WHERE DO WE STAND IN PROTON DECAY PREDICTIONS IN SU(5) SUPERSYMMETRIC MODELS?

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Mainly based on 1902.04888 (J. Ellis, J. Evans, N. Nagata K. Olive, L. V-S)

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- Model Framework
- Proton decay basics in SU(5)
- Results
- Messages

MOTIVATION

- Global symmetries may not be fundamental: Lepton and Baryon numbers may not represent exact symmetries $\longrightarrow$ we need a proof.
- + 40 years of Grand Unified Theories with sound predictions: Neutrino Masses, Proton Decay, Unification.
- Supersymmetry (and supergravity) $\longrightarrow$ Yes, up to now not a better **overall** framework.
- PD, EDM and flavor observables can be far more powerful than collider observables.

- Global symmetries may not be fundamental:

  - At the classical level it is not possible to construct an operator in the SM that violates Lepton or Baryon number.

  - Beyond CL, we can construct

$$\mathcal{L}_1 = \frac{1}{M} H\ H\ L\ L, \quad H = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}, \quad L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}$$

  - when H acquires a VEV we then have Majorana neutrino masses

$$\mathcal{L}_1 = L\ \frac{v^2}{M} L, \quad m_\nu = \frac{v^2}{M}$$

  - we then say that at the scale $\mathcal{L}_1$ can be constructed the lepton numbers $L_e,\quad L_\mu \quad \text{and} \quad L_\tau$ are violated.

- In the same way we can construct

$$\mathcal{L}_2 = \frac{1}{M^2} Q\ Q\ Q\ L, \quad Q = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$$

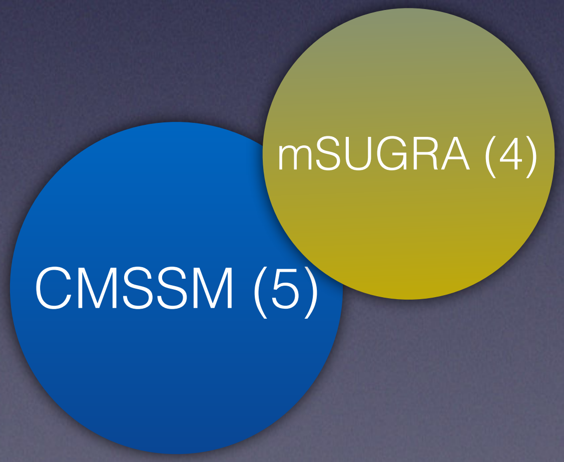
- Q does not acquire a VEV but id mediates proton decay. Hence $\mathcal{L}_2$ violates baryon number.

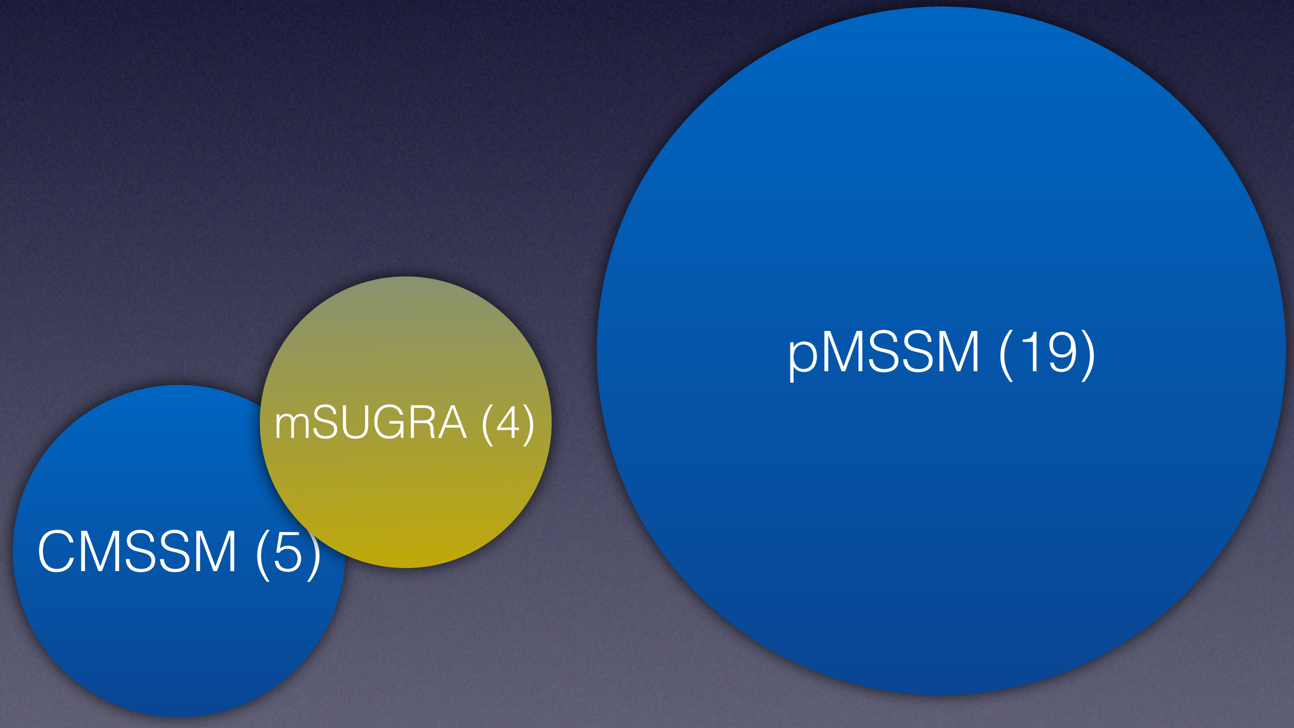
- If the scale M is the same as for $\mathcal{L}_1$, then $M = M_{\text{GUT}}$ and we then can connect M_{EW} with M_{GUT} through the normalisation group equations. This then would indicate the presence of a GUT theory ⟶ point out to not fundamental global symmetries.

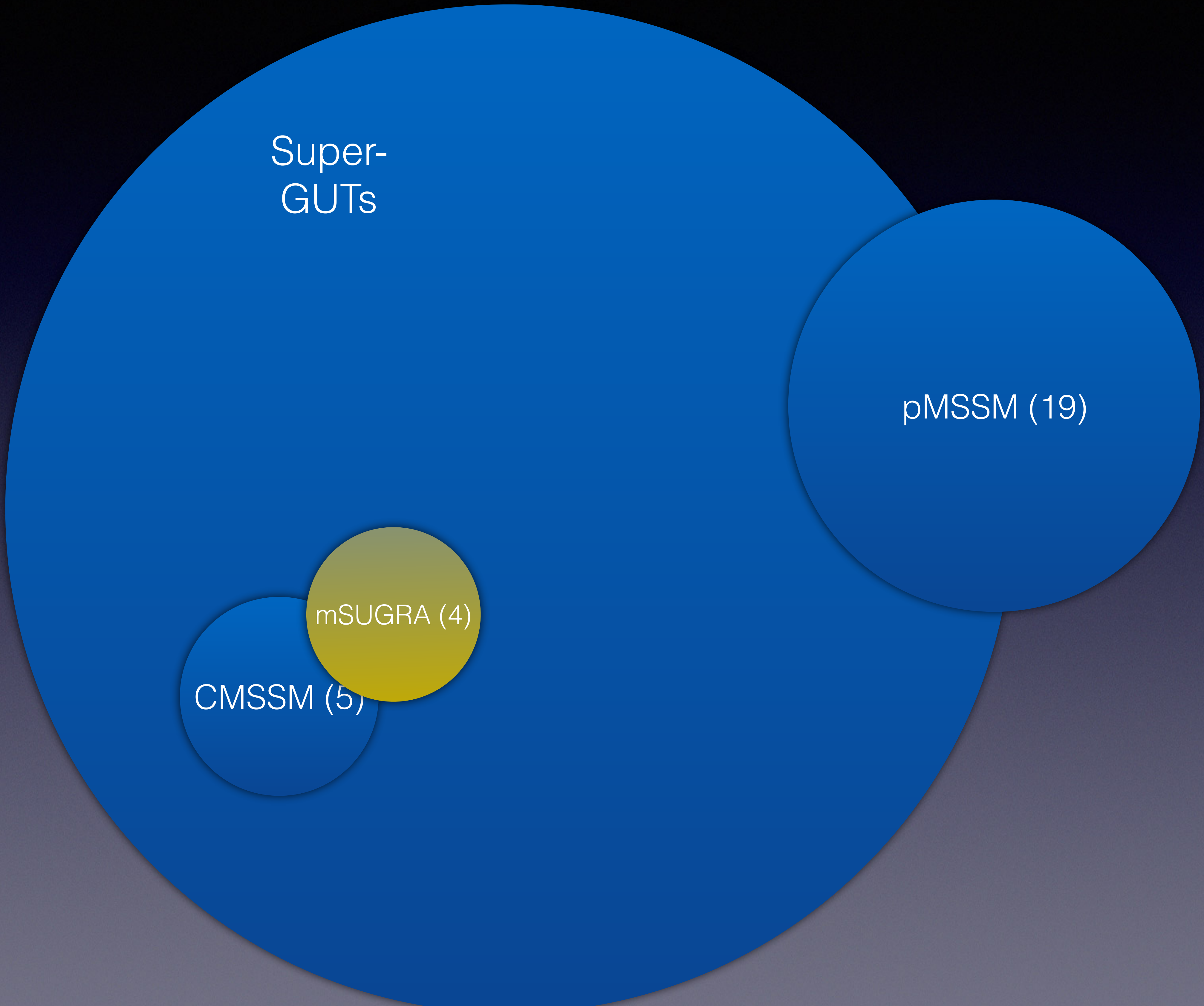
• + 40 years of Grand Unified Theories

- Georgi, Glasgow 1974, SU(5): Used only SU(5) operators, use of analytical 1-loop beta functions for couplings to match to EW
- 1980-1990 Matching to MSSM fields developed, use of numerical 1-loop beta functions for the CMSSM
- 1990-2000: Computation of 2-loops beta functions, matching at EW Scale developed
- 2000-2010: Computation of Higgs Observables, development of tools for probing SUSY at the LHC
- 2010: Adding of running above $M = M_{\rm GUT}$, use of supergravity: no- scale supergravity
- 2013: Code development of no-scale supergravity models
- 2015: Lattice calculation reduced to 30% uncertainty in hadronic parameters
- 2017: Lattice calculation reduced to 10% uncertainty in hadronic parameters
- 2019: Refinements in the theory and precision in calculations for PD

