# Quantum gravity

## Loop Quantum Gravity and Loop Quantum Cosmology

Bjorn Solheim

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University of Oslo

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1. Introduction to the problem of quantum gravity.

2. The theory of Loop quantum gravity.

3. A simple Loop quantum cosmology example

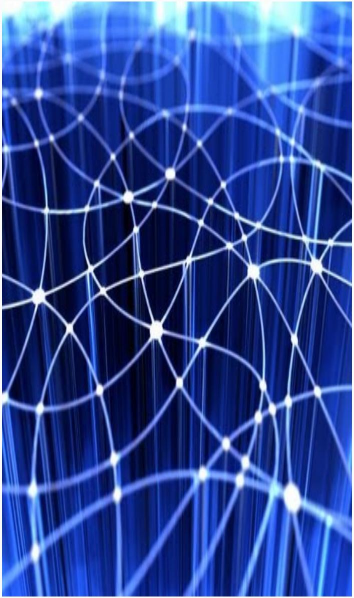
# 1. Introduction to quantum gravity

# Fundamental questions

What do space and time look like on the very smallest scales?

What is space made of? Is there such a thing as atoms of space?

Do space and time really exist, or are they just emergent concepts that serve as useful approximations in some physical domains?



# Microscopic Geometry

These are question about the microscopic structure of space and time. To answer such question we need a theory of the structure of space and time. So far our best such theory is General Relativity.

Let us look quickly about what GR tells us about the world.

# Equivalence principle

The unique property of gravity is that it is a force that affects all matter and energy in the same way. This property is summed up in the equivalence principle.

The equivalence principle enables us to describe gravity as geometry.

GR describes gravity as spacetime curvature.

"Matter tells geometry how to curve. Geometry tells matter how to move."

# Triple structure

The gravitational field in GR is a field of momentous importance. It gives a unified description of several key aspects of nature:

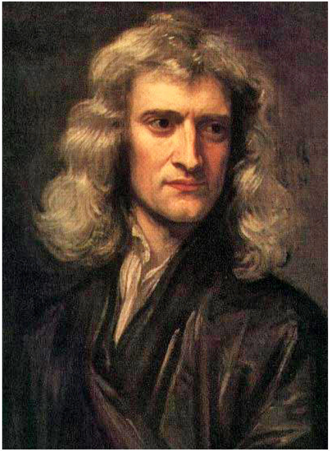
1. The force of gravity
2. The metric and causal structure of spacetime
3. The inertial structure of space

# A change of perspective

We need to fully embrace what GR tell us about the true nature of spacetime.

Newton was wrong. Spacetime is not a separate entity.
There is no spacetime in itself – spacetime IS the gravitational field.

Inertia is not movement with respect to some eternal spacetime, but movement relative to the gravitational field.



# A change of perspective – Non-dynamic

We often view Minkowski spacetime as spacetime without gravity. This is not helpful for the purpose of quantum gravity.

Minkowski spacetime is NOT spacetime without gravity. It is spacetime where we can ignore the dynamic aspects of gravity.

# A change of perspective – Nothingness

GR describes gravity as fields on a differentiable manifold. A differentiable manifold has no metric.

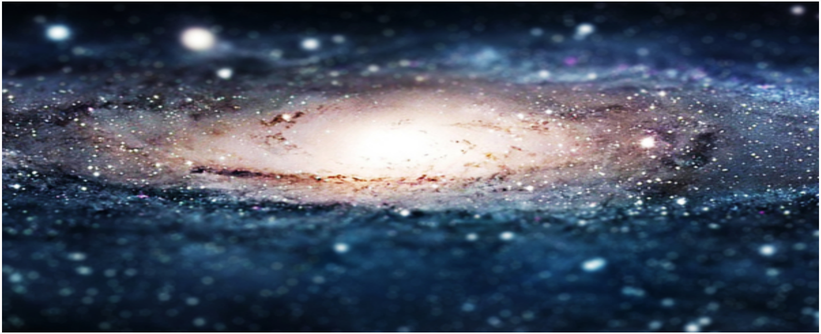
The natural physical meaning of a differentiable manifold is that it represents pure nothingness. It is a mathematical background, representing non-existence.

# A change of perspective – Start with nothing

Instead of regarding Minkowski space as perhaps the "default state" of GR. It is more useful for our purposes today to consider the default or basic state of GR to be a differentiable manifold without a metric. That is, no spacetime at all, only the non-physical mathematical emptiness.

*"In the beginning the world was an empty void without form. Then,... GOD created the gravitational field, and there was space, and time and spacetime, and all was right with the world."*

# Microscopic geometry



GR is very successful, but as a purely classical theory can it describe microscopic geometry?

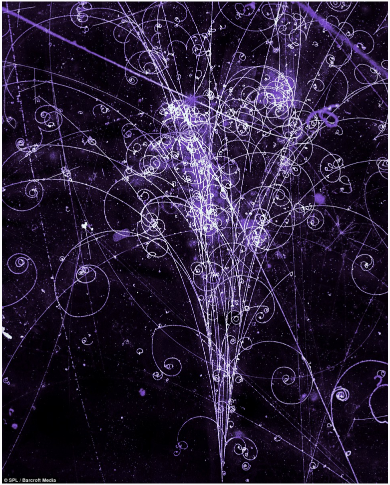
What does other theories of the microscopic domain tell us?

# Microscopic geometry

All other forces are described by quantum theories not classical theories.

This suggests that to answer question about microscopic geometry we need a quantum theory of geometry.

Can we apply QFT insights to GR?



# QFT vs GR

**Possible conflict**

Naively it is not so easy to combine GR and QFT. There seems to be several potential conflicts between the foundations of GR and QFT.

**QFT/SM** says matter is a purely **quantum** phenomena, described by operators and Hilbert space vectors, that lives on a smooth Minkowski space with a **fixed geometry**, and evolves according to a **global time** parameter.

**GR** says matter is described by a **classical** tensor field and lives on a curved manifolds with a **dynamic geometry** with **no global time** parameter available.

