# Prospects for detecting Majorana fermions

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Tel Aviv

# Outline

1. Introduction
   - Majorana fermions

2. Beam-splitter interferometry

   [HSR, S. H. Simon, PRB (2018)]
   - Protocol and key signatures
   - Vortex-edge coupling: lower size bound
   - Surface-bulk scattering: upper size bound

3. Finite temperature effects

   [HSR, R. Ilan, T. Meng, S. H. Simon, F. Flicker, arXiv:1901.09933 → SciPost]
   - Vortex core states
   - The parity disparity

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# Majorana fermions, key aspects

- Majorana fermions as quasiparticles in unconventional superconductors.

  A. Kitaev, Phys.-Uspekhi, (2001)

  M. Z. Hasan & C. L. Kane, RMP (2010)

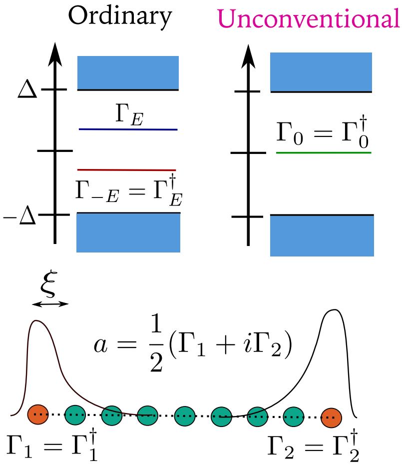
A key ingredient:

1. Particle-hole symmetry ($\pm E$).

Key properties:

1. Chargeless, $\{\Gamma_n, \Gamma_m\} = 2\delta_{n,m}$.
2. Non-Abelian anyons, $U_{n,n+1} = \exp\left(-\frac{\pi}{4}\Gamma_n\Gamma_{n+1}\right)$.

   D. A. Ivanov, PRL (2001)




$\{ |0\rangle, |1\rangle = a^\dagger |0\rangle \}$

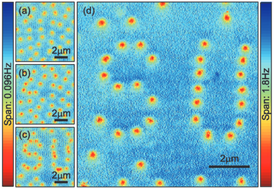
# Envisioning braiding with vortices

- Majorana fermions bind to vortex cores. N. Read & D. Green, PRB (2000)
- Hybridization:

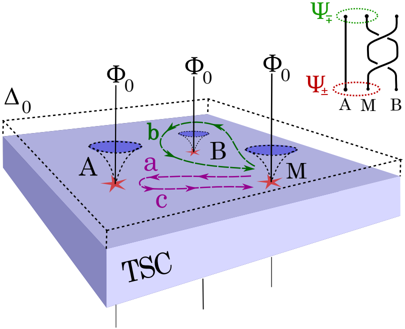
$$\varepsilon_{\pm} \propto \pm \Delta_0 \exp(-R/\xi),$$

$$\mathbf{\Psi}_{\pm} = \frac{1}{\sqrt{2}}(\mathbf{\Psi}_A \pm i\mathbf{\Psi}_M).$$

M. Cheng *et al*, PRL (2009)




E. W. J. Straver *et al*, Appl. Phys. Lett. (2008)




- Pinning force (MFM, STM, lasers) depends on parity.
- Limitations: poisoning time, temperature, intrinsic TSC.

# Proximity induced superconductivity

- Fu & Kane: topological insulator + conventional superconductor:

  L. Fu & C. L. Kane, PRL (2008)

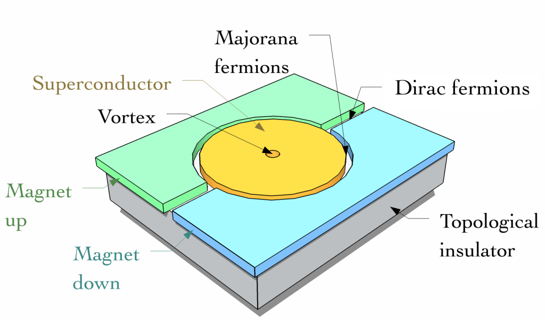
☑ Vortex Majoranas.

☑ Magnetic gap.

☑ Superconducting gap.

☑ $\Delta$–$M$ edge: chiral Majorana.

☑ $M_{\uparrow}$–$M_{\downarrow}$ edge: chiral Dirac fermion.



