CryoGrid 3 - Model description

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1 Introduction

CryoGrid 3 is a simple land-surface scheme dedicated to modeling of ground temperatures in permafrost environments. It builds up on the subsurface thermal model CryoGrid 2 (Westermann et al., 2013), supplemented by the calculation of the surface energy balance as upper boundary condition, as well as improvements in the snow scheme. CryoGrid 3 is implemented in MATLAB.
The model is driven by time series of (i) air temperature T_{air} $[K]$, relative or absolute humidity (RH [-] or q [-]) and wind speed u $[ms^{-1}]$ at a known height above ground h $[m]$, (ii) incoming short-wave and long-wave radiation (S_{in} and L_{in}) $[Wm^{-2}]$, (iii) air pressure p $[Pa]$, and (iv) rates of snowfall and rainfall (P_s and P_r) $[ms^{-1}]$. For forward time integration, the simplest possible scheme, first-order forward Euler, is employed. While not inherently stable or computationally efficient, it keeps the code simple and facilitates easy modifications, so that CryoGrid 3 can become a platform for testing different parameterizations for a wide variety of permafrost processes.

2 Model description

2.1 Definitions and constants

α: surface albedo [-]
ε: Kirchhoff emissivity [-]
σ: Stefan-Boltzmann constant $[W\,m^{-2}K^{-4}]$
ρ: density $[kg\,m^{-3}]$
c_p: specific heat capacity of air at constant pressure $[J\,kg^{-1}K^{-1}]$
ρ_a: density of air $[kg\,m^{-3}]$
$\kappa = 0.4$: von Kármán-constant (e.g. Foken, 2008)
u_*: friction velocity $[m\,s^{-1}]$
z_0: aerodynamic roughness length $[m]$
L_*: Obukhov length $[m]$
$\Psi_{M,H,W}(\zeta_1, \zeta_2)$: integrated stability functions for momentum, heat and water vapor [-]
g: gravitational constant $[m\,s^{-2}]$
r_s: surface resistance against evapotranspiration $[s\,m^{-1}]$
$L_{\text{sl}} = 0.33\,MJ\,kg^{-1}$: specific latent heat of fusion of water
$L_{\text{lg}} = 2.5\,MJ\,kg^{-1}$: specific latent heat of vaporization of water

2.2 The surface energy balance

The energy input to the uppermost grid cell (either soil or snow) is derived from the surface energy balance, i.e. the fluxes of short-wave radiation (S_{in}, S_{out}), long-wave radiation (L_{in}, L_{out}) and sensible and latent heat, Q_h and Q_e. The ground heat flux, Q_g, due to conductive heat transfer in the ground is presented in Sect. 2.3. The energy input to the uppermost grid is computed as:

$$\frac{\partial E_1}{\partial t} = S_{\text{in}} + S_{\text{out}} + L_{\text{in}} + L_{\text{out}} + Q_h + Q_e + Q_g \quad (1)$$

While the incoming short-wave and long-wave radiation are provided as driving data, the other fluxes are parameterized based on the driving data sets. With the albedo α, the outgoing short-wave is calculated as

$$S_{\text{out}} \;=\; -\alpha\, S_{\text{in}}\,, \tag{2}$$

while the outgoing long-wave radiation L_{out} is derived from Kirchhoff's and Stefan-Boltzmann Law as

$$L_{\text{out}} \;=\; (\varepsilon - 1)\, L_{\text{in}} \;-\; \varepsilon\sigma\, \left(T_1 + 273.15\, K\right)^4\;. \tag{3}$$

Both albedo and surface emissivity ε are assigned different values for snow-free and snow-covered ground. The turbulent fluxes of sensible and latent heat, Q_h and Q_e, are parameterized in terms of gradients of air temperature and absolute humidity between the air at height h above ground and at the surface following Monin-Obukhov similarity theory:

$$Q_h \;=\; -\frac{\rho_a c_p}{r_a^{\text{H}}}\left(T_{\text{air}}(h) - T_1\right) \tag{4}$$

$$Q_e \;=\; -\frac{\rho_a L_{\text{lg}}}{r_a^{\text{W}} + r_s}\left(q(h) - q(z_0)\right) \tag{5}$$

