Thermal relics and Dark Matter

Plan

- Short introduction to Cosmology
  - FRW metric
  - Friedman equations
  - Thermal history
- Boltzmann equation
  - Freeze-out
  - Thermal relics
- Dark Matter
  - WIMP
  - Freeze in

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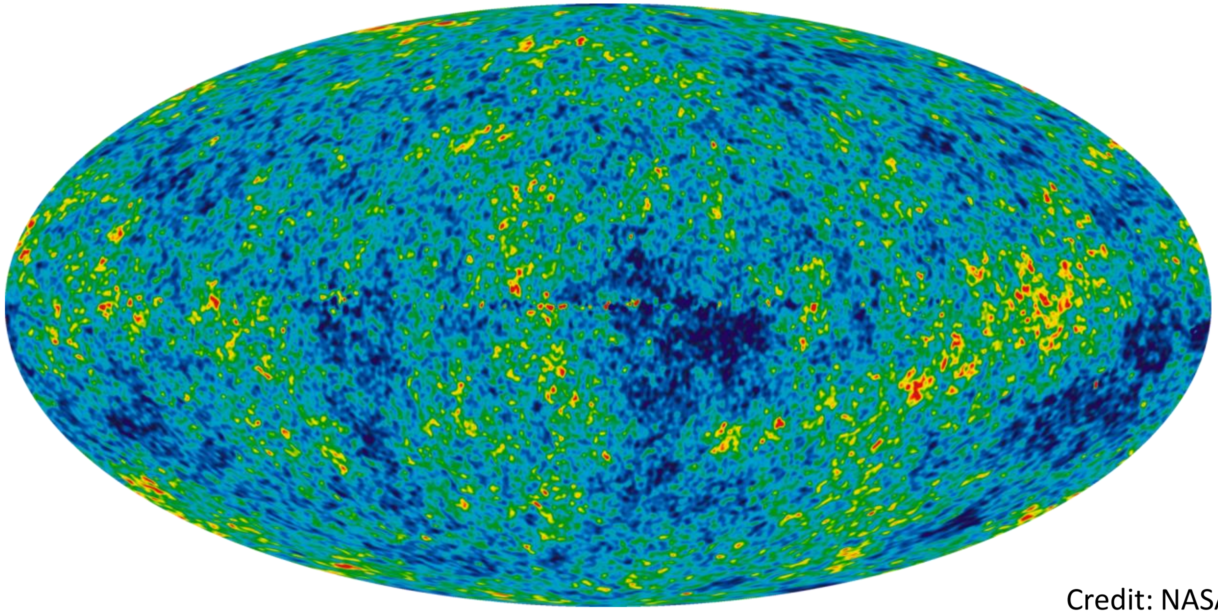
Friedman-Robertson-Walker metric

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

- Cosmological principle
- K=0, 1 or -1
- Scale factor



Credit: NASA

$$2.72548 \pm 0.00057K$$

Energy-momentum

- Choose perfect fluid S-E Tensor
- Equation of state
- Three examples

$$T^{\mu}_{\nu} = \mathrm{diag}(-\rho, p, p, p)$$

$$p = w\rho$$

$$\nabla_{\mu} T^{\mu\nu} = 0 \Rightarrow \frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a}$$

$$\rho \propto a^{-3(1+w)}$$

Dust

$$p = 0$$

$$\rho \propto a^{-3}$$

Radiation

Kinetic gas theory

$$p = \frac{1}{3}\rho$$

$$\rho_R \propto a^{-4}$$

Vacuum

$$T^{vaccum}_{\mu\nu} = -\frac{\Lambda g_{\mu\nu}}{8\pi G}$$

$$p = -\rho = \frac{\Lambda}{8\pi G}$$

Dust

$$p = 0$$

$$\rho \propto a^{-3}$$

Radiation

Kinetic gas theory

$$p = \frac{1}{3}\rho$$

$$\rho_R \propto a^{-4}$$

Vacuum

$$T_{\mu\nu}^{vaccum} = -\frac{\Lambda g_{\mu\nu}}{8\pi G}$$

$$p = -\rho = \frac{\Lambda}{8\pi G}$$

Friedman equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

- Time evolution of the scale factor
- Plug FRW-metric into EFE
- Density determines shape of universe

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$H \equiv \frac{\dot{a}}{a}$$

$$\Omega \equiv \frac{8\pi G}{3H^2}\rho \equiv \frac{\rho}{\rho_{crit}}$$

$$\Omega - 1 = \frac{k}{H^2 a^2}$$

$\Omega_m = 0.31,\ \Omega_\Lambda = 0.69$ and $\Omega_r \sim 10^{-4}\Omega_m$

$$\Omega = \sum_i \Omega_i \approx 1$$

$$\begin{array}{lllllll} \rho < \rho_{crit} & \Leftrightarrow & \Omega < 1 & \Leftrightarrow & k < 0 & \Leftrightarrow & \text{Open} \\ \rho = \rho_{crit} & \Leftrightarrow & \Omega = 1 & \Leftrightarrow & k = 0 & \Leftrightarrow & \text{Flat} \\ \rho > \rho_{crit} & \Leftrightarrow & \Omega > 1 & \Leftrightarrow & k > 0 & \Leftrightarrow & \text{Closed} \end{array}$$

Temperature

- Local Kinetic equilibrium and thermal equilibrium
- Phase-space distribution function
  - Fermi-Dirac & Bose-Einstein
- Relativistic approximation
- Non-relativistic approximation

