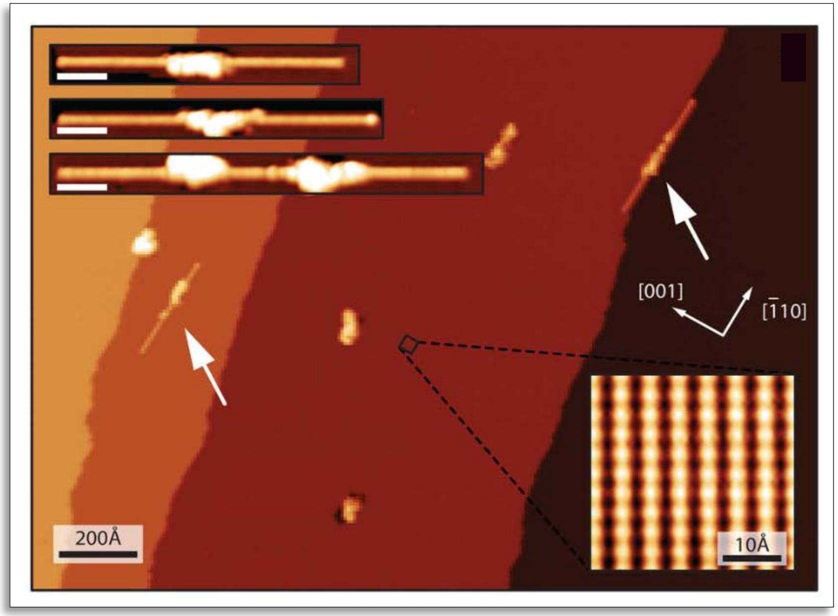


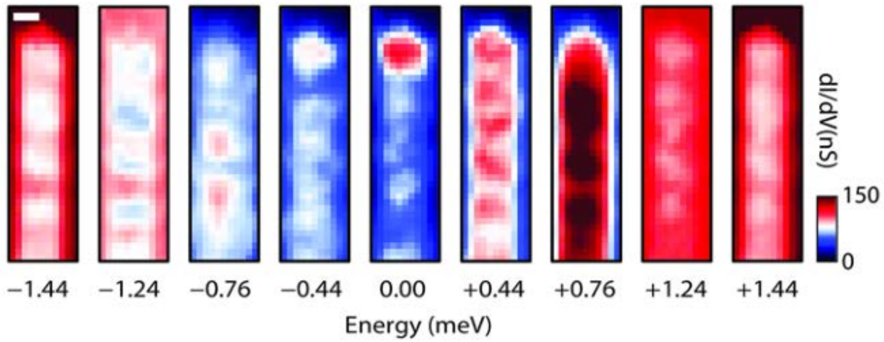
...013) PHYSICAL REVIEW LETTERS week ending 15 NOVEMBER 2013

Self-Organized Topological State with Majorana Fermions

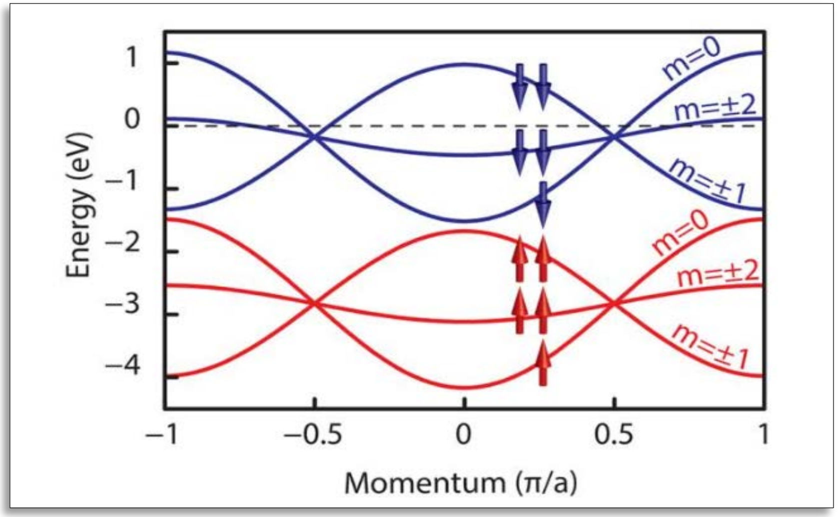
M. M. Vazifeh and M. Franz

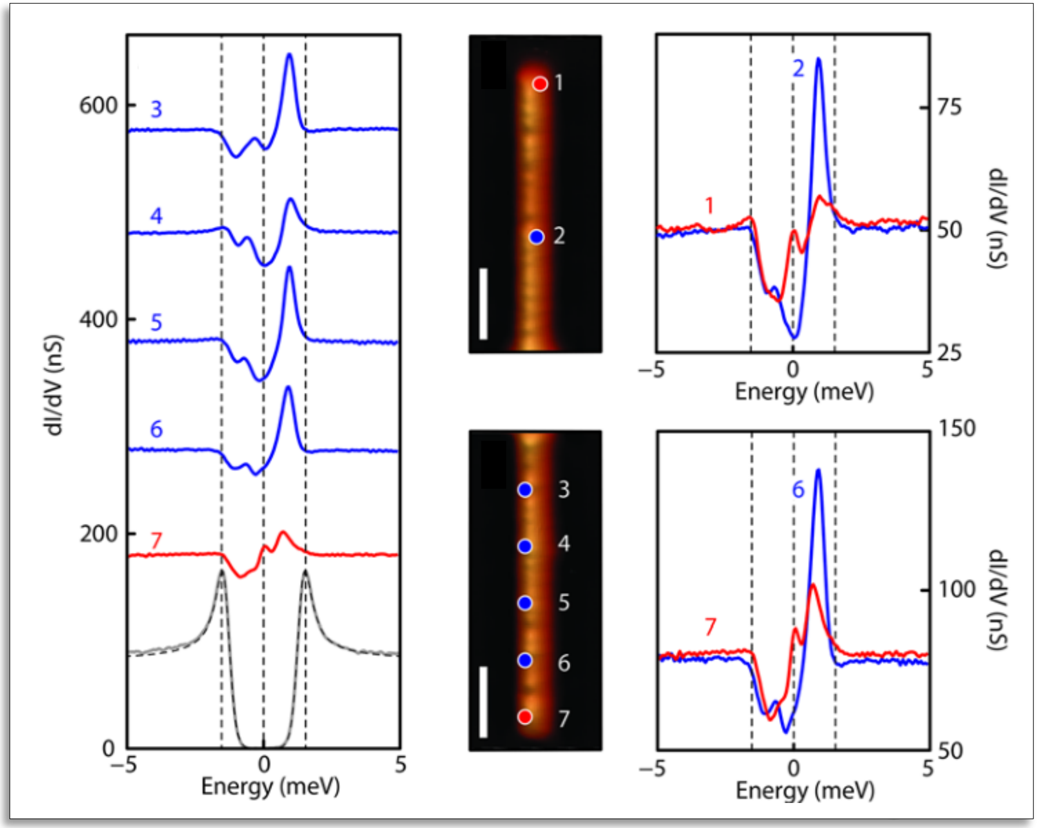
Majorana bound states from Fe chains on Pb(110)





Nadj-Perge et al., Science **346**, 602 (2014)





Yu-Shiba-Rusinov states

REVIEWS OF MODERN PHYSICS, VOLUME 78, APRIL–JUNE 2006

Impurity-induced states in conventional and unconventional superconductors

A. V. Balatsky*
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I. Vekhter†
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803, USA

Jian-Xin Zhu‡
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BOUND STATE IN SUPERCONDUCTORS WITH PARAMAGNETIC IMPURITIES

Yu Luh (Yu Lu)

Received July 10, 1963
Acta Physica Sinica 21, 75-91 (January, 1965)

*A generalized ...
to investigate ...
of superconduc...
exists around ...
gap. An analy...
wave function. ...
state should ap...
verifications ...
infrared absor...*

*Furthermore, th...
impurity and th...
considered.*

Progress of Theoretical Physics, Vol. 40, No. 3, September 1968

Classical Spins in Superconductors

Hiroyuki SHIBA*)

Dep...

It is shown tha...
with a classical s...
form an "impurity...
observable quantit...

SUPERCONDUCTIVITY NEAR A PARAMAGNETIC IMPURITY

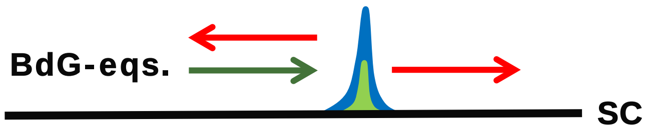
A. I. Rusinov
P. N. Lebedev Physics Institute, USSR Academy of Sciences

Submitted 28 November 1968
ZhETF Pis. Red. 9, No. 2, 146-149 (20 January 1969)

It is shown in [1] that introduction of a small amount ($\lesssim 1\%$) of paramagnetic impurities in a superconductor exerts a strong influence on its properties. In particular, the energy ...lloy no longer coincides with the magnitude of the ordering ... [1] was carried out in the Born approximation with respect ...ill be shown below that in the case of a superconductor al- ...of the scattering of the electrons by the magnetic impurity ... results.

$$\omega = \pm \omega_B \equiv \pm \Delta_0 \frac{1 - ((J/2) S \pi \rho)^2}{1 + ((J/2) S \pi \rho)^2}$$

Rusinov solution

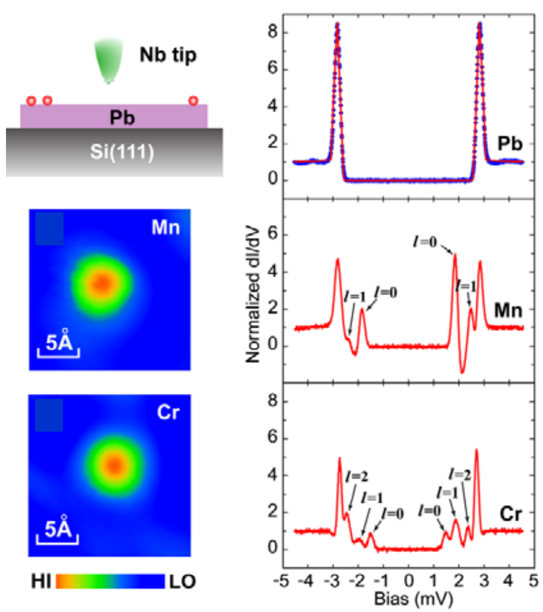


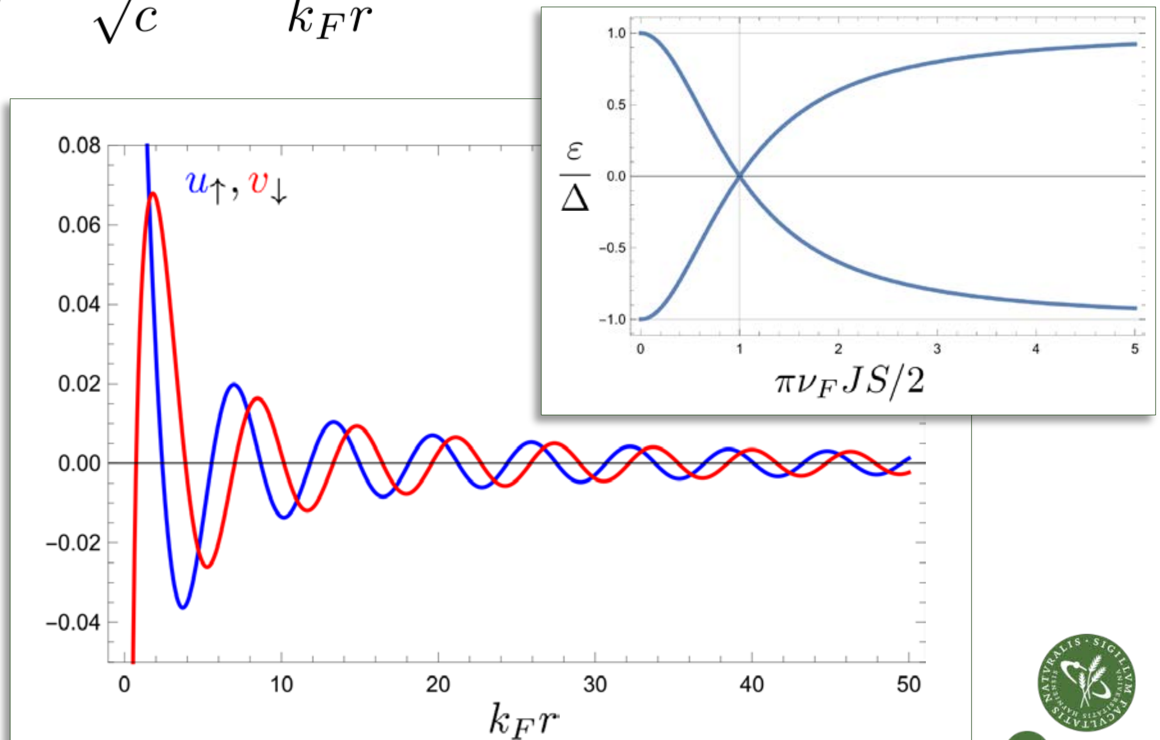
Spin-dependent δ-function potential: $V_\sigma(\mathbf{r}) = (U + JS\tau^z_{\sigma\sigma})\delta(\mathbf{r})$

Eigenenergies: $\varepsilon = \Delta \dfrac{1-(\pi\nu_F JS/2)^2}{1+(\pi\nu_F JS/2)^2} = \Delta\cos(\delta^+ - \delta^-),\;\; \tan(\delta^\pm) = \nu_F(U \pm JS)$

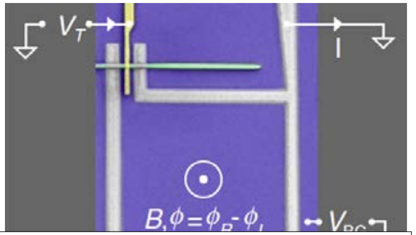
Eigenfunctions (3D): $\begin{pmatrix} u_\uparrow(\mathbf{r}) \\ v_\downarrow(\mathbf{r}) \end{pmatrix} = \dfrac{1}{\sqrt{c}} \dfrac{\sin(k_F r + \delta^\pm)}{k_F r} e^{-r/(\xi/|\sin(\delta^+-\delta^-)|)}$

Ji et al., Science **100**, 226801 (2008)