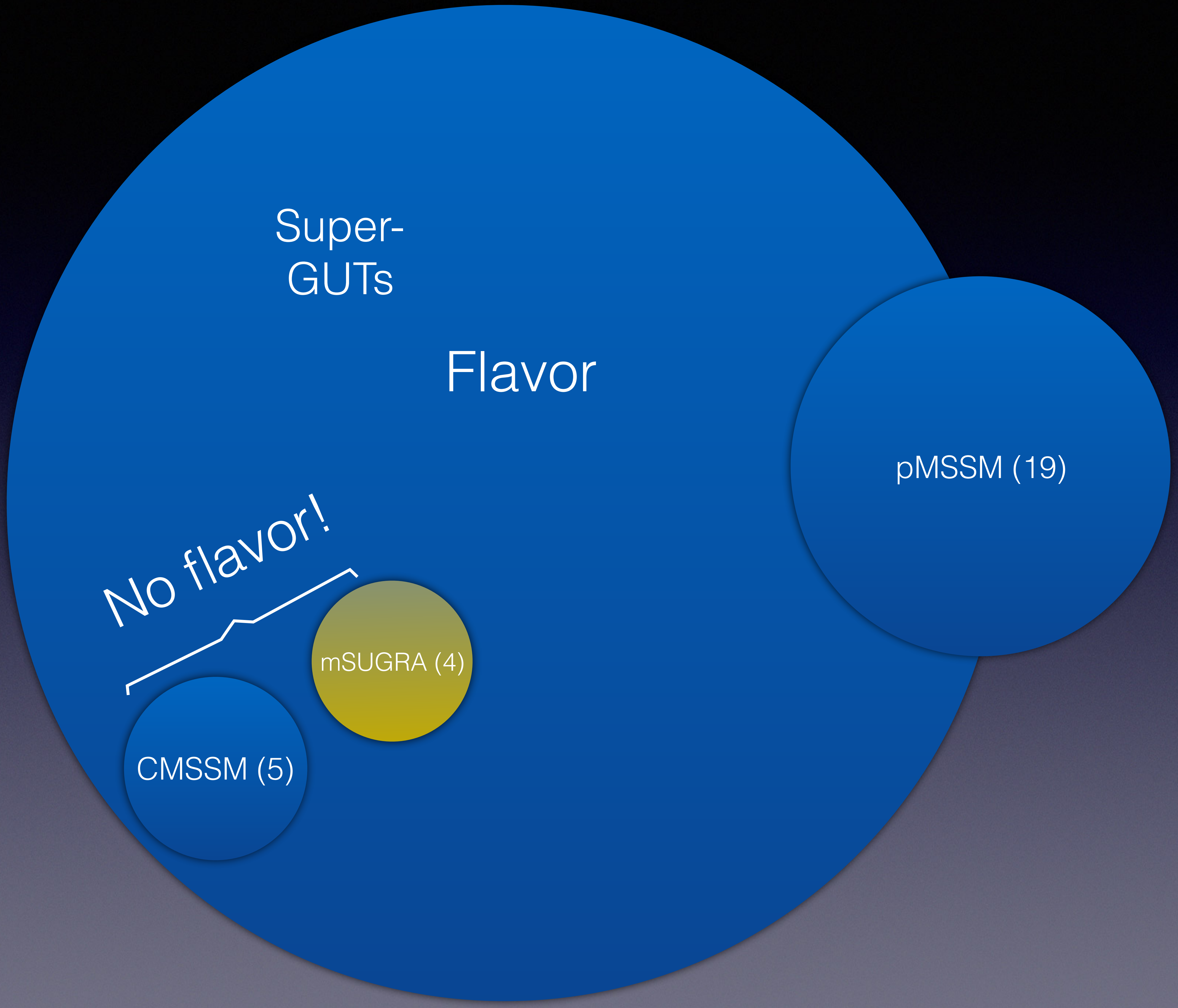
- Supersymmetry

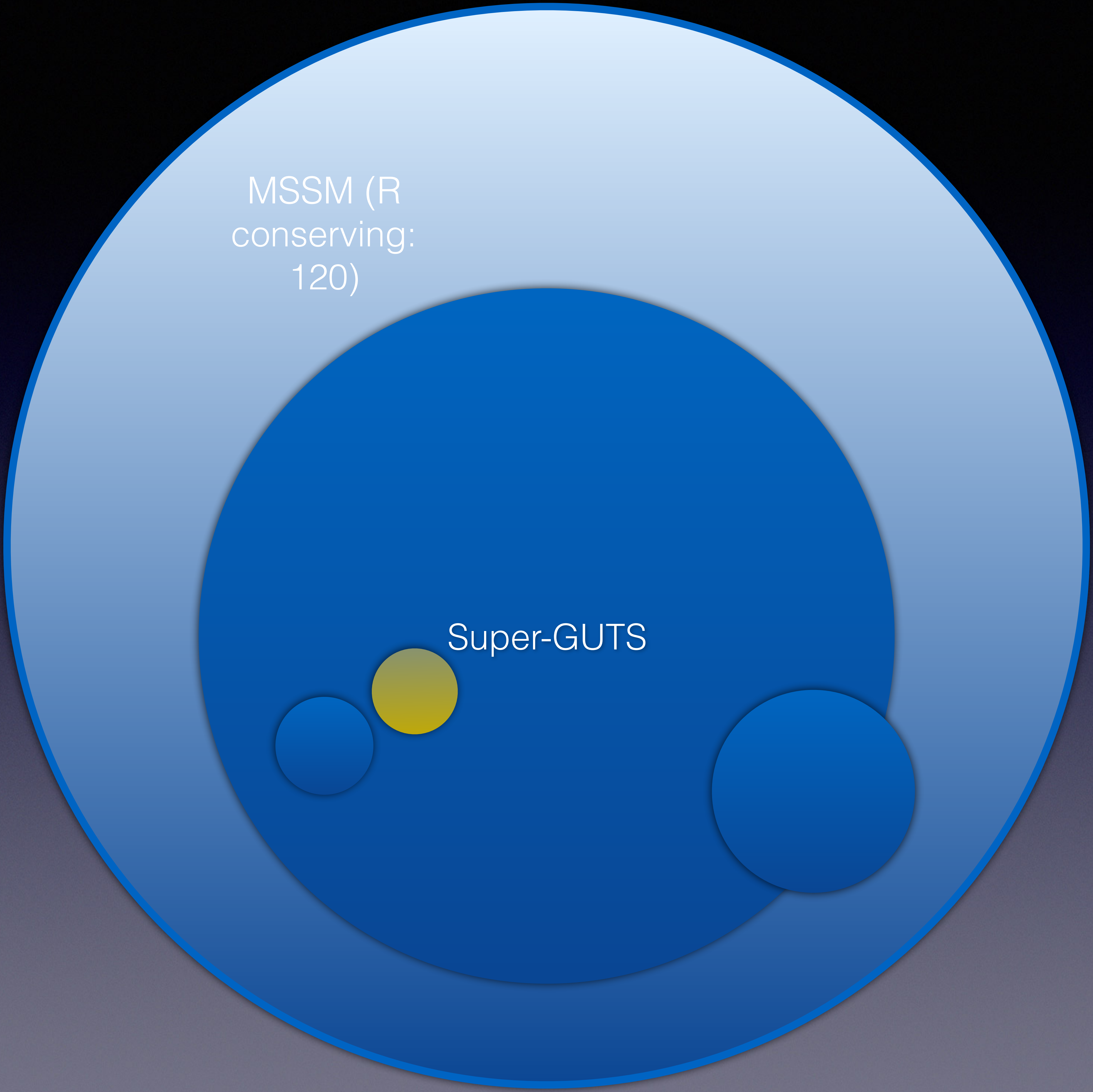



CMSSM (5)
mSUGRA (4)
pMSSM (19)



Super-
GUTs
pMSSM (19)
mSUGRA (4)
CMSSM (5)


Super-
GUTs
Flavor
pMSSM (19)
No flavor!
mSUGRA (4)
CMSSM (5)


MSSM (R conserving: 120)
Super-GUTS

MSSM studies

- Bottom up
  - Select parameters from the MSSM which capture the physics you want to study:
    - CMSSM (simplest model)
    - pMSSM (captures difference of 3rd family, most important contributions to m_h)
    - MSSM-30 (test MFV at M_{GUT})

- Top down
  - Test a well defined model
    - mSUGRA (simplest theoretical model)
    - SU(5), SO(10) models with flavor structures
    - SU(5), SO(10) with flavor-invariants

MSSM studies

- Bottom up

- Select parameters from the MSSM which capture the physics you want to study:
  - CMSSM (simplest predictive model)
  - pMSSM (captures difference of 3rd family, most important contributions to m_h)
  - MSSM-30 (test MFV at $M_{\rm GUT}$)

- Top down

- Test a well defined model
  - mSUGRA (simplest theoretical model)
  - SU(5), S0(10) models with flavor structures
  - SU(5), S0(10) with flavor-invariants + supergravity

CMSSM: Heavy reduction of parameters

$$
\begin{aligned}
-\mathcal{L}_{\rm soft} \;=\; & \tilde{q}^{\dagger}_{Li}(m^2_{\tilde{Q}})^{ij}\tilde{q}_{Lj} + \tilde{u}_{Rj}(m^2_{\tilde{u}})^{ji}\tilde{u}^*_{Ri} + \tilde{d}_{Rj}(m^2_{\tilde{d}})^{ji}\tilde{d}^*_{Ri} \\
& + \tilde{l}^{\dagger}_{Li}(m^2_{\tilde{L}})^{ij}\tilde{l}_{Lj} + \tilde{e}_{Rj}(m^2_{\tilde{e}})^{ji}\tilde{e}^*_{Ri} + \tilde{\nu}_{Rj}(m^2_{\tilde{\nu}})^{ji}\tilde{\nu}^*_{Ri} \\
& + m^2_{h_d}h^{\dagger}_d h_d + m^2_{h_u}h^{\dagger}_u h_u + (B\mu\, h_d h_u + \frac{1}{2}B^{ij}_{\nu}M^{ij}_{\nu}\tilde{\nu}^*_{Ri}\tilde{\nu}^*_{Rj} + \text{h.c.}) \\
& + \Big(-a^{ij}_d h_d \tilde{d}^*_{Ri}\tilde{q}_{Lj} + a^{ij}_u h_u \tilde{u}^*_{Ri}\tilde{q}_{Lj} - a^{ij}_l h_d \tilde{e}^*_{Ri}\tilde{l}_{Lj} + a^{ij}_{\nu} h_u \tilde{\nu}^*_{Ri}\tilde{l}_{Lj} \\
& \quad + \frac{1}{2}M_1\widetilde{B}\widetilde{B} + \frac{1}{2}M_2\widetilde{W}^a\widetilde{W}^a + \frac{1}{2}M_3\widetilde{G}^a\widetilde{G}^a + \text{h.c.}\Big),
\end{aligned}
$$

$M_{1/2} = M_i$ $\quad$ μ

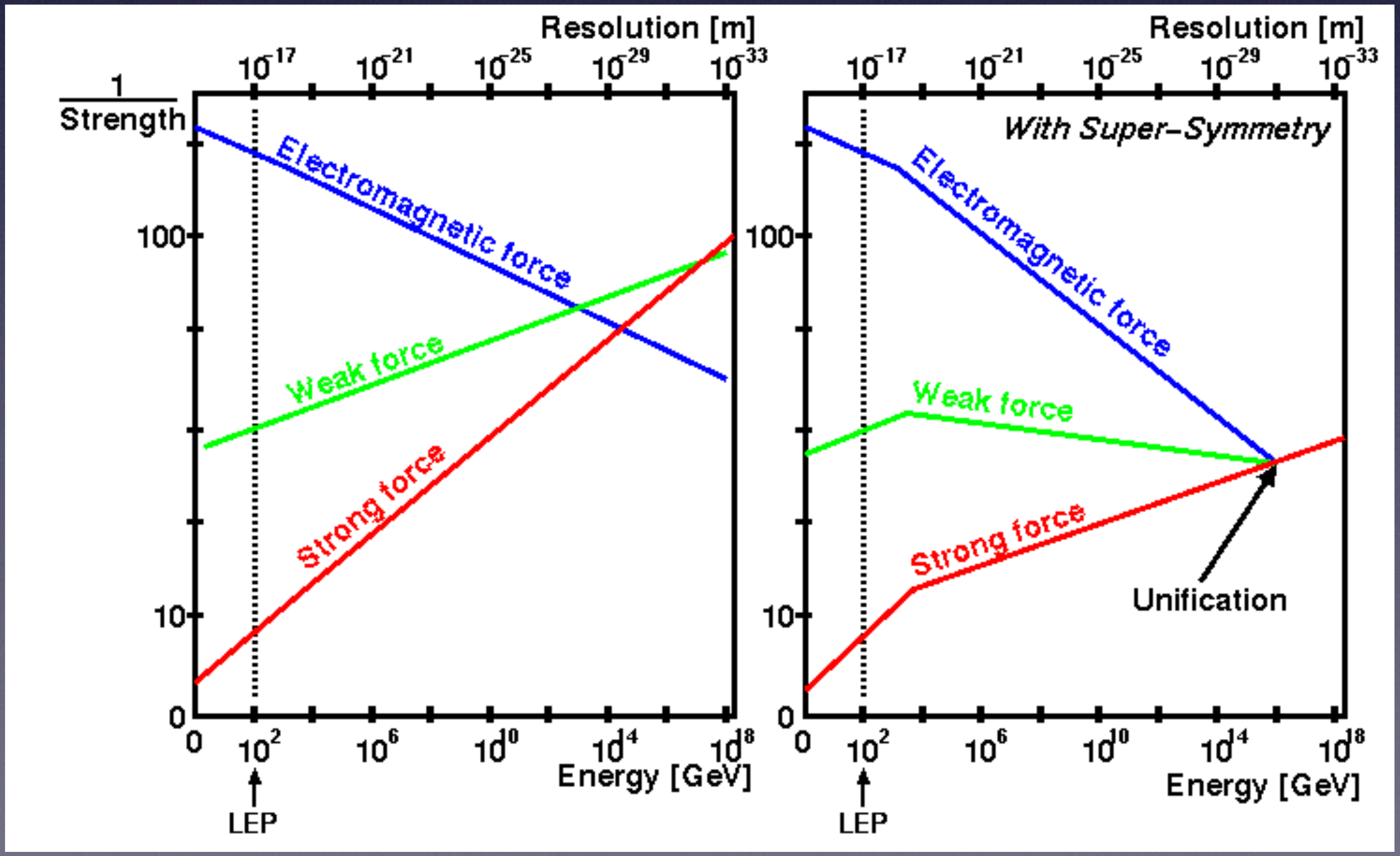
$m_0 = m^2_{\tilde{Q}_i}$ $\quad$ A_0

$\tan\beta$ $\quad$ $m_{3/2}$

• Non collider observables

- Exploit the use of Renormalization Group Equations which connect EW and GUT scales
- Most famous prediction: unification of gauge couplings

SM $MSSM$



MODEL FRAMEWORK

No-scale supergravity

$SU(5)$ $\quad$ $MSSM$ $\quad$ SM

$M_{\rm in} \leq M_{\rm P} \Longrightarrow M_{\rm GUT} \Longrightarrow M_{\rm EW}$

1702.00379, Ellis, et al.

Matching to MSSM fields

Low Energy Phenomenolog y

N=1 Supergravity: Local N=1 Supersymmetry

Ingredients:

- Superpotential
- Kähler potential
- Gauge Kinetic Functions

Superpotential

$$W_5 = \mu_\Sigma \mathrm{Tr}\Sigma^2 + \frac{1}{6}\lambda' \mathrm{Tr}\Sigma^3 + \mu_H \overline{H} H + \lambda \overline{H} \Sigma H + (Y_{\mathbf{10}})_{ij}\, \epsilon_{\alpha\beta\gamma\delta\zeta} \Psi_i^{\alpha\beta} \Psi_j^{\gamma\delta} H^\zeta + (Y_{\bar{\mathbf{5}}})_{ij}\, \Psi_i^{\alpha\beta} \Phi_{j\alpha} \overline{H}_\beta \ ,$$

Kähler potential (no-scale)

$$K = -3 \ln \left(T + \bar{T} - \frac{1}{3} \sum_i |\phi_i|^2 \right) + \sum_a \frac{|\varphi_a|^2}{(T + \bar{T})^{n_a}} ,$$

SU(5) SUSY breaking Lagrangian: (Computable)

Soft-breaking in global supersymmetry

$$\mathcal{L}_{\rm soft} = -\left(m^2_{\mathbf{10}}\right)_{ij}\widetilde{\psi}^*_i\widetilde{\psi}_j - \left(m^2_{\overline{\mathbf{5}}}\right)_{ij}\widetilde{\phi}^*_i\widetilde{\phi}_j - m^2_H|H|^2 - m^2_{\overline{H}}|\overline{H}|^2 - m^2_\Sigma{\rm Tr}\left(\Sigma^\dagger\Sigma\right)$$
$$-\left[\frac{1}{2}M_5\widetilde{\lambda}^A\widetilde{\lambda}^A + A_{\mathbf{10}}\left(h_{\mathbf{10}}\right)_{ij}\epsilon_{\alpha\beta\gamma\delta\zeta}\widetilde{\psi}_i^{\alpha\beta}\widetilde{\psi}_j^{\gamma\delta}H^\zeta + A_{\overline{\mathbf{5}}}\left(h_{\overline{\mathbf{5}}}\right)_{ij}\widetilde{\psi}_i^{\alpha\beta}\widetilde{\phi}_{j\alpha}\overline{H}_\beta\right.$$
$$\left. + B_\Sigma\mu_\Sigma{\rm Tr}\Sigma^2 + \frac{1}{6}A_{\lambda'}\lambda'{\rm Tr}\Sigma^3 + B_H\mu_H\overline{H}H + A_\lambda\lambda\overline{H}\Sigma H + {\rm h.c.}\right],$$

SU(5) soft-Lagrangian

$$\mathcal{L}_{\rm soft} = -\left(m^2_{\mathbf{10}}\right)_{ij} \widetilde{\psi}_i^* \widetilde{\psi}_j - \left(m^2_{\overline{\mathbf{5}}}\right)_{ij} \widetilde{\phi}_i^* \widetilde{\phi}_j - m^2_H |H|^2 - m^2_{\overline{H}} |\overline{H}|^2 - m^2_\Sigma \mathrm{Tr}\left(\Sigma^\dagger \Sigma\right)$$
$$- \left[\frac{1}{2} M_5 \widetilde{\lambda}^A \widetilde{\lambda}^A + A_{\mathbf{10}} \left(h_{\mathbf{10}}\right)_{ij} \epsilon_{\alpha\beta\gamma\delta\zeta} \widetilde{\psi}_i^{\alpha\beta} \widetilde{\psi}_j^{\gamma\delta} H^\zeta + A_{\overline{\mathbf{5}}} \left(h_{\overline{\mathbf{5}}}\right)_{ij} \widetilde{\psi}_i^{\alpha\beta} \widetilde{\phi}_{j\alpha} \overline{H}_\beta \right.$$
$$\left. + B_\Sigma \mu_\Sigma \mathrm{Tr}\Sigma^2 + \frac{1}{6} A_{\lambda'} \lambda' \mathrm{Tr}\Sigma^3 + B_H \mu_H \overline{H} H + A_\lambda \lambda \overline{H} \Sigma H + \mathrm{h.c.} \right] ,$$

No-scale Kähler potential

$$K \;=\; -3 \ln \left(T + \bar{T} - \frac{1}{3} \sum_i |\phi_i|^2 \right) + \sum_a \frac{|\varphi_a|^2}{(T + \bar{T})^{n_a}} ,$$

Twisted sector: non-zero modular weights

Superpotential

$$W = (T+c)^{\beta} W_2(\phi_i) + (T+c)^{\alpha} W_3(\phi_i) + (T+c)^{\sigma} W_2(\varphi_a) + (T+c)^{\rho} W_3(\varphi_a) + \mu_{\Lambda} ,$$

- The effective potential for T is flat at tree level, (undetermined vev) and the gravitino mass
- For $\alpha = \beta = 0 \Rightarrow m_0 = A_0 = B_0 = 0$

Pure no scale

- For $\alpha = \beta \neq 0 \Rightarrow$

$$\phi_i : \quad m_0 = 0 , \quad B_0 = -\beta m_{3/2} , A_0 = -\alpha m_{3/2} ,$$

$$\varphi_a : \quad m_0 = m_{3/2} , \quad B_0 = 2 m_{3/2} \left(1 - \frac{\sigma}{2}\right) , A_0 = 3 m_{3/2} \left(1 - \frac{\rho}{3}\right) ,$$

