Low-scale seesaw models, lepton number conservation and Higgs physics

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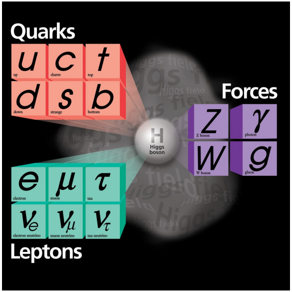
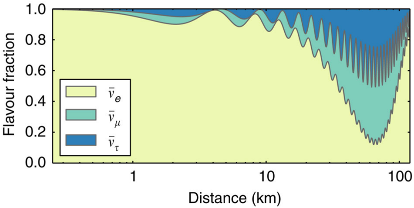
Why study neutrinos ?

- Most common fermion:
  10^{10} per electron in the Universe
  more than 300 per cm^3
- Sensitive to the weak interaction only
- Only electrically neutral elementary fermion: can be Dirac or Majorana
- Exhibit quantum properties over macrospic distances: neutrino oscillations
- Central role in supernovae explosion and probe of solar physics

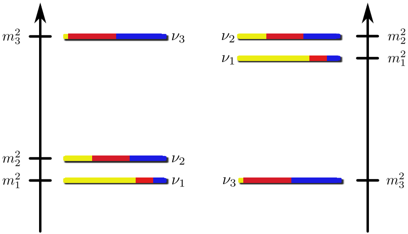




Neutrino phenomena

- Neutrino oscillations (best fit from nu-fit.org):

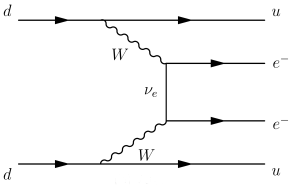
solar	$\theta_{12} \simeq 34^\circ$	$\Delta m^2_{21} \simeq 7.5 \times 10^{-5}\mathrm{eV}^2$
atmospheric	$\theta_{23} \simeq 42^\circ$	$\lvert\Delta m^2_{23}\rvert \simeq 2.5 \times 10^{-3}\mathrm{eV}^2$
reactor	$\theta_{13} \simeq 8.5^\circ$	



- Absolute mass scale:
  cosmology $\Sigma m_{\nu_i} < 0.23$ eV [Planck, 2016]
  β decays $m_{\nu_e} < 2.05$ eV [Mainz, 2005; Troitsk, 2011]

- Different mixing pattern from CKM, ν lightness $\overset{?}{\longleftarrow}$ Majorana ν

- Neutrino nature (Dirac or Majorana):
  Neutrinoless double β decays
  $m_{2\beta} < 0.061 - 0.165$ eV [KamLAND-ZEN, 2016]



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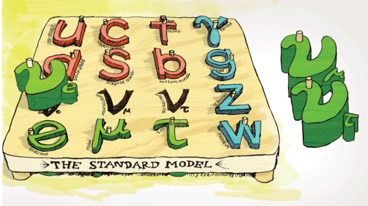
Massive neutrinos and New Physics

- Standard Model $L = \binom{\nu_L}{\ell_L}, \tilde{\phi} = \binom{H^{0*}}{H^-}$
  - No right-handed neutrino
    $\nu_R \rightarrow$ No Dirac mass term

$$\mathcal{L}_{\text{mass}} = -Y_\nu \bar{L}\tilde{\phi}\nu_R + \text{h.c.}$$

  - No Higgs triplet T
    $\rightarrow$ No Majorana mass term

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} f \bar{L} T L^c + \text{h.c.}$$



- Necessary to go beyond the Standard Model for ν mass
  - Radiative models
  - Extra-dimensions
  - R-parity violation in supersymmetry
  - Seesaw mechanisms $\rightarrow$ ν mass at tree-level
    + BAU through leptogenesis

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Dirac neutrinos ?

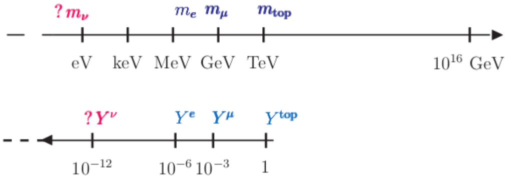
- Add gauge singlet (sterile), right-handed neutrinos ν_R $\Rightarrow \nu = \nu_L + \nu_R$

  $\mathcal{L}^{\text{leptons}}_{\text{mass}} = -Y_\ell \bar{L}\phi\ell_R - Y_\nu \bar{L}\tilde{\phi}\nu_R + \text{h.c.}$

  $\Rightarrow$ After electroweak symmetry breaking $\langle\phi\rangle = \binom{0}{\text{v}}$

  $\mathcal{L}^{\text{leptons}}_{\text{mass}} = -m_\ell \bar{\ell}_L \ell_R - m_D \bar{\nu}_L \nu_R + \text{h.c.}$

  $\Rightarrow$ 3 light active neutrinos: $m_\nu \lesssim 1\text{eV} \Rightarrow Y^\nu \lesssim 10^{-11}$



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Majorana neutrinos ?

- Add gauge singlet (sterile), right-handed neutrinos ν_R
  $\mathcal{L}^{\text{leptons}}_{\text{mass}} = -Y_\ell \bar{L}\phi\ell_R - Y_\nu \bar{L}\tilde{\phi}\nu_R - \frac{1}{2}M_R\overline{\nu_R}\nu_R^c + \text{h.c.}$

  $\Rightarrow$ After electroweak symmetry breaking $\langle\phi\rangle = \binom{0}{\text{v}}$
  $\mathcal{L}^{\text{leptons}}_{\text{mass}} = -m_\ell \ell_L \ell_R - m_D \bar{\nu}_L \nu_R - \frac{1}{2}M_R\overline{\nu_R}\nu_R^c + \text{h.c.}$

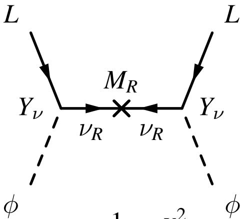
  $3\,\nu_R \Rightarrow$ 6 mass eigenstates: $\nu = \nu^c$

- ν_R gauge singlets
  $\Rightarrow M_R$ not related to SM dynamics, not protected by symmetries
  $\Rightarrow M_R$ between 0 and M_P

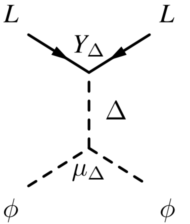
- $M_R\overline{\nu_R}\nu_R^c$ violates lepton number conservation $\Delta L = 2$