$$f(\vec{p}) = [\exp((E-\mu)/T) \pm 1]^{-1}$$

$$\rho = \frac{g}{(2\pi)^3}\int E(\vec{p}) f(\vec{p}) d^3p$$

$$n = \frac{g}{(2\pi)^3}\int f(\vec{p}) d^3p$$

$$p = \frac{g}{(2\pi)^3}\int \frac{|\vec{p}|^2}{3E} f(\vec{p}) d^3p$$

$$\rho_R^{Fermions} = \frac{7}{8}\rho_R^{Bosons} \propto T^4$$

$$p_R = \rho_R/3$$

$$n_R^{Fermions} = \frac{3}{4} n_R^{Bosons} \propto T^3$$

$$n_{NR} \propto T^{3/2}\exp[-(m-\mu)/T]$$

$$p_{NR} = n_{NR}T \ll \rho$$

$$\rho_{NR} = m n_{NR} \propto T^{3/2}\exp[-(m-\mu)/T]$$

Entropy

- Energy conservation and 2nd law of thermo dynamics

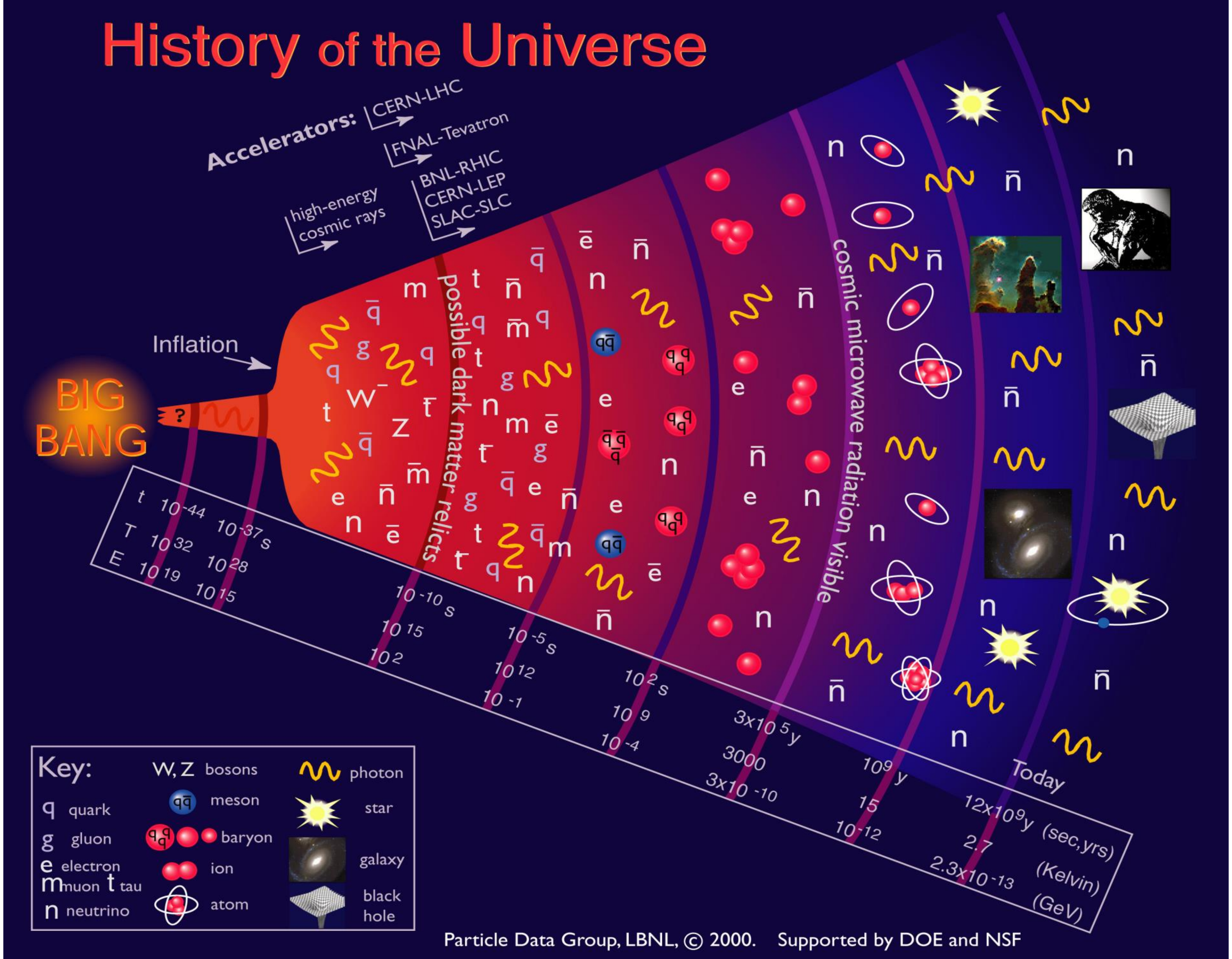
$$dS = d\left[\frac{(\rho+p)V}{T}\right] = 0$$

- Entropy (per comoving volume) concerved:
- Entropy density:

$$s = \frac{S}{V} = \frac{2\pi^2}{45} g_{*S} T^3$$

$$s \propto a^{-3} \Rightarrow g_{*S} T^3 a^3 = const$$

$$g_{*S} = \sum_{i=Bosons} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=Fermions} g_i \left(\frac{T_i}{T}\right)^3$$


History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10^-44 10^-37 s
T 10^32 10^28
E 10^19 10^15
10^-10 s
10^15
10^2
10^-5 s
10^12
10^-1
10^2 s
10^9
10^-4
3x10^5 y
3000
3x10^-10
10^9 y
15
10^-12
Today
12x10^9 y (sec,yrs)
2.7 (Kelvin)
2.3x10^-13 (GeV)
Key:
q quark
g gluon
e electron
m muon
t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

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Boltzmann equation

- Describe **departure** from thermal equilibrium
- Follow the microscopic evolution of the particle's phase space distribution

$$\hat{\mathbf{L}}[f] = \frac{df}{dt} = \mathbf{C}[f]$$

$$\hat{\mathbf{L}} = \frac{d}{d\tau} = \frac{\partial x^\mu}{\partial \tau}\frac{\partial}{\partial x^\mu} + \frac{\partial p^\mu}{\partial \tau}\frac{\partial}{\partial p^\mu} = p^\mu \frac{\partial}{\partial x^\mu} - \Gamma^\mu_{\rho\sigma} p^\rho p^\sigma \frac{\partial}{\partial p^\mu}$$

From geodesic Eq. $\frac{\partial p^\mu}{\partial \tau} + \Gamma^\mu_{\sigma\rho} p^\sigma p^\rho = 0$

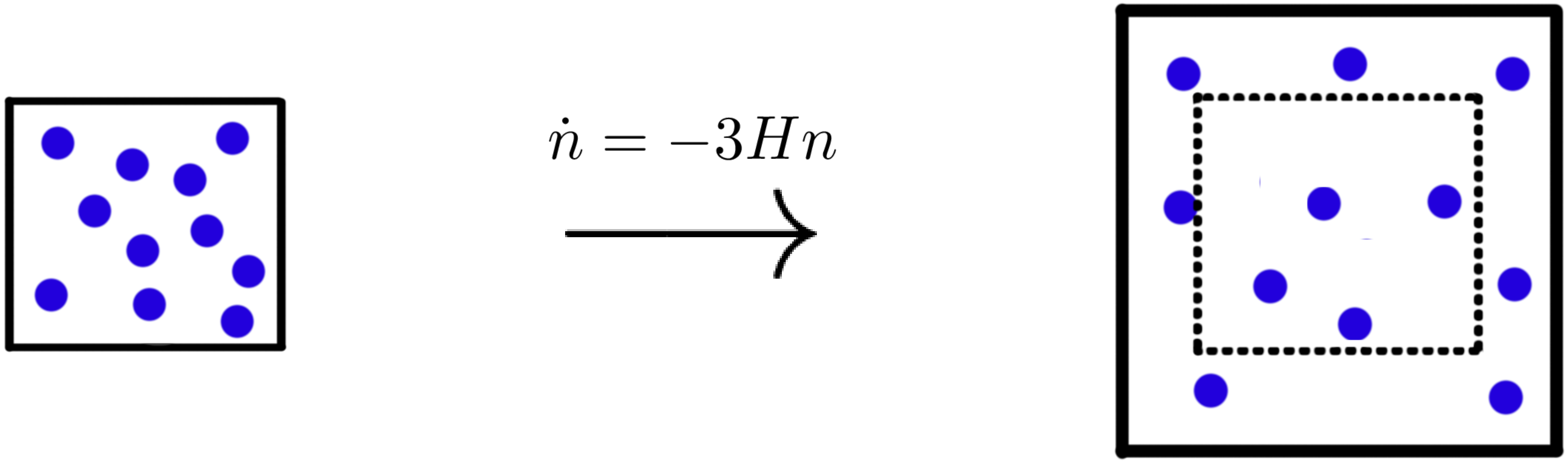
- For the FRW metric $\hat{L}[f(E,t)] = E\frac{\partial f}{\partial t} - H|\vec{\mathbf{p}}|^2 \frac{\partial f}{\partial E}$

$$n = \frac{g}{(2\pi)^3}\int f(\vec{p}) d^3p$$

$$\frac{1}{a^3}\frac{d}{dt}(na^3) = \dot{n} + 3Hn = \frac{g}{(2\pi)^3}\int \mathbf{C}[f]\frac{d^3p}{E}$$