- At the end of the day: Family of models with nice phenomenological possibilities depending on the values of α_F, β_S

$$(m^2_{\mathbf{10}})_{ij} = (m^2_{\overline{\mathbf{5}}})_{ij} = 0$$

$$m_H = p\ m_{3/2}, \quad m_{\bar{H}} = q\ m_{3/2},\ p, q = 0, 1$$

$$A_F = (r_F - \alpha_F) m_{3/2}, \quad F = \mathbf{10}, \overline{\mathbf{5}}, \lambda', \lambda,$$

$$B_S = (p_S - \beta_S) m_{3/2}, \quad S = H, \Sigma,$$

$$(a_{\mathbf{10}})_{ij} = (A_{\mathbf{10}})_{ij} (h_{\mathbf{10}})_{ij},$$

$$(a_{\overline{\mathbf{5}}})_{ij} = (A_{\overline{\mathbf{5}}})_{ij} (h_{\overline{\mathbf{5}}})_{ij}$$

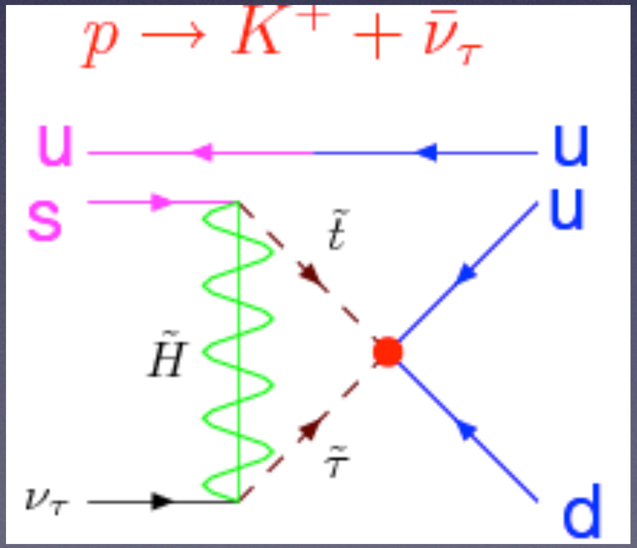
PD BASICS IN SU(5)

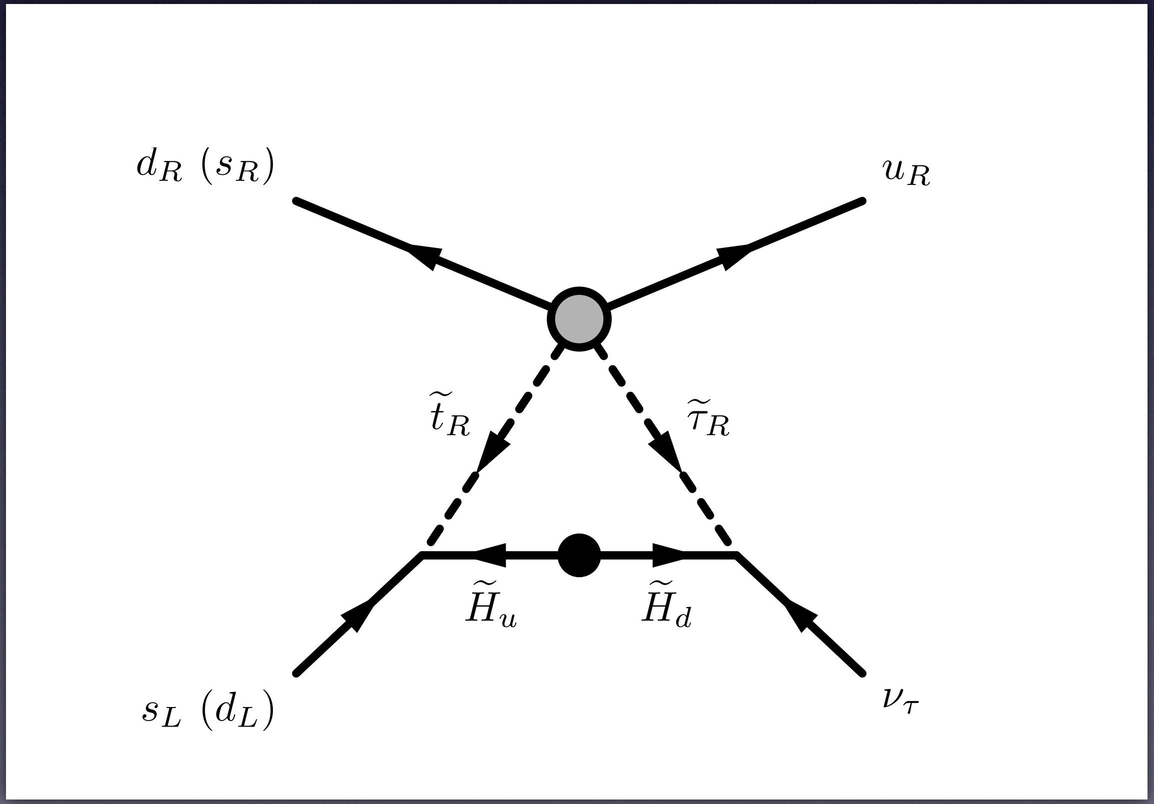
- Most constraining observable

$$\tau(p \to K^+ \nu) > 6.6 \times 10^{33} \text{ yrs}$$

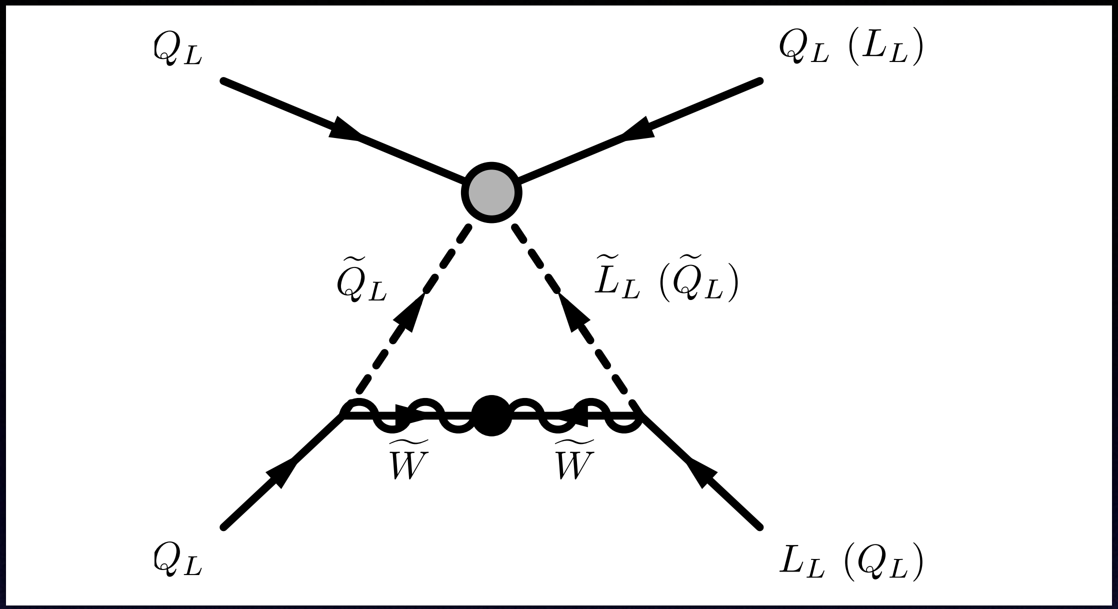
$p(u\ u\ d)$

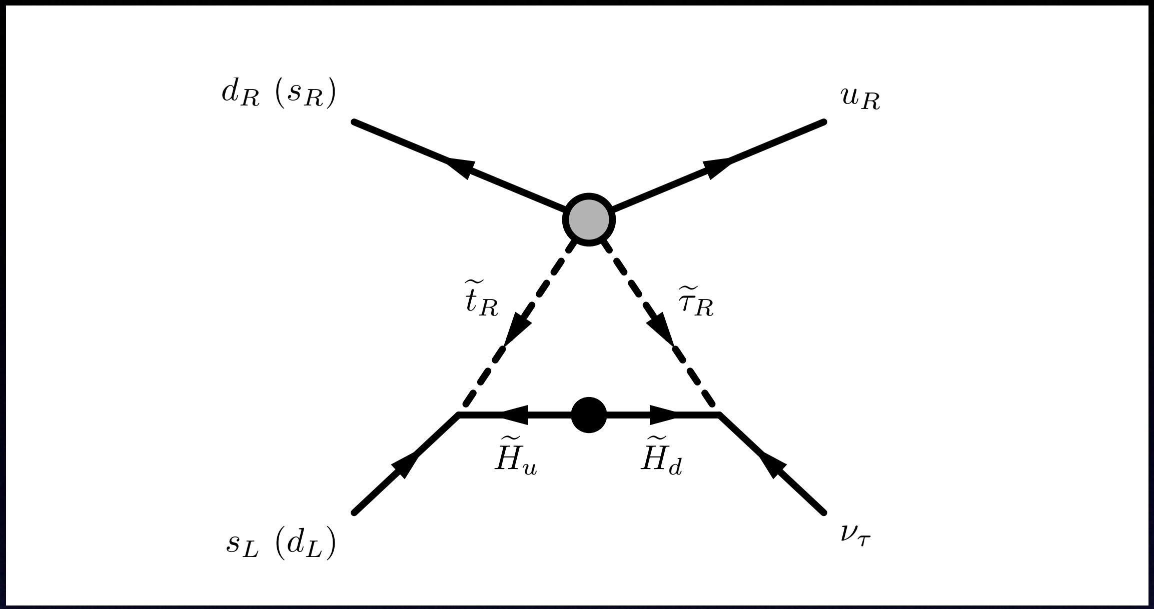
$K(u\bar{s})$





Writing an effective Lagrangian





$$\mathcal{O}^{5L}_{ijkl} \equiv \int d^2\theta \; \frac{1}{2}\epsilon_{abc}(Q^a_i \cdot Q^b_j)(Q^c_k \cdot L_l) \; ,$$

$$\mathcal{O}^{5R}_{ijkl} \equiv \int d^2\theta \; \epsilon^{abc}\overline{U}_{ia}\overline{E}_j\overline{U}_{kb}\overline{D}_{lc} \; ,$$

$$\mathcal{L}^{\rm eff}_5 = C^{ijkl}_{5L}\mathcal{O}^{5L}_{ijkl} + C^{ijkl}_{5R}\mathcal{O}^{5R}_{ijkl} \quad + \quad \text{h.c.} \; ,$$

$$C^{ijkl}_{5L}(M_{\rm GUT}) = \frac{1}{M_{H_C}} Y^{Q_i Q_j} Y^{Q_k L_l} ,$$

$$C^{ijkl}_{5R}(M_{\rm GUT}) = \frac{1}{M_{H_C}} Y^{U_i E_j} Y^{U_k D_l} ,$$

Write coefficients in terms of SU(5) parameters

$$W_{\mathrm{Y}}^{5} = (\mathbf{Y_{10}})_{ij} 10_i 10_j H + (\mathbf{Y_{\overline{5}}})_{ij} 10_i 5_j \bar{H}.$$

$$10_i = \left\{ Q_i, \mathrm{e}^{-i\phi_i} U_i^c, (V_{\mathrm{CKM}} E^c)_i \right\}, \quad 5_i = \left\{ D_i^c, L_i \right\}.$$

$$W_{\mathrm{Y}}^{5} \supset Q_i (Y_{10})_{ij} Q_j H + Q_i (Y_5)_{ij} L_j \overline{H} + Q_i (h_{10})_{ij} E_j^c H + E_i^c (Y_5)_{ij} L_j \overline{H}$$
$$+ U_i^c (Y_5)_{ij} D_j^c \overline{H} + Q_i (Y_{10})_{ij} U_i^c + U_i^c (Y_{10})_{ij} E_j^c.$$

$$C_{5L}^{ijkl}(M_{\mathrm{GUT}}) = \frac{1}{M_{H_C}} Y^{Q_i Q_j} Y^{Q_k L_l},$$

$$C_{5R}^{ijkl}(M_{\mathrm{GUT}}) = \frac{1}{M_{H_C}} Y^{U_i E_j} Y^{U_k D_l},$$

Run coefficients to the EW scale

$$C_{5L}^{ijkl}(M_{\rm GUT}) \quad \Rightarrow \quad C_{5L}^{ijkl}(M_{\rm EW})$$
$$C_{5R}^{ijkl}(M_{\rm GUT}) \quad \Rightarrow \quad C_{5R}^{ijkl}(M_{\rm EW})$$

Write SM effective Lagrangian

$$\mathcal{L}_{\rm SM}^{\rm eff}(M_{\rm EW}) = C_i^{\widetilde{H}}\mathcal{O}_{1i33} + C_{jk}^{\widetilde{W}}\widetilde{\mathcal{O}}_{1jjk} + C_{jk}^{\widetilde{W}}\widetilde{\mathcal{O}}_{j1jk} + \overline{C}_{jk}^{\widetilde{W}}\widetilde{\mathcal{O}}_{jj1k} \ ,$$

$$\mathcal{O}_{ijkl} \equiv \epsilon_{abc}(u_{Ri}^a d_{Rj}^b)(Q_{Lk}^c \cdot L_{Ll}) \ ,$$
$$\widetilde{\mathcal{O}}_{ijkl} \equiv \epsilon_{abc}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}(Q_{Li\alpha}^a Q_{Lj\gamma}^b)(Q_{Lk\delta}^c L_{Ll\beta}) \ ,$$

$$\mathcal{L}(p \to K^+\bar{\nu}_i) =$$
$$C_{RL}(usd\nu_i)\left[\epsilon_{abc}(u_R^a s_R^b)(d_L^c \nu_i)\right] + C_{RL}(uds\nu_i)\left[\epsilon_{abc}(u_R^a d_R^b)(s_L^c \nu_i)\right]$$
$$+C_{LL}(usd\nu_i)\left[\epsilon_{abc}(u_L^a s_L^b)(d_L^c \nu_i)\right] + C_{LL}(uds\nu_i)\left[\epsilon_{abc}(u_L^a d_L^b)(s_L^c \nu_i)\right] \ ,$$