# QFT vs GR

**Figure 1:** QFT vs GR

# QFT vs GR

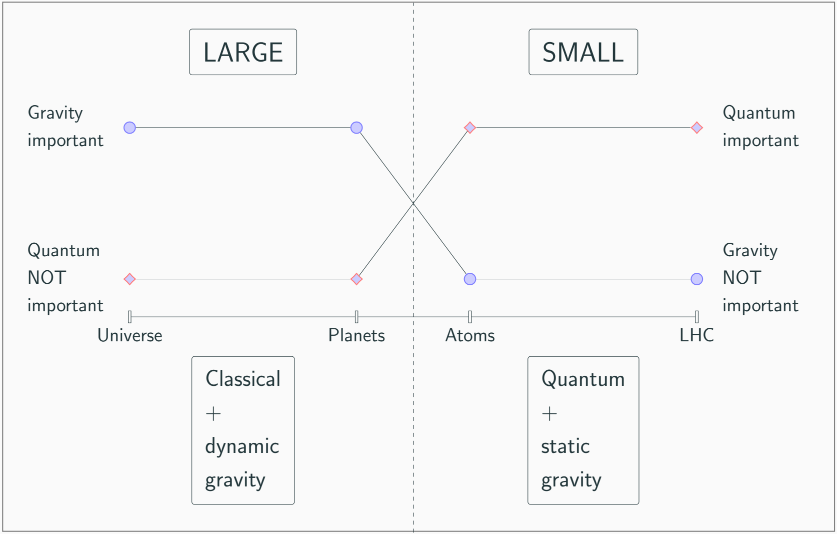
**Empirical success**

The spectacular phenomenological success of GR + SM/QFT has lead to a long period in physics where the differences between GR and QFT have been de-emphasized.

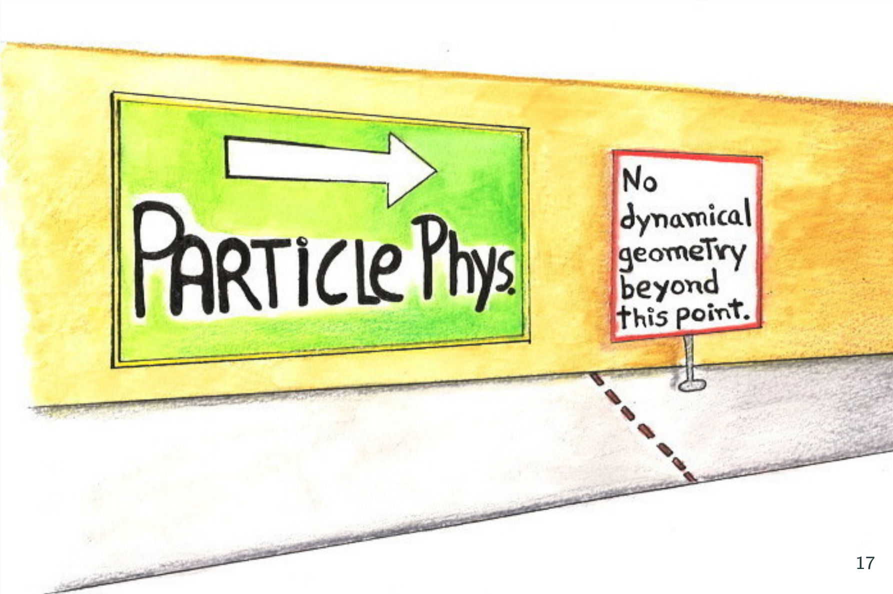
Although the underlying conflict between QFT and GR at the level of principles is well known, this doesn't always come to the surface, perhaps because in practical applications they can be friends at a distance.

