Quantum gravitational corrections to the Cosmic Microwave Background anisotropy spectrum

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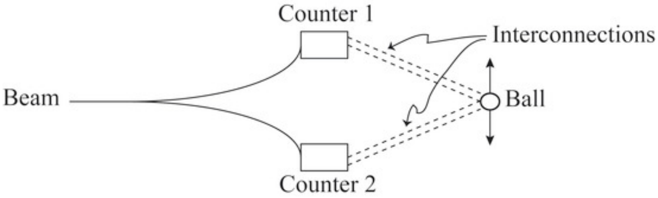
Richard Feynman 1957:

...if you believe in quantum mechanics up to any level then you have to believe in gravitational quantization in order to describe this experiment. ...It may turn out, since we've never done an experiment at this level, that it's not possible ...that there is something the matter with our quantum mechanics when we have too much *action* in the system, or too much mass—or something. But that is the only way I can see which would keep you from the necessity of quantizing the gravitational field. It's a way that I don't want to propose. ...



Why quantum gravity?

- Superposition principle
- Unification of all interactions
- Singularity theorems
  - Black holes
  - ‘Big Bang’
- Problem of time
- Absence of viable alternatives

Max Planck, Über irreversible Strahlungsvorgänge, *Sitzungsberichte der königlich-preußischen Akademie der Wissenschaften zu Berlin, phys.-math. Klasse*, Seiten 440–80 (1899)

Planck units

$$
\begin{aligned}
l_{\mathrm{P}} &= \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-35}\ \mathrm{m} \\
t_{\mathrm{P}} &= \frac{l_{\mathrm{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\ \mathrm{s} \\
m_{\mathrm{P}} &= \frac{\hbar}{l_{\mathrm{P}} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\ \mathrm{g} \approx 1.22 \times 10^{19}\ \mathrm{GeV}/c^2
\end{aligned}
$$

Max Planck (1899):

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

Main approaches to quantum gravity

No question about quantum gravity is more difficult than the question, “What is the question?”
(John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals, ...)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, ...)
- String theory
- Fundamental discrete approaches
  (quantum topology, causal sets, group field theory, ...);
  have partially grown out of the other approaches

See e.g. C. Kiefer, *Quantum Gravity*, 3rd ed. (Oxford 2012).

Linearized quantum gravity

Pioneered by Matvei Bronstein (1936)

Perturbation theory:

$$\boxed{g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}}$$

- $\bar{g}_{\mu\nu}$: classical background
- Perturbation theory with respect to $h_{\mu\nu}$ (Feynman rules)
- 'Particle' of quantum gravity: graviton (massless[1] spin-2 particle)
- perturbative non-renormalizability

[1] $m_{\rm g} \lesssim 7.7 \times 10^{-23}\ {\rm eV}/c^2$ from LIGO and Virgo, cf. $m_\gamma \leq 10^{-18} {\rm eV}/c^2$

Tests of quantum gravity in the laboratory?

Example: Transition rate from the $3d$ level to the $1s$ level in the hydrogen atom due to the emission of a graviton:

$$\Gamma_{\rm g} = \frac{G m_{\rm e}^3 c \alpha^6}{360\hbar^2} \approx 5.7 \times 10^{-40}\ {\rm s}^{-1}$$

This corresponds to a life-time of

$$\tau_{\rm g} \approx 5.6 \times 10^{31}\ {\rm years}\ .$$

Other possibility: Test of the superposition principle à la Feynman ('gravcat states')?

Gravitons from the early Universe

Gravitons are created out of the vacuum during an inflationary phase of the early Universe ($\sim 10^{-34}$ s after the big bang); the quantized gravitational mode functions $h_{\mathbf{k}}$ in de Sitter space obey

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'} \rangle = \frac{4}{k^3} \, (t_{\mathrm{P}} H)^2 \, \delta(\mathbf{k} + \mathbf{k}') \equiv P_{\mathsf{T}} \, \delta(\mathbf{k} + \mathbf{k}')$$

Power spectrum:

$$\mathcal{P}_{\mathsf{T}}(k) := \frac{4k^3}{\pi} P_{\mathsf{T}} = \frac{16}{\pi} \, (t_{\mathrm{P}} H)^2$$

(H is evaluated at Hubble-horizon exit)

The BICEP2 experiment

"Background Imaging of Cosmic Extragalactic Polarization"

Figure credit: BICEP2 Collaboration

Most likely, the observed signal comes from a dust foreground

BICEP2/KECK, PLANCK Collaborations, *Phys. Rev. Lett.* **114**, 101301 (2015).