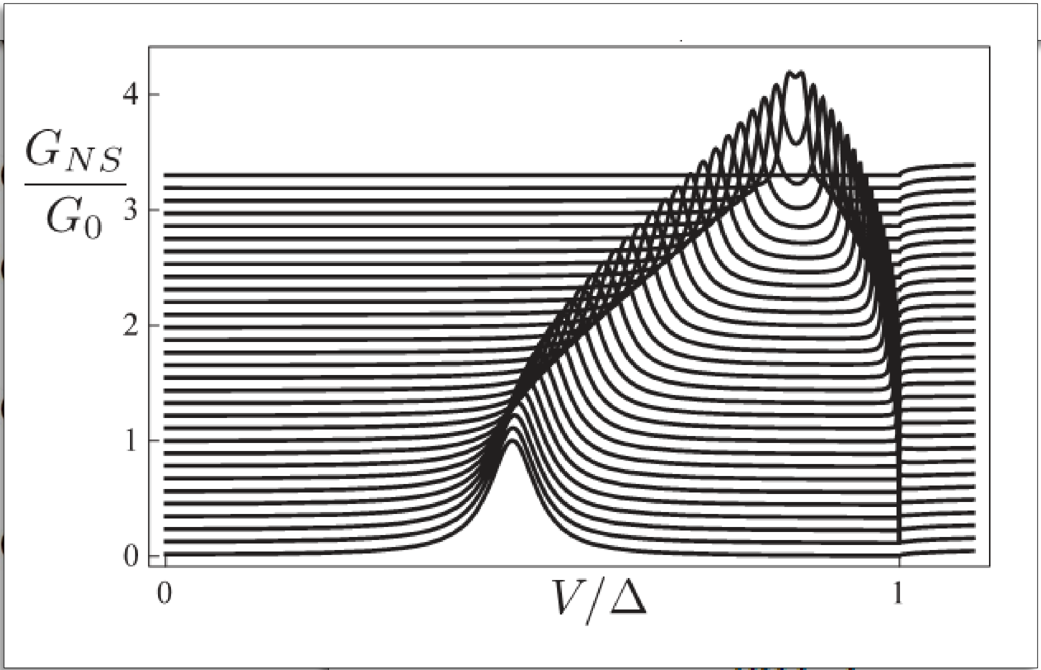


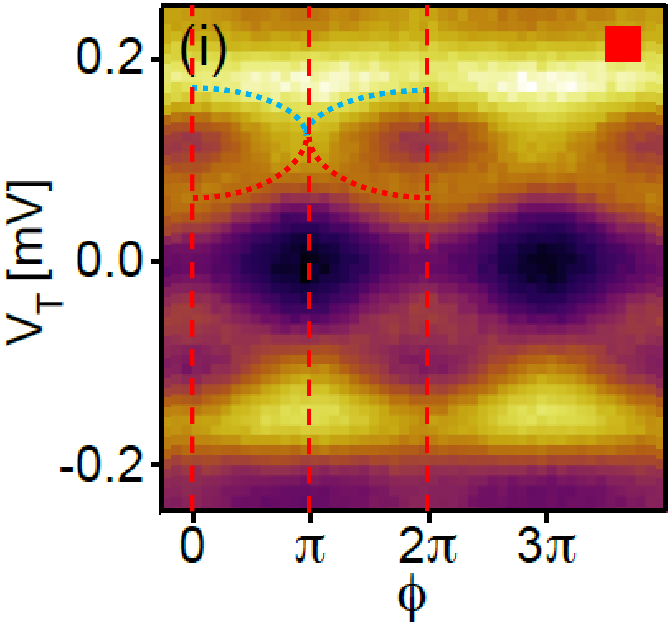


YSR in quantum dots (2-channels, phase-bias)



$$E^2 = \frac{|\Delta|^2}{(1+u^2)^2+4g^2}\left(1-u^2(1+u^2)\sin^2(2\theta)\sin^2\frac{\phi}{2}+2w^2+u^4 \pm g\sqrt{4g^2+4u^2[1+u^2\cos^2(2\theta)]\sin^2(2\theta)\sin^2\frac{\phi}{2}+u^4\sin^4(2\theta)\sin^2\phi}\right)$$

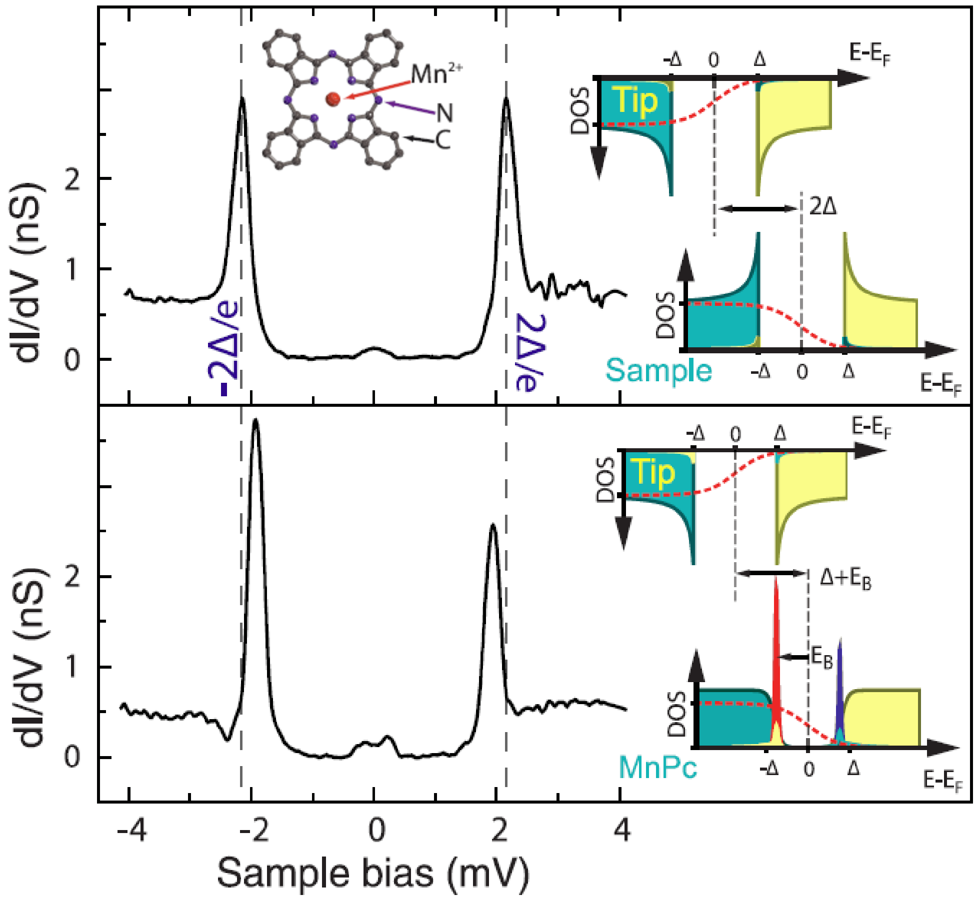


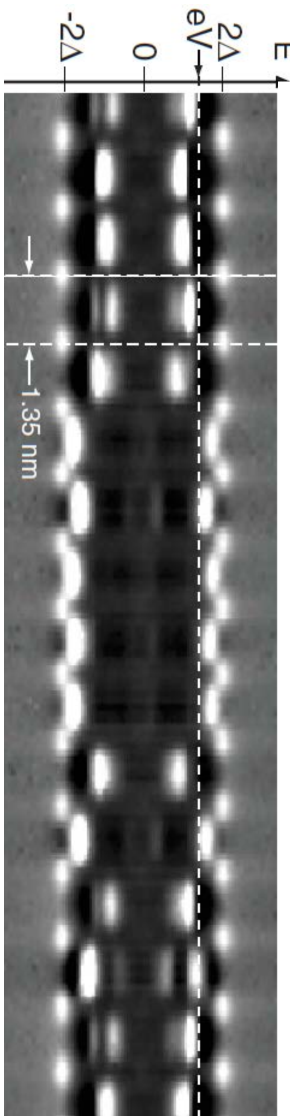


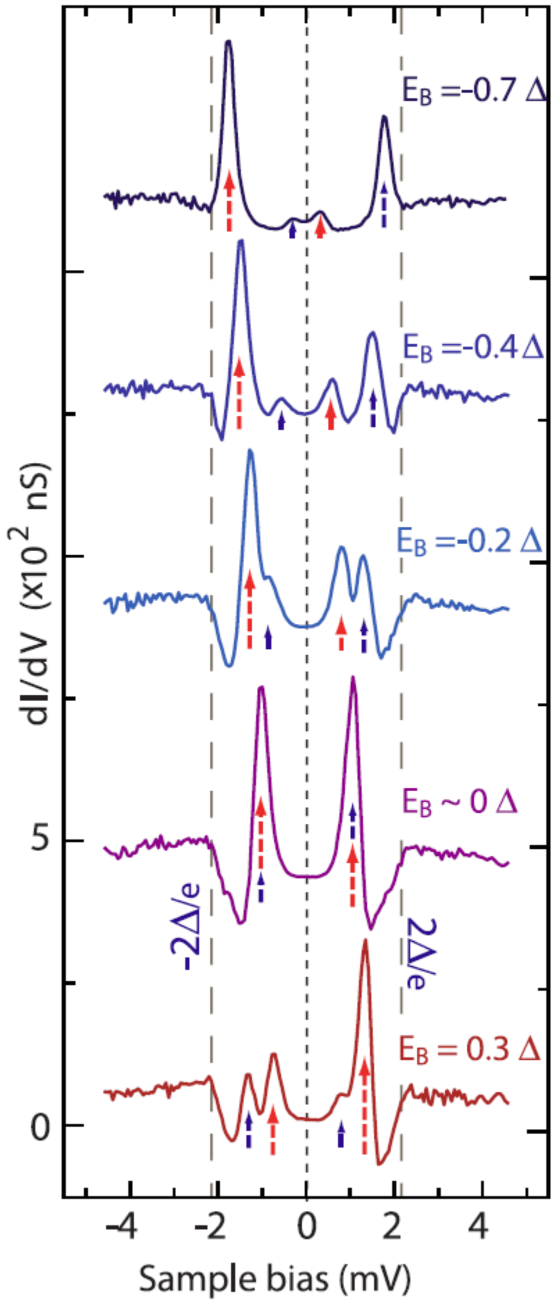
W. Chang et al., PRL **110**, 217005 (2013); G. Kirsanskas et al., PRB **92**, 235422 (2015)

Pb-Mn*Pc*-Pb (STM, 2D spin-lattice)

K. J. Franke et al., Science **332**, 940 (2011)

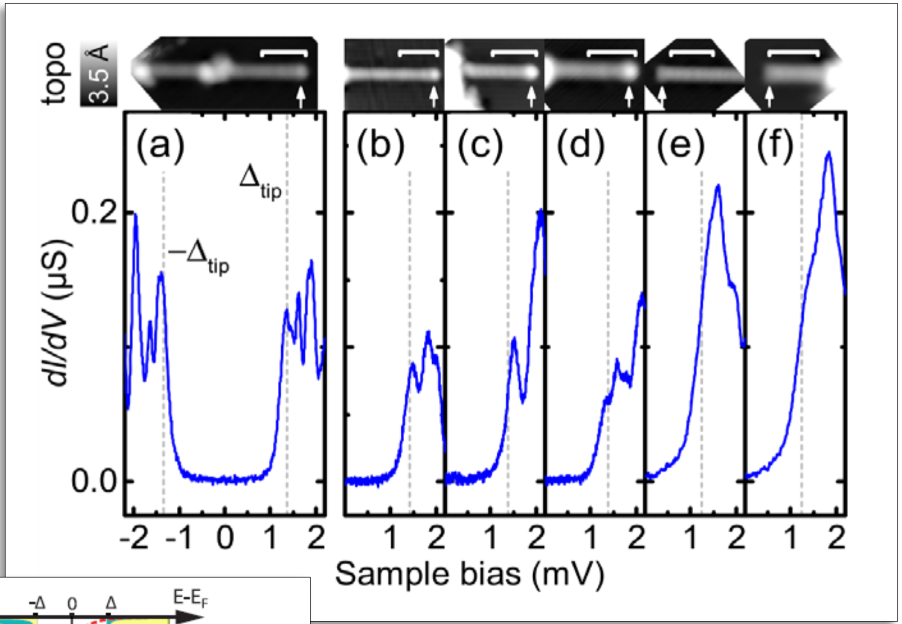


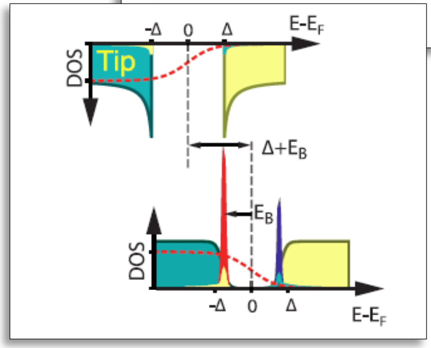




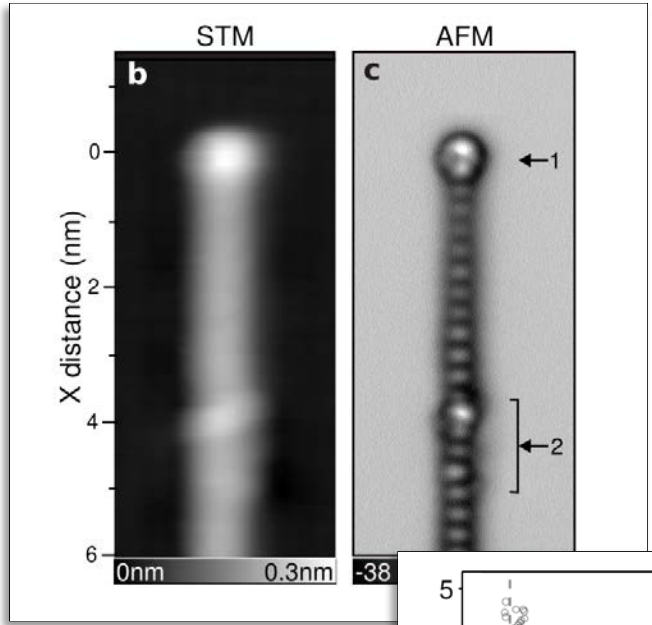
More zero-energy sub-gap states from **Fe** chains on **Pb**(110)

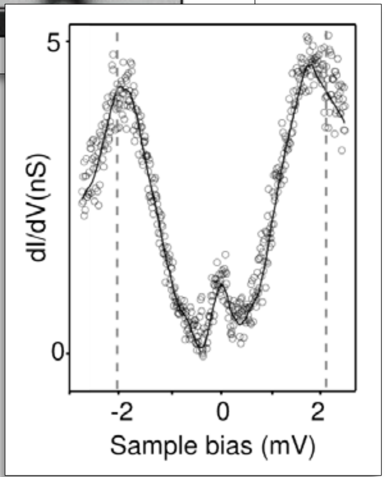
Ruby et al., PRL **115**, 197204 (2015)



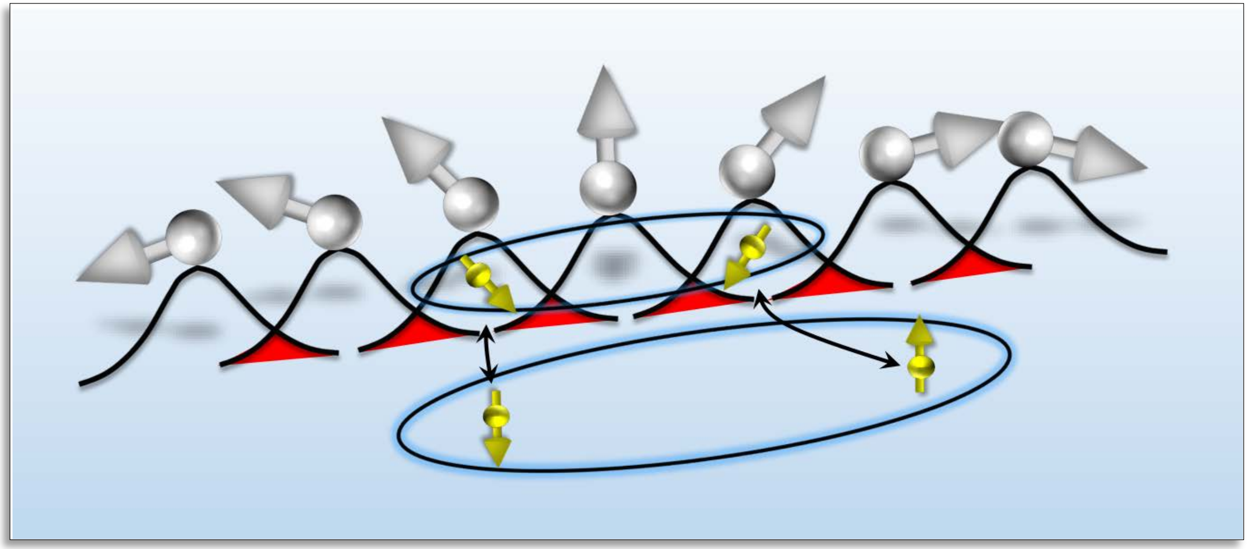


Pawlak et al., arXiv:1505.06078





The 1D classical spin chain in a 3D superconductor



$$H = \frac{1}{2}\sum_{\mathbf{k}} \Psi^\dagger_{\mathbf{k}} \left(\xi_{\mathbf{k}}\tau_z + \Delta\tau_x\right) \Psi_{\mathbf{k}} + \frac{1}{2} J \int_{\mathbf{r}} \Psi^\dagger_{\mathbf{r}} \vec{S}_{\mathbf{r}} \cdot \boldsymbol{\sigma} \Psi_{\mathbf{r}}$$

$$\xi(\mathbf{k}) = \frac{\mathbf{k}^2 - k_F^2}{2m}$$

$$\Psi = (\psi_\uparrow, \psi_\downarrow, \psi^\dagger_\downarrow, -\psi^\dagger_\uparrow)^{\mathrm{T}}$$

$$\vec{S}_r = S\sum_j \delta(r - ja)\,(\cos qr, \sin qr, 0)$$

The calculation

$$\mathcal{G}(ik_n, 0) = -\frac{\pi\nu_F}{2}\frac{ik_n\tau_0 + \Delta\tau_x}{\sqrt{k_n^2+\Delta^2}}\sigma_0$$

$$\mathcal{T}(ik_n) = J\sigma_x\left(1 + \frac{\pi J\nu_F}{2}\frac{ik_n\tau_0 + \Delta\tau_x}{\sqrt{k_n^2+\Delta^2}}\sigma_x\right)^{-1}$$

