

# EARLY KINETIC DECOUPLING OF DARK MATTER

WHEN THE STANDARD WAY OF CALCULATING THE THERMAL RELIC DENSITY FAILS

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University of Oslo



based on: **T. Binder, T. Bringmann, M. Gustafsson and AH,**  
**Phys.Rev. D96 (2017) 115010, [astro-ph.co/1706.07433](https://arxiv.org/abs/1706.07433)**

Oslo, 7th February 2018

# OUTLINE

## 1. Introduction

- standard approach to **thermal relic density**
- recent novel models/ideas

## 2. Kinetic decoupling

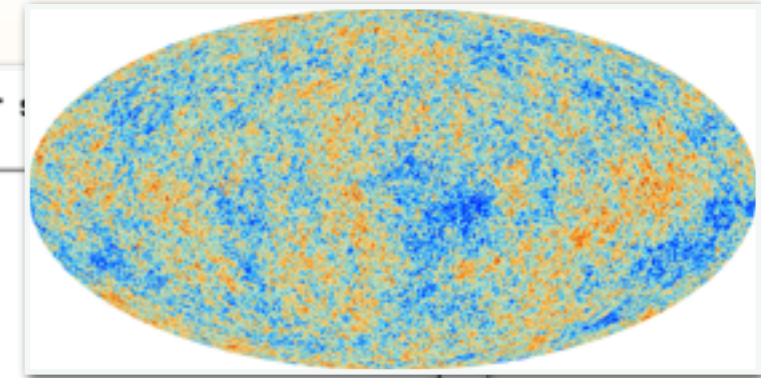
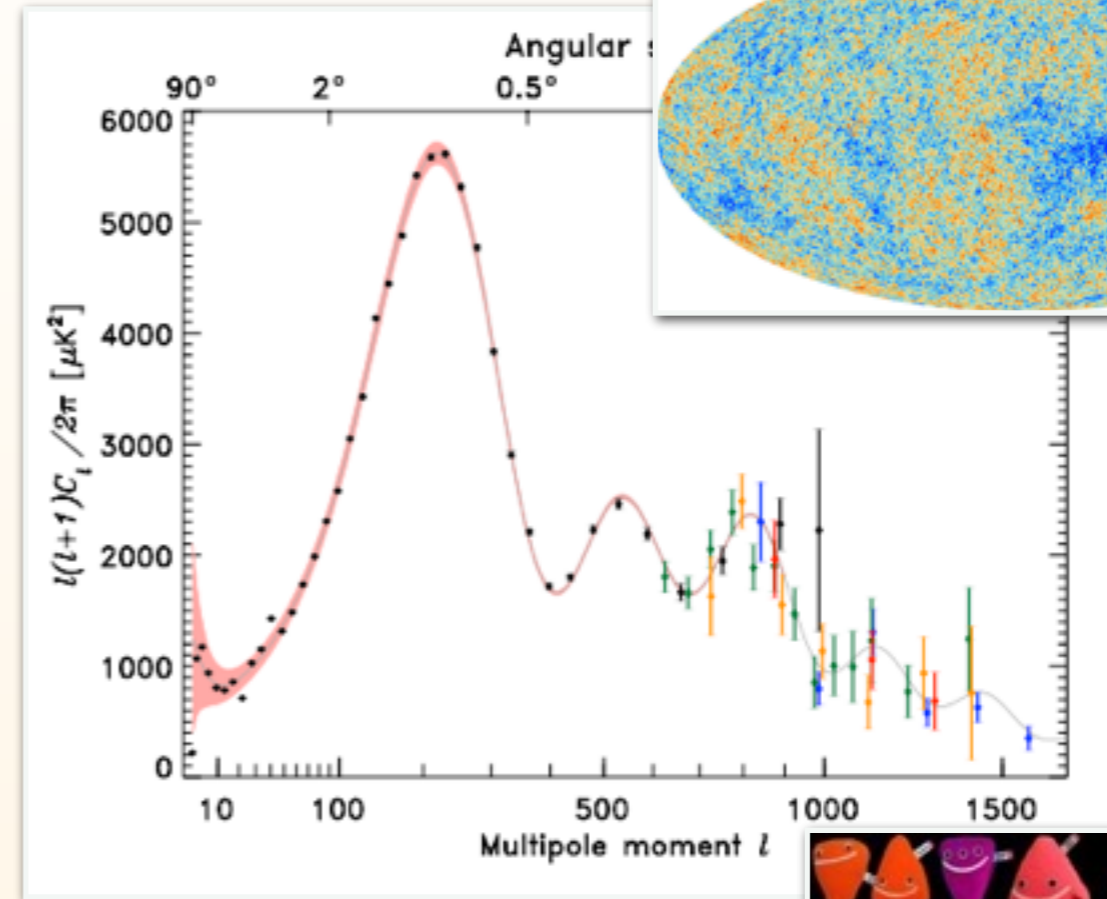
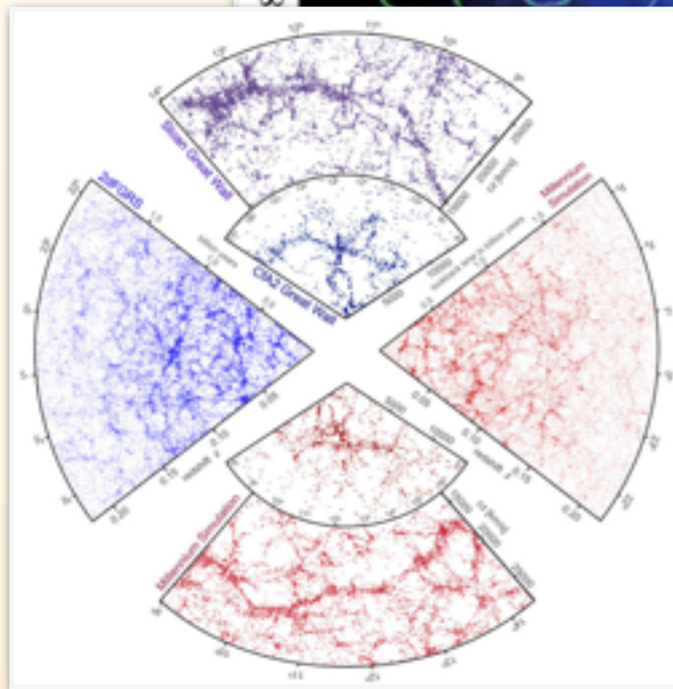
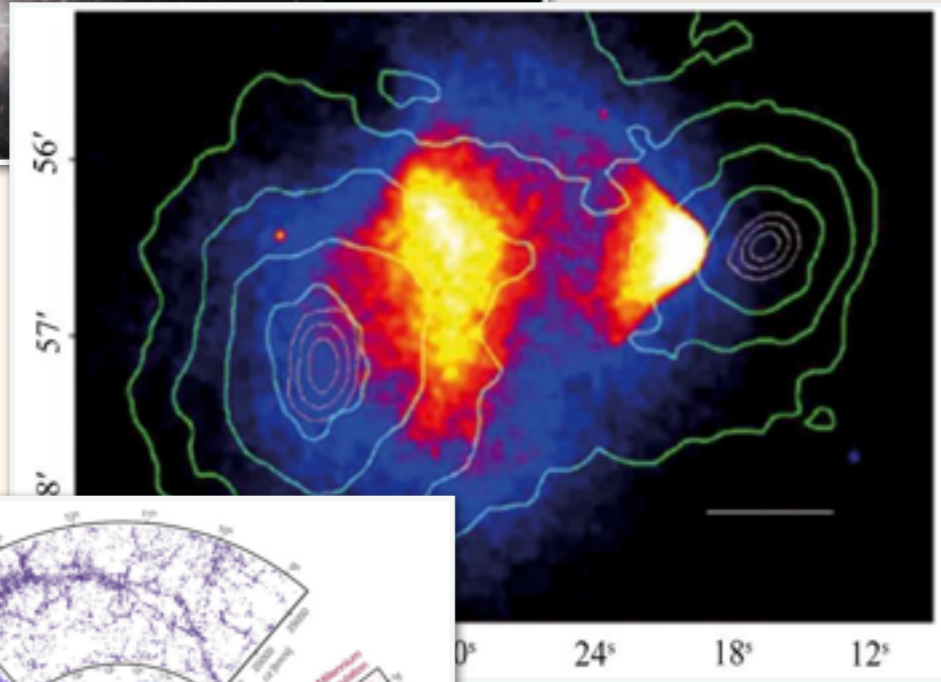
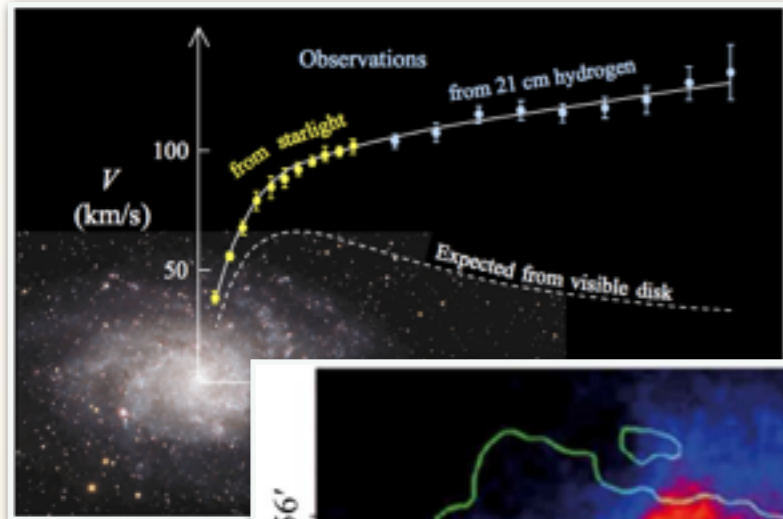
- **freeze-out vs. decoupling**
- significance for cosmology

## 3. Our work

- **early kinetic decoupling** with
- **velocity dependent** annihilation

## 4. Summary

# DARK MATTER IS EVERYWHERE!



⇒ Evidence on all scales!



# THE ORIGIN OF DARK MATTER

Dark matter could be created in many different ways...

...but every massive particle with not-too-weak interactions with the SM will be produced thermally, with relic abundance:

Lee, Weinberg '77; + others

$$\Omega_\chi h^2 \approx 0.1 \frac{3 \times 10^{-26} \text{cm}^3 \text{s}^{-1}}{\langle \sigma v \rangle}$$

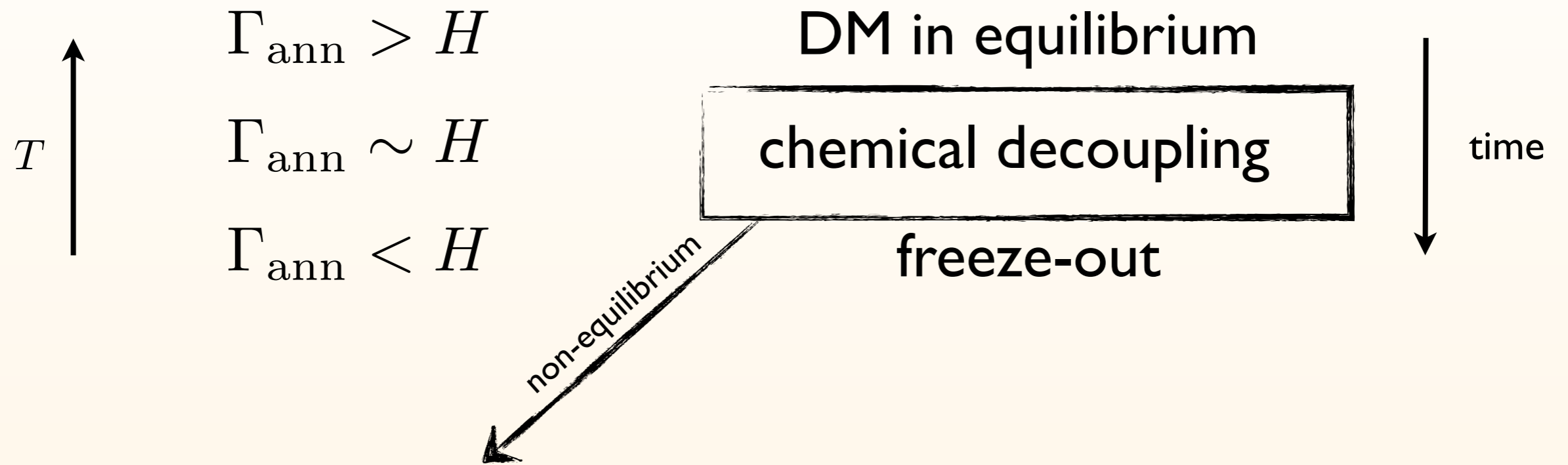
It is very natural to expect that this mechanism is responsible for the origin of **all of dark matter**

...but **even if not, it still is present nevertheless** and it's important to be able to correctly determine thermal population abundance



# THERMAL RELIC DENSITY

## STANDARD APPROACH



time evolution of  $f_\chi(p)$  in kinetic theory:

$$E (\partial_t - H \vec{p} \cdot \nabla_{\vec{p}}) f_\chi = \mathcal{C}[f_\chi] \implies \frac{dn_\chi}{dt} + 3Hn_\chi = C$$

Liouville operator in FRW background

the collision term

integrated

assumptions for using Boltzmann eq: classical limit, molecular chaos,...

...for derivation from thermal QFT see e.g., M. Beneke, F. Dighera, AH; JHEP 1410 (2014) 45 5

# THERMAL RELIC DENSITY

## THE COLLISION TERM

for  $2 \leftrightarrow 2$  CP invariant process:

$$C_{\text{LO}} = -h_{\chi}^2 \int \frac{d^3 \vec{p}_{\chi}}{(2\pi)^3} \frac{d^3 \vec{p}_{\bar{\chi}}}{(2\pi)^3} \sigma_{\chi\bar{\chi} \rightarrow ij} v_{\text{rel}} [f_{\chi} f_{\bar{\chi}} (1 \pm f_i)(1 \pm f_j) - f_i f_j (1 \pm f_{\chi})(1 \pm f_{\bar{\chi}})]$$

assuming kinetic equilibrium at chemical decoupling:  $f_{\chi} \sim a(\mu) f_{\chi}^{\text{eq}}$

$$C_{\text{LO}} = -\langle \sigma_{\chi\bar{\chi} \rightarrow ij} v_{\text{rel}} \rangle^{\text{eq}} (n_{\chi} n_{\bar{\chi}} - n_{\chi}^{\text{eq}} n_{\bar{\chi}}^{\text{eq}})$$

where the thermally averaged cross section:

$$\langle \sigma_{\chi\bar{\chi} \rightarrow ij} v_{\text{rel}} \rangle^{\text{eq}} = -\frac{h_{\chi}^2}{n_{\chi}^{\text{eq}} n_{\bar{\chi}}^{\text{eq}}} \int \frac{d^3 \vec{p}_{\chi}}{(2\pi)^3} \frac{d^3 \vec{p}_{\bar{\chi}}}{(2\pi)^3} \sigma_{\chi\bar{\chi} \rightarrow ij} v_{\text{rel}} f_{\chi}^{\text{eq}} f_{\bar{\chi}}^{\text{eq}}$$

# THERMAL RELIC DENSITY

## BOLTZMANN EQ.

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar{\chi}\rightarrow ij}\sigma_{\text{rel}}\rangle^{\text{eq}} (n_\chi n_{\bar{\chi}} - n_\chi^{\text{eq}} n_{\bar{\chi}}^{\text{eq}})$$