# The official story – QFT and GR are friends (sort of!)

# Large and small nicely separated

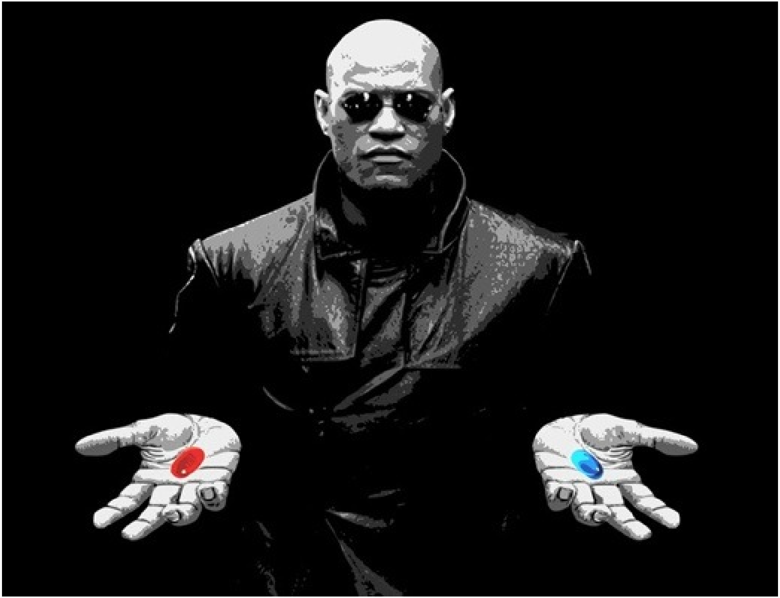


# You take that part...

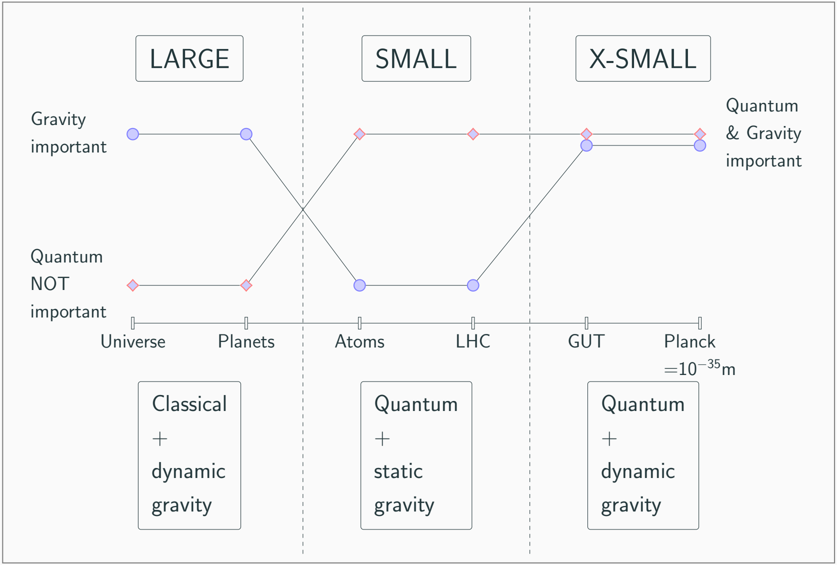


# ...and I take this part

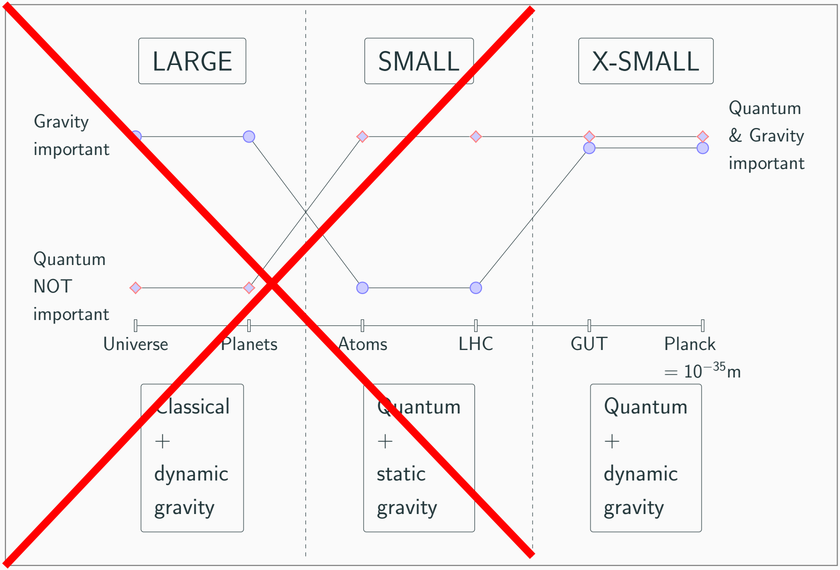
# The truth?



# No such separation!



# Fundamental physics belongs in the X-small domain



# Conceptual failure

**Empirical success, conceptual failure**

QFT (as formulated for the SM) depends crucially on features of spacetime which GR tell us only applies in limited domains.

GR depends on features of matter which SM tells us is only true as a large scale approximation.

The peaceful coexistence of SM and GR is an illusion stemming from the fact that we live in a very big universe with very light fundamental particles.

**Conceptual healing**

We must look for a theory that is conceptually unified and internally consistent (as well as being empirically correct).

Can we combine the principles of GR and QM in one theory?
And what would such a theory look like?
Can spacetime survive?

# QFT vs GR – Core Principles

**QFT**

- Basic structure
  - Noncommutative algebra (operators)
  - States (Hilbert space)
  - Global time parameter (Hamiltonian)
- Geometry and symmetry
  - Fixed geometry
  - Global O(1,3) symmetry
  - Not DIFF invariant

**GR**

- Basic structure
  - Commutative algebra (functions)
  - States (Phase space)
  - No time parameter (Hamiltonian vanishes)
- Geometry and symmetry
  - No (prior) geometry
  - No isometries
  - DIFF invariant

# Quantum theory + Dynamic geometry

**QFT**

- Basic structure
  - Noncommutative algebra (operators)
  - States (Hilbert space)
  - ~~Global time parameter (Hamiltonian)~~
- Geometry and symmetry
  - ~~Fixed geometry~~
  - ~~Global O(1,3) symmetry~~
  - ~~Not DIFF invariant~~

**GR**

- Basic structure
  - ~~Commutative algebra (functions)~~
  - ~~States (Phase space)~~
  - No time parameter (Hamiltonian vanishes)
- Geometry and symmetry
  - No (prior) geometry
  - No isometries
  - DIFF invariant

**Methods**

There are no known quantum gravity phenomena. Such phenomena likely resides at the Planck scale way beyond current experimental reach. The likelihood of getting decisive empirical input soon is small. Empirical confirmation is of course still crucial to stay in the domain of science, but it seems unlikely that experimental results will be the driving force of the development of quantum gravity theories.

To make progress we have to rely on consistency, conceptual clarity, enforcing the principles of known physics, and reproducing the empirical success on known theories.

# Quantum gravity – What do we need?

We need to build a theory with the following properties

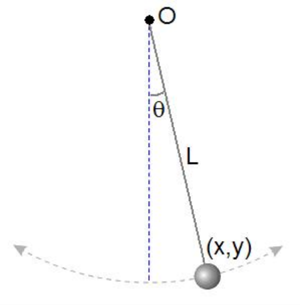
- Non-perturbative and Quantum
- Background independent and Diff. invariant
- Supports matter and geometric degrees of freedom
- Does not depend on any time parameter
- Space, time and spacetime emerge in the proper limit

# 2. Loop Quantum Gravity

# Loop Quantum Gravity

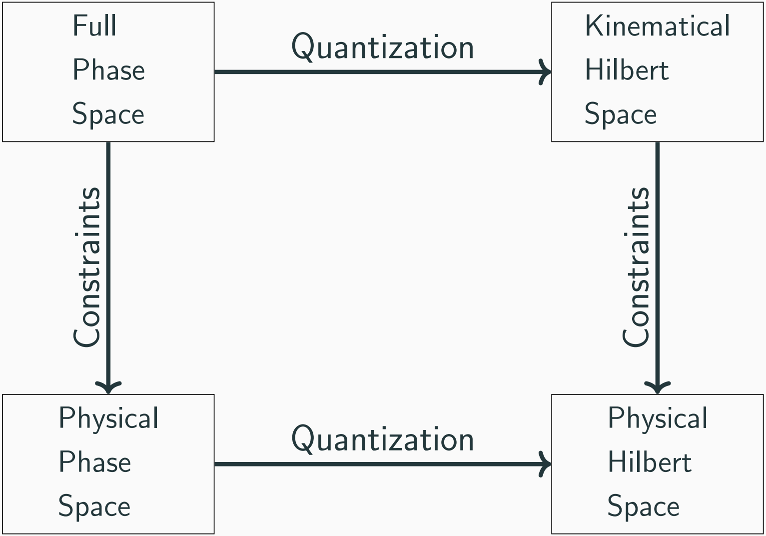
Loop Quantum Gravity (for the purposes of this talk) is a canonical quantization of a gauge theory formulation of GR, with the connection as the fundamental variable instead of the metric.