- Scale out effect of expanding universe $$Y \equiv \frac{n}{s} \Rightarrow s\dot{Y} = \dot{n} + 3Hn \qquad sa^3 = const$$
- Change variable $$x \equiv \frac{m}{T} \to s\dot{Y} = xH\frac{dY}{dx} \text{ since } T \propto a^{-1}$$

$$\frac{x}{s}\frac{dY}{dx} = \frac{1}{H}\frac{g}{(2\pi)^3}\int \mathbf{C}[f]\frac{d^3p}{E}$$



$$\dot{n} = -3Hn$$

Collision term

$$d\Pi \equiv g\frac{1}{2E}\frac{d^3p}{(2\pi)^3}$$

- From thermal QFT: $$\frac{g}{(2\pi)^3}\int \mathbf{C}[f]\frac{d^3p}{E} = -\int d\Pi_\psi d\Pi_a d\Pi_b ... d\Pi_i d\Pi_j ...$$ $$\times(2\pi)^4\delta^4(p_\psi + p_a + p_b + ... - p_i - p_j - ...)$$ $$\times\Big[|\mathcal{M}|^2_{\psi+a+b+...\to i+j+..} f_a f_b ... f_\psi (1\pm f_i)(1\pm f_j)...$$ $$-|\mathcal{M}|^2_{i+j+...\to\psi+a+b+...} f_i f_j ... (1\pm f_a)(1\pm f_b)...(1\pm f_\psi)\Big]$$

- Approximations: T (or CP) invariance and Maxwell-Boltzmann distribution

$$\dot{n} + 3Hn = -\int d\Pi_\psi d\Pi_a d\Pi_b ... d\Pi_i d\Pi_j ...$$
$$\times (2\pi)^4\delta^4(p_\psi + p_a + p_b + ... - p_i - p_j - ...)|\mathcal{M}|^2[f_\psi f_a f_b ... - f_i f_j ...]$$

Freeze out

- Suppose thermal equilibrium and only creation and annihilation
- Energy conservation
- Boltzmann Equation
- Cross section times velocity
- Rule of thumb

$$\bar{\psi}\psi \leftrightarrow \bar{X}X$$

$$f_X f_{\bar{X}} = \exp[-(E_X + E_{\bar{X}})/T] = \exp[-(E_\psi + E_{\bar{\psi}})/T] = f_\psi^{EQ} f_{\bar{\psi}}^{EQ}$$

$$\dot{n_\psi} + 3Hn_\psi = -\langle \sigma_{\psi\bar{\psi}\to X\bar{X}} |v| \rangle [n_\psi^2 - (n_\psi^{EQ})^2]$$

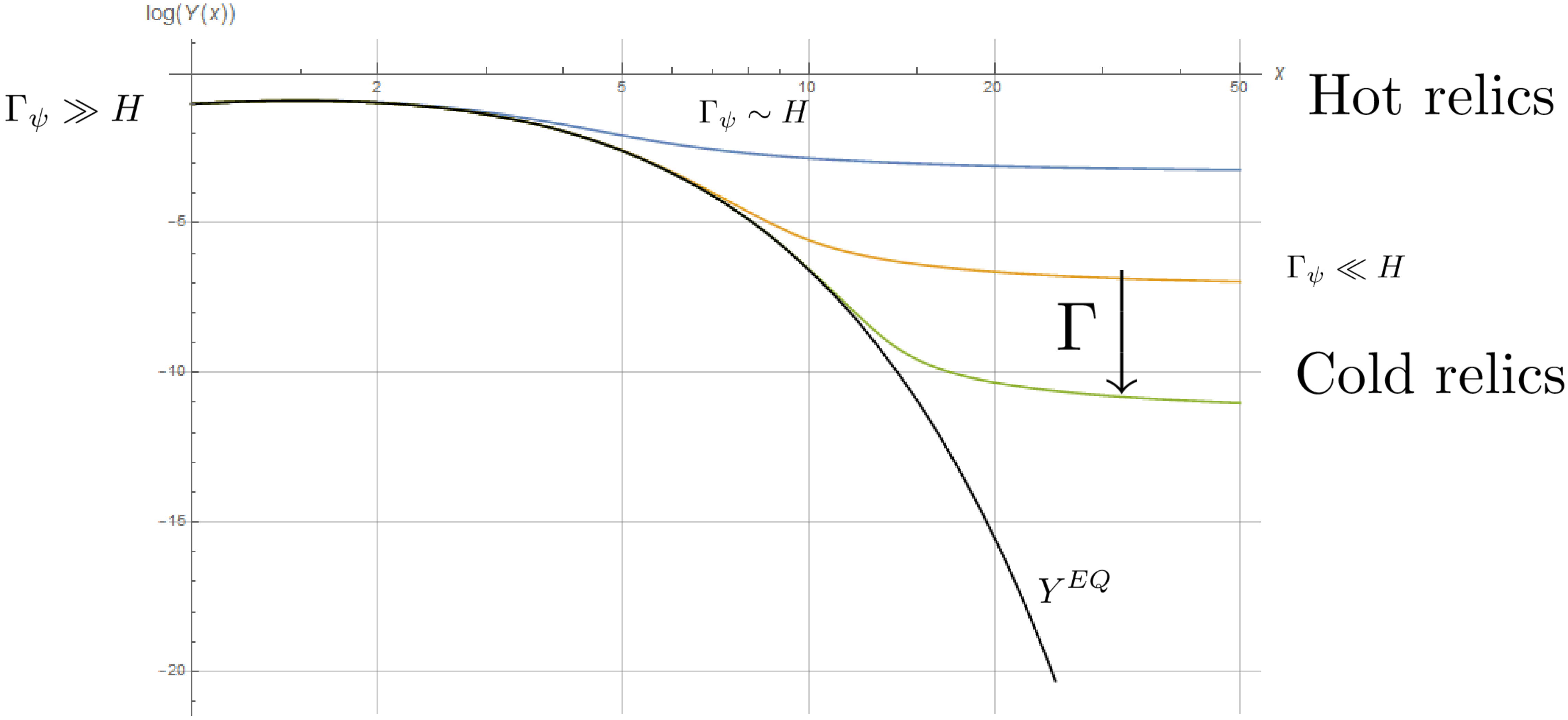
$$\langle \sigma_{\psi\bar{\psi}\to X\bar{X}} |v| \rangle = (n_\psi^{EQ})^{-2} \int d\Pi_\psi d\Pi_{\bar{\psi}} d\Pi_X d\Pi_{\bar{X}} (2\pi)^4 \times \delta^{(4)}(p_\psi + p_{\bar{\psi}} - p_X - p_{\bar{X}}) |\mathcal{M}|^2 \exp(-E_\psi/T) \exp(-E_{\bar{\psi}/T})$$

$$\frac{x}{Y_\psi^{EQ}} \frac{dY_\psi}{dx} = -\frac{\Gamma_\psi}{H} \left[\left(\frac{Y_\psi}{Y_\psi^{EQ}} \right)^2 - 1 \right]$$

$$\Gamma_\psi = n_\psi^{EQ} \langle \sigma_{\psi\bar{\psi}\to X\bar{X}} |v| \rangle$$

$$\frac{x}{Y_\psi^{EQ}}\frac{dY_\psi}{dx} = -\frac{\Gamma_\psi}{H}\left[\left(\frac{Y_\psi}{Y_\psi^{EQ}}\right)^2 - 1\right]$$

$$\Gamma_\psi = n_\psi^{EQ}\langle\sigma_{\psi\psi\to\bar{X}\bar{X}}|v|\rangle$$



Hot relics

- Highly relativistic $x_f = \frac{m}{T_f} \ll 3$
- Comoving number density
- Example: Neutrinos
  - Temperature when decoupling
  - Temperature today

$$s = \frac{S}{V} = \frac{2\pi^2}{45} g_{*S} T^3$$

$$Y_\infty = Y(x \to \infty) = Y_{EQ}(x_f) = \frac{45\zeta(3)}{2\pi^4} \frac{g_{eff}}{g_{*s}}$$

$$\frac{\Gamma}{H} \simeq \left(\frac{T}{1\ \mathrm{MeV}}\right)^3$$

$$sa^3 = const \Rightarrow g_{*s}^{before} T_{before}^3 = g_{*s}^{after} T_{after}^3$$