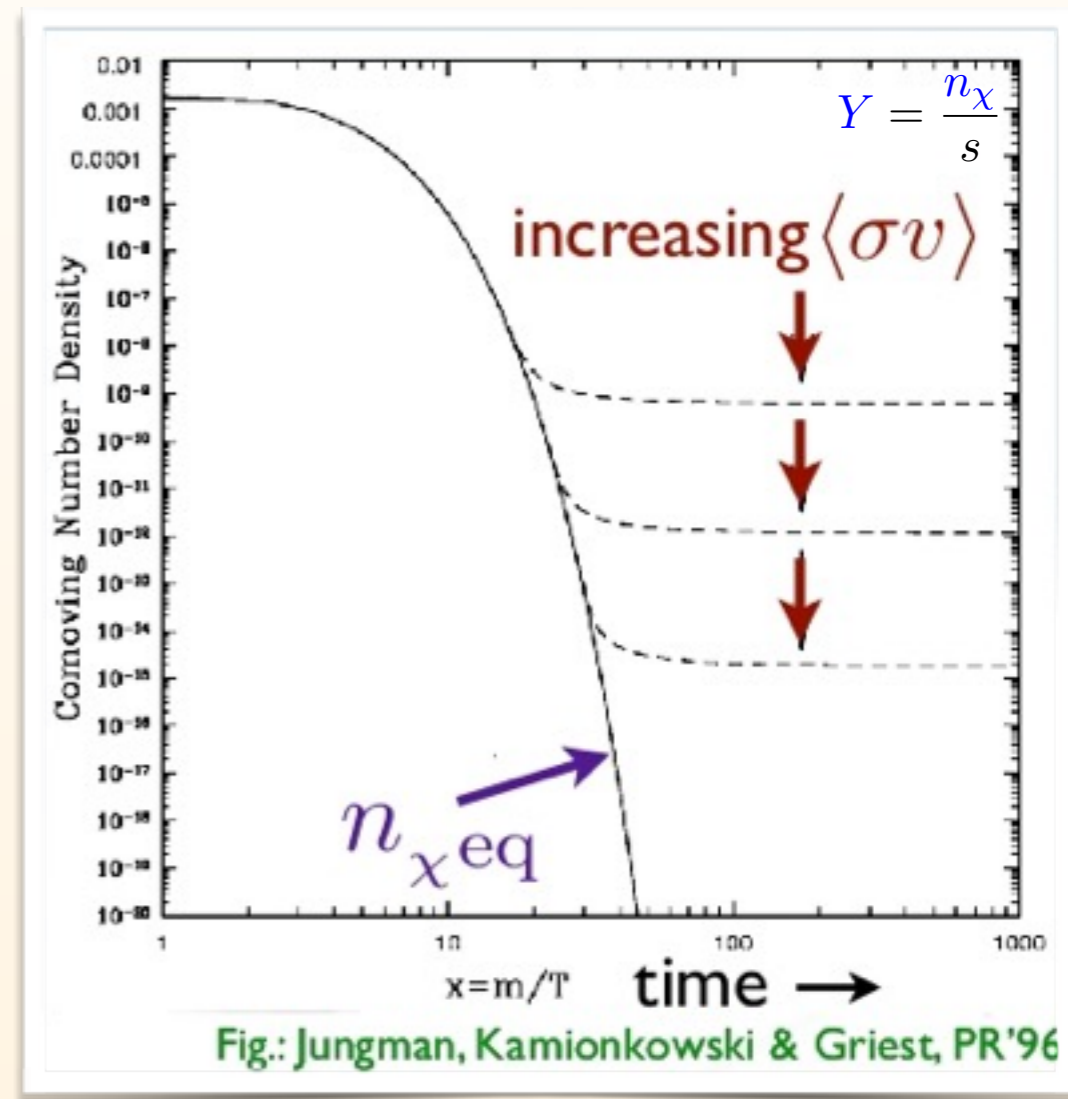
Re-written for the comoving number density:

$$\frac{dY}{dx} = \sqrt{\frac{g_* \pi m_\chi^2}{45G}} \frac{\langle\sigma_{\chi\bar{\chi}\rightarrow ij}\sigma_{\text{rel}}\rangle^{\text{eq}}}{x^2} (Y^2 - Y_{\text{eq}}^2)$$

$$\lim_{x\rightarrow 0} Y = Y_{\text{eq}} \quad \lim_{x\rightarrow\infty} Y = \text{const}$$

Recipe:

compute annihilation **cross-section**,  
 take a **thermal bath average**,  
 throw it into **BE**... and voilà



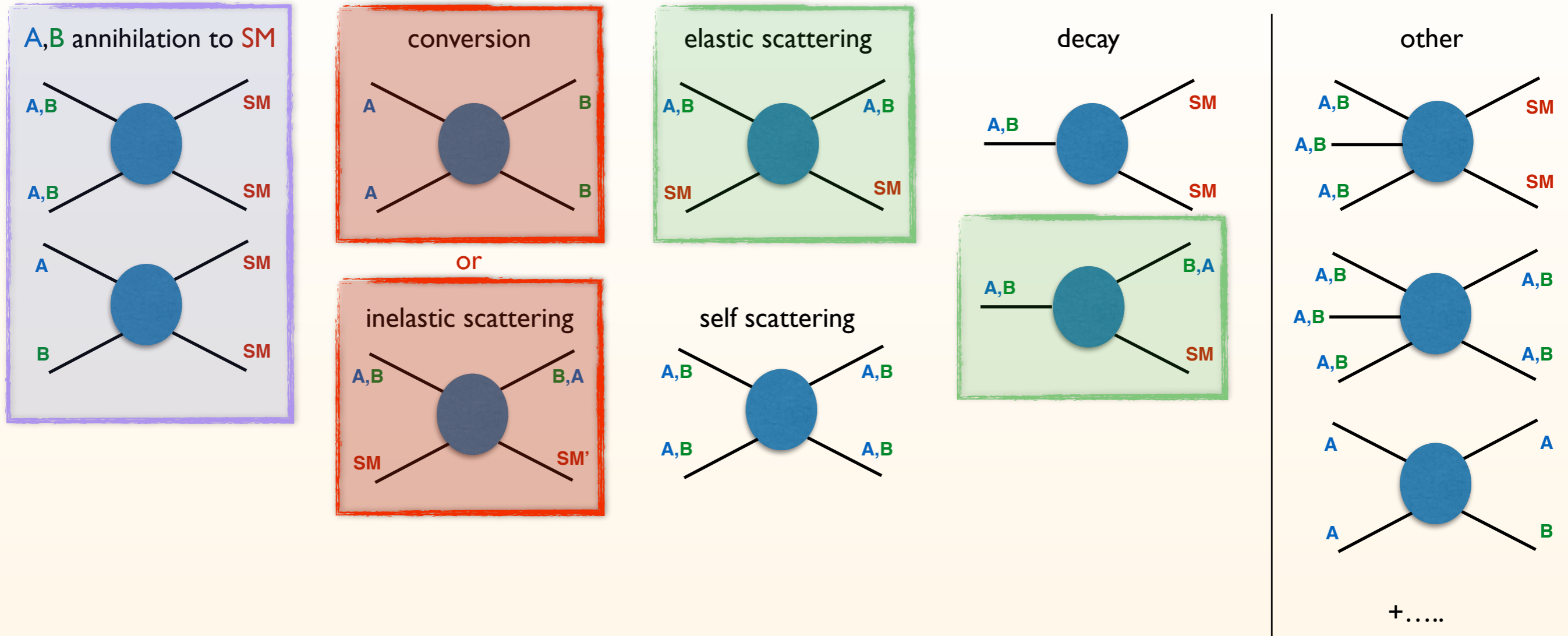
# THERMAL RELIC DENSITY

## “EXCEPTIONS”

1. Three “exceptions”  
Griest, Seckel '91
2. Non-standard cosmology  
many works... very recent e.g., D'Eramo, Fernandez, Profumo '17
3. Bound State Formation  
recent e.g., Petraki et al. '15, '16; An et al. '15, '16; Cirelli et al. '16; ...
4.  $3 \rightarrow 2$  and  $4 \rightarrow 2$  annihilation  
e.g., D'Agnolo, Ruderman '15; Cline et al. '17; Choi et al. '17; ...
5. Second era of annihilation  
Feng et al. '10; Bringmann et al. '12; ...
6. Semi-annihilation  
D'Eramo, Thaler '10; ...
7. Cannibalization  
e.g., Kuflik et al. '15; Pappadopulo et al. '16; ...
8. ...

In other words: whenever studying non-minimal scenarios “exceptions” appear —  
but most of them come from interplay of **new added effects**,  
while do **not affect the foundations** of modern calculations

# WHAT IF NON-MINIMAL SCENARIO?



Co-annihilation  $\longrightarrow$   
 Griest, Seckel '91

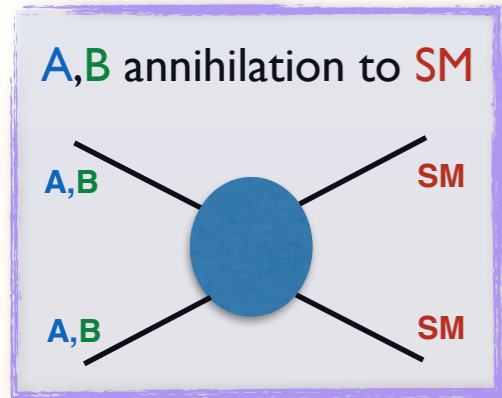
due to **efficient conversion processes** one can trace only number density of sum of the states with shared conserved quantum number using **weighted annihilation cross section**

+.....  
 $\hookrightarrow$  typically forbidden by symmetry

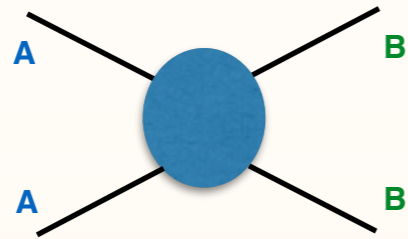
- needed to be efficient for mechanism to work
- setting the relic density
- assumed in computation



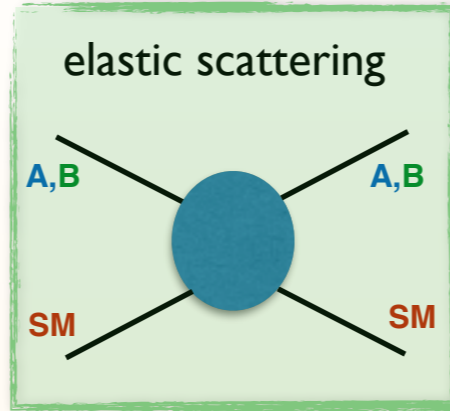
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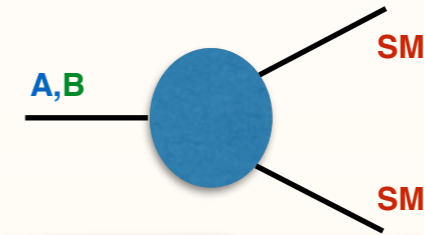
conversion



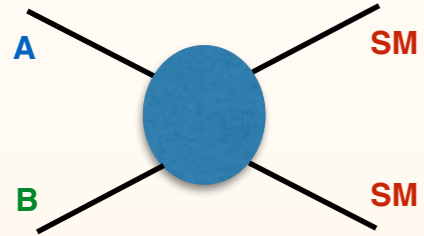
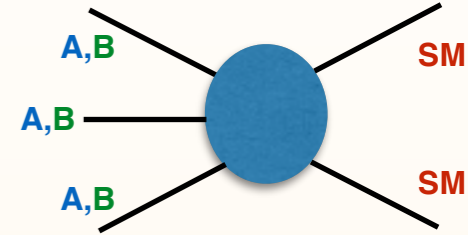
elastic scattering



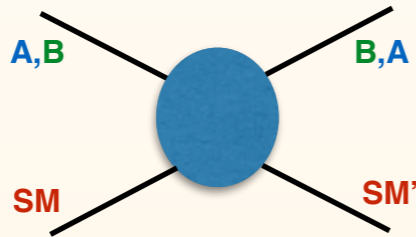
decay



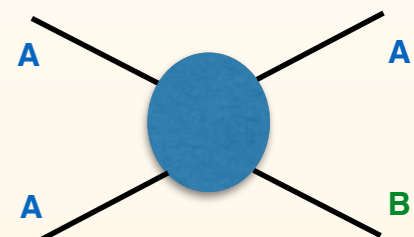
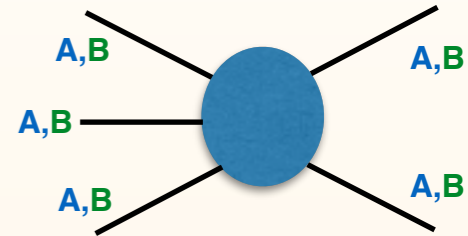
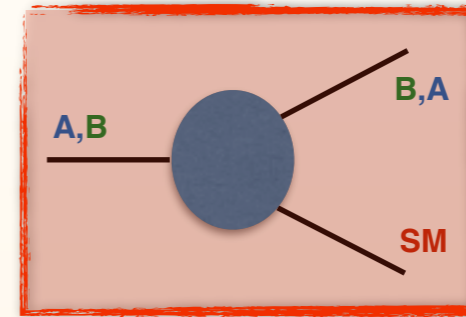
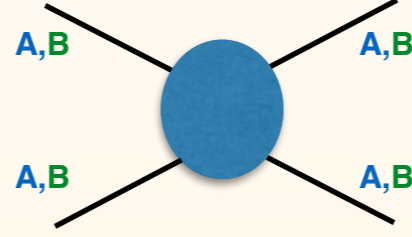
other



inelastic scattering



self scattering



+.....