(YSR's as poles in T-matrix)

$$\mathcal{G}(ik_n, r) = -\frac{\pi\nu_F}{2}\frac{e^{-r\sqrt{k_n^2+\Delta^2}/v_F}}{k_F r}\sigma_0\left[\frac{ik_n\tau_0 + \Delta\tau_x}{\sqrt{k_n^2+\Delta^2}}\sin(k_F r) + \tau_z\cos(k_F r)\right]$$

$$\tilde{\mathcal{G}}(i\omega, k) = \sum_{j\neq 0}\mathcal{G}(i\omega, ja)e^{-ikaj}$$

(Fourier series along transl. inv. chain)

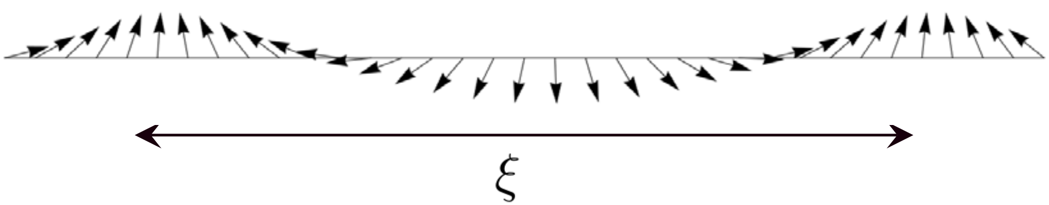
$$E(q) = -\frac{Na}{2}\int\frac{d\omega}{2\pi}\frac{dk}{2\pi}\mathrm{Tr}\ln\left[1 - \tilde{\mathcal{G}}(i\omega, k - \frac{1}{2}q\sigma_z)\mathcal{T}\right]$$

$$\approx \frac{1}{4}\sum_{i\neq j}\int\frac{d\omega}{2\pi}\mathrm{Tr}\left[\mathcal{T}_i\mathcal{G}_{0,ij}\mathcal{T}_j\mathcal{G}_{0,ji}\right]$$

(Effective Heisenberg exchange)

To 2nd order in J: Weak instability of FM

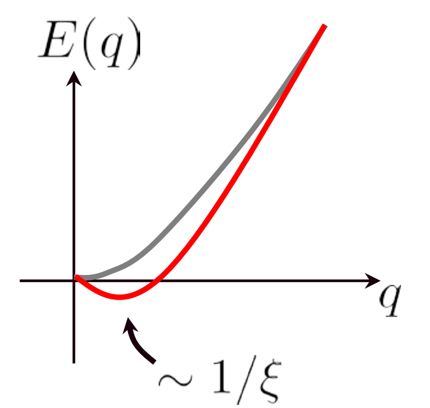
Anderson & Suhl, Phys. Rev. **116**, 898–900 (1959)
Abrikosov, “Fundamentals of the theory of metals”, (1988)
Aristov et al., Z. Phys. B **102**, 467–471 (1997)



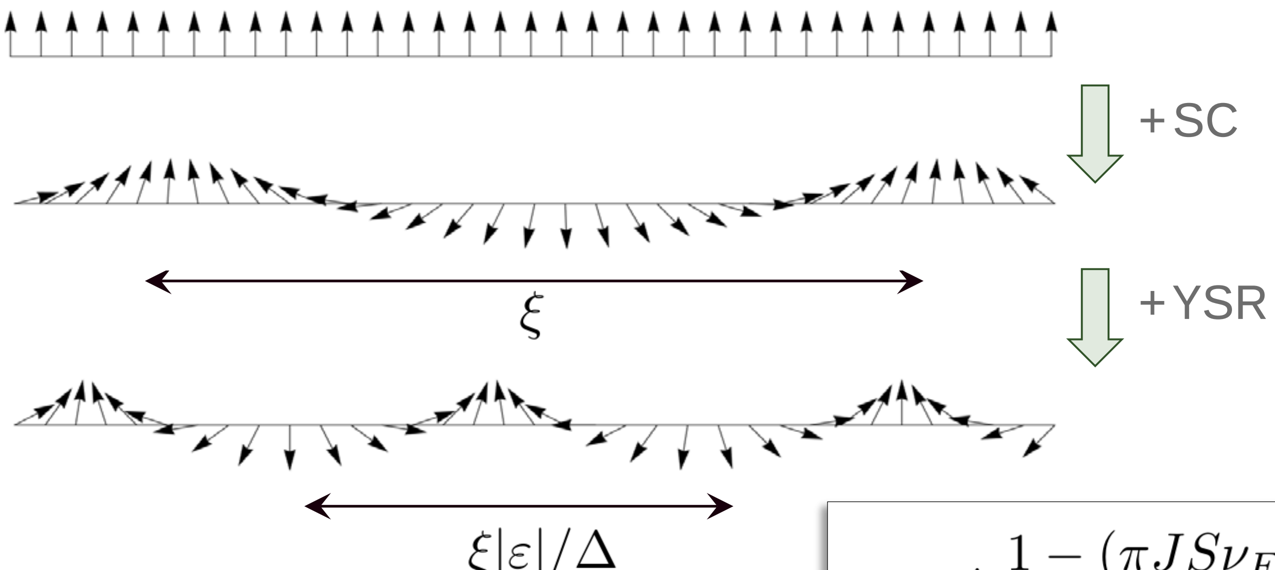
BCS-modified RKKY-interaction:

$$v_F = \xi\Delta$$

$$I(r) \propto J^2 e^{-2r/\xi}\left[\frac{v_F}{2\pi r^3}\cos(2k_F r) + \frac{\Delta}{r^2}\sin^2(k_F r)\right]$$



To 2nd order in $\mathcal{T}$: More pronounced spiral order



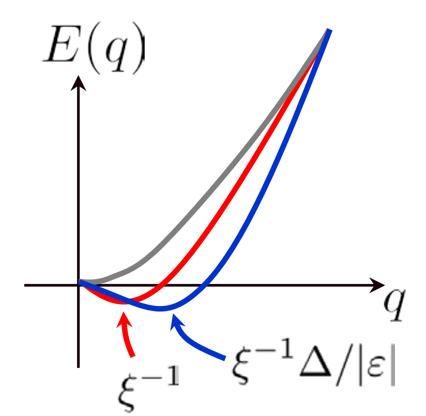
$$\varepsilon = \Delta \frac{1-(\pi J S \nu_F/2)^2}{1+(\pi J S \nu_F/2)^2}$$

Yao et al., PRL **113**, 087202 (2014)

YSR/BCS–modified interaction: $I(r) \propto J^2 e^{-2r/\xi}\left[\frac{v_F}{2\pi r^3}\cos(2k_F r) + \frac{\Delta}{r^2}\sin^2(k_F r)\right]$

$\varepsilon \to \Delta$

$$I(r) \propto \left(1-\frac{\varepsilon^2}{\Delta^2}\right) e^{-2r/\xi}\left[\frac{v_F}{2\pi r^3}\cos(2k_F r) + \frac{\Delta^2\cos^2(k_F r)}{2|\varepsilon| r^2} + \frac{|\varepsilon|}{4r^2}\left[1-3\cos(2k_F r)\right]\right]$$



YSR band-structure

YSR-band from 4 poles of Green fct.:

$$0 = \det\left[\tilde{\mathcal{G}}(\omega, k - q\sigma_z/2) - \mathcal{T}^{-1}(\omega)\right]$$
$$= \det\left[\tilde{B}\sigma_x + \frac{\omega + \Delta\tau_x}{\sqrt{\Delta^2 - \omega^2}}(\tilde{\Delta}_s + \tilde{\Delta}_t\sigma_z) + (\tilde{\xi} + \tilde{\alpha}\sigma_z)\tau_z\right]$$

$$\tilde{\xi} = \mathrm{Re}\, g_e(\omega, k) \qquad \tilde{\alpha} = \mathrm{Re}\, g_o(\omega, k)$$
$$\tilde{\Delta}_s = 1 + \mathrm{Im}\, g_e(\omega, k) \qquad \tilde{\Delta}_t = \mathrm{Im}\, g_o(\omega, k)$$

$$\tilde{B} = (\pi J \nu_F/2)^{-1} = \sqrt{(\varepsilon + \Delta)/(\varepsilon - \Delta)}$$

$$g_{e/o}(\omega, k) = -\frac{1}{2k_F a}\Big[\ln(1 - e^{-(a/\xi)\sqrt{1-\omega^2/\Delta^2} + i(k_F + k - q/2)a}) + \ln(1 - e^{-(a/\xi)\sqrt{1-\omega^2/\Delta^2} + i(k_F - k + q/2)a}) \pm (k \to -k)\Big]$$

Compare to Y. Oreg et al., PRL **105**, 177002 (2010):

$$H = (\xi_k + \alpha k \sigma_z)\tau_z + \Delta\tau_x + B\sigma_x$$

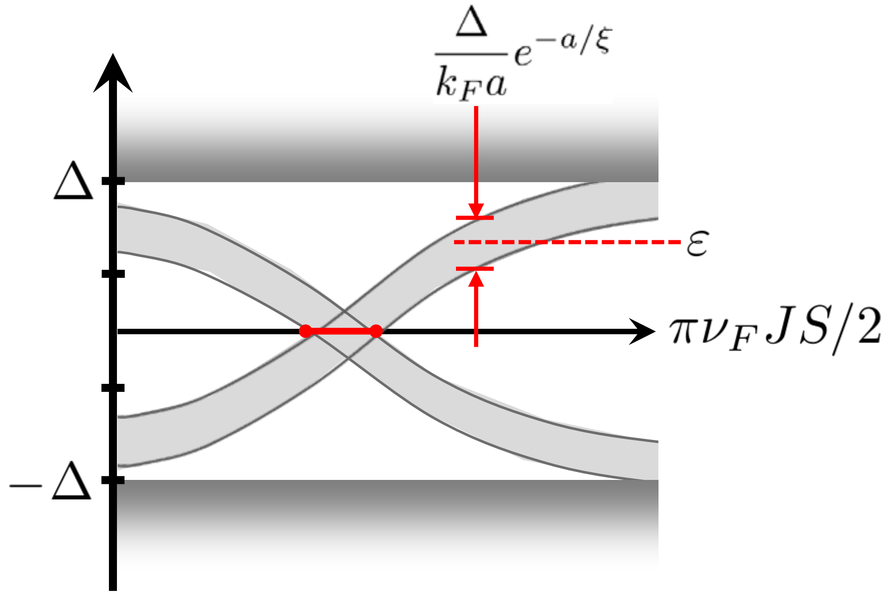
Limit of low-lying YSR states ($\varepsilon \ll \Delta$)

Expand to 2nd order in $\varepsilon/\Delta \ll 1 \ll k_F a$ gives the two lowest-lying bands:

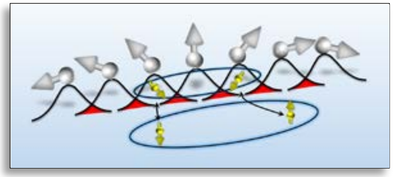
$$E_k = \sqrt{(h_k - \varepsilon)^2 + \Delta_k^2}$$

$$h_k = \Delta \operatorname{Im} g_e(0,k)$$
$$\Delta_k = -\Delta \operatorname{Re} g_o(0,k)$$

(Pientka et al. PRB **88** 155420 (2013))



Limit of low-lying YSR states ($|\varepsilon| < \Delta/(k_F a)$)



YSR contribution to total energy: $E_{\text{YSR}} = -\frac{1}{2}\sum_k E_k$ (Minimize, varying q)

Two competing effects:

I. *Double-exchange* (FM): Maximize kinetic energy by aligning spins, gapless

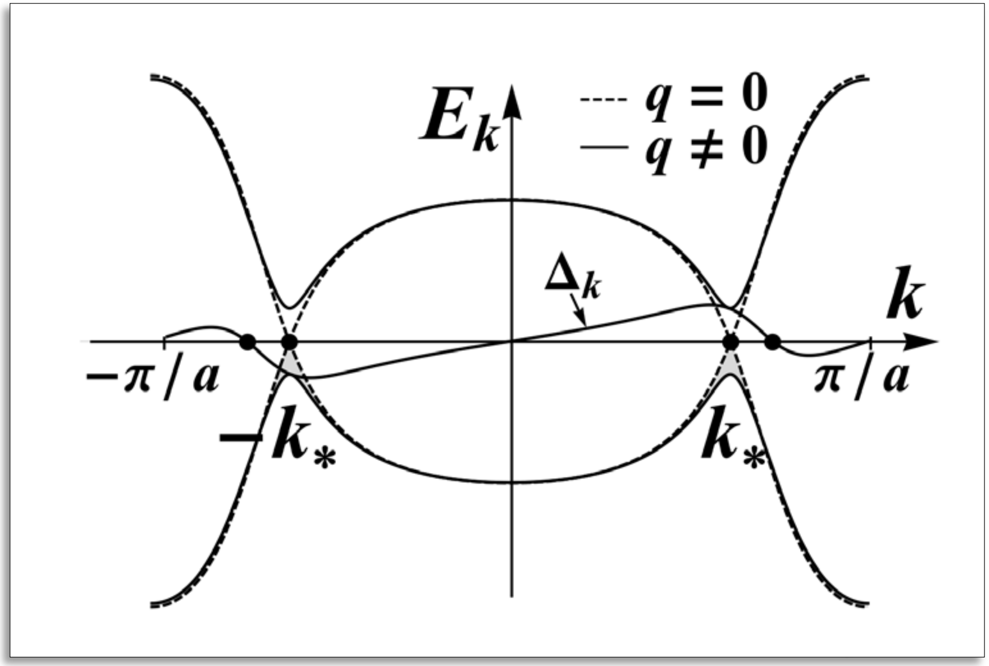
II. *Cooper-pair tunneling* (spiral): Needs non-aligned spins, gap $\propto q$

I. (FM) only wins at critical line:

$$\varepsilon_c = \frac{\Delta}{k_F a}\left[\pi/2 - (k_F a \bmod \pi)\right]$$

II. (Spiral) wins elsewhere with:

$$\frac{qa}{\pi} = \left|\frac{\Delta/k_F a}{\varepsilon - \varepsilon_c}\right| e^{-\left(\frac{\Delta/k_F a}{\varepsilon - \varepsilon_c}\right)^2}$$



Topological YSR superconductor ($|\varepsilon| < \Delta/(k_F a)$)

Topological Hamiltonian:
$$H_t(k) = \tilde{\mathcal{G}}(0, k - \frac{1}{2}q\sigma_z) - \mathcal{T}^{-1}(0)$$
$$= \tilde{B}\sigma_x + \tilde{\xi}\tau_z + \tilde{\alpha}\sigma_z\tau_z + \tilde{\Delta}_s\tau_x + \tilde{\Delta}_t\sigma_z\tau_x$$