# What is a constraint?




A constraint is a phase space function that we require to be zero. This usually comes from redundancies in the description of the system. Let say we have a pendulum that is restricted to swing in the xy-plane. If we choose to describe the system by points in a three-dimensional phase space $(x, y, z) \in \mathbb{R}^3$, we get two constraint equations, $x^2 + y^2 - L^2 = 0$ and $z = 0$. These are not dynamical equations, they just restrict the proper physical phase space to a one-dimensional $(3 - 2 = 1)$ submanifold of the original (redundant and un-physical) three-dimensional manifold.

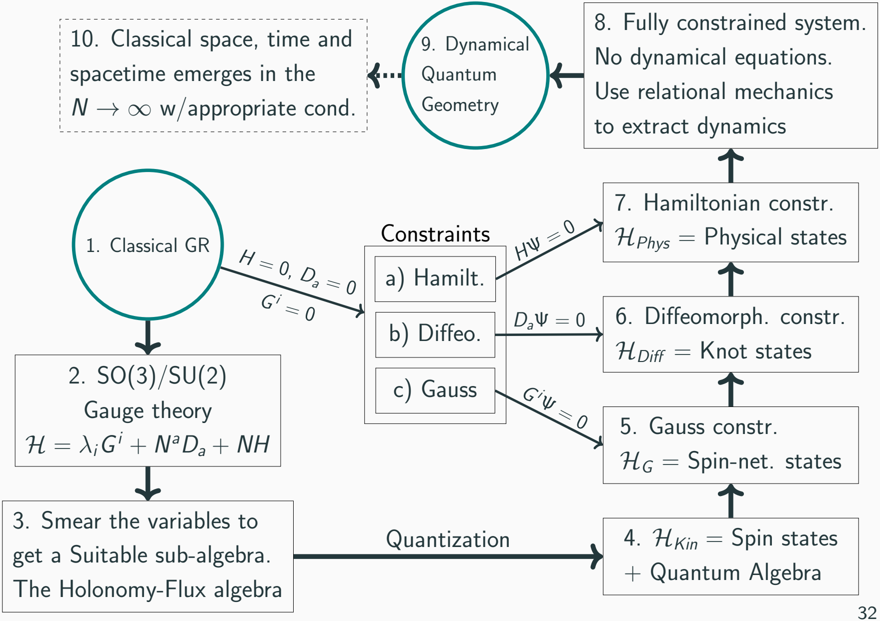
# Dirac quantization of constrained systems



# Caution

Take a deep breath.. Heavy slide coming up.

# The road to reality - Loop Quantum Gravity



# Idea 1: GR as SO(3)/SU(2) Gauge theory

$$(g_{\mu\nu}, \dot{g}_{\mu\nu}) \xrightarrow[\text{SU(2) gauge theory}]{\text{Rewrite as}} (A^i_a, E^a_i)$$

Replace the metric (and its derivative) by an SU(2) connection $A^i_a$ and a (densitized) triad field $E^a_i$.

Think of this as a standard SU(2) Yang-Mills theory (with some extra constraint equations).

Note that we are now in the Hamiltonian field theory formulation. This theory is defined on a three-dimensional (spatial) hypersurface $\Sigma$.

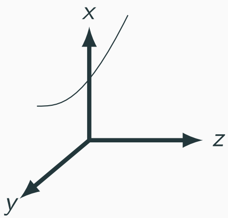
# Idea 2: Smear the Gauge variables – Holonomy-Flux

$$(A_a^i, E_i^a) \xrightarrow[\text{edges and surfaces}]{\text{Smear along}} (g, \mathfrak{g}) \in SU(2) \times \mathfrak{su}(2)$$

The SU(2) connection is a one form and (the dual of) densitized triad is a two form. We can integrate these variable along edges (1-D) and surfaces (2-D ) (without using a metric!).

From this integration we get an SU(2) group element $g$ (in the spin-J representation) for each edge, and an $\mathfrak{su}(2)$ algebra element $\mathfrak{g}$ for each surface.

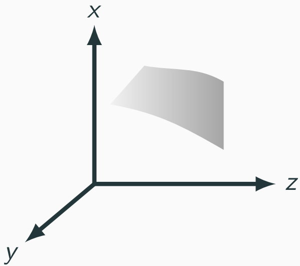
# Idea 2: Smear the Gauge variables – Holonomy-Flux




**Holonomy**

Edge $e \to g$ $SU(2)$ element

$e \to h_e[\mathcal{A}] = \exp \int_e A d\lambda = g \in SU(2)$

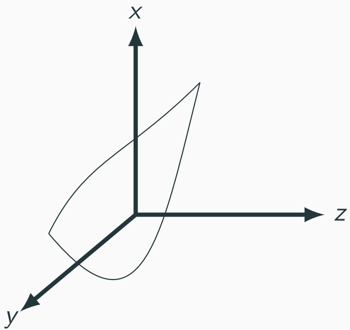



**Flux**

Surface $S \to \mathfrak{g}$ $\mathfrak{su}(2)$ element

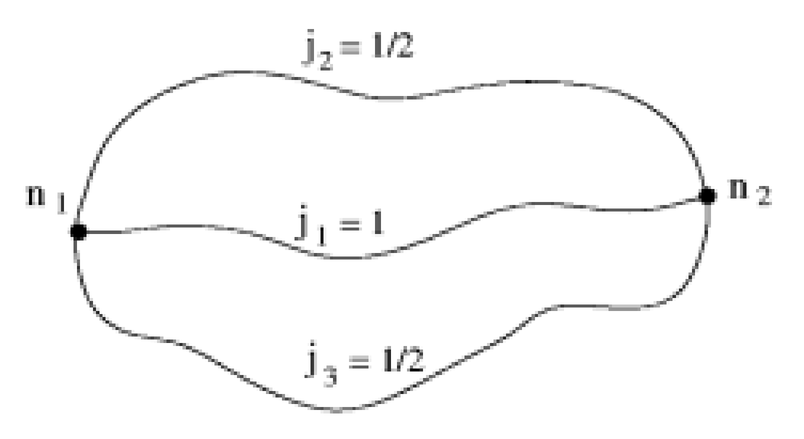
$S \to F_S[\mathcal{E}] = \int_S E dS = \mathfrak{g} \in \mathfrak{su}(2)$