Quantum origin of perturbations

Power spectrum for the scalar modes (inflaton plus metric):

$$\mathcal{P}_{\mathsf{S}} = \frac{1}{\pi}\,(t_{\mathrm{P}} H)^2\,\epsilon^{-1} \approx 2 \times 10^{-9}$$

ϵ: slow-roll parameter

Tensor-to-scalar ratio: $r := \frac{\mathcal{P}_{\mathsf{T}}}{\mathcal{P}_{\mathsf{S}}} = 16\epsilon$

The CMB spectrum from the PLANCK mission

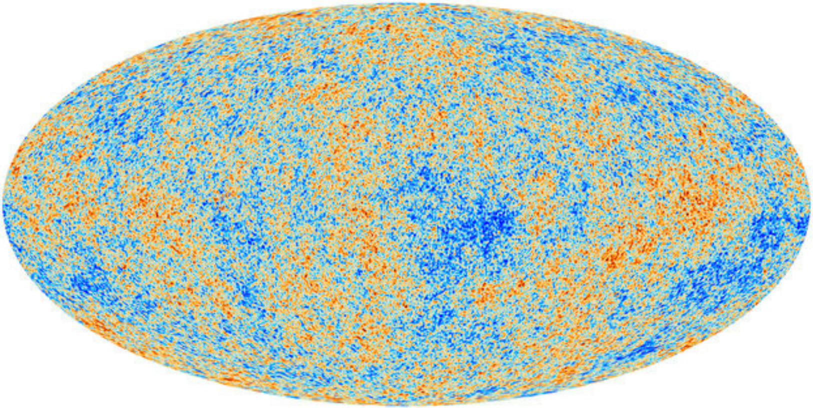


Figure credit: ESA/PLANCK Collaboration

First observational test of quantum gravity

- Within the inflationary scenario, the observed CMB fluctuations can only be understood from quantized metric plus scalar field modes.
- This is an indirect test of linearized quantum gravity.
- The observation of primordial B-modes would be an indirect confirmation of the existence of gravitons.
- The difference in the duration of inflation between the ‘cold spots’ and the ‘hot spots’ in the CMB spectrum is only of the order of the Planck time.

Erwin Schrödinger 1926:

We know today, in fact, that our classical mechanics fails for very small dimensions of the path and for very great curvatures. Perhaps this failure is in strict analogy with the failure of geometrical optics ... that becomes evident as soon as the obstacles or apertures are no longer large compared with the real, finite, wavelength. ... Then it becomes a question of searching for an undulatory mechanics, and the most obvious way is by an elaboration of the Hamiltonian analogy on the lines of undulatory optics.[2]

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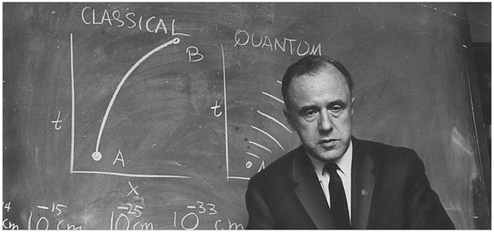
[2] *wir wissen doch heute, daß unsere klassische Mechanik bei sehr kleinen Bahndimensionen und sehr starken Bahnkrümmungen versagt*. Vielleicht ist dieses Versagen eine volle Analogie zum Versagen der geometrischen Optik ..., das bekanntlich eintritt, sobald die 'Hindernisse' oder 'Öffnungen' nicht mehr groß sind gegen die wirkliche, endliche Wellenlänge. ... Dann gilt es, eine 'undulatorische Mechanik' zu suchen – und der nächstliegende Weg dazu ist wohl die wellentheoretische Ausgestaltung des Hamiltonschen Bildes.