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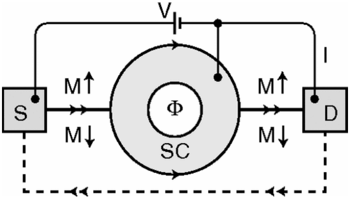
# Beam-splitter interferometry

- Interferometry on a topological insulator surface.
- Electron/hole transfer:

$$\begin{pmatrix} \psi_e \\ \psi_h \end{pmatrix}_{\text{drain}} = \underbrace{\begin{pmatrix} \mathcal{T}_{ee} & \mathcal{T}_{eh} \\ \mathcal{T}_{he} & \mathcal{T}_{hh} \end{pmatrix}}_{\mathcal{T}} \begin{pmatrix} \psi_e \\ \psi_h \end{pmatrix}_{\text{source}},$$

$$\mathcal{T}(E) = S^{\dagger}(E) \begin{pmatrix} e^{in\pi + ikl_1} & 0 \\ 0 & e^{ikl_2} \end{pmatrix} S(E),$$

$$S(E \ll \Delta_0) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} + \mathcal{O}(E^2).$$




L. Fu & C. L. Kane, PRL (2009)

A. R. Akhmerov *et al.*, PRL (2009)

# Differential conductance signature

$$\left.\frac{\mathrm{d}I_{\mathrm{SC}}}{\mathrm{d}V}\right|_{T=0} = \frac{2e^2}{h}|\mathcal{T}_{eh}(eV)|^2 = \frac{2e^2}{h}\sin^2\left(\frac{n\pi}{2} + \frac{eV\delta L}{2v_m}\right)$$

$n$: # flux quanta, $v_m$: Majorana velocity, $\delta L = l_1 - l_2$, $V$: bias voltage.

Remarks:

- A $\mathbb{Z}_2$ interferometer.
- No oscillations for $\delta L = 0$.
- Assumptions: no dephasing.

Our work:

- Add vortex-edge coupling.
- Add surface-bulk scattering.
- Spoiler alert: stringent size limitations.

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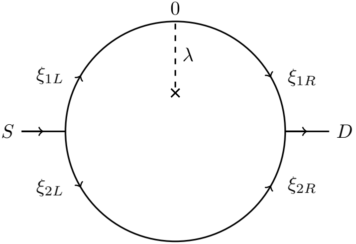
# Vortex-edge coupling: one arm

Method:

- Add tunnel coupling.
- Equations of motion $\Rightarrow$ phase acquired P. Fendley *et al.*, Ann. Phys. (2009)

$$\delta\mathcal{L} = 2i\lambda\xi_1(0)\xi_0$$

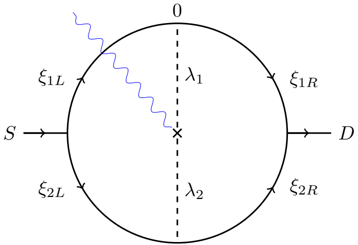



$$\xi_{1R} = \frac{E + i\frac{\lambda^2}{2v_m}}{E - i\frac{\lambda^2}{2v_m}}\, e^{ik\ell}\xi_{1L}.$$

- $E \gg \lambda^2/(2v_m)$: Vortex-Majorana "not felt".
- $E \ll \lambda^2/(2v_m)$: Vortex-Majorana "absorbed" by the edge.

# Vortex-edge coupling: both arms

$$\delta\mathcal{L} = 2i\left[\lambda_1\xi_1(0) + \lambda_2\xi_2(0)\right]\xi_0$$



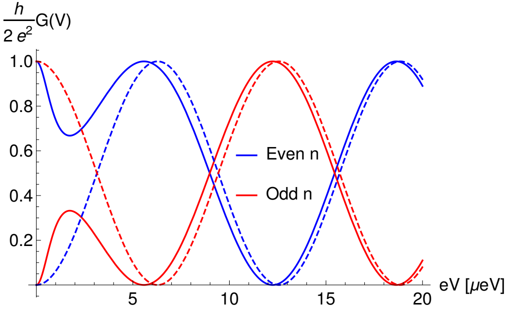

$$\begin{pmatrix}\xi_1\\ \xi_2\end{pmatrix}_{\text{drain}} = \mathcal{U}(E)\begin{pmatrix}\xi_1\\ \xi_2\end{pmatrix}_{\text{source}}$$

- When $\lambda_1 = \lambda_2$: $\mathcal{U}(E \to 0) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.
- Cross-tunneling assisted by the vortex! This mechanism was suggested to achieve braiding. Y-F. Zhou *et al.*, arXiv:1811:03230