↳ typically forbidden by symmetry

SuperWIMP



Feng, Rajaraman, Takayama '03

DM abundance is an effect of complete decay of heavier state which freezes-out as standard WIMP



needed to be efficient for mechanism to work

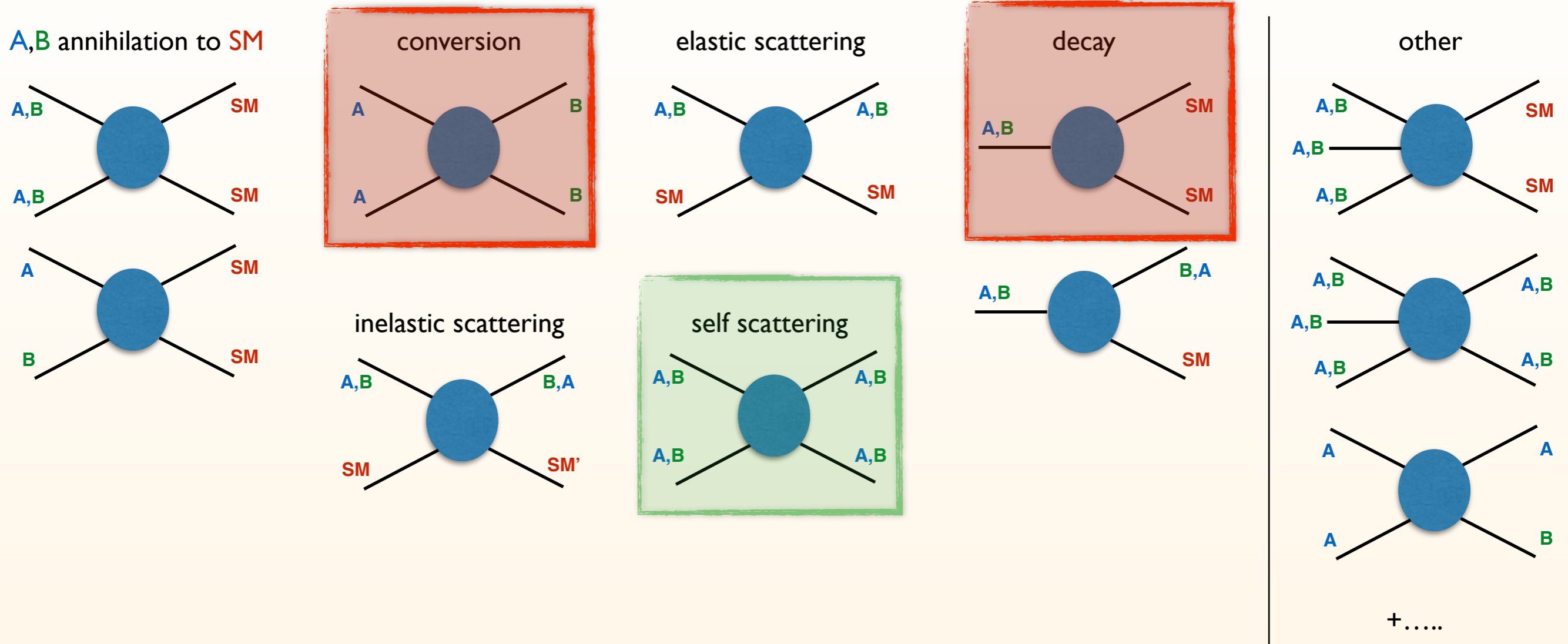


setting the relic density



assumed in computation

# WHAT IF NON-MINIMAL SCENARIO?



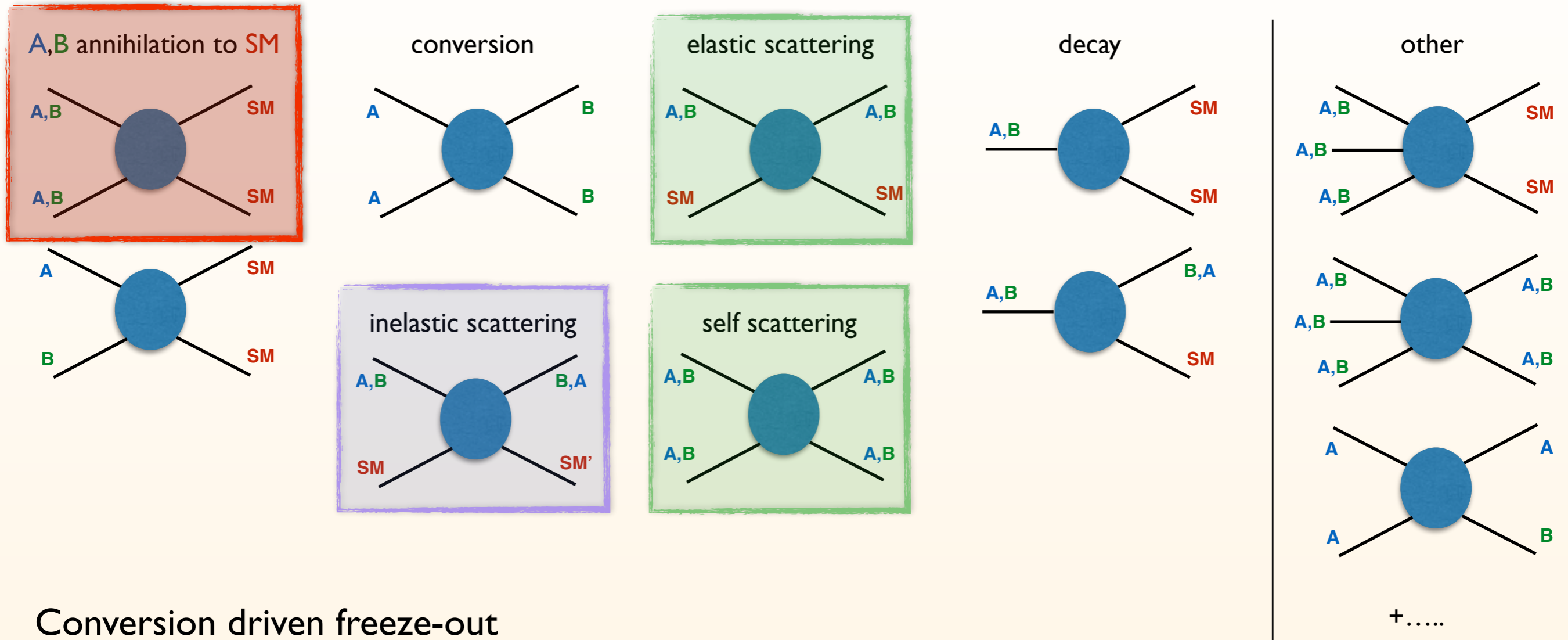
Co-decaying  
Dror, Kuflik, Ng '16

DM decouples when relativistic but then one of the dark sector states **decays** and this effect important as long as **conversions** are

+.....  
↳ typically forbidden by symmetry

- needed to be efficient for mechanism to work
- setting the relic density
- assumed in computation

# WHAT IF NON-MINIMAL SCENARIO?



Conversion driven freeze-out

Garny, Heisig, Lulf, Vogl '17

Co-scattering




D'Agnolo, Pappadopulo, Ruderman '17



only one of the dark sector states annihilates efficiently, but also conversions stop being efficient which blocks co-annihilation

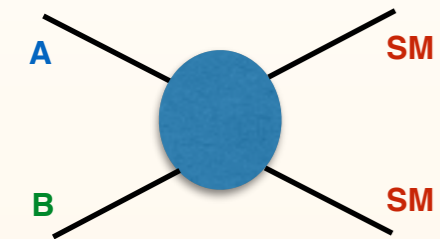
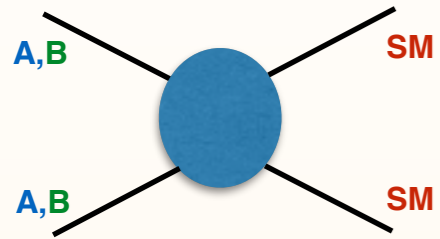
+.....

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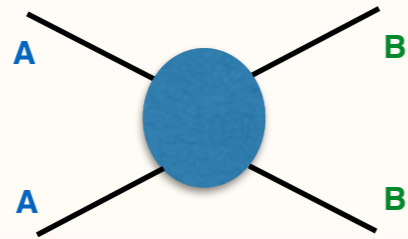
-  needed to be efficient for mechanism to work
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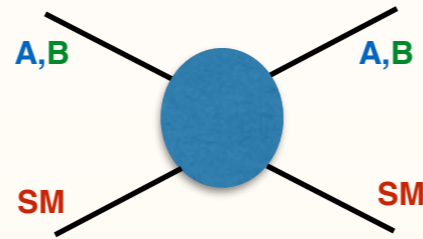
A,B annihilation to SM



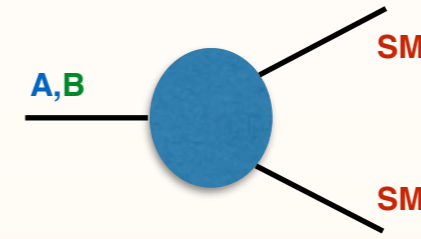
conversion



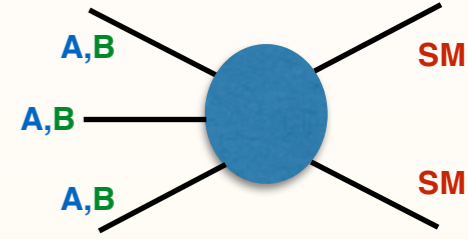
elastic scattering



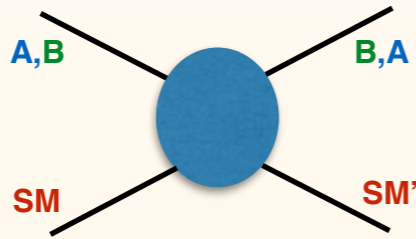
decay



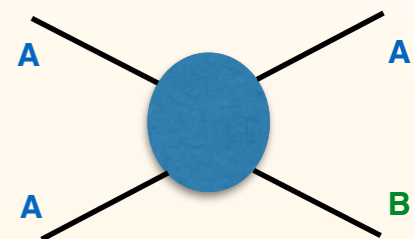
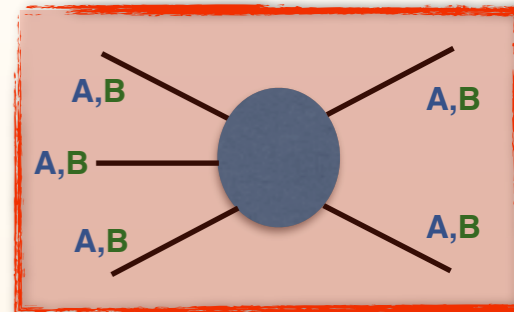
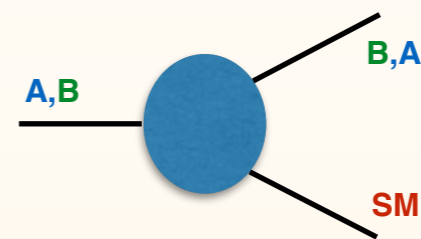
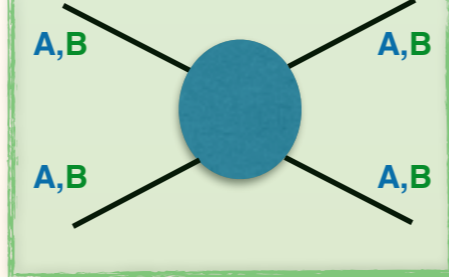
other



inelastic scattering



self scattering



Cannibalization →

Kuflik et al. '15

secluded dark sector, with efficient 3-2 annihilation leading to **DM zero chemical potential** and keeping DM at **much higher temperature** than SM plasma

+.....

↳ typically forbidden by symmetry



needed to be efficient for mechanism to work



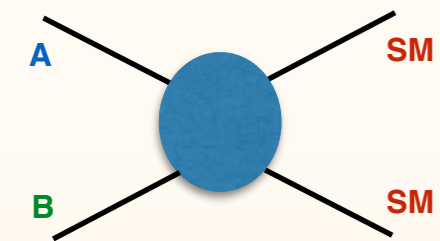
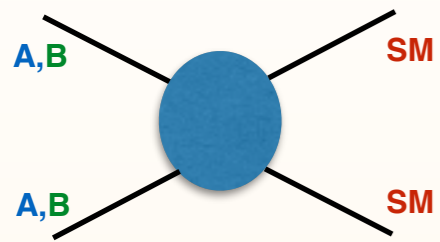
setting the relic density



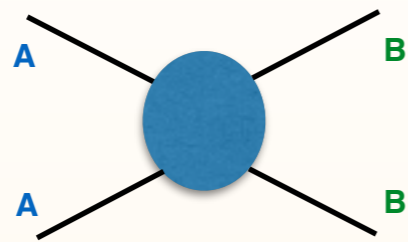
assumed in computation

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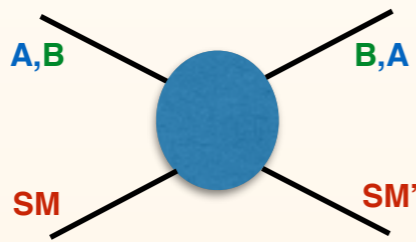
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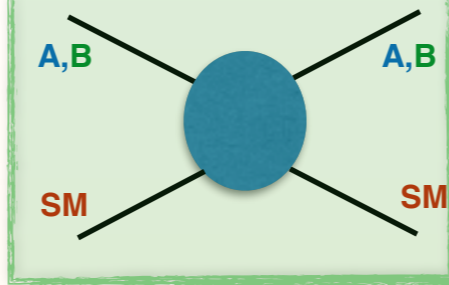
conversion



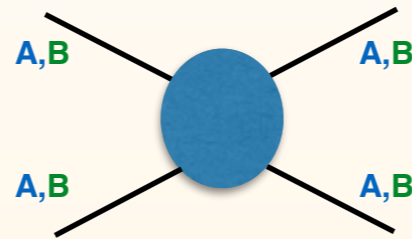
inelastic scattering



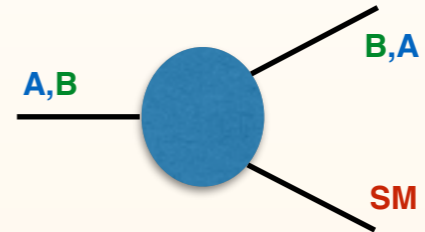
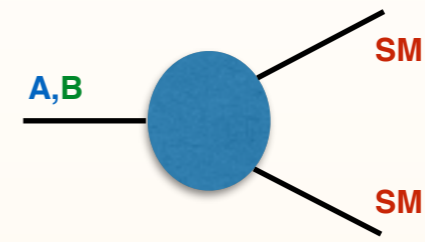
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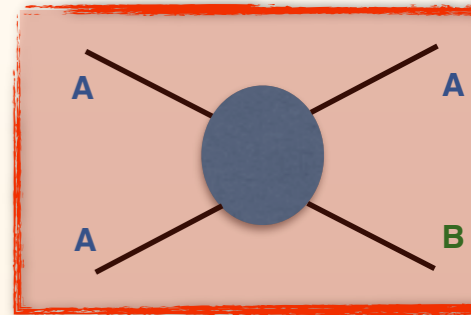
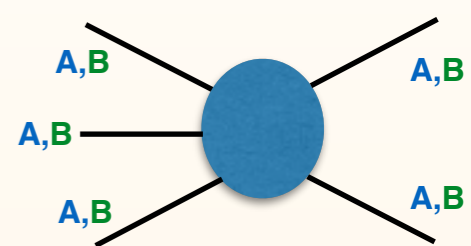
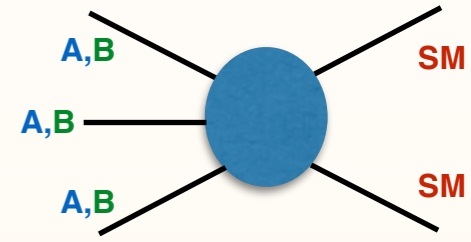
self scattering



decay



other



+.....