The seesaw mechanisms

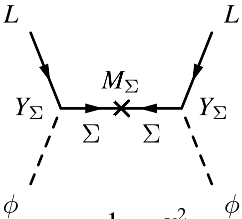
- Seesaw mechanism: new fields + lepton number violation
  $\Rightarrow$ Generate m_ν in a renormalizable way and at tree-level
- 3 minimal tree-level seesaw models $\Rightarrow$ 3 types of heavy fields
  - type I: right-handed neutrinos, SM gauge singlets
  - type II: scalar triplets
  - type III: fermionic triplets



$$\mathrm{m}_\nu = -\frac{1}{2} Y_\nu \frac{\mathrm{v}^2}{M_R} Y_\nu^T$$



$$\mathrm{m}_\nu = -2 Y_\Delta \mathrm{v}^2 \frac{\mu_\Delta}{M_\Delta^2}$$



$$\mathrm{m}_\nu = -\frac{1}{2} Y_\Sigma \frac{\mathrm{v}^2}{M_\Sigma} Y_\Sigma^T$$

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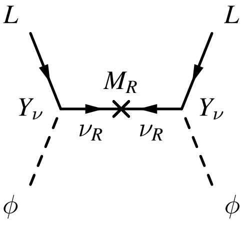
Towards testable Type I variants



- Taking $M_R \gg m_D$ gives the "vanilla" type 1 seesaw

$$\mathrm{m}_\nu = -m_D M_R^{-1} m_D^T$$

$$\mathrm{m}_\nu \sim 0.1\,\mathrm{eV} \Rightarrow \left| \begin{array}{l} Y_\nu \sim 1 \quad \text{and} \quad M_R \sim 10^{14}\,\mathrm{GeV} \\ Y_\nu \sim 10^{-6}\ \text{and}\ M_R \sim 10^{2}\ \ \mathrm{GeV} \end{array} \right.$$

- m_ν suppressed by small active-sterile mixing m_D/M_R

- Cancellation in matrix product to get large m_D/M_R
  - Lepton number, e.g. low-scale type I [Ilakovac and Pilaftsis, 1995] and others
    inverse seesaw [Mohapatra and Valle, 1986, Bernabéu et al., 1987]
    linear seesaw [Akhmedov et al., 1996, Barr, 2004, Malinsky et al., 2005]
  - Flavour symmetry, e.g. $A_4 \times \mathbb{Z}_2$ [Chao et al., 2010]
    A_4 or $\Sigma(81)$ [Chattopadhyay and Patel, 2017]
    $\mathbb{Z}(3)$ [Gu et al., 2009]
  - Gauge symmetry, e.g. $\mathrm{U}(1)_{B-L}$ [Pati and Salam, 1974] and others

$m_\nu = 0$ equivalent to conserved L for models with 3 ν_R
or less of equal mass [Kersten and Smirnov, 2007]

Extending the Kersten-Smirnov theorem

- Can the result of Kersten and Smirnov be generalized ?
- Are lepton number violating processes suppressed in all low-scale seesaw models ?

Theorem

If: - no cancellation between different orders of the seesaw expansion[a]
- no cancellations between different radiative orders[b]

Then $m_\nu = 0$ equivalent to having the neutrino mass matrix, in the basis $(\nu_L^C, \{\nu_{R,1}^{(1)} \ldots \nu_{R,n}^{(1)}\}, \{\nu_{R,1}^{(2)} \ldots \nu_{R,n}^{(2)}\}, \{\nu_{R,1}^{(3)} \ldots \nu_{R,m}^{(3)}\})$

$$\tilde{M} = \begin{pmatrix} 0 & \alpha & \pm i\alpha & 0 \\ \alpha^T & M_1 & 0 & 0 \\ \pm i\alpha^T & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix}, \tag{1}$$

for an arbitrary number of ν_R and to all radiative orders, with M_1 and M_2 diagonal matrices with positive entries and α a generic complex matrix.

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[a] This is a necessary requirement to satisfy phenomenological constraints

[b] These are highly fine-tuned solution that cannot be achieved solely by specific textures of the neutrino mass matrix

Corollary on lepton number violation

Using a unitary matrix D, let us construct

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \pm\frac{i}{\sqrt{2}}D & \frac{1}{\sqrt{2}}D & 0 \\ 0 & \frac{1}{\sqrt{2}}D & \pm\frac{i}{\sqrt{2}}D & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

then through a change of basis

$$Q^T\tilde{M}Q = \begin{pmatrix} 0 & \pm i\sqrt{2}(D^T\alpha^T)^T & 0 & 0 \\ \pm i\sqrt{2}D^T\alpha^T & 0 & \pm iD^T M_1 D & 0 \\ 0 & \pm iD^T M_1 D & 0 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix} \sim \begin{pmatrix} 0 & M_D^T & 0 & 0 \\ M_D & 0 & M_R & 0 \\ 0 & M_R^T & 0 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix}$$

- Similar to the L conserving limit of inverse and/or linear seesaw
- Explicitly L conserving taking the L assignment $(+1, -1, +1, 0)$

Corollary

The most general gauge-singlet neutrino extensions of the SM with no cancellation between different orders of the seesaw expansion, no fine-tuned cancellations between different radiative orders and which lead to three massless neutrinos are L conserving.

Eq. (1) as a sufficient condition

- Directly obtained from the corollary[1]

[1] In the seesaw limit, light neutrinos are Majorana fermions whose mass violate L conservation. Eq. (1) being equivalent to L conservation implies that the light neutrinos are massless.

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Necessary condition: tree level

- At tree-level and for the first order of the seesaw expansion

$$\mathrm{m}_\nu \approx -m_D M_R^{-1} m_D^T$$

- If $m_D M_R^{-1} m_D^T = 0$ and using $Z = M_R^{-1} m_D^T$, then the exact block-diagonalisation of the full neutrino mass matrix gives
  [Korner et al., 1993, Grimus and Lavoura, 2000]

$$\begin{aligned}\mathrm{m}_\nu = &- \left(1 + Z^* Z^T\right)^{-\frac{1}{2}} Z^T m_D^T \left(1 + Z^\dagger Z\right)^{-\frac{1}{2}} \\ &- \left(1 + Z^T Z^*\right)^{-\frac{1}{2}} m_D Z \left(1 + Z Z^\dagger\right)^{-\frac{1}{2}} \\ &+ \left(1 + Z^* Z^T\right)^{-\frac{1}{2}} Z^T M_R Z \left(1 + Z Z^\dagger\right)^{-\frac{1}{2}}\end{aligned}$$

- All terms contain $m_D M_R^{-1} m_D^T$ thus

$$\mathrm{m}_\nu = 0 \Rightarrow m_D M_R^{-1} m_D^T = 0$$

  to all orders of the seesaw expansion

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An aside on the Kersten-Smirnov theorem