(a) John Archibald Wheeler

(b) Bryce DeWitt

- *Question*: what is the quantum wave equation that immediately gives Einstein's equations in the semiclassical limit?
- *Answer*: the Wheeler–DeWitt equation

$$\boxed{\hat{H}\Psi = 0}$$

Constraints of this type also occur in loop quantum gravity

In explicit form, the constraint equations read as follows ($c = 1$)

$$\hat{\mathcal{H}}_\perp \Psi \equiv \left(-16\pi G \hbar^2 G_{abcd} \frac{\delta^2}{\delta h_{ab} \delta h_{cd}} - (16\pi G)^{-1} \sqrt{h} \big({}^{(3)}R - 2\Lambda \big) + \hat{\mathcal{H}}_\perp^{\mathrm{m}} \right) \Psi = 0$$

Wheeler–DeWitt equation

$$\hat{\mathcal{H}}_a \Psi \equiv -2 D_b h_{ac} \frac{\hbar}{\mathrm{i}} \frac{\delta \Psi}{\delta h_{bc}} + \hat{\mathcal{H}}_a^{\mathrm{m}} \Psi = 0$$

quantum diffeomorphism (momentum) constraint

- External time t has vanished from the formalism
- This holds also for loop quantum gravity and probably for string theory
- Wheeler–DeWitt equation has the structure of a wave equation (hyperbolic equation) and may therefore allow the introduction of an "intrinsic time"

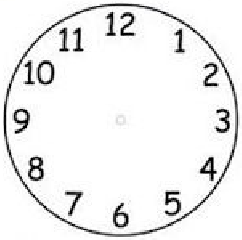


Figure: Absence of time in full quantum gravity

Figure from: C.K, B. Nikolić, *J.Phys.Conf.Ser.* **880**, 012002 (2017).

Semiclassical expansion

Quantum field theory in curved spacetime can be obtained from canonical quantum general relativity (GR) by a Born-Oppenheimer type of expansion with respect to the Planck-mass squared, $m_{\rm P}^2 = \hbar/G$.

$$\Psi[h_{ab},\phi] \equiv \exp\left(\frac{\rm i}{\hbar} S[h_{ab},\phi]\right)$$

Expansion of S:

$$S[h_{ab},\phi] = m_{\rm P}^2 S_0 + S_1 + m_{\rm P}^{-2} S_2 + \ldots$$

Insert this into the Wheeler-DeWitt equation and compare different orders of $m_{\rm P}^2$.

- m_{P}^4: S_0 is independent of ϕ
- m_{P}^2: Hamilton-Jacobi equation for S_0
- m_{P}^0: Equation for S_1 that can be simplified by introducing

  $$f \equiv D[h_{ij}] \exp\left(\frac{\mathrm{i}}{\hbar} S_1\right)$$

  and demanding the "WKB prefactor equation" for D.
- Introduce a local "bubble" (Tomonaga-Schwinger) time functional by

  $$\frac{\delta}{\delta\tau(\mathbf{x})} := G_{abcd} \frac{\delta S_0}{\delta h_{ab}} \frac{\delta}{\delta h_{cd}}$$

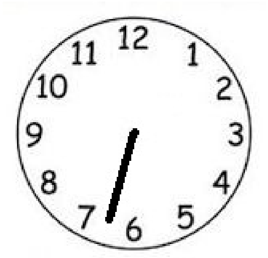


Figure: Emergence of semiclassical time

Figure from: C.K, B. Nikolić, *J.Phys.Conf.Ser.* **880**, 012002 (2017).

$$\mathrm{i}\hbar\frac{\delta f}{\delta\tau} = \hat{\mathcal{H}}_{\perp}^{\mathrm{m}} f$$

- τ is not a scalar function.
- This equation can be integrated to yield a (functional) Schrödinger equation.
- It describes the limit of quantum field theory in curved spacetime.
- Next order (m_{P}^{-2}): quantum gravitational corrections (modify the power spectrum of the CMB anisotropies)

Conformally invariant gravity

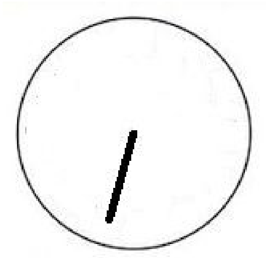


Figure: Emergence of scale-less time

Figure from: C.K, B. Nikolić, *J.Phys.Conf.Ser.* **880**, 012002 (2017).