↳ typically forbidden by symmetry

Semi-annihilation  $\longrightarrow$   
D'Eramo, Thaler '10

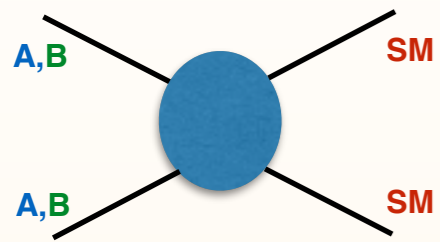
new **type of annihilation** process that can dominate the freeze-out dynamics; occurs when new „flavour” or „baryon” structure in dark sector

- needed to be efficient for mechanism to work
- setting the relic density
- assumed in computation

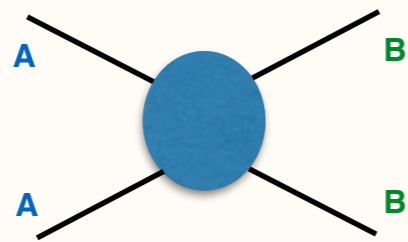


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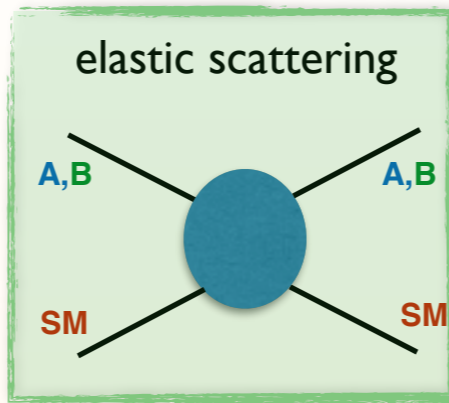
A,B annihilation to SM



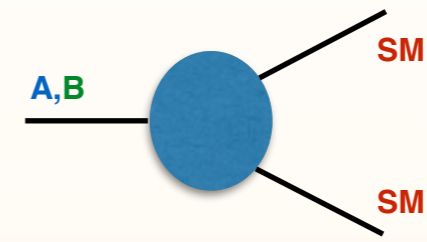
conversion



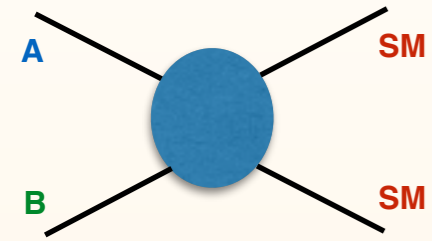
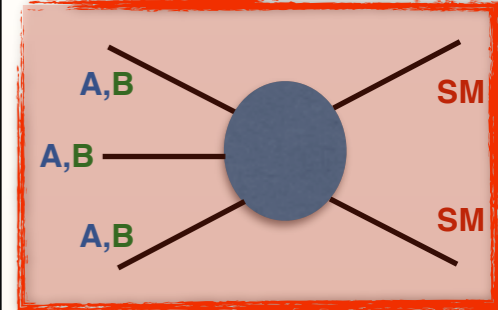
elastic scattering



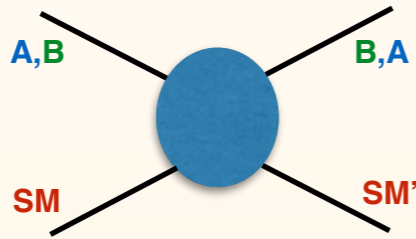
decay



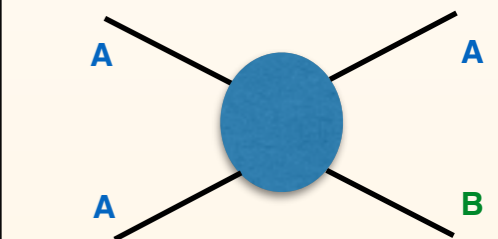
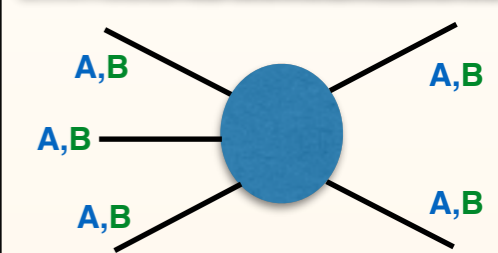
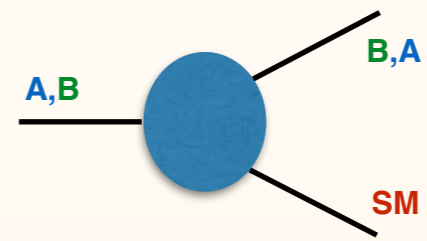
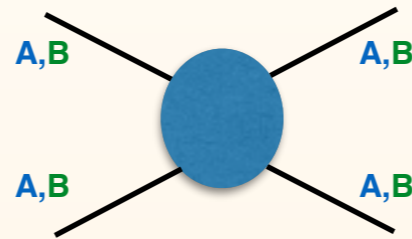
other



inelastic scattering



self scattering




Forbidden-like DM  $\longrightarrow$

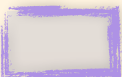
e.g., D'Agnolo, Ruderman '15;  
Cline et al. '17; Choi et al. '17;


annihilation to a state heavier than DM,  
possible from tail of the Boltzmann  
distribution or when 3-2 processes are  
non-negligible

+.....

$\longleftarrow$  typically  
forbidden by  
symmetry

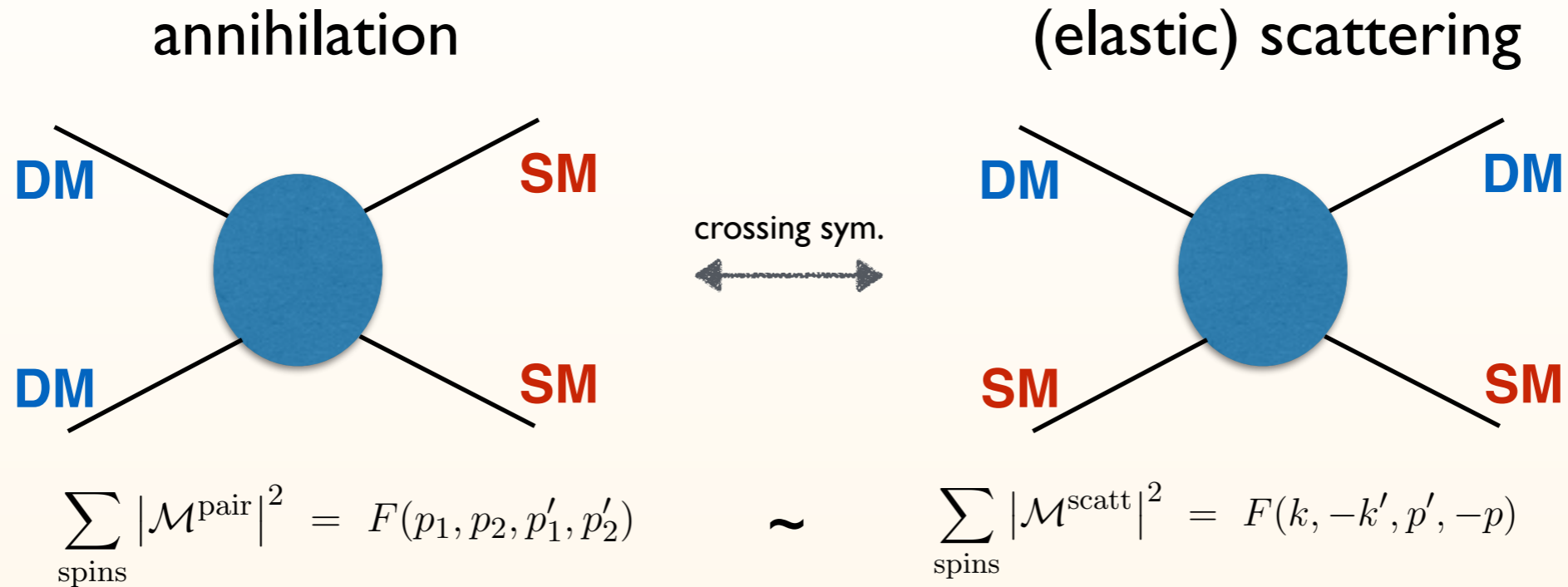
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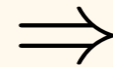
 assumed in computation

# KINETIC DECOUPLING

# FREEZE-OUT vs. DECOUPLING



Boltzmann suppression of **DM** vs. **SM**



scatterings typically more frequent

dark matter frozen-out but typically still kinetically coupled to the plasma

$$\tau_r(T_{\text{kd}}) \equiv N_{\text{coll}}/\Gamma_{\text{el}} \sim H^{-1}(T_{\text{kd}})$$

Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

Two consequences:

1. During freeze-out (chemical decoupling) typically:  $f_\chi \sim a(\mu) f_\chi^{\text{eq}}$
2. If kinetic decoupling much, much later: possible impact on the matter power spectrum  
i.e. kinetic decoupling can have observable consequences and affect e.g. missing satellites problem

see e.g., Bringmann, Ihle, Karsten, Valia '16

# IMPLICATIONS OF KINETIC DECOUPLING

E.g. for SUSY neutralino:

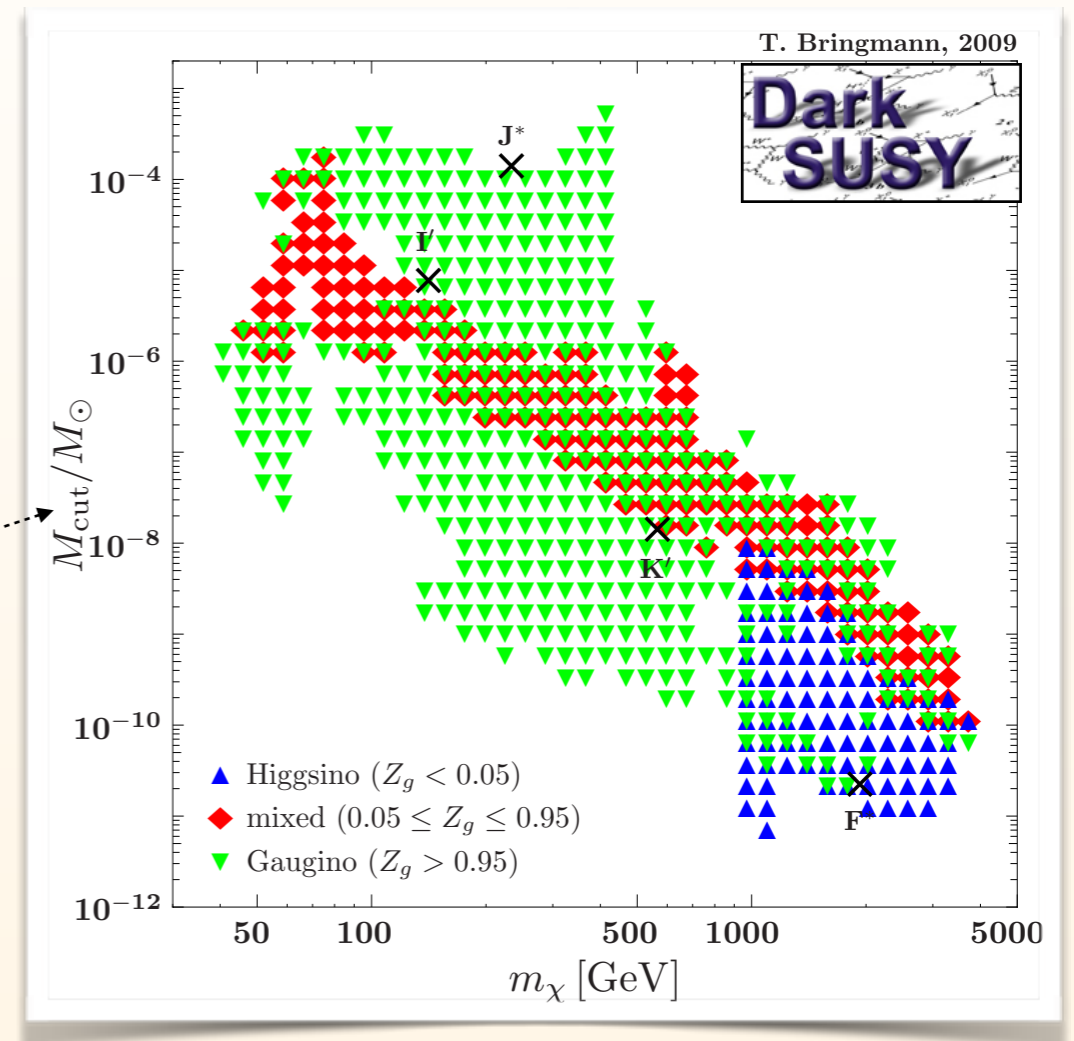
Bringmann '09

Free-streaming of DM after KD washes out **density contrasts at small scales** (similarly to baryonic oscillations)

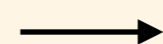
Green, Hofmann, Schwarz '05



Cut-off in the power spectrum corresponding to **smallest gravitationally bound objects**



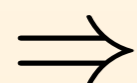
„Typical” values for **WIMPs** are relatively small



small substructures expected

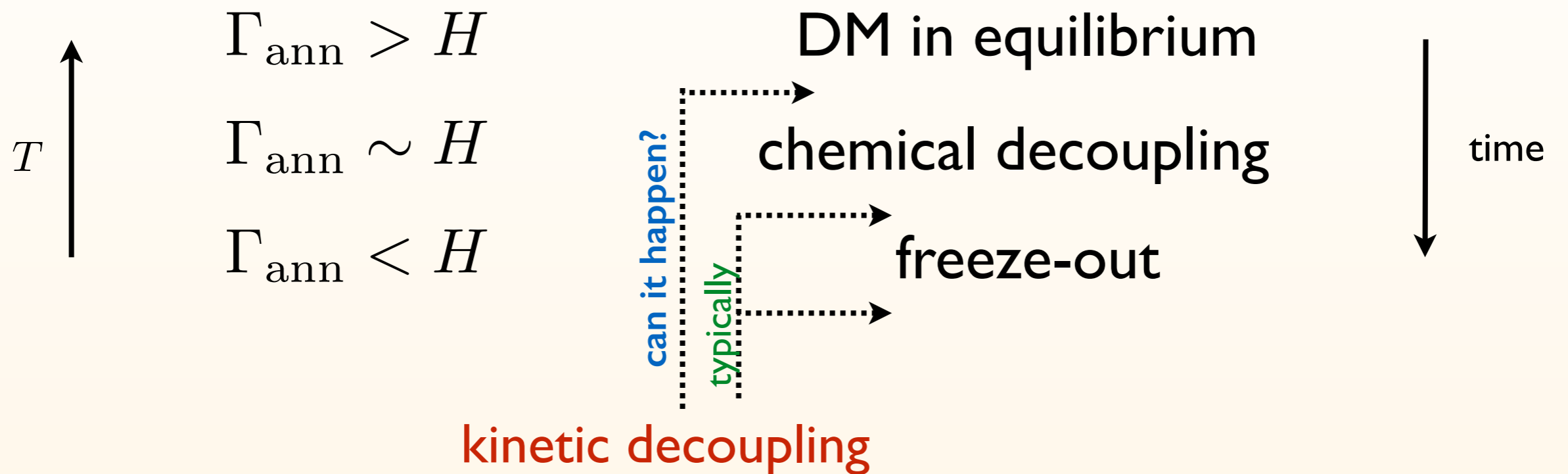


but bad for missing satellites problem



moment of KD leaves important imprint on the Universe

# A PITFALL IN A NUTSHELL



If **KD** happens around CD  $\longrightarrow$  what would be the relic density?

assuming kinetic equilibrium at chemical decoupling:  $f_\chi \sim a(\mu) f_\chi^{\text{eq}}$

$$C_{\text{LO}} = -\langle \sigma_{\chi\bar{\chi} \rightarrow ij} v_{\text{rel}} \rangle^{\text{eq}} (n_\chi n_{\bar{\chi}} - n_\chi^{\text{eq}} n_{\bar{\chi}}^{\text{eq}})$$

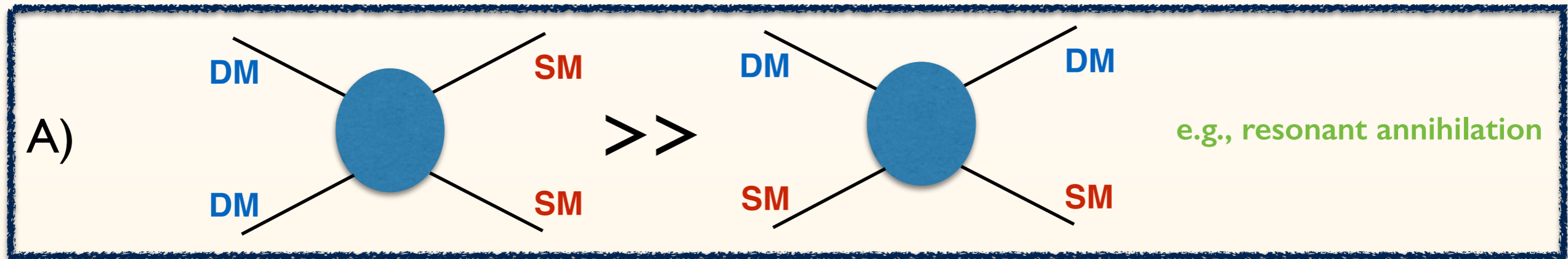
$\downarrow$   
 how to even compute that?  $\implies$  need for refined treatment of solving the Boltzmann eq.