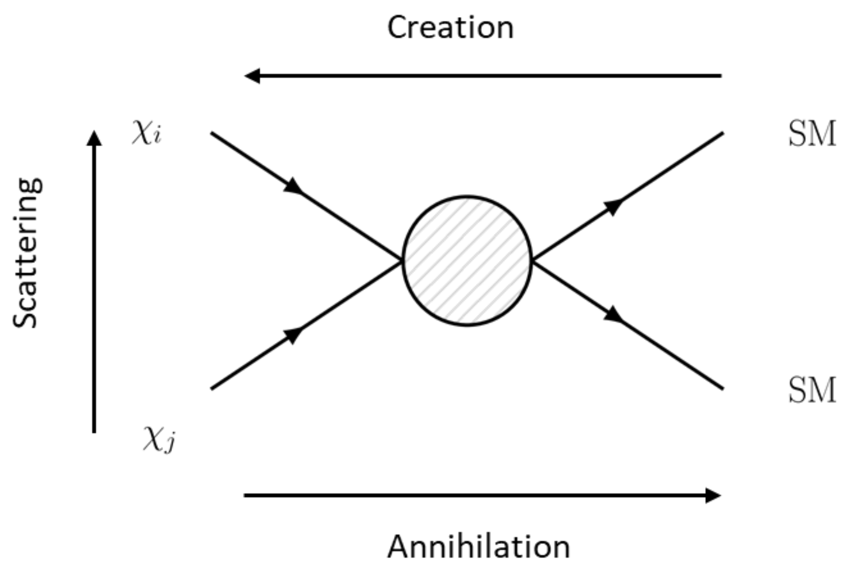
$$g_{*s}^{before} = 2_\gamma + \frac{7}{8} \times 2_{e^+e^-} \times 2_{\uparrow\downarrow} = \frac{11}{2}$$

$$g_{*s}^{after} = 2_\gamma$$

$$T_\nu = \left(\frac{11}{4}\right)^{-\frac{1}{3}} T_\gamma = \left(\frac{11}{4}\right)^{-\frac{1}{3}} \times 2.73K = 1.95K$$

Co-annihilations

- More **general** Boltzmann equation
- If $\Delta_m \gg T_f, \chi_2$ particle play **no** significant role.
- Particle close in mass → also present when freeze-out



$$\sigma_{ij} = \sum_X \sigma(\chi_i \chi_j \to X)$$

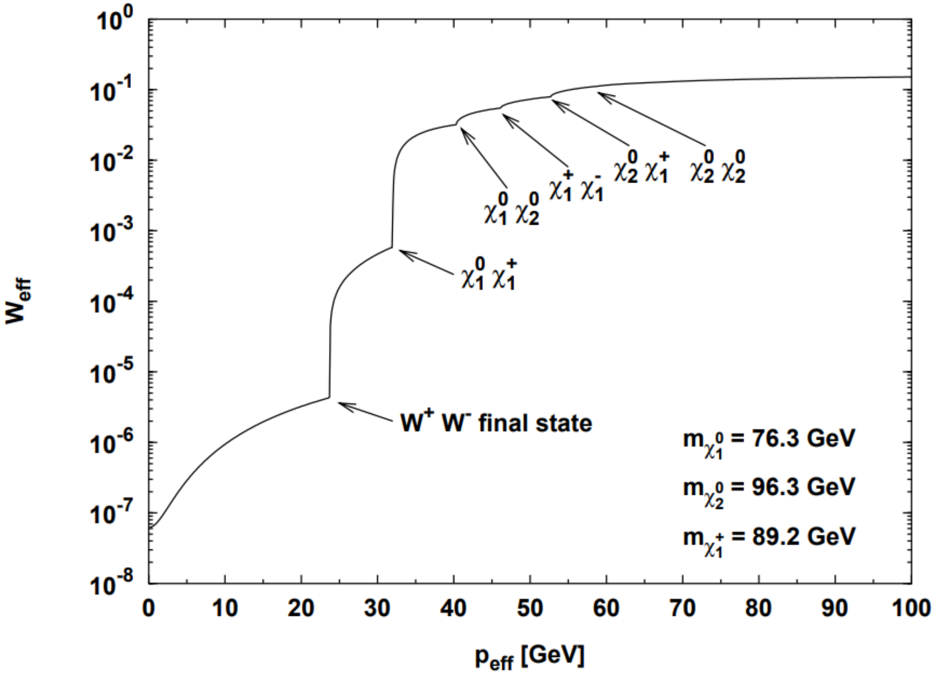
$$\sigma'_{Xij} = \sum_Y \sigma(X\chi_i \to \chi_j Y)$$

$$\dot{n}_i + 3Hn_i = -\sum_j \langle \sigma_j v_j \rangle (n_i n_j - n_i^{EQ} n_j^{EQ})$$
$$- \sum_{i \neq j} [\langle \sigma'_{Xi} v_j \rangle (n_i n_X - n_i^{EQ} n_X^{EQ}) - \langle \sigma'_{Xj} v_j \rangle (n_j n_X - n_j^{EQ} n_X^{EQ})]$$
$$- \sum_{i \neq j} [\Gamma_{ij}(n_i - n_i^{EQ}) - \Gamma_{ji}(n_j - n_j^{EQ})]$$

$$\Gamma_{ij} = \sum_X \Gamma(\chi_i \to \chi_j X)$$

Edsjö, J. hep-ph/9704384

Effective invariant annhiliation rate



- Rewrite into usual Boltzmann equation by summing over all particles
- All particles decay down
- Neutralinos in SUSY
- Smaller relic density