# Distorted conductance

$$\left.\frac{\mathrm{d}I_{\mathrm{SC}}}{\mathrm{d}V}\right|_{T=0} = \frac{2e^2}{h}\sin^2\left(\frac{n\pi}{2} + \frac{eV\delta L}{2v_m} + \arctan\left[\frac{\lambda^2}{2v_m eV}\right]\right)$$

- Voltage range:
  $\lambda^2/(2v_m) \lesssim eV \lesssim \min\{M_0, \Delta_0\}$.
- $\lambda \propto \mu \exp(-R/\xi)$, $\xi = v_F/\Delta_0$.
- Require $R \gg \xi$.




- $Bi_2Se_3$: $\Delta_0 \sim 0.1$ meV, $\xi \sim 1\ \mu$m, $\mu \sim 100$ meV by doping.

# Outline

# Scattering on a conducting lead

Motivation:

- Most topological insulators are poor bulk insulators.

  B. Skinner *et al*, J. Exp. Theor. Phys (2013)

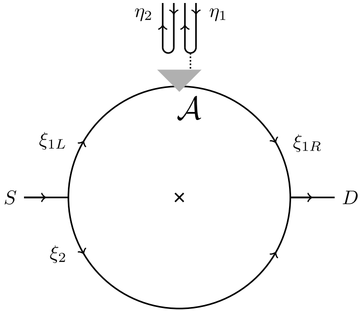
- Surface states not protected from surface-bulk scattering.

  K. Saha & I. Garate, PRB (2014)

Scattering:

$$(\xi_{1R}, \eta_1, \eta_2)^\intercal = \mathcal{A}(E)(\xi_{1L}, \eta_1, \eta_2)^\intercal,$$

$$\mathcal{A}(E \ll \Delta_0) = \begin{pmatrix} r_1 & -t_1 & 0 \\ t_1 & r_1 & 0 \\ 0 & 0 & r_2 \end{pmatrix} \in \mathrm{SO}(3).$$




- Low energy: $r_2 = 1$. J. Li *et al*, PRB (2012)
- Drain current: $I_D(V) = r_1 I_{D,0}(V)$.

# From multiple leads to the continuum

- Scattering onto a collection of leads: $I_D(V) = I_{D,0}(V) \prod_{j=1}^{N} r_{1,j}$.
- Continuum: $\lim_{N\to\infty} r_1^N = \exp(-\ell/\ell_S)$:

$$\boxed{\frac{\mathrm{d}I_{\mathrm{SC}}}{\mathrm{d}V} = \frac{\mathrm{d}I_{\mathrm{SC},0}}{\mathrm{d}V} e^{-\ell/\ell_S} + \frac{e^2}{h}(1 - e^{-\ell/\ell_S})}$$

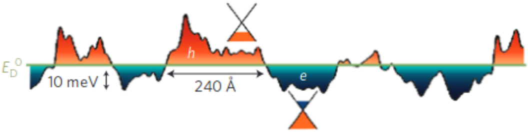
- Estimate $\ell_S$ with Fermi's golden rule:

$$\Gamma_{\mathrm{SB}}^{\mathrm{imp}}(\epsilon_F) = 2\pi \sum_{\boldsymbol{k}',n'} |g^{\mathrm{imp}}_{\boldsymbol{k}_\parallel - \boldsymbol{k}'}|^2 |F_{S,\boldsymbol{k}_\parallel;B,\boldsymbol{k}',n'}|^2 \delta(\xi_{B,n',\boldsymbol{k'}} - \epsilon_F)$$

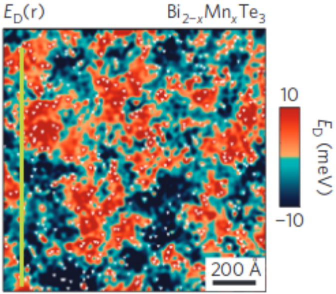
# Tuning of the chemical potential

- From numerics: $\ell_S^{(e)} = v_F/\Gamma_{\rm SB}^{\rm imp} \approx 0.1$ mm.
- The Majorana scattering length: $\ell_S^{(m)} \sim (v_m/v_F)^4 \ell_S^{(e)}$.
- Impose $\ell_S^{(m)} \gtrsim \xi \Rightarrow v_m/v_F \gtrsim 0.4$. Analytically: $v_m/v_F = \frac{\sqrt{1-(\mu/M_0)^2}}{1+(\mu/\Delta_0)^2}$.