Compute decay amplitude, decay width and lifetime

$$C_{RL}(usd\nu_\tau) = -V_{td} C_2^{\widetilde{H}}(M_Z) \ ,$$

$$C_{RL}(uds\nu_\tau) = -V_{ts} C_1^{\widetilde{H}}(M_Z) \ ,$$

$$C_{LL}(usd\nu_k) = \sum_{j=2,3} V_{j1} V_{j2} C_{jk}^{\widetilde{W}}(M_Z) \ ,$$

$$C_{LL}(uds\nu_k) = \sum_{j=2,3} V_{j1} V_{j2} C_{jk}^{\widetilde{W}}(M_Z) \ .$$

$$\mathcal{A}(p \to K^+ \bar{\nu}_i) =$$

$$C_{RL}(usd\nu_i)\langle K^+|(us)_R d_L|p\rangle + C_{RL}(uds\nu_i)\langle K^+|(ud)_R s_L|p\rangle$$

$$+ C_{LL}(usd\nu_i)\langle K^+|(us)_L d_L|p\rangle + C_{LL}(uds\nu_i)\langle K^+|(ud)_L s_L|p\rangle \ .$$

$$\Gamma(p \to K^+ \bar{\nu}_i) = \frac{m_p}{32\pi}\left(1 - \frac{m_K^2}{m_p^2}\right)^2 |\mathcal{A}(p \to K^+ \bar{\nu}_i)|^2 \ , \ .$$

$$\tau_P = \frac{1}{\Gamma(p \to K^+ \bar{\nu}_i)}$$

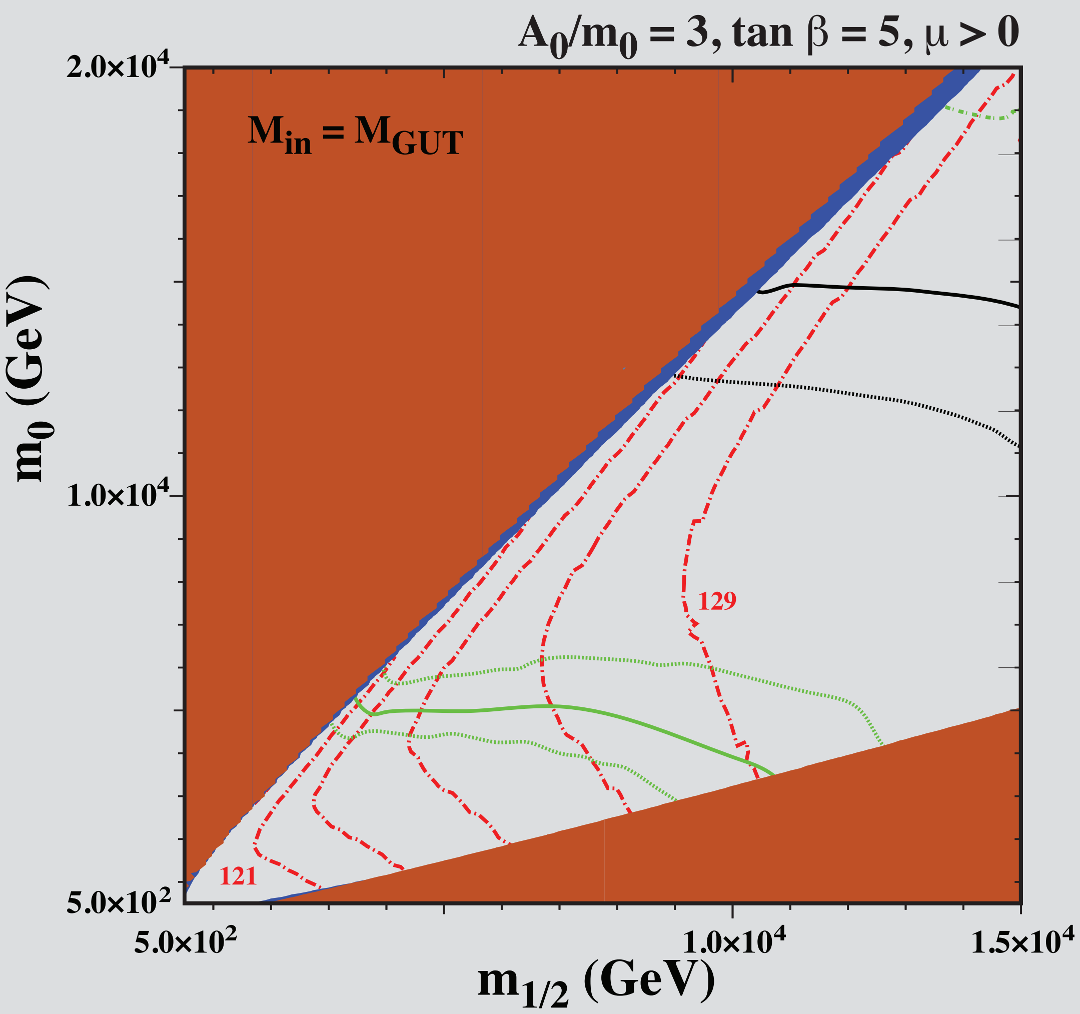
Current and future lifetime values (10^{33} yr)

Decay Mode	Current (90% CL)	Future (Discovery)	Future (90% CL)
$p \to K^+\bar{\nu}$	6.6 [6]	JUNO: 12 (20) [3] DUNE: 30 (50) [3] Hyper-K: 20 (30) [3]	JUNO: 19 (40) [1] DUNE: 33 (65) [2] Hyper-K: 32 (50) [3]
$p \to \pi^+\bar{\nu}$	0.39 [29]		
$p \to e^+\pi^0$	16 [40]	DUNE: 15 (25) [3] Hyper-K: 63 (100) [3]	DUNE: 20 (40) [3] Hyper-K: 78 (130) [3]
$p \to \mu^+\pi^0$	7.7 [40]	Hyper-K: 69 [3]	Hyper-K: 77 [3]
$n \to K_S^0\bar{\nu}$	0.26 [25]		
$n \to \pi^0\bar{\nu}$	1.1 [29]		
$n \to e^+\pi^-$	5.3 [48]	Hyper-K: 13 [3]	Hyper-K: 20 [3]
$n \to \mu^+\pi^-$	3.5 [48]	Hyper-K: 11 [3]	Hyper-K: 18 [3]

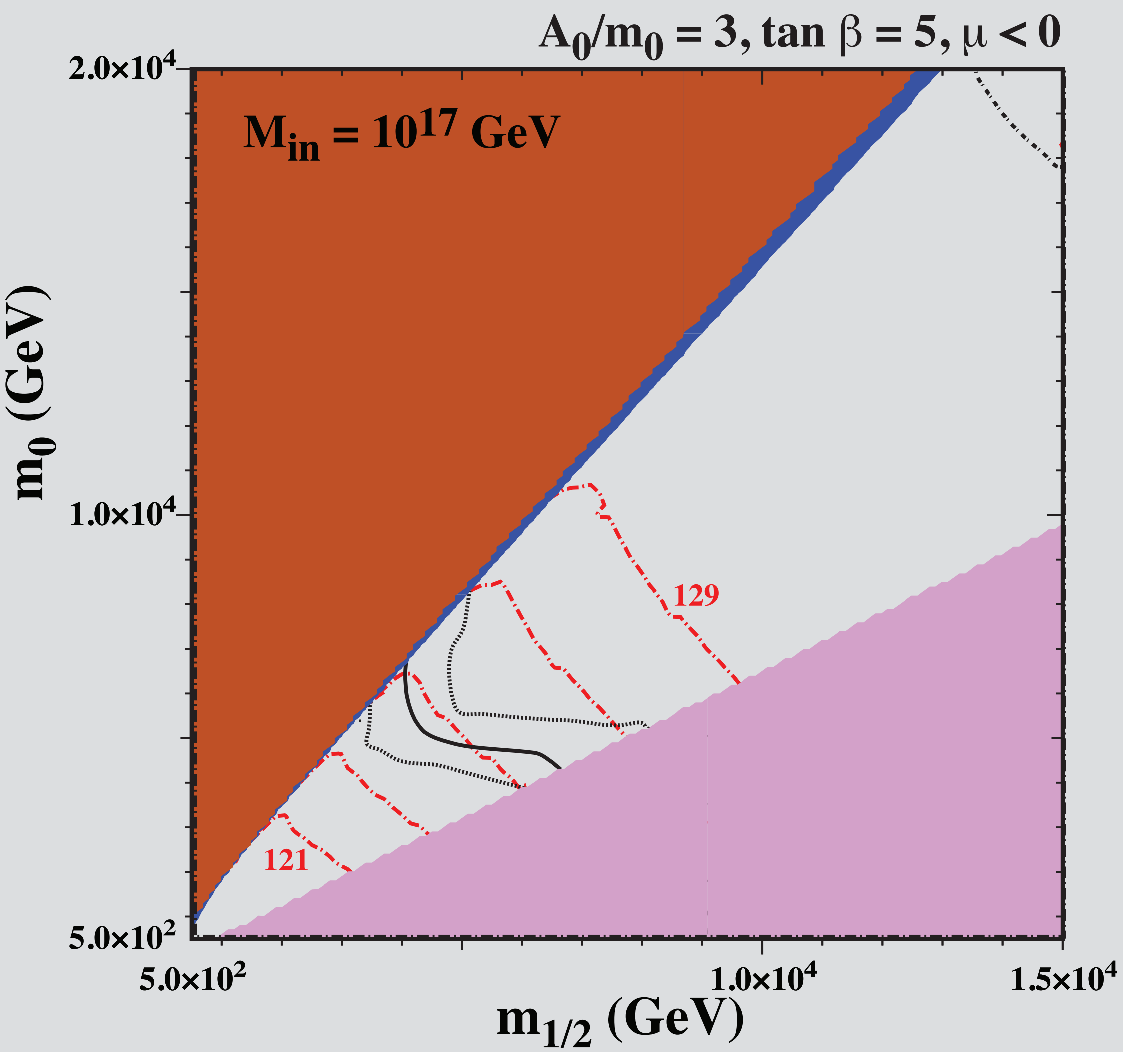
Deep Underground Neutrino Experiment (DUNE), FERMILAB, Operational: 2027