- Using tree-level contributions ($\mathrm{m}_\nu = 0 \Leftrightarrow m_D M_R^{-1} m_D^T = 0$), they get the general result if $\#\nu_R \leqslant 3$

$$m_D = m \begin{pmatrix} y_1 & y_2 & y_3 \\ a y_1 & a y_2 & a y_3 \\ b y_1 & b y_2 & b y_3 \end{pmatrix}, \quad \text{and} \quad \frac{y_1^2}{M_{R,1}} = \frac{y_2^2}{M_{R,2}} = \frac{y_3^2}{M_{R,3}}$$

- For $\#\nu_R > 3$, the system of linear equations in their proof is under-constrained
- In general, no symmetry is present. Necessary to assume degenerate heavy neutrinos to make a statement.
- Justify this by requiring radiative stability but approach based on running of the Weinberg operator
  → Works only if Higgs boson lighter than all heavy neutrinos

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Necessary condition: one-loop level

- When $m_\nu = 0$ at tree-level, the one-loop induced masses are

$$\delta m_{ij} = \Re\left[\frac{\alpha_W}{16\pi^2 m_W^2} C_{ik} C_{jk} f\left(m_k\right)\right]$$

with C the mixing matrix in the neutral current and Higgs couplings and f the loop function

- In the basis where M_R is diagonal, the full neutrino mass matrix M is

$$M = \begin{pmatrix} 0 & m_{D1} & \dots & m_{Dn} \\ m_{D1}^T & \mu_1 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ m_{Dn}^T & 0 & \dots & \mu_n \end{pmatrix}$$

and at the first order in the seesaw expansion

$$\delta m = 0 \Rightarrow \sum_{i=1}^{n} \mu_i^{-2} m_{Di} m_{Di}^T f\left(\mu_i\right) = 0$$

Necessary condition: one-loop level

- Cancellation could still come from summation of non-zero terms ☹
- But a rescaling $M \to \Lambda M$ does not affect the condition $m_\nu = \delta m = 0$
- $f(x)$ being monotonically increasing and strictly convex,

$$\sum_{i=1}^{n} \mu_i^{-2} m_{Di} m_{Di}^T f(\mu_i) = 0 \to \Lambda^{-2} \sum_{i=1}^{n} \mu_i^{-2} m_{Di} m_{Di}^T f(\Lambda \mu_i) = 0$$

generate linearly independent equations from which

$$m_\nu = 0 \Rightarrow m_{Di} m_{Di}^T = 0$$

since $\mu_i > 0, f(\mu_i) > 0$

From the necessary one-loop condition to the theorem

- We write $m_{Di}^T = (u^i, v^i, w^i)$, then

$$m_{Di} m_{Di}^T = \begin{pmatrix} u^{iT} u^i & u^{iT} v^i & u^{iT} w^i \\ v^{iT} u^i & v^{iT} v^i & v^{iT} w^i \\ w^{iT} u^i & w^{iT} v^i & w^{iT} w^i \end{pmatrix} = 0$$

- We construct $Y^i = u^{i*} u^{iT} + u^i u^{i\dagger}$. Imposing $u^{iT} u^i = 0$ and excluding the trivial solution $u^i = 0$, $\mathrm{rank}(Y^i) = 2$
- Y^i symmetric and real: we can build a basis of real orthogonal eigenvectors $b^i_{1\ldots n_i}$. For the zero $n_i - 2$ eigenvalues,

$$Y^i b^i_k = 0 \Rightarrow ||u^i||^2 (u^{iT} b^i_k) = 0 \Rightarrow u^{iT} b^i_k = 0$$

- Then

$$u^{i'} = R^i_u u^i = \begin{pmatrix} b_1^{iT} u^i \\ b_2^{iT} u^i \\ b_3^{iT} u^i \\ \vdots \\ b_{n_i}^{iT} u^i \end{pmatrix} = \begin{pmatrix} u_1^{i'} \\ u_2^{i'} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

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From the necessary one-loop condition to the theorem

- Finally $u^{iT}u^{i} = 0 \Rightarrow u_2^{i'} = \pm i u_1^{i'}$
- Rinse and repeat for the other vectors, leaving M_R unaffected in the process, to get

$$m_{Di} = \begin{pmatrix} u_1^{i'} & \pm i u_1^{i'} & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ v_1^{i'} & \pm i v_1^{i'} & v_3^{i''} & \pm i v_3^{i''} & 0 & 0 & 0 & \dots & 0 \\ w_1^{i'} & \pm i w_1^{i'} & w_3^{i''} & \pm i w_3^{i''} & w_5^{i'''} & \pm i w_5^{i'''} & 0 & \dots & 0 \end{pmatrix}$$

- By rearranging the columns and rows, flavour-basis mass matrix becomes

$$M = \begin{pmatrix} 0 & \alpha & \pm i\alpha & 0 \\ \alpha^T & M_1 & 0 & 0 \\ \pm i\alpha^T & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix} = \tilde{M} \quad \square$$

Consequences for phenomenology and model building

- Any symmetry that leads to massless light neutrinos contains L as a subgroup or an accidental symmetry
- Prove the requirement of a nearly conserved L in low-scale seesaw models, baring fine-tuned solutions involving different radiative orders
- Smallness of the light neutrino mass related to the smallness of the L breaking parameter, or equivalently to the degeneracy of the heavy neutrinos in pseudo-Dirac pairs
- Naively expects L violating signatures to be suppressed
  → Needs to be assessed

A new opportunity

- How to search for heavy neutrino with $m_\nu > \mathcal{O}(1\,\mathrm{TeV})$?

Use the Higgs sector to probe neutrino mass models

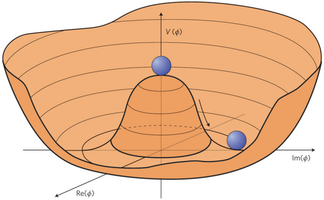
- $H\bar{\ell}_i\ell_j$:
  - Contribution negligible in the SM → evidence of new physics if observed
  - Complementary to other LFV searches
  - Large branching ratios are possible:
    $\mathrm{Br}(H \to \tau\mu) \sim 10^{-5}$ in ISS [Arganda, Herrero, Marcano, CW, 2015]
    $\mathrm{Br}(H \to \tau\mu) \sim 1\%$ in SUSY-ISS [Arganda, Herrero, Marcano, CW, 2016]

- *HHH*:
  - Reconstruct the scalar potential
    → validate the Higgs mechanism as the origin of EWSB
  - Sizeable SM 1-loop corrections ($\mathcal{O}(10\%)$)
    → Quantum corrections cannot be neglected
  - One of the main motivations for future colliders

The triple Higgs coupling

- Scalar potential before EWSB:

$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$



- After EWSB: $m_H^2 = 2\mu^2\,,\quad \mathrm{v}^2 = \mu^2/\lambda$

$$\phi = \begin{pmatrix} 0 \\ \frac{\mathrm{v}+H}{\sqrt{2}} \end{pmatrix} \rightarrow V(H) = \frac{1}{2}m_H^2 H^2 + \frac{1}{3!}\lambda_{HHH}H^3 + \frac{1}{4!}\lambda_{HHHH}H^4$$

and

$$\lambda^0_{HHH} = -\frac{3M_H^2}{\mathrm{v}}\,,\quad \lambda^0_{HHHH} = -\frac{3M_H^2}{\mathrm{v}^2}$$

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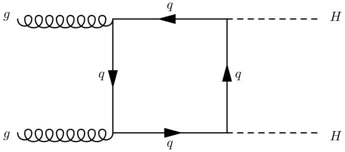
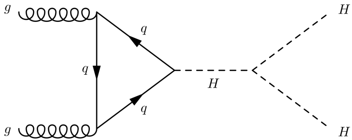
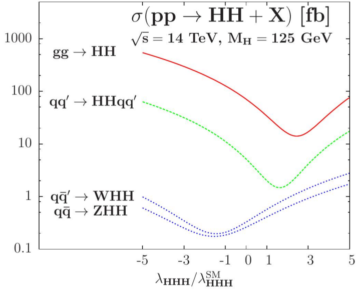
Experimental measurement of the HHH coupling

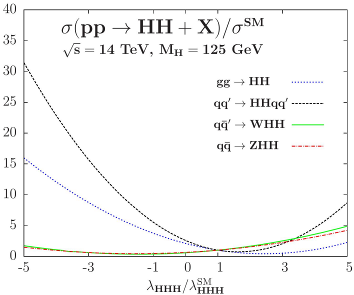
- Extracted from HH production





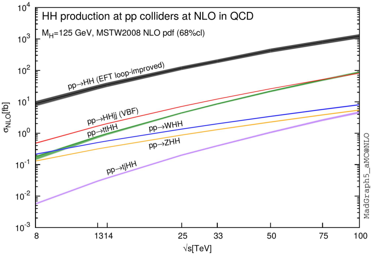
- Destructive interference between diagrams with and without λ_{HHH}



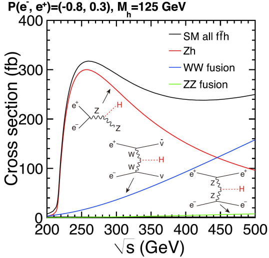


- Most sensitive channel in the SM: VBF [Baglio et al., 2013]

Future sensitivities to the SM HHH coupling



[Contino et al., 2017]



[Fujii et al., 2015]

- At hadron colliders
  - Production: gg dominates, VBF cleanest
  - HL-LHC: $\sim 50\%$ for ATLAS or CMS [CMS-PAS-FTR-15-002] and [Baglio et al., 2013]
    $\sim 35\%$ combined
  - FCC-hh: 8% per experiment with 3 ab^{-1} using only $b\bar{b}\gamma\gamma$ [He et al., 2016]
    $\sim 5\%$ combining all channels
- At e^+e^- collider
  - Main production channels: Higgs-strahlung and VBF
  - ILC: 27% at 500 GeV with 4 ab^{-1} [Fujii et al., 2015]
    10% at 1 TeV with 5 ab^{-1} [Fujii et al., 2015]

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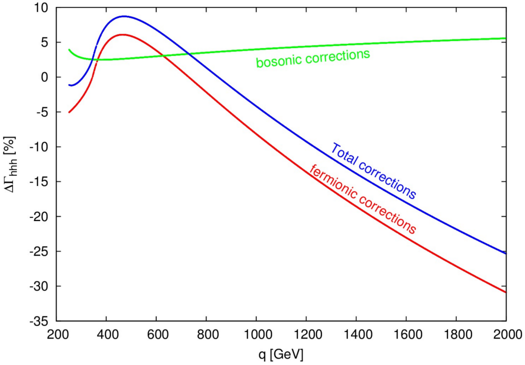
SM 1-loop corrections



taken from [Arrhib et al., 2015]

- tree-level: $\lambda^0_{HHH} \simeq 190\,\mathrm{GeV}$
- Dominant contribution from top-quark loops
  [Kanemura et al., 2004]

$$\lambda_{HHH}(q^2, m_H^2, m_H^2) = -\frac{3m_H^2}{v}\left[1 - \frac{1}{16\pi^2}\frac{16m_t^4}{v^2 m_H^2} \times \left\{1 + \mathcal{O}\left(\frac{m_H^2}{m_t^2}, \frac{q^2}{m_t^2}\right)\right\}\right]$$

- Opposite sign for the threshold ($\sqrt{q^2} = 2m_t$) and m_t^2 contributions

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A model independent approach

- A generic approach to illustrate the impact of new heavy sterile neutrinos
- Simplified model with 3 light active and 1 heavy sterile neutrinos, with masses $m_1, ..., m_4$ and mixing B
- Modified couplings to $W^{\pm}$, Z^0, H

$$\begin{aligned}\mathcal{L} \ni & -\frac{g_2}{\sqrt{2}}\bar{\ell}_i\gamma^\mu W_\mu^- B_{ij} P_L \nu_j \\ & -\frac{g_2}{2\cos\theta_W}\bar{\nu}_i\gamma^\mu Z_\mu (B^\dagger B)_{ij} P_L \nu_j \\ & -\frac{g_2}{2M_W}\bar{\nu}_i (B^\dagger B)_{ij} H (m_i P_L + m_j P_R)\nu_j\end{aligned}$$

$$B_{3\times 4} = \begin{pmatrix} B_{e1} & B_{e2} & B_{e3} & B_{e4} \\ B_{\mu 1} & B_{\mu 2} & B_{\mu 3} & B_{\mu 4} \\ B_{\tau 1} & B_{\tau 2} & B_{\tau 3} & B_{\tau 4} \end{pmatrix}$$

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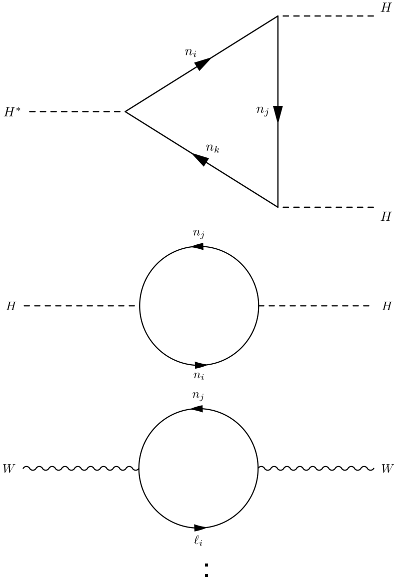
Beyond SM: simplified 3+1 model (PRD94(2016)013002)