  L. Fu & C. L. Kane, PRL (2009)
- Assume $M_0 \gg |\mu|$. Then $|\mu| \lesssim 0.1$ meV.
- Charge puddle fluctuations: $\delta\mu \simeq 10 - 20$ meV!



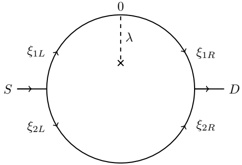

H. Beidenkopf *et al*, Nat. Phys. (2011)



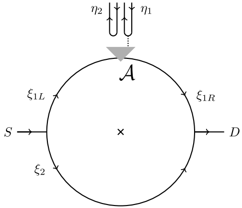
# Summary part I

- Lower size bound: $R \gtrsim \xi$.
- Upper size bound: $R \lesssim \ell_S^{(m)}$.
- Well-studied topological insulators, *e.g.* $Bi_2Se_3$, $Bi_2Te_3$, $Sb_2Te_3$, unsuited.
- Possible future directions: iron based superconductors (large gap). D. Wang *et al*, Science (2018)
- Putative topological Kondo insulator $SmB_6$.

  X. Zhang *et al*, PRX (2013)




$R \gtrsim \xi$



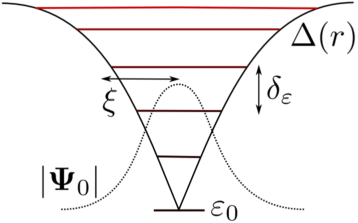

$R \lesssim \ell_S^{(m)}$

# Outline

# Vortex core states in chiral superconductors

- Mean-field treatment:

$$\mathcal{H}_{\mathrm{BdG}} \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = \varepsilon_n/2 \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix}.$$




- Quasiparticles:

$$\hat{\Gamma}_n = \int \mathrm{d}^2 r \left( u_n^* \hat{\psi}(\boldsymbol{r}) + v_n^* \hat{\psi}^\dagger(\boldsymbol{r}) \right).$$

- Majorana solution $u_0 = v_0^*$ in vortex core, $\Delta = \Delta(r) e^{i\varphi}$:

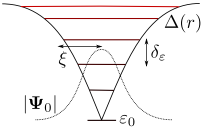
$$u_0(r, \varphi) = \mathcal{N} J_1(k_F r) e^{i\varphi - \frac{1}{v_F}\int_0^r \mathrm{d}r' \Delta(r')}.$$

N. Read & D. Green, PRB (2000); M. Cheng *et al*, PRB (2010)

# Hybridization in a two-vortex system

- Tower of bound states with spacing $\delta_\varepsilon \approx \Delta_0^2/E_F$ (the "minigap").

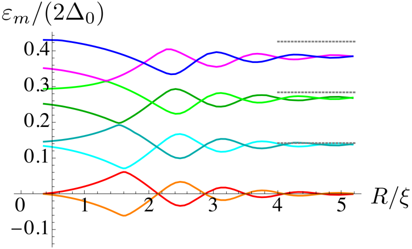
  C. Caroli P. D. Gennes, J. Matricon, Phys. Lett. (1964)



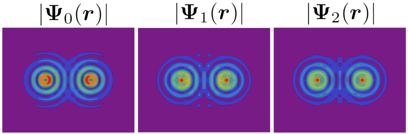

- Two-vortex hybridization:

$$\varepsilon_0 \to \varepsilon_\pm \approx \mp \frac{2\Delta_0}{\pi^{3/2}} \frac{\cos\left(k_F R + \frac{\pi}{4}\right)}{\sqrt{k_F R}} e^{-\frac{R}{\xi}}$$

M. Cheng *et al*, PRL (2009)




- Q: What is the effect of thermal occupation of the CdGM states?