RESULTS

CMSSM

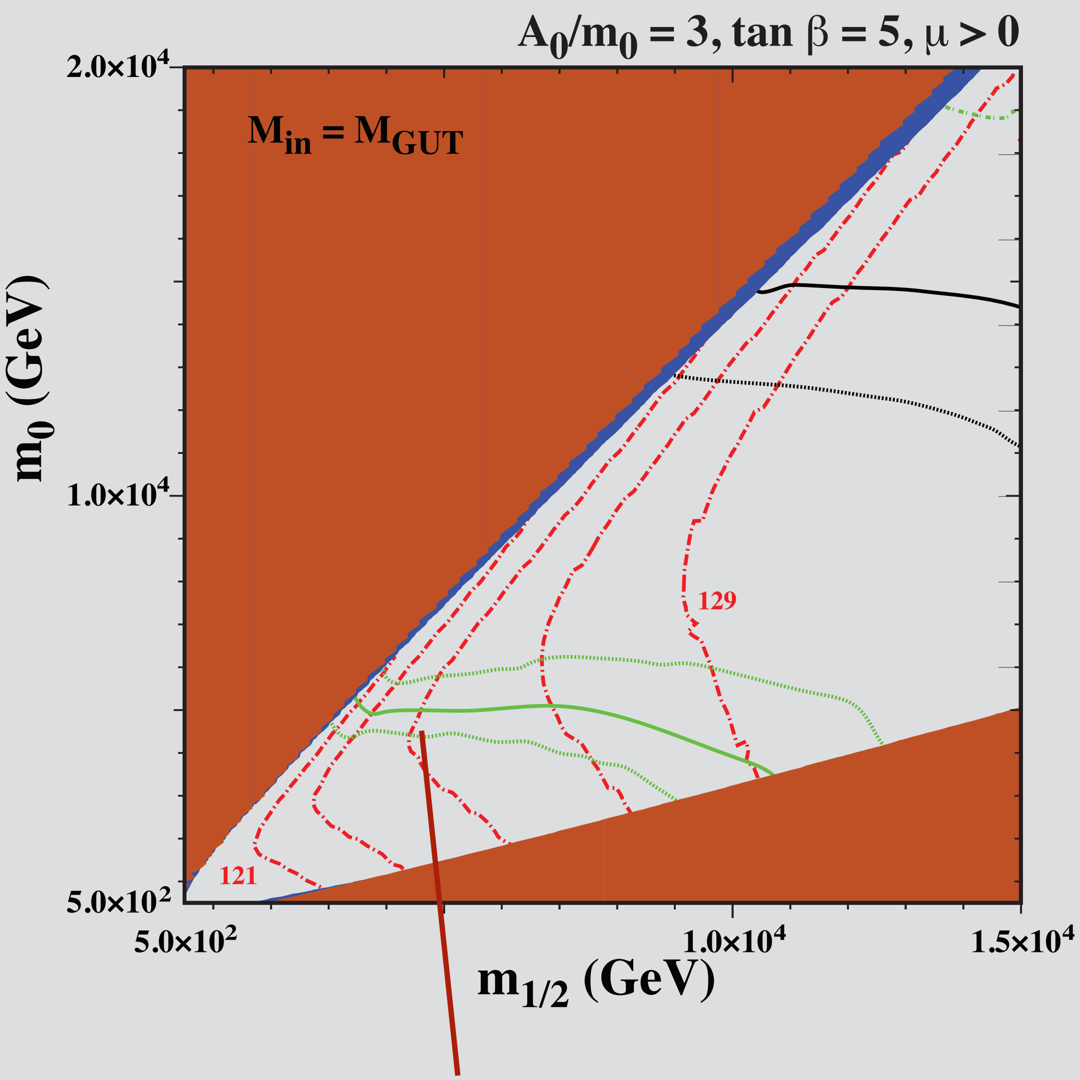


super-GUT



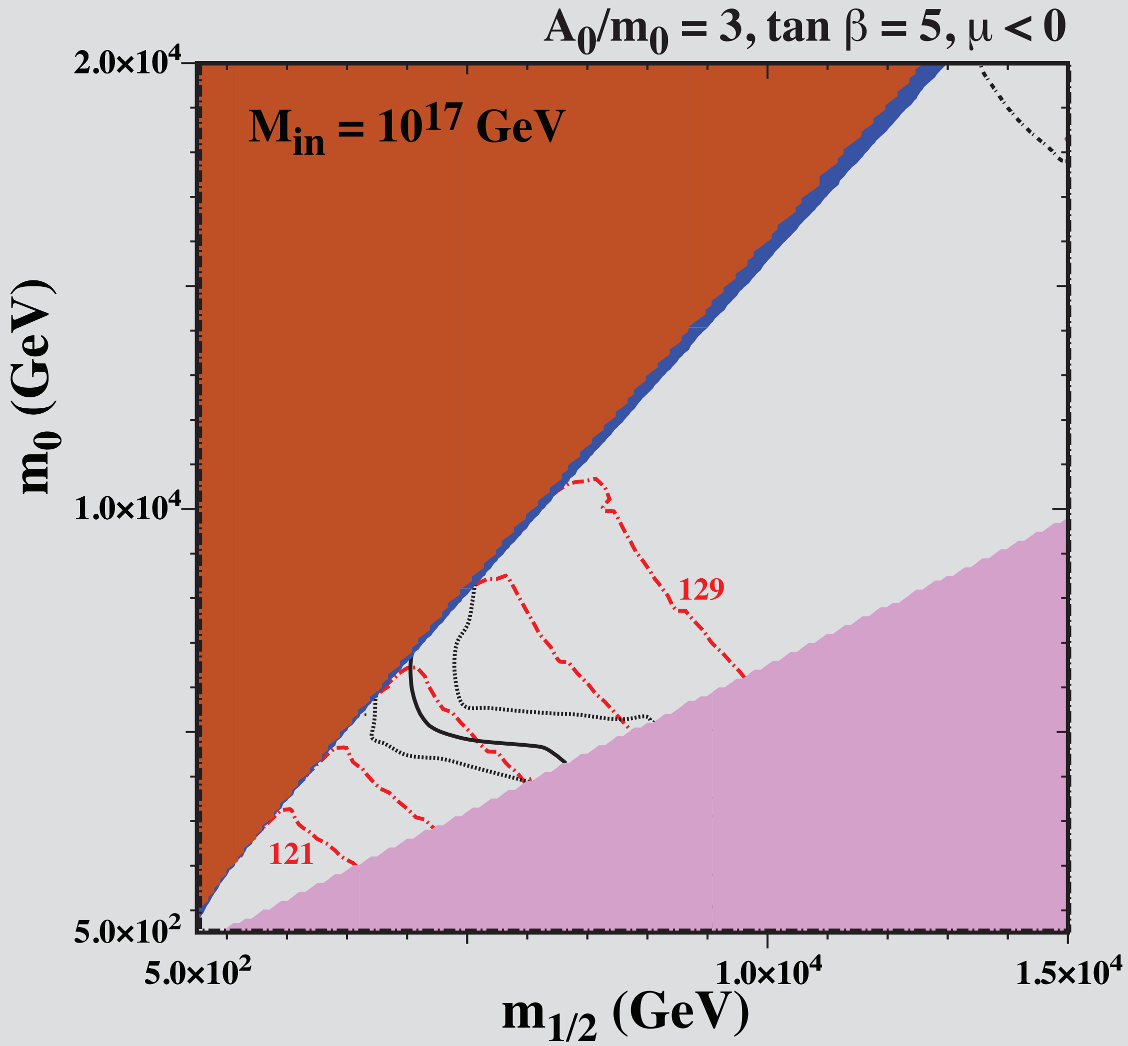
RESULTS

CMSSM



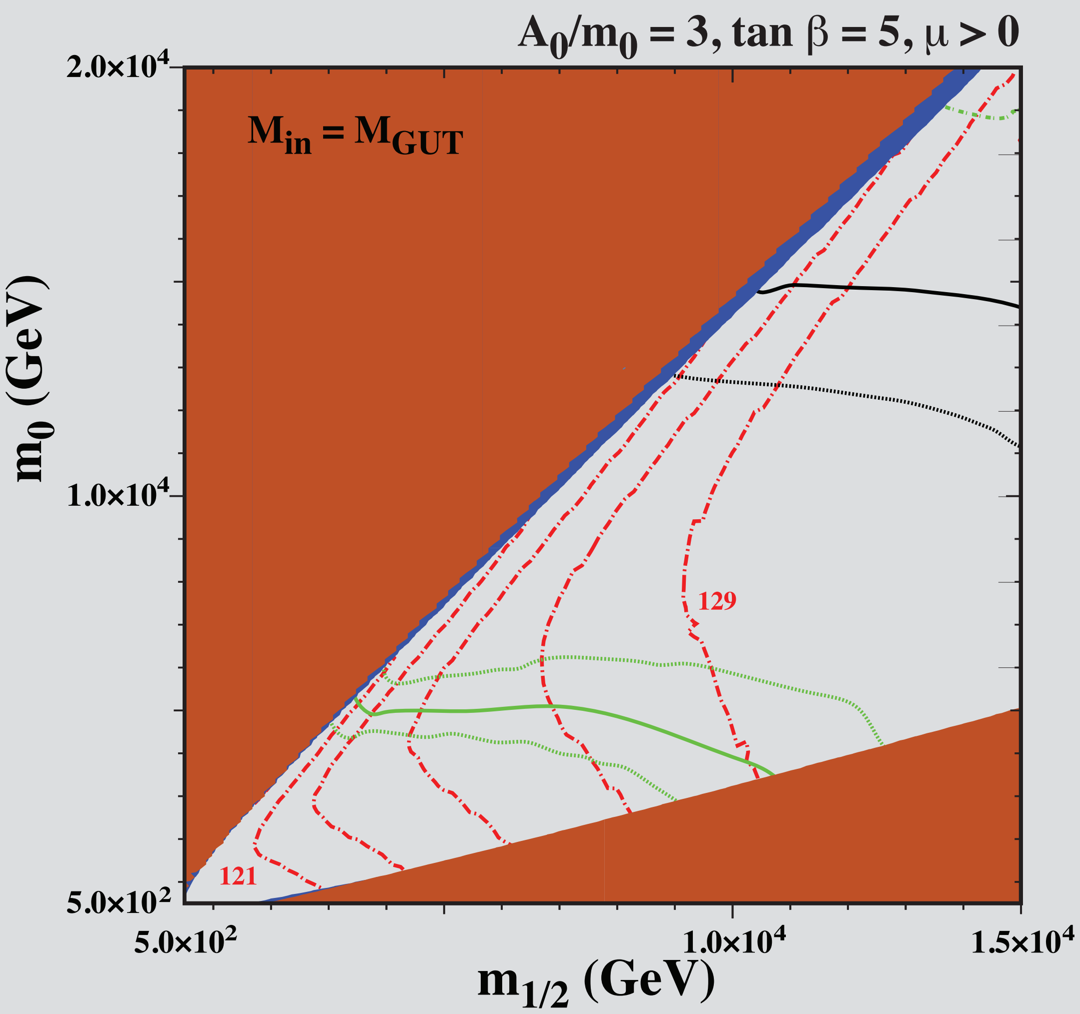
$m_h = 125$ GeV

super-GUT

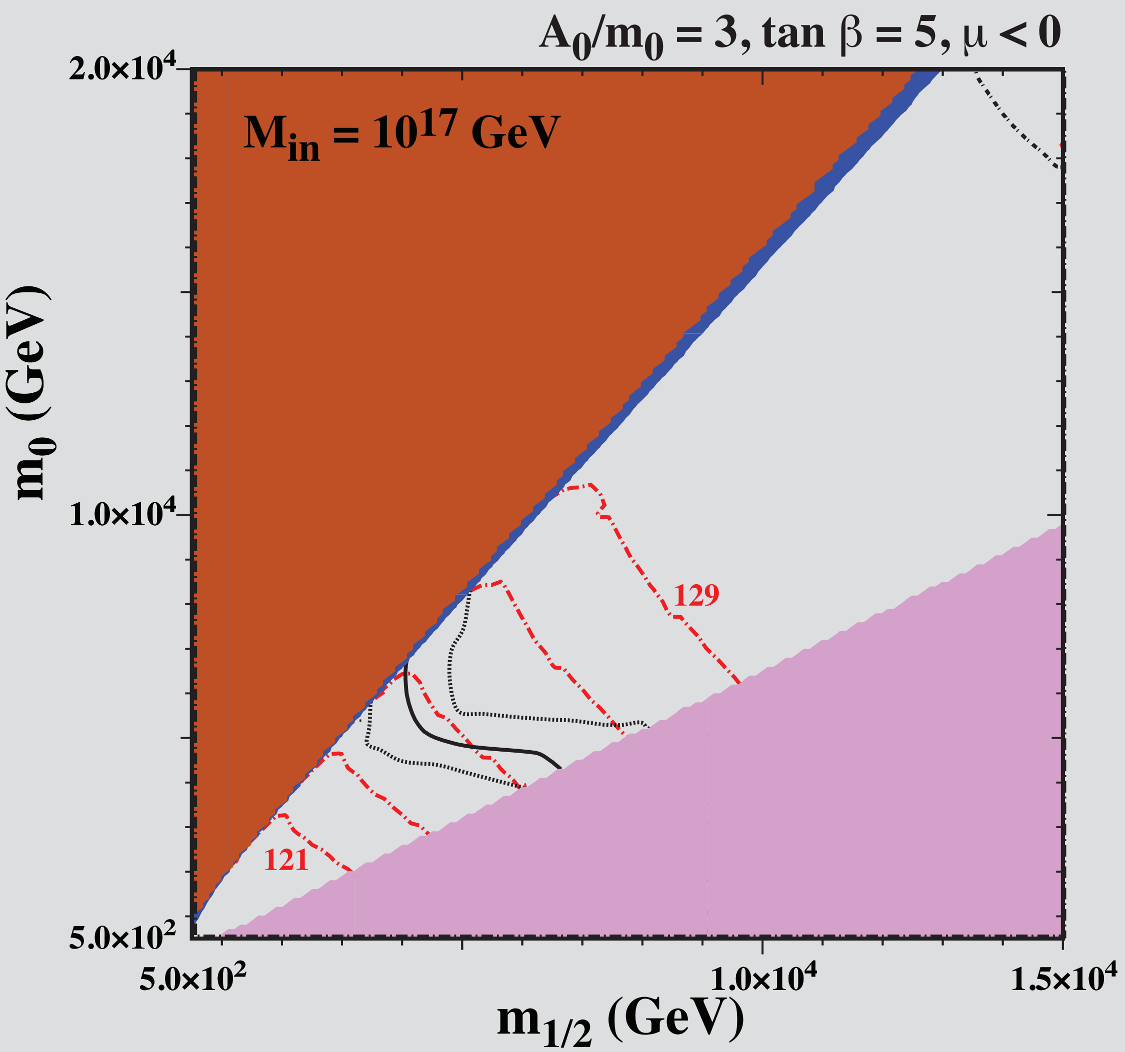


RESULTS

CMSSM

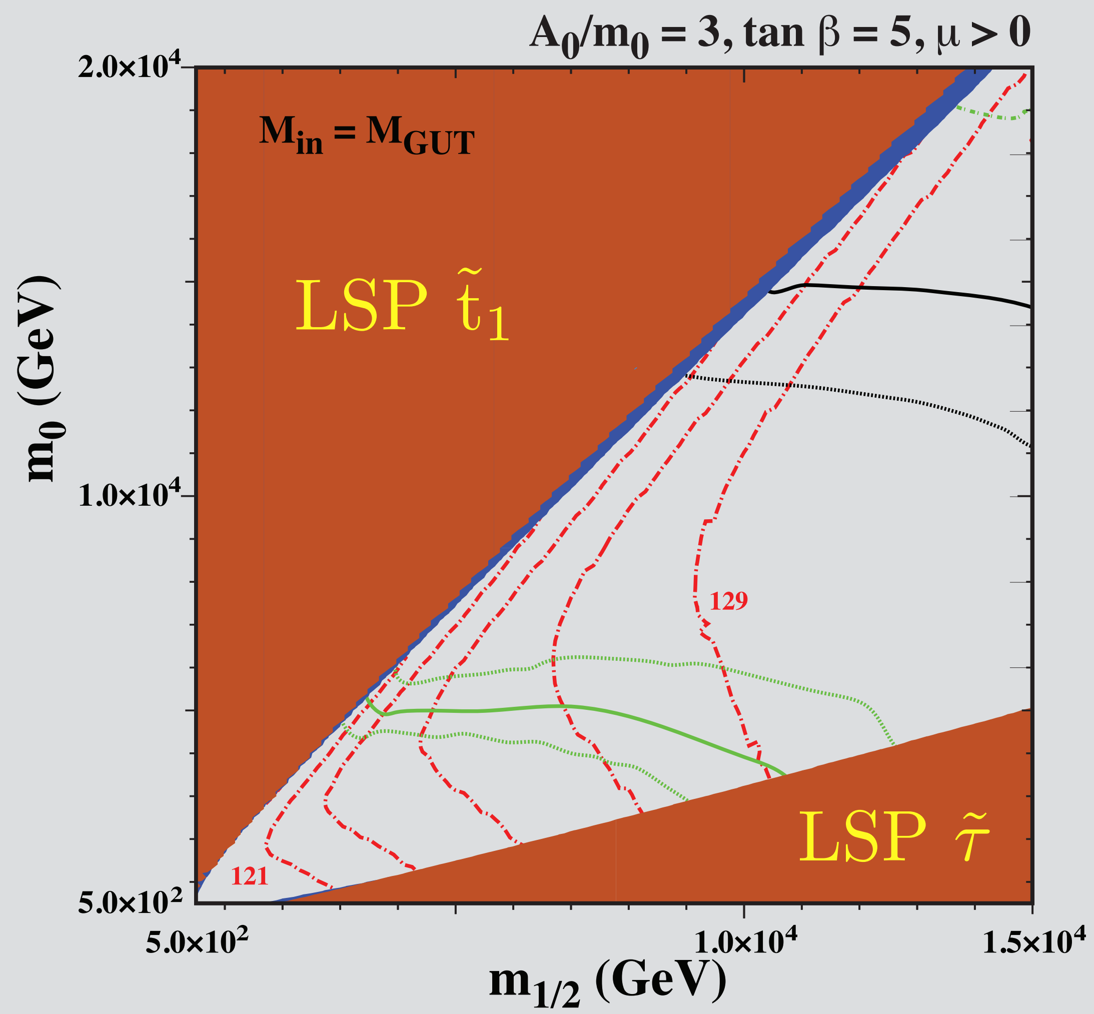


super-GUT

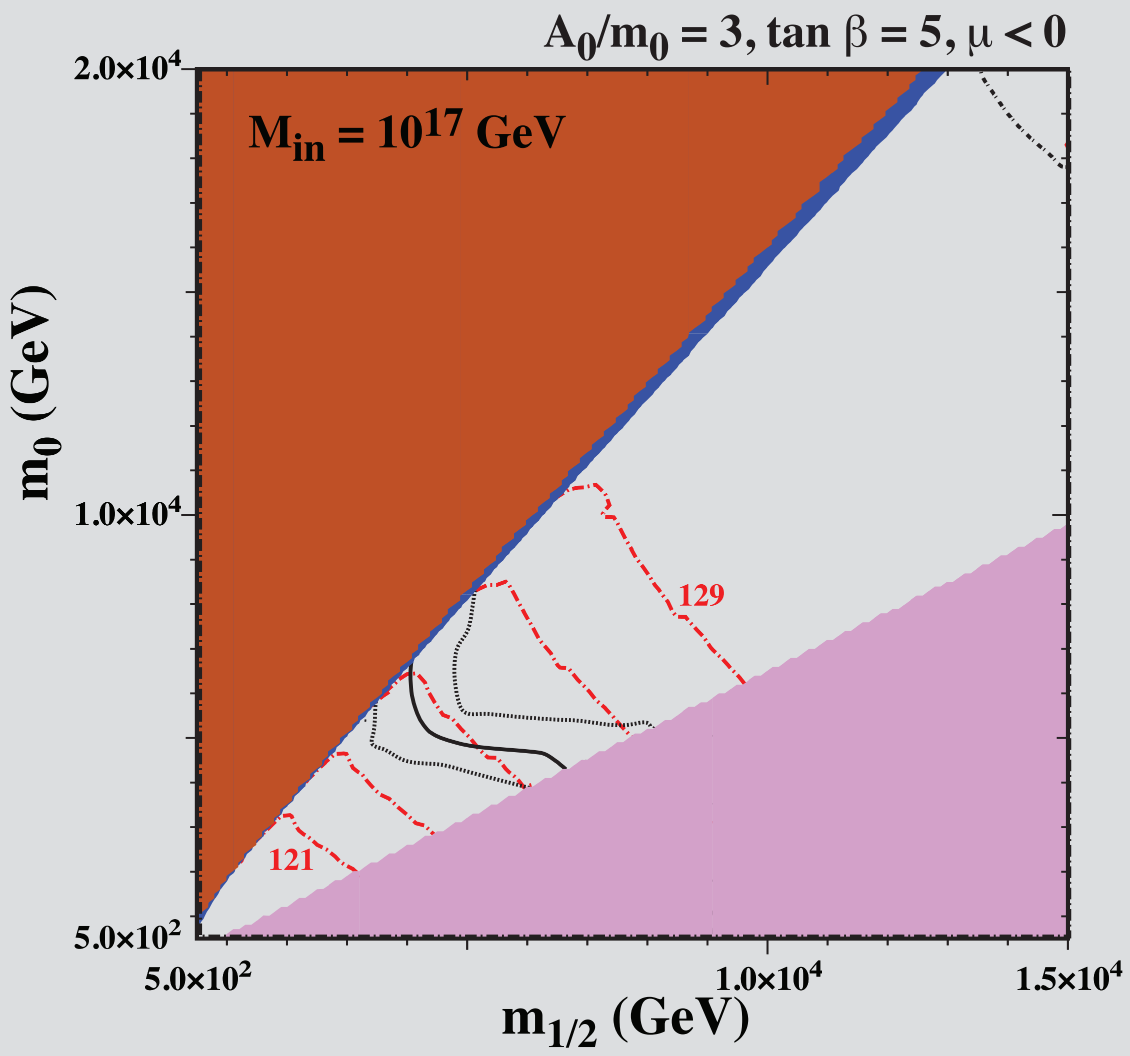


RESULTS

CMSSM

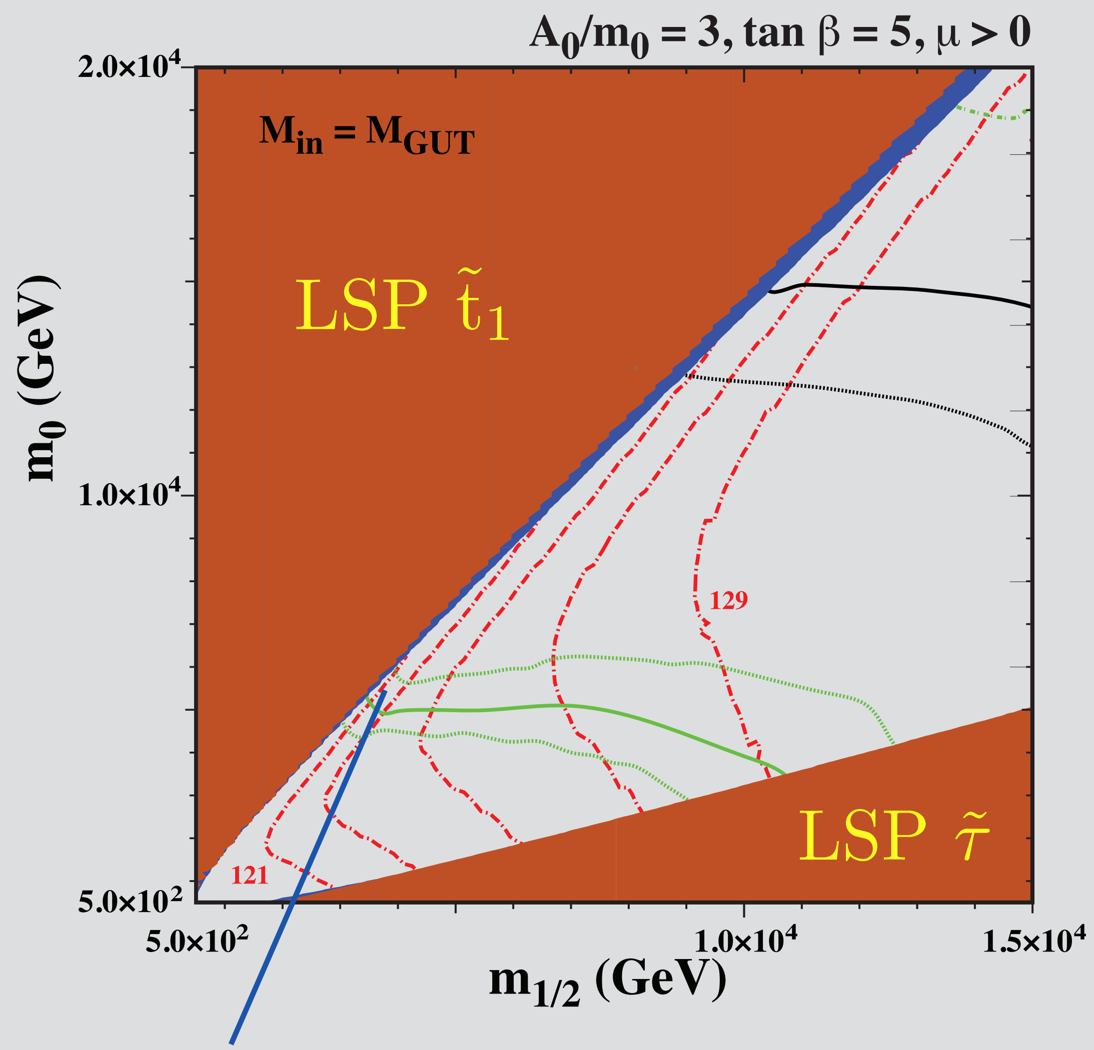


super-GUT



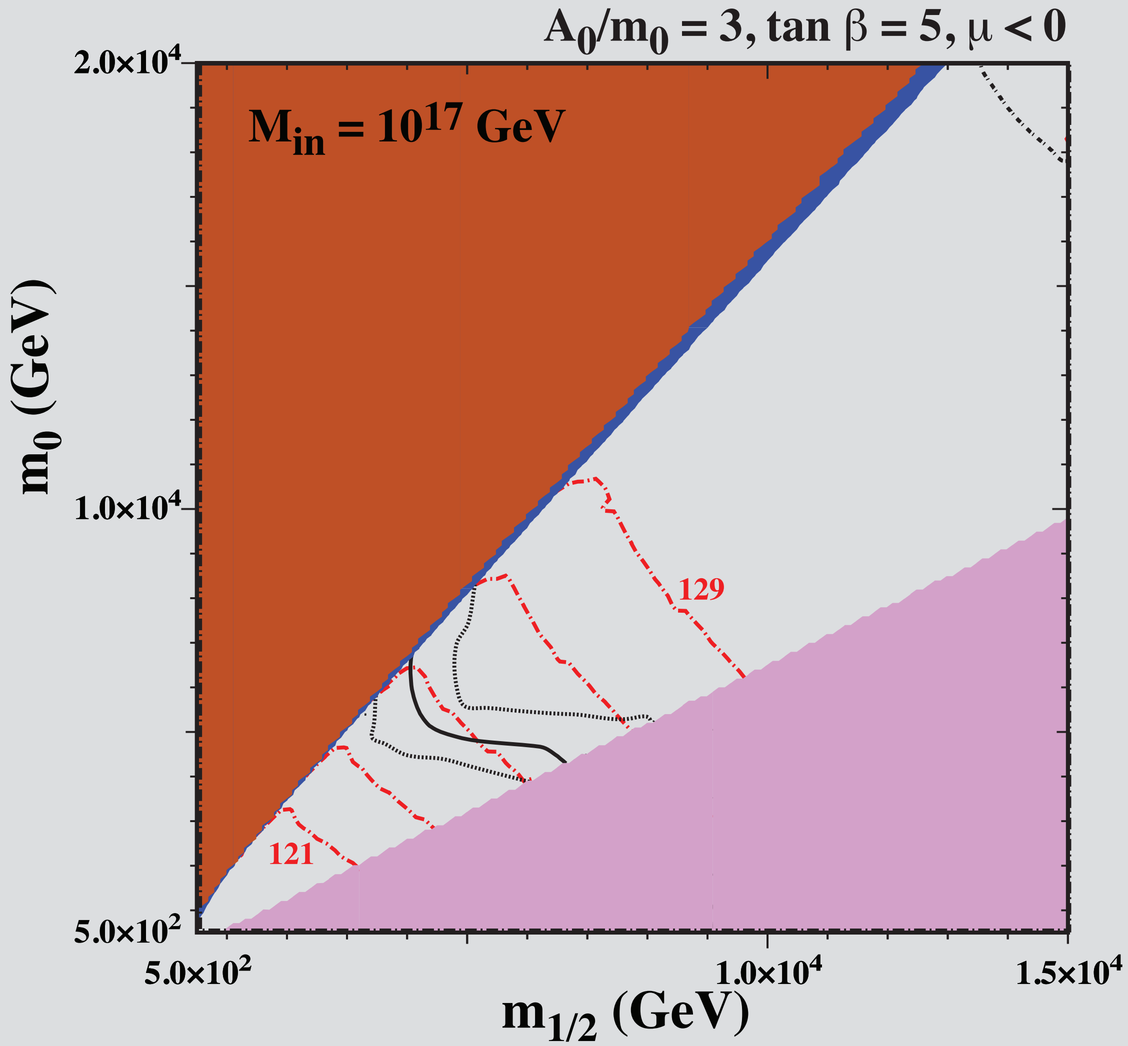
RESULTS

CMSSM



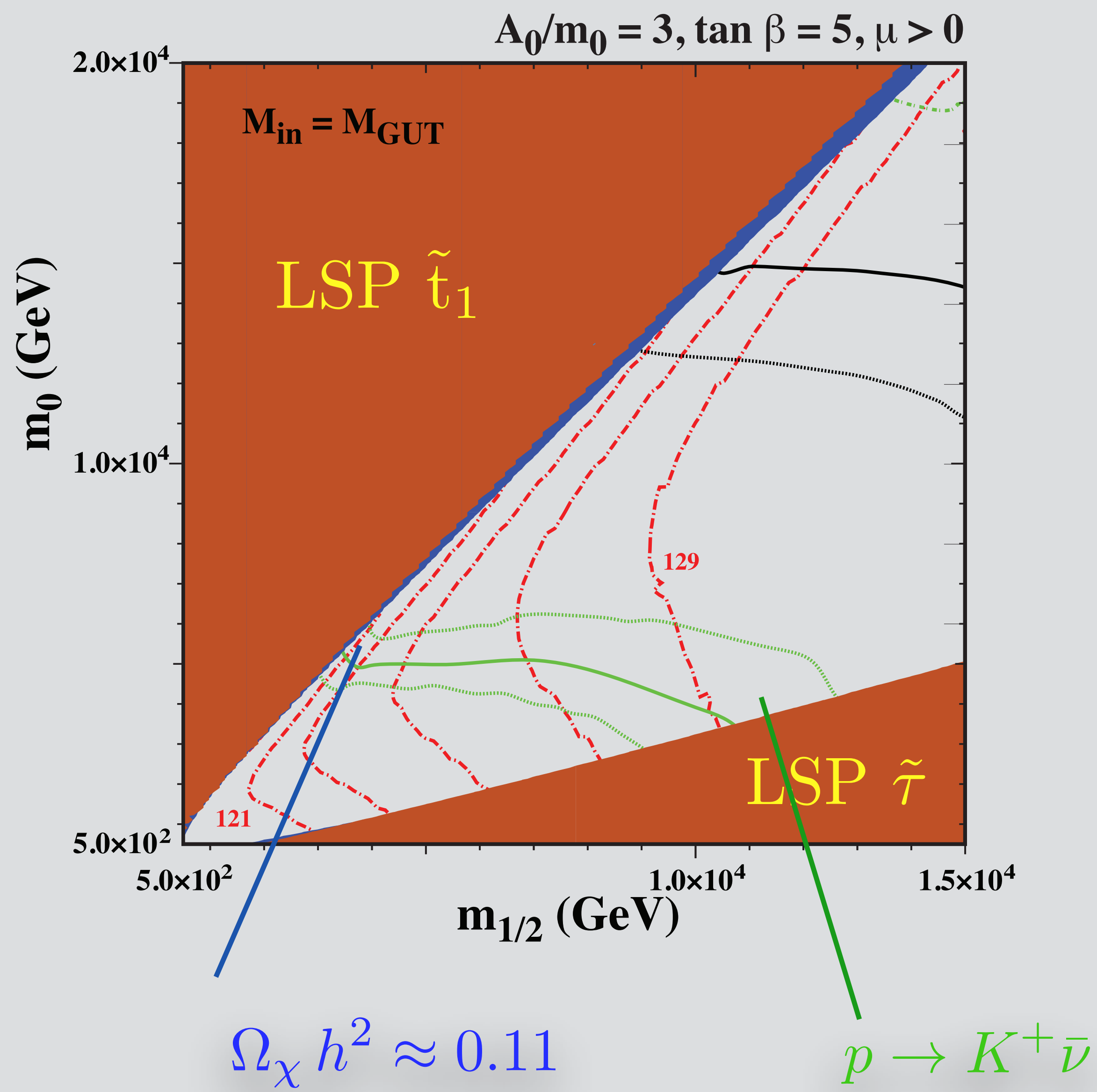
$\Omega_\chi h^2 \approx 0.11$

super-GUT

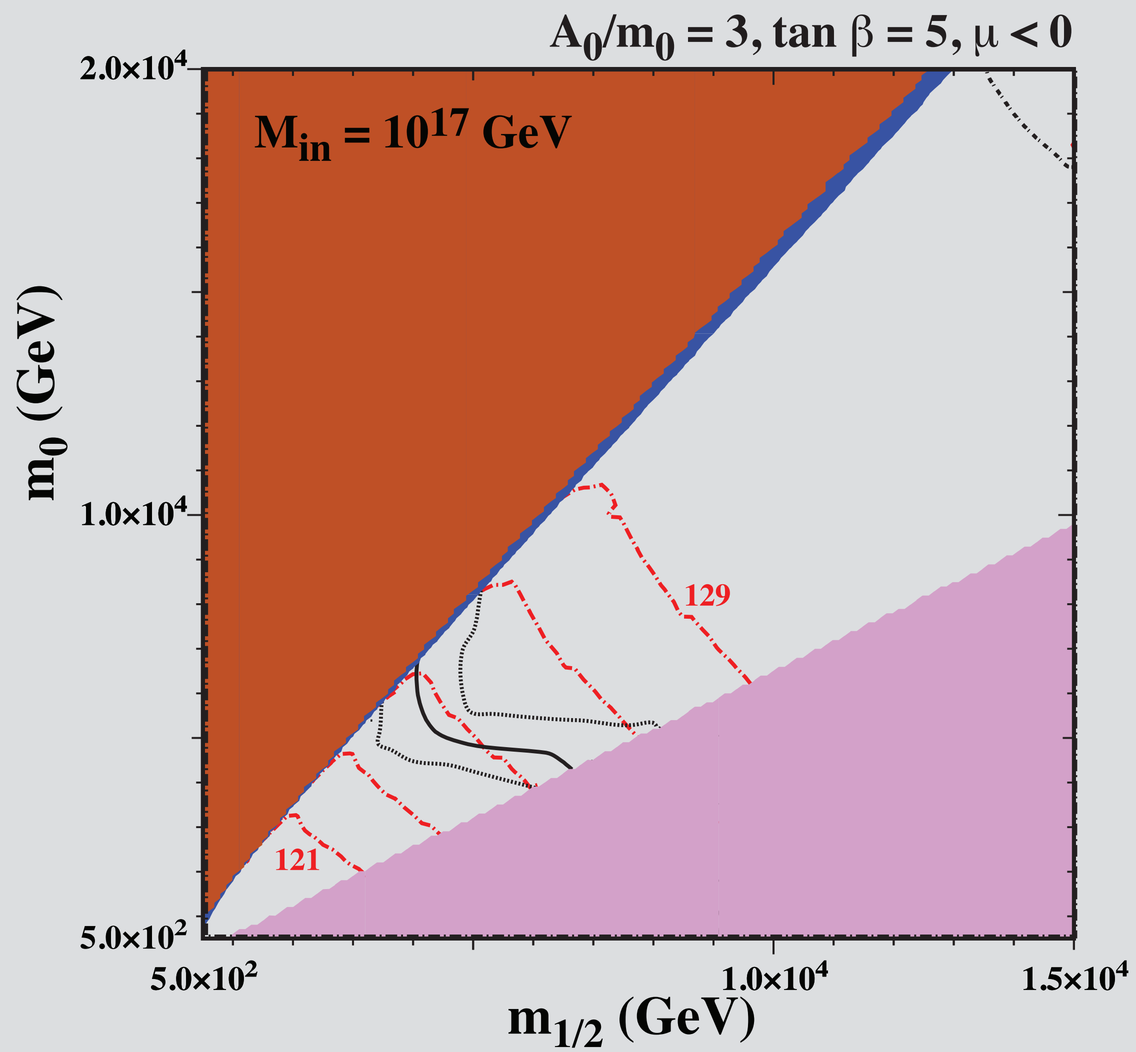