Quantum gravitational corrections

The next order in the Born–Oppenheimer approximation gives

$$\hat{H}^{\mathrm{m}} \to \hat{H}^{\mathrm{m}} + \frac{1}{m_{\mathrm{P}}^2} \times (\text{various terms})$$

(C.K. and T.P. Singh (1991); A. O. Barvinsky and C.K. (1998))

Example: Quantum gravitational correction to the trace anomaly in de Sitter space:

$$\delta\epsilon \approx -\frac{2G\hbar^2 H_{\mathrm{dS}}^6}{3(1440)^2\pi^3 c^8}$$

(C.K. 1996)

Observations

Does the anisotropy spectrum of the cosmic background radiation contain information about quantum gravity?

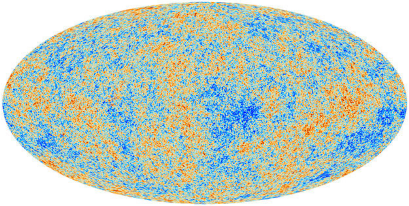


Figure credit: ESA/PLANCK Collaboration

C.K. and M. Krämer, *Phys. Rev. Lett.*, **108**, 021301 (2012); D. Bini, G. Esposito, C.K., M. Krämer, and F. Pessina, *Phys. Rev. D*, **87**, 104008 (2013); D. Brizuela, C.K., M. Krämer, *ibid.* **93**, 104035 (2016); *ibid.* **94**, 123527 (2016); D. Brizuela, C.K., M. Krämer, S. Robles-Pérez, *ibid.* **99**, 104007 (2019).

Perturbed inflationary universe

- flat Friedmann-Lemaître universe plus fluctuations
- massive scalar field ϕ with potential $\mathcal{V}(\phi)$
- use conformal time, $\mathrm{d}\eta/\mathrm{d}t = a^{-1}$
- $\hbar = c = 1$; $m_{\mathrm{P}} = \sqrt{3/4\pi G} \approx 0.60 \times 10^{19}$ GeV
- metric:

$$\begin{aligned} \mathrm{d}s^2 &= a^2(\eta)\Big\{ -(1-2A)\,\mathrm{d}\eta^2 + 2\,(\partial_i B)\,\mathrm{d}x^i \mathrm{d}\eta \\ &\quad + \left[(1-2\psi)\,\delta_{ij} + 2\partial_i\partial_j E + h_{ij}\right] \mathrm{d}x^i \mathrm{d}x^j \Big\} \end{aligned}$$

- combine scalar perturbations $\varphi(\eta, \vec{x})$ with scalar metric perturbations to get the gauge-invariant Mukhanov-Sasaki variable:

$$v(\eta, \vec{x}) := a\left[\varphi + \frac{\phi'}{\mathscr{H}}\left(A + 2\,\mathscr{H}\,(B - E') + [B - E']'\right)\right],$$

where $\mathscr{H} := a'/a$

$$
\begin{aligned}
S = \frac{1}{2}\int \mathrm{d}\eta \Bigg\{ & \mathfrak{L}^3 \left[-\frac{3}{4\pi G}\,(a')^2 + a^2\,(\phi')^2 - 2\,a^4\,\mathcal{V}(\phi) \right] \\
& + \frac{1}{\mathfrak{L}^3} \sum_{\vec{k}} \left[v'_{\vec{k}} v^{*\prime}_{\vec{k}} + {}^{\mathsf{S}}\omega^2_{\vec{k}}\, v_{\vec{k}} v^*_{\vec{k}} \right] \\
& + \frac{1}{\mathfrak{L}^3} \sum_{\lambda=+,\times} \sum_{\vec{k}} \left[v^{(\lambda)\prime}_{\vec{k}} v^{(\lambda)*\prime}_{\vec{k}} + {}^{\mathsf{T}}\omega^2_{\vec{k}}\, v^{(\lambda)}_{\vec{k}} v^{(\lambda)*}_{\vec{k}} \right] \Bigg\}
\end{aligned}
$$