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# The projected partition function

- Fermion parity conserved.
- Grand canonical ensemble: project unconstrained partition function,

$$\mathcal{Z}_{\pm} = \frac{1}{2}\left[1 \pm \prod_m \tanh(\beta\varepsilon_m/2)\right]\mathcal{Z}_0$$

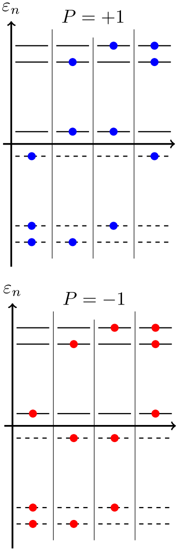
M. T. Tuominen *et al*, PRL (1992) B. Jankó *et al*, PRB (1994)

- Even parity, $P = +1$:

$$\mathcal{Z}_+ = e^{\frac{\beta}{2}(\varepsilon_0+\varepsilon_1+\varepsilon_2)} + e^{\frac{\beta}{2}(-\varepsilon_0-\varepsilon_1+\varepsilon_2)} + e^{\frac{\beta}{2}(-\varepsilon_0+\varepsilon_1-\varepsilon_2)} + e^{\frac{\beta}{2}(\varepsilon_0-\varepsilon_1-\varepsilon_2)}$$

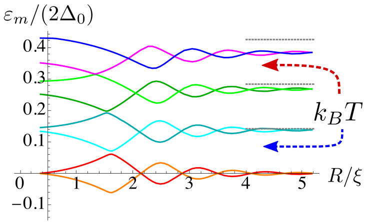
- Odd parity, $P = -1$:

$$\mathcal{Z}_- = e^{\frac{\beta}{2}(-\varepsilon_0+\varepsilon_1+\varepsilon_2)} + e^{\frac{\beta}{2}(\varepsilon_0-\varepsilon_1+\varepsilon_2)} + e^{\frac{\beta}{2}(\varepsilon_0+\varepsilon_1-\varepsilon_2)} + e^{\frac{\beta}{2}(-\varepsilon_0-\varepsilon_1-\varepsilon_2)}$$



# The parity disparity

- The free energy: $F_\pm = -\frac{1}{\beta}\log \mathcal{Z}_\pm$.
- "Parity disparity": $\Delta F = F_- - F_+$.
- Define $\varepsilon_0$, $\varepsilon_1 = \delta_\varepsilon - w_1$, $\varepsilon_2 = \delta_\varepsilon + w_2$.



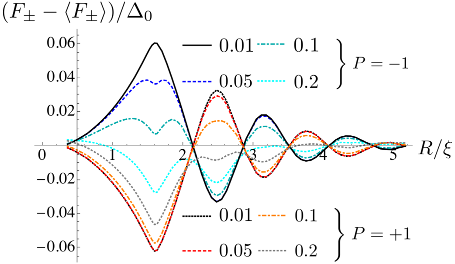

- $k_B T < \delta_\varepsilon$:

$$\Delta F \approx \varepsilon_0 - \frac{4}{\beta}\cosh\left(\beta\frac{w_1 + w_2}{2}\right)\sinh(\beta\varepsilon_0)e^{-\beta(\delta_\varepsilon + [w_2 - w_1]/2)}$$

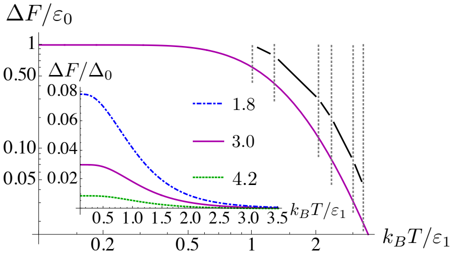
- $\varepsilon_n \ll k_B T \ll \Delta_0$:

$$\Delta F \approx \varepsilon_0(\beta/2)^n \prod_{m=1}^{n} \varepsilon_m \prod_{l>n} \tanh\left(\beta\varepsilon_l/2\right)$$

# Numerics




Labels: values of $k_B T/\Delta_0$.




In black: $\Delta F \approx \varepsilon_0 (\beta/2)^n \prod_{m=1}^{n} \varepsilon_m$.

# Summary part II

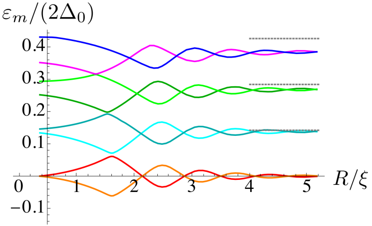
- Below the minigap: $\sim \exp(-\beta\delta_\varepsilon)$.
- Above the minigap: $\Delta F \sim T^{-n}$.
- Spatial Fourier transform of $\Delta F$.
- Promising compounds:

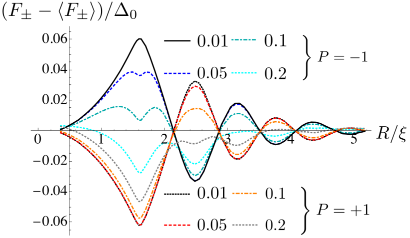
  (LiFe)OHFeSe with $\delta_\varepsilon \simeq 11$ K,

  Q. Liu *et al*, PRX (2018)

  $FeTe_{0.55}Se_{0.45}$ with $\delta_\varepsilon \simeq 2$ K.