# Quantum excitations along the edges of an embedded graph.

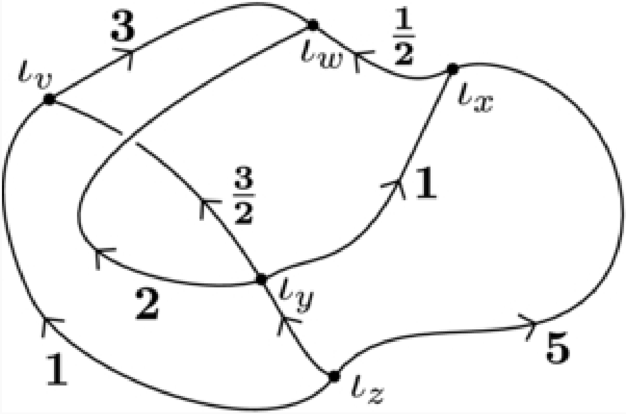



For quantum states we only need to focus on the configuration space. So informally only "half" of the phase space we just described. The quantum states are build from the basic excitations of the connection field. In this case the excitation are similar to Faraday lines of the electric field. However in this case, these are not excitations of some field IN space, but excitations OF space itself. The excitations goes along the edges of an embedded graph.

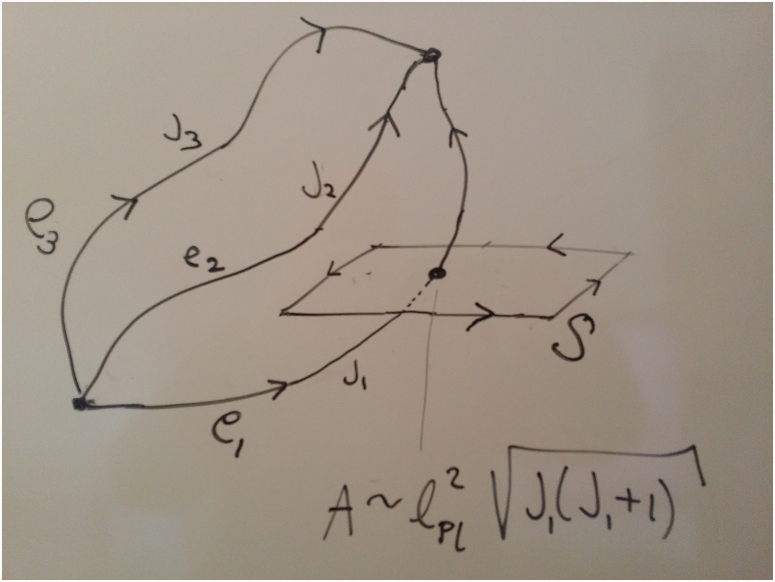
# Quantum states – Group elements on each edge



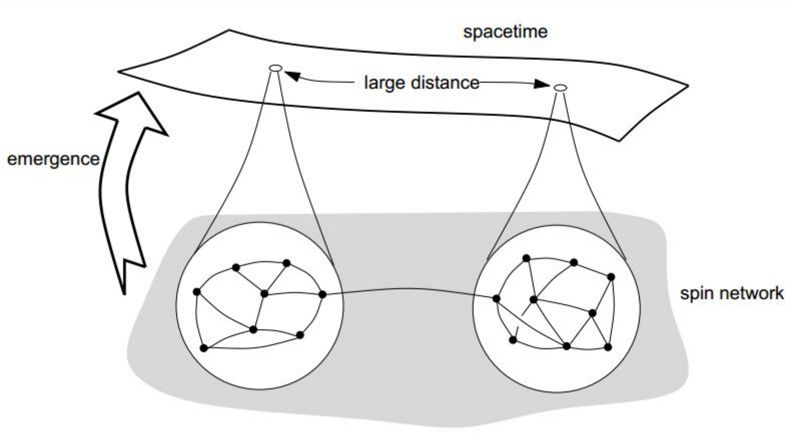
# Quantum states – Group elements on each edge



# Area is discrete



# Space emerges in a limit



# Problem of time – Relational mechanics

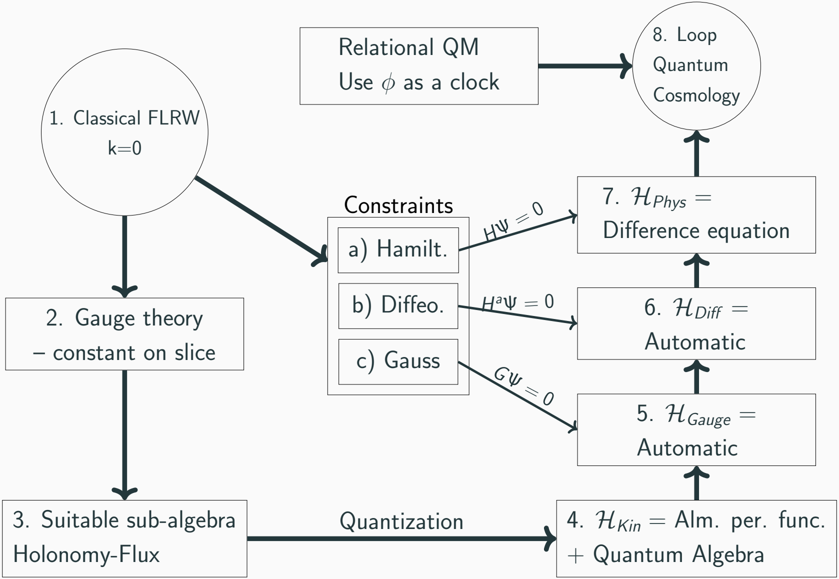
What is the problem of time in loop quantum gravity? The Hamiltonian of the theory is a linear combination of constraints, and all (Dirac) observables must commute with the constraints. Therefore the Dirac observables must commute with the Hamiltonian. This means they do not change under the flow generated by the Hamiltonian.

Superficially the observables are frozen in time and there is no change. One can say that the "time translations" generated by the Hamiltonian are "pure gauge".

Several solution using relational mechanics exists. We will see an example in the cosmology section.