The absolute humidity q is derived from the relative humidity RH and the saturation vapor pressure e^* accessible through the empirical August-Roche-Magnus formula:

$$q(z_0) \;=\; 0.622\, \frac{e^*(T_1)}{p} \tag{6}$$

$$q(h) \;=\; 0.622\, \frac{RH(h)\, e^*(T(h))}{p} \tag{7}$$

$$e^*(T) \;=\; a_1 \exp\left(\frac{a_2 T}{T + a_3}\right)\;. \tag{8}$$

We assign a_1 =611 Pa, a_2 =17.62 and a_3 =243.12 K for the saturation vapor pressure over water surfaces and a_1 =611 Pa, a_2 =22.46 and a_3 =272.62 K over ice surfaces ($T_1 < 0°C$) (Sonntag, 1990).
The resistance terms r constitute the aerodynamic resistance, r_a, characterizing the strength of the turbulent exchange, and the surface resistance against evapotranspiration, r_s. The latter is an empirical parameter that can be adjusted to account for the fact that the water vapor pressure above soil surfaces is lower than the saturation vapor pressure above a water surface (Eq. 8) for non-saturated surface soils.
The aerodynamic resistances r_a for sensible heat H and latent heat W,

$$r_a^{\text{H,W}} \;=\; (\kappa u_*)^{-1}\left[\ln\frac{h}{z_0} - \Psi_{\text{H,W}}\left(\frac{h}{L_*}, \frac{z_0}{L_*}\right)\right]\,, \tag{9}$$

account for the atmospheric stability by including the integrated atmospheric stability functions $\Psi_{\text{H,W}}$ which describe deviations from the logarithmic profile of the neutral atmospheric stratification. Furthermore, they are inversely proportional to the friction velocity

$$u_* \;=\; u(h)\kappa\left[\ln\frac{h}{z_0} - \Psi_M\left(\frac{h}{L_*}, \frac{z_0}{L_*}\right)\right]^{-1}\,, \tag{10}$$

which depends on the wind speed u and the integrated stability function for momentum, Ψ_M. The variable L_* is the Obukhov length, which in itself is a function of the turbulent fluxes Q_h and Q_e as well as the friction velocity u_*:

$$L_* = \frac{\rho_a c_p T(h)}{\kappa g} \frac{u_*^3}{Q_h + 0.61 \frac{c_p}{L_{\mathrm{lg}}} (T_{\mathrm{air}}(h) + 273.15\,K)\, Q_e} \,. \tag{11}$$

Eqs. 4 to 11 thus constitute a coupled non-linear equation system, for which unique solutions Q_h, Q_e, u_*, and L_* exists.

The integrated stability functions $\Psi_{M,H,W}$ are determined by integrating the universal functions $\varphi_{M,H,W}(\zeta)$ as

$$\Psi_{M,H,W}(\zeta_1, \zeta_2) = \int_{\zeta_2}^{\zeta_1} d\zeta\, \varphi_{M,H,W}(\zeta)\,. \tag{12}$$

For neutral and unstable atmospheric stratifications ($h/L_* \leq 0$), we assume the commonly employed functions by Høgstrøm (1988)

$$\varphi_M(\zeta) = (1 - 19\zeta)^{-1/4} \tag{13}$$

$$\varphi_{H,W}(\zeta) = 0.95\,(1 - 11.6\zeta)^{-1/2} \;, \tag{14}$$

while functions compiled from data of the SHEBA campaign (Uttal et al., 2002) over Arctic sea ice are employed for stable stratification conditions (Grachev et al., 2007), $h/L_* > 0$

$$\varphi_M(\zeta) = 1 + \frac{6.5\zeta(1+\zeta)^{1/3}}{1.3 + \zeta} \tag{15}$$

$$\varphi_{H,W}(\zeta) = 1 + \frac{5\zeta(1+\zeta)}{1 + 3\zeta + \zeta^2} \,. \tag{16}$$