- Impact of a new Dirac fermion coupled through the neutrino portal
- New 1-loop diagrams and new counterterms
- Strongest experimental constraints on active-sterile mixing: EWPO

  [de Blas, 2013]

  $$|B_{e4}| \leqslant 0.041$$
  $$|B_{\mu 4}| \leqslant 0.030$$
  $$|B_{\tau 4}| \leqslant 0.087$$

- Loose (tight) perturbativity of λ_{HHH}:

  $$\left(\frac{\max|(B^\dagger B)_{i4}|\, g_2\, m_{n_4}}{2M_W}\right)^3 < 16\pi\ (2\pi)$$

- Width limit: $\Gamma_{n_4} \leqslant 0.6\, m_{n_4}$

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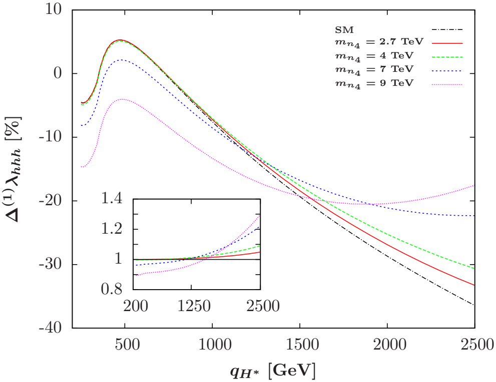
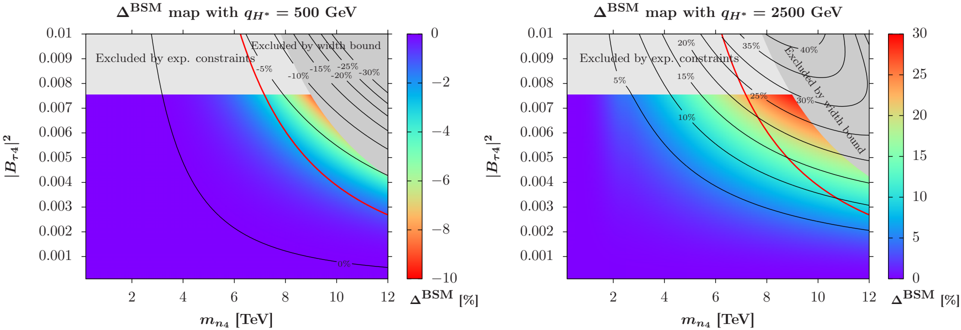
Momentum dependence



- $\Delta^{(1)}\lambda_{HHH} = \frac{1}{\lambda^0}\left(\lambda^{1r}_{HHH} - \lambda^0\right)$
- Assume $B_{\tau 4} = 0.087$, $B_{e4} = B_{\mu 4} = 0$
- Deviation of the BSM correction with respect to the SM correction in the insert
- $\max\left|(B^\dagger B)_{i4}\right| m_{n_4} = m_t$ $\to m_{n_4} = 2.7\,\text{TeV}$
  tight perturbativity of λ_{HHH} bound: $m_{n_4} = 7\,\text{TeV}$
  width bound: $m_{n_4} = 9\,\text{TeV}$

- Largest positive correction at $q_H^* \simeq 500\,\text{GeV}$, heavy ν decreases it
- Large negative correction at large q_H^*, heavy ν increases it

Results in 3+1 simplified model



- $\Delta^{\rm BSM} = \frac{1}{\lambda^{1r,{\rm SM}}_{HHH}}\left(\lambda^{1r,{\rm full}}_{HHH} - \lambda^{1r,{\rm SM}}_{HHH}\right)$
- Red line: tight perturbativity of λ_{HHH} bound
- Heavy ν effects at the limit of HL-LHC sensitivity (35%)
- Heavy ν effects clearly visible at the ILC (10%) and FCC-hh (5%)
- Similar behaviour for active-sterile mixing B_{e4} and $B_{\mu 4}$

From the 3+1 Dirac model to the ISS

- TeV-scale neutrino induces sizeable corrections to λ_{HHH}
  - Decrease at $q_H^* \simeq 500\,\mathrm{GeV}$
  - Increase at large q_H^*
- Effects could be used to constrain the active-sterile mixing at the ILC and FCC-hh
- What are the effects in an appealing low-scale seesaw model ?
  - Inverse seesaw → Additional constraints need to be included

The inverse seesaw mechanism

- Lower seesaw scale from approximately conserved lepton number
- Add fermionic gauge singlets ν_R $(L = +1)$ and X $(L = -1)$

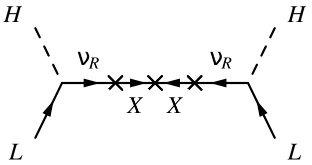
  [Mohapatra and Valle, 1986]

$$\mathcal{L}_{inverse} = -Y_\nu \overline{L}\tilde{\phi}\nu_R - M_R \overline{\nu_R^c} X - \frac{1}{2}\mu_X \overline{X^c} X + \text{h.c.}$$

with $m_D = Y_\nu \mathrm{v}\,, M^\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu_X \end{pmatrix}$

$$m_\nu \approx \frac{m_D^2}{M_R^2}\mu_X$$

$$m_{N_1,N_2} \approx \mp M_R + \frac{\mu_X}{2}$$



2 scales: μ_X and M_R

- Decouple neutrino mass generation from active-sterile mixing
- Inverse seesaw: $Y_\nu \sim \mathcal{O}(1)$ and $M_R \sim 1\,\mathrm{TeV}$

  $\Rightarrow$ within reach of the LHC and low energy experiments

Most relevant constraints for the ISS

- Accommodate low-energy neutrino data using parametrization

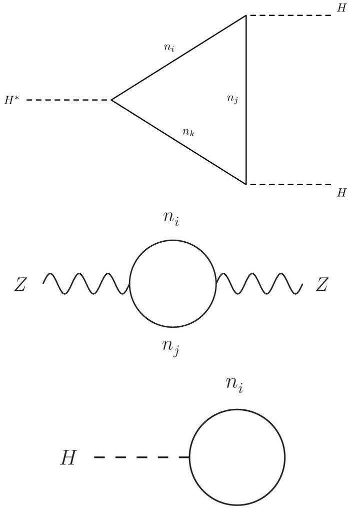
$$\mathrm{v}Y_\nu^T = V^\dagger \mathrm{diag}(\sqrt{M_1}\,,\sqrt{M_2}\,,\sqrt{M_3})\;R\;\mathrm{diag}(\sqrt{m_1}\,,\sqrt{m_2}\,,\sqrt{m_3})U_{PMNS}^\dagger$$
$$M = M_R \mu_X^{-1} M_R^T$$

or

$$\mu_X = M_R^T\;Y_\nu^{-1}\;U_{\mathrm{PMNS}}^* m_\nu U_{\mathrm{PMNS}}^\dagger\;Y_\nu^{T^{-1}} M_R \mathrm{v}^2 \qquad \text{and beyond}$$