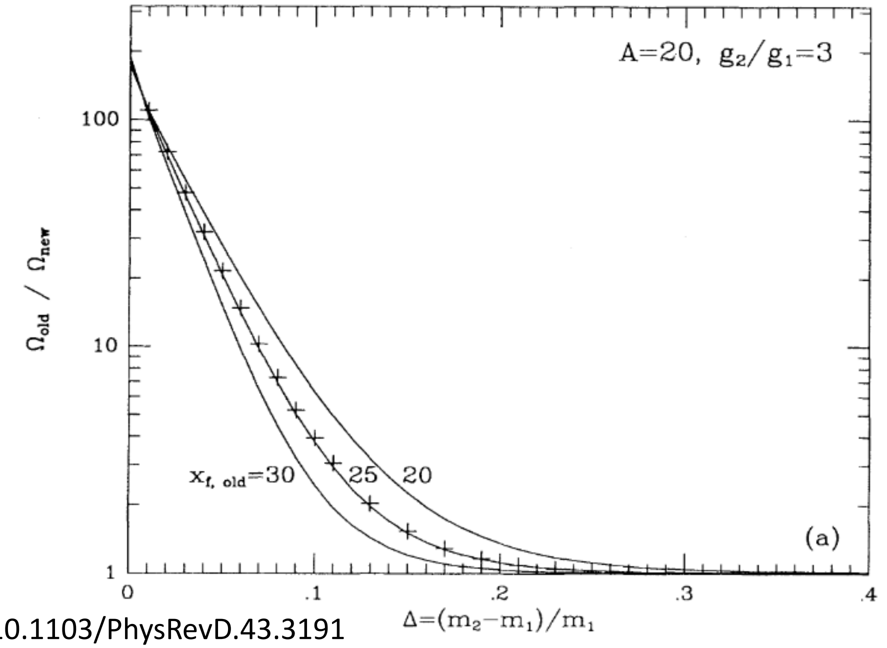
$$n = \sum_i n_i$$

$$\frac{dn}{dt} + 3Hn = -\langle \sigma_{eff} v \rangle (n^2 - n_{eq}^2)$$

$$\sigma_{eff} = \sum_{ij}^{N} \sigma_{ij} \frac{g_i g_j}{g_{eff}^2} (1+\Delta_i)^{3/2} (1+\Delta_j)^{3/2}$$
$$\times \exp[-x(\Delta_i + \Delta_j)]$$

$$\Delta_i = (m_i - m_1)/m_1$$

$$\Omega_\chi h_2 = 0.18 \rightarrow 0.030$$



Griest. K, Seckel, D. 10.1103/PhysRevD.43.3191

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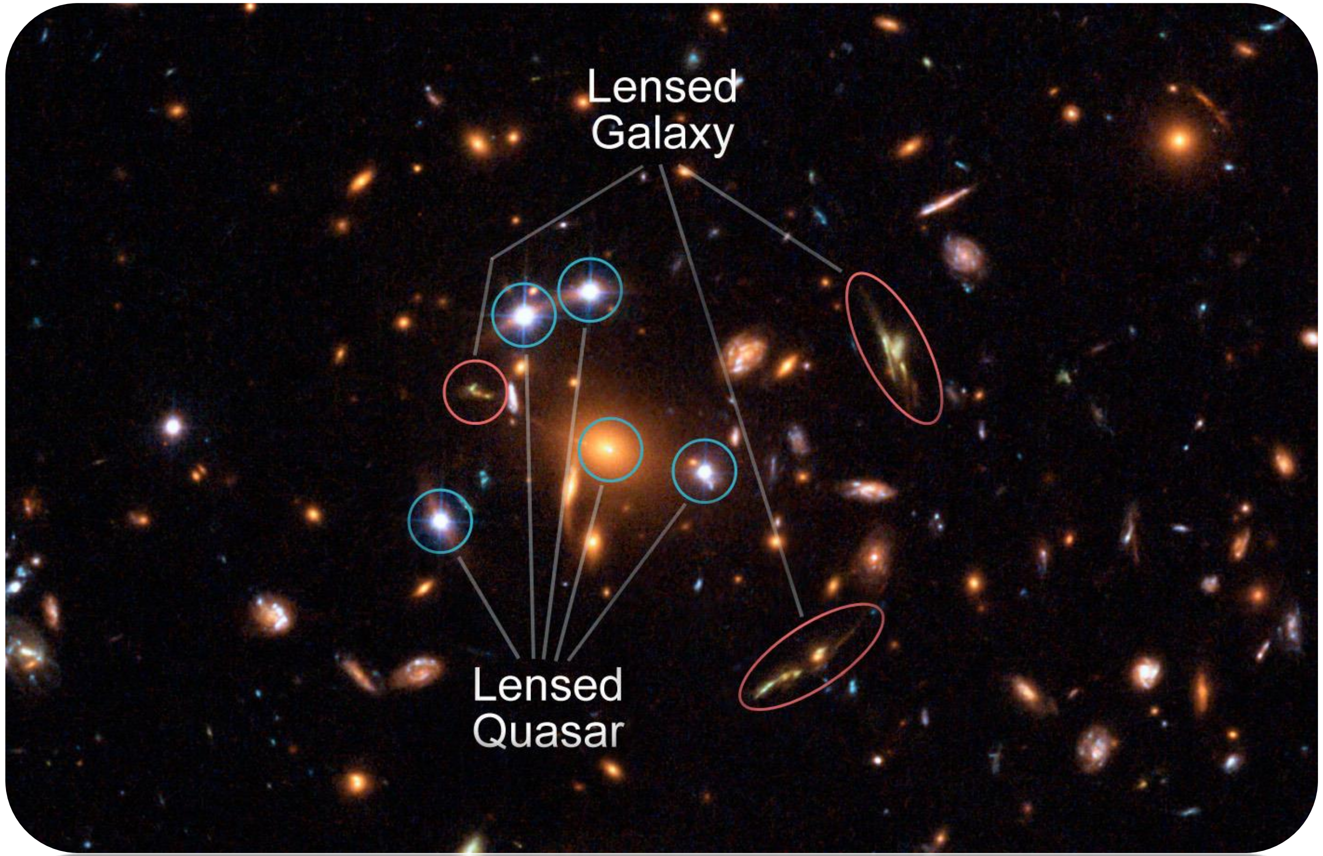
Introduction to Dark Matter

- Rotation curves of spiral galaxies
- Structure formation in the universe
- Structure of CMB
- Gravitational lensing



Credits: ESA, NASA, K. Sharon (Tel Aviv University) and E. Ofek (Caltech)

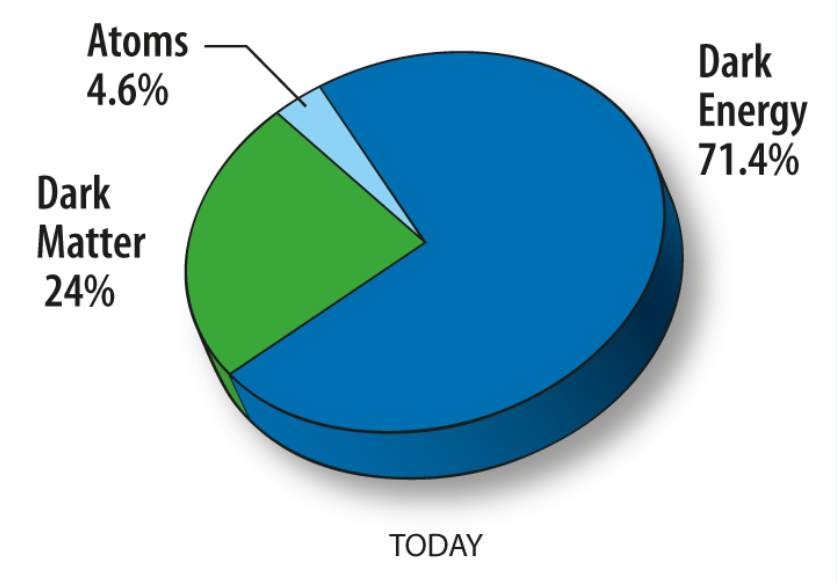
What we know

$$\Omega_{DM}h^2 \approx 0.12$$

$$\Omega_b h^2 \approx 0.02$$

- Massive
- Interacts through gravity
- Majority is cold, (hot DM suppress clustering)
- Stable on cosmological scales