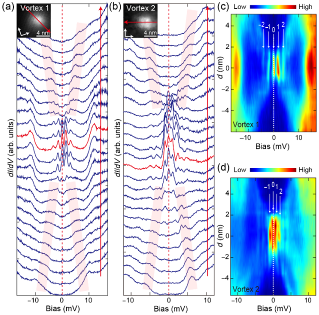
  D. Wang *et al*, Science (2018)



# Two recent experiments

Vortex cores in (LiFe)OHFeSe:




Q. Liu *et al*, PRX (2018)

- Nature of vortices?
- Large (mini)gap: $\Delta_0 \simeq 10$ meV, $\delta_\varepsilon \simeq 1$ meV!

News flash:

Article | Published: 04 March 2019

Aharonov–Bohm interference of fractional quantum Hall edge modes

J. Nakamura, S. Fallahi, H. Sahasrabudhe, R. Rahman, S. Liang, G. C. Gardner & M. J. Manfra

*Nature Physics* (2019) | Download Citation

News & Views | Published: 04 March 2019

QUANTUM HALL EFFECT

Fractional oscillations

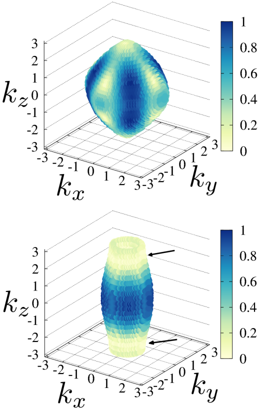
Steven H. Simon

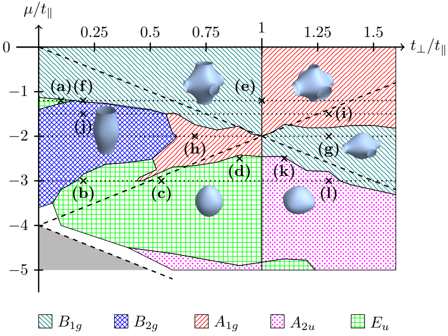
*Nature Physics* (2019) | Download Citation

**An electrical interferometer device has detected interference patterns that suggest anyons could be conclusively demonstrated in the near future.**

# Extra: Unconventional superconductivity

Weak-coupling superconductivity in an anisotropic Hubbard model:



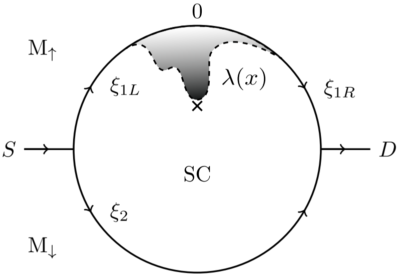




HSR, F. Flicker, T. Scaffidi, S. H. Simon, PRB (2018)

Future applications to the multiband superconductor $Sr_2RuO_4$ (work in progress).

# Extra: smeared tunnel coupling

$$\delta\mathcal{L} = 2i \int \mathrm{d}x \; \lambda(x)\xi_1(x)\xi_0$$




$$\xi_{1R} = \frac{E + \frac{i}{v_m}\int \mathrm{d}x\lambda(x)e^{ikx}\int_x \mathrm{d}x'\lambda(x')e^{-ikx'}}{E - \frac{i}{v_m}\int \mathrm{d}x\lambda(x)e^{ikx}\int^x \mathrm{d}x'\lambda(x')e^{-ikx'}}\,\xi_{1L}.$$

# Extra: scattering rate

$$\Gamma^{\mathrm{imp}}_{\mathrm{SB}}(\epsilon_F) = n_{3\mathrm{D}} \left( \frac{e^2}{2\pi\epsilon_0\epsilon_r k_{\parallel}^2} \right)^2 \sum\nolimits_{n' \in B} \frac{k'_{\parallel}}{|\nabla \xi_{B,n',k'_{\parallel}}|} \int_0^\infty \mathrm{d}k'_z \frac{\mathrm{d}\sigma^{(n')}_{\mathrm{long}}(\epsilon_F)}{\mathrm{d}k'_z}$$