- $v_{\vec{k}}$: Fourier transform of $v(\eta, \vec{x})$; Fourier-transformed perturbation variable of the gauge-invariant tensor perturbations h_{ij} with polarization $\lambda \in \{+, \times\}$: $v^{(\lambda)}_{\vec{k}} := \frac{a\, h^{(\lambda)}_{\vec{k}}}{\sqrt{16\pi G}}$
- $$ {}^{\mathsf{S}}\omega^2_{\vec{k}}(\eta) := k^2 - \frac{z''}{z}\,, \quad {}^{\mathsf{T}}\omega^2_{\vec{k}}(\eta) := k^2 - \frac{a''}{a}\,, $$

  where $z := a\,\phi'/\mathscr{H}$.

$\mathfrak{L}$: maximum length scale (IR cutoff); remove this by redefinitions after which a has the dimension of a length, whereas η, k, and $v_{\vec{k}}$ are dimensionless.

Canonical quantization and choosing a product ansatz for the wave function leads to

$$\frac{1}{2}\left\{\mathrm{e}^{-2\alpha}\left[\frac{1}{m_{\mathsf{P}}^2}\,\partial^2\alpha^2-\partial^2\phi^2+2\,\mathrm{e}^{6\alpha}\,\mathcal{V}(\phi)\right]-\frac{\partial^2}{\partial v_{\vec{k}}^2}+\omega_{\vec{k}}^2(\eta)\,v_{\vec{k}}^2\right\}\Psi_{\vec{k}}(\alpha,\phi,v_{\vec{k}})=0,$$

where $\alpha := \ln(a/a_0)$

Born-Oppenheimer approximation

Rescale $\tilde{\phi} := m_{\mathrm{P}}^{-1}\phi$ and perform the expansion

$$\Psi_{\vec{k}} = \exp\Big[\mathrm{i}\left(m_{\mathrm{P}}^{2}\, S_0 + m_{\mathrm{P}}^{0}\, S_1 + m_{\mathrm{P}}^{-2}\, S_2 + \dots\right)\Big]$$

- $\mathcal{O}(m_{\mathrm{P}}^{4})$: S_0 is independent of $v_{\vec{k}}$
- $\mathcal{O}(m_{\mathrm{P}}^{2})$: S_0 obeys the Hamilton–Jacobi equation of the minisuperspace background
- $\mathcal{O}(m_{\mathrm{P}}^{0})$: After the definition of WKB time according to

$$\partial\eta := \mathrm{e}^{-2\alpha}\left[-\,\partial S_0 \alpha \partial\alpha + m_{\mathrm{P}}^{2}\,\partial S_0 \phi \partial\phi\right],$$

one finds that each $\psi_{k}^{(0)}$ obeys a Schrödinger equation

$$\mathcal{H}_{\vec{k}}\psi_{\vec{k}}^{(0)} = \mathrm{i}\,\partial\eta\,\psi_{\vec{k}}^{(0)}.$$

- $\mathcal{O}(m_{\mathrm{P}}^{-2})$: corrected Schrödinger equation

$$\mathrm{i}\,\frac{\partial}{\partial\eta}\,\psi_{\vec{k}}^{(1)} = \mathcal{H}_{\vec{k}}\psi_{\vec{k}}^{(1)} - \frac{\psi_{\vec{k}}^{(1)}}{2\,m_{\mathrm{P}}^2\,\psi_{\vec{k}}^{(0)}}\left[\frac{\left(\mathcal{H}_{\vec{k}}\right)^2}{V}\,\psi_{\vec{k}}^{(0)} + \mathrm{i}\,\frac{\partial}{\partial\eta}\left(\frac{\mathcal{H}_{\vec{k}}}{V}\right)\psi_{\vec{k}}^{(0)}\right],$$

  where

$$V(a,\phi) := \frac{2\,a^4}{m_{\mathrm{P}}^2}\,\mathcal{V}(\phi),$$

  which has the dimension of a length squared.