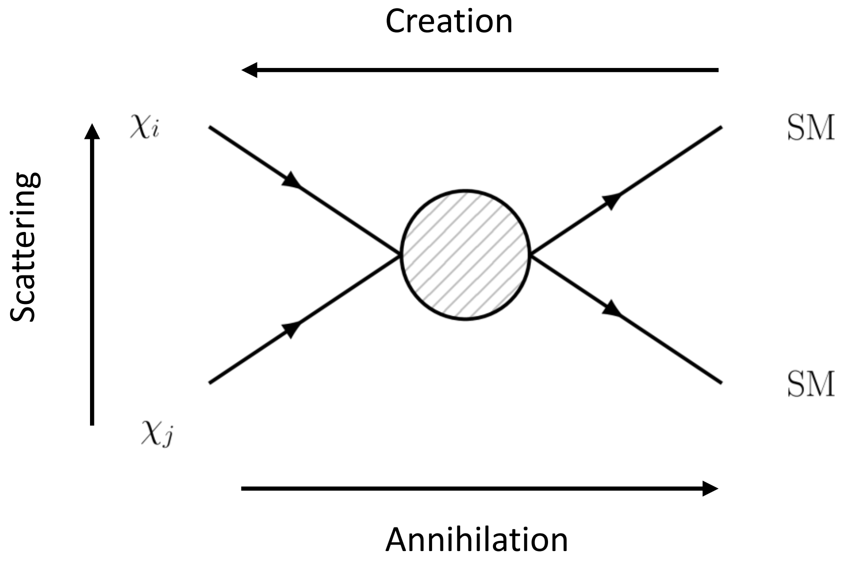
$$H_0 = 100h\text{km s}^{-1}\text{Mpc}^{-1}$$



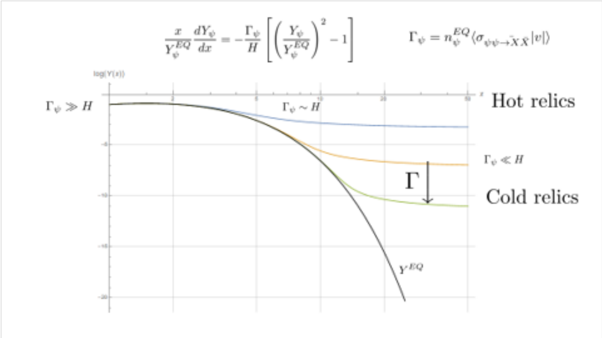
https://wmap.gsfc.nasa.gov/universe/uni_matter.html

WIMP Magic

- **Weak** scale Dark Matter
- Freeze-out when non-relativistic $x_f \ll 3$
- Assume $\chi\chi \leftrightarrow SMSM$
- Co-annihilation
- **WIMP Miracle!**
- Neutralino if LSP, assuming R-Parity.
- No show ☹

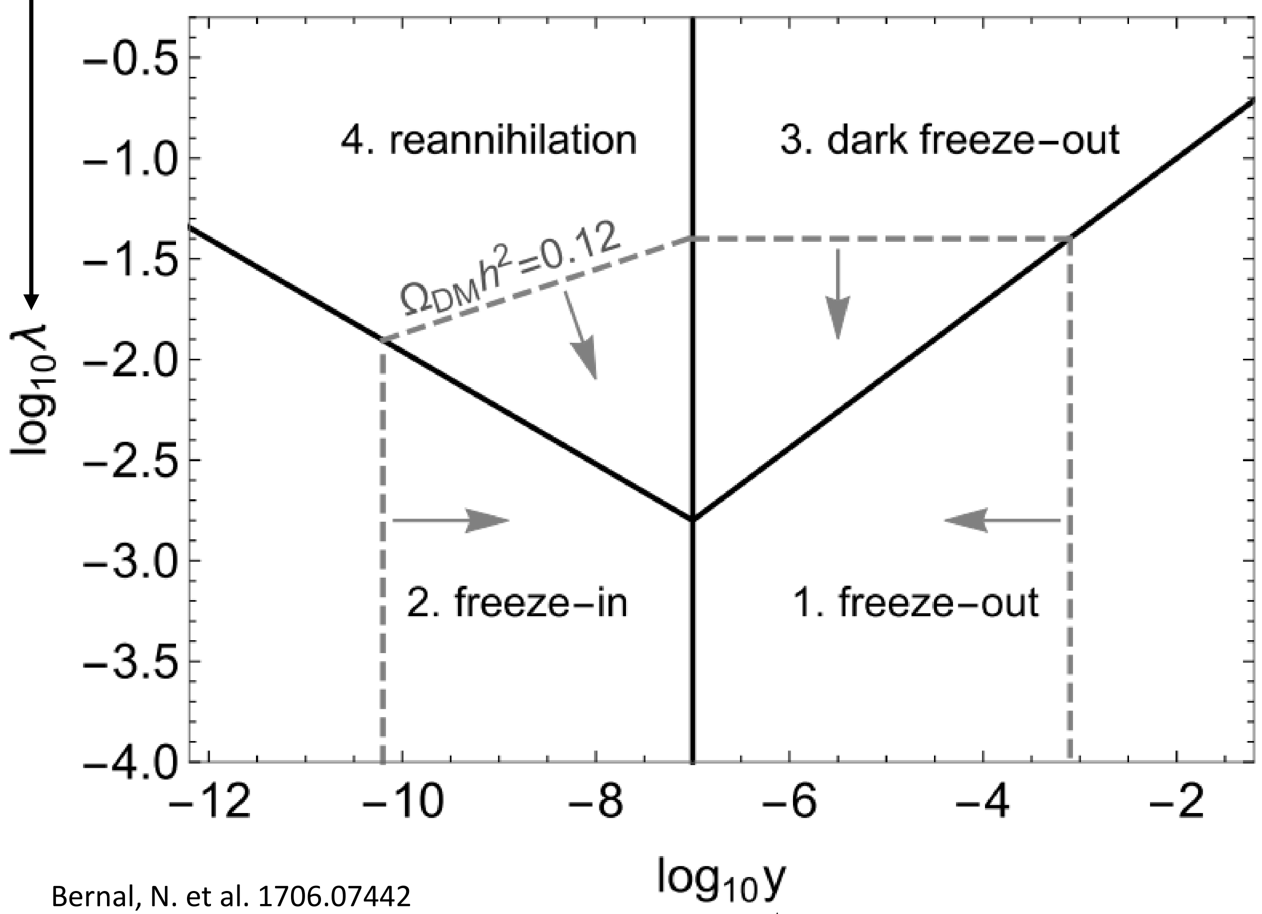


$$\Omega_\chi h^2 \approx 0.12 \times \frac{3 \times 10^{-26} cm^3 s^{-1}}{\langle \sigma v \rangle} \approx 0.12 \times \left(\frac{g_{weak}}{g_\chi} \right)^4 \left(\frac{m_\chi}{m_{weak}} \right)^2$$