- Charged lepton flavour violation
  → For example: $\mathrm{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ [MEG, 2016]
- Global fit to EWPO and lepton universality tests [Fernandez-Martinez et al., 2016]
- Electric dipole moment: 0 with real PMNS and mass matrices
- Invisible Higgs decays: $M_R > m_H$, does not apply
- Yukawa perturbativity: $|\frac{Y_\nu^2}{4\pi}| < 1.5$

Calculation in the ISS



- Similar diagrams to the 3+1 Dirac scenario but with Majorana neutrinos
- Evaluated with `FeynArts`, `FormCalc` and `LoopTools`
- More heavy neutrinos
  $\Rightarrow$ effects generically larger than 3+1

> Formulas for both Dirac and Majorana fermions coupling through the neutrino portal are available

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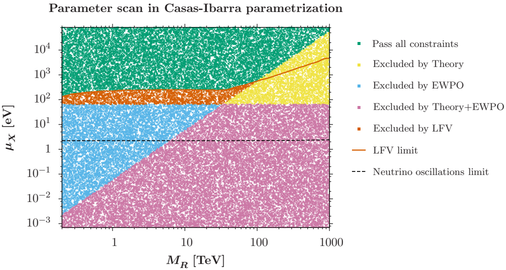
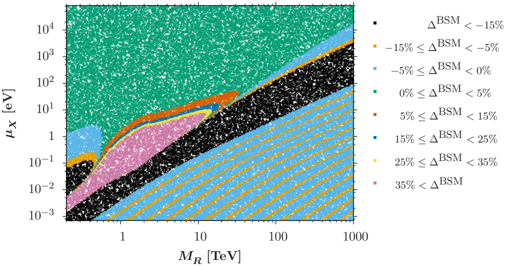
Results using the Casas-Ibarra parametrization





- Random scan: 180000 points with degenerate M_R and μ_X

$$\begin{aligned} 0 &\leqslant \theta_i &\leqslant 2\pi,\ (i=1,2,3) \\ 0.2\text{ TeV} &\leqslant M_R &\leqslant 1000\text{ TeV} \\ 7\times 10^{-4}\text{ eV} &\leqslant \mu_X &\leqslant 8.26\times 10^{4}\text{ eV} \end{aligned}$$

- $\Delta^{\rm BSM} = \frac{1}{\lambda_{HHH}^{1r,\rm SM}} \left(\lambda_{HHH}^{1r,\rm full} - \lambda_{HHH}^{1r,\rm SM} \right)$
- Strongest constraints:
  - Lepton flavour violation, mainly $\mu \to e\gamma$
  - Yukawa perturbativity (and neutrino width)
- Large effects necessarily excluded by LFV constraints ?

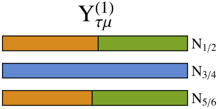
Suppressing LFV constraints

- How to evade the LFV constraints ?
- Approximate formulas for large Y_ν [Arganda, Herrero, Marcano, **CW**, 2015]:

$$\mathrm{Br}^{\mathrm{approx}}_{\mu\to e\gamma} = 8 \times 10^{-17}\mathrm{GeV}^{-4}\frac{m_\mu^5}{\Gamma_\mu}\left|\frac{v^2}{2M_R^2}(Y_\nu Y_\nu^\dagger)_{12}\right|^2$$

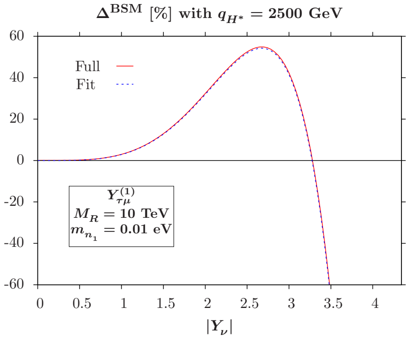
- Solution: Textures with $(Y_\nu Y_\nu^\dagger)_{12} = 0$

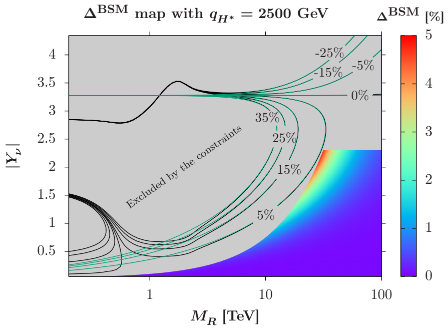
$$Y_{\tau\mu}^{(1)} = |Y_\nu| \begin{pmatrix} 0 & 1 & -1 \\ 0.9 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$



- Or even take Y_ν diagonal

Results for $Y^{(1)}_{\tau\mu}$





- $\Delta^{\text{BSM}} = \frac{1}{\lambda^{1r,\text{SM}}_{HHH}} \left(\lambda^{1r,\text{full}}_{HHH} - \lambda^{1r,\text{SM}}_{HHH} \right)$
- Right: Full calculation in black, approximate formula in green
- Well described at $M_R > 3$ TeV by approximate formula

$$\Delta^{\text{BSM}}_{\text{approx}} = \frac{(1\ \text{TeV})^2}{M_R^2} \left(8.45\, \text{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) - 0.145\, \text{Tr}(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) \right)$$

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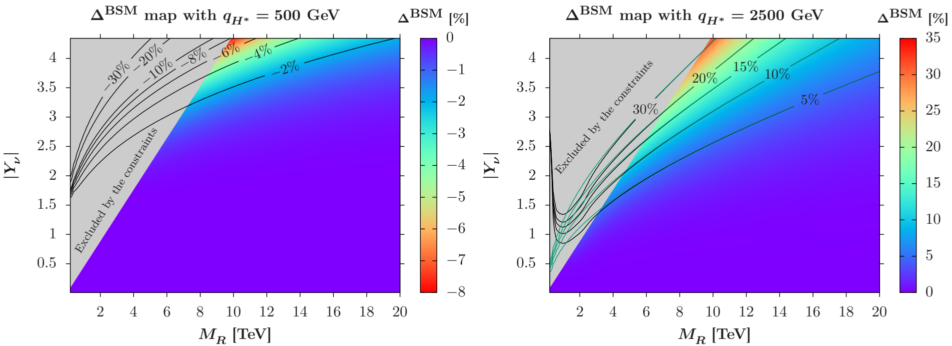
- Can maximize Δ^{BSM} by taking $Y_\nu \propto \text{I}_3$