- Gaussian ansatz:

$$\psi_{\vec{k}}^{(0,1)}(\eta, v_{\vec{k}}) = N_{\vec{k}}^{(0,1)}(\eta)\,\mathrm{e}^{-\frac{1}{2}\,\Omega_{\vec{k}}^{(0,1)}(\eta)\,v_{\vec{k}}^2}$$

- Initial condition: adiabatic vacuum (resembles Minkowski vacuum locally)

Power spectrum

The power spectrum is obtained from the two-point function of the quantum state

- Scalar perturbations

$$\mathcal{P}_{\mathsf{S}}^{(1)}(k) = \frac{4\pi G}{a^2\,\epsilon}\,\frac{k^3}{2\pi^2}\,\frac{1}{2\,\mathfrak{Re}\,{}^{\mathsf{S}}\Omega_{\vec{k}}^{(1)}} \approx \mathcal{P}_{\mathsf{S}}^{(0)}(k)\Big\{1+\Delta_{\mathsf{S}}\Big\},$$

- Tensor perturbations

$$\mathcal{P}_{\mathsf{T}}^{(1)}(k) = \frac{64\pi G}{a^2}\,\frac{k^3}{2\pi^2}\,\frac{1}{2\,\mathfrak{Re}\,{}^{\mathsf{T}}\Omega_{\vec{k}}^{(1)}} \approx \mathcal{P}_{\mathsf{T}}^{(0)}(k)\Big\{1+\Delta_{\mathsf{T}}\Big\}$$

- Tensor-to-scalar ratio:

$$r^{(1)} := \frac{\mathcal{P}_{\mathsf{T}}^{(1)}(k)}{\mathcal{P}_{\mathsf{S}}^{(1)}(k)} \approx r^{(0)}\,(1+\Delta_{\mathsf{T}}-\Delta_{\mathsf{S}})$$

Uncorrected power spectra

The calculation gives the standard result for the power spectra:

- Scalar modes:

$$\mathcal{P}_{\mathsf{S}}^{(0)}(k) = \frac{G\,H_k^2}{\pi\,\epsilon}\left[1 - 2\epsilon + \gamma(4 - 2\gamma_{\mathsf{E}} - 2\ln(2))\right]$$

- Tensor modes:

$$\mathcal{P}_{\mathsf{T}}^{(0)}(k) = \frac{16\,G\,H_k^2}{\pi}\left[1 - 2\epsilon + \epsilon(4 - 2\gamma_{\mathsf{E}} - 2\ln(2))\right],$$

where $\gamma_{\mathsf{E}} \simeq 0.5772$ is the Euler–Mascheroni constant, $H_k = k/a$, and the result should be evaluated at the horizon exit of the modes. Note that this is already a quantum gravitational effect (tree level)!

Corrected power spectra

The quantum gravitational corrections to the power spectra can be put into the form

$$\Delta_{\mathsf{S}} = \frac{H_k^2}{k^3 m_{\mathsf{P}}^2}\left[\beta_{\mathsf{dS}} + \epsilon\,\beta_\epsilon + \gamma\,\beta_\gamma\right]$$

and

$$\Delta_{\mathsf{T}} = \frac{H_k^2}{k^3 m_{\mathsf{P}}^2}\left[\beta_{\mathsf{dS}} + \epsilon\,(\beta_\epsilon + \beta_\gamma)\right],$$

where the β's are k-independent numbers that have to be determined numerically (except β_{dS}, which can be calculated analytically). Note the breaking of scale invariance!

Calculation of the β's shows that there is an enhancement of power at large scales of the order 10^{-10}, which is too small to be observable.

Summary and outlook

- Concrete prediction from a conservative approach to quantum gravity (Wheeler-DeWitt equation); consistent with existing observational limits
- Enhancement of power on largest scales
- Corrections: k^{-3}-dependence resp. ℓ^{-3}-dependence
- In the present case, the effect is too small to be observable (main limit for accuracy: cosmic variance)
- Similar results (but different in detail) from a modified scheme put forward by Kamenshchik, Tronconi, and Venturi (2013–2016)
- More general initial states lead to additional correction terms
- Quantum gravitational corrections for galaxy–galaxy correlation functions?
- Non-slow roll models of inflation?