RESULTS

CMSSM



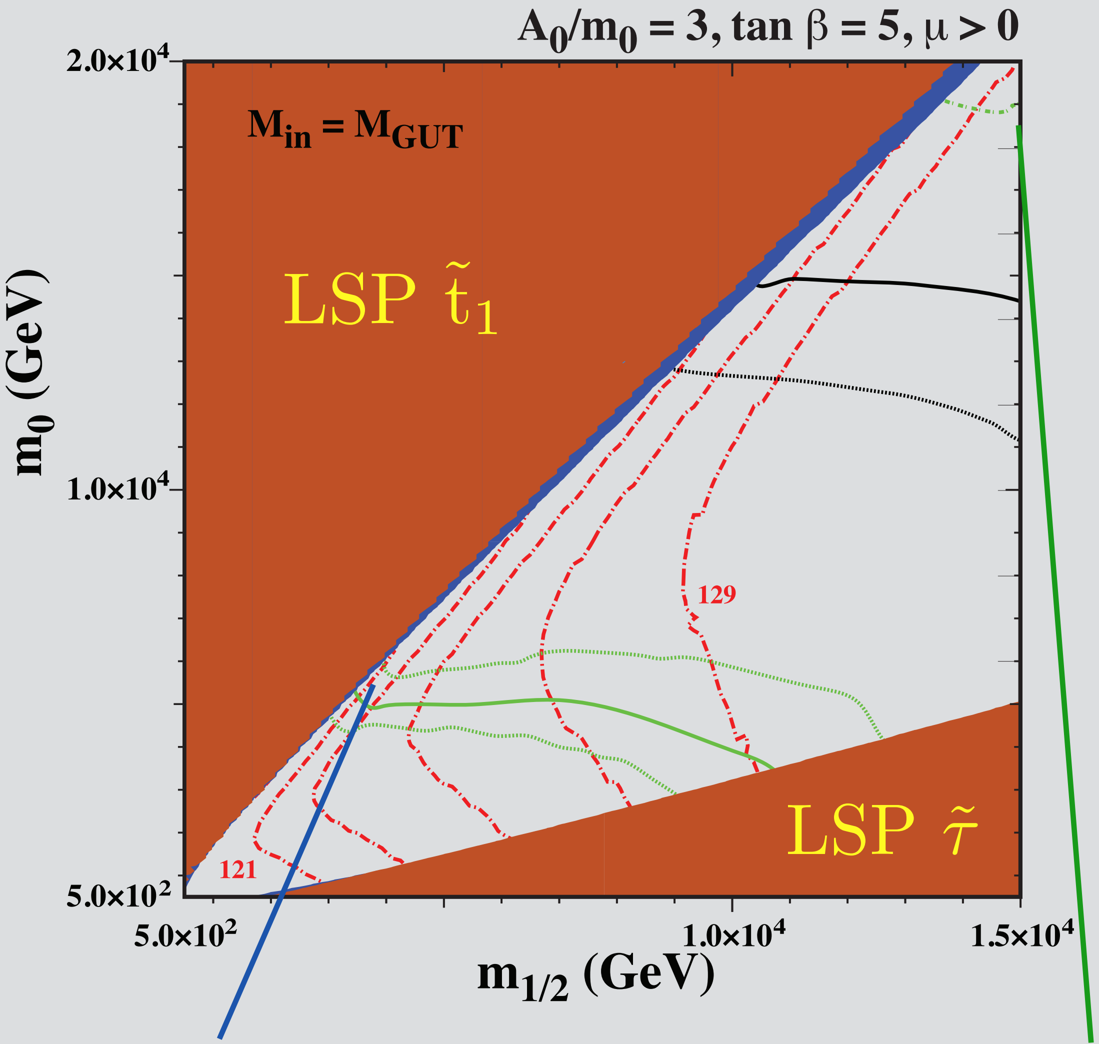
super-GUT



PRESENT LIMIT

RESULTS

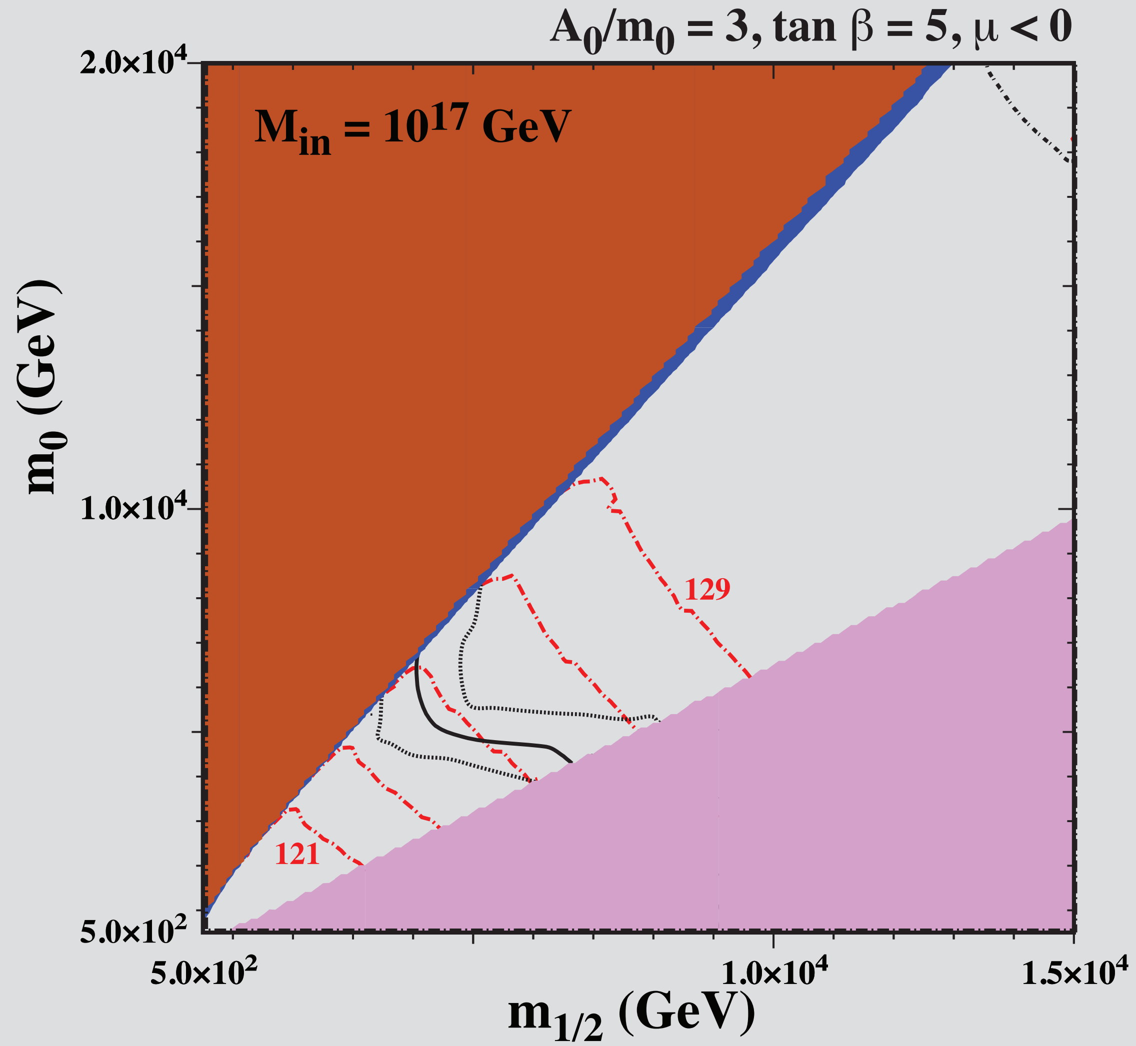
CMSSM



$\Omega_\chi h^2 \approx 0.11$

$p \to K^+ \bar{\nu}$

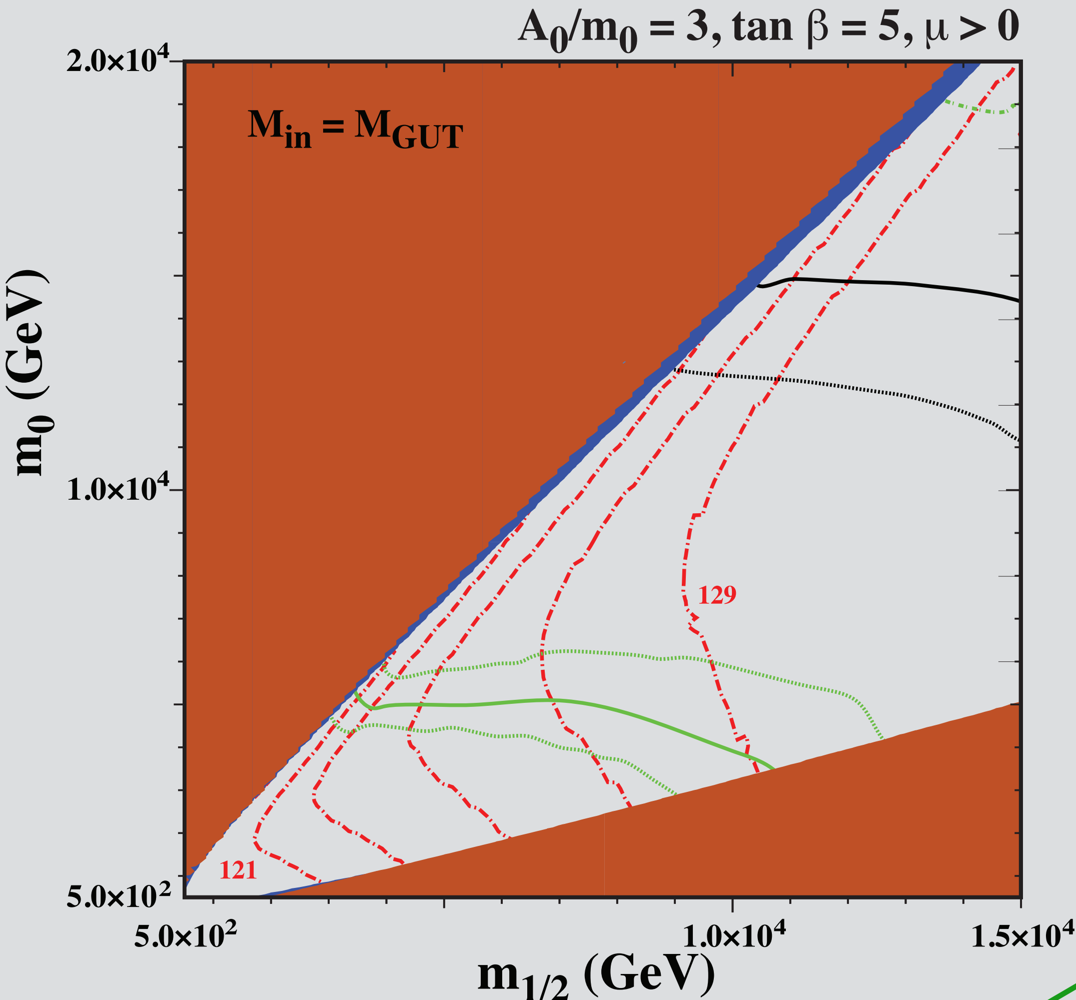
super-GUT



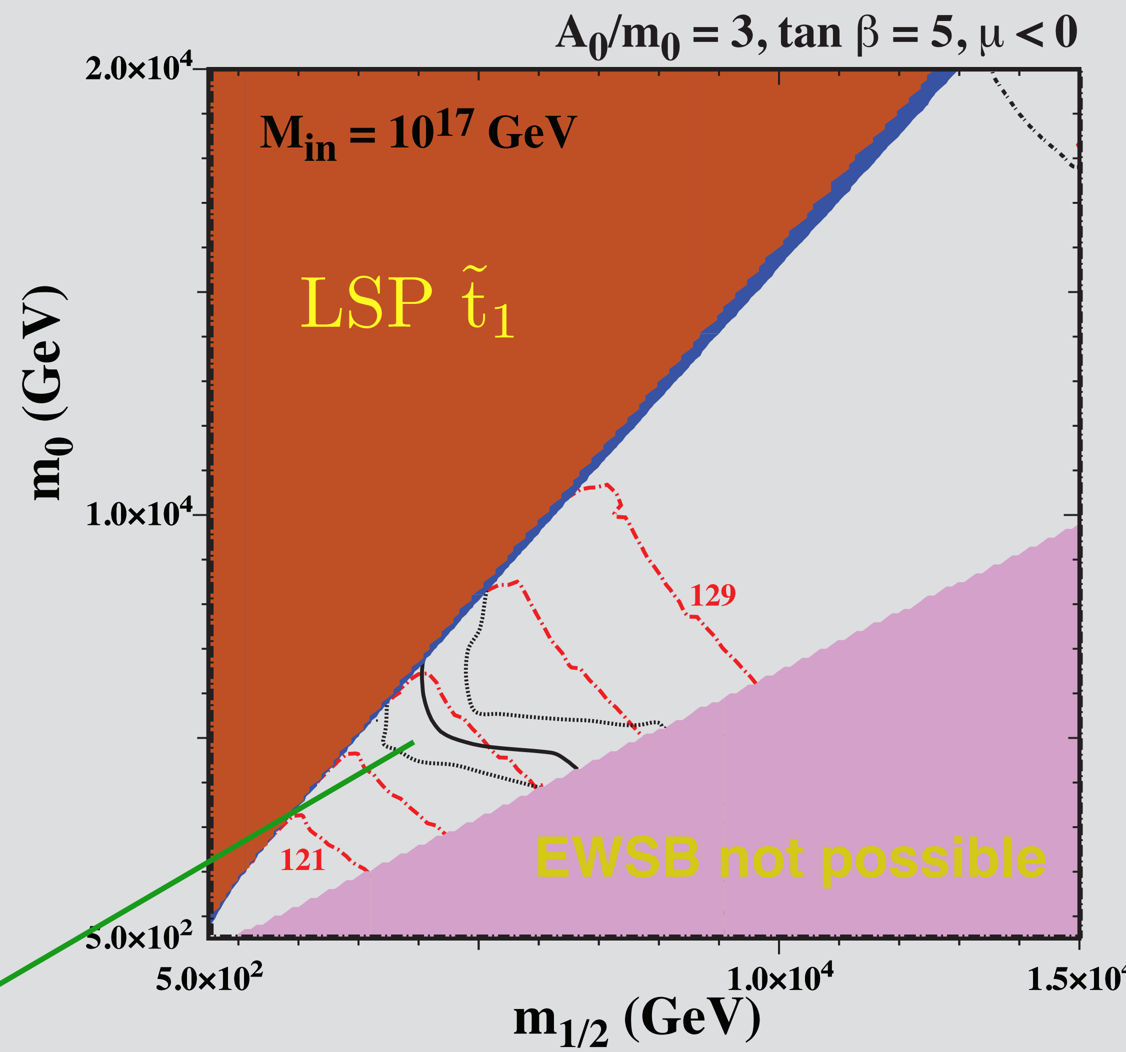
DUNE

RESULTS

CMSSM



super-GUT

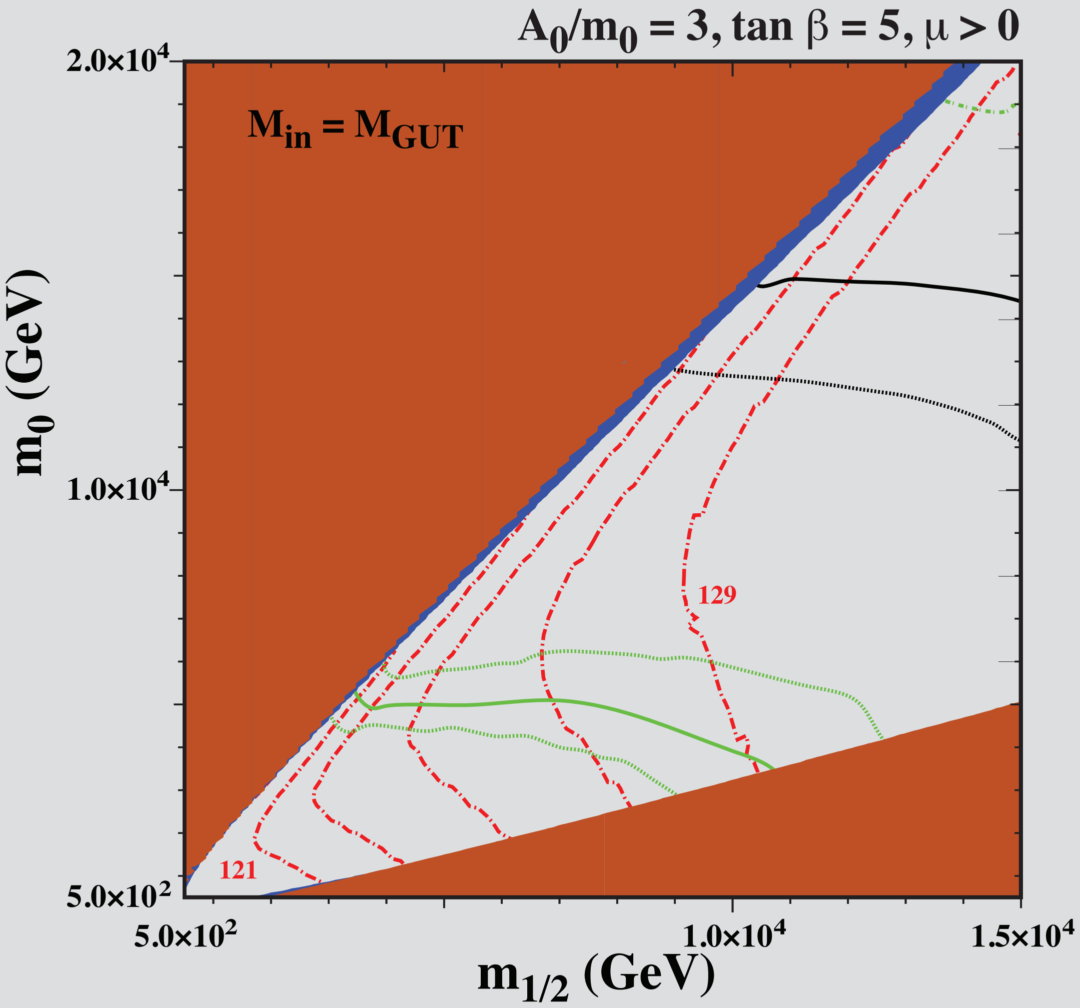


$p \to K^+ \bar{\nu}$

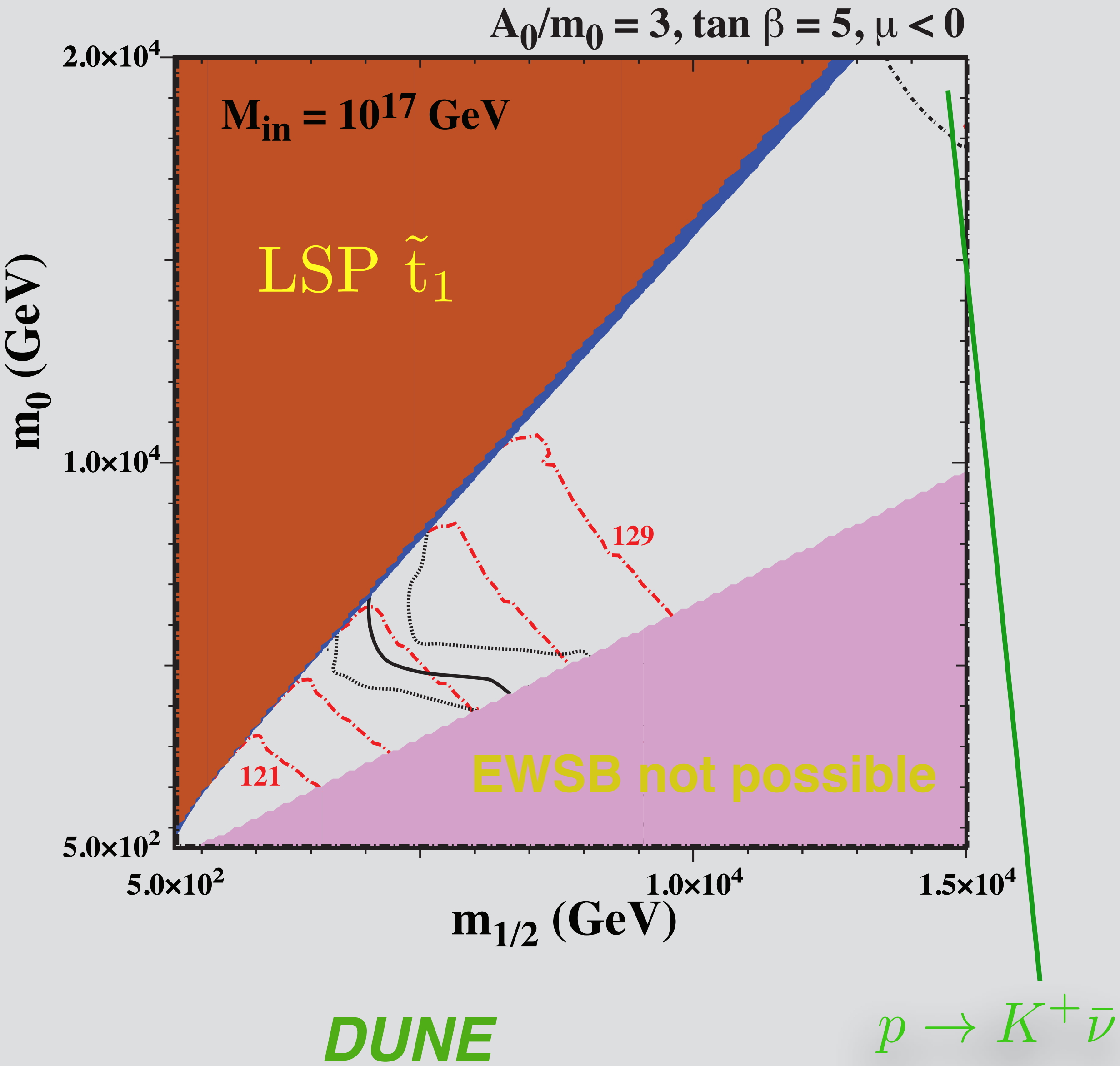
PRESENT LIMIT

RESULTS

CMSSM



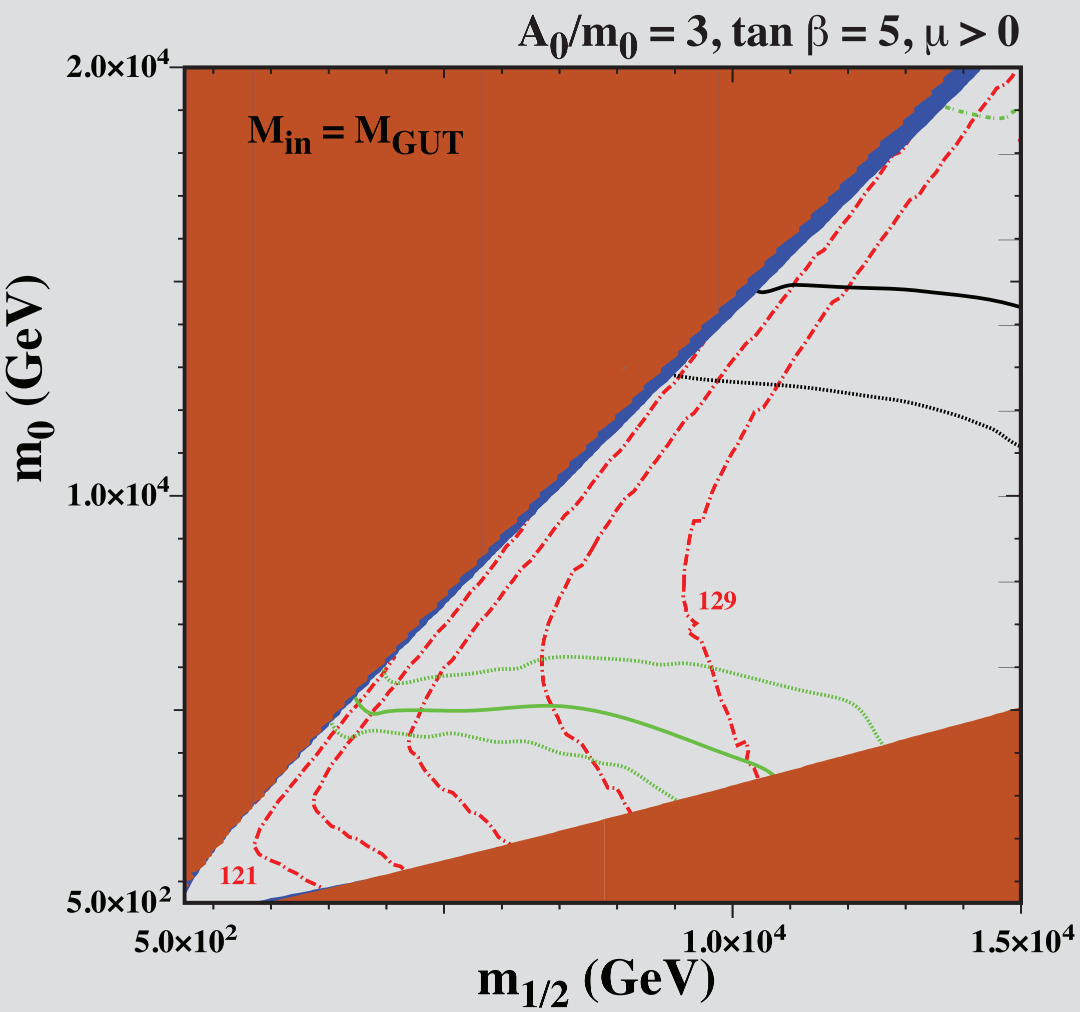
super-GUT



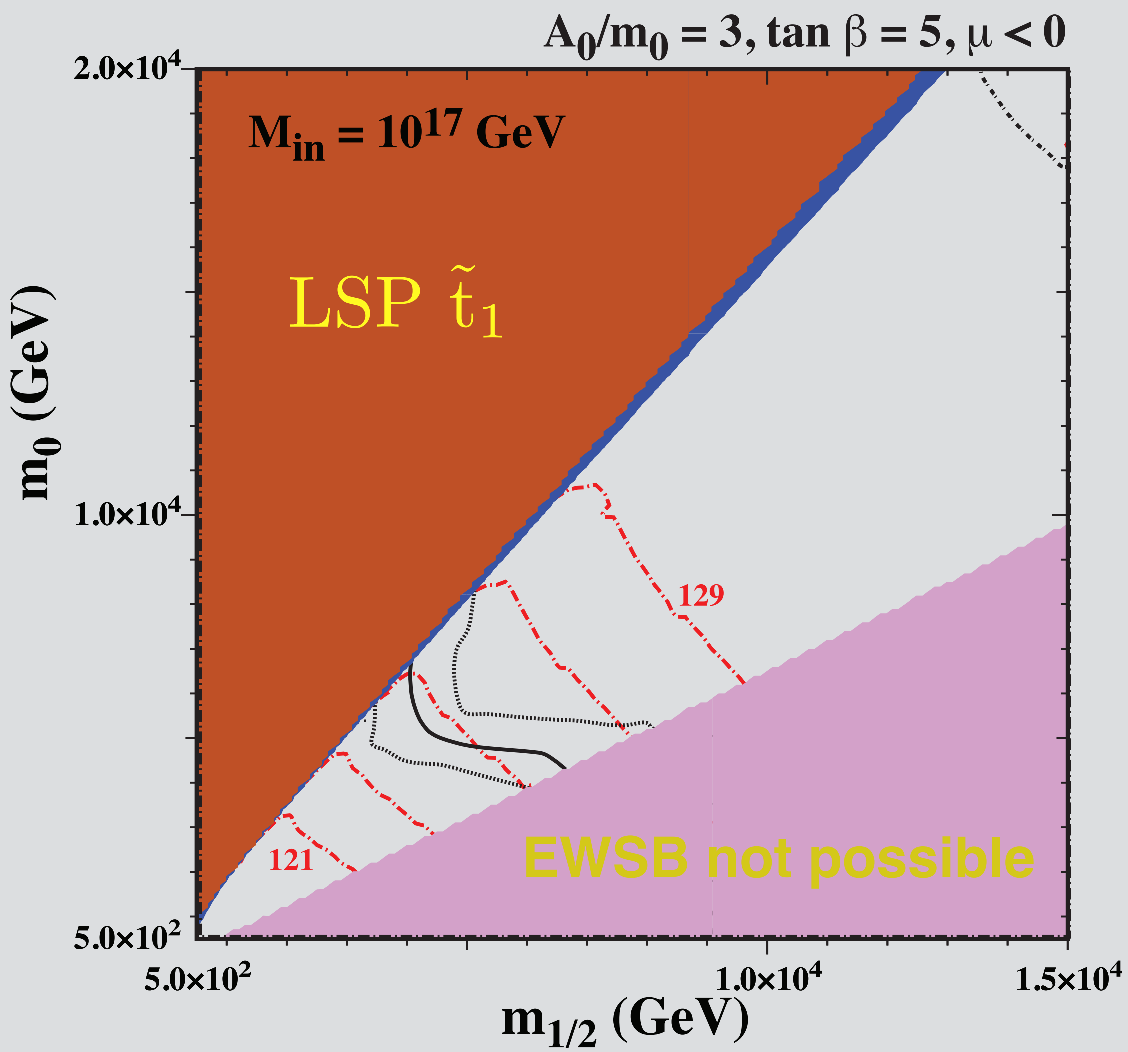
DUNE

RESULTS

CMSSM



super-GUT



Promising

MESSAGES

- More than 40 years of SU(5) models: sound predictions: **Gauge coupling unification, Proton Decay**

- **Optimistic view**: Supergravity and supersymmetry are very rich frameworks which are predictive and allow for the majority of computed observables to be in agreement with measured quantities.

MESSAGES

- More than 40 years of SU(5) models: sound predictions: **Gauge coupling unification, Proton Decay**
- **Optimistic view**: **Proton decay discovery!!**

MESSAGES

- More than 40 years of SU(5) models: sound predictions: **Gauge coupling unification, Proton Decay**
- **Optimistic view**: Supergravity and supersymmetry are very rich frameworks which are predictive and allow for the majority of computed observables to be in agreement with measured quantities.
- **Pessimistic view**: **PD, EDMs and flavor observables are** a quicker way to exclude models!

- We have found models where actually all measured observables agree & we even have predictions!

- Predictions of ELECTRIC DIPOLE MOMENTS, flavor observables: not independent form PD

- No-scale supergravity: Can accommodate models for current cosmological observations!