More mechanisms

DM Self coupling



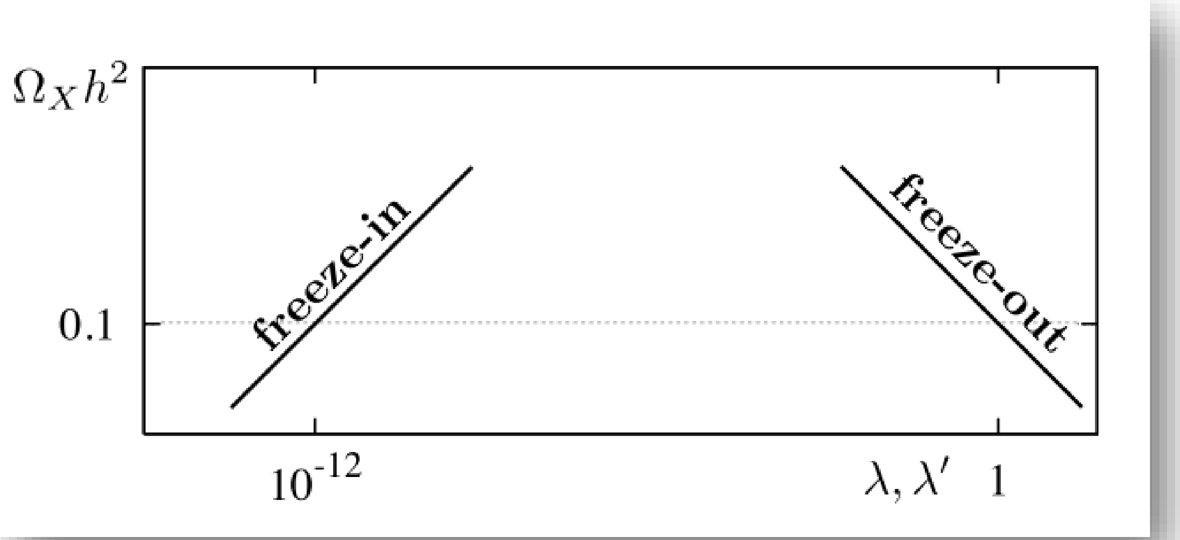
Bernal, N. et al. 1706.07442

Coupling to the visble sector

Arrows indicate increase in relic density

Freeze-in mechanism

- **Lower coupling** → no thermal equilibrium
- Not WIMP but **FIMP** (Feebly Interacting Massive Particle)
- Abundance produced by
  - Decay $\sigma \to \chi\chi$, $\sigma \to X\chi$, etc.
  - 2→2 $\sigma\sigma \to \chi\chi$, etc.
- Much more difficult to search for.



Hall, L. et al. 0911.1120

Freeze in from 2→2

- Suppose scalar DM
- Assume
  - Initial DM abundance negligible
  - B masses negligible
  - Matrix element $|M|^2=y^2$
- Calculate relic and get coupling

$$\dot{n_\psi} + 3Hn_\psi = -\langle\sigma_{\psi\bar{\psi}\to X\bar{X}}|v|\rangle[n_\psi^2 - (n_\psi^{EQ})^2]$$

$$\mathcal{L}_{4-scalar} = y\chi B_1 B_2 B_3$$

Due to $B_2 B_3 \to B_1\chi$ and $B_1 B_2 \to B_2\chi$

$$\dot{n_\chi} + 3n_\chi H \approx 3\int d\Pi_{B1} d\Pi_{B2} d\Pi_{B3} d\Pi_\chi (2\pi)^4\delta^4(p_{B_1} + p_{B_2} - p_{B_3} - p_\chi)|\mathcal{M}|^2_{B_1 B_2\to B_3\chi} f_{B_1} f_{B_2}$$

$$y \simeq 1\times 10^{-11}\left(\frac{\Omega_\chi h^2}{0.1}\right)^{1/2}\left(\frac{g_*(m_\chi)}{100}\right)^{3/4}$$

Freeze-in from decay

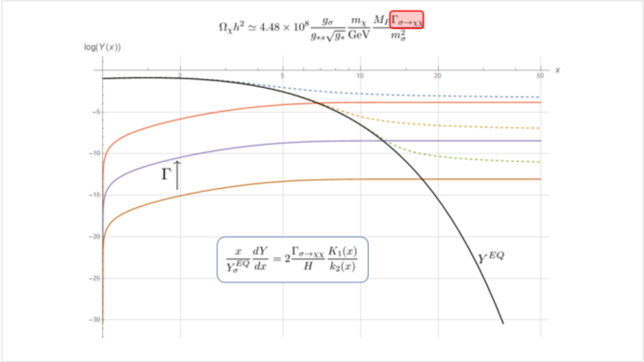
- Decay $\sigma \to \chi\chi$
- Boltzmann equation
- Coupling

$$\dot{n}_\chi + 3Hn_\chi \approx g_\sigma \int d\Pi_\sigma \Gamma_{\sigma\to\chi\chi} m_\sigma f_\sigma = 2\Gamma_{\sigma\to\chi\chi}\frac{K_1(m_\sigma/T)}{K_2(m_\sigma/T)} n_\sigma^{EQ}$$

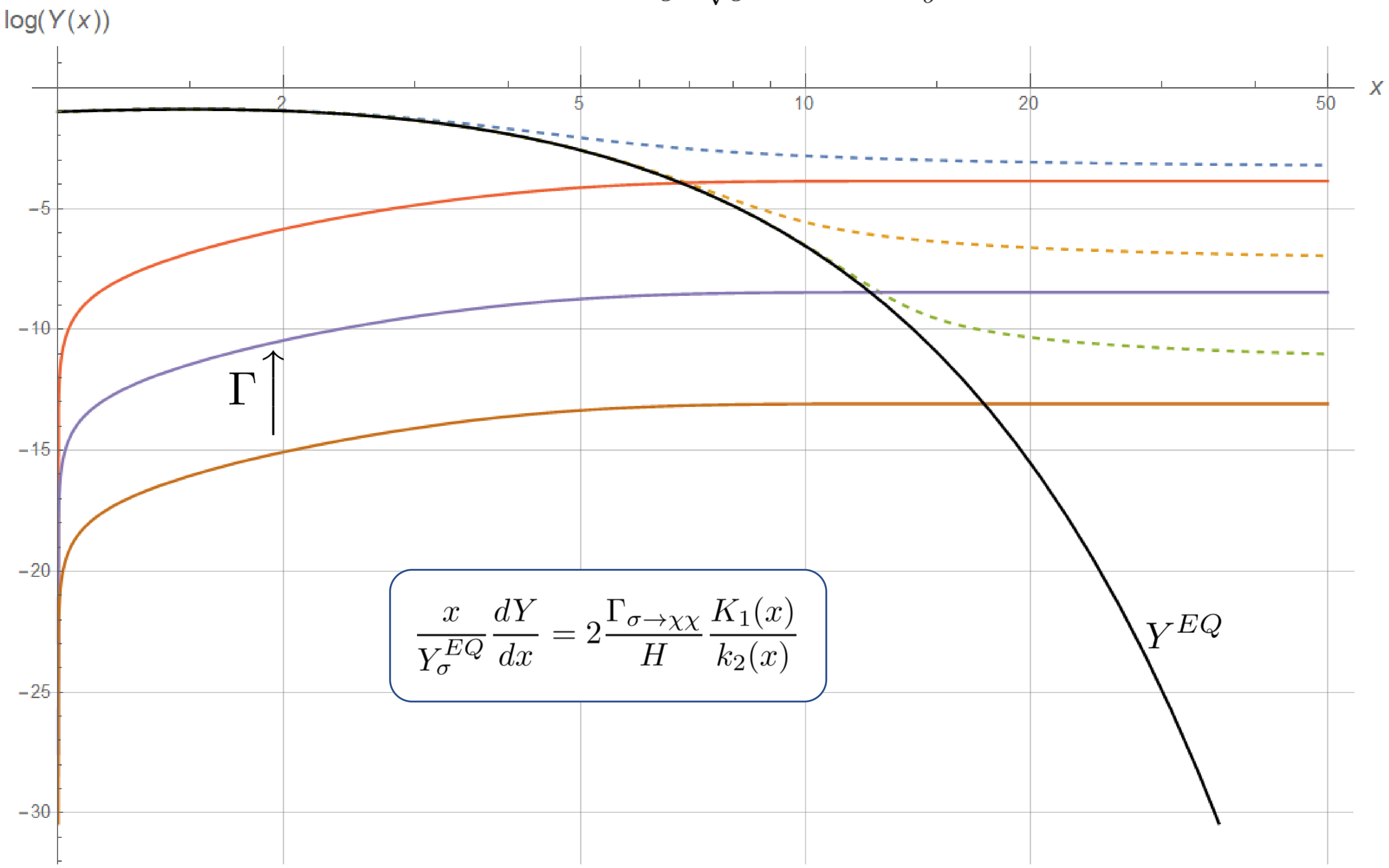
$$\frac{x}{Y_\sigma^{EQ}}\frac{dY}{dx} = 2\frac{\Gamma_{\sigma\to\chi\chi}}{H}\frac{K_1(x)}{k_2(x)}$$