# EARLY KINETIC DECOUPLING?

A **necessary** and **sufficient** condition: scatterings weaker than annihilation  
i.e. rates around freeze-out:  $H \sim \Gamma_{\text{ann}} \gtrsim \Gamma_{\text{el}}$

Possibilities:



B) Boltzmann suppression of **SM** as strong as for **DM**  
e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure  
e.g., semi-annihilation, 3 to 2 models,...

# EARLY KD AND RESONANCE

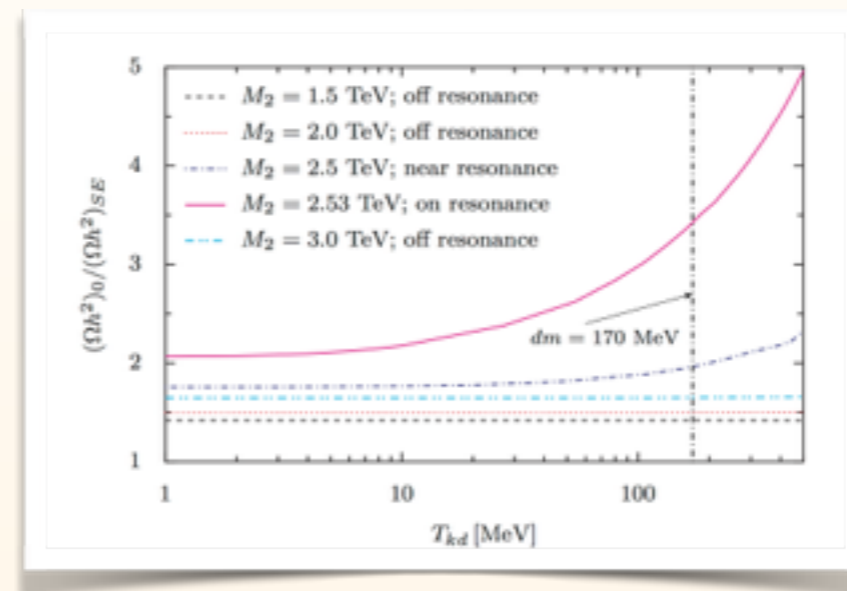
our work wasn't the first to realize that resonant annihilation can lead to early kinetic decoupling...

Feng, Kaplinghat, Yu '10 — noted that for Sommerfeld-type resonances KD can happen early

Dent, Dutta, Scherrer '10 — looked at potential effect of KD on thermal relic density

Since then people were aware of this effect and sometimes tried to estimate it assuming **instantaneous KD**, e.g., in the case of Sommerfeld effect in the MSSM:

but **no systematic studies** of decoupling process were performed, until...



AH, Iengo, Ullio '11

...models with very late KD become popular, in part to solve „missing satellites” problem  
van den Aarssen et al '12; Bringmann et al '16, x2; Binder et al '16

this progress allowed for **better approach to early KD** scenarios as well and was applied to the **resonant annihilation case** in

Duch, Grządkowski '17

... but we developed a **dedicated accurate method/code** to deal with this and other similar situations

# HOW TO DESCRIBE KD?

All information is in full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E (\partial_t - H\vec{p} \cdot \nabla_{\vec{p}}) f_x = \mathcal{C}[f_x]$$

contains both scatterings and annihilation

Two possible approaches:

solve numerically  
for full  $f_x(p)$

have insight on the distribution  
no constraining assumptions

numerically challenging  
typically overkill

consider system of equations  
for moments of  $f_x(p)$

partially analytic/much easier numerically  
manifestly captures all of the relevant physics

finite range of validity  
no insight on the distribution

0-th moment:  $n_x$   
2-nd moment:  $T_x$

...

# SCATTERING

The **elastic scattering** collision term:

$$\begin{aligned}
 C_{\text{el}} = & \frac{1}{2g_\chi} \int \frac{d^3k}{(2\pi)^3 2\omega} \int \frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}} \int \frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{E}} \\
 & \times (2\pi)^4 \delta^{(4)}(\tilde{p} + \tilde{k} - p - k) |\mathcal{M}|_{\chi f \leftrightarrow \chi f}^2 \\
 & \times \left[ (1 \mp g^\pm)(\omega) g^\pm(\tilde{\omega}) f_\chi(\tilde{\mathbf{p}}) - (\omega \leftrightarrow \tilde{\omega}, \mathbf{p} \leftrightarrow \tilde{\mathbf{p}}) \right] \\
 & \quad \quad \quad \downarrow \\
 & \quad \quad \quad \text{equilibrium functions for SM particles}
 \end{aligned}$$

Expanding in **NR** and small momentum transfer: [Bringmann, Hofmann '06](#)

$$C_{\text{el}} \simeq \frac{m_\chi}{2} \gamma(T) \left[ T m_\chi \partial_p^2 + \left( p + 2T \frac{m_\chi}{p} \right) \partial_p + 3 \right] f_\chi$$

More generally, Fokker-Planck scattering operator  
(relativistic, but still small momentum transfer): [Binder et al. '16](#)

physical interpretation:  
**scattering rate**

$$C_{\text{el}} \simeq \frac{E}{2} \nabla_{\mathbf{p}} \cdot \left[ \gamma(T, \mathbf{p}) (ET \nabla_{\mathbf{p}} + \mathbf{p}) f_\chi \right]$$

Semi-relativistic: assume that scattering  $\gamma(T, \mathbf{p})$  is momentum independent

# KINETIC DECOUPLING 101

DM temperature  
Definition:

$$T_\chi \equiv \frac{g_\chi}{3m_\chi n_\chi} \int \frac{d^3 p}{(2\pi)^3} p^2 f_\chi(p)$$

$$y \equiv \frac{m_\chi T_\chi}{s^{2/3}}$$

actually: normalized average NR energy - equals temperature at equilibrium

First take late KD scenario and consider only **temperature evolution** -  
i.e. leave out feedback **on/from** changing **number density**:

then 2nd moment of full BE (up to terms  $p^2/m_\chi^2$ ) gives:

$$\frac{y'}{y} = -\frac{Y'}{Y} \left( 1 - \frac{\langle \sigma v_{\text{rel}} \rangle_2}{\langle \sigma v_{\text{rel}} \rangle} \right) - \left( 1 - \frac{x}{3} \frac{g'_{*S}}{g_{*S}} \right) \frac{2m_\chi c(T)}{Hx} \left( 1 - \frac{y_{\text{eq}}}{y} \right)$$

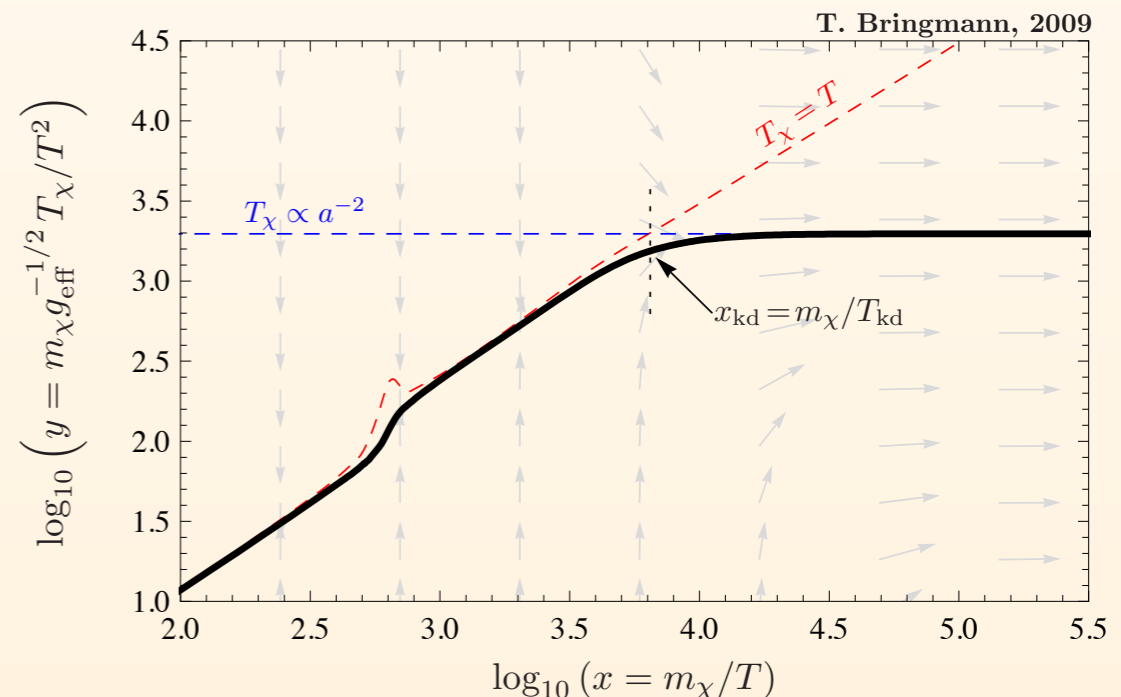
where:

$$\langle \sigma v_{\text{rel}} \rangle_2 \equiv \frac{g_\chi^2}{3Tm_\chi n_\chi^2} \int \frac{d^3 p}{(2\pi)^3} \int \frac{d^3 \tilde{p}}{(2\pi)^3} p^2 v_{\text{rel}} \sigma_{\bar{\chi}\chi \rightarrow \bar{X}X} f(E) f(\tilde{E})$$

impact of **annihilation**

$$c(T) = \frac{1}{12(2\pi)^3 m_\chi^4 T} \sum_X \int dk k^5 \omega^{-1} g^\pm (1 \mp g^\pm) \int_{-4k^2}^0 (-t) \frac{1}{8k^4} |\mathcal{M}_{\text{el}}|^2$$

impact of elastic  
**scatterings**





# ONE STEP FURTHER...

Now consider general KD scenario, i.e. coupled **temperature** and **number density** evolution:

annihilation and production thermal averages done at different  $T$  — feedback of modified  $y$  evolution

$$\frac{Y'}{Y} = -\frac{1 - \frac{x}{3} \frac{g'_{*S}}{g_{*S}}}{Hx} sY \left( \langle \sigma v_{\text{rel}} \rangle \Big|_{x=m_\chi^2/(s^{2/3}y)} - \frac{Y_{\text{eq}}^2}{Y^2} \langle \sigma v_{\text{rel}} \rangle \Big|_x \right)$$

$$\frac{y'}{y} = -\frac{1 - \frac{x}{3} \frac{g'_{*S}}{g_{*S}}}{Hx} \left[ 2m_\chi c(T) \left( 1 - \frac{y_{\text{eq}}}{y} \right) - sY \left( \left( \langle \sigma v_{\text{rel}} \rangle - \langle \sigma v_{\text{rel}} \rangle_2 \right) \Big|_{x=m_\chi^2/(s^{2/3}y)} - \frac{Y_{\text{eq}}^2}{Y^2} \left( \langle \sigma v_{\text{rel}} \rangle - \frac{y_{\text{eq}}}{y} \langle \sigma v_{\text{rel}} \rangle_2 \right) \Big|_x \right) \right]$$

$$+ \frac{1 - \frac{x}{3} \frac{g'_{*S}}{g_{*S}}}{3m_\chi} \langle p^4/E^3 \rangle_{x=m_\chi^2/(s^{2/3}y)}$$

"relativistic" term

elastic scatterings term

impact of annihilation

$$T_\chi \equiv \frac{g_\chi}{3n_\chi} \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{E} f_\chi(p)$$

These equations still assume the equilibrium shape of  $f_\chi(p)$  — but with variant temperature

or more accurately: that the thermal averages computed with true non-equilibrium distributions don't differ much from the above ones

# NUMERICAL APPROACH

... or one can just solve full phase space Boltzmann eq.

$$\begin{aligned}
 \partial_x f_\chi(x, q) &= \frac{m_\chi^3}{\tilde{H}x^4} \frac{g_{\bar{\chi}}}{2\pi^2} \int d\tilde{q} \tilde{q}^2 \frac{1}{2} \int d\cos\theta v_{M\phi l} \sigma_{\bar{\chi}\chi \rightarrow \bar{f}f} \\
 &\times [f_{\chi,eq}(q) f_{\chi,eq}(\tilde{q}) - f_\chi(q) f_\chi(\tilde{q})] \\
 &+ \frac{2m_\chi c(T)}{2\tilde{H}x} \left[ x_q \partial_q^2 + \left( q + \frac{2x_q}{q} + \frac{q}{x_q} \right) \partial_q + 3 \right] f_\chi \\
 &+ \tilde{g} \frac{q}{x} \partial_q f_\chi,
 \end{aligned}$$

fully general

expanded in NR and small momentum transfer (semi-relativistic!)

discretization,  
~1000 steps

$$\begin{aligned}
 \partial_x f_i &= \\
 &\frac{m_\chi^3}{\tilde{H}x^4} \frac{g_{\bar{\chi}}}{2\pi^2} \sum_{j=1}^{N-1} \frac{\Delta\tilde{q}_j}{2} \left[ \tilde{q}_j^2 \langle v_{M\phi l} \sigma_{\bar{\chi}\chi \rightarrow \bar{f}f} \rangle_{i,j}^\theta (f_i^{\text{eq}} f_j^{\text{eq}} - f_i f_j) \right. \\
 &+ \left. \tilde{q}_{j+1}^2 \langle v_{M\phi l} \sigma_{\bar{\chi}\chi \rightarrow \bar{f}f} \rangle_{i,j+1}^\theta (f_i^{\text{eq}} f_{j+1}^{\text{eq}} - f_i f_{j+1}) \right] \\
 &+ \frac{2m_\chi c(T)}{2\tilde{H}x} \left[ x_{q,i} \partial_q^2 + \left( q_i + \frac{2x_{q,i}}{q_i} + \frac{q_i}{x_{q,i}} \right) \partial_q + 3 \right] f_i \\
 &+ \tilde{g} \frac{q_i}{x} \partial_q f_i,
 \end{aligned}$$