Results in the ISS



- $\Delta^{\text{BSM}} = \frac{1}{\lambda_{HHH}^{1r,\text{SM}}}\left(\lambda_{HHH}^{1r,\text{full}} - \lambda_{HHH}^{1r,\text{SM}}\right)$
- Diagonal Y_ν: full calculation in black, approximate formula in green
- Results agree with 3+1 Dirac analysis despite stronger constraints
- Heavy ν effects at the limit of HL-LHC (35%) and ILC (10%) sensitivities
- Heavy ν effects clearly visible at the FCC-hh (5%)

Conclusions

- ν oscillations → New physics is needed to generate masses and mixing
- One of the simplest ideas: Add right-handed, sterile neutrinos
- Nearly conserved L is a cornerstone of low-scale type I seesaw variants
- Corrections to the HHH coupling from heavy ν as large as 30%: measurable at future colliders
  - Maximal for diagonal Y_ν
  - Provide a new probe of the $\mathcal{O}(10)$ TeV region
  - Complementary to existing observables
- Generic effect, expected in all models with TeV fermions and large Higgs couplings
- Next Step: Assess impact on LNV processes
  Corrections to the di-Higgs production cross-section
  Consider a UV completion to generate masses (e.g. $U(1)_{B-L}$)

Backup slides

Cancellation between different seesaw orders

- To second order in the expansion

$$m_\nu^{(2)} = -m_\nu^{(1)} + \frac{1}{2}\left(m_n^{(1)} u\theta + \theta^T m_\nu^{(1)}\right)$$

with $m_\nu^{(1)}$ the first order expression and θ is $Z^\dagger Z$ up to a unitary transformation

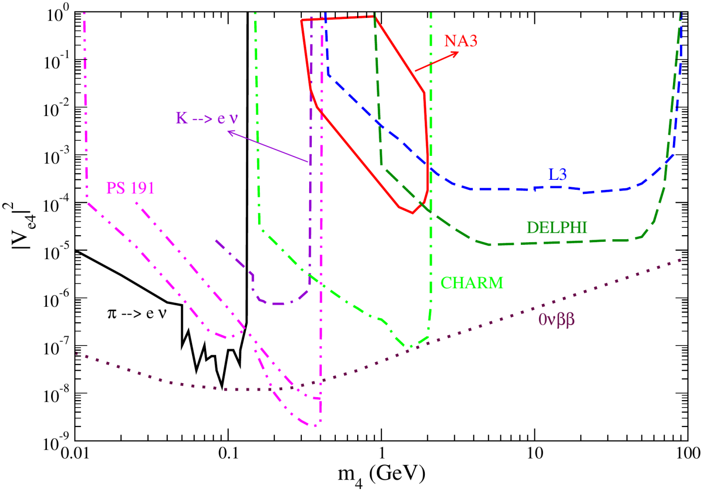
- Then

$$(m_\nu^{(2)})_{ii} = 0 \Leftrightarrow -\hat{m}_{lii}^{(1)} + \hat{m}_{lii}^{(1)}\theta_{ii} = 0$$

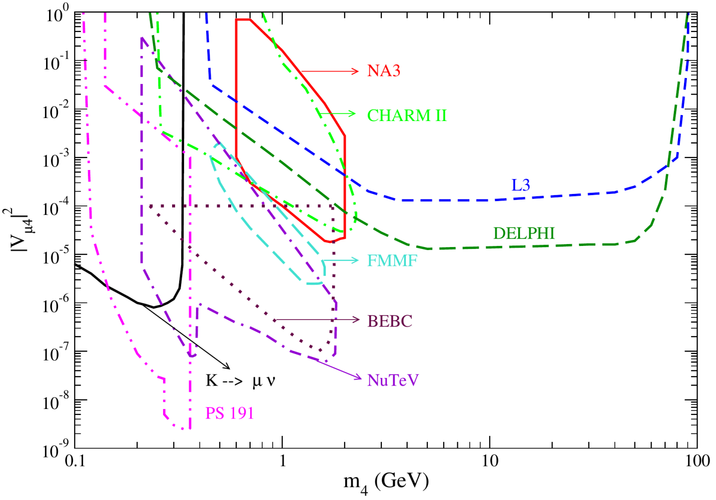
and $\theta_{ii} = 1$

- This contradicts [Fernandez-Martinez et al., 2016] which gives $||\theta|| \leqslant 0.0075$

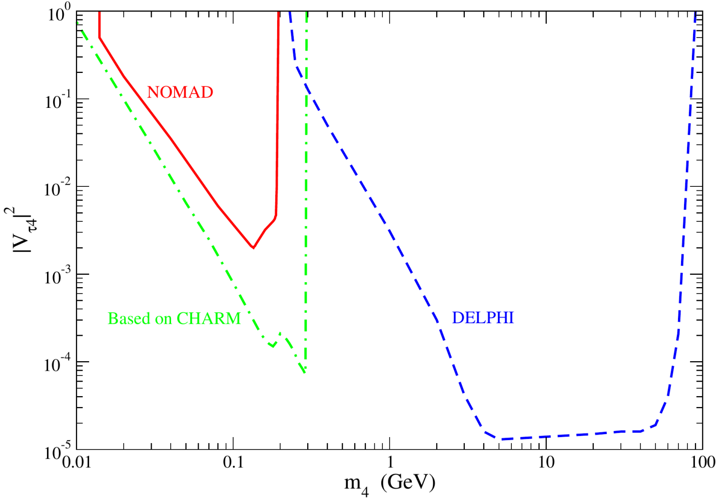
Direct constraints from JHEP05(2009)030



Direct constraints from JHEP05(2009)030



Direct constraints from JHEP05(2009)030



Details of one-loop proof I

- The loop function is

$$f(m_k) = m_k \left(3m_Z^2 g_{kZ} + m_H^2 g_{kH}\right)$$

where

$$g_{ab} = \frac{m_a^2}{m_a^2 - m_b^2} \log \frac{m_a^2}{m_b^2}$$

which gives

$$U_l^T \left(1 + Z^T Z^*\right)^{-1} Z^T U_h^* f_h U_h^\dagger Z \left(1 + Z^\dagger Z\right)^{-1} U_l = 0$$

$$Z^T U_h^* f_h U_h^\dagger Z = 0$$

to the first order in the seesaw expansion

$$U_h \approx 1$$

$$Z^T F_h Z = 0$$

Details of one-loop proof II

- Once we have

$$u^{i'} = \left(u_1^{i'}, \pm i u_1^{i'}, 0, \ldots, 0\right)^T$$

Under this transformation, we have

$$u^{iT} v^i = 0 \to u'^{iT} v'^i = 0$$

leading us to conclude that

$$v^{i'} = \left(v_1^{i'}, \pm i v_1^{i'}, v_3^{i'}, v_4^{i'}, \ldots, v_{n_i}^{i'}\right)^T$$