Symmetries:

PH	$\Xi H_t(k)\Xi^{-1} = -H_t(-k)$	$\Xi = \sigma_y\tau_y\mathcal{K}$	$\Xi^2 = 1$
T (hidden)	$\mathcal{O}H_t(k)\mathcal{O}^{-1} = H_t(-k)$	$\mathcal{O} = -i\sigma_x\mathcal{K}$	$\mathcal{O}^2 = 1$
CHIRAL	$\{\mathcal{C}, H_t(k)\} = 0$	$\mathcal{C} = \mathcal{O}\Xi = \sigma_z\tau_y$	$\mathcal{C}^2 = 1$

→ Class **BDI**, with $\mathbb{Z}$-invariant $W = \frac{1}{2\pi i}\oint_{BZ} dk\partial_k \ln z(k)$

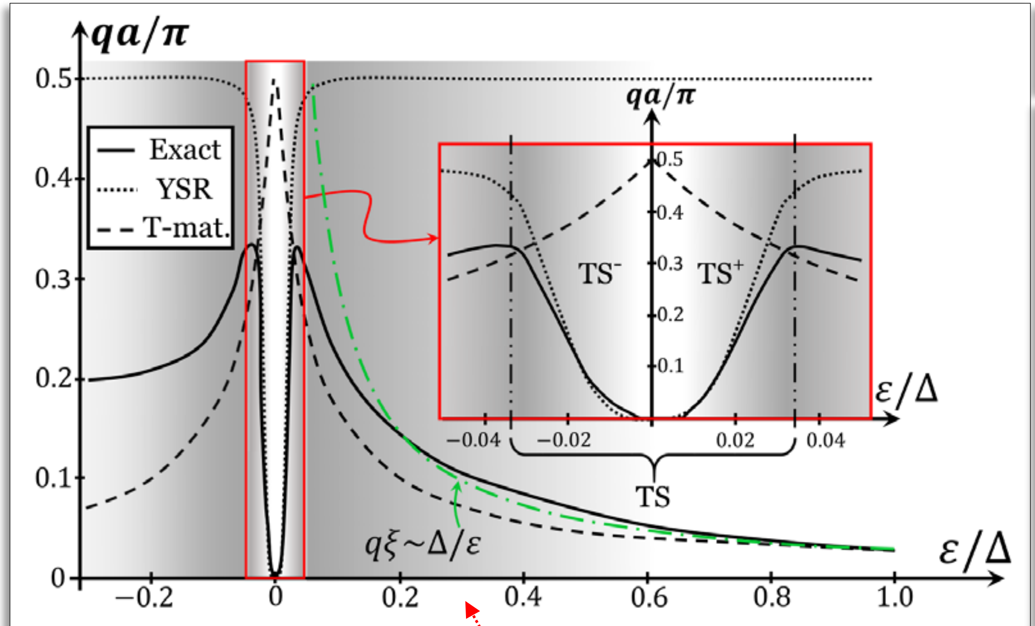
Gap closings at TRI momenta:

$$\frac{\varepsilon^{\pm}}{\Delta} \approx -(k_F a \bmod \mp\pi)/k_F a$$

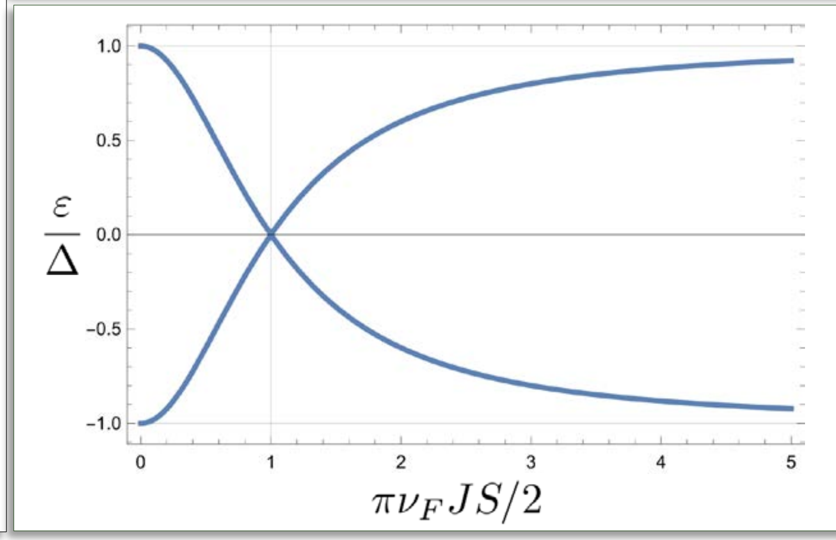
$$z(k) = \det A/|\det A|$$

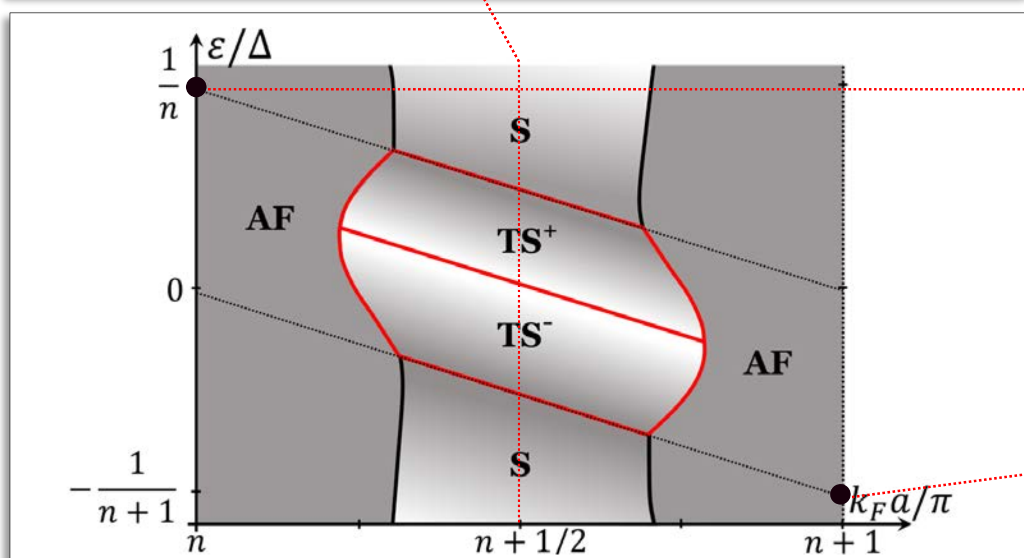
$$U^{\dagger}H_tU = \begin{pmatrix} 0 & A \\ A^{\dagger} & 0 \end{pmatrix}$$

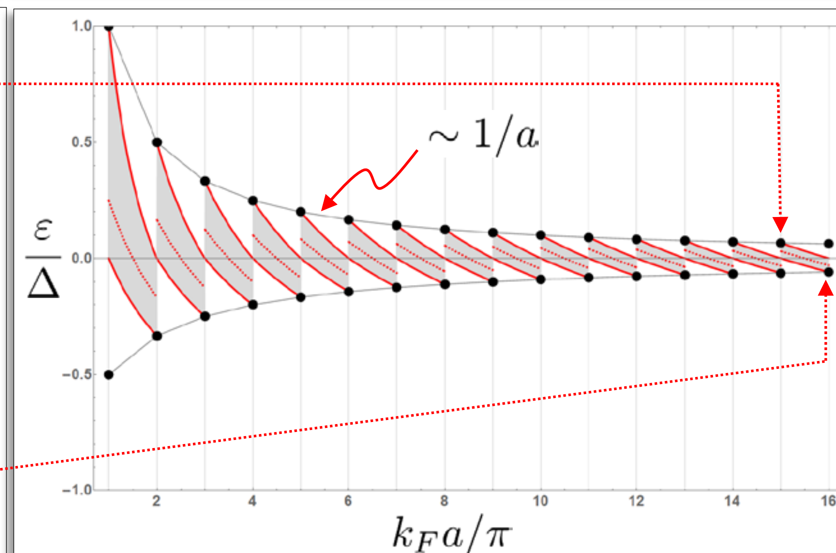
Summarizing



$\Delta = 1 \qquad \xi = 2000 k_F^{-1} \qquad a = 15.5\pi k_F^{-1}$







Experimental considerations

Optimizing the topological gap:

$$\Delta_{top} \sim \Delta/50 \sim 200\mathrm{mK}$$

$$\Delta_{top} \sim qa\frac{\Delta}{k_F a} \lesssim \frac{\Delta}{k_F a}$$ max for $qa \sim 1$

$$q \sim \xi^{-1}\frac{\Delta}{\varepsilon} \lesssim \frac{k_F a}{\xi}$$ ⇒ $$k_F a \sim \sqrt{k_F \xi}$$ ⇒

Broad max at

$$\xi \sim 2000 k_F^{-1}$$
$$a \sim 14.5\pi k_F^{-1}$$

Q&A:

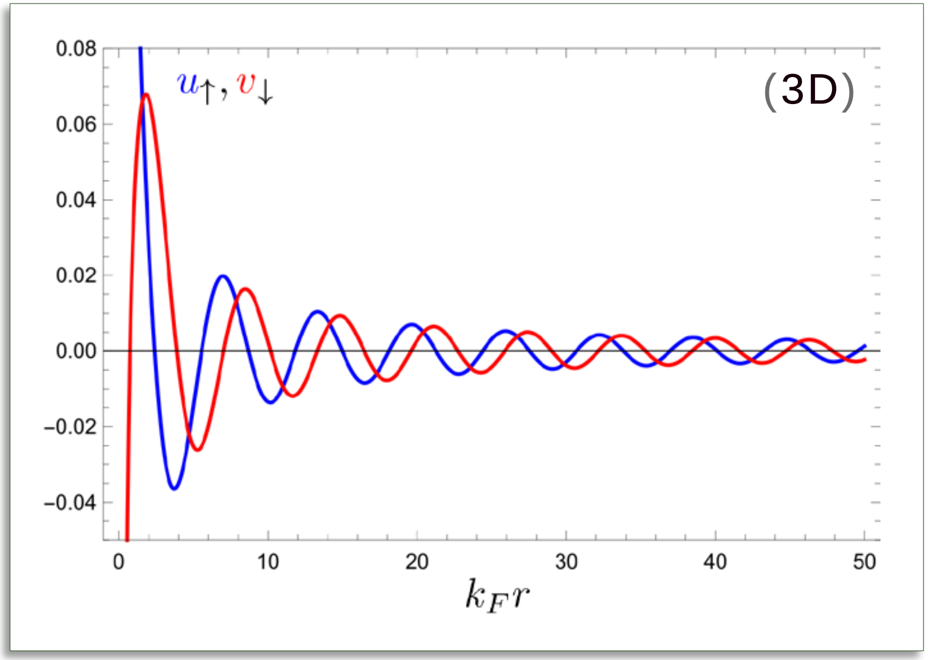
1. Magnetic anisotropy: $H_{\mathrm{D}} = -D\sum_{j=1}^{N}\left(S_j^z\right)^2$, harmless for $D < 0$
2. Spin-orbit coupling: 'Gauges away into the spiral'
3. Self-consistent pairing potential: Not relevant for $k_F \gg 1/a, 1/\xi$
4. Fluctuations: Frozen out for $T \lesssim \max\left\{D, \frac{S}{\ln N}\frac{\Delta}{k_F a}\right\}$
5. Quantum spins: From YSR, to Kondo lattice only when $\Delta \lesssim T_K$

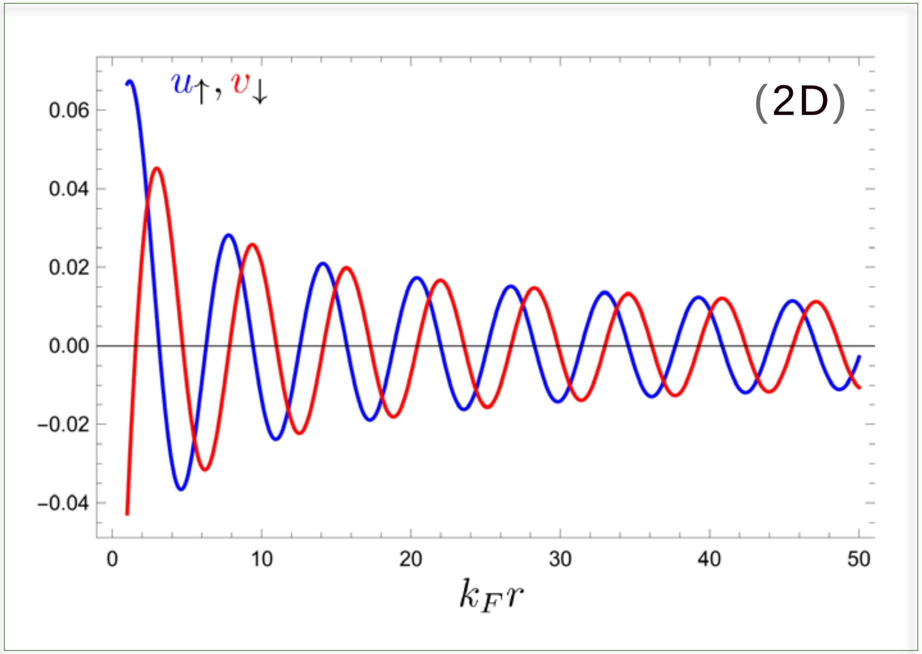
1D spin chain in 2D superconductor

Eigenfunctions (3D): $$\begin{pmatrix} u_\uparrow(\mathbf{r}) \\ v_\downarrow(\mathbf{r}) \end{pmatrix} = \frac{1}{\sqrt{c}} \frac{\sin(k_F r + \delta^\pm)}{k_F r} e^{-r/(\xi/|\sin(\delta^+ - \delta^-)|)}$$

Eigenfunctions (2D): $$\begin{pmatrix} u_\uparrow(\mathbf{r}) \\ v_\downarrow(\mathbf{r}) \end{pmatrix} = \frac{1}{\sqrt{c}} \frac{\sin(k_F r - \frac{\pi}{4} + \delta^\pm)}{\sqrt{\pi k_F r}} e^{-r/(\xi/|\sin(\delta^+ - \delta^-)|)}$$

Larger YSR hybridization expected in 2D:





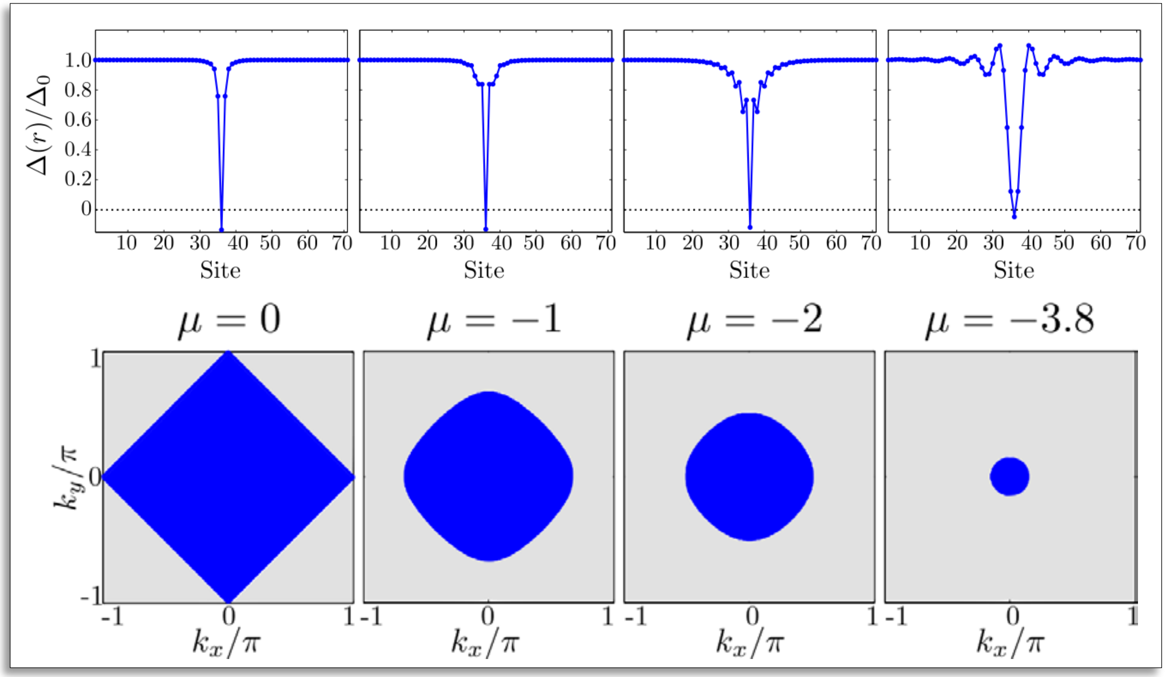
1D spin chain in 2D superconductor

$$H = -\sum_{\langle ij \rangle} (t_{ij} + \mu\delta_{ij})\, c^\dagger_{i\alpha} c_{j\alpha} - V \sum_i n_{i\uparrow} n_{i\downarrow} + J \sum_{\substack{i\in I \\ \alpha\beta}} \mathbf{S}_i \cdot c^\dagger_{i\alpha} \boldsymbol{\tau}_{\alpha\beta} c_{i\beta}$$

$$H^{\mathrm{MF}}_{\mathrm{SC}} = -\sum_i \left[\Delta_i c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} + h.c - \frac{|\Delta_i|^2}{V} \right]$$

$$\Delta_i = V \langle c_{i\downarrow} c_{i\uparrow} \rangle$$

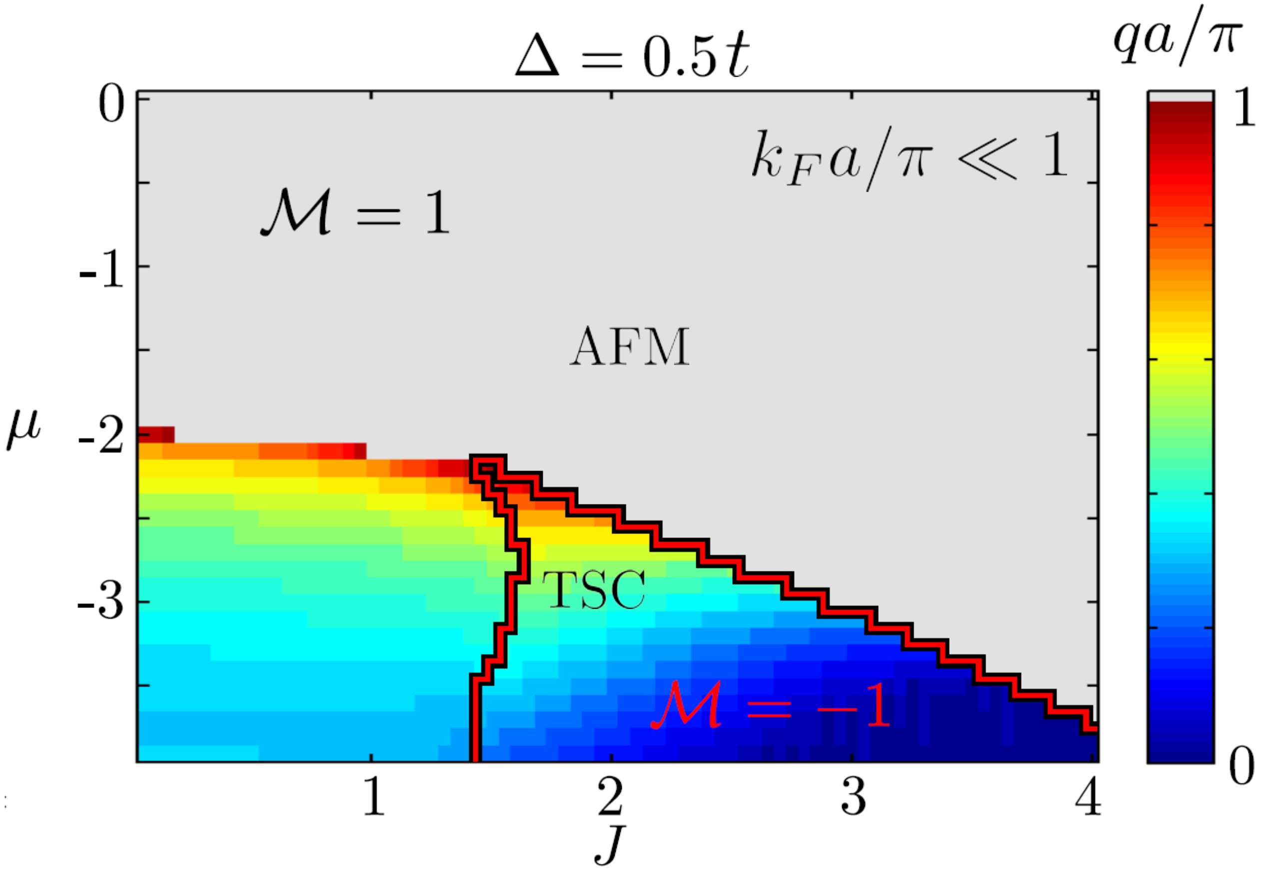
BCS mean-field decoupling

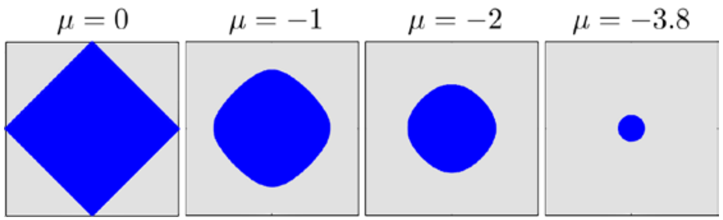


Local pairing Potential

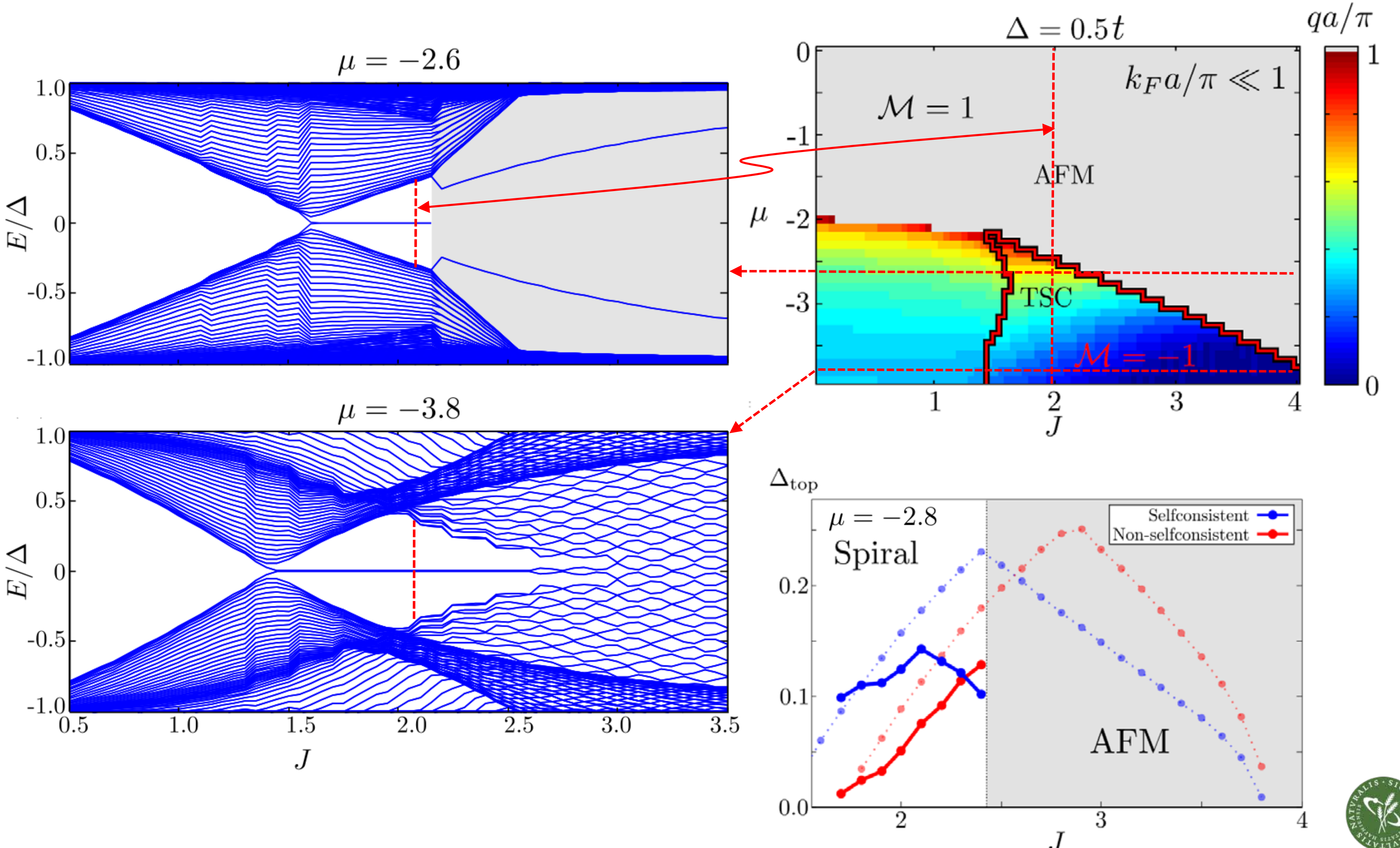
Evolution of Fermi surfaces with μ

1D spin chain in 2D superconductor: Phase-diagram

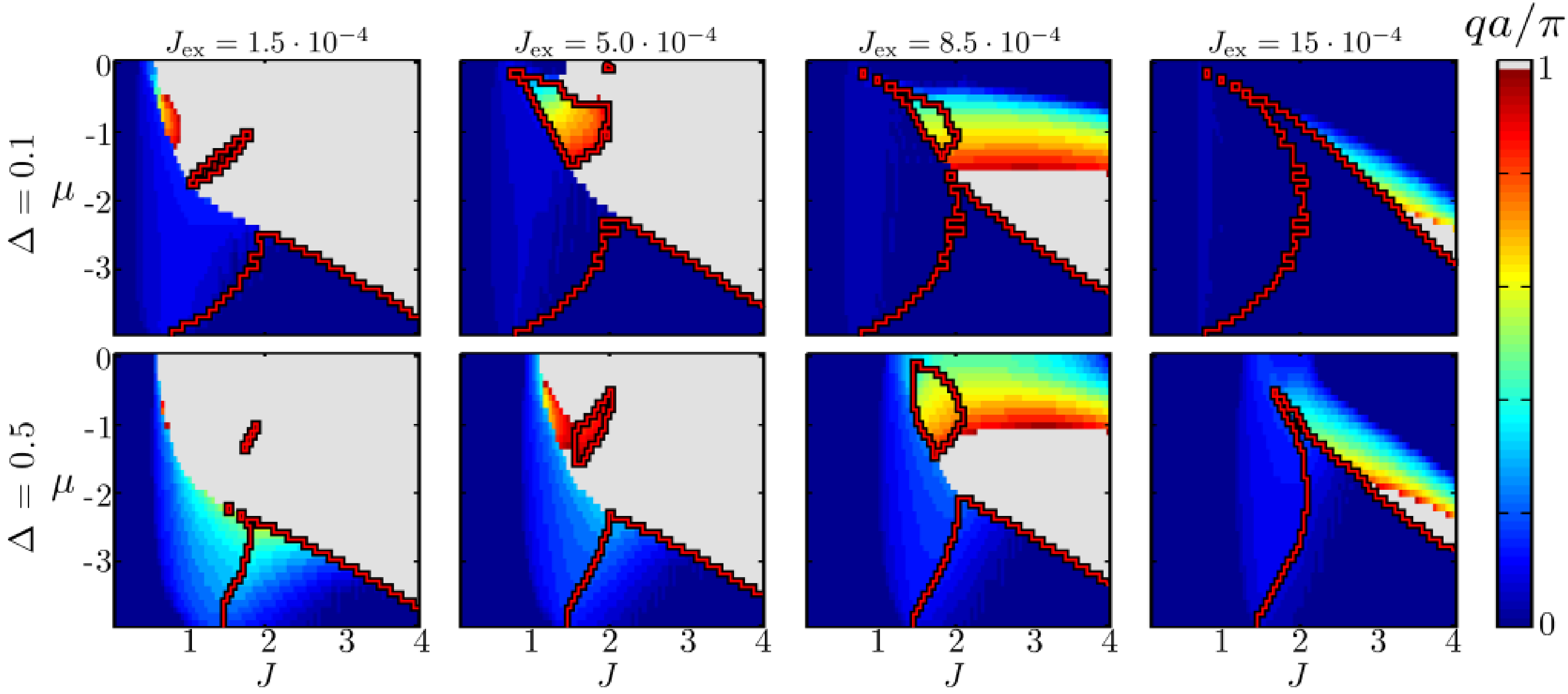




YSR bands and Majorana bound states



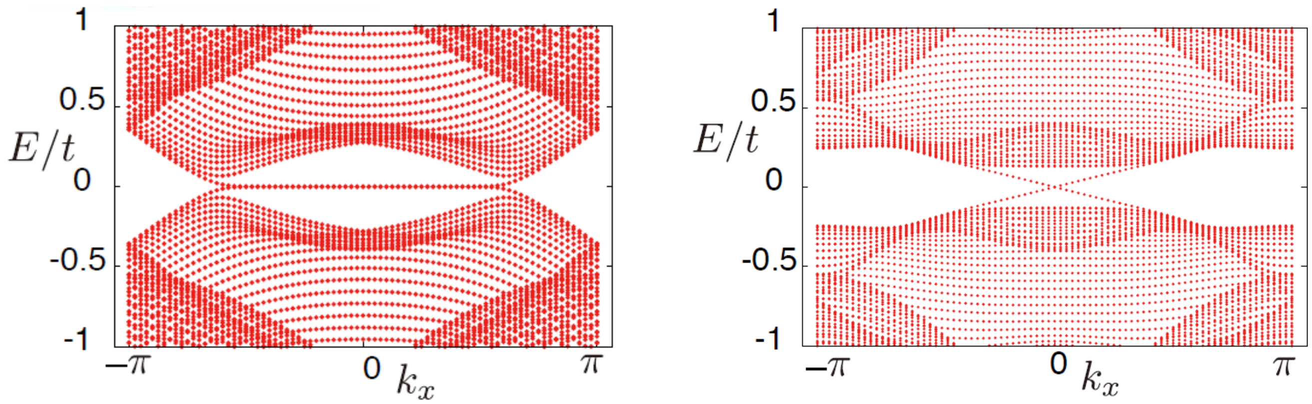
Including direct nearest neighbor FM exchange



2D spin lattice in 3D superconductor

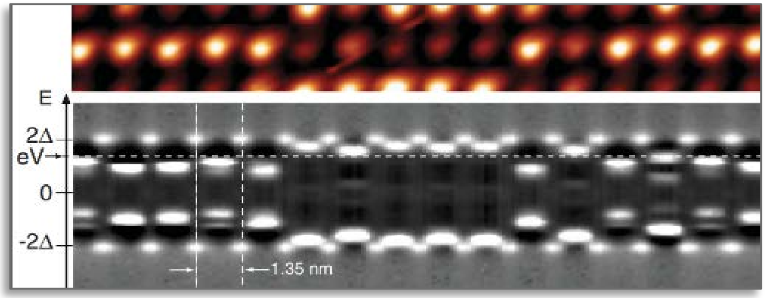
Noncollinear/noncoplanar magnetism may give rise to a 2D nodal/chiral p-wave YSR superconductor