Solved numerically with MatLab

Note:

can be extended to e.g. self-scatterings

very stiff, care needed with numerics

**EXAMPLE:**  
**SCALAR SINGLET DM**

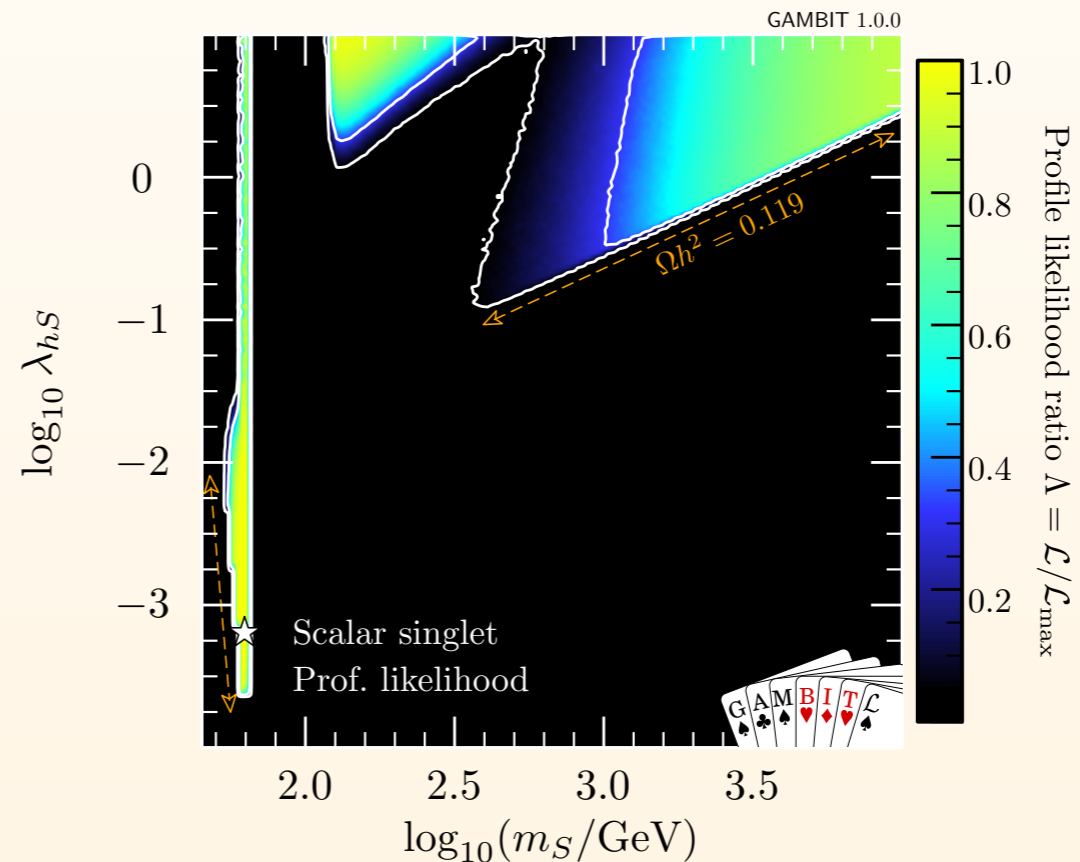
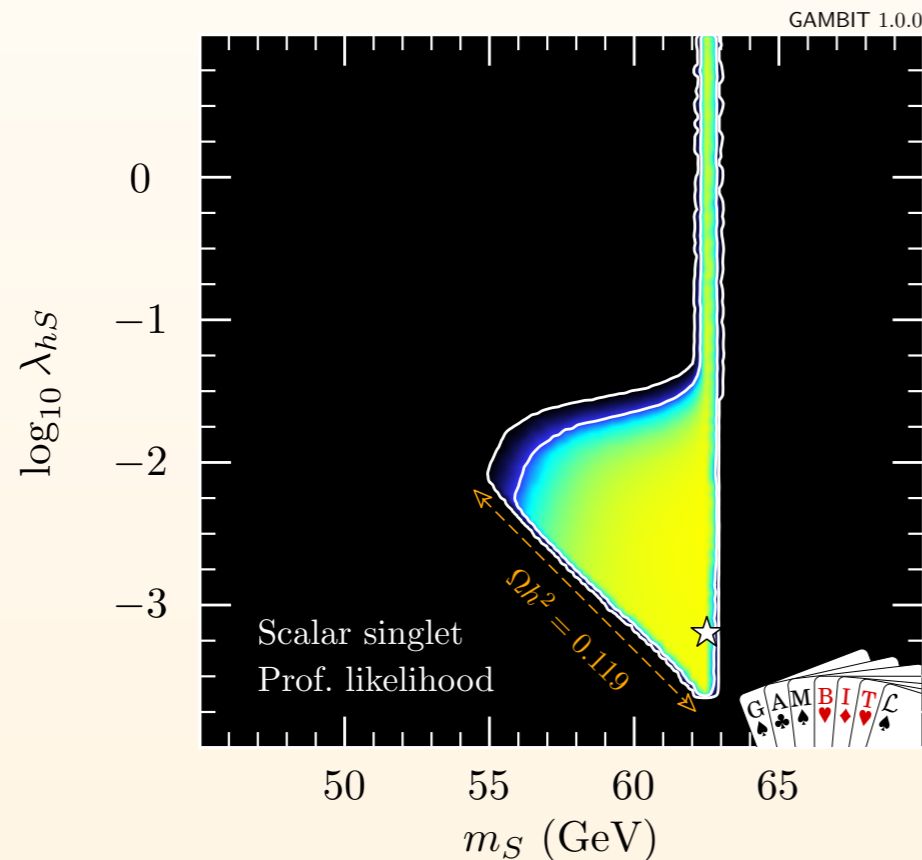
# SCALAR SINGLET DM

## VERY SHORT INTRODUCTION

To the SM Lagrangian add one singlet scalar field  $S$  with interactions with the Higgs:

$$\mathcal{L}_S = \frac{1}{2} \partial_\mu S \partial^\mu S - \frac{1}{2} \mu_S^2 S^2 - \frac{1}{2} \lambda_s S^2 |H|^2$$

$$m_s = \sqrt{\mu_S^2 + \frac{1}{2} \lambda_s v_0^2}$$



GAMBIT collaboration  
1705.07931

Most of the parameter space excluded, but... even such a simple model is hard to kill

# SCALAR SINGLET DM

## ANNIHILATION VS. SCATTERINGS

$$\sigma v_{\text{rel}} = \frac{2\lambda_s^2 v_0^2}{\sqrt{s}} |D_h(s)|^2 \Gamma_h(\sqrt{s})$$

with:

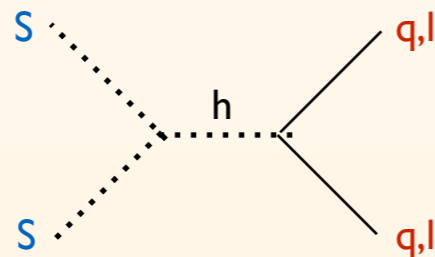
$$|D_h(s)|^2 \equiv \frac{1}{(s - m_h^2)^2 + m_h^2 \Gamma_h^2(m_h)}$$

tabulated  
Higgs width

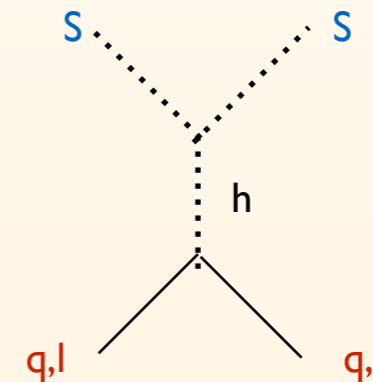
$$\langle |\mathcal{M}|^2 \rangle_t = \sum_f \frac{N_f \lambda_S^2 m_f^2}{8k^4} \left[ \frac{2k_{\text{cm}}^2 - 2m_f^2 + m_h^2}{1 + m_h^2/(4k_{\text{cm}}^2)} - (m_h^2 - 2m_f^2) \log(1 + 4k_{\text{cm}}^2/m_h^2) \right]$$

**Hierarchical Yukawa couplings:** strongest coupling to more Boltzmann suppressed quarks/leptons

Annihilation  
processes:  
**resonant**



El. scattering  
processes:  
**non-resonant**



Freeze-out at few GeV → what is the abundance of heavy quarks in QCD plasma?

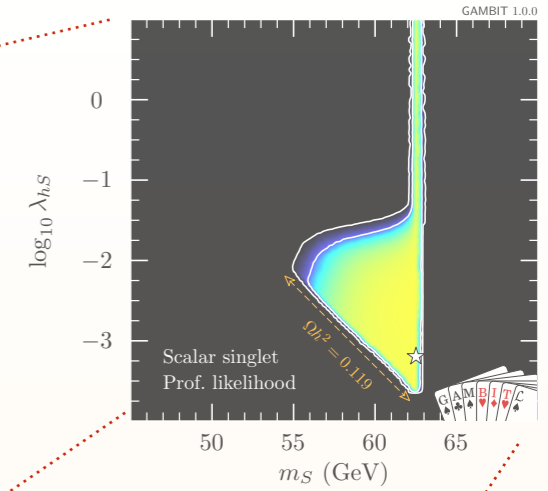
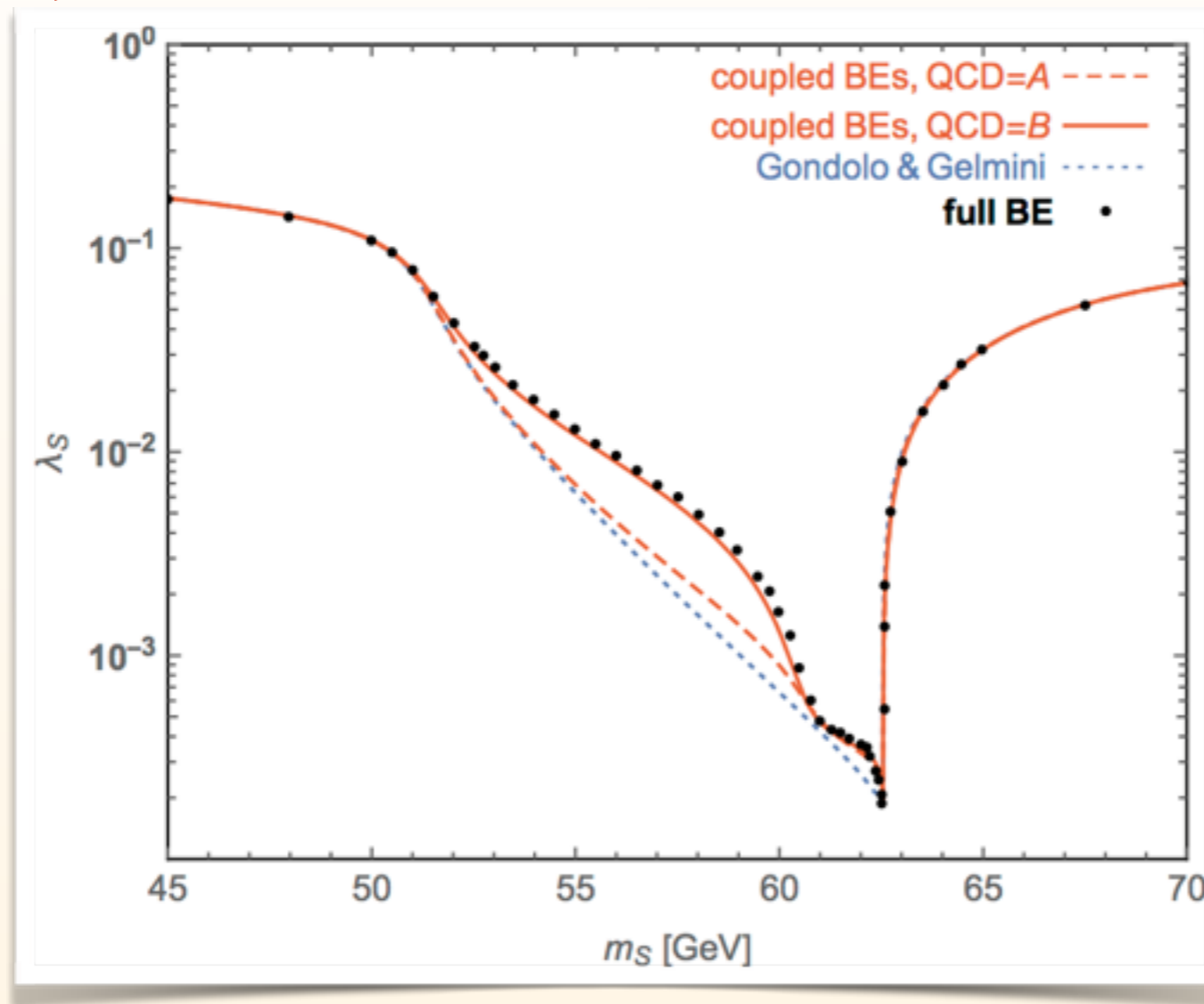
two scenarios:

QCD = A - all quarks are free and present in the plasma down to  $T_c = 154$  MeV

QCD = B - only light quarks contribute to scattering and only down to  $4T_c$

# RESULTS

## RD CONTOURS



essentially the only region left for this model

Significant modification of the **observed relic density contour** in the Scalar Singlet DM model

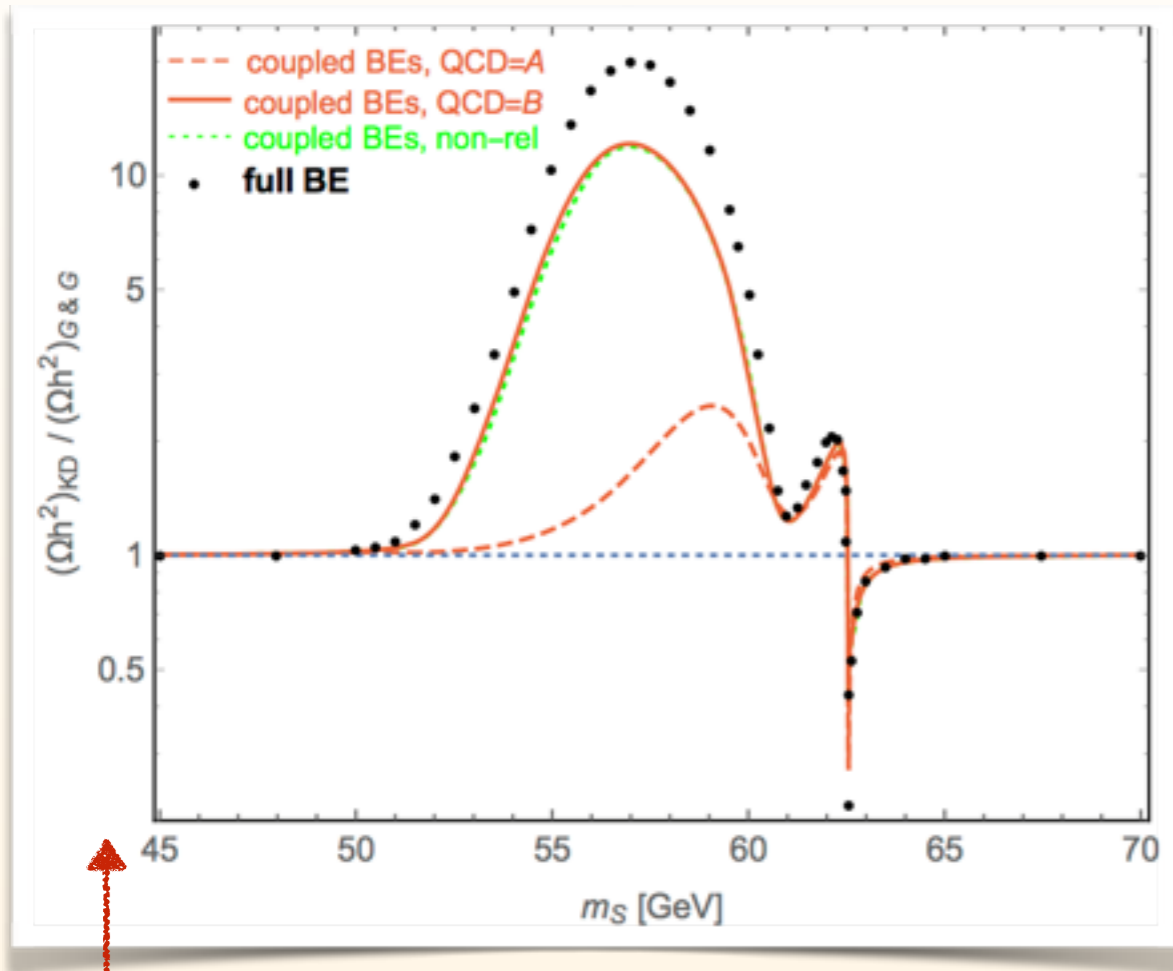
→ **larger coupling** needed → better chance for closing the last window