# 3. Loop quantum cosmology

# The road to reality - Loop Quantum Cosmology



# Loop quantum cosmology

We can now make an explicit expression of the Hamiltonian constraint.

$$\begin{aligned} C_{gravity} &= Tr\left(h_i^\mu h_j^\mu (h_i^\mu)^{-1} (h_j^\mu)^{-1} h_k^\mu \left\{(h_k^\mu)^{-1}, V\right\}\right) \\ &= \sin^2(\bar{\mu}c)\left[\sin(\bar{\mu}c) V \cos((\bar{\mu}c)) - \cos((\bar{\mu}c) V \sin(\bar{\mu}c))\right] \end{aligned}$$

This translates into the following difference equation

$$C^-(v)\Psi(v-4,\phi) + C^0(v)\Psi(v,\phi) + C^+(v)\Psi(v+4,\phi) = 0$$

We add a homogenous scalar field to act as clock variable.

$$\partial_\phi^2 \Psi(v,\phi) = C^-(v)\Psi(v-4,\phi) + C^0(v)\Psi(v,\phi) + C^+(v)\Psi(v+4,\phi)$$

# Numerical solutions and effective equations

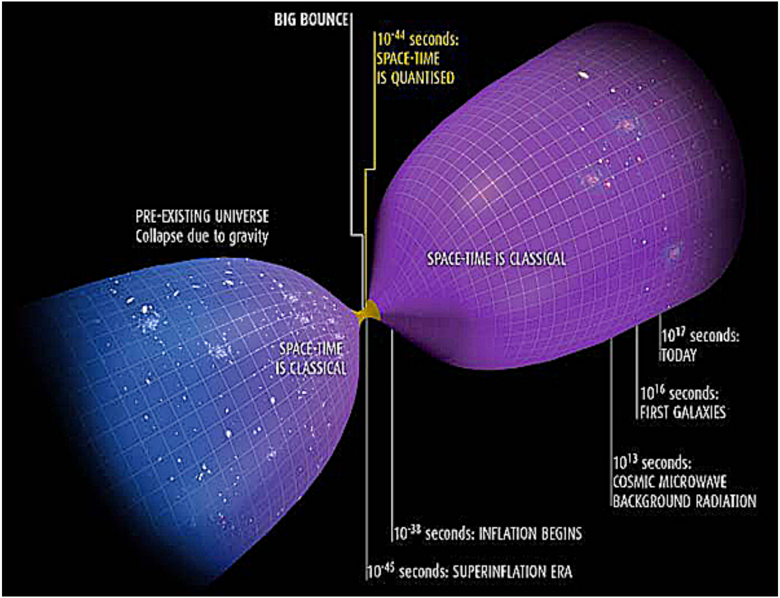
The Hamiltonian constraint can be solved numerically. We can also develop effective equations to represent the dynamics. We shall only present the effective Friedmann equation. The classical Friedmann equation is given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

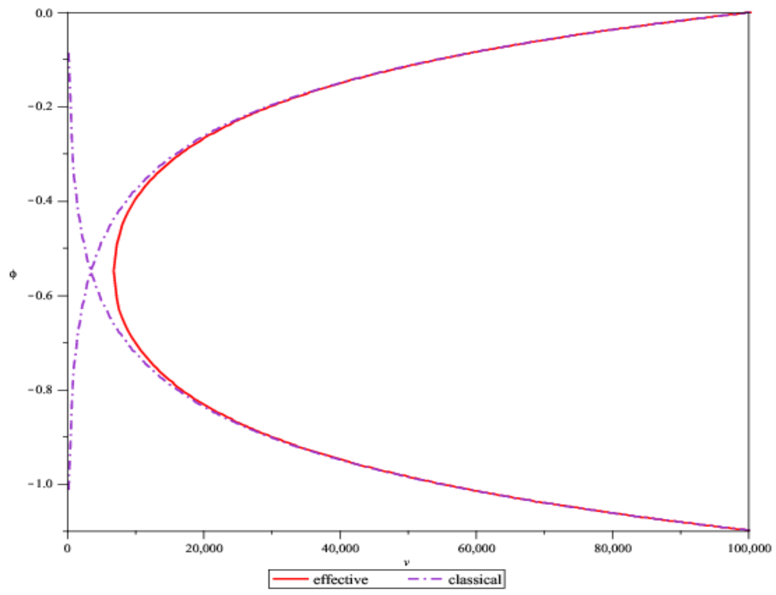
The modified effective equation is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{Crit}}\right), \quad \rho_{Crit} = 0.41\rho_{Pl}$$

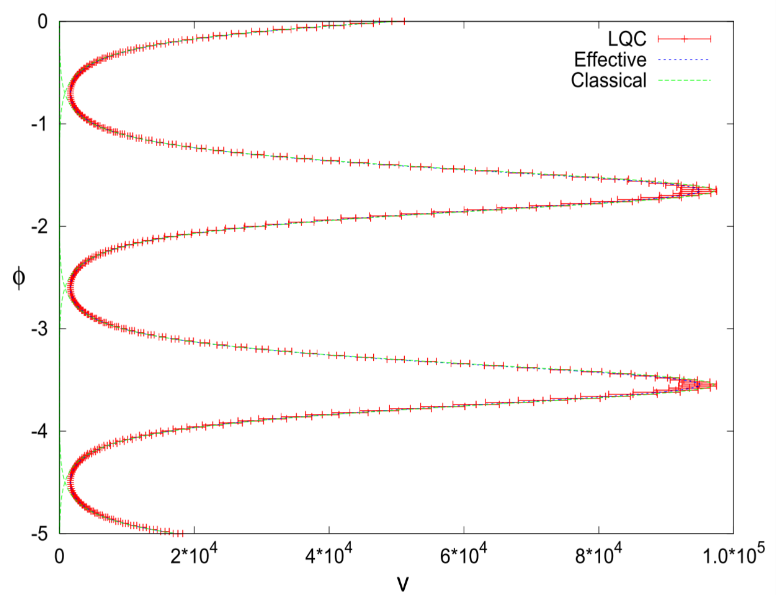
# Quantum bounce



# Quantum bounce



# Quantum bounce, k = 1



# Summary

# Summary – Part I

- Naively QFT + GR appears to separate nicely into small and big, but this is an artifact of living in a big universe with very light elementary particles.
- The extra-small domain is the proper domain of fundamental physics. No such separation exists there, and the current model is inadequate.
- Uniting QFT and GR likely requires developing some sort of non-perturbative background independent quantum (field) theories.
- Space, and time, and spacetime are all emergent phenomena and we must use relational methods to define dynamics.