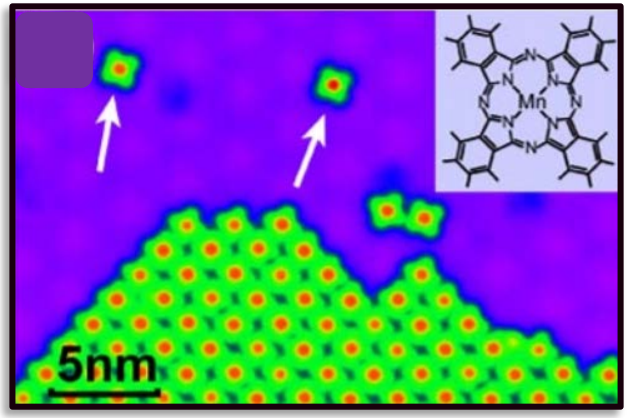
S. Nakosai, Y. Tanaka, N. Nagaosa, PRB **88**, 180503(R) (2013)



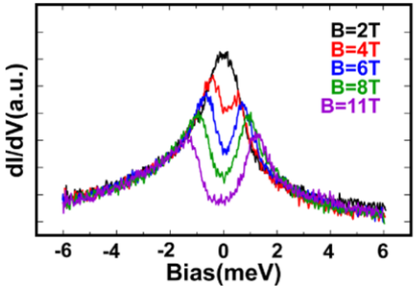
Square lattice of Mn-PC on Pb(111)

K. J. Franke et al., Science **332**, 940 (2011)





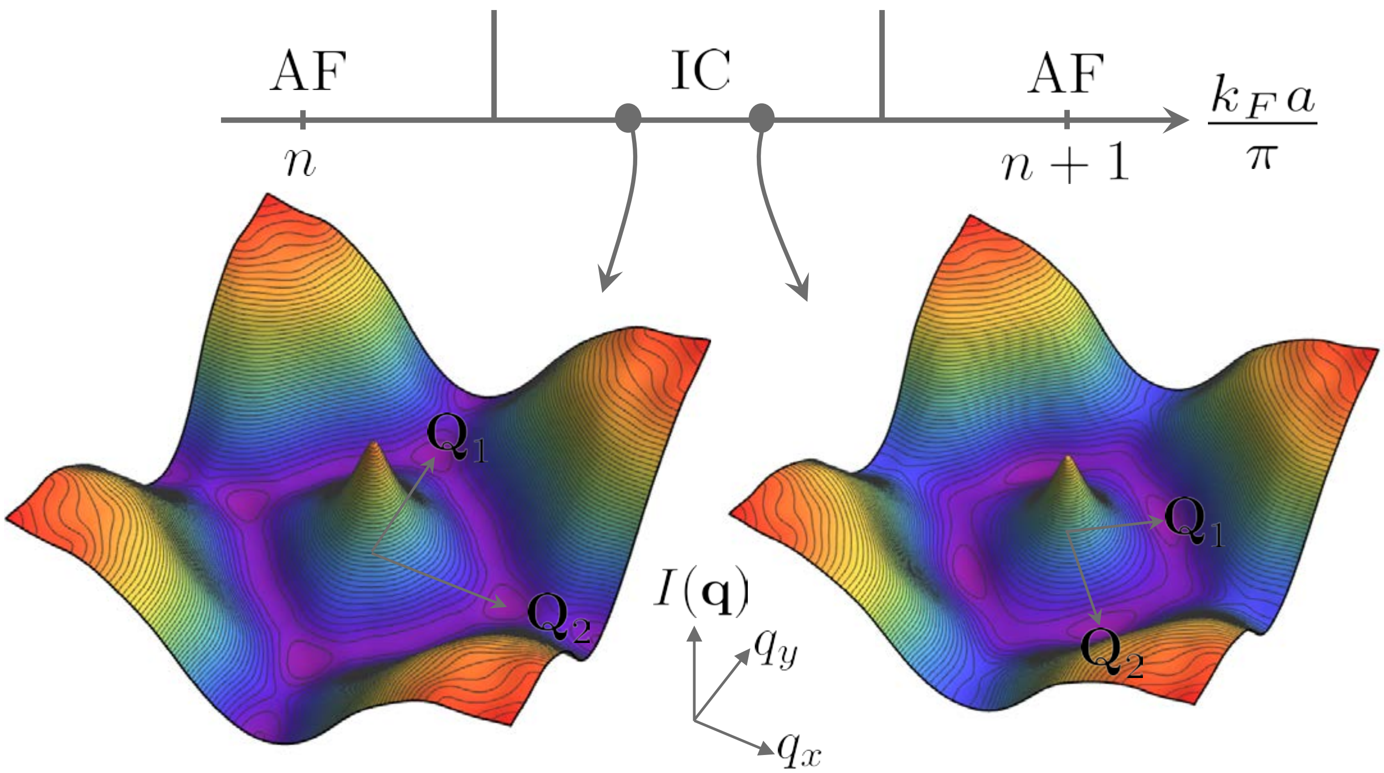
Y.S. Fu et al., PRL **99**, 256602 (2007)



Isotropic BCS+YSR-modified RKKY interaction (2D Fourier Transf.)

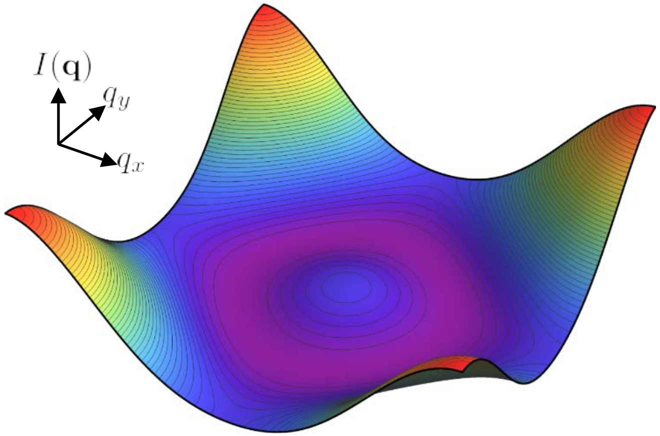
$$I(r) \propto \left(1-\frac{\varepsilon^2}{\Delta^2}\right)\frac{e^{-2r/\xi}}{(k_F r)^{d-1}}\left[\frac{\Delta^2+\varepsilon^2}{4|\varepsilon|}+\left(\frac{v_F}{2\pi r}+\frac{\Delta^2}{4|\varepsilon|}-\frac{3|\varepsilon|}{4}\right)\cos(2k_F r+(3-d)\pi/2)\right]$$

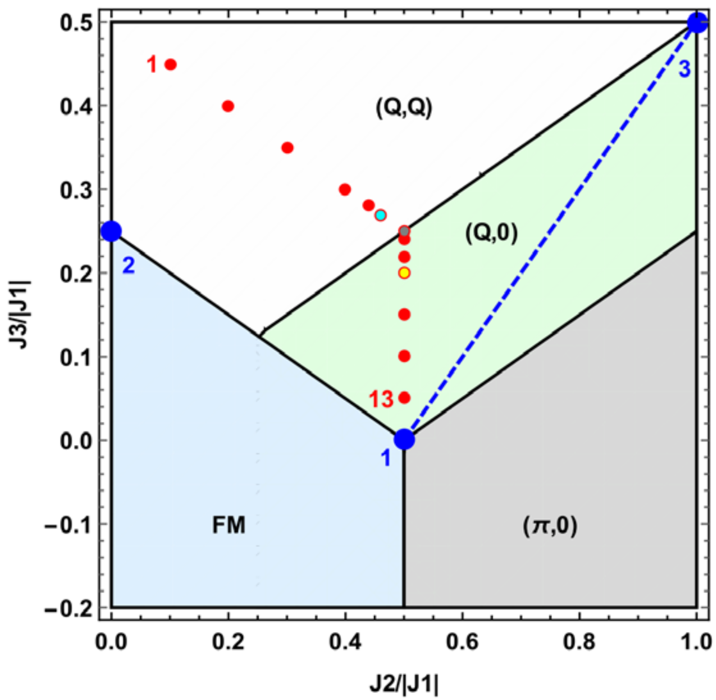
$$\mathcal{H}_{\text{Heis}} = \sum_{i,j} I_{(i-j)}\mathbf{S}_i\cdot\mathbf{S}_j = \sum_Q I_Q|\mathbf{S}_Q|^2$$

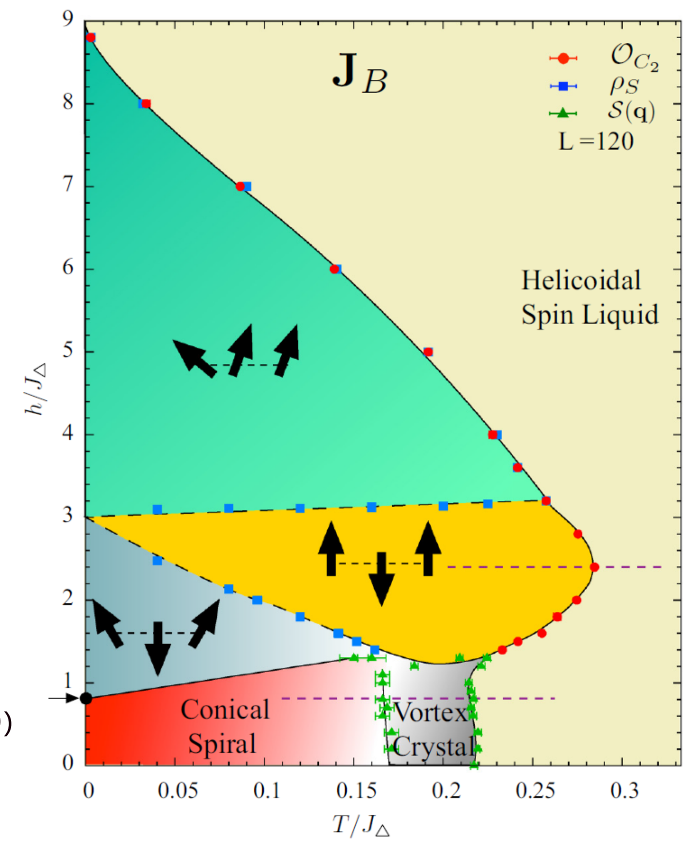


Ferromagnetic J_1-J_2-J_3 Toy Model

$$H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j + J_3 \sum_{\langle\langle\langle ij \rangle\rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$



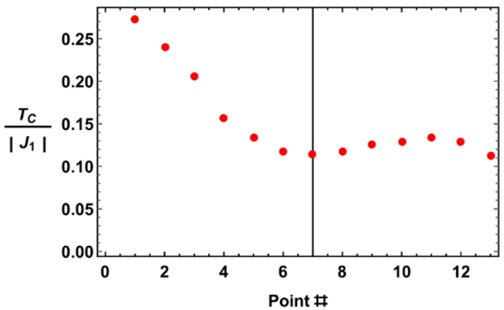


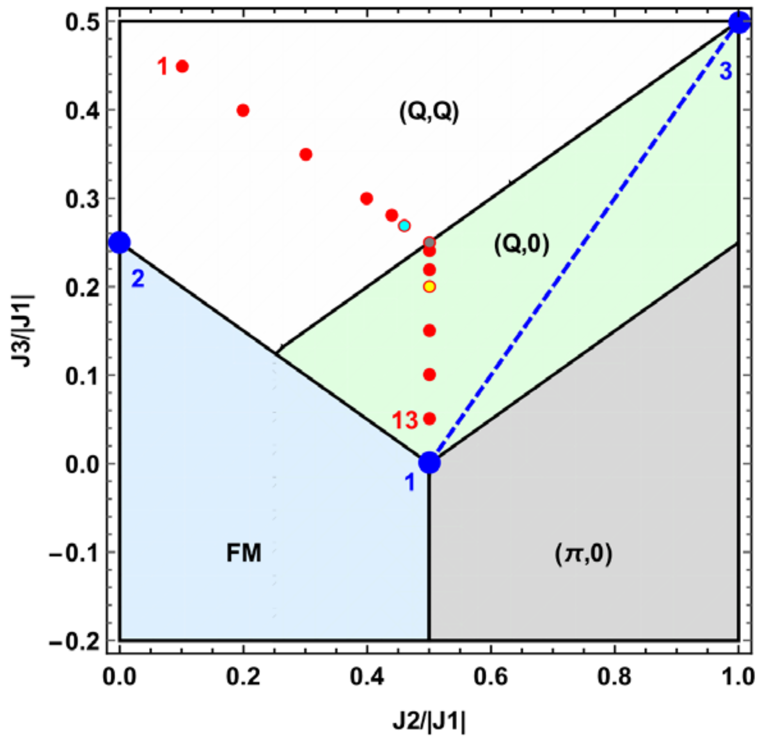


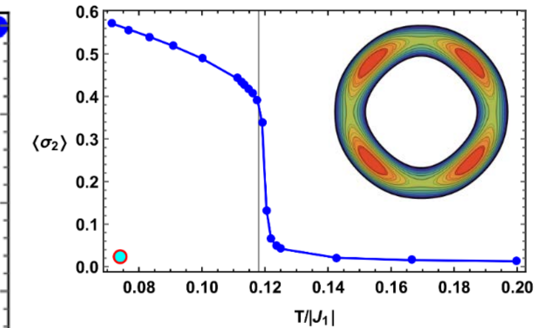
1) P. Chandra, P. Coleman, A.I. Larkin et al. Phys. Rev. Lett. **64**, 88 (1990)
2) L. Capriotti and S. Sachdev, Phys. Rev. Lett. **93**, 257206 (2004)
3) L. Seabra et al., Phys. Rev. B **93**, 085132 (2016)

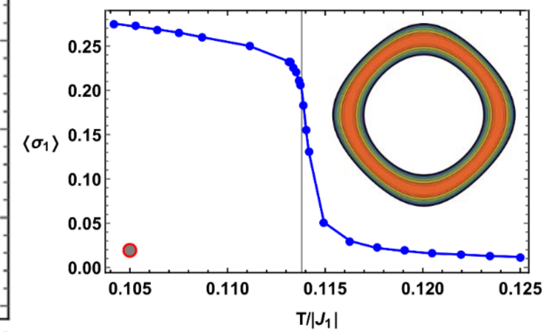
Finite-temperature $\mathbb{Z}_2$ (Ising) transition (Monte-Carlo)

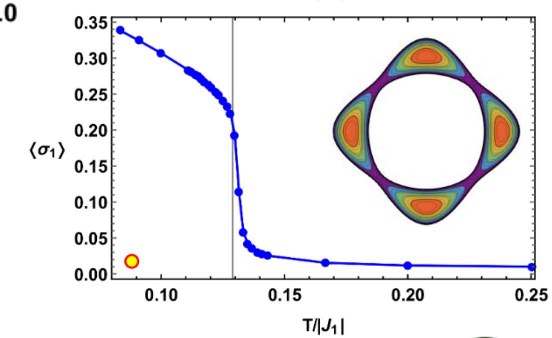
Critical temperatures:



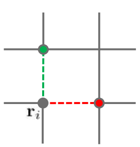




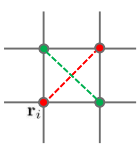




$\mathbb{C}_4$-breaking order-parameters:

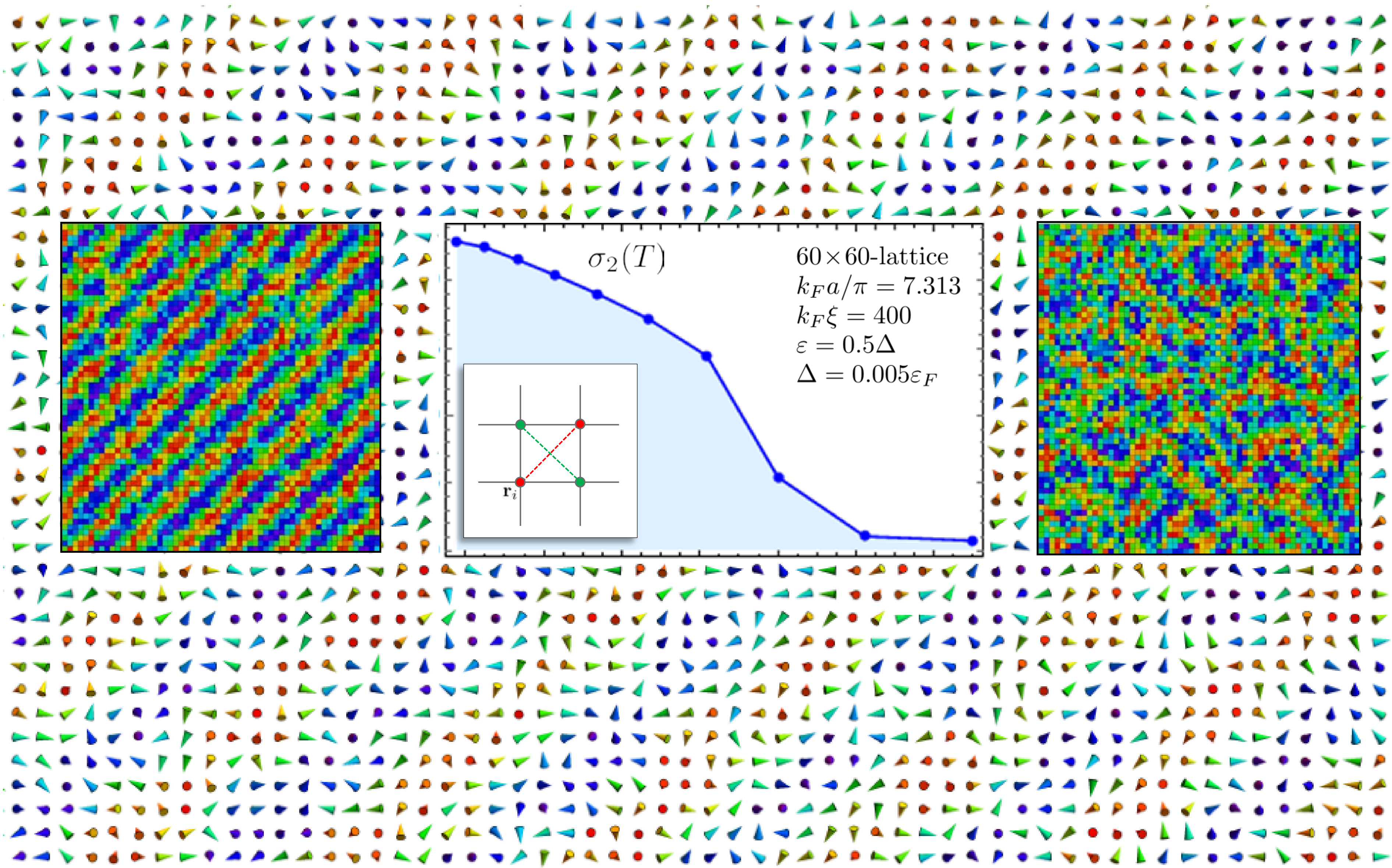


$$\sigma_1 = \frac{1}{N}\sum_i \left(\vec{S}_{\mathbf{r}_i}\cdot\vec{S}_{\mathbf{r}_i+(0,a)} - \vec{S}_{\mathbf{r}_i}\cdot\vec{S}_{\mathbf{r}_i+(0,a)}\right)$$

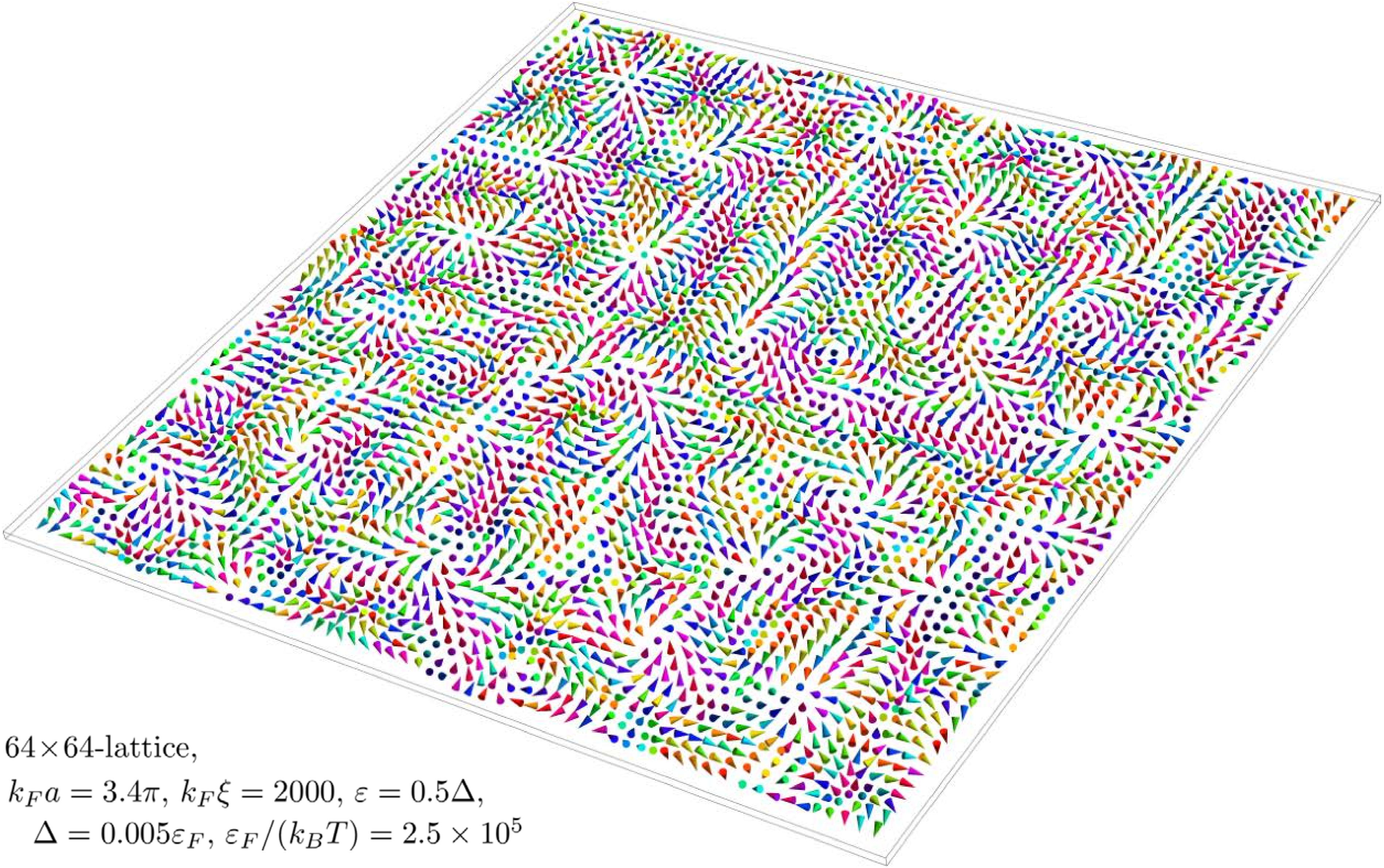


$$\sigma_2 = \frac{1}{N}\sum_i \left(\vec{S}_{\mathbf{r}_i}\cdot\vec{S}_{\mathbf{r}_i+(a,a)} - \vec{S}_{\mathbf{r}_i+(0,a)}\cdot\vec{S}_{\mathbf{r}_i+(a,0)}\right)$$

Finite-temperature $\mathbb{Z}_2$ (Ising?) transition (BCS+YSR) (Monte-Carlo)



Frustrated spin configuration (BCS+YSR) (Monte-Carlo)

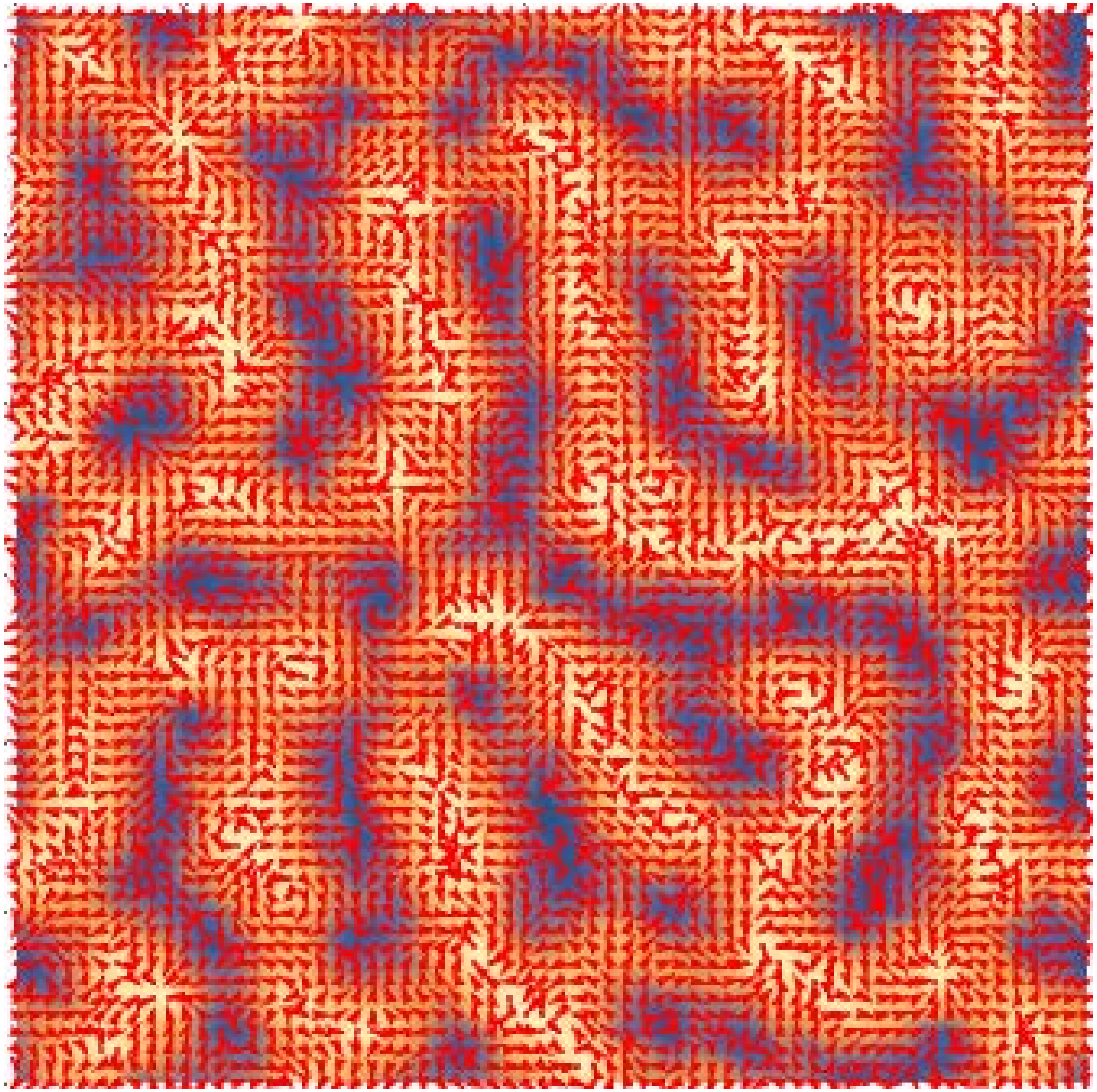


64×64-lattice,

$k_F a = 3.4\pi$, $k_F \xi = 2000$, $\varepsilon = 0.5\Delta$,

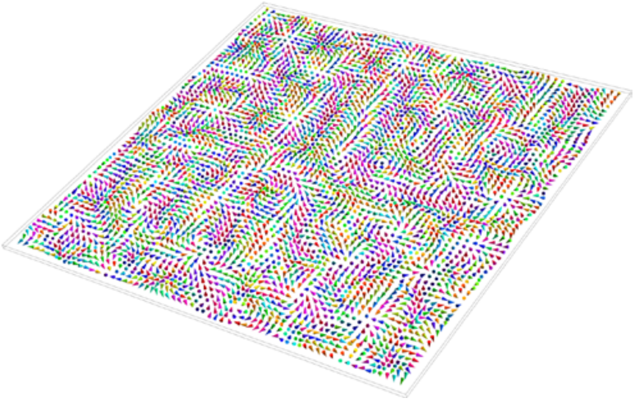
$\Delta = 0.005\varepsilon_F$, $\varepsilon_F/(k_B T) = 2.5 \times 10^5$

Frustrated spin configuration (BCS+YSR) (Monte-Carlo)



Z-axis projection (density)

XY-projection (red arrows)



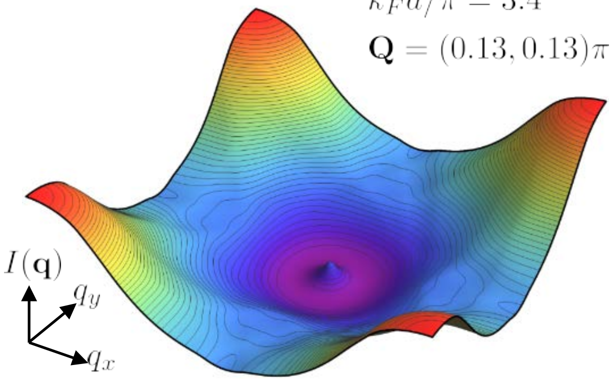
Energy-landscapes and spin configurations (BCS+YSR)

$\Delta = 0.005 E_F$

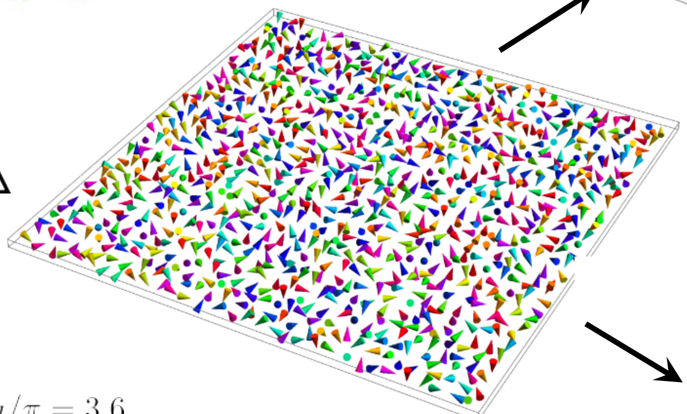
$k_F \xi = 400,\ \varepsilon = 0.5\Delta$