2.3 Subsurface heat transfer

The subsurface thermal scheme of CryoGrid 3 is based on conductive heat transfer as given by Fourier's Law,

$$c_{\mathrm{eff}}(z,T)\,\frac{\partial T}{\partial t} - \frac{\partial}{\partial z}\left(k(z,T)\,\frac{\partial T}{\partial z}\right) = 0\,, \tag{17}$$

where the generation and consumption of latent heat due to the phase change of soil water is taken into account through an effective volumetric heat capacity $c_{\mathrm{eff}}(z,T)$ $[Jm^{-3}K^{-1}]$ featuring a sharp peak in the freezing range of the soil water. Both the effective heat capacity and the thermal conductivity $k(z,T)$ $[Wm^{-1}K^{-1}]$ of the soil are functions of the volumetric fractions of the soil constituents water, ice, air, mineral and organic (Cosenza et al., 2003; Westermann et al., 2013). Vertical or lateral movement of water is not considered in CryoGrid 3, so that the sum of the volumetric contents of ice and water remains constant (for simplicity, the densities of ice and water are assumed equal). The phase change of the soil water is determined by the freezing characteristic (see Dall'Amico et al., 2011) in dependency of the soil type (three classes, sand, silt

and clay). The employed parameterizations for $c_{\text{eff}}(z,T)$, $k(z,T)$ and the freezing characteristic are identical to the ones employed in CryoGrid 2, documented in detail in Westermann et al. (2013).
Movement of soil water is not accounted for, so that the sum of the soil water and ice contents are constant in CryoGrid 2.

2.4 Energy transfer in the snow pack

Similar to the the soil domain, the principal means of energy transfer within the snow pack is conductive heat transfer

$$c_{\text{snow}}(z,T)\,\frac{\partial T^*}{\partial t} \;-\; \frac{\partial}{\partial z}\left(k_{\text{snow}}(z,T)\,\frac{\partial T^*}{\partial z}\right) \;=\; 0\,, \tag{18}$$

with the snow heat capacity c_{snow} and thermal conductivity k_{snow} being functions of the snow density ρ_{snow} as

$$c_{\text{snow}} \;=\; c_{\text{ice}}\frac{\rho_{\text{snow}}}{\rho_{\text{ice}}} \tag{19}$$

and (Yen, 1981)

$$k_{\text{snow}} \;=\; c_{\text{ice}}\left(\frac{\rho_{\text{snow}}}{\rho_{\text{water}}}\right)^{1.88}\,. \tag{20}$$

In contrast to the soil domain, where latent heat effects are accounted for by the effective heat capacity, two state variables are required to describe the energy content of a snow grid cell, its temperature T and the volumetric water content θ_w. Eq. 18 allows for snow temperatures T^* above the melting point of ice, $T_m = 0°C$, or $T^* < T_m$ for non-zero water content $\theta_w^* \neq 0$. The temperature T and water content θ_w of a cell are thus adjusted according to the energy content $E = c_{\text{snow}}(T^* - T_m) + L\theta_w$,

$$(T|\theta_w) \;=\; \begin{cases} \left(T_m \,\middle|\, \theta_w^* + (T^* - T_m)\frac{c_{\text{snow}}}{L}\right) & \text{for } E > 0 \\[2ex] \left(T^* + \theta_w^*\frac{L}{c_{\text{snow}}} \,\middle|\, 0\right) & \text{for } E \leq 0 \end{cases} \tag{21}$$

Since a decrease in the water content, $\theta_w < \theta_w^*$, corresponds to refreezing of the corresponding amounts of water, the ice content θ_i is increased by $\theta_w^* - \theta_w$. If the water content in a grid cell exceeds the field capacity of the snow, θ_w^{fc} (i.e. the maximum volumetric water content) following melt or rainfall, the water is infiltrated in the snow cover in a routing scheme similar to the one employed in Westermann et al. (2011). The water flux at the upper boundary is given by the rainfall rate, which is added to the water content of the first grid cell. Hence, a snow grid cell with temperature T can hold the field capacity, plus the amount of water required to increase its temperature to T_m, i.e. $\theta_w^{\text{fc}} - (T - T_m)\,c_{\text{snow}}/L$. Excess water is routed to the next cell, until the cell at the bottom of the snow pack is reached, where infiltration is allowed until saturation. The water then starts pooling up from bottom to top up, until it reaches the top of the snow pack, where excess water is removed from the system as runoff. The infiltration routine updates the water contents θ_w for each grid cell. In a final step, the temperature T is recalculated by applying Eq. 21 to account for the refreezing of the infiltrated water.

To account for the built-up and disappearance of the snow cover, the position of the upper boundary is allowed to change dynamically by adding or removing grid cells, as described in (Westermann et al., 2013).

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