## Summary – Part II

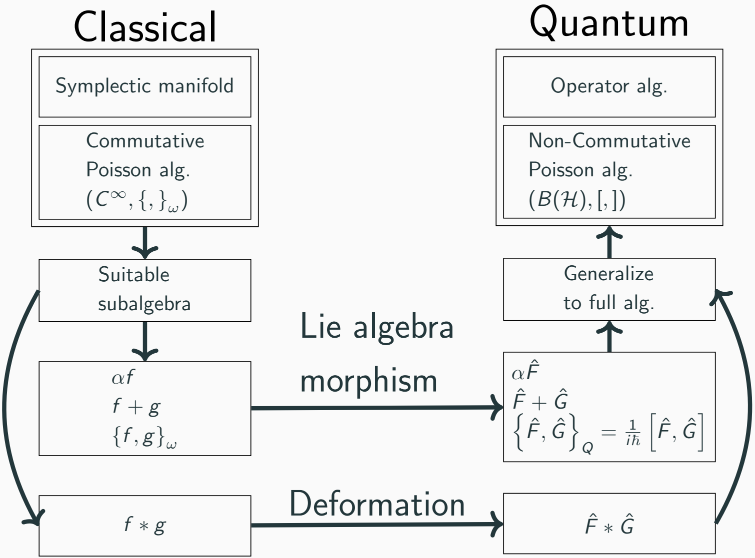
- LQG is a non-perturbative background independent quantization of GR.
- LQG implements the core principles of QM and GR.
- LQG is mathematically rigorous and based on a clear set of ideas, and results in a picture of space based on quantum geometry.
- Time does not exist as a fundamental entity, and dynamics is implemented by relational methods.

# Summary – Part III

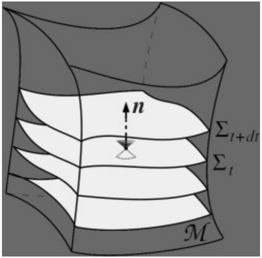
- LQC is an application of LQG is simplified setting.
- The quantum FLRW model eliminates the big bang singularity and replaces it with a smooth bounce.
- At super high density a very strong repulsive quantum geometric force appears.
- At lower density the model behaves the same as classical FLRW cosmology
- The model demonstrate the use of relational time and how smooth spacetime emerges from the quantum geometric picture.

# Extra Material

# Quantization



# Rewriting EH-action




Rewriting EH-action $\rightarrow$ hypersurface variables:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R$$
$$= \frac{1}{2\kappa}\int dx^0\int d^3x\, N\sqrt{q}(\bar{R} + \left(q^{ik}q^{jl} - q^{ij}q^{kl}\right)K_{ij}K_{kl})$$

Legendre transfromation:

$$\Pi = \frac{\partial L}{\partial \dot{N}} = 0, \quad \Pi_a = \frac{\partial L}{\partial \dot{N}^a} = 0,$$
$$P^{ab} = \frac{\partial L}{\partial \dot{q}_{ab}} = \frac{1}{\kappa}\sqrt{q}\left(q^{ac}q^{bd} - q^{ab}q^{cd}\right)K_{cd}$$

The action becomes

$$S = \frac{1}{2\kappa}\int dx^0\int d^3x\left\{\dot{q}_{ab}P^{ab} - [N^aC_a + NC]\right\}$$

# First class constraints

We already saw two primary constraints coming directly from the Legendre transformation

$$\Pi = \frac{\partial L}{\partial \dot{N}} = 0, \quad \Pi_a = \frac{\partial L}{\partial \dot{N}^a} = 0,$$

Preserving these two primary constraints under Hamiltonian ("time") evolution leads to two secondary constraints.

$$\dot{\Pi} = C = 0, \quad \dot{\Pi}_a = C_a = 0$$

$C$ and $C_a$ are called the Hamiltonian constraint and the diffeomorphism constraint. The Hamiltonian is a linear combination of these constraints.

$$H = \frac{1}{2\kappa} \int d^3x \left\{ N^a C_a + NC \right\}$$

# Triads and Extrinsic curvature

We now exchange the metric for a set of triads (orthonormal frame fields). The triads specify orthonormal basis in terms of the coordinate basis.

$$\vec{e}_i = e_i^a \vec{e}_a, \quad \vec{e}_a = e_a^i \vec{e}_i$$

This gives

$$q_{ab} = \vec{e}_a \cdot \vec{e}_b = e_a^i \vec{e}_i \cdot e_b^j \vec{e}_j = e_a^i e_b^j \left(\vec{e}_i \cdot \vec{e}_j\right) = e_a^i e_b^j \left(\delta_{ij}\right)$$

The conjugate momenta of the triad is the extrinsic curvature with mixed indices.

$$K_a^i = \delta^{ab} e_b^j K_{ij}$$

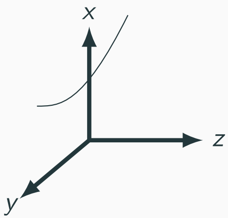
# Ashtekar variables

We move straight on to the final modification. Introducing the Ashtekar-Barbero connection and the densitized triads. Notice the arbitray Barbero-Immirzi parameter $\gamma$.

$$A_a^i := \omega_a^i + \gamma K_a^i, \quad E_i^a := \sqrt{q} e_i^a$$

The Poisson brackets are simply

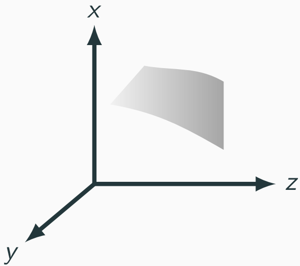
$$\left\{A_a^i(x), A_b^j(y)\right\} = 0, \quad \left\{E_i^a(x), E_j^b(y)\right\} = 0,$$
$$\left\{A_a^i(x), E_j^b(y)\right\} = \gamma \delta_j^i \delta_a^b \delta^3(x, y)$$