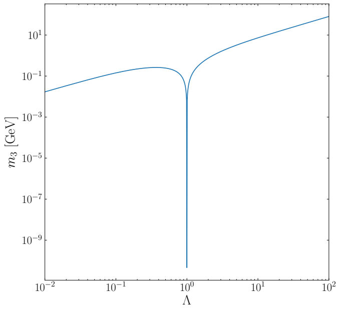
- Similarly, we construct a second matrix R_v acting on $\left(v_3^{i'}, v_4^{i'}, \ldots, v_{n_i}^{i'}\right)^T$ such that $v^{i'}$ is reduced to

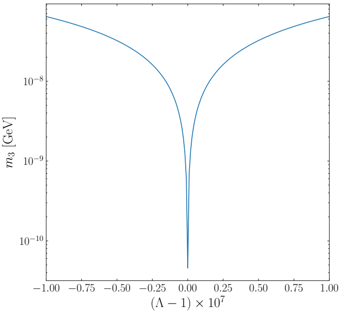
$$v^{i''} = \left(v_1^{i'}, \pm i v_1^{i'}, v_3^{i''}, \pm i v_3^{i''}, 0, \ldots, 0\right)^T$$

- Rinse and repeat for w

Fine-tuning

We adopt here the idea of [Lopez-Pavon et al., 2015], where the tree-level and one-loop contributions cancel.





Evolution of m_3 as a function of the rescaling parameter Λ. Input masses and couplings where chosen to give $m_\nu = m_{\text{tree}} + m_{\text{1-loop}} = 0.046$ eV at $\Lambda = 1$.
A deviation of less then 10^{-7} here, is enough to spoil the cancellation and contradict experimental limits.

Renormalization procedure for the HHH coupling I

- No tadpole: $t_H^{(1)} + \delta t_H = 0 \Rightarrow \delta t_H = -t_H^{(1)}$
- Counterterms:

$$
\begin{aligned}
M_H^2 &\to M_H^2 + \delta M_H^2 \\
M_W^2 &\to M_W^2 + \delta M_W^2 \\
M_Z^2 &\to M_Z^2 + \delta M_Z^2 \\
e &\to (1 + \delta Z_e) e \\
H &\to \sqrt{Z_H} = (1 + \frac{1}{2}\delta Z_H) H
\end{aligned}
$$

- Full renormalized 1–loop triple Higgs coupling: $\lambda_{HHH}^{1r} = \lambda^0 + \lambda_{HHH}^{(1)} + \delta\lambda_{HHH}$

$$
\begin{aligned}
\frac{\delta\lambda_{HHH}}{\lambda^0} = \frac{3}{2}\delta Z_H + \delta t_H \frac{e}{2M_W \sin\theta_W M_H^2} + \delta Z_e + \frac{\delta M_H^2}{M_H^2} \\
- \frac{\delta M_W^2}{2M_W^2} + \frac{1}{2}\frac{\cos^2\theta_W}{\sin^2\theta_W}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right)
\end{aligned}
$$

Renormalization procedure for the HHH coupling II

- OS scheme

$$\delta M_W^2 = Re\Sigma_{WW}^T(M_W^2)$$
$$\delta M_Z^2 = Re\Sigma_{ZZ}^T(M_Z^2)$$
$$\delta M_H^2 = Re\Sigma_{HH}(M_H^2)$$

- Electric charge:

$$\delta Z_e = \frac{\sin\theta_W}{\cos\theta_W}\frac{\mathrm{Re}\Sigma_{\gamma Z}^T(0)}{M_Z^2} - \frac{\mathrm{Re}\Sigma_{\gamma\gamma}^T(M_Z^2)}{M_Z^2}$$

- Higgs field renormalization

$$\delta Z_H = -\mathrm{Re}\frac{\partial\Sigma_{HH}(k^2)}{\partial k^2}\bigg|_{k^2=M_H^2}$$

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Next-order terms in the μ_X-parametrization

- Weaker constraints on diagonal couplings
  $\rightarrow$ Large active-sterile mixing $m_D M_R^{-1}$ for diagonal terms
- Previous parametrizations built on the 1st term in the $m_D M_R^{-1}$ expansion
  $\rightarrow$ Parametrizations breaks down
- Solution: Build a parametrization including the next order terms
- The next-order μ_X-parametrization is then

$$\mu_X \simeq \left(\mathbf{1} - \frac{1}{2} M_R^{*-1} m_D^\dagger m_D M_R^{T-1}\right)^{-1} M_R^T m_D^{-1} U_{\text{PMNS}}^* m_\nu U_{\text{PMNS}}^\dagger m_D^{T-1} M_R \times \left(\mathbf{1} - \frac{1}{2} M_R^{-1} m_D^T m_D^* M_R^{\dagger-1}\right)^{-1}$$

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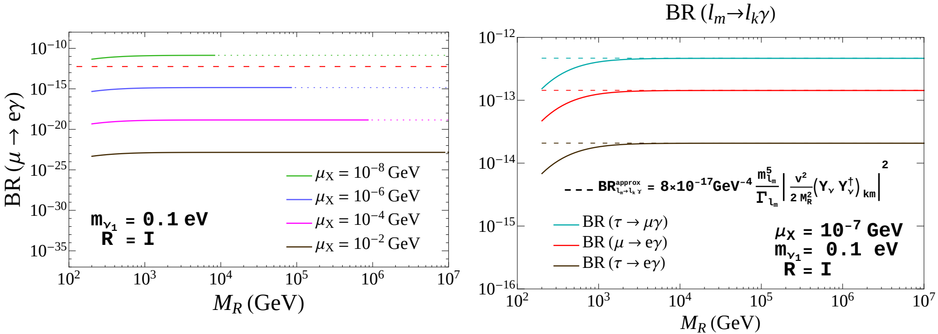
Constraints: focus on $\mu \to e\gamma$



- M_R and μ_X real and degenerate, Casas-Ibarra (C-I) parametrization
- Constrains μ_X
- Perturbativity $\to \left|\frac{Y_\nu^2}{4\pi}\right| < 1.5$ (Dotted line = non-perturbative couplings)
- $\frac{v^2 (Y_\nu Y_\nu^\dagger)_{km}}{M_R^2} \approx \frac{1}{\mu_X} \frac{(U_{\text{PMNS}} \Delta m^2 \; U_{\text{PMNS}}^T)_{km}}{2 m_{\nu_1}}$

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