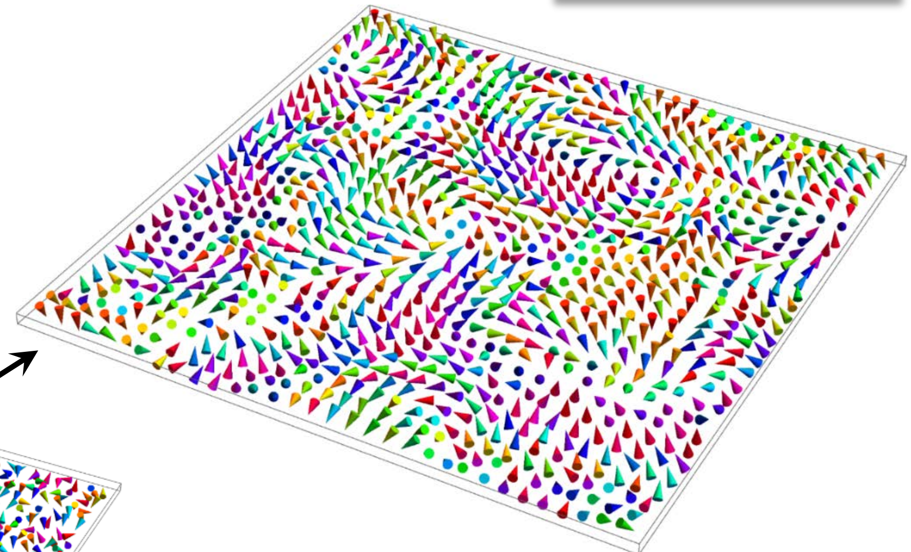
$k_F a/\pi = 3.4$

$\mathbf{Q} = (0.13, 0.13)\pi$



$T = 2\Delta$

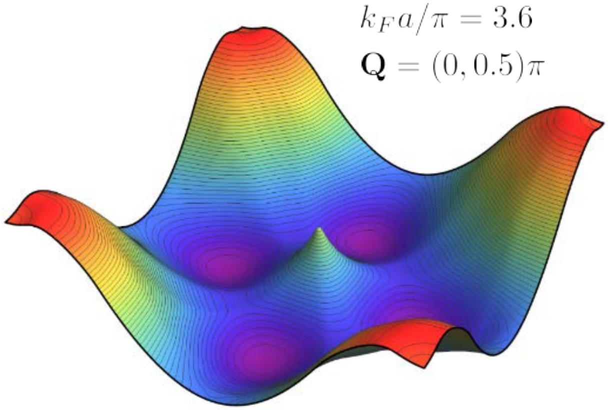


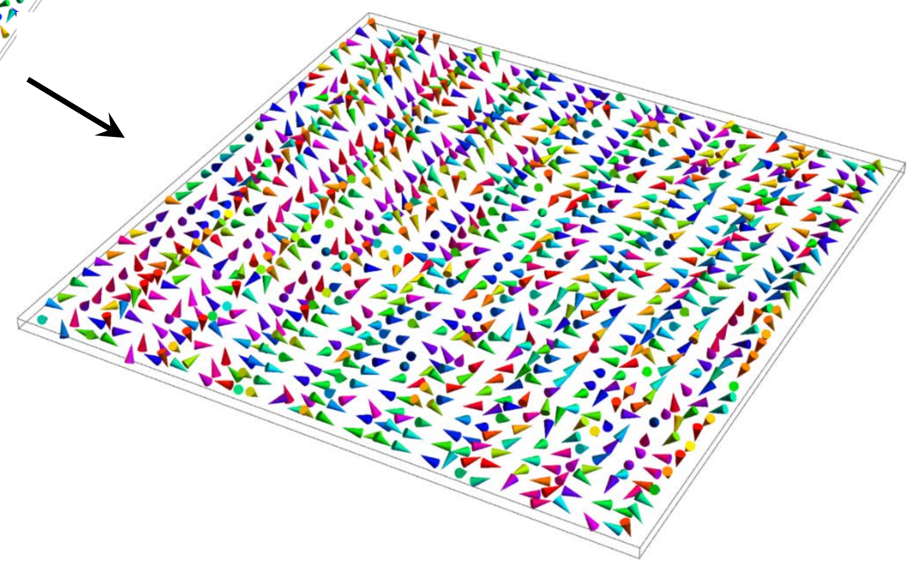


$T = 0.2\Delta$

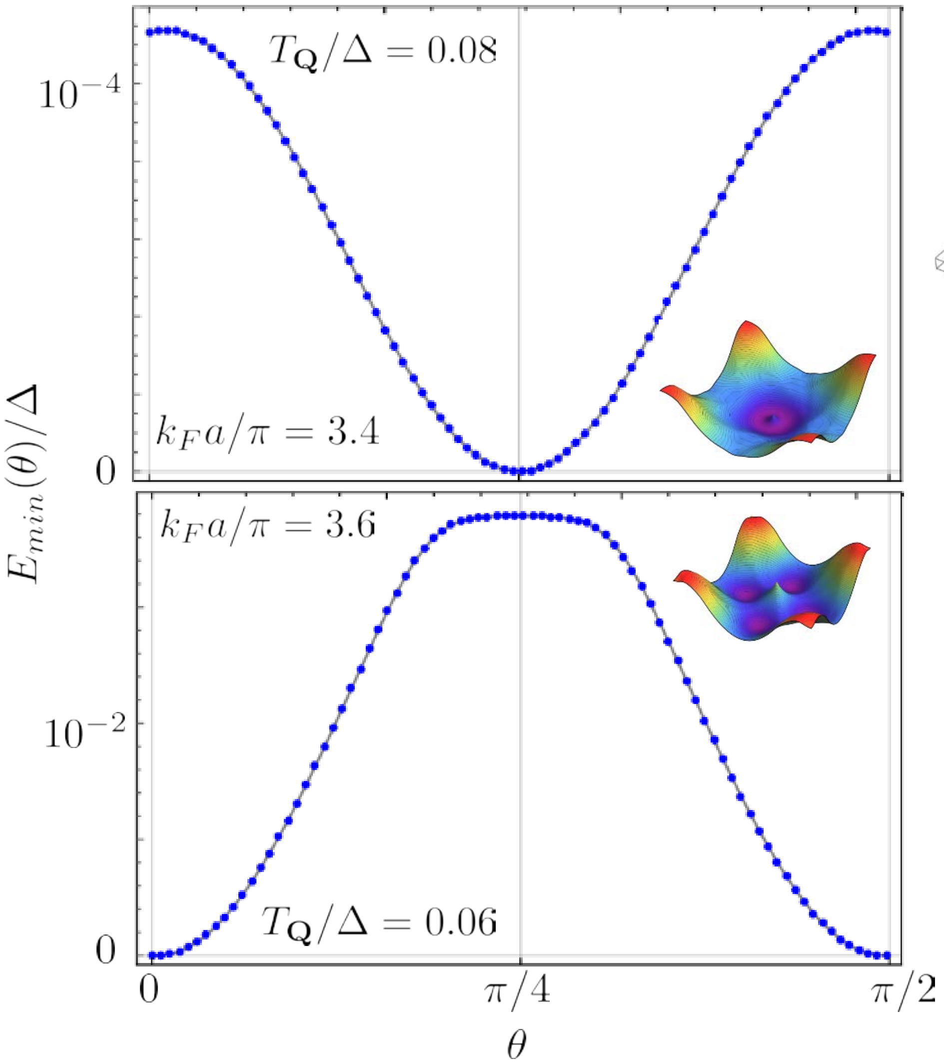
$k_F a/\pi = 3.6$

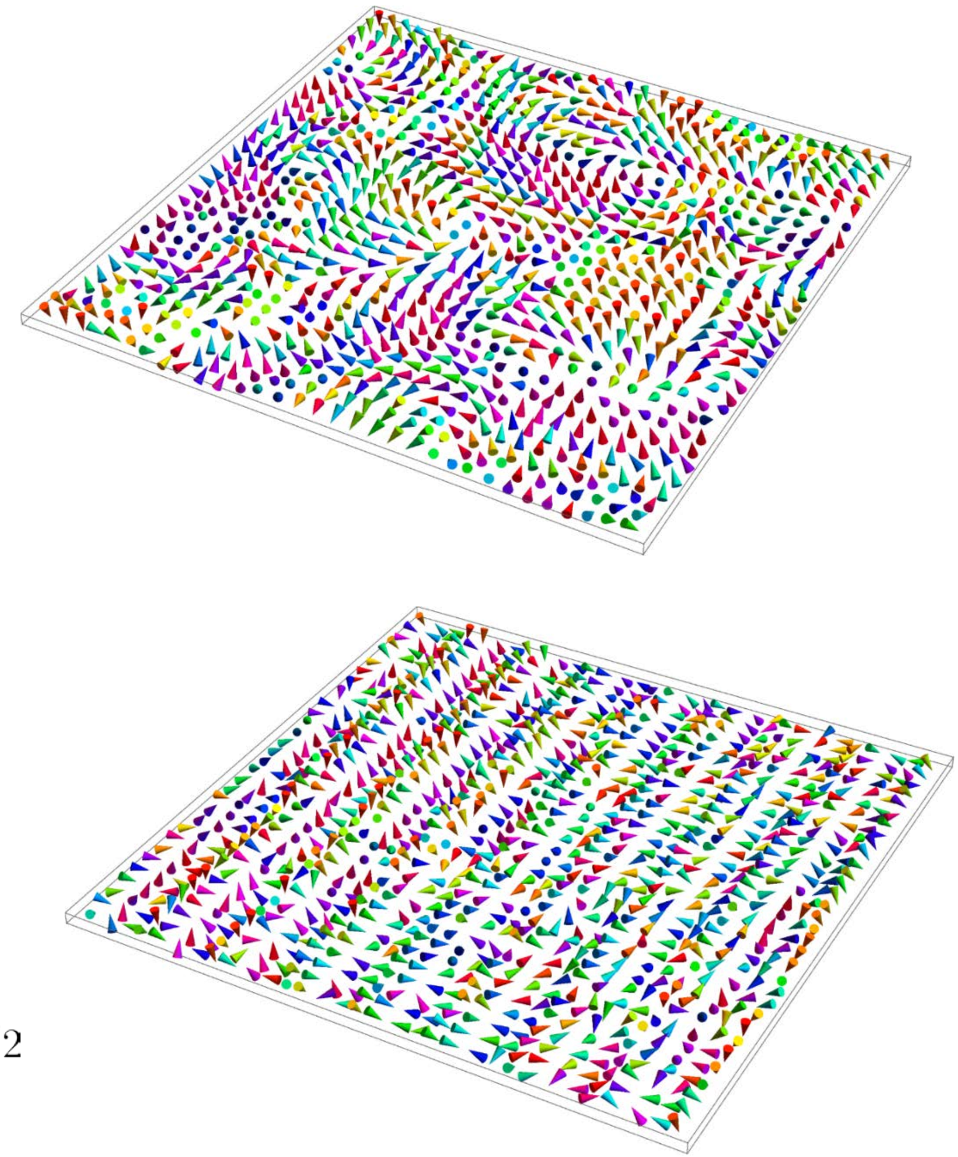
$\mathbf{Q} = (0, 0.5)\pi$



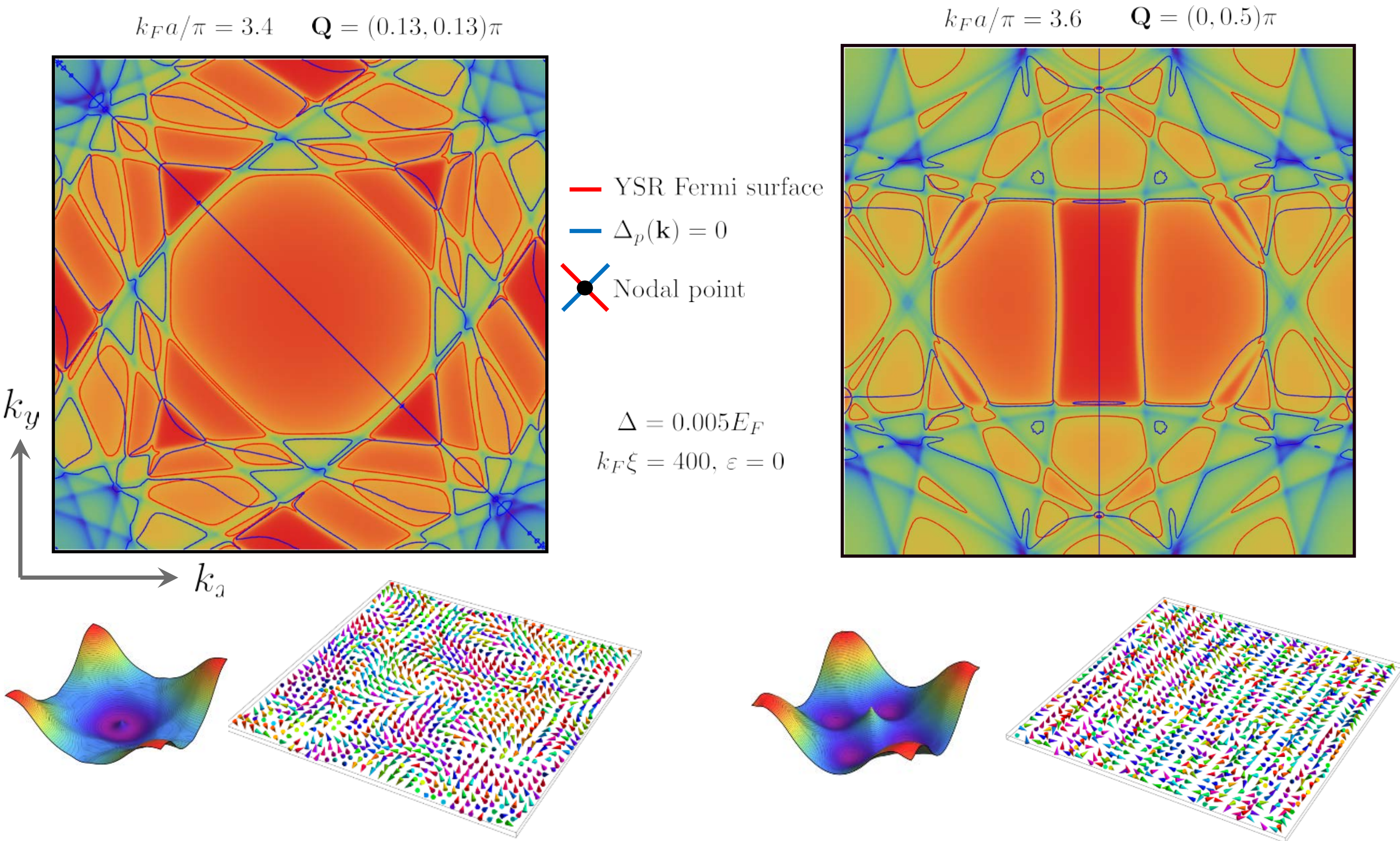


Angular energy barriers at radial minima: Frustration



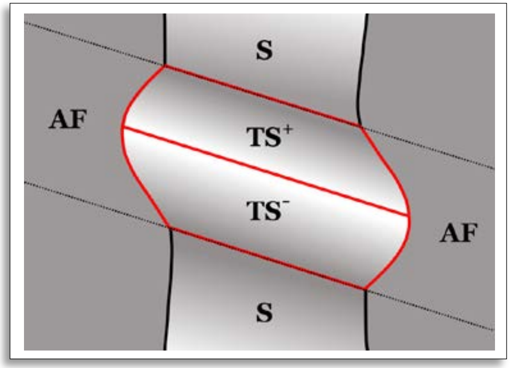


YSR sub-gap band-structure

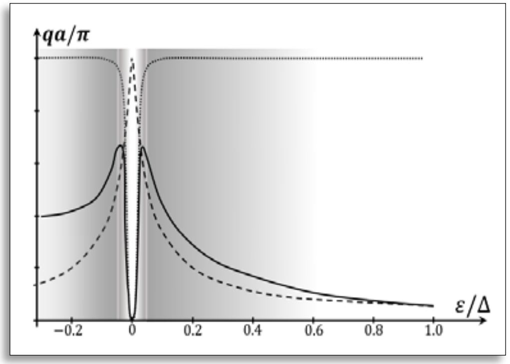


Schecter et al., PRB **93**, 140503(R) (2016)

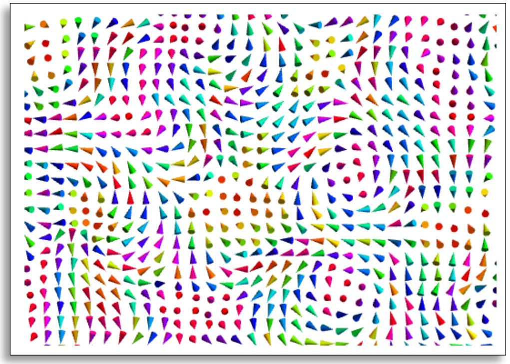
Summary & outlook



- Ferromagnetism destabilized by SC
- Even more so with YSR bound states
- Self-organized p-wave SC with Majoranas



- Deep YSR states give high-pitch spiral...
- ...but very deep YSR can give FM
- Double-exchange vs. Cooper-pair tunneling



- SC provides frustrated long-ranged exch.
- Rich magnetism for 2D in 3D SC...
- Multiple sub-gap top-band crossings...