$$\Omega_\chi h^2 \simeq 4.48\times 10^8 \frac{g_\sigma}{g_{*s}\sqrt{g_*}}\frac{m_\chi}{\text{GeV}}\frac{M_P\Gamma_{\sigma\to\chi\chi}}{m_\sigma^2}$$

$$y \simeq 10^{-12}\left(\frac{\Omega_\chi h^2}{0.12}\right)^{1/2}\left(\frac{g_*}{100}\right)^{3/4}\left(\frac{m_\sigma}{m_\chi}\right)^{1/2}$$



$$\Omega_\chi h^2 \simeq 4.48\times 10^8 \frac{g_\sigma}{g_{*s}\sqrt{g_*}} \frac{m_\chi}{\mathrm{GeV}} \frac{M_P \Gamma_{\sigma\to\chi\chi}}{m_\sigma^2}$$



$$\frac{x}{Y_\sigma^{EQ}}\frac{dY}{dx} = 2\frac{\Gamma_{\sigma\to\chi\chi}}{H}\frac{K_1(x)}{k_2(x)}$$

Summary

- Bolzmann equation describes **departure** from thermal equilibrium.
- If coupling is “large” → freeze-out
- More mechanisms than freeze-out
- If coupling is small → freeze-in
- Observed relic can be explained
- Large self-interaction coupling: reannihilation or dark freeze-out

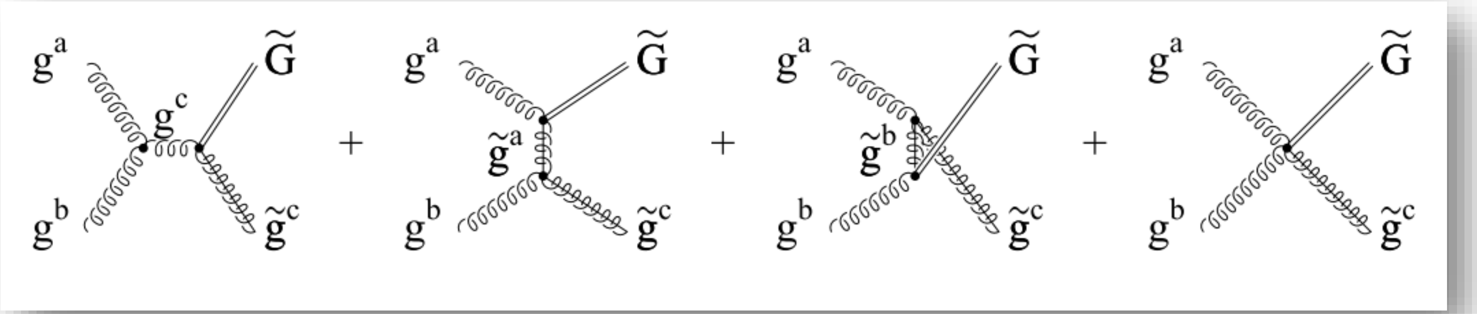
Thank you for listening

Gravitino

- SUGRA: make SUSY Local
- Spin-3/2
- Gravitino problems
  1. Overclose the universe
  2. Late decays affecting BBN
- Three mechanisms
  - superWIMP
  - Thermal scattering of superpartners in the early universe (UV Freeze-in)
  - Decays of superpartners which are in thermal equilibrium (IR Freeze-in)

UV Freeze-in

Bolz, M. et al. hep-ph/0701104



- Dimension 5 operator
- Dependent on reheating temperature
- 2→2 QCD scattering processes dominates

$$\Omega_{3/2}h^2 \simeq 0.5 \left(\frac{T_R}{10^{10}\mathrm{GeV}}\right)\left(\frac{100\mathrm{GeV}}{m_{3/2}}\right)\left(\frac{m_{\tilde{g}}(\mu)}{1\mathrm{TeV}}\right)^2$$

IR Freeze-in

- Abundance from decay
- Depends solely on superpartner spectrum and $m_{3/2}$
- Lower temperature
- Corresponds to vertical lines
- Size depends on scalar and gaugino masses

Scalar masses

$$\frac{Y_{3/2}^{decay}}{Y_{3/2}^{scatt}} \propto \frac{\sum_i m_i^3}{T_R \sum_a g_a^2 m_a^2}$$

Gaugino masses

$$\Omega_{3/2}h^2 \simeq \frac{1.09 \cdot 10^{27}}{g_*^{3/2}} m_{3/2} \sum_i g_i \frac{\Gamma_i}{m_i^2}$$

Hall, L. et al. 1103.4394

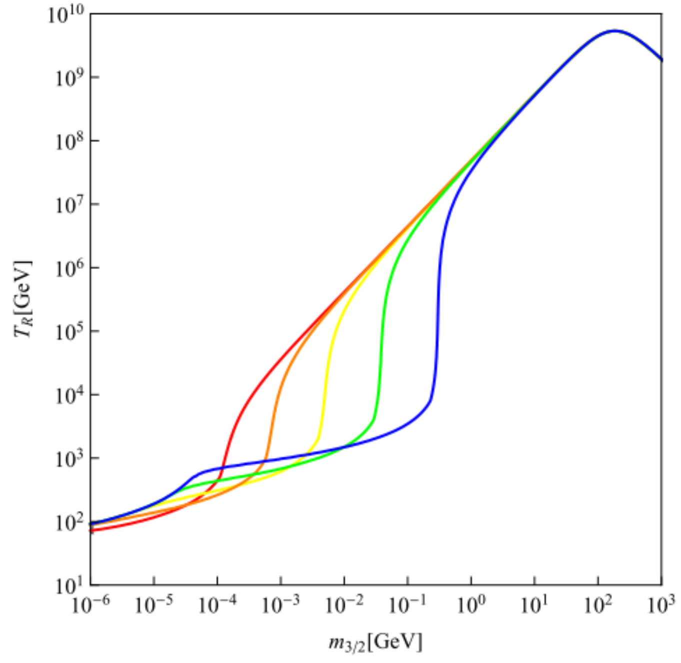


FIG. 1: Contours of $\Omega_{3/2}h^2 = 0.11$ for gaugino masses fixed to $\{m_{\tilde{b}}, m_{\tilde{w}}, m_{\tilde{g}}\} = \{100, 210, 638\}$ GeV. The {red, orange, yellow, green, blue} contours correspond to universal scalar masses {500 GeV, 1 TeV, 2 TeV, 4 TeV, 8 TeV}.