# RESULTS

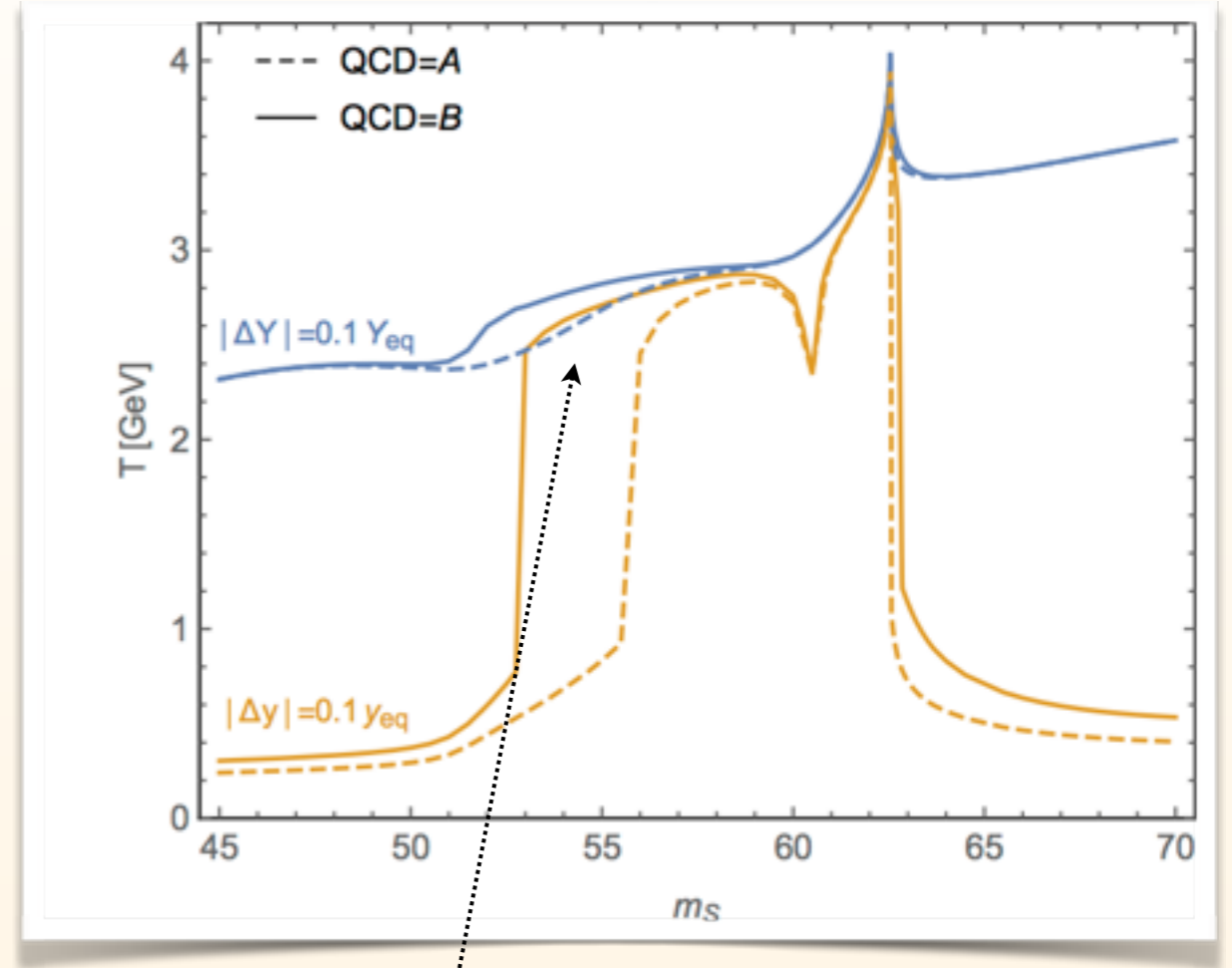
## EFFECT

effect on relic density:



effect on relic density:  
up to  $O(\sim 10)$

kinetic and chemical decoupling:



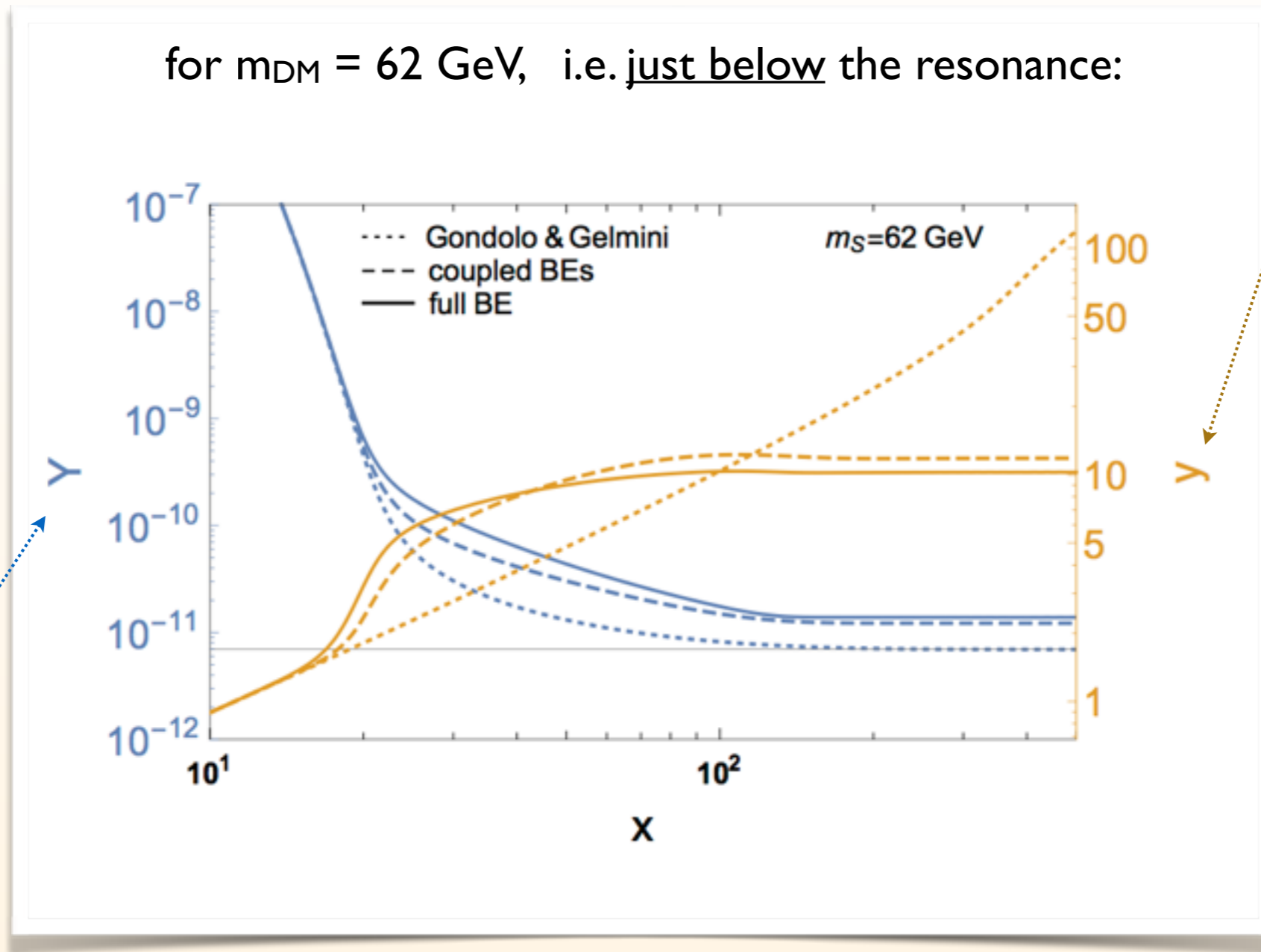
ratio approaches 1,  
but does not reach it!

Why such **non-trivial shape** of the effect of early kinetic decoupling?

↳ we'll inspect the  $y$  and  $Y$  evolution...

# DENSITY AND $T_{\text{DM}}$ EVOLUTION

for  $m_{\text{DM}} = 62 \text{ GeV}$ , i.e. just below the resonance:

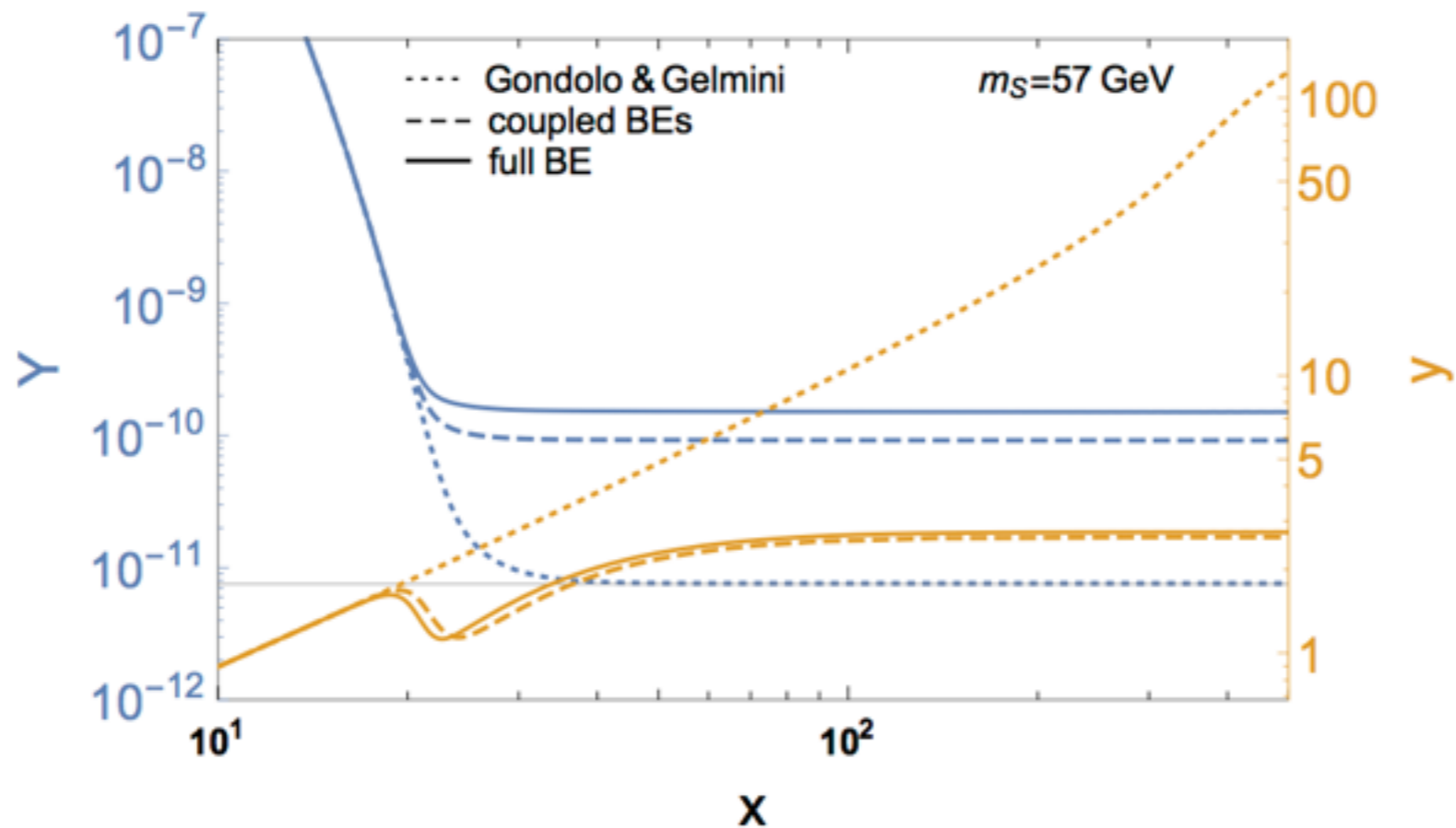


**Resonant annihilation** most effective for low momenta

→ DM fluid goes through "heating" phase before leaves kinetic equilibrium

# DENSITY AND $T_{\text{DM}}$ EVOLUTION

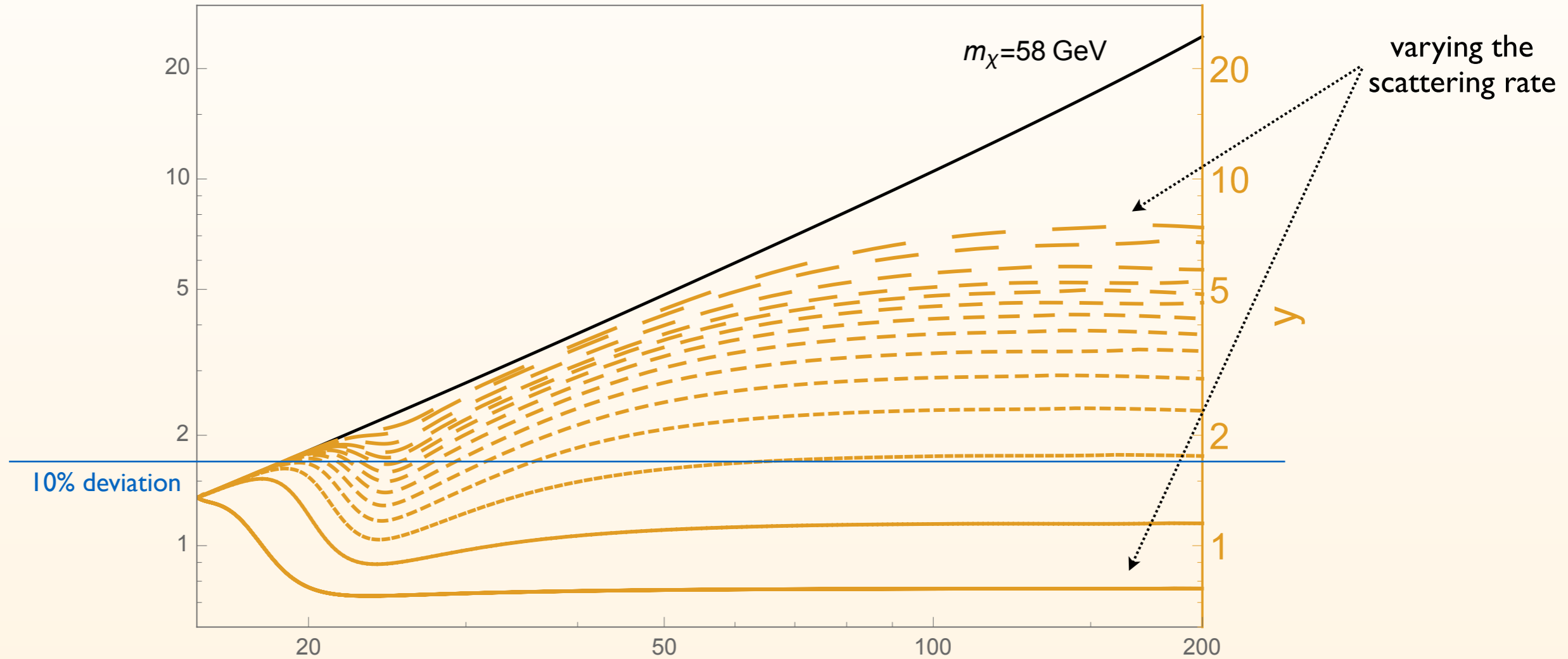
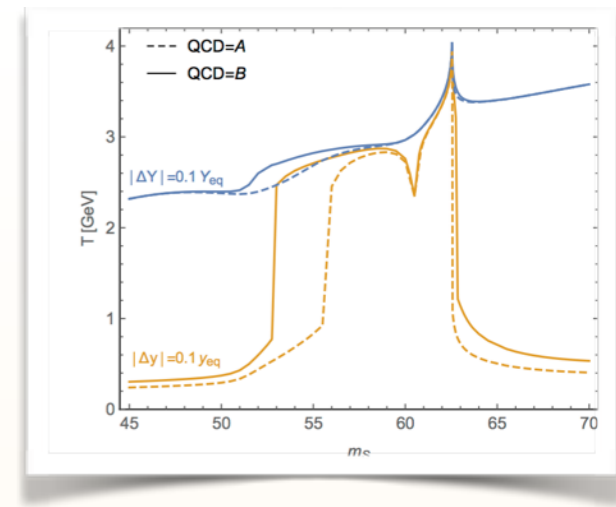
for  $m_{\text{DM}} = 57 \text{ GeV}$ , i.e. further away from the resonance:



**Resonant annihilation** most effective for high momenta

→ DM fluid goes through fast "cooling" phase  
after that when  $T_{\text{DM}}$  drops to much annihilation not effective anymore

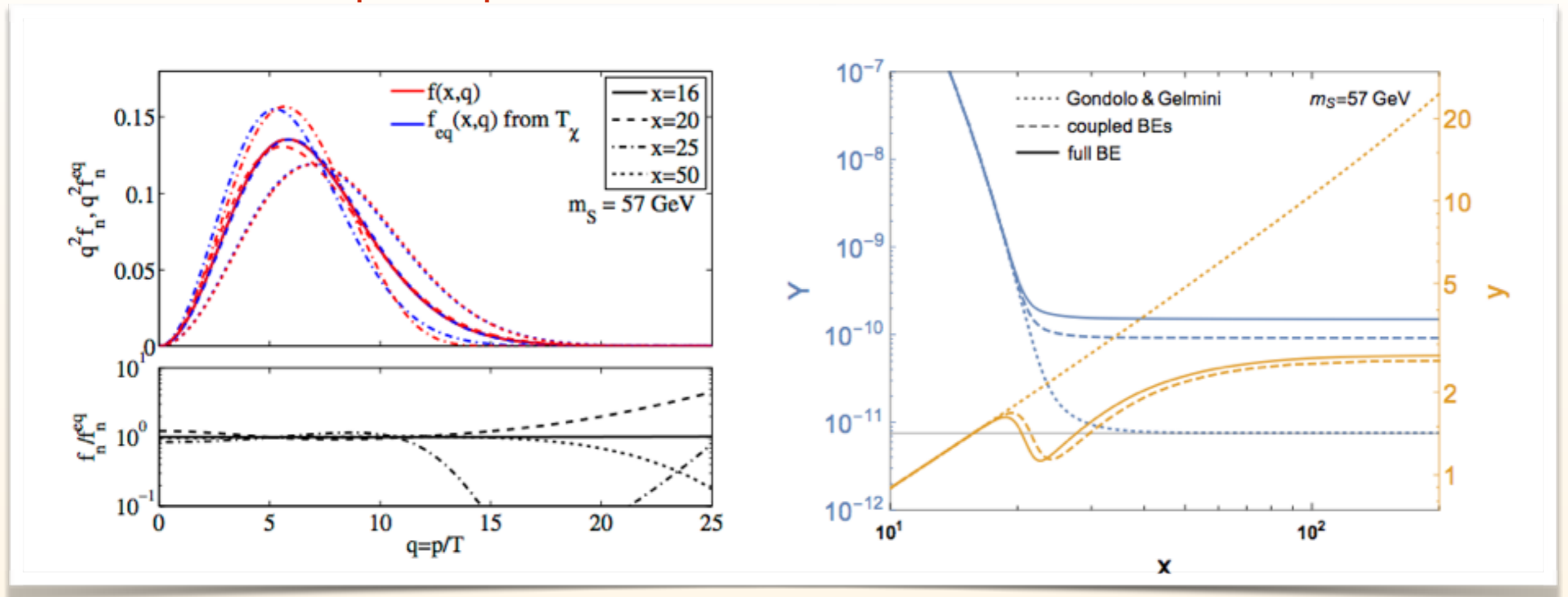
# WHY SPIKES IN $T_{KD}$ ?



Effect resembling first order „phase transition” —  
**artificial** as dependent on a particular choice of  $T_{KD}$  definition

# FULL PHASE-SPACE BE SOLVER

Solutions for full **phase-space distribution function**:



Results of both approaches compatible:  
 some **deviation from equilibrium shape** mildly affects the  $Y$  and  $y$  evolution

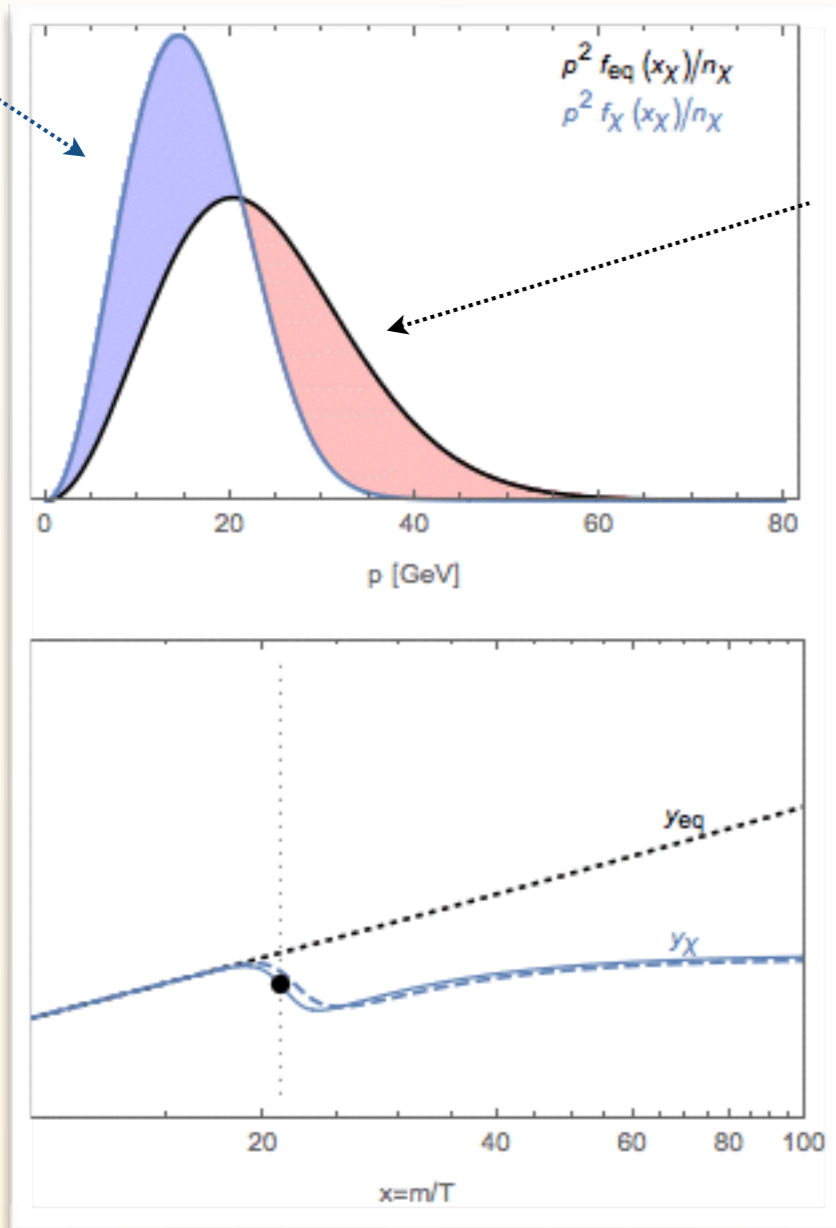
Allows to study the evolution of  $f_\chi(p)$  and  
 the interplay between scatterings and annihilation!

# FULL PHASE-SPACE EVOLUTION

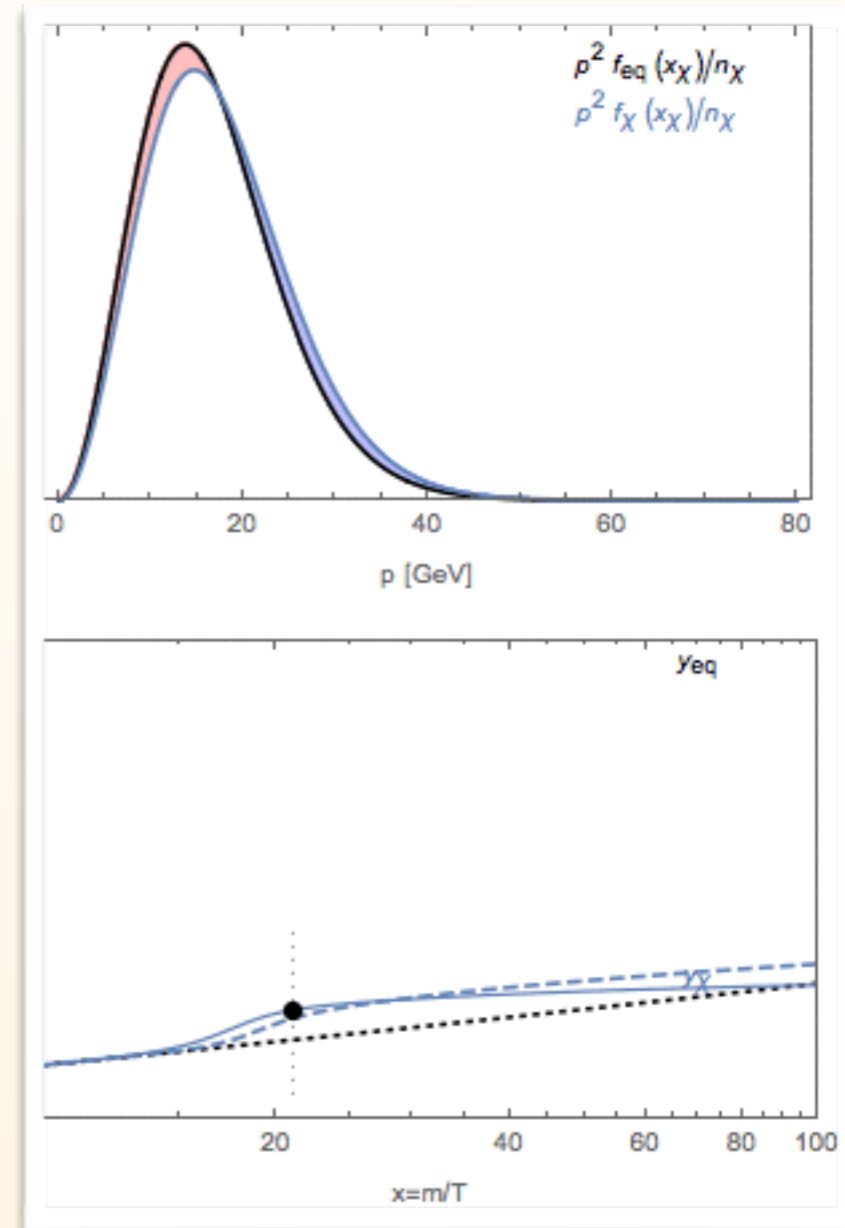
$m_{DM} = 58 \text{ GeV}$

$m_{DM} = 62.5 \text{ GeV}$

blue - full solution for  $f_{DM}$  at  $T_{DM}$



black - equilibrium at  $T_{DM}$



significant deviation from equilibrium shape **already around freeze-out**

large deviations **at later times**, around freeze-out not far from eq. shape

→ effect on relic density largest, **both from different T and  $f_{DM}$**

→ effect on relic density **~only from different T**



# KD BEFORE CD?

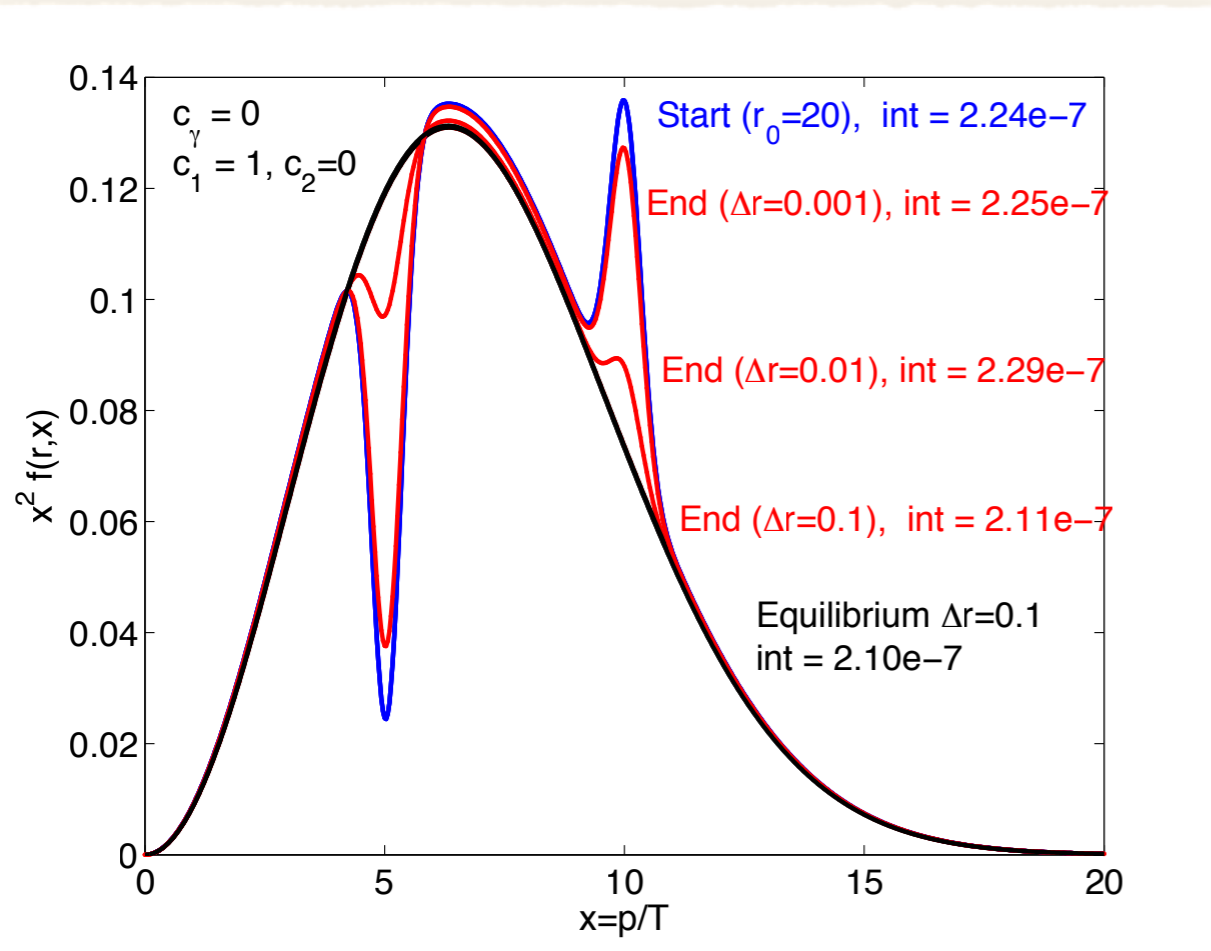
Obvious issue:

How to define exactly the **kinetic** and **chemical** decouplings and what is the significance of such definitions?



Improved question:

Can **kinetic** decoupling happen much earlier than **chemical**?



we have already seen that even if scatterings were very inefficient compared to annihilation, departure from equilibrium for both  $Y$  and  $y$  happened **around the same time...**

← turn off scatterings and take s-wave annihilation;  
look at local disturbance

annihilation/production processes drive to  
restore **kinetic equilibrium!**

# WHAT NEXT?

1. Extend the **numerical full phase space BE code** to the case of **scattering on heavy particles**

(no small momentum transfer approximation!)

2. Prepare a **public release** and study some more examples

3. Work on extension to **self-scattering**

(none of the particles in scattering term has equilibrium phase space density)

4. Maybe: in-elastic scatterings, semi-annihilation, cannibal, ...

# CONCLUSIONS

1. One needs to remember that **kinetic equilibrium** is a necessary assumption for standard relic density calculations
2. Coupled **system of Boltzmann equations for 0th and 2nd moments** allow for a very accurate treatment of the kinetic decoupling and its effect on relic density
3. In special cases the **full phase space Boltzmann equation** can be necessary — especially if one wants to trace DM temperature as well