**Holonomy**

Edge $e \rightarrow g$ $SU(2)$ element

$e \rightarrow h_e[\mathcal{A}] = \exp \int_e A d\lambda = g \in SU(2)$




**Flux**

Surface $S \rightarrow g$ $su(2)$ element

$S \rightarrow F_S[\mathcal{E}] = \int_S E dS = g \in su(2)$

# Holonomy-Flux

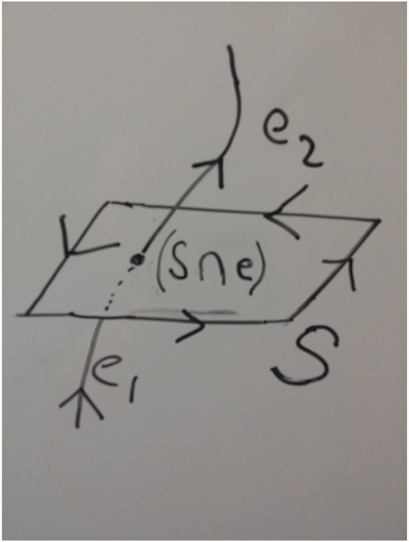


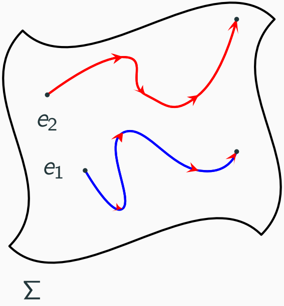

**Figure 2:** Holonomy line piercing a flux surface.

The Poisson bracket of the holonomies and the fluxes can now be calculated. The Flux splits the holonomy edge into two holonomies and "takes down" a SU(2) generator at the intersection point. In simplified notation

$$\{h_e[A], F\} = h_{e_1}[A]\,\tau^a h_{e_2}[A]$$

# Cylindrical functions

A (embedded) graph $\Gamma$ is a collection of edges $e_i$ in$\Sigma$. Example $\Gamma = \{e_1, e_2\}$.




**Phase space functions**

$f : \mathcal{A} \to \mathbb{C}$

**Cylindrical functions**

Depend on the connection only along holonomy edges of a graph $\Gamma$.

Made by composition $f = k \circ h$ of a holonomy map

$$h_\Gamma : \mathcal{A} \to \left(h_{e_1}\left[\mathcal{A}\right], h_{e_2}\left[\mathcal{A}\right]\right)$$

and

$$k : SU(2) \times SU(2) \to \mathbb{C}$$

## Action on basis states

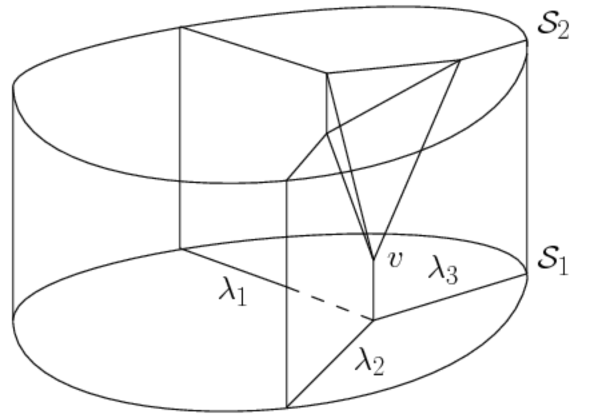
Action of SU(2) gauge constraint. A gauge transformation on $\Sigma$ is given by a map $\theta : \Sigma \to G$. The effect of such a transformation on a holonomy is given by

$$h_e [\mathcal{A}] \to \theta(e_0) h_e [\mathcal{A}] \, \theta(e_1).$$

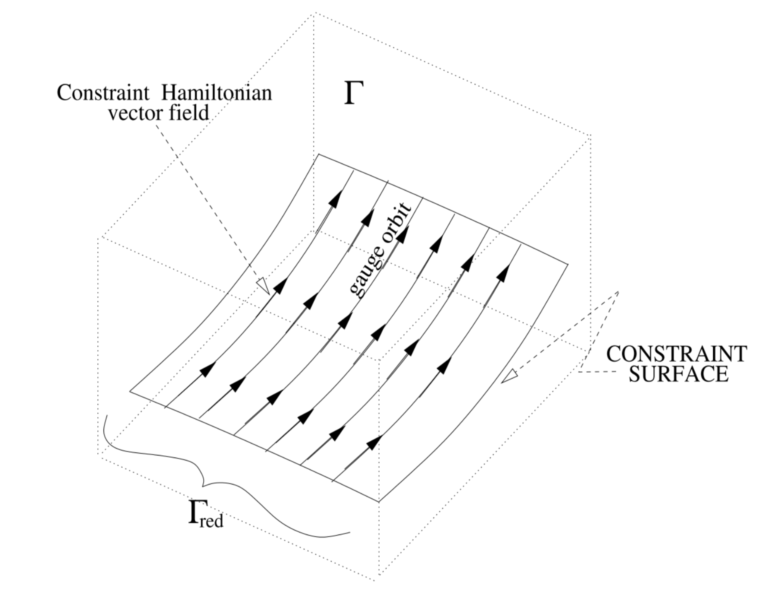
Action of a diffeomorphism $\phi : \Sigma \to \Sigma$ on a holonomy is

$$\phi h_e [\mathcal{A}] = h_{\phi^\star e} [\mathcal{A}]$$

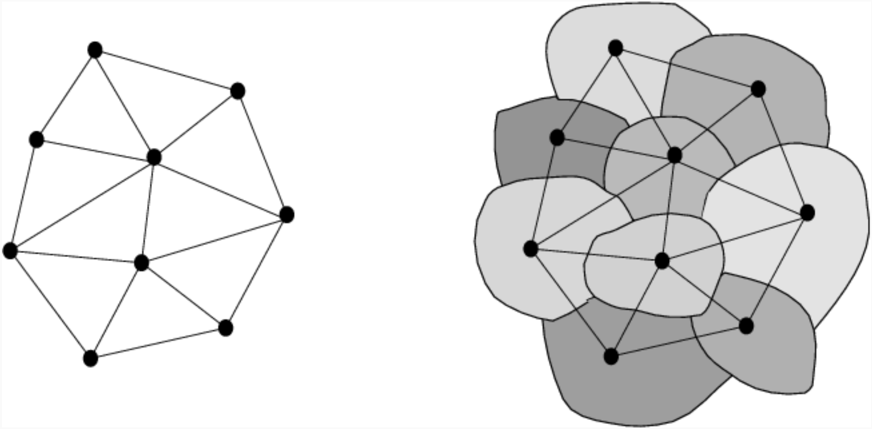
# Spin-network states – Evolution



# Constraints



# Physics: Area and Volume



# Idea 4: Use knots to make it DIFF invariant.